Mathematica Labs | iLearnMath.net | Denis Shubleka

Subject: Calculus

Topic: Inflection Points

- Goal: Use *Mathematica* to identify inflection points.

Task 1

By definition, f(x) has an inflection point at (a, f(a)) as long as three conditions are satisfied:
(1) f(x) is continuous at x=a.
(2) f′(x) does not change sign at x=a.
(3) f″(x) changes sign at x=a.

We define a cubic function:

```
f[x_] := 2 x^2 - x^3
```

The second derivative is also a polynomial (degree 1), hence any sign change of f″(x) would occur at a point where f″(x)=0. We find the root(s), by searching near x=1, as we suspect the inflection is somewhere between x=0 and x=2.

```
FindRoot[f''[x] == 0, {x, 1}]
```

We assign a variable to the (x, y) pair that describes the inflection point.

```
inflectionpt = {x, f[x]} /. %
```

, and then plot the original function, as well as the inflection point:

```
Plot[f[x], {x, -1, 3}, Epilog → {PointSize[0.03], Blue, Point[inflectionpt]}]
```

From the graph we confirm that the identified point is in fact an inflection point. To confirm a sign change in the second derivative, test whether the product of the second deratives evaluated on either side is negative:

```
f''[0.65] * f''[0.67] < 0
```

Although not necessary in this example, we can also verify that the first derivative maintains its sign (‘- to -‘ or ‘+ to +‘):

```
f'[0.65] * f'[0.67] > 0
```

We conclude this task by plotting f(x),f′(x), and f″(x) in the same window:

```
Plot[{f[x], f'[x], f''[x]}, {x, -2, 3}, PlotStyle → {Red, Black, Blue}]
```

After plotting, complete the following sentences:
f(x) is concave up whenever f″(x) is ____________.
f(x) is concave down whenever f″(x) is __________.
f(x) is increasing whenever f′(x) is ____________.

f(x) is decreasing whenever f’(x) is ____________.

Related Exercises/Notes:
