Mathematica Labs | iLearnMath.net | Denis Shubleka

Subject: Calculus

Topic: Computing Limits

- Goal: Investigate Limits

Task 1

Below we define three functions f(x), g(x), and h(x):

```
f[x_] := (Cos[x] - 1) / x
g[x_] := 1 / x
h[x_] := Sin[x]
```

Graphically, investigate the behavior of the following near the origin (as x approaches 0):

```
Plot[f[x], {x, -5, 5}]
```

```
Plot[g[x], {x, -5, 5}]
```

```
Plot[g[x] * h[x], {x, -5, 5}]
```

Q : What happens to the y-values of each function as x approaches 0?

One can also use the Limit command in Mathematica to investigate. Try it:

```
Limit[f[x], x → 0]
```

```
Limit[g[x], x → 0]
```

```
Limit[g[x] * h[x], x → 0]
```

One-sided limits can be tested as well. "Direction -> 1" implies "approach from the left", whereas "Direction -> -1" implies "approach from the right". Try it:

```
Limit[g[x], x → 0, Direction → -1]
```

```
Limit[g[x], x → 0, Direction → 1]
```

Based on the graph obtained earlier, these answers should not be surprising.

Interpret Mathematica's output when you compute the following limit:

```
Limit[Cos[x], x → ∞]
```

Task 2

■ Goal: Investigate the Limit of a Piece-Wise Function

Below we define a piece-wise functions s(x):

$$s[x\_] := \begin{cases} x^2 - 2x + 1 & x \leq 1 \\ e^x & x > 1 \end{cases}$$

```
Plot[s[x], {x, -1, 3}]
```

Numerically, investigate the left-sided limit by constructing a table:

```
dataleft = Table[{N[1 - 10^-n], N[s[1 - 10^-n], 15]}, {n, 1, 5}];
Text@Grid[Prepend[dataleft, {"x", "s(x)"}],
  Alignment → Left, Dividers → {Center, 2 → True}]
```

Next, investigate the right-sided limit by constructing a table:

```
dataright = Table[{N[1 + 10^-n], N[s[1 + 10^-n], 15]}, {n, 1, 5}];
Text@Grid[Prepend[dataright, {"x", "s(x)"}],
  Alignment → Left, Dividers → {Center, 2 → True}]
```

Compare the one-sided limits. Does the overall limit exist?

Now confirm the one-sided limits using Mathematica:

```
Limit[s[x], x → 1, Direction → 1]
Limit[s[x], x → 1, Direction → -1]
```

One can also Manipulate to investigate the function values as x approaches 1.
What happens to function value when x < 1? When x > 1?
Does the overall limit exist?

```
Manipulate[s[x], {x, 0.8, 1.2}]
```

Related Exercises/Notes: