Classroom Voting Questions: Multivariable Calculus

12.1 Functions of Two Variables

1. A function $f(x, y)$ can be an increasing function of x with y held fixed, and be a decreasing function of y with x held fixed.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

2. You awaken one morning to find that you have been transferred onto a grid which is set up like a standard right-hand coordinate system. You are at the point (-1, 3,-3), standing upright, and facing the xz-plane. You walk 2 units forward, turn left, and walk for another 2 units. What is your final position?

   (a) (-1,1,-1)
   (b) (-3,1,-3)
   (c) (-3,5,-3)
   (d) (1,1,-3)

3. Starting at the origin, if you move 3 units in the positive y-direction, 4 units in the negative x-direction, and 2 units in the positive z-direction: you are at:

   (a) (3,4,2)
   (b) (3,-4,2)
   (c) (4,3,2)
   (d) (-4,3,2)

4. Which of the following points lies closest to the xy-plane?

   (a) (3,0,3)
   (b) (0,4,2)
   (c) (2,4,1)

(d) (2,3,4)

5. Which of the following points lies closest to the origin?

   (a) (3,0,3)
   (b) (0,4,2)
   (c) (2,4,1)
   (d) (2,3,4)

6. Which of the following points lies closest to the y-axis?

   (a) (3,0,3)
   (b) (0,4,2)
   (c) (2,4,1)
   (d) (2,3,4)

7. The point (2,1,3) is closest to:

   (a) the xy plane
   (b) the xz plane
   (c) the yz plane
   (d) the plane z=6

8. Which of the following points lies closest to the point (1,2,3)?

   (a) (3,0,3)
   (b) (0,4,2)
   (c) (2,4,1)
   (d) (2,3,4)

9. Sphere A is centered at the origin and the point (0,0,3) lies on it. Sphere B is given by the equation $x^2 + y^2 + z^2 = 3$. Which of the following is true?

   (a) A encloses B
   (b) A and B are equal
   (c) B encloses A

(d) none of the above

10. The points (1,0,1) and (0,-1,1) are the same distance from the origin.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

11. The point (2, -1, 3) lies on the graph of the sphere $(x-2)^2+(y+1)^2+(z-3)^2=25$.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

12. In a table of values for a linear function, the columns must have the same slope as the rows.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

13. The set of all points whose distance from the z-axis is 4 is the:

    (a) sphere of radius 4 centered on the z-axis
    (b) line parallel to the z-axis 4 units away from the origin
    (c) cylinder of radius 4 centered on the z-axis
    (d) plane $z=4$

12.2 Graphs of Functions of Two Variables

14. What does a graph of the function $f(x, y) = x$ look like?

    (a) A line in the xy plane
    (b) A line in three dimensions
    (c) A horizontal plane
    (d) A tilted plane

15. Let $h(x, t) = 3 + 3\sin\left(\frac{\pi}{10}x\right)\cos(2\pi t)$ be the distance above the ground (in feet) of a jump rope x feet from one and after t seconds. The two people turning the rope stand 10 feet apars. Then $h(x, 1/4)$ is

    (a) Concave up
    (b) Concave down
    (c) Flat
    (d) Changes concavity in the middle

16. The object in 3-space described by $x = 2$ is

    (a) A point
    (b) A line
    (c) A plane
    (d) Undefined

17. The set of points whose distance from the z-axis equals the distance from the xy-plane describes a

    (a) Plane
    (b) Cylinder
    (c) Sphere
    (d) Cone
    (e) Double cone (two cones joined at their vertices)

18. The graph of $f(x, y) = 2^{-x^2-y^2}$ will look most like

(a) a bowl opening up, but more shallow than $x^2 + y^2$
(b) a bowl opening up, but more steep than $x^2 + y^2$
(c) a bowl opening down
(d) a small hill in a large plane

19. The cross sections of $g(x, y) = \sin(x) + y + 1$ with x fixed are

(a) lines
(b) parabolas
(c) sinusoidal curves
(d) none of the above

20. The graph of the equation $f(x, y) = 2$ is a plane parallel to the xz-plane.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

21. The cross section of the function $f(x, y) = x + y^2$ for $y = 1$ is a line.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

22. The graphs of $f(x, y) = x^2 + y^2$ and $g(x, y) = 1 - x^2 - y^2$ intersect in a circle.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

23. The equation $Ax + By + Cz + D = 0$ represents a line in space.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

24. In three space, $x^2 + y^2 = 1$ represents

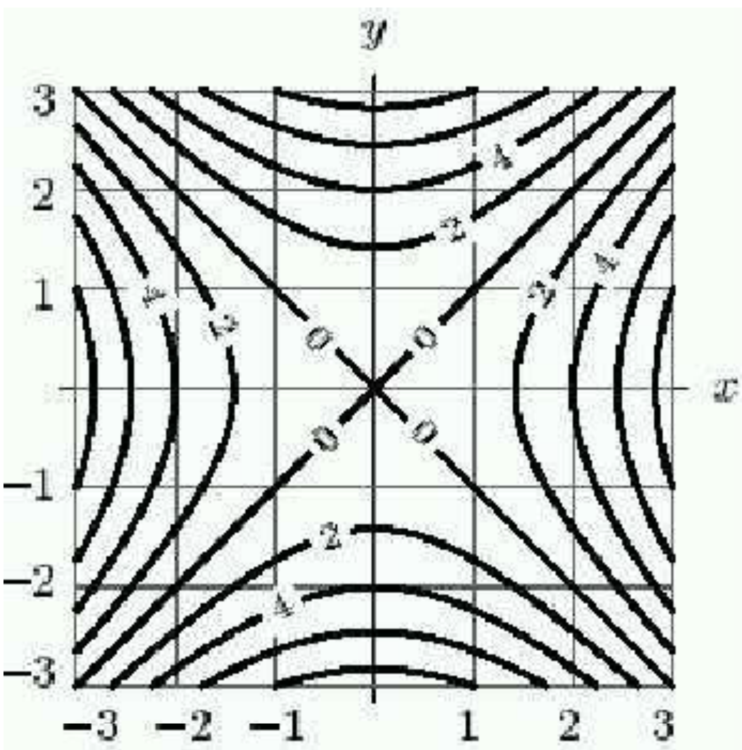
(a) a circle

(b) a cylinder

(c) a sphere

12.3 Contour Diagrams

25. Which of the following terms best describes the origin in the contour diagram in the figure below?



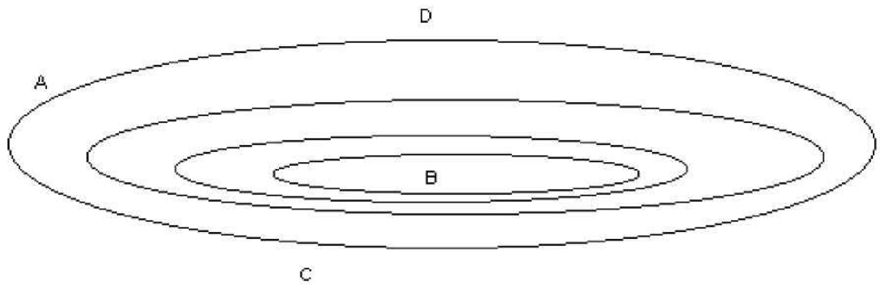
(a) A mountain pass

(b) A dry river bed

(c) A mountain top

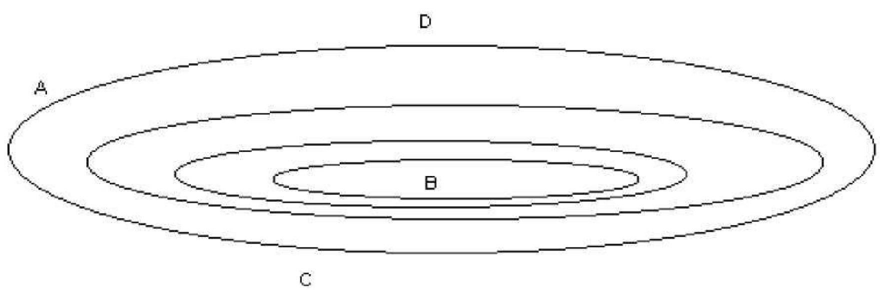
(d) A ridge

26. Using the contour plot pictured, which path will result in the greatest change in altitude?



(a) From A to B
(b) From C to B
(c) From D to B
(d) All changes in altitude are approximately equal.

27. Using the contour plot pictured, which path is the steepest?



(a) From A to B
(b) From C to B
(c) From D to B
(d) All changes in altitude are approximately equal.

28. The contour lines of $z = \exp(\sin(x^2 + y^2))$ will be

(a) lines
(b) circles
(c) exponential curves

(d) none of the above

29. The contours of graphs of $f(x, y) = y^2 + (x - 2)^2$ are either circles or a single point.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

30. If all the contours for $f(x, y)$ are parallel lines, then the graph of f is a plane.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

31. On a weather map, there can be two isotherms (contour lines) which represent the same temperature but do not intersect.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

12.4 Linear Functions

32. A plane has a z-intercept of 3, a slope of 2 in the x direction, and a slope of -4 in the y direction. The height of the plane at (2,3) is

    (a) -2
    (b) -8
    (c) -5
    (d) not given by this information

33. Which of the following planes is parallel to the plane $z = -2 - 2x - 4y$?

(a) $z = -1 - 2x - 2y$
(b) $(z-1) = -2 - 2(x-1) - 4(y-1)$
(c) $z = 2 + 2x + 4y$

34. Any three points in 3 space determine a unique plane.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

35. Any two distinct lines in 3-space determine a unique plane.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

36. If the graph of $z = f(x, y)$ is a plane, then each cross section is a line.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

12.5 Functions of Three Variables

37. Level surfaces of the function $f(x, y, z) = \sqrt{x^2 + y^2}$ are

(a) Circles centered at the origin
(b) Spheres centered at the origin
(c) Cylinders centered around the z-axis
(d) Upper-hemispheres centered at the origin

38. Any level surface of a function of 3 variables can be thought of as a surface in 3 space.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

39. Any surface that is a graph of a 2-variable function $z = f(x, y)$ can be thought of as a level surface of a function of 3 variables.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

40. Any level surface of a function of 3 variables can be thought of as the graph of a function $z = f(x, y)$.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

41. Suppose the temperature at time t at the point (x, y) is given by the function $T(x, y, t) = 5t - x^2 - y^2$. Which of the following will not cause temperature to decrease?

    (a) moving away from the origin in the positive x direction
    (b) moving away from the origin in the positive y direction
    (c) moving away from the origin in the direction of the line $y = x$
    (d) standing still and letting time pass

42. If temperature at the point (x, y, z) is given by $T(x, y, z) = \cos(z - x^2 - y^2)$, the level surfaces look like:

    (a) spheres
    (b) Pringles brand potato chips
    (c) planes
    (d) bowls

43. The level surfaces of the function $f(x, y, z) = x^2 + y^2 + z^2$ are cylinders with axis along the y-axis.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

13.1 Displacement Vectors

44. The length of the sum of two vectors is always strictly larger than the sum of the lengths of the two vectors

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

45. $||\vec{v}|| = |v_1| + |v_2| + |v_3|$, where $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

46. $\vec{v}$ and $\vec{w}$ are parallel if $\vec{v} = \lambda\vec{w}$ for some scalar λ

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

47. Any two parallel vectors point in the same direction.

(a) True, and I am very confident
(b) True, but I am not very confident

(c) False, but I am not very confident
(d) False, and I am very confident

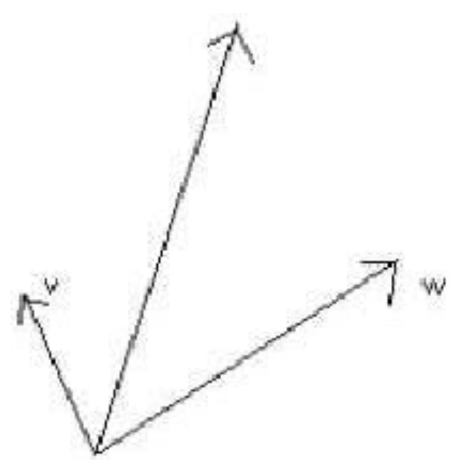
48. Any two points determine a unique displacement vector.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

49. $2\vec{v}$ has twice the magnitude as $\vec{v}$

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

50. In the picture, the unlabelled vector is closest to



(a) $v + w$
(b) $v - w$
(c) $v + 2w$
(d) $2v + w$

51. A "unit vector" is a vector with a magnitude of one. The vectors $\hat{\imath} = \langle 1, 0, 0\rangle, \hat{\jmath} = \langle 0, 1, 0\rangle$ and $\hat{k} = \langle 0, 0, 1\rangle$ are unit vectors that point in the x, y, and z directions, respectively.

    True or False: The vector $\langle \frac{1}{2}, \frac{1}{2}\rangle$ is a unit vector.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

52. **True or False:** The vector $\frac{1}{\sqrt{3}}\hat{\imath} - \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{2}{\sqrt{3}}\hat{k}$ is a unit vector.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

53. Which of the following is a unit vector that is parallel to the vector $\langle 1, -2, 3\rangle$?

    (a) $\langle \frac{1}{6}, -\frac{2}{6}, \frac{3}{6}\rangle$
    (b) $\langle \frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$
    (c) $\langle \frac{1}{14}, -\frac{2}{14}, \frac{3}{14}\rangle$
    (d) $\langle -\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}\rangle$
    (e) More than one of the above

54. The vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + \hat{k}$ are parallel.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

55. Find a vector that points in the same direction as the vector $\langle 2, 1, 2\rangle$, but has a magnitude of 5.

    (a) $\langle \frac{10}{3}, \frac{5}{3}, \frac{10}{3}\rangle$

(b) $\langle \frac{10}{\sqrt{3}}, \frac{5}{\sqrt{3}}, \frac{10}{\sqrt{3}} \rangle$

(c) $\langle \frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}} \rangle$

(d) $\langle 10, 5, 10 \rangle$

(e) $\langle 30, 15, 30 \rangle$

(f) More than one of the above

13.2 Vectors in General

56. A plane is flying due south. There is a strong wind from the west. In what direction does the pilot have to point the plane to stay on course?

    (a) South

    (b) East

    (c) West

    (d) Southeast

    (e) Southwest

57. A boat is traveling with a velocity of 30 mph due West relative to the water. The current is flowing 10 mph at an angle of 45° West of North. What is the boat's net velocity?

    (a) 37.7 mph at 10.8° South of West

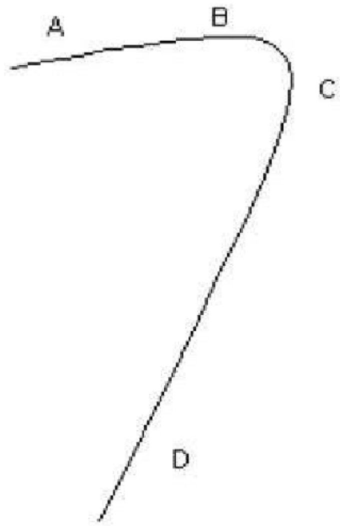
    (b) 37.7 mph at 79.2° West of North

    (c) 24.0 mph at 17.1° North of West

    (d) 24.0 mph at 17.1° West of North

    (e) None of the above

58. A car is traveling along the path from point A towards point D. When is the velocity vector closest to being parallel to $\hat{j}$ (assuming this path is in the xy plane)?



(a) A
(b) B
(c) C
(d) D

13.3 The Dot Product

59. The only way that $\vec{v} \cdot \vec{w} = 0$ is if $\vec{v} = 0$ or $\vec{w} = 0$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

60. The zero vector $\vec{0}$ (with magnitude $||\vec{0}|| = 0$) is perpendicular to all other vectors.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

61. Any plane has only two distinct normal vectors

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

62. Parallel planes share a same normal vector.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

63. Perpendicular planes have perpendicular normal vectors.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

64. What is the angle between the vectors $\langle \sqrt{3}, 1 \rangle$ and $\langle -\sqrt{3}, 1 \rangle$?

(a) 30 degrees
(b) 60 degrees
(c) 90 degrees
(d) 120 degrees
(e) 150 degrees
(f) None of the above

65. The angle between the vectors $-x\hat{\imath} - \hat{\jmath} + \hat{k}$ and $x\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$:

(a) is 0 degrees
(b) is less than 90 degrees
(c) is greater than 90 degrees
(d) can be any of the above depending on the value of x.

66. Two vectors have a dot product of 14. To guarantee the dot product is equal to 28, you could:

    (a) double the angle between the vectors
    (b) double the length of both vectors
    (c) double the length of one vector
    (d) none of the above

67. Which of the following is a point in the plane parallel to $3x + 4y - 2z = 6$ containing the origin?

    (a) (1,1,1)
    (b) (1,2,3)
    (c) (3,2,1)
    (d) none of the above

68. In 2 space, consider the vector $\vec{v} = 5\hat{i} + 7\hat{j}$. For which unit vector below will the component of $\vec{v}$ perpendicular to that unit vector be largest?

    (a) $\hat{i}$
    (b) $(\hat{i} - \hat{j})/\sqrt{2}$
    (c) $\hat{j}$
    (d) $(\hat{i} + \hat{j})/\sqrt{2}$

69. A 100-meter dash is run on a track heading northeasterly. If the wind is blowing out of the south at a speed of 8 km/hr. The rules say that a legal wind speed measured in the direction of the dash must not exceed 5 km/hr. Will the race results be disqualified due to an illegal wind?

    (a) Yes, the race results will be disqualified because the wind exceeds 5 km/hr in the direction of the race.
    (b) No, the race results will not be disqualified because the wind does not exceed 5 km/hr in the direction of the race.
    (c) There is not enough information to answer this question.

70. A 100-meter dash is run on a track in the direction of the unit vector $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$. If the wind velocity $\vec{w}$ is $\vec{w} = 5\hat{i} + 1\hat{j}$ km/hr. The rules say that a legal wind speed measured in the direction of the dash must not exceed 5 km/hr. Will the race results be disqualified due to an illegal wind?

(a) Yes, the race results will be disqualified because the wind exceeds 5 km/hr in the direction of the race.

(b) No, the race results will not be disqualified because the wind does not exceed 5 km/hr in the direction of the race.

(c) There is not enough information to answer this question.

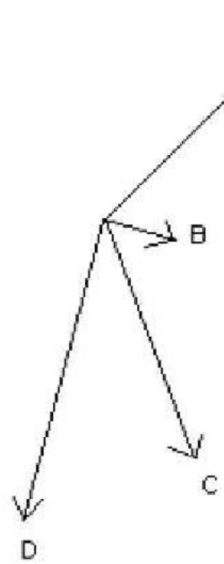
71. A 100-meter dash is run on a track in the direction of the vector $\vec{v} = 2\hat{\imath} + 6\hat{\jmath}$. The wind velocity is $\vec{w} = 5\hat{\imath} + \hat{\jmath}$ km/hr. The rules say that a legal wind speed measured in the direction of the dash must not exceed 5 km/hr. Will the race results be disqualified due to an illegal wind?

(a) Yes, the race results will be disqualified because the wind exceeds 5 km/hr in the direction of the race.

(b) No, the race results will not be disqualified because the wind does not exceed 5 km/hr in the direction of the race.

(c) There is not enough information to answer this question.

72. The picture shown is in 2 space. If the force vector is $\vec{F} = -4\hat{\jmath}$, the total work to which point will be the most positive?



(a) A

(b) B

(c) C

(d) D

73. An equation of the plane with normal vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ containing the point (1, 2, 3) is $z = x + y$.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

74. Which of the following vectors is normal to the plane $z = -3x + 4y + 25$?

   (a) $\langle 3, -4, 1\rangle$
   (b) $\langle -3, 4, 1\rangle$
   (c) $\langle -3, 4, 25\rangle$
   (d) $\langle -3, 4, -1\rangle$
   (e) More than one of the above

75. For any two vectors $\vec{u}$ and $\vec{v}$, $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

76. For any two vectors $\vec{u}$ and $\vec{v}$ and any scalar k, it is true that $k(\vec{u} \cdot \vec{v}) = (k\vec{u}) \cdot \vec{v}$.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

13.4 The Cross Product

77. The cross product of $2\hat{\imath}$ and $3\hat{\jmath}$ is

    (a) $6\hat{k}$
    (b) $-6\hat{k}$
    (c) 0
    (d) $6\hat{\imath}\hat{\jmath}$

78. For the vectors $\vec{a} = 4\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$, the cross product $\vec{a} \times \vec{b}$ is

    (a) $-13\hat{\imath} + 14\hat{\jmath} + 19\hat{k}$
    (b) $13\hat{\imath} + 14\hat{\jmath} - 19\hat{k}$
    (c) $-13\hat{\imath} - 14\hat{\jmath} + 19\hat{k}$
    (d) $13\hat{\imath} - 14\hat{\jmath} - 19\hat{k}$

79. A vector that is normal to the plane containing the vectors $\vec{a} = 4\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$ is

    (a) $-13\hat{\imath} + 14\hat{\jmath} + 19\hat{k}$
    (b) $13\hat{\imath} + 14\hat{\jmath} - 19\hat{k}$
    (c) $-13\hat{\imath} - 14\hat{\jmath} + 19\hat{k}$
    (d) $13\hat{\imath} - 14\hat{\jmath} - 19\hat{k}$

80. If $\vec{d} = \vec{a} \times \vec{b}$, then $\vec{a} \cdot \vec{d} =$

    (a) $\vec{a} \times (\vec{b} \cdot \vec{b})$
    (b) 0
    (c) $\vec{a} \times \vec{a} \cdot \vec{b}$
    (d) $(\vec{a} \cdot \vec{b}) \times \vec{b}$

81. For any vectors $\vec{u}$ and $\vec{v}$, $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$

    (a) True, and I am very confident
    (b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

82. For any vectors $\vec{u}$ and $\vec{v}$, $(\vec{u} \times \vec{v}) \times (\vec{v} \times \vec{u}) = (\vec{v} \times \vec{u}) \times (\vec{u} \times \vec{v})$

(a) True, and I am very confident

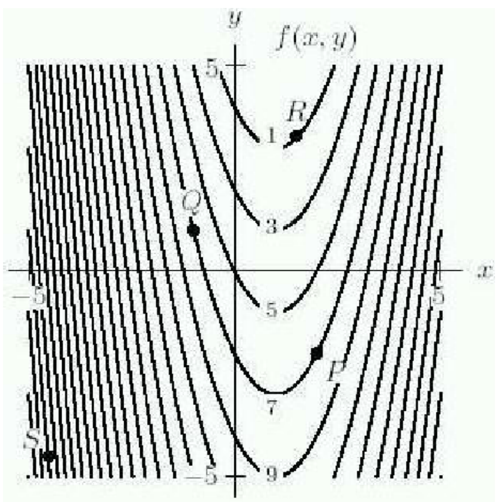
(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

14.1 The Partial Derivative

83. At point Q in the diagram below, which of the following is true?



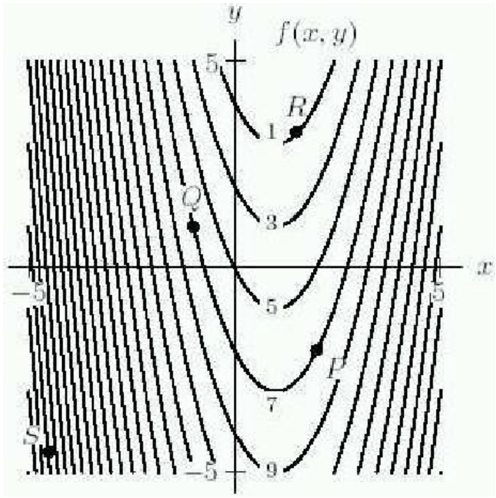
(a) $f_x > 0, f_y > 0$

(b) $f_x > 0, f_y < 0$

(c) $f_x < 0, f_y > 0$

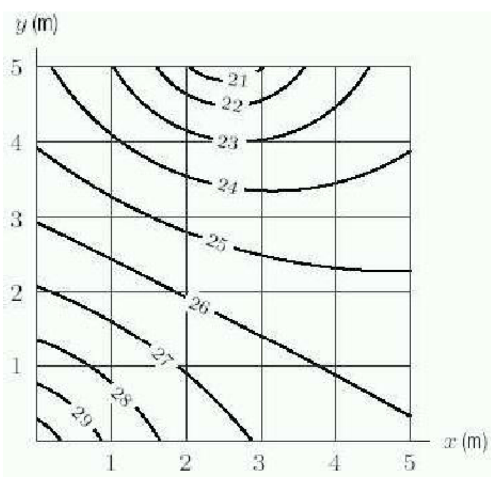
(d) $f_x < 0, f_y < 0$

84. List the points P, Q, R in order of decreasing f_x.



(a) $P > Q > R$
(b) $P > R > Q$
(c) $R > P > Q$
(d) $R > Q > P$
(e) $Q > R > P$

85. Using the level curves of $f(x, y)$ given in the figure below, which is larger, $f_x(2, 1)$ or $f_y(1, 2)$.



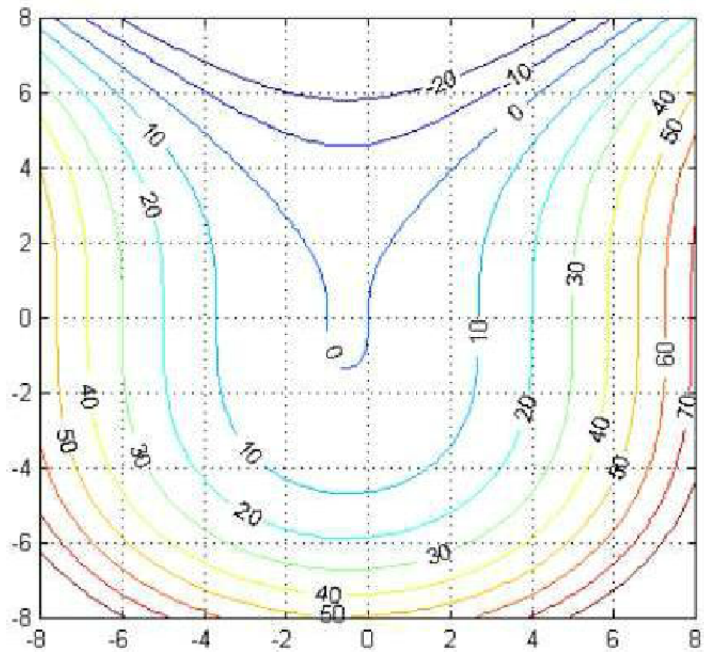
(a) $f_x(2, 1) > f_y(1, 2)$.
(b) $f_x(2, 1) < f_y(1, 2)$.

86. Suppose that the price P (in dollars), to purchase a used car is a function of C, its original cost (also in dollars), and its age A (in years). So $P = f(C, A)$. The sign of $\frac{\partial P}{\partial C}$ is

(a) Positive

(b) Negative

87. Using the contour plot of $f(x, y)$, which of the following is true at the point (4,2)?



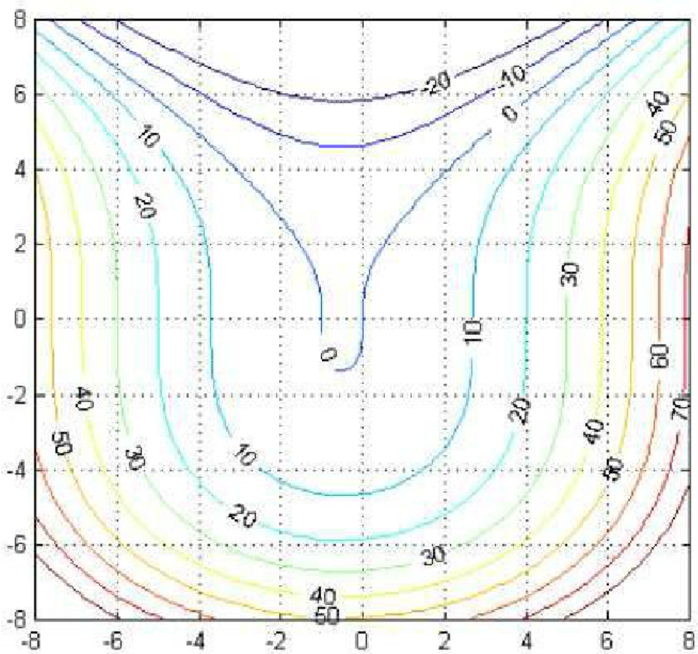
(a) $f_x > 0$ and $f_y > 0$

(b) $f_x > 0$ and $f_y < 0$

(c) $f_x < 0$ and $f_y > 0$

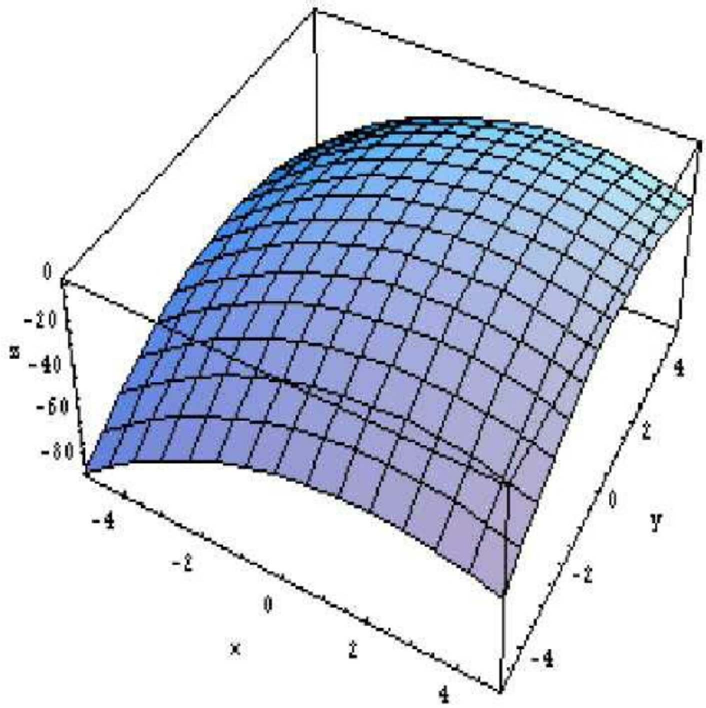
(d) $f_x < 0$ and $f_y < 0$

88. Using the contour plot of $f(x, y)$, which of the following is closest to the partial derivative of f with respect to x at (4,2)?



(a) 40
(b) 20
(c) 10
(d) 4

89. At which point above the xy plane will both partial derivatives be positive?



(a) (-5,-5)
(b) (5,-5)
(c) (5,5)
(d) (-5,5)

14.2 Computing Partial Derivatives Algebraically

90. Which of the following functions satisfy Euler's Equation, $xf_x + yf_y = f$?

(a) $f = x^2y^3$
(b) $f = x + y + 1$
(c) $f = x^2 + y^2$
(d) $f = x^{0.4}y^{0.6}$

91. If $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ everywhere, then $f(x, y)$ is constant.

(a) True, and I am very confident
(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

92. There exists a function $f(x, y)$ with $f_x = 2y$ and $f_y = 2x$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

14.3 Local Linearity and the Differential

93. Let $f(2, 3) = 7$, $f_x(2, 3) = -1$, and $f_y(2, 3) = 4$. Then the tangent plane to the surface $z = f(x, y)$ at the point $(2, 3)$ is

(a) $z = 7 - x + 4y$

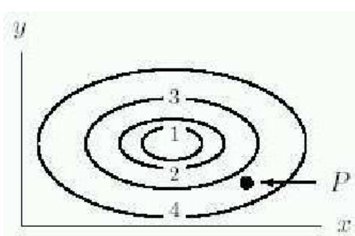
(b) $x - 4y + z + 3 = 0$

(c) $-x + 4y + z = 7$

(d) $-x + 4y + z + 3 = 0$

(e) $z = 17 + x - 4y$

94. The figure below shows level curves of the function $f(x, y)$. The tangent plane approximation to $f(x, y)$ at the point $P = (x_0, y_0)$ is $f(x, y) \approx c + m(x - x_0) + n(y - y_0)$. What are the signs of c, m, and n?



(a) $c > 0$, $m > 0$, $n > 0$

(b) $c < 0$, $m > 0$, $n < 0$

(c) $c > 0$, $m < 0$, $n > 0$

(d) $c < 0$, $m < 0$, $n < 0$

(e) $c > 0$, $m > 0$, $n < 0$

95. Suppose that $f(x, y) = 2x^2y$. What is the tangent plane to this function at $x = 2$, $y = 3$?

(a) $z = 4xy(x - 2) + 2x^2(y - 3) + 24$
(b) $z = 4x(x - 2) + 2(y - 3) + 24$
(c) $z = 8(x - 2) + 2(y - 3) + 24$
(d) $z = 24(x - 2) + 8(y - 3) + 24$
(e) $z = 24x + 8y + 24$

96. The differential of a function $f(x, y)$ at the point (a, b) is given by the formula $df = f_x(a, b)dx + f_y(a, b)dy$. Doubling dx and dy doubles df.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

97. The differential of a function $f(x, y)$ at the point (a, b) is given by the formula $df = f_x(a, b)dx + f_y(a, b)dy$. Moving to a different point (a, b) may change the formula for df.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

98. The differential of a function $f(x, y)$ at the point (a, b) is given by the formula $df = f_x(a, b)dx + f_y(a, b)dy$. If dx and dy represent small changes in x and y in moving away from the point (a, b), then df approximates the change in f.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

99. The differential of a function $f(x, y)$ at the point (a, b) is given by the formula $df = f_x(a, b)dx + f_y(a, b)dy$. The equation of the tangent plane to $z = f(x, y)$ at the point (a, b) can be used to calculate values of df from dx and dy.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

100. A small business has \$300,000 worth of equipment and 100 workers. The total monthly production, P (in thousands of dollars), is a function of the total value of the equipment, V (in thousands of dollars), and the total number of workers, N. The differential of P is given by $dP = 4.9dN + 0.5dV$. If the business decides to lay off 3 workers and buy additional equipment worth \$20,000, then

(a) Monthly production increases.

(b) Monthly production decreases.

(c) Monthly production stays the same.

101. Which of the following could be the equation of the tangent plane to the surface $z = x^2 + y^2$ at a point (a, b) in the first quadrant?

(a) $z = -3x + 4y + 7$

(b) $z = 2x - 4y + 5$

(c) $z = 6x + 6y - 18$

(d) $z = -4x - 4y + 24$

102. Suppose $f_x(3, 4) = 5$, $f_y(3, 4) = -2$, and $f(3, 4) = 6$. Assuming the function is differentiable, what is the best estimate for $f(3.1, 3.9)$ using this information?

(a) 6.3

(b) 9

(c) 6

(d) 6.7

103. We need to figure out the area of the floor of a large rectangular room, however our measurements aren't very precise. We find that the room is 52.3 ft by 44.1 ft so we get an area of 2,306.43 square feet, but our measurements are only good to within a couple of inches, roughly an error of 0.2 feet in both directions, so our estimate of the area is probably off by a bit. Use differentials to determine the likely error in our estimate of the floor's area.

(a) 0.04 square feet

(b) 19.24 square feet

(c) 19.28 square feet

(d) 19.32 square feet

(e) 19.56 square feet

104. A giant stone cylinder suddenly appears on the campus lawn outside, and we of course ask ourselves: What could its volume be? We send out a student with a meter stick to measure the cylinder, who reports that the cylinder has a height of 3.34 meters (with a measurement error of 2 centimeters) and has a radius of 2.77 meters (give or take 3 centimeters). We know that the volume of a cylinder is $V = \pi r^2 h$, so we estimate its volume as 80.5 m^3. What is our measurement error on this volume?

(a) 5 cm

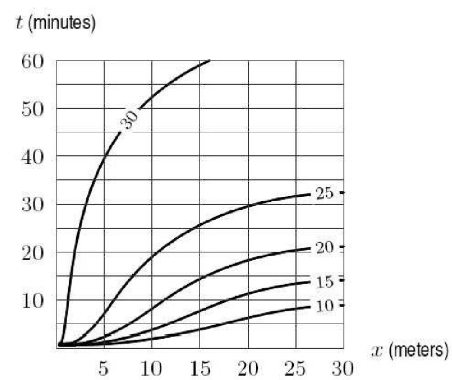
(b) 1.00 m^3

(c) 1.89 m^3

(d) 2.23 m^3

14.4 Gradients and Directional Derivatives in the Plane

105. The figure shows the temperature T $°C$ in a heated room as a function of distance x in meters along a wall and time t in minutes. Which of the following is larger?



(a) $||\nabla T(15, 15)||$

(b) $||\nabla T(25, 25)||$

106. The table below gives values of the function $f(x,y)$ which is smoothly varying around the point (3, 5). Estimate the vector $\nabla(f(3,5))$. If the gradient vector is placed with its tail at the origin, into which quadrant does the vector point?

x \ y	4.9	5	5.1
2.9	18.12	17.42	16.73
3	18.42	17.74	17.04
3.1	18.71	18.04	17.35

(a) I
(b) II
(c) III
(d) IV
(e) Can't tell without more information

107. Let $\nabla f(1,1) = 3\hat{\imath} - 5\hat{\jmath}$. What is the sign of the directional derivative of f in the direction of the vector $\nwarrow$ and in the direction of the vector $\uparrow$?

(a) positive and positive
(b) positive and negative
(c) negative and positive
(d) negative and negative

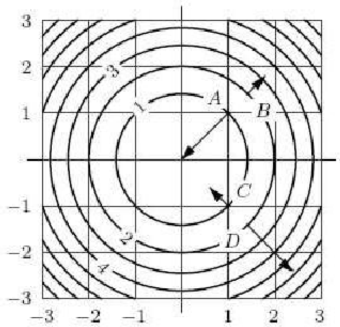
108. Let $\nabla f(1,1) = 3\hat{\imath} - 5\hat{\jmath}$. What is the sign of the directional derivative of f in the direction of the vector $\leftarrow$ and in the direction of the vector $\searrow$?

(a) positive and positive
(b) positive and negative
(c) negative and positive
(d) negative and negative

109. In which direction is the directional derivative of $z = x^2 + y^2$ at the point (2,3) most positive?

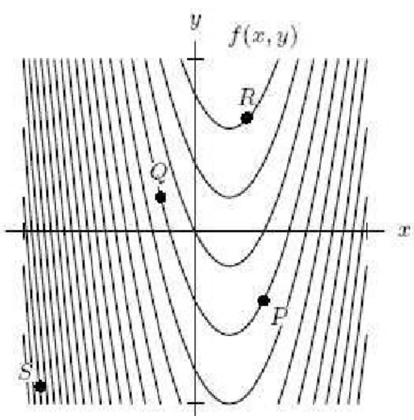
(a) $\hat{\imath}$
(b) $-\hat{\imath} - \hat{\jmath}$
(c) $-\hat{\imath} + \hat{\jmath}$
(d) $\hat{\imath} + \hat{\jmath}$

110. Which of the vectors shown on the contour diagram of $f(x, y)$ in the figure below could be ∇f at the point at which the tail is attached?



(a) A
(b) B
(c) C
(d) D

111. At which of the points P, Q,R, S in the figure below does the gradient have the largest magnitude?



(a) P
(b) Q
(c) R
(d) S

112. The surface of a hill is modeled by $z = 25 - 2x^2 - 4y^2$. When a hiker reaches the point (1, 1, 19), it begins to rain. She decides to descend the hill by the most rapid way. Which of the following vectors points in the direction in which she starts her descent?

(a) $-4x\hat{i} - 8y\hat{j}$

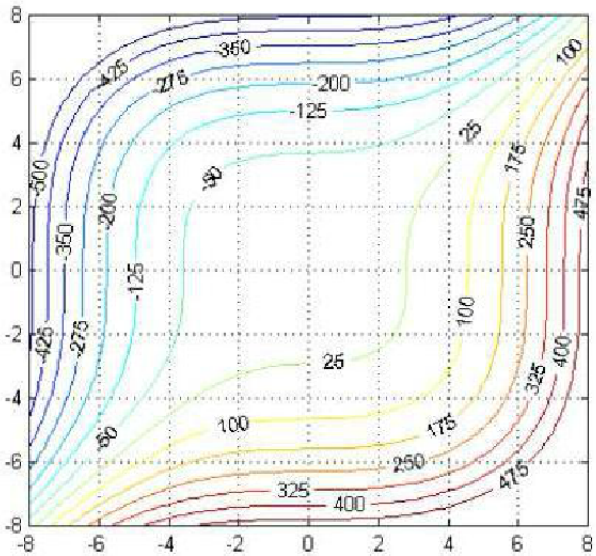
(b) $4x\hat{i} + 8y\hat{j}$

(c) $-4\hat{i} - 8\hat{j}$

(d) $4\hat{i} + 8\hat{j}$

(e) None of the above

113. At which point will the gradient vector have the largest magnitude?



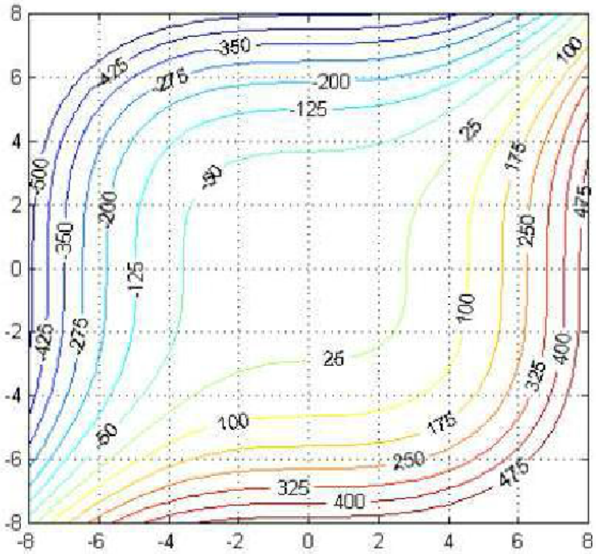
(a) (0,2)

(b) (-4,-4)

(c) (0,0)

(d) (6,-2)

114. At which point will the gradient vector be most parallel to $\hat{j}$?



(a) (0,4)
(b) (-4,-4)
(c) (0,0)
(d) (6,-2)

115. $\nabla f(1,1) = 3\hat{\imath} - 5\hat{\jmath}$. What is the sign of the directional derivative of f in the direction $\vec{v} = 4\hat{\imath} + 2\hat{\jmath}$?

(a) Positive
(b) Negative

116. Suppose that the temperature at a point (x, y) on the floor of a room is given by $T(x, y)$. Suppose heat is being radiated out from a hot spot at the origin. Which of the following could be $\nabla T(a, b)$, where $a, b > 0$?

(a) $2\hat{\imath} + 2\hat{\jmath}$
(b) $-2\hat{\imath} - 2\hat{\jmath}$
(c) $-2\hat{\imath} + 2\hat{\jmath}$
(d) $2\hat{\imath} - 2\hat{\jmath}$

14.5 Gradients and Directional Derivatives in Space

117. Suppose the temperature at a point (x, y, z) in a room is given by $T(x, y, z)$. Suppose heat is being radiated out from a hot spot at the origin. Which of the following could be $\nabla T(a, b, c)$ where a, b, c are all positive?

   (a) $2\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$
   (b) $-3\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$
   (c) $-2\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$
   (d) $3\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

118. Let $f(x, y, z) = x^2 + y^2 + z^2$. Which statement best describes the vector $\nabla f(x, y, z)$? It is always perpendicular to:

   (a) vertical cylinder passing through (x, y, z).
   (b) a horizontal plane passing through (x, y, z).
   (c) a sphere centered on the origin passing through (x, y, z).
   (d) None of the above

119. For $f(x, y, z)$, suppose $\nabla f(a, b, c) \cdot \hat{\imath} > \nabla f(a, b, c) \cdot \hat{\jmath} > \nabla f(a, b, c) \cdot \hat{k} > 0$. The tangent plane to the surface $f(x, y, z) = 0$ through the point (a, b, c) is given by $z = p + mx + ny$. Which of the following is correct?

   (a) $m > n > 0$
   (b) $n > m > 0$
   (c) $m < n < 0$
   (d) $n < m < 0$

120. The function $f(x, y)$ has gradient ∇f at the point (a, b). The vector ∇f is perpendicular to the level curve $f(x, y) = f(a, b)$.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

121. The function $f(x, y)$ has gradient ∇f at the point (a, b). The vector ∇f is perpendicular to the surface $z = f(x, y)$ at the point $(a, b, f(a, b))$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

122. The function $f(x, y)$ has gradient ∇f at the point (a, b). The vector $f_x(a, b)\hat{\imath} + f_y(a, b)\hat{\jmath} + \hat{k}$ is perpendicular to the surface $z = f(x, y)$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

14.6 The Chain Rule

123. A company sells regular widgets for \$4 apiece and premium widgets for \$6 apiece. If the demand for regular widgets is growing at a rate of 200 widgets per year, while the demand for premium widgets is dropping at the rate of 80 per year, the company's revenue from widget sales is:

(a) staying constant
(b) increasing
(c) decreasing

124. Suppose $R = R(u, v, w)$, $u = u(x, y, z)$, $v = v(x, y, z)$, $w = w(x, y, z)$. In the chain rule, how many terms will you have to add up to find the partial derivative of R with respect to x?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

125. Let $z = z(u, v)$ and $u = u(x, y, t)$; $v = v(x, y, t)$ and $x = x(t)$; $y = y(t)$. Then the expression for $\frac{dz}{dt}$ has

(a) Three terms

(b) Four terms

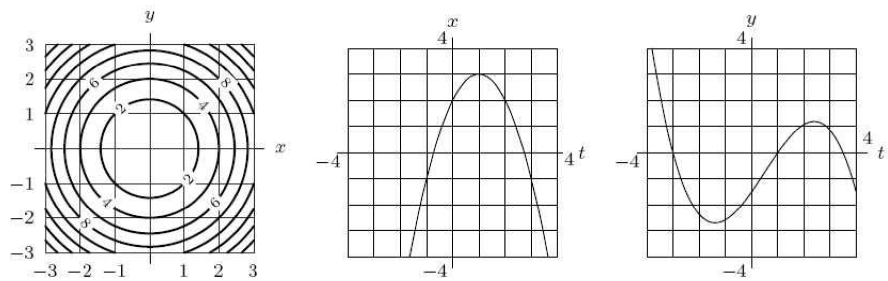
(c) Six terms

(d) Seven terms

(e) Nine terms

(f) None of the above

126. The figures below show contours of $z = z(x, y)$, x as a function of t, and y as a function of t. Decide if $\left.\frac{dz}{dt}\right|_{t=2}$ is



(a) Positive

(b) Negative

(c) Approximately zero

(d) Can't tell without further information

127. Let $s = f(x; y; z)$ and $x = x(u; v; w)$; $y = y(u; v; w)$; $z = z(u; v; w)$. To calculate $\frac{\partial s}{\partial u}(u = 1, v = 2, w = 3)$, which of the following pieces of information do you **not** need?

I $f(1, 2, 3) = 5$

II $f(7, 8, 9) = 6$

III $x(1, 2, 3) = 7$

IV $y(1, 2, 3) = 8$

V $z(1, 2, 3) = 9$

VI $f_x(1, 2, 3) = 20$

VII $f_x(7, 8, 9) = 30$

VIII $x_u(1, 2, 3) = -5$

IX $x_u(7, 8, 9) = -7$

(a) III, IV, VII, VIII

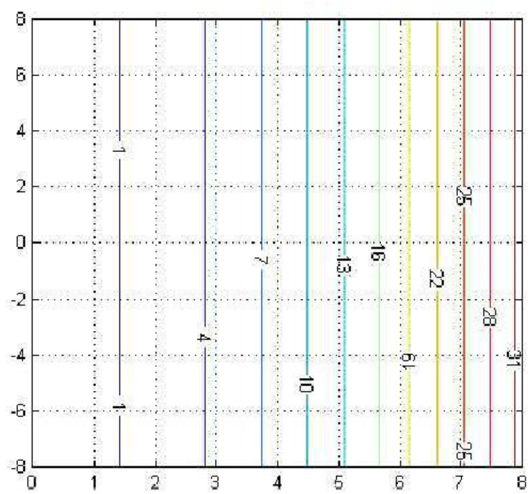
(b) I, IV, VI, VII

(c) II, V, VI, IX

(d) I, II, VI, IX

14.7 Second-Order Partial Derivatives

128. At the point (4,0), what is true of the second partial derivatives of $f(x, y)$?



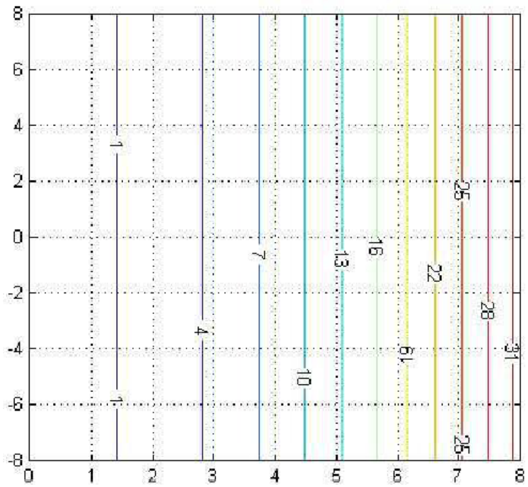
(a) $f_{xx} > 0$ and $f_{yy} > 0$

(b) $f_{xx} < 0$ and $f_{yy} < 0$

(c) $f_{xx} > 0$ and $f_{yy} = 0$

(d) $f_{xx} < 0$ and $f_{yy} = 0$

129. At the point (4,0), what is true of the second partial derivatives of $f(x, y)$?

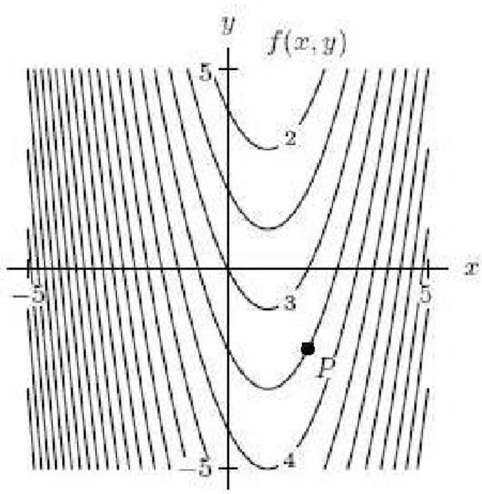


(a) $f_{xy} > 0$

(b) $f_{xy} < 0$

(c) $f_{xy} = 0$

130. The figure below shows level curves of $f(x, y)$. What are the signs of $f_{xx}(P)$ and $f_{yy}(P)$?



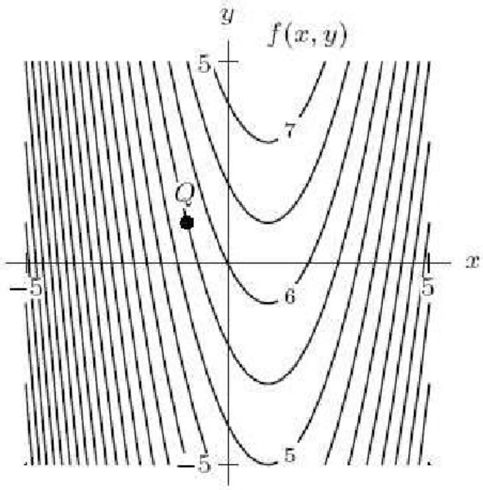
(a) $f_{xx}(P) > 0, f_{yy}(P) \approx 0$

(b) $f_{xx}(P) > 0, f_{yy}(P) < 0$

(c) $f_{xx}(P) \approx 0, f_{yy}(P) \approx 0$

(d) $f_{xx}(P) < 0, f_{yy}(P) > 0$

131. The figure below shows level curves of $f(x, y)$. What are the signs of $f_{xx}(Q)$ and $f_{yy}(Q)$?



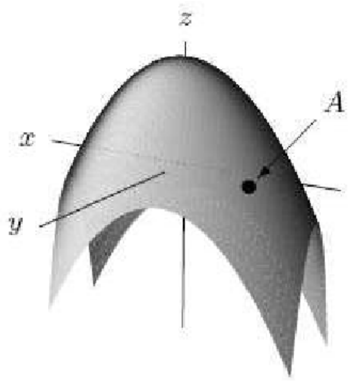
(a) $f_{xx}(Q) > 0, f_{yy}(Q) < 0$

(b) $f_{xx}(Q) < 0, f_{yy}(Q) < 0$

(c) $f_{xx}(Q) \approx 0, f_{yy}(Q) \approx 0$

(d) $f_{xx}(Q) < 0, f_{yy}(Q) \approx 0$

132. The figure below shows the surface $z = f(x, y)$. What are the signs of $f_{xx}(A)$ and $f_{yy}(A)$?



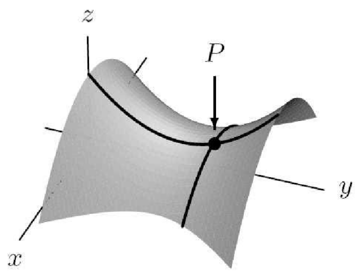
(a) $f_{xx}(A) > 0, f_{yy}(A) < 0$

(b) $f_{xx}(A) < 0, f_{yy}(A) < 0$

(c) $f_{xx}(A) \approx 0, f_{yy}(A) \approx 0$

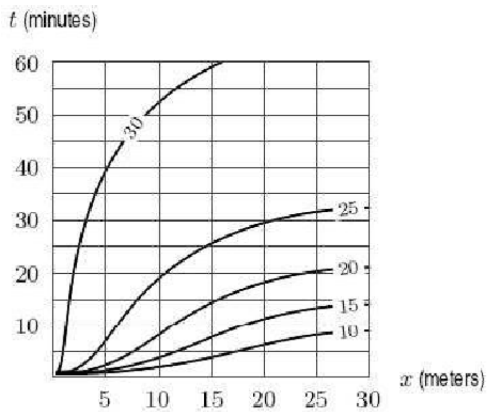
(d) $f_{xx}(A) < 0, f_{yy}(A) \approx 0$

133. The figure below shows the surface $z = f(x, y)$. What are the signs of $f_{xx}(P)$ and $f_{yy}(P)$?



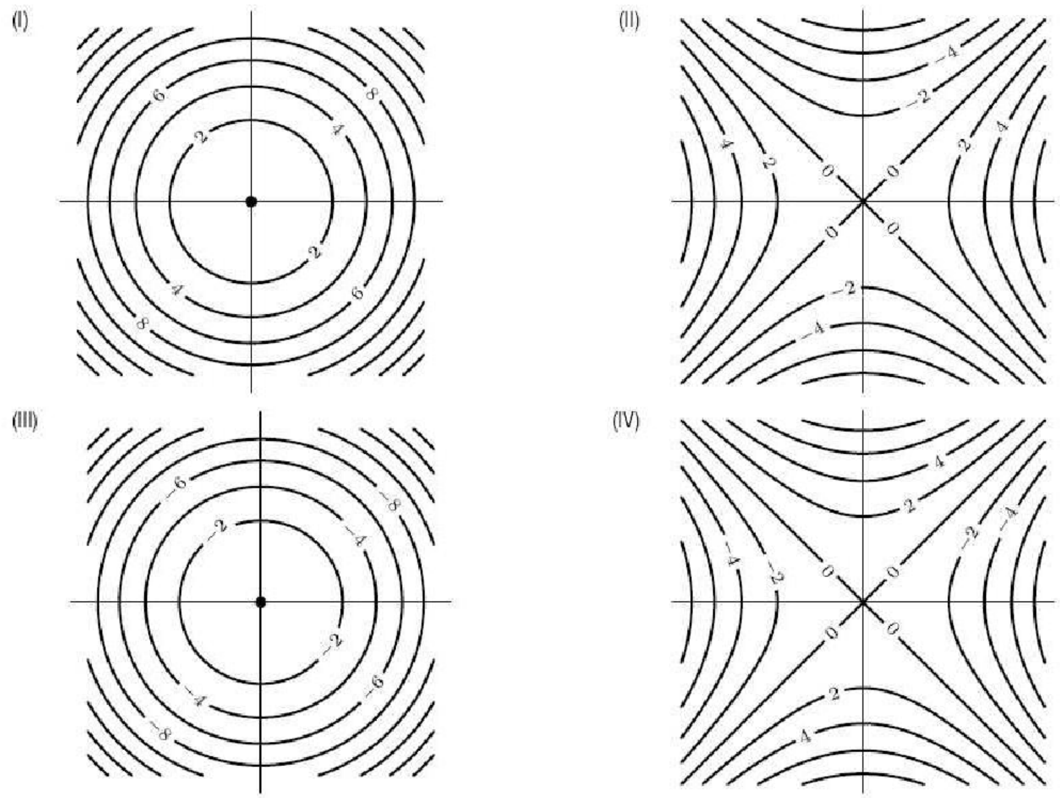
(a) $f_{xx}(P) > 0, f_{yy}(P) \approx 0$
(b) $f_{xx}(P) > 0, f_{yy}(P) < 0$
(c) $f_{xx}(P) \approx 0, f_{yy}(P) \approx 0$
(d) $f_{xx}(P) < 0, f_{yy}(P) > 0$

134. The figure below shows the temperature T °C as a function of distance x in meters along a wall and time t in minutes. Choose the correct statement and explain your choice without computing these partial derivatives.



(a) $\frac{\partial T}{\partial t}(t, 10) < 0$ and $\frac{\partial^2 T}{\partial t^2}(t, 10) < 0$.
(b) $\frac{\partial T}{\partial t}(t, 10) > 0$ and $\frac{\partial^2 T}{\partial t^2}(t, 10) > 0$.
(c) $\frac{\partial T}{\partial t}(t, 10) > 0$ and $\frac{\partial^2 T}{\partial t^2}(t, 10) < 0$.
(d) $\frac{\partial T}{\partial t}(t, 10) < 0$ and $\frac{\partial^2 T}{\partial t^2}(t, 10) > 0$.

135. The quadratic Taylor Polynomials (A)-(D) each approximate a function of two variables near the origin. Figures (I)-(IV) are contours near the origin. Match (A)-(D) to (I)-(IV).



A $-x^2 + y^2$

B $x^2 - y^2$

C $-x^2 - y^2$

D $x^2 + y^2$

(a) A - I, B - III, C - II, D - IV

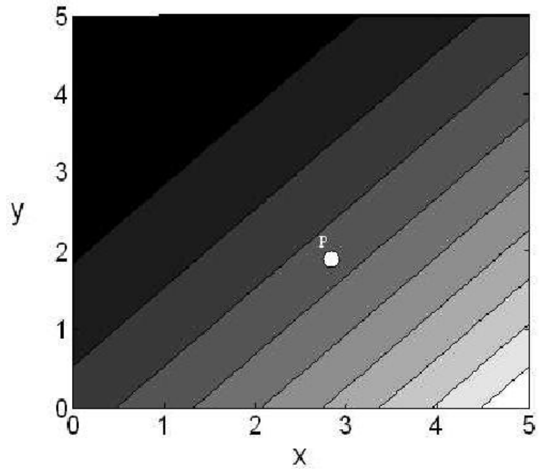
(b) A - II, B - IV, C - I, D - III

(c) A - IV, B - II, C - III, D - I

(d) A - III, B - I, C - IV, D - II

(e) A - II, B - IV, C - III, D - I

136. In the countour plot below dark shades represent small values of the function and light shades represent large values of the function. What is the sign of the mixed partial derivative?



(a) $f_{xy} > 0$

(b) $f_{xy} < 0$

(c) $f_{xy} \approx 0$

(d) This cannot be determined from the figure.

15.1 Local Extrema

137. Which of these functions has a critical point at the origin?

(a) $f(x, y) = x^2 + 2y^3$

(b) $f(x, y) = x^2y + 4xy + 4y$

(c) $f(x, y) = x^2y^3 - x^4 + 2y$

(d) $f(x, y) = x \cos y$

(e) All of the above

138. True or False? The function $f(x, y) = x^2y + 4xy + 4y$ has a local maximum at the origin.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

139. Which of these functions does not have a critical point?

(a) $f(x, y) = x^2 + 2y^3$
(b) $f(x, y) = x^2y + 4xy + 4y$
(c) $f(x, y) = x^2y^3 - x^4 + 2y$
(d) $f(x, y) = x \cos y$
(e) All have critical points

140. Which of these functions has a critical point at the origin?

(a) $f(x, y) = x^2 + 2x + 2y^3 - y^2$
(b) $f(x, y) = x^2y + xy$
(c) $f(x, y) = x^2y^2 - (1/2)x^4 + 2y$
(d) $f(x, y) = x^4y - 7y$

141. How would you classify the function $f(x, y) = x^2y + xy$ at the origin?

(a) This is a local maximum.
(b) This is a local minimum.
(c) This is a saddle point.
(d) We cannot tell.
(e) This is not a critical point.

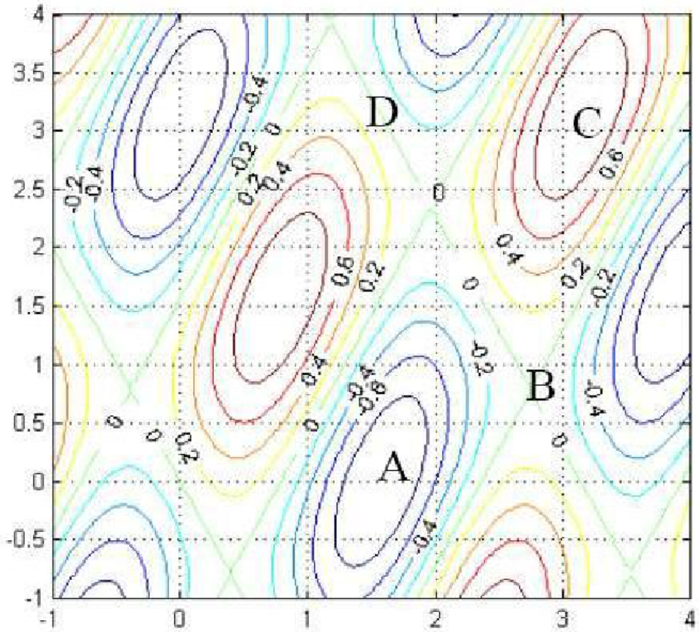
142. Which of these functions does not have a critical point with $y = 0$?

(a) $f(x, y) = x^2 + 2x + 2y^3 - y^2$
(b) $f(x, y) = x^2y + xy$
(c) $f(x, y) = x^2y^2 - (1/2)x^4 + 2y$
(d) $f(x, y) = x^4y - 7y$

143. Which of these functions does not have a critical point with $x = -1$?

(a) $f(x, y) = x^2 + 2x + 2y^3 - y^2$
(b) $f(x, y) = x^2y + xy$
(c) $f(x, y) = x^2y^2 - (1/2)x^4 + 2y$
(d) $f(x, y) = x^4y - 7y$

144. Which of the following points are critical points?



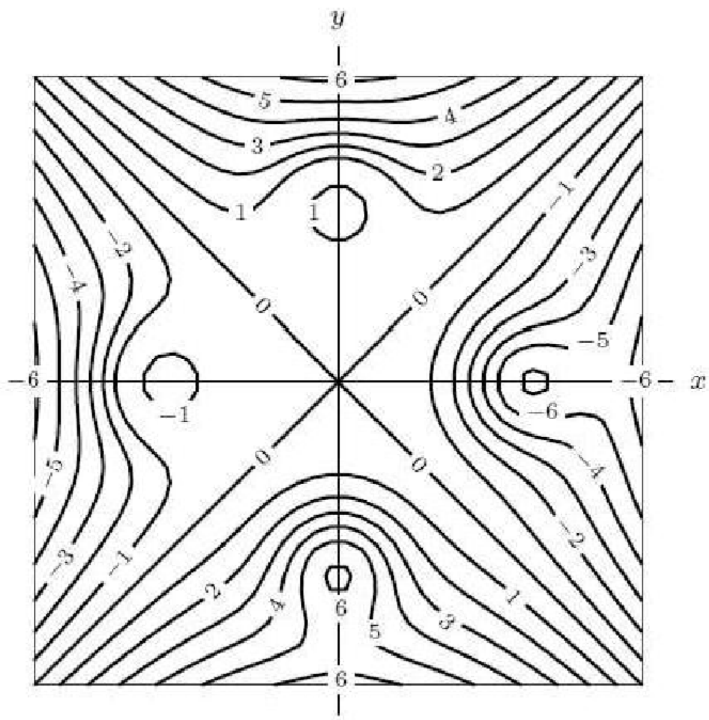
(a) A and C
(b) A, C, and D
(c) A, B, and C
(d) A, B, C, and D

145. Which of the following guarantees a saddle point of the function $f(x, y)$ at (a, b)?

(a) f_{xx} and f_{yy} have the same sign at (a, b).
(b) f_{xx} and f_{yy} have opposite signs at (a, b).
(c) f_{xy} is negative at (a, b).
(d) none of the above

15.2 Optimization

146. Estimate the global maximum and minimum of the functions whose level curves are given below. How many times does each occur?



(a) Max ≈ 6, occurring once; min ≈ -6, occurring once

(b) Max ≈ 6, occurring once; min ≈ -6, occurring twice

(c) Max ≈ 6, occurring twice; min ≈ -6, occurring twice

(d) Max ≈ 6, occurring three times; min ≈ -6, occurring three times

(e) None of the above

147. What are the global maximum and minimum values of $f(x, y) = x^2 + y^2$ on the triangular region in the first quadrant bounded by $x + y = 2$, $x = 0$, $y = 0$?

(a) Maximum = 2, Minimum = -2

(b) Maximum = 2, Minimum = 0

(c) Maximum = 4, Minimum = 2

(d) Maximum = 4, Minimum = 0

148. The function $f(x, y) = x^3 + 12xy + y^4$ has:

(a) no global maxes or mins

(b) a global max, but no global min

(c) a global min, but no global max

(d) both a global min and a global max

149. Which of the following would be enough evidence to conclude that a smooth function $f(x, y)$ has a global min?

(a) D is always positive

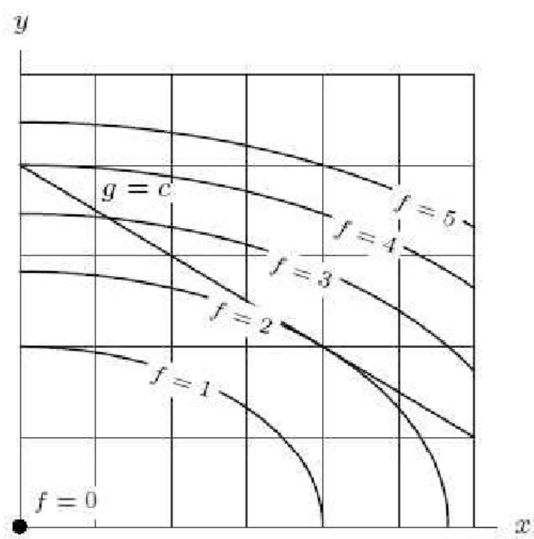
(b) $f_{xx} > 0$ and $f_{yy} > 0$

(c) $f(x, y)$ has no saddle points or local maxes

(d) none of the above

15.3 Constrained Optimization: Lagrange Multipliers

150. Find the maximum and minimum values of f on $g = c$.



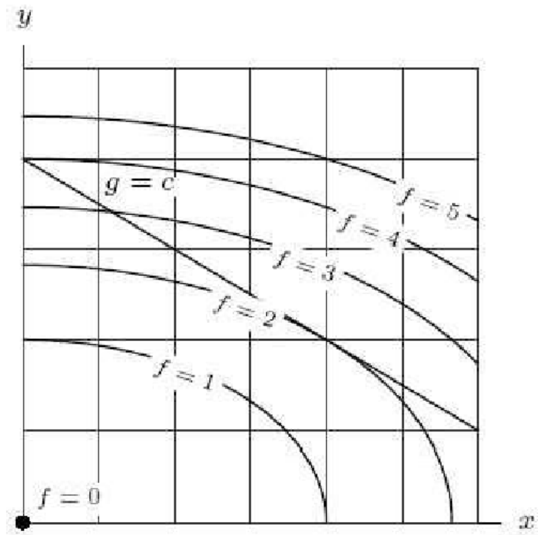
(a) max = 5, min = 0

(b) max = 4, min = 0

(c) max = 3, min = 2

(d) max = 4, min = 2

151. Find the maximum and minimum values of f on the trapezoidal region below $g = c$ in the first quadrant.



(a) max = 5, min = 0

(b) max = 5, min = 2

(c) max = 4, min = 1

(d) max = 4, min = 0

(e) max = 3, min = 2

(f) max = 3, min = 0

(g) max = 4, min = 2

(h) max = 5, min = 2

(i) max = 2, min = 0

152. Find the maximum of the production function $f(x, y) = xy$ in the first quadrant subject to each of the three budget constraints. Arrange the x coordinates of the optimal point in increasing order.

I $x + y = 12$

II $2x + 57y = 12$

III $3x + y/2 = 12$

(a) $I < II, < III$

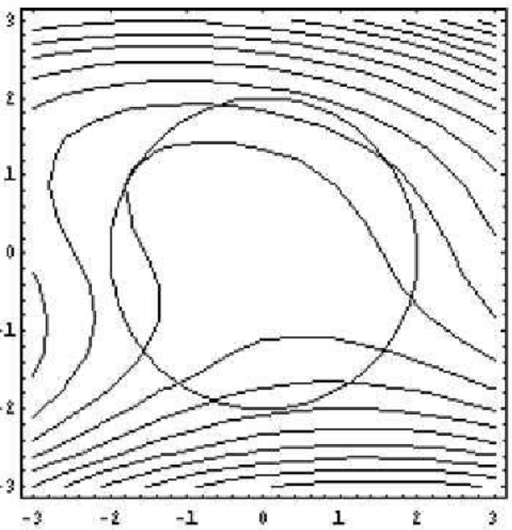
(b) $III < II < I$

(c) $II < III < I$

(d) $II < I < III$

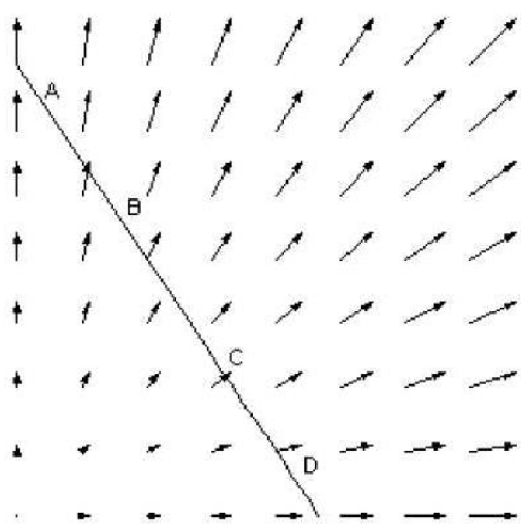
(e) $III < I < II$

153. This contour plot of $f(x, y)$ also shows the circle of radius 2 centered at (0,0). If you are restricted to being on the circle, how many local maxes and mins does $f(x, y)$ have?



(a) 1
(b) 2
(c) 3
(d) 4

154. This plot shows the gradient vectors for a (hidden) function $f(x, y)$ and a linear constraint. Which point is closest to the global min of $f(x, y)$ on this constraint?



(a) A
(b) B
(c) C
(d) D

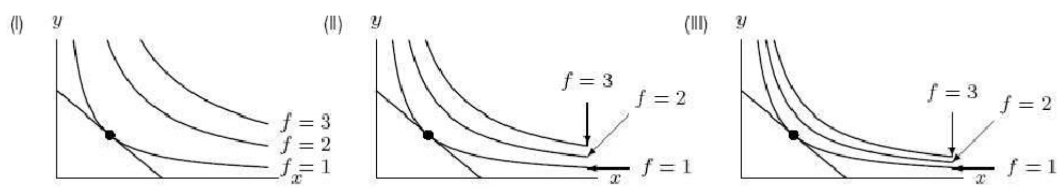
155. How many local maxs and mins does the function $f(x, y, z) = ax + by + cz$ have on the sphere $x^2 + y^2 + z^2 = 1$?

(a) 1
(b) 2
(c) 3
(d) 4
(e) None

156. How many points will produce local max/min of $f(x, y) = x^2 - y^2$ over the region $x^2 + y^2 <= r^2$?

(a) 1
(b) 2
(c) 3
(d) 4

157. The figure below shows the optimal point (marked with a dot) in three optimization problems with the same constraint. Arrange the corresponding values of λ in increasing order. (Assume λ is positive.)



(a) $I < II < III$
(b) $II < III < I$
(c) $III < II < I$
(d) $I < III < II$

158. Minimize $x^2 + y^2$ subject to $x^2y^2 = 4$

(a) 1
(b) 2
(c) $2\sqrt{2}$
(d) 4
(e) 8
(f) 16

159. Maximize x^2y^2 subject to $x^2 + y^2 = 4$.

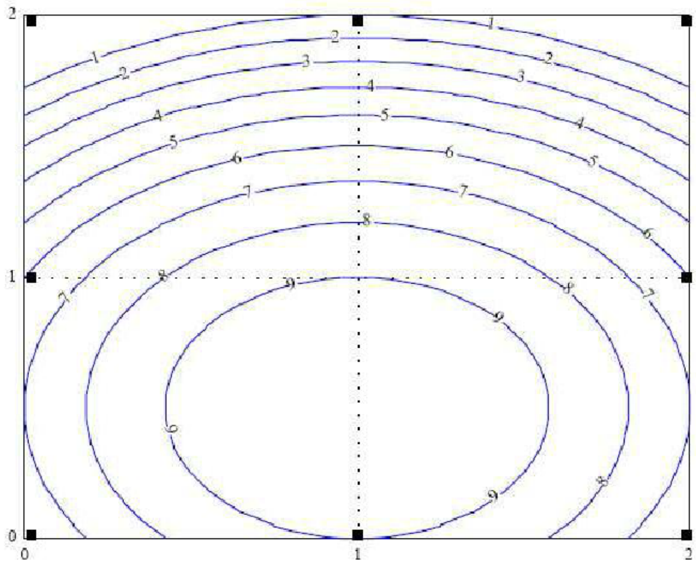
(a) 1
(b) 2
(c) $2\sqrt{2}$
(d) 4
(e) 8
(f) 16

160. Maximize x^2y^2 subject to $x + y = 4$ with $x, y \geq 0$.

(a) 1
(b) 2
(c) $2\sqrt{2}$
(d) 4
(e) 8
(f) 16

16.1 The Definite Integral of a Function of Two Variables

161. Suppose the contour plot shown shows the height of a pile of dirt in feet. Which of the following is clearly a lower bound for the volume of dirt?



(a) 0*1+0*1+6*1+6*1

(b) 9*1+9*1+9*1+9*1

(c) 9

(d) 6*1+6*1+9*1+9*1

162. Let R be the region $10 \le x \le 14$; $20 \le y \le 30$. The table below gives values of $f(x, y)$. Using upper and lower Riemann sums, what are the best possible upper and lower estimates for the integral

$$I = \int_R f(x, y) dxdy$$

x \ y	20	25	30
10	2.3	4.2	7.3
12	3.7	5.8	8.1
14	4.3	6.2	9.9

(a) $23 < I < 990$

(b) $92 < I < 300$

(c) $160 < I < 396$

(d) $160 < I < 300$

(e) $92 < I < 396$

163. Let R be the square defined by $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. The sign of the definite integral of x^4 over R is:

(a) positive
(b) negative
(c) zero
(d) cannot be determined

164. The value of $(1/\pi)$ times the integral of $1+x$ over the unit circle R is:

(a) 0
(b) 1
(c) π
(d) $\pi/2$

165. The integral $\int_R x\,dA$ over the region where R is the rectangle $-1 \leq x \leq 1$, $-1 \leq y \leq 1$ is

(a) positive
(b) negative
(c) zero

166. The integral $\int_T y dA$ over the region where T is the rectangle $-1 \leq x \leq 1$, $0 \leq y \leq 1$ is

(a) positive
(b) negative
(c) zero

167. The integral $\int_R (x - x^2) dA$ over the region where R is the rectangle $-1 \leq x \leq 1$, $-1 \leq y \leq 1$ is

(a) positive
(b) negative
(c) zero

168. The integral $\int_T (y - y^2) dA$ over the region where T is the rectangle $-1 \leq x \leq 1$, $0 \leq y \leq 1$ is

(a) positive
(b) negative
(c) zero

169. The integral $\int_L (x^2 - x)dA$ over the region where L is the rectangle $-1 \leq x \leq 0$, $-1 \leq y \leq 1$ is

(a) positive
(b) negative
(c) zero

170. The integral $\int_L (y + y^3)dA$ over the region where L is the rectangle $-1 \leq x \leq 0$, $-1 \leq y \leq 1$ is

(a) positive
(b) negative
(c) zero

171. The integral $\int_R (2x + 3y)dA$ over the region where R is the rectangle $-1 \leq x \leq 1$, $-1 \leq y \leq 1$ is

(a) positive
(b) negative
(c) zero

16.2 Iterated Integrals

172. The integral $\int_0^1 \int_0^1 x^2 dxdy$ represents the

(a) Area under the curve $y = x^2$ between $x = 0$ and $x = 1$.
(b) Volume under the surface $z = x^2$ above the square $0 \leq x, y \leq 1$ on the xy-plane.
(c) Area under the curve $y = x^2$ above the square $0 \leq x, y \leq 1$ on the xy-plane.

173. The integral $\int_0^1 \int_x^1 dydx$ represents the

(a) Area of a triangular region in the xy-plane.

(b) Volume under the plane $z = 1$ above a triangular region of the plane.
(c) Area of a square in the xy-plane.

174. Let $f(x, y)$ be a positive function. Rank the following integrals from smallest to largest.

$$I_1 = \int_0^1 \int_{x^2}^1 f(x,y)dydx \qquad I_2 = \int_0^1 \int_{x^3}^1 f(x,y)dydx \qquad I_3 = \int_0^1 \int_0^1 f(x,y)dydx$$

(a) $I_1 < I_2 < I_3$
(b) $I_1 < I_3 < I_2$
(c) $I_2 < I_1 < I_3$
(d) $I_2 < I_3 < I_1$
(e) $I_3 < I_2 < I_1$
(f) $I_3 < I_1 < I_2$

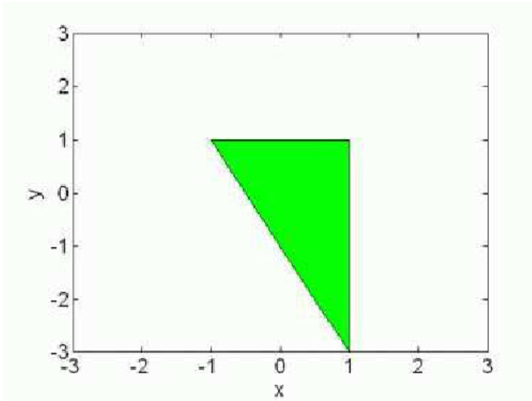
175. $\int_0^1 \int_0^{2-2x} f(x,y)dydx$ is an integral over which region?

(a) The triangle with vertices (0,0), (2,0), (0,1).
(b) The triangle with vertices (0,0), (0,2), (1,0).
(c) The triangle with vertices (0,0), (2,0), (2,1).
(d) The triangle with vertices (0,0), (1,0), (1,2).

176. $\int_0^1 \int_{2y}^2 f(x,y)dxdy$ is an integral over which region?

(a) The triangle with vertices (0,0), (2,0), (0,1).
(b) The triangle with vertices (0,0), (0,2), (1,0).
(c) The triangle with vertices (0,0), (2,0), (2,1).
(d) The triangle with vertices (0,0), (1,0), (1,2).

177. Which of the following integrals has the proper limits to integrate the shaded region below?



(a) $\int_{-1}^{1}\int_{-3}^{-2x-1} f(x,y)dydx$

(b) $\int_{-3}^{1}\int_{-\frac{1}{2}y-\frac{1}{2}}^{1} f(x,y)dxdy$

(c) $\int_{-1}^{1}\int_{-\frac{1}{2}x-1}^{1} f(x,y)dydx$

(d) $\int_{-3}^{1}\int_{-1}^{-\frac{1}{2}y-\frac{1}{2}} f(x,y)dxdy$

(e) None of the above

178. Which of the following integrals is equal to $\int_0^3\int_0^{4x} f(x,y)dydx$?

(a) $\int_0^{4x}\int_0^3 f(x,y)dxdy$

(b) $\int_0^{12}\int_{y/4}^3 f(x,y)dxdy$

(c) $\int_0^{12}\int_3^{y/4} f(x,y)dxdy$

(d) $\int_0^{12}\int_0^{y/4} f(x,y)dxdy$

(e) $\int_0^3\int_0^{4x} f(x,y)dxdy$

179. The region of integration in the integral $\int_0^2\int_0^{2x} f(x,y)dydx$ is a

(a) rectangle

(b) triangle with width 2 and height 4

(c) triangle with width 4 and height 2

(d) none of the above

180. The value of $\int_{-r}^{r}\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} xdydx$ is

(a) πr

(b) $\pi/2$

(c) πr^2

(d) 0

16.3 Triple Integrals

181. Which of the following is the mass of the solid defined by $0 \leq x \leq 2$, $0 \leq y \leq 3$, and $0 \leq z \leq 4$ with density function $\delta(x, y, z) = x + y$?

   (a) $\int_0^2 \int_0^4 \int_0^3 1 dydzdx$
   (b) $\int_0^2 \int_0^4 \int_0^3 (x+y) dydxdz$
   (c) $\int_0^2 \int_0^4 \int_0^3 (x+y) dxdzdy$
   (d) $\int_0^2 \int_0^4 \int_0^3 (x+y) dydzdx$

182. The region of integration for the integral $\int_{-r}^{r} \int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} \int_0^{10} f(x, y, z) dzdydx$ is a

   (a) sphere
   (b) cylinder
   (c) cone
   (d) none of the above

183. What does the integral $\int_0^1 \int_0^1 \int_0^1 zdzdydx$ represent?

   (a) The volume of a cube of side 1.
   (b) The volume of a sphere of radius 1.
   (c) The area of a square of side 1.
   (d) None of the above.

184. Which of the following integrals is equal to $\int_0^3 \int_0^2 \int_0^y f(x, y, z) dzdydx$?

   (a) $\int_0^2 \int_0^3 \int_0^y f(x, y, z) dzdxdy$
   (b) $\int_0^2 \int_0^3 \int_0^y f(x, y, z) dzdydx$
   (c) $\int_0^3 \int_0^2 \int_0^y f(x, y, z) dxdydz$
   (d) $\int_0^3 \int_0^2 \int_0^z f(x, y, z) dydzdx$

185. $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2) dzdydx$ describes the mass of

   (a) a cone that gets heavier toward the outside.

(b) a cone that gets lighter toward the outside.
(c) a ball that gets heavier toward the outside.
(d) a ball that gets lighter toward the outside.

186. Which of the following integrals does not make sense?

(a) $\int_1^3 \int_{y-1}^2 \int_0^y f(x,y,z)dzdxdy$
(b) $\int_1^3 \int_0^y \int_2^{y-1} f(x,y,z)dxdydz$
(c) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} f(x,y,z)dzdydx$
(d) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_0^{\sqrt{1-x^2-y^2}} f(x,y,z)dzdxdy$

16.4 Double Integrals in Polar Coordinates

187. A point is at coordinates $(r,\theta) = (1,\pi)$. What are the rectangular coordinates of this point?

(a) $(x,y) = (1,0)$
(b) $(x,y) = (0,1)$
(c) $(x,y) = (-1,0)$
(d) $(x,y) = (0,-1)$
(e) More than one of the above

188. A point is at coordinates $(x,y) = (0,-1)$. What are the polar coordinates of this point?

(a) $(r,\theta) = (1, \frac{3\pi}{4})$
(b) $(r,\theta) = (1, \frac{3\pi}{2})$
(c) $(r,\theta) = (1, -\pi)$
(d) $(r,\theta) = (1, -\frac{\pi}{2})$
(e) More than one of the above

189. Which of the following regions resembles a quarter of a doughnut?

(a) $0 \leq r \leq 5$, $0 \leq \theta \leq \pi/2$

(b) $3 \leq r \leq 5$, $0 \leq \theta \leq 2\pi$
(c) $3 \leq r \leq 5$, $\pi \leq \theta \leq 2\pi$
(d) $3 \leq r \leq 5$, $\pi \leq \theta \leq 3\pi/2$

190. Which of the following integrals is equivalent to $\int_0^3 \int_\pi^{2\pi} r d\theta dr$?

(a) $\int_0^3 \int_{-\sqrt{9-x^2}}^0 1 dy dx$
(b) $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} 1 dy dx$
(c) $\int_{-3}^3 \int_0^{\sqrt{9-y^2}} 1 dx dy$
(d) $\int_{-3}^0 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} 1 dx dy$

191. What geometric shape is describe by the equation $r = \theta$?

(a) line
(b) circle
(c) spiral
(d) none of the above

192. What geometric shape is describe by the equation $r = 4$?

(a) line
(b) circle
(c) spiral
(d) none of the above

193. What geometric shape is describe by the equation $\theta = 4$?

(a) line
(b) circle
(c) spiral
(d) none of the above

194. What geometric shape is describe by the equation $r = \sin\theta$?

(a) line
(b) circle
(c) spiral
(d) none of the above

195. What geometric shape is describe by the equation $r = 1/\sin\theta$?

(a) line
(b) circle
(c) spiral
(d) none of the above

196. Which of the following describes the upper half of the xy-plane?

(a) $0 \le r \le 2$, $0 \le \theta \le 2\pi$
(b) $0 \le r \le \infty$, $0 \le \theta \le \pi$
(c) $0 \le r \le \theta$, $0 \le \theta \le \pi$
(d) $2 \le r \le 4$, $\pi \le \theta \le 3\pi/2$

197. Which integral gives the area of the unit circle?

(a) $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x dy dx$
(b) $\int_0^{2\pi} \int_0^1 r dr d\theta$
(c) $\int_0^{2\pi} \int_0^1 dr d\theta$
(d) $\int_0^1 \int_0^{2\pi} d\theta dr$

16.5 Integrals in Cylindrical and Spherical Coordinates

198. What are the Cartesian coordinates of the point with cylindrical coordinates $(r, \theta, z) = (4, \pi, 6)$?

(a) $(x, y, z) = (0, -4, 4)$
(b) $(x, y, z) = (0, 4, 6)$

(c) $(x, y, z) = (-4, 4, 4)$
(d) $(x, y, z) = (4, 0, 4)$
(e) $(x, y, z) = (-4, 0, 6)$

199. What are the cylindrical coordinates of the point with Cartesian coordinates $(x, y, z) = (3, 3, 7)$?

(a) $(r, \theta, z) = (3, \pi, 7)$
(b) $(r, \theta, z) = (3, \pi/4, 3)$
(c) $(r, \theta, z) = (3\sqrt{2}, \pi/4, 7)$
(d) $(r, \theta, z) = (3\sqrt{2}, \pi, 7)$
(e) $(r, \theta, z) = (3\sqrt{2}, \pi, 3)$

200. What are the Cartesian coordinates of the point with spherical coordinates $(\rho, \phi, \theta) = (4, \pi, 0)$?

(a) $(x, y, z) = (0, 0, -4)$
(b) $(x, y, z) = (0, 0, 4)$
(c) $(x, y, z) = (4, 0, 0)$
(d) $(x, y, z) = (-4, 0, 0)$
(e) $(x, y, z) = (0, 4, 0)$

201. What are the spherical coordinates of the point with Cartesian coordinates $(x, y, z) = (0, -3, 0)$?

(a) $(\rho, \phi, \theta) = (3, \pi, \frac{\pi}{2})$
(b) $(\rho, \phi, \theta) = (3, \pi, -\frac{\pi}{2})$
(c) $(\rho, \phi, \theta) = (3, \frac{\pi}{2}, \frac{\pi}{2})$
(d) $(\rho, \phi, \theta) = (3, \frac{\pi}{2}, -\frac{\pi}{2})$
(e) $(\rho, \phi, \theta) = (3, \frac{\pi}{2}, \pi)$

202. Which of the following regions represents the portion of a cylinder of height 4 and radius 3 above the 3rd quadrant of the xy plane?

(a) $1 \leq r \leq 3$, $0 \leq z \leq 4$, $0 \leq \theta \leq \pi/2$
(b) $0 \leq r \leq 3$, $0 \leq z \leq 4$, $\pi \leq \theta \leq 3\pi/2$

(c) $0 \le r \le 4,\ 0 \le z \le 3,\ \pi \le \theta \le 3\pi/2$

(d) $0 \le r \le 3,\ 0 \le z \le 4,\ 0 \le \theta \le \pi/2$

203. Which of the following is equivalent to

$$\int_{-5}^{5}\int_{0}^{3}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} x\ dydzdx$$

(a) $\int_0^3\int_0^3\int_0^\pi r^2\cos\theta\ d\theta dzdr$

(b) $\int_0^5\int_0^3\int_0^\pi r^2\cos\theta\ d\theta dzdr$

(c) $\int_0^3\int_0^5\int_0^{2\pi} r\cos\theta\ d\theta dzdr$

(d) $\int_0^5\int_0^3\int_0^{2\pi} r^2\cos\theta\ d\theta dzdr$

204. Which of the following describes the bottom half of a sphere of radius 4 centered on the origin?

(a) $0 \le \rho \le 4,\ \pi/2 \le \phi \le \pi,\ 0 \le \theta \le 2\pi$

(b) $0 \le \rho \le 4,\ 0 \le \phi \le \pi/2,\ 0 \le \theta \le 2\pi$

(c) $0 \le \rho \le 4,\ 0 \le \phi \le \pi,\ 0 \le \theta \le \pi$

(d) $0 \le \rho \le 4,\ 0 \le \phi \le \pi,\ 0 \le \theta \le 2\pi$

205. Which of the following describes the surface of the cylinder of radius 3 centered on the z-axis?

(a) $0 \le \rho < \infty,\ \theta = \pi,\ 0 \le \phi \le \pi$

(b) $r = 3,\ \theta = \frac{\pi}{2},\ -\infty < z < \infty$

(c) $1 \le r \le 4,\ 0 \le \theta \le 2\pi,\ -5 \le z \le 2$

(d) $r = 3,\ 0 \le \theta \le 2\pi,\ -\infty < z < \infty$

206. Which of the following describes the solid cylinder of radius 4, centered on the z-axis, with the central cylindrical core removed?

(a) $0 \le \rho < \infty,\ \theta = \pi,\ 0 \le \phi \le \pi$

(b) $r = 3,\ \theta = \frac{\pi}{2},\ -\infty < z < \infty$

(c) $1 \le r \le 4,\ 0 \le \theta \le 2\pi,\ -5 \le z \le 2$

(d) $r = 3,\ 0 \le \theta \le 2\pi,\ -\infty < z < \infty$

207. Which of the following integrals give the volume of the unit sphere?

(a) $\int_0^{2\pi} \int_0^{2\pi} \int_0^1 d\rho d\theta d\phi$
(b) $\int_0^{\pi} \int_0^{2\pi} \int_0^1 d\rho d\theta d\phi$
(c) $\int_0^{\pi} \int_0^{2\pi} \int_0^1 \rho^2 \sin\phi d\rho d\theta d\phi$
(d) $\int_0^{\pi} \int_0^{2\pi} \int_0^1 \rho^2 \sin\phi d\rho d\phi d\theta$
(e) $\int_0^{\pi} \int_0^{2\pi} \int_0^1 \rho d\rho d\phi d\theta$

17.1 Parameterized Curves

208. Which of the following is an equation of a line in three dimensions (x, y, z)?

(a) $x = 4$
(b) $y = 2x + 3$
(c) $z = 3x + 2y + 7$
(d) All of the above
(e) None of the above

209. Which of the following best describes the path of a particle defined by the parametric equations $x(t) = \cos(t^2)$, $y(t) = \sin(t^2)$?

(a) a circle around which the particle moves faster and faster
(b) a parabola on which the particle travels at constant speed
(c) a parabola on which the particle travels faster and faster
(d) a circle on which the particle moves slower and slower

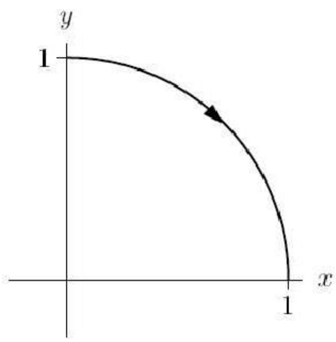
210. Which of the following is not a parametrization of the entire curve $y = x^3$?

(a) $x(t) = t; y(t) = t^3$
(b) $x(t) = t^2; y(t) = t^6$
(c) $x(t) = t^3; y(t) = t^9$
(d) $x(t) = 2t; y(t) = 8t^3$

211. What does the path of the particle described by $x(t) = \cos(t)$, $y(t) = \sin(t)$, $z(t) = -t$ look like?

(a) a circle in the xz plane
(b) a helix on which the particle is traveling up
(c) a helix on which the particle is traveling down
(d) a sine wave in the xz plane

212. Which of the following parameterizations does not describe the quarter circle in the figure below?



(a) $(\cos t, \sin t), 0 \leq t \leq \pi/2$
(b) $(\sin t, \cos t), 0 \leq t \leq \pi/2$
(c) $(-\cos t, \sin t), \pi/2 \leq t \leq \pi$
(d) $(\cos t, -\sin t), 3\pi/2 \leq t \leq 2\pi$

213. Let $(\cos at, \sin at)$ be the position at time t seconds of a particle moving around a circle, where $a > 0$. If a is increased,

(a) The radius of the circle increases.
(b) The speed of the particle increases.
(c) The center of the circle changes.
(d) The path ceases to be a circle.

214. Let $(a \cos t, a \sin t)$ be the position at time t seconds of a particle moving around a circle, where $a > 0$. If a is increased,

(a) The radius of the circle increases.
(b) The speed of the particle increases.
(c) The center of the circle changes.
(d) The path ceases to be a circle.

215. Which of the following parametric curves does not trace out the unit circle?

(a) $(\cos t, \sin t), 0 \leq t \leq 2\pi$

(b) $(\sin^2 t, \cos^2 t), 0 \leq t \leq 2\pi$

(c) $(\sin(t^2), \cos(t^2)), 0 \leq t \leq 2\pi$

(d) $(\sin(2t), \cos(2t)), 0 \leq t \leq 2\pi$

216. Which of the following parametric paths describe particles that are not traveling along a straight line in 3-space?

(a) $(1-t, 2+2t, 3-t)$

(b) $(1-t^2, 2+2t^2, 3-t^2)$

(c) $(1, 2, 1-t)$

(d) $(1, t, 1-t^2)$

(e) More than one of the above

217. The value of c for which the lines $l(t) = (c+4t, 2-t, 3+t)$ and $m(t) = (4t, 1-8t, 4+4t)$ intersect is

(a) 4

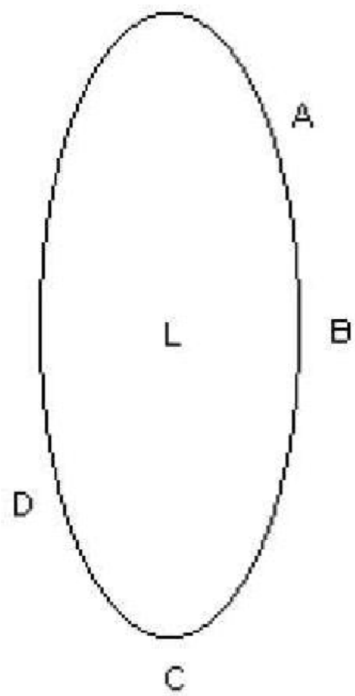
(b) 0

(c) -4

(d) There is no such c.

17.2 Motion, Velocity, and Acceleration

218. A lighthouse at position L is in the middle of a lake. Its beam is turning counterclockwise with constant angular velocity. At which point is the velocity vector of the beam largest?



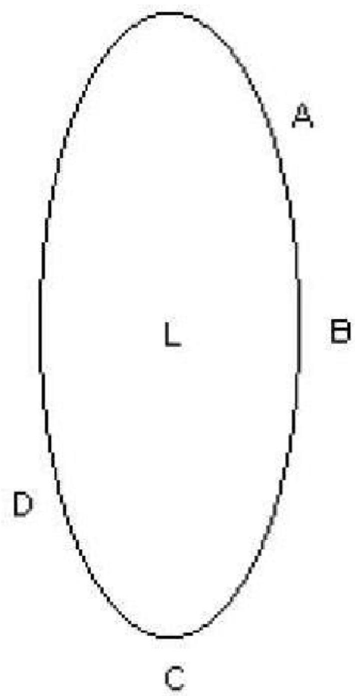
(a) A

(b) B

(c) C

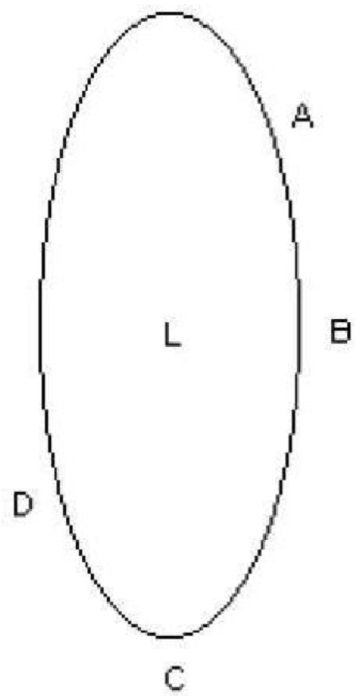
(d) D

219. A lighthouse at position L is in the middle of a lake. Its beam is turning counterclockwise with constant angular velocity. At which point is the velocity vector of the beam most parallel to $\hat{j}$?



(a) A
(b) B
(c) C
(d) D

220. A lighthouse at position L is in the middle of a lake. Its beam is turning counterclockwise with constant angular velocity. At which point is the acceleration vector of the beam most parallel to $\hat{j}$?



(a) A
(b) B
(c) C
(d) D

221. Which of the following describes the motion of a particle that is moving along a straight line and slowing down?

(a) $\vec{a}$ and $\vec{v}$ are parallel and point in the same direction.
(b) $\vec{a}$ and $\vec{v}$ are parallel and point in opposite directions.
(c) $\vec{a}$ and $\vec{v}$ are perpendicular.
(d) None of the above.

222. True or False: If the speed of a particle is zero, its velocity must be zero.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

223. A particle that is not accelerating must have zero velocity.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

224. A particle with constant speed must have zero acceleration.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

225. A particle with zero acceleration must have constant speed.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

226. A particle with constant speed must have constant velocity.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

227. The functions $x(t)$ and $y(t)$ describe the coordinates of a jeep in miles as it drives around the desert from noon ($t = 0$ hrs) until 2 pm ($t = 2$ hrs), when the jeep returns to its starting location. We want to use these functions to predict how many miles will be recorded on the odometer during this interval by doing an integral of some function $\int_0^2 f(t)\,dt$. What units must the function $f(t)$ have?

(a) miles
(b) miles/hr
(c) miles/hr^2

(d) miles^2/hr^2

(e) None of the above

228. The functions $x(t)$ and $y(t)$ describe the coordinates of a jeep in miles as it drives around the desert from noon ($t = 0$ hrs) until 2 pm ($t = 2$ hrs), when the jeep returns to its starting location. Which of the following must be true?

(a) $\int_0^2 x(t)\,dt = 0$

(b) $\int_0^2 x'(t)\,dt = 0$

(c) $\int_0^2 x''(t)\,dt = 0$

(d) More than one of the above

(e) None of the above

229. The functions $x(t) = 2\sin \pi t$ and $y(t) = 2\cos \pi t$ describe the coordinates of a jeep in miles as it drives around the desert from noon ($t = 0$ hrs) until 2 pm ($t = 2$ hrs), when the jeep returns to its starting location. According to the jeep's odometer, how far will it have traveled?

(a) 4 miles

(b) 2π miles

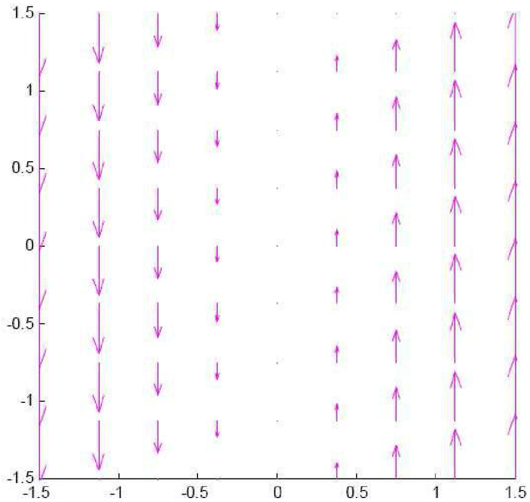
(c) 4π miles

(d) $8\pi^2$ miles

(e) None of the above

17.3 Vector Fields

230. Which of the following could be a formula for the vector field pictured?



(a) $\vec{F}(x, y) = x\hat{i}$

(b) $\vec{F}(x, y) = y\hat{i}$

(c) $\vec{F}(x, y) = x\hat{j}$

(d) $\vec{F}(x, y) = y\hat{j}$

231. Which of the following formulas will produce a vector field where all vectors point away from the y axis and all vectors on a vertical line have the same length?

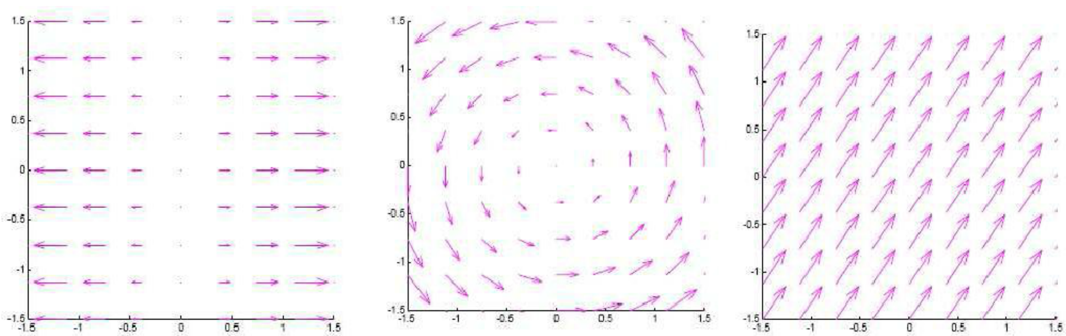
(a) $\vec{F}(x, y) = x^3\hat{i}$

(b) $\vec{F}(x, y) = x^2\hat{i}$

(c) $\vec{F}(x, y) = x^3\hat{j}$

(d) $\vec{F}(x, y) = x^2\hat{j}$

232. Which of the following vector fields cannot be a gradient vector field?

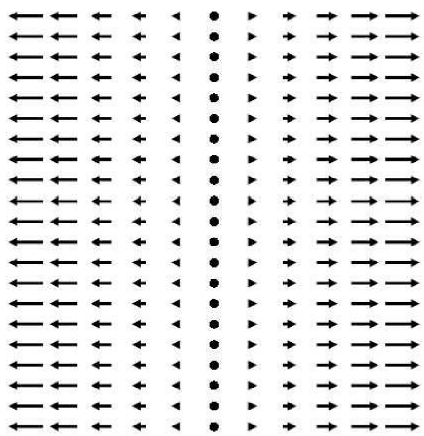


(a) the one on the left

(b) the one in the middle

(c) the one on the right

233. Which formula below could produce the graph of the vector field:



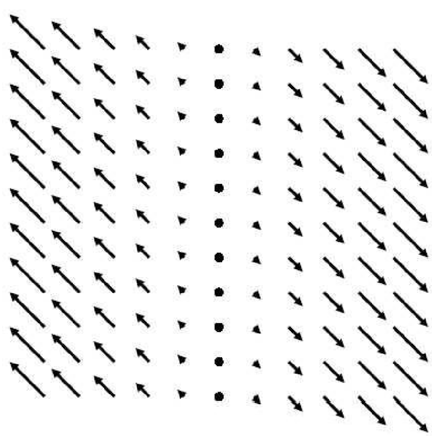
(a) $f(x)\hat{\imath}$

(b) $g(x)\hat{\jmath}$

(c) $h(y)\hat{\imath}$

(d) $k(y)\hat{\jmath}$

234. Which formula below could produce the graph of the vector field:



(a) $f(x)\hat{\imath} + f(x)\hat{\jmath}$

(b) $g(x)\hat{\imath} - g(x)\hat{\jmath}$

(c) $h(y)\hat{\imath} + h(y)\hat{\jmath}$

(d) $k(y)\hat{\imath} - k(y)\hat{\jmath}$

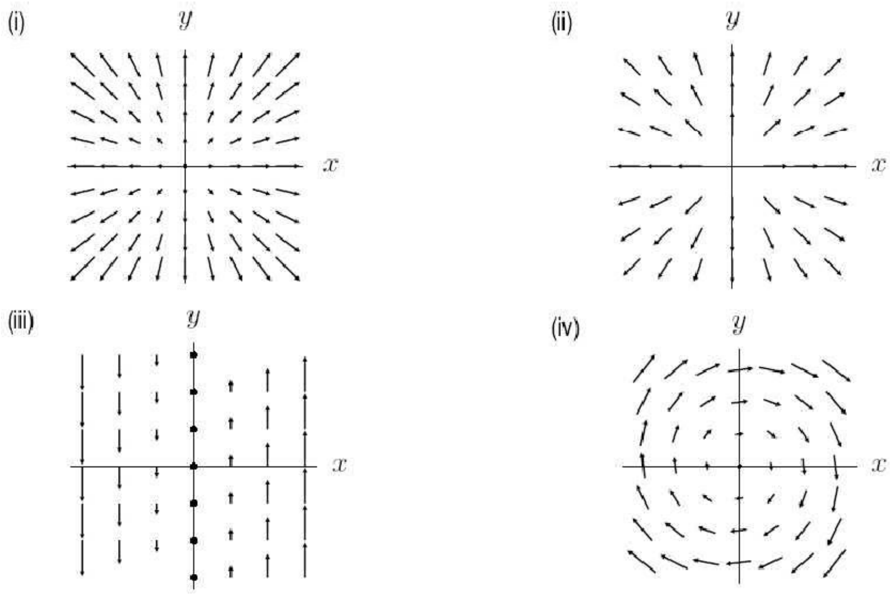
235. Match the vector fields with the appropriate graphs.

1 $\vec{F}_1 = \frac{\vec{r}}{||\vec{r}||}$

2 $\vec{F}_2 = \vec{r}$

3 $\vec{F}_3 = y\hat{\imath} - x\hat{\jmath}$

4 $\vec{F}_4 = x\hat{\jmath}$



(a) 1 and III, 2 and I, 3 and IV, 4 and II

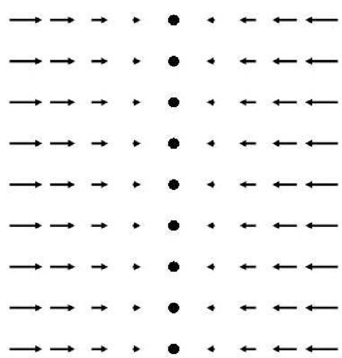
(b) 1 and IV, 2 and I, 3 and II, 4 and III

(c) 1 and II, 2 and I, 3 and IV, 4 and III

(d) 1 and II, 2 and IV, 3 and I, 4 and III

(e) 1 and I, 2 and II, 3 and IV, 4 and III

236. The figure shows the vector field $\vec{F} = \nabla f$. Which of the following are possible choices for $f(x, y)$?



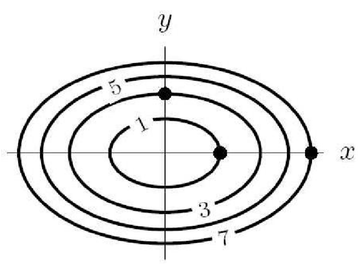
(a) x^2

(b) $-x^2$

(c) $-2x$

(d) $-y^2$

237. Rank the length of the gradient vectors at the points marked on the contour plot below.



(a) $7 > 5 > 3 > 1$

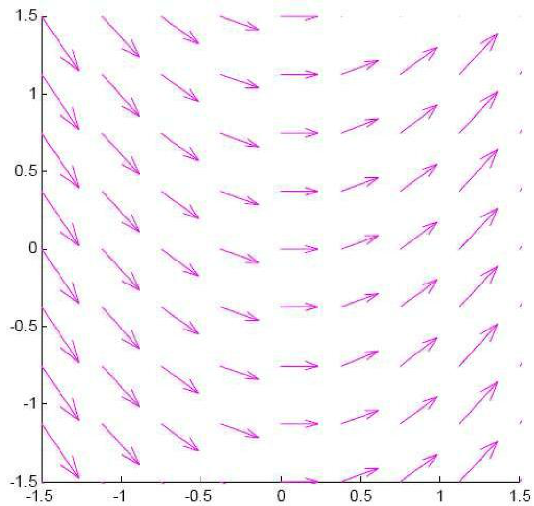
(b) $1 > 3 > 5 > 7$

(c) $7 > 1 > 3 > 5$

(d) $3 > 1 > 7 > 5$

17.4 The Flow of a Vector Field

238. The flow lines for the vector field pictured will be:



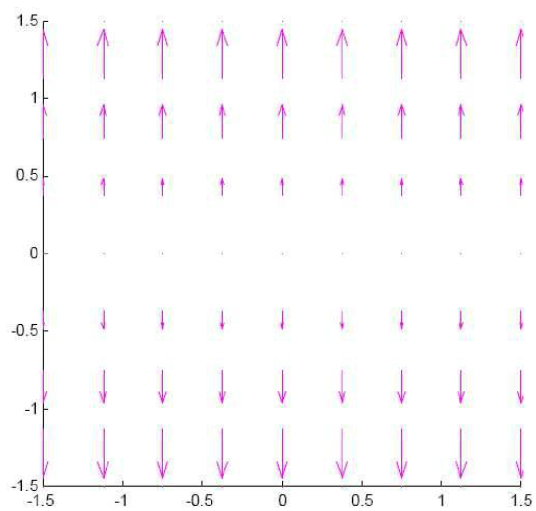
(a) straight lines

(b) circles

(c) ellipses

(d) parabolas

239. The function that describes the distance of a particle from the x-axis as it follows a flow line is:



(a) linear

(b) exponential

(c) sinusoidal

(d) logarithmic

240. Which parameterized curve is not a flow line of the vector field $\vec{F} = x\hat{i} + y\hat{j}$?

(a) $\vec{r}(t) = e^t\hat{i} + e^t\hat{j}$
(b) $\vec{r}(t) = e^t\hat{i} + 2e^t\hat{j}$
(c) $\vec{r}(t) = 3e^t\hat{i} + 3e^t\hat{j}$
(d) $\vec{r}(t) = 2e^t\hat{i} + e^{2t}\hat{j}$

241. Which parameterized curves are not flow lines of the vector field $\vec{F} = -y\hat{i} + x\hat{j}$.

(a) $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j}$
(b) $\vec{r}(t) = \cos t\hat{i} - \sin t\hat{j}$
(c) $\vec{r}(t) = \sin t\hat{i} - \cos t\hat{j}$
(d) $\vec{r}(t) = 2\cos t\hat{i} + 2\sin t\hat{j}$

242. The path $x = t$, $y = e^t$ is a flow line of which vector field?

(a) $\hat{i} + y\hat{j}$
(b) $\hat{i} + x^2\hat{j}$
(c) $x\hat{i} + x\hat{j}$
(d) $y\hat{i} + \hat{j}$

243. An object flowing in the vector field $\vec{F} = y\hat{i} + x\hat{j}$ is at the point $(1, 2)$ at time $t = 5.00$. Estimate the approximate position of the object at time $t = 5.01$.

(a) $(1.02, 2)$
(b) $(1.02, 2.01)$
(c) $(1.01, 2)$
(d) $(1.01, 2.02)$

244. Two different curves can be flow lines for the same vector field.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

245. If one parameterization of a curve is a flow line for a vector field, then all its parameterizations are flow lines for the vector field.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

246. If $\vec{r}(t)$ is a flow line for a vector field $\vec{F}$, then $\vec{r}_1(t) = \vec{r}(t-5)$ is a flow line of the same vector field $\vec{F}$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

247. If $\vec{r}(t)$ is a flow line for a vector field $\vec{F}$, then $\vec{r}_1(t) = \vec{r}(2t)$ is a flow line of the vector field $2\vec{F}$.

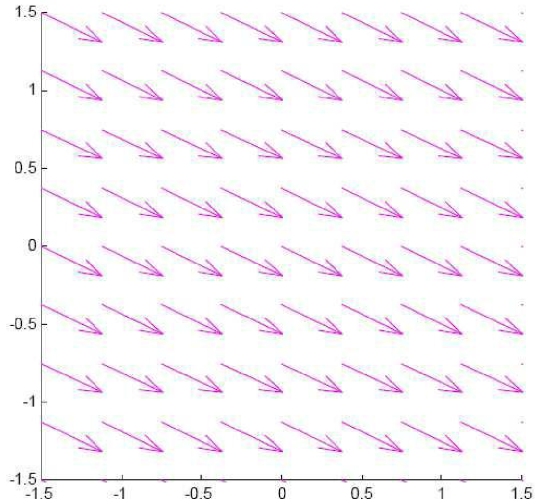
(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

248. If $\vec{r}(t)$ is a flow line for a vector field $\vec{F}$, then $\vec{r}_1(t) = 2\vec{r}(t)$ is a flow line of the vector field $2\vec{F}$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

18.1 The Idea of a Line Integral

249. Suppose C is the path consisting of a straight line from (-1,0) to (1,0) followed by a straight line from (1,0) to (1,1). The line integral along this path is

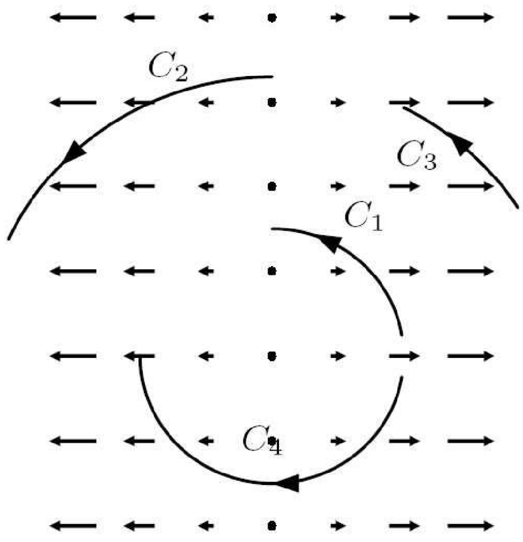


(a) positive.
(b) zero.
(c) negative.

250. Given three curves, C_1 (a straight line from (0,0) to (1,1)), C_2 (a straight line from (1,-1) to (1,1)), and C_3 (the portion of the circle of radius $\sqrt{2}$ centered at the origin moving from (1,-1) to (1,1)), rank the curves according to the value of the line integral of $\vec{F} = -y\hat{\imath} + x\hat{\jmath}$ on each curve.

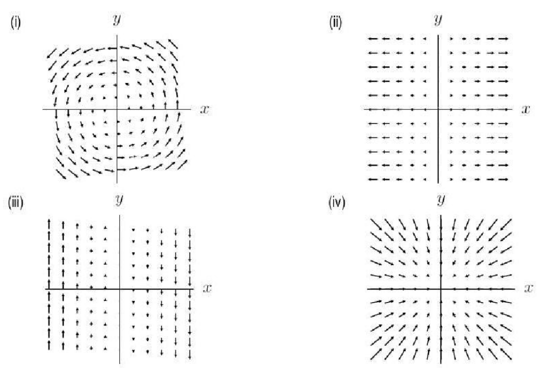
(a) $C_1 < C_2 < C_3$
(b) $C_2 < C_1 < C_3$
(c) $C_3 < C_1 < C_2$

251. The vector field $\vec{F}$ and several curves are shown below. For which of the paths is the line integral positive?



(a) C_1
(b) C_2
(c) C_3
(d) C_4

252. If the path C is a circle centered at the origin, oriented clockwise, which of the vector fields below has a positive circulation?



(a) i
(b) ii
(c) iii
(d) iv

253. True or false? Given two circles centered at the origin, oriented counterclockwise, and any vector field $\vec{F}$, then the path integral of $\vec{F}$ is larger around the circle with larger radius.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

254. True or false? If $\vec{F}$ is any vector field and C is a circle, then the integral of $\vec{F}$ around C traversed clockwise is the negative of the integral of $\vec{F}$ around C traversed counterclockwise.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

255. The work done by the force field $\vec{F} = y\hat{\imath}$ as an object moves along a straight line joining (1, 1) to (1,-1) is

(a) positive

(b) negative

(c) zero

256. How much work does it take to move in a straight line from coordinates (1,3) to (5,3) in the vector field $\vec{F} = -4\hat{\imath} + 3\hat{\jmath}$? Assume that coordinates are in meters and force is in Newtons.

(a) -25 Joules

(b) -16 Joules

(c) 7 Joules

(d) 16 Joules

(e) 25 Joules

18.2 Computing Line Integrals Over Parameterized Curves

257. Which of the following is equivalent to the line integral of $\vec{F}(x,y)$ on the line segment from (1,1) to (3,4)?

   (a) $\int_0^1 \vec{F}(1+2t, 1+3t)dt$
   (b) $\int_0^1 \vec{F}(1+2t, 1+3t) \cdot (2\hat{\imath}+3\hat{\jmath})dt$
   (c) $\int_0^1 \vec{F}(3,4) \cdot (2\hat{\imath}+3\hat{\jmath})dt$
   (d) $\int_0^1 \vec{F}(1+t, 1+t) \cdot (2\hat{\imath}+3\hat{\jmath})dt$

258. Which of the following is equivalent to the line integral of $\vec{F}(x,y)$ on the line segment from (1,1) to (3,4)?

   (a) $\int_0^2 \vec{F}(1+t, 1+1.5t) \cdot (\hat{\imath}+1.5\hat{\jmath})dt$
   (b) $\int_0^2 \vec{F}(1+t, 1+1.5t) \cdot (2\hat{\imath}+3\hat{\jmath})dt$
   (c) $\int_0^1 \vec{F}(1+t, 1+1.5t) \cdot (\hat{\imath}+1.5\hat{\jmath})dt$
   (d) $\int_0^1 \vec{F}(1+t, 1+1.5t) \cdot (2\hat{\imath}+3\hat{\jmath})dt$

259. If C_1 is the path parameterized by $\vec{r}_1(t) = \langle t, t\rangle$, $0 \le t \le 1$, and if C_2 is the path parameterized by $\vec{r}_2(t) = \langle t^2, t^2\rangle$, $0 \le t \le 1$, and if $\vec{F} = x\hat{\imath} + y\hat{\jmath}$, which of the following is true?

   (a) $\int_{C_1} \vec{F} \cdot d\vec{r} > \int_{C_2} \vec{F} \cdot d\vec{r}$
   (b) $\int_{C_1} \vec{F} \cdot d\vec{r} < \int_{C_2} \vec{F} \cdot d\vec{r}$
   (c) $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$

260. If C_1 is the path parameterized by $\vec{r}_1(t) = \langle t, t\rangle$, $0 \le t \le 1$, and if C_2 is the path parameterized by $\vec{r}_2(t) = \langle 1-t, 1-t\rangle$, $0 \le t \le 1$, and if $\vec{F} = x\hat{\imath} + y\hat{\jmath}$, which of the following is true?

   (a) $\int_{C_1} \vec{F} \cdot d\vec{r} > \int_{C_2} \vec{F} \cdot d\vec{r}$
   (b) $\int_{C_1} \vec{F} \cdot d\vec{r} < \int_{C_2} \vec{F} \cdot d\vec{r}$
   (c) $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$

261. If C_1 is the path parameterized by $\vec{r}_1(t) = \langle t, t \rangle$, $0 \leq t \leq 1$, and if C_2 is the path parameterized by $\vec{r}_2(t) = \langle \sin t, \sin t \rangle$, $0 \leq t \leq 1$, and if $\vec{F} = x\hat{\imath} + y\hat{\jmath}$, which of the following is true?

(a) $\int_{C_1} \vec{F} \cdot d\vec{r} > \int_{C_2} \vec{F} \cdot d\vec{r}$

(b) $\int_{C_1} \vec{F} \cdot d\vec{r} < \int_{C_2} \vec{F} \cdot d\vec{r}$

(c) $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$

262. Consider the path C_1 parameterized by $\vec{r}_1(t) = (\cos t, \sin t)$, $0 \leq t \leq 2\pi$ and the path C_2 paramterized by $\vec{r}_2(t) = (2\cos t, 2\sin t)$, $0 \leq t \leq 2\pi$. Let $\vec{F}$ be a vector field. Is it always true that $\int_{C_2} \vec{F} \cdot d\vec{r} = 2\int_{C_1} \vec{F} \cdot d\vec{r}$?

(a) Yes

(b) No

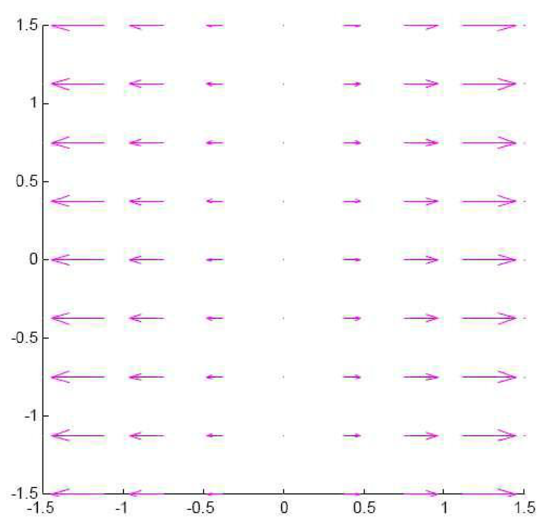
263. Consider the path C_1 parameterized by $\vec{r}_1(t) = (\cos t, \sin t)$, $0 \leq t \leq 2\pi$ and the path C_2 paramterized by $\vec{r}_2(t) = (\cos 2t, \sin 2t)$, $0 \leq t \leq 2\pi$. Let $\vec{F}$ be a vector field. Is it always true that $\int_{C_2} \vec{F} \cdot d\vec{r} = 2\int_{C_1} \vec{F} \cdot d\vec{r}$?

(a) Yes

(b) No

18.3 Gradient Fields and Path-Independent Fields

264. The vector field shown is the gradient vector field of $f(x, y)$. Which of the following are equal to $f(1, 1)$?



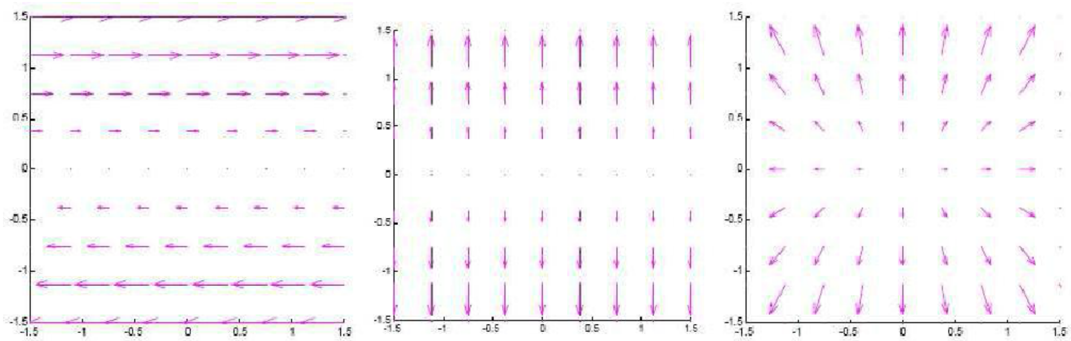
(a) $f(1,-1)$

(b) $f(-1,1)$

(c) both of the above

(d) none of the above

265. Which of the vector fields below is not path independent?

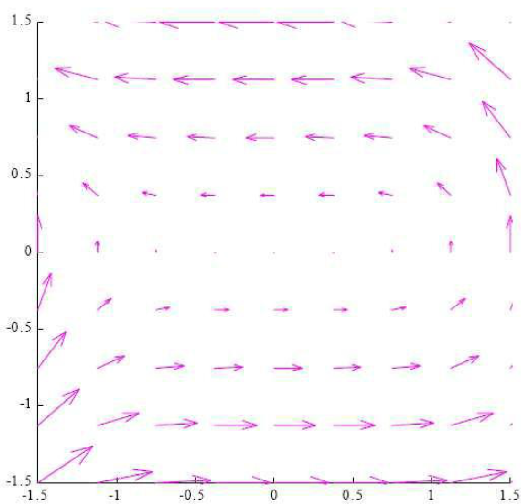


(a) the one on the left

(b) the one in the middle

(c) the one on the right

266. Which of the following explains why this vector field is not a gradient vector field?



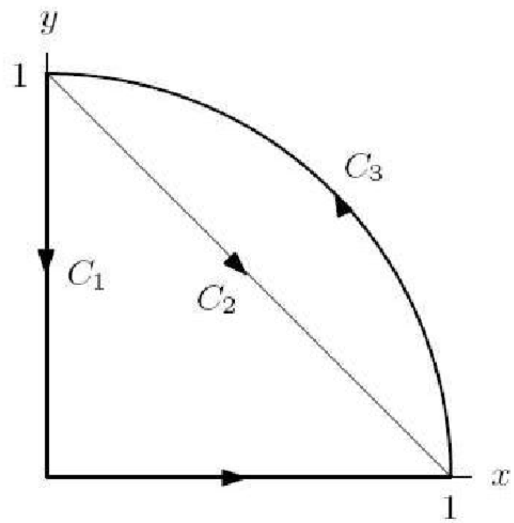
(a) The line integral from (-1,1) to (1,1) is negative.

(b) The circulation around a circle centered at the origin is zero.

(c) The circulation around a circle centered at the origin is not zero.

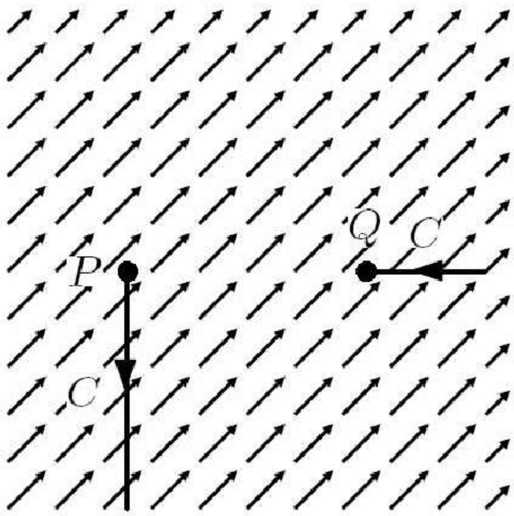
(d) None of the above.

267. The line integral of $\vec{F} = \nabla f$ along one of the paths shown below is different from the integral along the other two. Which is the odd one out?



(a) C_1
(b) C_2
(c) C_3

268. The figure below shows the vector field ∇f, where f is continuously differentiable in the whole plane. The two ends of an oriented curve C from P to Q are shown, but the middle portion of the path is outside the viewing window. The line integral $\int_C \nabla f \cdot d\vec{r}$ is



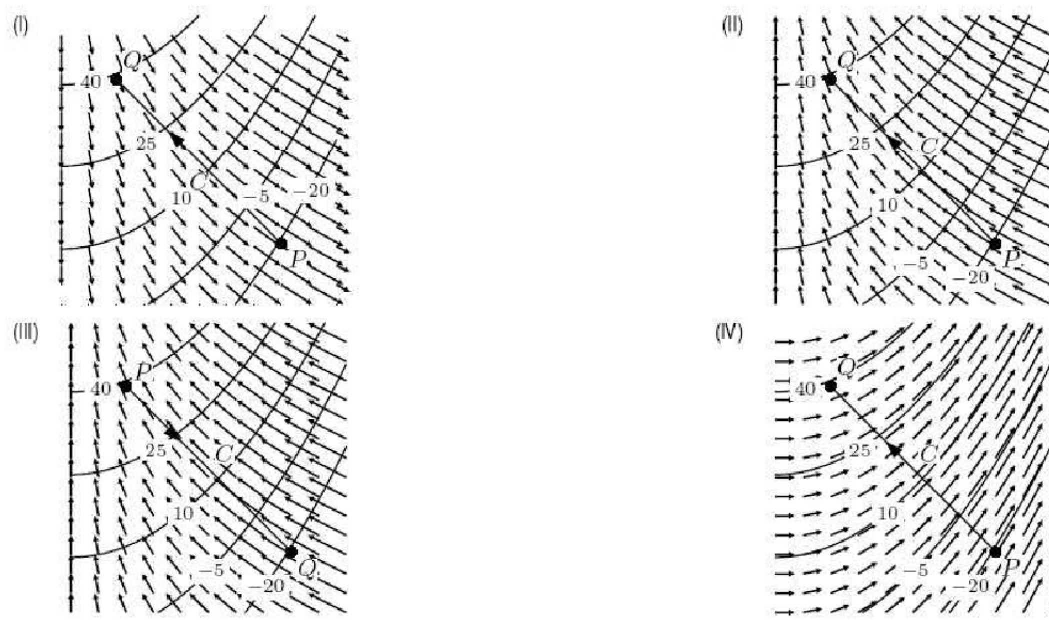
(a) Positive

(b) Negative

(c) Zero

(d) Can't tell without further information

269. Which of the diagrams contain all three of the following: a contour diagram of a function f, the vector field ∇f of the same function, and an oriented path C from P to Q with $\int_C \nabla F \cdot d\vec{r} = 60$?



(a) I
(b) II
(c) III
(d) IV

270. If f is a smooth function of two variables that is positive everywhere and $\vec{F} = \nabla f$, which of the following can you conclude about $\int_C \vec{F} \cdot d\vec{r}$?

(a) It is positive for all smooth paths C.
(b) It is zero for all smooth paths C.
(c) It is positive for all closed smooth paths C.
(d) It is zero for all closed smooth paths C.

271. What is the potential function for the vector field $\vec{F} = 2y\hat{\imath} + 2x\hat{\jmath}$?

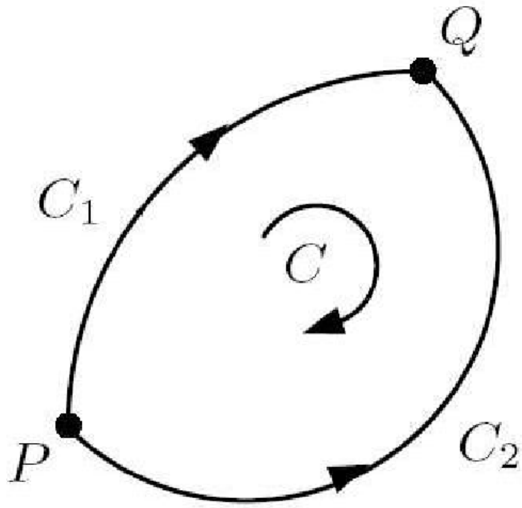
(a) $f(x, y) = 4xy$
(b) $f(x, y) = 2x^2 + 2y^2$
(c) $f(x, y) = 2xy$
(d) This is not a conservative vector field.

18.4 Path-Dependent Vector Fields and Green's Theorem

272. What will guarantee that $\vec{F}(x, y) = y\hat{\imath} + g(x, y)\hat{\jmath}$ is not a gradient vector field?

(a) $g(x, y)$ is a function of y only
(b) $g(x, y)$ is a function of x only
(c) $g(x, y)$ is always larger than 1
(d) $g(x, y)$ is a linear function

273. The figure shows a curve C broken into two pieces C_1 and C_2. Which of the following statements is true for any smooth vector field $\vec{F}$?



(a) $\int_C \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r}$

(b) $\int_C \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} - \int_{C_2} \vec{F} \cdot d\vec{r}$

(c) $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$

(d) $\int_C \vec{F} \cdot d\vec{r} = 0$

(e) More than one of the above is true.

274. A smooth two dimensional vector field $\vec{F} = F_1\hat{\imath} + F_2\hat{\jmath}$, with $\vec{F} \neq \vec{0}$ satisfies $\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$ at every point in the plane. Which of the following statments is not true?

(a) $\int_C \vec{F} \cdot d\vec{r} = 0$ for all smooth paths C.

(b) $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$ for any two smooth paths C_1 and C_2 with the same starting and ending points.

(c) $\vec{F} = \nabla f$ for some function f.

(d) If C_1 is the straight line from -1 to 1 on the y-axis and if C_2 is the right half of the unit circle, traversed counter-clockwise, then $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$.

(e) More than one of the above is not true.

19.1 The Idea of a Flux Integral

275. A river is flowing downstream at a constant rate of 5 ft/s. We take a rectangular net that is 6 ft wide and 3 ft deep and place it in the river so that a vector perpendicular

to the net (a normal vector) is parallel to the velocity of the water. What is the rate at which water flows through the net?

(a) 0 ft^3/s
(b) 15 ft^2/s
(c) 30 ft^2/s
(d) 90 ft^3/s
(e) None of the above

276. A river is flowing downstream at a constant rate of 5 ft/s. We take a rectangular net that is 6 ft wide and 3 ft deep and place it in the river so that there is a 30 degree angle between a vector perpendicular to the net (a normal vector) and the velocity of the water. What is the rate at which water flows through the net?

(a) 0 ft^3/s
(b) 90 ft^3/s
(c) 45 ft^3/s
(d) ≈ 78 ft^3/s
(e) None of the above

277. Through which surface is the flux of $\vec{F}(x, y, z) = 2\hat{\imath}$ negative?

(a) A square of side length 2 in the yz plane, oriented in the negative x direction
(b) A square of side length 2 in the xz plane, oriented in the positive y direction
(c) A square of side length 2 in the yz plane, oriented in the positive x direction
(d) A square of side length 2 in the xz plane, oriented up.

278. Through which surface is the flux of $\vec{F}(x, y, z) = x\hat{\imath}$ the most positive?

(a) A square of side length 2 in the yz plane, oriented in the positive x direction
(b) A square of side length 2 in the plane $x = 4$, oriented in the positive x direction
(c) A square of side length 4 in the plane $x = 2$, oriented in the positive x direction
(d) A square of side length 1 in the plane $x = 8$, oriented in the positive x direction

279. Consider the flux of $\vec{F} = x\hat{\imath}$ through a disk of radius 1 oriented as described below. In which case is the flux positive?

(a) In the yz-plane, centered at the origin and oriented in the direction of increasing x.

(b) In the plane $x = 2$, centered on the x-axis and oriented away from the origin.

(c) In the plane $y = 2$, centered on the y-axis and oriented away from the origin.

(d) In the plane $x + y = 2$, centered on the x-axis and oriented away from the origin.

(e) More than one of the above has positive flux.

(f) None of the above.

280. Consider the flux of $\vec{F} = y\hat{i}$ through a disk of radius 1 oriented as described below. In which case is the flux positive?

(a) In the yz-plane, centered at the origin and oriented in the direction of increasing x.

(b) In the plane $x = 2$, centered on the x-axis and oriented away from the origin.

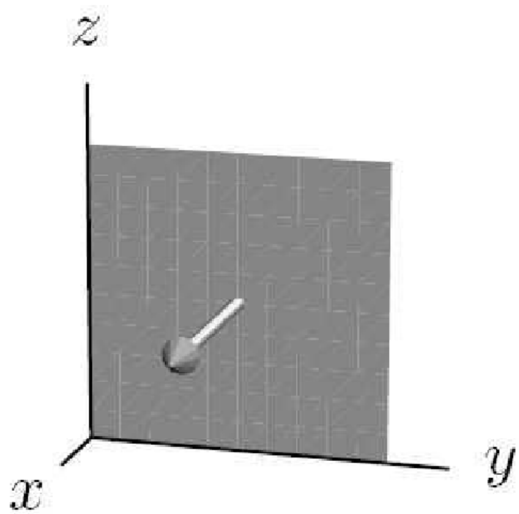
(c) In the plane $y = 2$, centered on the y-axis and oriented away from the origin.

(d) In the plane $x + y = 2$, centered on the x-axis and oriented away from the origin.

(e) More than one of the above has positive flux.

(f) None of the above.

281. Which vector field has a positive flux through the surface below?



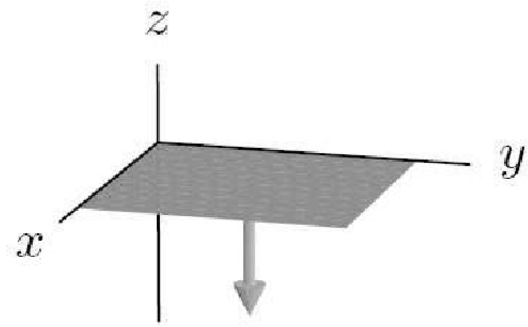
(a) $\vec{F} = x\hat{j}$

(b) $\vec{F} = y\hat{j}$

(c) $\vec{F} = -z\hat{i}$

(d) $\vec{F} = (z + x)\hat{i}$

282. Which vector field has a positive flux through the surface below?

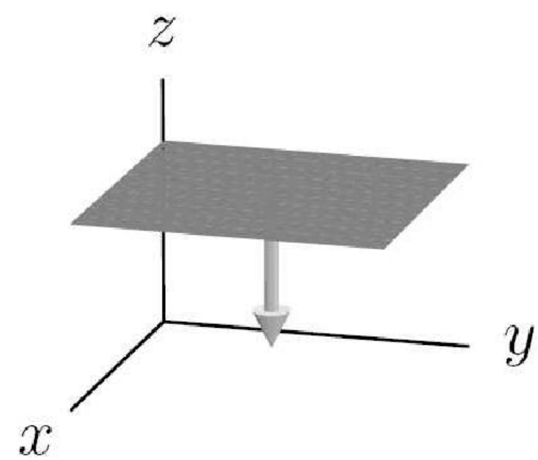


(a) $\vec{F} = -y\hat{k}$
(b) $\vec{F} = y\hat{j}$
(c) $\vec{F} = -z\hat{i}$
(d) $\vec{F} = x\hat{k}$

283. Let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$. Which of the surfaces below has positive flux?

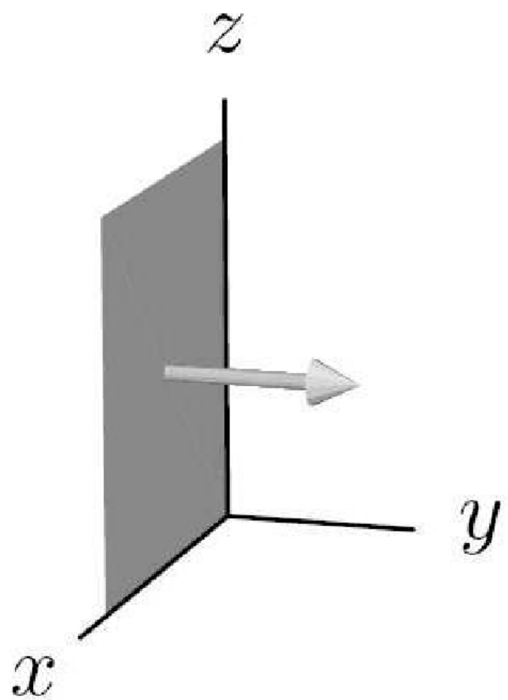
(a) Sphere of radius 1 centered at the origin, oriented outward.
(b) Unit disk in the xy-plane, oriented upward.
(c) Unit disk in the plane $x = 2$, oriented toward the origin.
(d) None of the above.

284. Choose the vector field with the largest flux through the surface below.



(a) $\vec{F}_1 = 2\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$
(b) $\vec{F}_2 = \hat{\imath} - 2\hat{\jmath} + 7\hat{k}$
(c) $\vec{F}_3 = -7\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$
(d) $\vec{F}_4 = -11\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$
(e) $\vec{F}_5 = -5\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

285. Choose the vector field with the largest flux through the surface below.



(a) $\vec{F}_1 = 2\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$
(b) $\vec{F}_2 = \hat{\imath} - 2\hat{\jmath} + 7\hat{k}$
(c) $\vec{F}_3 = -7\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$
(d) $\vec{F}_4 = -11\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$
(e) $\vec{F}_5 = -5\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

286. Which of the following vector fields has the largest flux through the surface of a sphere of radius 2 centered at the origin?

(a) $\vec{F}_1 = \frac{\vec{\rho}}{||\vec{\rho}||}$
(b) $\vec{F}_2 = \frac{\vec{\rho}}{||\vec{\rho}||^2}$

(c) $\vec{F}_3 = x\hat{j}$
(d) $\vec{F}_4 = \vec{\rho}||\vec{\rho}||$

19.2 Flux Integrals For Graphs, Cylinders, and Spheres

287. The flux of the vector field $\vec{F} = 4\hat{\rho}$ through a sphere of radius 2 centered on the origin is:

    (a) 0
    (b) 8π
    (c) 16π
    (d) 32π
    (e) 64π
    (f) None of the above

288. The flux of the vector field $\vec{F} = 3\hat{\rho} + 2\hat{\theta} + \hat{\phi}$ through a sphere of radius 1/2 centered on the origin is:

    (a) 0
    (b) 3π
    (c) 9π
    (d) 27π
    (e) 72π
    (f) None of the above

289. The flux of the vector field $\vec{F} = 4\hat{\theta} + 2\hat{z}$ through a cylinder of radius 1 centered on the z axis, between $z = 0$ and $z = 2$:

    (a) 0
    (b) π
    (c) 2π
    (d) 3π
    (e) 4π

290. The flux of the vector field $\vec{F} = r\hat{r} + r\hat{\theta}$ through a cylinder of radius 2 centered on the z axis, between $z = 0$ and $z = 2$:

(a) 0
(b) 2π
(c) 4π
(d) 8π
(e) 16π

291. The flux of the vector field $\vec{F} = z\hat{r}$ through a cylinder of radius $1/2$ centered on the z axis, between $z = 0$ and $z = 3$ is

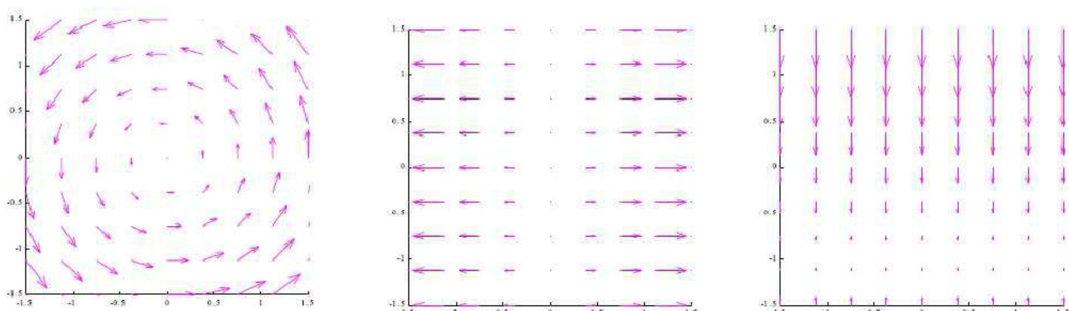
(a) 0
(b) 2π
(c) 3π
(d) $\frac{9}{2}\pi$
(e) 9π

292. All but one of the flux calculations below can be done with just multiplication, but one requires an integral. Which one?

(a) $\vec{F} = 3\hat{\rho} + 2\hat{\phi}$ through a sphere of radius 4.
(b) $\vec{F} = \rho\hat{\rho} + \theta\hat{\phi}$ through a sphere of radius 3.
(c) $\vec{F} = r\hat{r} + r\hat{z}$ through a disk of radius 2, centered on the z axis, in the $z = 2$ plane.
(d) $\vec{F} = r^2\hat{r} + z\hat{\theta}$ through a cylinder of radius 1, between $z = 1$ and $z = 3$.

20.1 The Divergence of a Vector Field

293. Moving from the picture on the left to the picture on the right, what are the signs of $\nabla \cdot \vec{F}$?



(a) positive, positive, negative

(b) zero, positive, negative

(c) positive, negative, zero

(d) zero, negative, positive

294. If $\vec{F}(x, y, z)$ is a vector field and $f(x, y, z)$ is a scalar function, which of the following is not defined?

(a) ∇f

(b) $\nabla \cdot \vec{F} + f$

(c) $\vec{F} + \nabla f$

(d) $\nabla \cdot \vec{F} + \nabla f$

(e) More than one of the above

(f) None of the above

295. If $\vec{F}(x, y, z)$ is a vector field and $f(x, y, z)$ is a scalar function, which of the following quantities is a vector?

(a) $\nabla \cdot \vec{F}$

(b) $\nabla f \cdot \vec{u}$

(c) $\nabla \cdot \nabla f$

(d) $(\nabla \cdot \vec{F})\vec{F}$

296. True or False? If all the flow lines of a vector field $\vec{F}$ are parallel straight lines, then $\nabla \cdot \vec{F} = 0$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

297. True or False? If all the flow lines of a vector field $\vec{F}$ radiate outward along straight lines from the origin, then $\nabla \cdot \vec{F} > 0$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident
(d) False, and I am very confident

298. In Cartesian coordinates given the vector field $\vec{F} = F_1\hat{\imath} + F_2\hat{\jmath} + F_3\hat{k}$,

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

.

Which of the following vector fields has zero divergence, so that it could represent the flow of a liquid which does not expand or contract?

(a) $\vec{F} = 2\sin(3z^2)\hat{\imath} + 5xyz\hat{\jmath} + 3e^{7x}\hat{k}$
(b) $\vec{F} = 3\ln(yz)\hat{\imath} + 2x^3z^7\hat{\jmath} + 4\cos(2x)\hat{k}$
(c) $\vec{F} = 6e^{2y}\hat{\jmath} + 3\sin(4z)\hat{k}$
(d) None of the above

299. In cylindrical coordinates given the vector field $\vec{F} = F_1\hat{r} + F_2\hat{\theta} + F_3\hat{z}$,

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{r}\frac{\partial (rF_1)}{\partial r} + \frac{1}{r}\frac{\partial F_2}{\partial \theta} + \frac{\partial F_3}{\partial z}$$

.

What is the divergence of the vector field $\vec{F} = 2\theta\hat{r} + 3z\hat{\theta} + 4r\hat{z}$?

(a) $\vec{\nabla} \cdot \vec{F} = 2\theta + 3z + 4r$
(b) $\vec{\nabla} \cdot \vec{F} = 9$
(c) $\vec{\nabla} \cdot \vec{F} = \frac{2\theta}{r}$
(d) $\vec{\nabla} \cdot \vec{F} = 0$
(e) None of the above

300. In spherical coordinates given the vector field $\vec{F} = F_1\hat{\rho} + F_2\hat{\theta} + F_3\hat{\phi}$,

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{\rho^2}\frac{\partial (\rho^2 F_1)}{\partial \rho} + \frac{1}{\rho\sin\phi}\frac{\partial F_2}{\partial \theta} + \frac{1}{\rho\sin\phi}\frac{\partial (F_3\sin\phi)}{\partial \phi}$$

.

What is the divergence of the vector field $\vec{F} = \frac{3}{\rho^2}\hat{\rho} + 2r\hat{\theta}$?

(a) $\vec{\nabla} \cdot \vec{F} = \frac{2r}{\rho\sin\phi}$
(b) $\vec{\nabla} \cdot \vec{F} = \frac{3}{\rho^2}$
(c) $\vec{\nabla} \cdot \vec{F} = \frac{\cos\phi}{\rho\sin\phi}$
(d) $\vec{\nabla} \cdot \vec{F} = 0$
(e) None of the above

20.2 The Divergence Theorem

301. Given a small cube resting on the xy plane with corners at $(0,0,0)$, $(a,0,0)$, $(a,a,0)$, and $(0,a,0)$, which vector field will produce positive flux through that cube?

    (a) $\vec{F} = 3\hat{\imath}$
    (b) $\vec{F} = x\hat{\imath} - y\hat{\jmath}$
    (c) $\vec{F} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$
    (d) $\vec{F} = z\hat{k}$

302. Let $\vec{F} = (5x + 7y)\hat{\imath} + (7y + 9z)\hat{\jmath} + (9z + 11x)\hat{k}$. This vector field produces the largest flux through which of the following surfaces?

    (a) S_1, a sphere of radius 2 centered at the origin.
    (b) S_2, a cube of side 2 centered at the origin, with sides parallel to the axes.
    (c) S_3, a sphere of radius 1 centered at the origin.
    (d) S_4, a pyramid contained inside S_3.

303. True or False? The vector field $\vec{F}$ is defined everywhere in a region W bounded by a surface S. If $\nabla \cdot \vec{F} > 0$ at all points of W, then the vector field $\vec{F}$ points outward at all points of S.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

304. True or False? The vector field $\vec{F}$ is defined everywhere in a region W bounded by a surface S. If $\nabla \cdot \vec{F} > 0$ at all points of W, then the vector field $\vec{F}$ points outward at some points of S.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

305. True or False? The vector field $\vec{F}$ is defined everywhere in a region W bounded by a surface S. If $\int_S \vec{F} \cdot d\vec{A} > 0$, then $\nabla \cdot \vec{F} > 0$ at some points of W.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

306. Which of the following vector fields produces the largest flux out of the unit sphere centered at the origin?

(a) $\vec{F_1} = (e^z + x^3)\hat{i} + e^x\hat{j} + y^3\hat{k}$

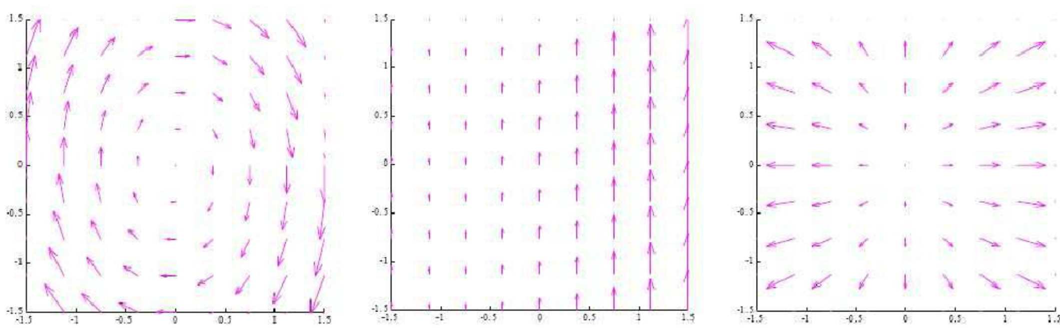
(b) $\vec{F_2} = (z^2 + \cos y)\hat{i} - y^3\hat{j} + x^3y^3\hat{k}$

(c) $\vec{F_3} = z^2\hat{i} - (x^2 + z^2)\hat{j} + (z^3 + zy^2)\hat{k}$

(d) $\vec{F_4} = (x^4 - y^4)\hat{i} - (z^4 - 2x^3y)\hat{j} + (y^4 - 2x^3z)\hat{k}$

20.3 The Curl of a Vector Field

307. The pictures below show top views of three vector fields, all of which have no z component. Which one has the curl pointing in the positive $\hat{k}$ direction at the origin?



(a) the one on the left

(b) the one in the middle

(c) the one on the right

(d) none of them

308. Let $\vec{F}(x, y, z)$ be a vector field and let $f(x, y, z)$ be a scalar function. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, which of the following is not defined?

(a) $\nabla \times f$

(b) $\nabla \times \vec{F} + \nabla f$

(c) $\nabla \times (\vec{r} \times \nabla f)$

(d) $f + \nabla \cdot \vec{F}$

(e) More than one of the above

309. Which one of the following vector fields has a curl which points purely in the $\hat{j}$?

(a) $y\hat{i} - x\hat{j} + z\hat{k}$

(b) $y\hat{i} + z\hat{j} + x\hat{k}$

(c) $-z\hat{i} + y\hat{j} + x\hat{k}$

(d) $x\hat{i} + z\hat{j} - y\hat{k}$

310. True or False? If all the flow lines of a vector field $\vec{F}$ are straight lines, then $\nabla \times \vec{F} = 0$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

311. True or False? If all the flow lines of a vector field $\vec{F}$ lie in planes parallel to the xy-plane, then the curl of $\vec{F}$ is a multiple of $\hat{k}$ at every point.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

20.4 Stokes' Theorem

312. Which of the following facts about $\vec{F} = \rho\hat{\rho}$ is implied by Stokes' Theorem?

(a) The line integral from (0,0,0) to (1,1,1) is equal to 3/2.

(b) $\vec{F}$ has positive divergence everywhere.

(c) The line integral on any closed curve is zero.

(d) The curl of $\vec{F}$ is non-zero.

313. What can be said about the vector field ∇f in terms of curl?

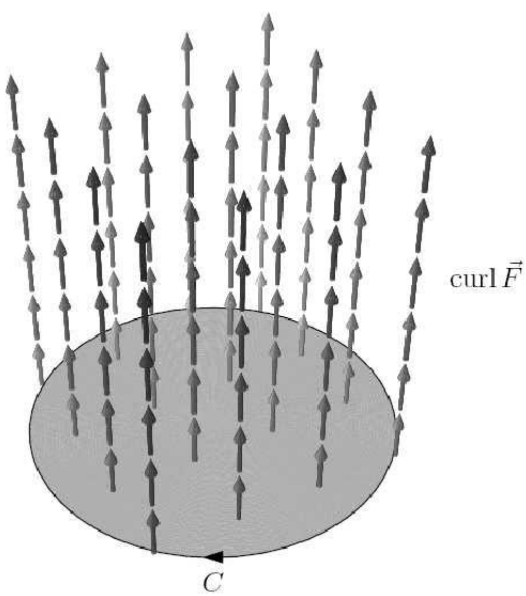
(a) Its curl is negative.

(b) Its curl is zero.

(c) Its curl is positive.

(d) Its curl depends on the function f.

314. The figure below shows the vector field $\nabla \times \vec{F}$. No formula for the vector field $\vec{F}$ is given. The oriented curve C is a circle, perpendicular to $\nabla \times \vec{F}$. The sign of the line integral $\int_C \vec{F} \cdot d\vec{r}$



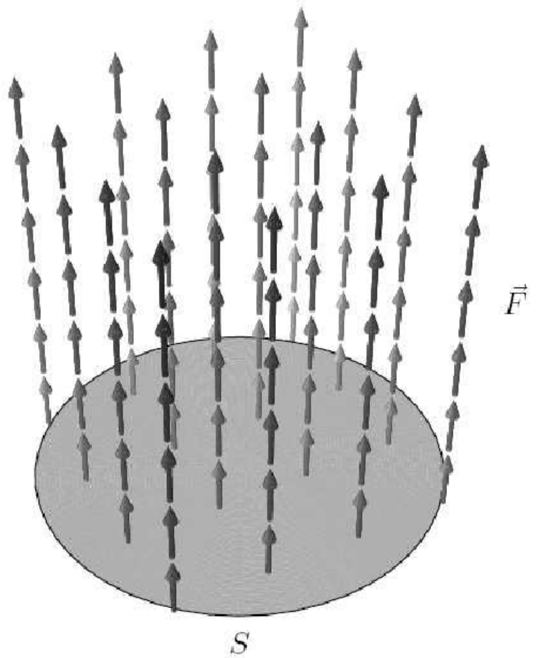
(a) is positive.

(b) is negative.

(c) is zero.

(d) can't be determined without further information.

315. The figure below shows the vector field $\vec{F}$. The surface S is oriented upward and perpendicular to $\vec{F}$ at every point. The sign of the flux of $\nabla \times \vec{F}$ through the surface



(a) is positive.
(b) is negative.
(c) is zero.
(d) can't be determined without further information.

316. The vector field $\vec{F}$ has curl $\nabla \times \vec{F} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$. What is the circulation of $\vec{F}$ around the perimeter of the square with corners at coordinates (1,2,3), (4,2,3), (4,2,6), and (1,2,6)?

(a) 0
(b) 18
(c) 27
(d) 36
(e) 81
(f) None of the above