

Subject: Calculus

Topic: Chain Rule

- Goal: Use *Mathematica* to find partial derivatives of compositions of several variables.

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#### Task 1

In *Mathematica* we define three function  $z(x, y)$ ,  $x(s, t)$ , and  $y(s, t)$ .

```
z[x_, y_] := E^(x + 2 y);  
x[s_, t_] := s / t;  
y[s_, t_] := t / s;
```

To compute the partial derivatives  $\frac{\partial z}{\partial t}$  and  $\frac{\partial z}{\partial s}$ , we use the derivative command.

```
D[z[x[s, t], y[s, t]], t]  
D[z[x[s, t], y[s, t]], s]
```

To evaluate the partial derivative  $\frac{\partial z}{\partial s}$ , which we computed last, at  $\{s=1, t=2\}$  use the substitution rule:

```
% /. {s -> 1, t -> 2}
```

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#### Related Exercises/Notes:

- 1. Use the Chain Rule to find the indicated partial derivatives. Use *Mathematica* to confirm your answers by first finding the derivative(s) and then evaluating it.

a)  $P = \sqrt{u^2 + v^2 + w^2}$ ,  $u = x e^y$ ,  $v = y e^x$ ,  $w = e^{xy}$ ;  $\frac{\partial P}{\partial x}$ ,  $\frac{\partial P}{\partial y}$  when  $x = 0$ ,  $y = 2$ .

b)  $u = x e^{t^y}$ ,  $x = \alpha^2 \beta$ ,  $y = \beta^2 \gamma$ ,  $t = \gamma^2 \alpha$ ;  $\frac{\partial u}{\partial \alpha}$ ,  $\frac{\partial u}{\partial \beta}$ ,  $\frac{\partial u}{\partial \gamma}$  when  $\alpha = -1$ ,  $\beta = 2$ ,  $\gamma = 1$ .