Mathematica Labs | iLearnMath.net | Denis Shubleka

Subject: Calculus

Topic: Multivariable Functions

- Goal: Use *Mathematica* to visualize functions in two variables.

Task 1

In *Mathematica* we define a real-valued function in two variables:

```
f[x_, y_] := Cos[x] + Sin[y];
```

Or more conveniently:

```
Clear[f, x, y];
f = Cos[x] + Sin[y];
```

To evaluate the function at a given point (a, b), we use the replacement rules:

```
f /. {x → Pi / 4, y → Pi / 3}
```

Generally, if the answer needs simplification, use the Simplify command:

```
Simplify[%]
```

Note that the percentage sign refers to the result of the previous command.

Task 2

To sketch the domain of a function in two variables, we use the RegionPlot command. For example, the function:

$$f(x, y) = \frac{\sqrt{x - y^2}}{y - x}$$

will exist whenever the denominator is nonzero and the expression inside th square root is greater than or equal to zero. We specify these inequalities in the Region-plot command:

```
RegionPlot[{x - y^2 ≥ 0, y - x ≠ 0}, {x, -5, 5}, {y, -5, 5}, Axes → True]
```

Task 3

To plot a function of two variables in 3-space, we use the Plot3D command. Remember to include the bounds for each independent variable:

```
Plot3D[f, {x, -1, 3}, {y, -3, 3}]
```

To get a vertical cross-section, we simply set one the independent variables equal to a specific value:

```
Plot[f /. y → Pi / 3, {x, -1, 3}]
```

Your turn: Set the x-coordinate to a specific value, to get a cross section of the surface that shows f(x,y) versus y.

Your turn: In the Documentation Center, look up the following options of Plot3D:- PlotStyle, Lighting,PlotRange,BoxRatios,Boxed, and Axes. Explore at your own pace with a function of your own.

Task 4

To plot the level curves (contours) of a funtion of two variables, we use the Contour-Plot command in Mathemaica. By default, it will produce ten contour regions separated by nine contour lines. The regions are shaded according to relative height in the z-direction. To set a custom number of contour lines, use the Contours property:

```
ContourPlot[x * y, {x, -3, 3}, {y, -3, 3}, Contours → 20]
```

To plot level surfaces of functions in three variables (w = f(x, y, z)), we use the ContourPlot3D command. Here is an example:

```
ContourPlot3D[x^2 + y^2 + z^2, {x, -1, 1}, {y, -1, 1},
 {z, 0, 1}, BoxRatios → {2, 2, 1}, Contours → 5, Mesh → None]
```

Related Exercises/Notes:

- 1. Find and sketch the domains of the following functions by hand, and then use *Mathematica*'s RegionPlot command to confirm your answers.

$$f(x, y) = \frac{\ln(x + y)}{\sqrt{x^2 + y^2 - 1}}$$

$$g(x, y) = \frac{\text{arcctan}(x + y)}{\sqrt{x^2 + y^2}}$$

- 2. Use the Plot3D command to graph the functions in the previous exercise. Visually confirm that the z-values only exist for pairs (x, y) in the domain you found earlier.
- 3. Use *Mathematica* to draw level curves (countour lines) for each function below. Describe in words the contours and how they are spaced.

$$h(x, y) = \cos\sqrt{x^2 + y^2}$$

$$l(x, y) = \sqrt{x^2 + 2y^2}$$