Problem 1

a)

$$\int_{t=1}^{t=5} 450\sqrt{\sin(0.65t)}dt$$

The integral describes the number of vehicles arriving between 6AM and 10AM.

b)

$$\frac{1}{5-1}\int_{t=1}^{t=5} 450\sqrt{\sin(0.65t)}dt = 375.537$$

The average rate from 6AM to 10AM is approximately 375.537 vehicles per hour.

c)

$$A'(t) = \frac{139.5\mathrm{Cos}[0.62t]}{\sqrt{\mathrm{Sin}[0.62t]}}$$

A'(1) = 148.947 > 0 (calculator), therefore the rate A(t) is increasing at 6AM.

d)

Find the value of *a* by solving A(t) = 400 with the aid of a graphing calculator.
Find $a = 1.469$
By the Fundamental Theorem of Calculus, we first find the derivative of N(t) and set it equal to zero or undefined. With technology (TI-84), we find two critical numbers:

$$N'(t) = A(t) - 400 = 0 \text{ or undefined} \rightarrow t = 1.469, t = 3.598$$

N(3.598) = 71.254 → 71 cars is the global maximum on [1.469, 4]. We can justify by the First Derivative Test for Global Extrema: N'(t) changes sign once at t=3.598 from positive to negative, therefore at this point N(t) has a local and global maximum. Alternatively, one can justify using the Closed Interval Method by computing and comparing N(t) values at the endpoints t=1.469 and t=4 with N(3.598).

Problem 2

A = -2
B= 0.781975 (Find this intersection point with the calculator).

a)

$$\int_A^B f(x) - g(x)dx = 3.604$$

b)

$$h(x) = f(x) - g(x)$$

$$h'(-0.5) = f'(-0.5) - g'(-0.5) = -0.600$$

With the aid of a calculator, since h'(-0.5) is negative, h is decreasing at x = -0.5.

c)

$$A(x) = (f(x) - g(x))^2$$

$$V = \int_A^B (f(x) - g(x))^2 dx = 5.340$$

d)

$$A(x) = (h(x))^2$$

$$dx/dt = 7$$

$$dA/dt = dA/dx * dx/dt$$

$$dA/dt = 2 * h(-0.5) * h'(-0.5) * 7 = -9.272 \text{ square units per second.}$$

Problem 3

a)

$$f(4) - f(0) = \int_0^4 f'(x)dx \to f(0) = 3 - \frac{-\pi 2^2}{2} = 3 + 2\pi$$
$$f(5) - f(4) = \int_4^5 f'(x)dx \to f(5) = 3 + \frac{1 * 1}{2} = 7/2$$

b)

The inflection points of *f* occur at x=2 and x=6 because at these two points the second derivative *f''* changes sign, the first derivative *f'* does NOT change sign, and the original function *f* is continuous (since *f* is differentiable).

c)

$$g'(x) = f'(x) - 1 < 0 \to f'(x) < 1$$
$$0 < x < 5$$

d)

$$g'(x) = f'(x) - 1 = 0 \text{ or undefined}$$
$$x = 5 \text{ is a critical number.}$$

We use the Closed Interval Method:

$$g(0) = f(0) - 0 = 3 + 2\pi$$
$$g(5) = f(5) - 5 = 7/2 - 5 = -3/2$$
$$g(7) = f(7) - 7 = 13/2 - 7 = -1/2$$

The absolute minimum equals -3/2 and it occurs at x=5.

Problem 4

a)

$$r"(8.5) \approx \frac{r'(10) - r'(7)}{10 - 7} = \frac{-3.8 + 4.4}{3} = \frac{1}{5} \text{ cm per day per day}$$

b)
Yes. By the Intermediate Value Theorem, since r'(t) is continuous on [0, 3] and -6 is in between r'(0) and r'(3), there must be a time during the interval (0, 3) such that r'(t) is precisely equal to -6.

c)

$$\int_0^{12} r'(t)dt \approx R_4 = 3f(3) + 4f(7) + 3f(10) + 2f(12) =$$

$$(-15) + (-17.6) + (-15.2) + (-14) = -61.800 \text{ cm.}$$

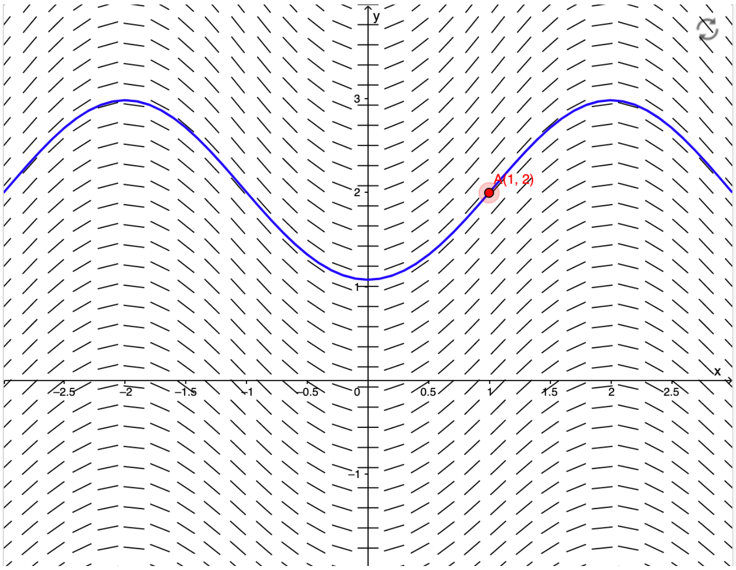
d)

$$V = \frac{\pi}{3} r^2 h \rightarrow dV/dt = \frac{\pi}{3}(r^2 dh/dt + 2r dr/dt h)$$

$$dV/dt = \frac{\pi}{3}(100^2(-2) + 2 * 100 * (-5.0) * 50) = \frac{-70000\pi}{3} \text{cubic cm per day}$$

Problem 5

a)



b)

$$x = 1, y = 2 \to dy/dx = (1/2)\sin \pi/2\sqrt{9} = 3/2$$

$$L(x) = 2 + (3/2)(x - 1) \to f(0.8) \approx L(0.8) = 1.7$$

c)

If the second derivative is positive, then the true solution will be concave upward on the given interval, which means any tangent line approximation like the one in part b) will provide an underestimate as the line falls below the curve.

d)

$$\frac{dy}{dx} = \frac{1}{2}\sin(\frac{\pi x}{2})\sqrt{y+7}$$

$$\frac{1}{\sqrt{y+7}}dy = \frac{1}{2}\sin(\frac{\pi x}{2})dx$$

$$\int \frac{1}{\sqrt{y+7}}dy = \int \frac{1}{2}\sin(\frac{\pi x}{2})dx$$

$$2\sqrt{y+7} = \frac{-1}{\pi}\cos(\frac{\pi}{2}x) + C \to C = 6$$

$$y = (\frac{-1}{2\pi}\cos(\frac{\pi}{2}x) + 3)^2 - 7$$

Problem 6

a)

$$v_P(t) = 4e^{-t}$$

b)

$$a_Q(t) = \frac{-2}{t^3}$$

The speed of particle Q is decreasing whenever its velocity and acceleration have opposite signs. The acceleration is always negative, whereas the velocity is always positive. Therefore, particle Q's speed is decreasing for all t > 0.

c)

$$y_Q(t) = \int \frac{1}{t^2} dt = \frac{-1}{t} + C \to C = 3$$

$$y_Q(t) = \frac{-1}{t} + 3$$

d)

Particle P will be farther from the origin because its position approaches 6 units as t goes to infinity, whereas the position of particle Q approaches 3.

$$\lim_{t \to \infty} y_P(t) = 6$$

$$\lim_{t \to \infty} y_Q(t) = 3$$