Mathematica Labs | iLearnMath.net | Denis Shubleka

Subject: Calculus

Topic: Improper Integrals

- Goal: Use *Mathematica* to determine whether or not an improper integral converges.

Task 1

There are two types of improper integrals:
(1) At least one of the integration bounds is infinite.
(2) Both integration bounds are finite, but the integrand is discontinous at least
Let us look at some examples of type (1).

Integrate$\left[\frac{1}{x}, \{x, 1, \infty\}\right]$

Mathematica confirms that the above interval is divergent. Here is another one, using the definite integral symbol from the palette:

$$\int_1^\infty \frac{1}{x^2}\, dx$$

Investigate a few more improper integrals of the form:

$$\int_1^\infty \frac{1}{x^p}\, dx$$

For what values of p does the improper integral converge? In those instances when it converges, can you generalize what it converges to?

Next, we look at examples of type (2). Before computing, verify that the following is in fact an improper integral.

$$\int_2^5 \frac{1}{\sqrt{x-2}}\, dx$$

To get a plot of the integrand, enter and execute:

Plot$\left[\frac{1}{\sqrt{x-2}}, \{x, 2, 5\}, \text{PlotRange} \to \{0, 4\}, \text{Filling} \to \text{Axis}\right]$

Here are a few more type (2) examples.

$$\int_{-1}^4 \frac{1}{x}\, dx$$

$$\int_0^\pi \text{Sec}[x]\, dx$$

The symmetry of the second function might suggest that the integral converges to zero. Note, however, that the sum of two divergent integrals [0 to $\frac{\pi}{2}$ and $\frac{\pi}{2}$ to π]

divergent, so *Mathematica* is correct.

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Plot[Sec[x], {x, 0, π}, Filling → Axis]
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Related Exercises/Notes:

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