

Subject: Calculus

Topic: Dot Product

- Goal: Use *Mathematica* to explore the operation of dot product between two vectors.
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Task 1

A vector in 2-space is represented as a list of length 2. For higher dimension spaces, simply define a vector as a longer list. We start with a simple vector addition in 2-space:

$$\{1, 9\} + \{-3, 1\}$$

Scalar multiplication in 3-space works as one would expect:

$$3 \{-1, 1, 1\}$$

We can also define a few vectors and perform operations on them, such as, for example, a linear combination:

```
u = {1, 1, 1};  
v = {-1, 2, 4};  
2 u + 3 v
```

Task 2

Mathematica computes the dot product operation between two vectors when we place a period in between them:

$$\{a, b\} \cdot \{c, d\}$$

Feel free to try it with two specific vectors in 3-space.

The norm (or length) of a vector is determined using the Norm command:

$$\text{Norm}[\{x, y, z\}]$$

To find the angle between two vectors we first define them, and then ask *Mathematica* to compute the arc cosine:

$$u = \{1, 1, 1\}; v = \{1, 0, 0\};$$
$$\text{ArcCos}\left[\frac{u \cdot v}{\text{Norm}[u] \text{Norm}[v]}\right] // N$$

The output is in radian measure, so we convert the result (%) to degrees:

$$\% / \text{Degree}$$

We conclude this task by showing how *Mathematica* displays vectors in 2-space:

```
Graphics[{Arrow[{{0, 0}, {1, 1}}], Arrow[{{0, 0}, {-1, 3}}]}, Axes → True]
```

Note that rendering vectors in 3-space requires the Graphics3D package. For example, to plot vectors $(1, 1, 1)$ and $(-1, 1, -1)$ execute the following:

```
Graphics3D[{Arrow[{{0, 0, 0}, {1, 1, 1}}], Arrow[{{0, 0, 0}, {-1, 1, -1}}]},  
  Axes → True, Boxed → False]
```

Related Exercises/Notes:
