Crop rotation constraints in agricultural production planning

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***Abstract*—In this paper mathematical model of optimal agricultural production plan is derived and applied on small farm which is located in west Vojvodina. In order to construct more realistic model of optimal crop distribution and deal with crop rotation, the special case of integer optimization, the knapsack problem is used. It is assumed that farmer on one plot sows only one type of crop.**

Index Terms: agricultural production planning, optimal crop distribution, linear program, integer program, binary model, combinatorial optimization.

I. INTRODUCTION

Let us consider production of wheat, soybean, maize, sugar beet, paprika and sunflower on small farm in west Vojvodina. Goal of this model is to find optimal agricultural plan of this farm in order to maximize total gross margin.

Vojvodina is typically agricultural region, which supplies whole region of Serbia with agricultural products and food. Unfortunately in last twenty years bad agricultural politics destroyed already existing agricultural system and forced smaller farmers to sell their plots to large agricultural firms. Model of optimal crop distribution and farm planning can improve agricultural politics of government and also can help agricultural producers to determine optimal crop distribution in next year and in this way to maximize total gross margin.

Optimal crop distribution is a real-world problem and in global level is of big importance for every state. State should have agricultural policy and in interest of both stimulate farmers to produce certain crops in certain volume. In order to make model as realistic as can, we should take into consideration predictions about yields and expected prices and costs.

Overview of the paper:
In subsection *A.* problem of agricultural production planning is introduced, and required historical data are presented. Subsection *B.* describes both linear and binary mathematical model of optimal crop distribution. Results of these models are also derived. In section II there are compared linear and binary models and their results.

A. Problem Description

Optimal farm plan will be applied as mentioned on small farm in west Vojvodina. It consists of six plots and each plot is size of $[2, 3, 4, 3, 3, 5]$ hectares, respectively. Total amount of cultivated land is 20 ha. Farmer produces wheat, soybean, maize, sugar beet, paprika and sunflower on his farm. Historical data of crop yields, crop prices and expenses of agricultural production are collected in period from year 2000 to 2006. Unfortunately, historical data for year 2007 are not collected. In order to find optimal crop distribution plan for year 2008, model requires estimation of crop yields, crop prices and expenses of agricultural production. There are statistical methods of estimation based on historical data such as statistical expectation, median and extrapolation. These methods all have both advantages and disadvantages. According to [22], one can approximate the probability distribution of crop yields and prices with normal probability distribution. In that case, one can approximate expectation with mean. Shortfall of mean is that outliers have large impact on the value of mean. Medians don't take into consideration outliers, but this is also shortfall, because they don't reflect real distribution of sample. Extrapolation is method which uses statistical tools like regression to calculate value of function out of interval where the function is determined. Shortfall of extrapolation is its unreliability far from interval where function is known.

Historical data for crop yields in period from 2000 to 2006 are given in table *(I)*. Measure is t/ha. Also, value of extrapolation, median and mean are calculated. One can ask, which method for prediction is best of these three methods. One can argue which method is better, but nobody can answer this question 100% correctly, because future is unpredictable. In this sense, model of optimal crop distribution carries some risk. Interesting topic for further investigation is to determine distribution of crop yields for given crops. This problem is studied in [22].

TABLE I
CROP YIELDS IN T/HA

	Wheat	Soyb.	Maize	Sugar b.	Papr.	Sunfl.
2000	3.370	1.219	2.938	24.709	1.341	*1.522*
2001	4.068	2.394	5.588	*42.687*	2.010	1.980
2002	3.441	2.470	4.987	*41.001*	1.781	*1.900*
2003	2.304	1.711	3.417	27.394	*1.594*	*1.791*
2004	4.793	2.723	5.882	*46.762*	*1.972*	2.338
2005	3.954	2.828	6.542	*48.489*	*2.303*	1.769
2006	3.989	2.771	5.942	44.857	*2.091*	*2.076*
Extrap.	3.921	3.639	7.617	55.5383	2.501	2.207
Median	3.954	2.470	5.588	42.687	1.972	1.900
Mean	3.7027	2.3023	5.0423	39.414	1.8703	1.9109

Historical data for crop prices in period from 2000 to 2006 are given in table *(II)*. These prices are given in Eur/t. Values of extrapolation, median and mean are also calculated. One can see that year 2000 is possible outlier, because prices were extremely high comparing other years. This is due to political situation and economical politics of government in year 2000.

TABLE II
CROP PRICES IN EUR/T

	Wheat	Soyb.	Maize	Sugar b.	Papr.	Sunfl.
2000	331.49	703.03	389.43	132.94	1090.75	*533.45*
2001	124	224.43	145.23	*29.72*	207.59	198.58
2002	112.18	212.76	90.65	*28.43*	188.16	*195.82*
2003	124.82	194.35	101.67	25.73	*197.33*	*179.12*
2004	93.74	161.70	117.47	*24.38*	*182.47*	150.92
2005	88.30	195.14	75.08	*23.87*	*197.59*	172.90
2006	111.78	195	50.03	31.57	*206.43*	*175*
Ekstrap.	271.06	104.80	139.14	53.44	246.55	114.57
Median	112.18	199.14	101.67	28.43	197.59	179.12
Mean	140.90	269.49	138.51	42.377	324.33	229.40

Historical data from table *(I)* and *(II)* are used in model of optimal crop distribution to calculate income of farmers. Unfortunately, farmer has limited resources, so he is constrained when planning agricultural production. In model, constraints of budget limit, land amount and constraints of crop rotation will be considered.

Budget limit is constraint which depends on financial resources of farmer. Sources of these expenses are ploughing, NPK fertilizer, sowing preparation, sowing, mineral fertilizer, pest and disease control, fallowing, combine harvesting. Total expenses of production of given crop on one hectare of land are given in table *(III)*. As budget limit farmer will take part of his income from last year which was 12916.98 Eur. One assumption is made for expenses: they are constant in every year, because they are connected to Eur. Because of this reason, there are no need to predict them, values in table *(III)* will be used in model.

TABLE III
EXPENSES OF CROP PRODUCTION ON ONE HECTARE OF LAND IN EUR

Crop	Expenses
Wheat	198.45
Soybean	161.81
Maize	226.45
Sugar beet	421.13
Paprika	296.12
Sunflower	199.81

Farmer has limited size of land, let us assume 20 hectares.

Crop rotations can be important for pest and disease control and for maintaining soil fertility. Crop rotation constraints are:

1) Wheat, maize and soybean can't be sown two years in the same plot.
2) If soybean was planted in some plot then sunflower can't be planted in four years at that parcel and vice versa.
3) In the same parcel sugar beet can be sawn once in five years.
4) In the same parcel paprika can be seeded once in five years.
5) If sugar beet was planted in some plot then paprika can't be planted in five years at that parcel and vice versa.
6) In the same parcel sunflower can be sawn once in four years.
7) After sowing maize, it is not recommended to sow nor wheat nor sugar beet for one year.

B. Mathematical Model of Agricultural Production Planning

In order to construct the model of optimal crop distribution, let us introduce next notation:

- Farmer has N hectares of land.
- Indexes $j = 1, 2, \dots, m$ stand for sowed crops, respectively, where m is number of crops.
- Selling prices $p_j,\ j = 1, \dots, m$, and yields $y_j,\ j = 1, \dots, m$, are unknown - they are predicted in terms of extrapolation from data given in tables *(I)* and *(II)*.
- Farmer's income from one hectare of cultivated land is denoted with $c_j,\ j = 1, 2, \dots m$ where $c_j = p_j \times y_j,\ j = 1, 2, \dots, m$.
- Resources required for agricultural production are denoted with $a_{ij},\ i = 1, 2, \dots, n,\ j = 1, 2,\ \dots, m$, where n is number of resources required.
- Available amount of resources are denoted with $b_i,\ i = 1, 2, \dots, n$.
- With $x_j,\ \ j = 1, \dots, m$, are denoted the number of hectares planted with j^{th} crop.

Implementation of rotation constraints requires historical data of crop sowing plan. These data are given in table *(IV)*. As mentioned earlier, sizes of plots are $[2, 3, 4, 3, 3, 5]$ hectares, respectively. From table *(IV)* one can calculate number of hectares sown with certain crop.

TABLE IV
SOWING PLAN IN PERIOD FROM 2000 TO 2006

	Plot 1	Plot 2	Plot 3	Plot 4	Plot 5	Plot 6
2000	Paprika	Sugar b.	Wheat	Soyb.	Wheat	Maize
2001	Wheat	Wheat	Sunfl.	Maize	Paprika	Soyb.
2002	Soyb.	Maize	Wheat	Soyb.	Wheat	Paprika
2003	Wheat	Soyb.	Maize	Sugar b.	Maize	Wheat
2004	Maize	Wheat	Sugar b.	Wheat	Sunfl.	Maize
2005	Soyb.	Maize	What	Maize	Wheat	Sunfl.
2006	Wheat	Soyb.	Sugar b.	Soyb.	Maize	Wheat

First, model of optimal crop distribution will be introduced, which is problem of linear programming. Later in this paper, integer programm will be applied on mentioned model and results will be compared.

1) Optimal crop distribution - problem of linear program: As starting model, a simple optimal crop distribution model will be derived. This model is problem of linear program which can be solved by simplex method. Objective is to maximize income of farmer, so objective function can be written as follows:

$$\sum_{j=1}^{m} y_j p_j x_j = \sum_{j=1}^{m} c_j x_j \rightarrow max, \tag{1}$$

As mentioned, constraints consists of land constraints, resource constraints and rotation constraints. Land constraint can be written in next way:

$$\sum_{j=1}^{m} x_j \leq N.$$

In general, constraints of resources can be mathematically written as follows:

$$\sum_{j=1}^{m} a_{ij} \leq b_i, \ i = 1, 2, \ldots n. \qquad (2)$$

In this paper, only budget constraints will be considered, so in *(2)* $n = 1$ and it becomes:

$$\sum_{j=1}^{m} a_j \leq b.$$

In order to implement constraints of rotation in this model, data in table *(IV)* must be modified as in table *(V)*. Using

TABLE V
SOWING PLAN IN HECTARES

	Wheat	Soyb.	Maize	Sugar b.	Papr.	Sunfl.
2000	7	3	5	3	2	0
2001	5	5	3	0	3	4
2002	7	5	3	0	5	0
2003	7	3	7	3	0	0
2004	6	0	7	4	0	3
2005	7	2	6	0	0	5
2006	7	6	3	4	0	0

table *(V)*, one can obtain next constraints for rotation:

$$\begin{aligned}
x_1 &\leq N - 7 \\
x_2 &\leq N - 6 \\
x_3 &\leq N - 3 \\
x_4 &\leq N - (3 + 4 + 4) \\
x_5 &\leq N - 5 \\
x_6 &\leq N - (3 + 5) \\
x_6 &\leq N - (5 + 3 + 2 + 6) \\
x_2 &\leq N - 8 \\
x_4 &\leq N - 5 \\
x_5 &\leq N - (3 + 4 + 4) \\
x_1 &\leq N - 6 \\
x_4 &\leq N - 6
\end{aligned}$$

Let us assume that budget limit of farmer is certain part of his income from last year which was 12916.98 Eur. Applying model to data collected from farm in west Vojvodina, one can obtain next problem of linear programming:

$$\sum_{j=1}^{6} c_j x_j \rightarrow max, \qquad (3)$$

subject to

$$\begin{aligned}
\sum_{j=1}^{6} x_j &\leq 20 \\
\sum_{j=1}^{6} a_j x_j &\leq b \\
0 \leq x_1 &\leq 13 \\
0 \leq x_2 &\leq 12 \\
0 \leq x_3 &\leq 17 \\
0 \leq x_4 &\leq 9 \\
0 \leq x_5 &\leq 9 \\
0 \leq x_6 &\leq 4
\end{aligned}$$

Using median as method of prediction for crop prices and yields, and varying budget b, one can get results given in table *(VI)*. One can see, that with smaller budget soybean is preferred, but with increasing of budget, maize and sugar beet is preferred. It means that income from maize and sugar beet is higher, but initial expenses of these crops are also higher.

TABLE VI
OPTIMAL CROP DISTRIBUTION PLAN FOR LINEAR MODEL

	30% b	40% b	50%b	100%b
Wheat	0	0	0	0
Soybean	12	12	0	0
Maize	0	0	11	11
Sugar beet	4.59	7.66	9	9
Paprika	0	0	0	0
Sunflower	0	0	0	0
Income	11355.45	15077.80	17171.77	17171.77

This model have some shortfalls concerning land allocation and constraints of rotation. Farmers usually have cultivated land which is consisted of several plots. On one plot they sow one type of crop, so they would like to get answer from model, on which plot what kind of crop to sow. Answer to this question will be answered with help of model described in next part of this paper.

2) Optimal crop distribution - problem of integer programming: In order to derive model of optimal crop distribution, let us introduce extra notation:

- Plot $i, \ i = 1, 2, \ldots q$ which is sowed with crop $j, \ j = 1, 2, \ldots, m$ is denoted with x_{ij}, where

$$x_{ij} = \begin{cases} 1, & i\text{-th plot is planted with } j\text{-th crop} \\ 0, & \text{else.} \end{cases}$$

Constraint which tells us that on one plot can be sown just one type of crop determines structure of matrix $X = [x_{ij}], \ i = 1, 2, \ldots q, \ j = 1, 2, \ldots m$. If we make an assumption that farmer newer leaves land on fallow, then row vectors of matrix X are unit vectors $e_i \in \Re^q$, where q stands for number of plots. Neglecting other constraints, there are q^m solutions of optimal crop distribution model.

- Elements of vector $v = [v_1\ v_2 \dots v_q]^T \in \Re^q$ are sizes of plots in hectares. In our case, we have $v = [2, 3, 4, 3, 3, 5]$.
- Sowing plan for period 2000 to 2006 given in table *(IV)* can be written separately for every year in matrix form $S^g = [s^g_{ij}] \in \{0,1\}^{q,m},\ g \in \{2001, 2002, \dots, 2006\}$, where g stands for considered year. Structure of these matrices are same as structure of matrix X. In terms of these matrices sowing plan for period 2000 can be written as follows:

$$S^{2000} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

For other years one can write these matrices in the same way. After introducing matrices of crop sowing plan, one can write constraints of rotation in next way:

Wheat:
- $s^{2006}_{i1} + x_{i1} \le 1,\ i = 1, 2, \dots, 6$ stands for constraint that wheat can't be sown two following years on the same plot;
- $s^{2006}_{i3} + x_{i1} \le 1,\ i = 1, 2, \dots, 6$ means that after sowing maize farmer can't sow wheat on the same plot;

Soyb.:
- $s^{2006}_{i2} + x_{i2} \le 1,\ i = 1, 2, \dots, 6$ tells us that soybean can't be sown two following years on the same plot;
- $s^g_{i6} + x_{i2} \le 1,\ i = 1, 2, \dots, 6,\ g = 2003, 2004, \dots, 2006$ means that after planting sunflower soybean cannot be planted for 4 years on the same plot;

Maize:
- $s^{2006}_{i3} + x_{i3} \le 1,\ i = 1, 2, \dots, 6$ stands for constraint that maize can't be sown two following years on the same plot;

Sugar b.:
- $s^g_{i4} + x_{i4} \le 1,\ i = 1, 2, \dots, 6,\ g = 2002, 2003, \dots, 2006$ means that sugar beet can't be planted for 5 following years on the same plot;
- $s^{2006}_{i3} + x_{i4} \le 1,\ i = 1, 2, \dots, 6$ tells us that after sowing maize it is not recommended to sow sugar beet on the same plot;
- $s^g_{i5} + x_{i4} \le 1,\ i = 1, 2, \dots, 6,\ g = 2002, 2003, \dots, 2006$ means that after planting paprika it is not allowed to sow sugar beet for 5 years;

Paprika:
- $s^g_{i5} + x_{i5} \le 1,\ i = 1, 2, \dots, 6,\ g = 2002, 2003, \dots, 2006$ is constraint which tells us that paprika cannot be planted for 5 following years on the same plot;
- $s^g_{i4} + x_{i5} \le 1,\ i = 1, 2, \dots, 6,\ g = 2002, 2003, \dots, 2006$ means that after planting sugar beet it is not allowed to sow paprika for 5 years;

Sunfl.:
- $s^g_{i2} + x_{i6} \le 1,\ i = 1, 2, \dots, 6,\ g = 2003, 2004, \dots, 2006$ means that after planting soybean, sunflower cannot be planted for 4 years on the same plot;
- $s^g_{i6} + x_{i6} \le 1,\ i = 1, 2, \dots, 6,\ g = 2003, 2004, \dots, 2006$ means that sunflower can't be planted for 4 following years on the same plot;.

Other constraints in model can be formulated in next way:

- $\sum_{j=1}^{m} x_{ij} \le 1,\ i = 1, 2, \dots, 6$ stands for assumption that on one plot only one type of crop can be planted.
- Constraint of cultivated land amount can be written as $\sum_{q=1}^{6} v_i \sum_{j=1}^{m} x_{ij} \le N$. This constraint allows farmer to leave some plot on fallow.
- $\sum_{i=1}^{q} v_i \sum_{j=1}^{m} a_j x_{ij} \le b$ stands for budget limit.
- Constraint $x_{ij} \in \{0,1\},\ i = 1, 2, \dots q,\ j = 1, 2, \dots, m$ ensure that output of model answer on question: "Which crop to sow on given plot?" Thanks to this constraint model of optimal crop distribution becomes problem of integer programming.

With this notation, applying model to data collected from farm in west Vojvodina one can obtain following integer program:

$$\begin{aligned}
\sum_{i=1}^{6} v_i \sum_{j=1}^{6} c_j x_{ij} &\quad \to \quad max, \\
\sum_{j=1}^{6} x_{ij} &\quad \le \quad 1,\ i = 1, \dots, 6 \\
\sum_{i=1}^{6} v_i \sum_{j=1}^{6} x_{ij} &\quad \le \quad 20 \\
\sum_{i=1}^{6} v_i \sum_{j=1}^{6} tr_j x_{ij} &\quad \le \quad b \\
s^{2006}_{i1} + x_{i1} &\quad \le \quad 1,\ i = 1, \dots, 6 \\
s^{2006}_{i3} + x_{i1} &\quad \le \quad 1,\ i = 1, \dots, 6 \\
s^{2006}_{i2} + x_{i2} &\quad \le \quad 1,\ i = 1, \dots, 6 \\
s^{g}_{i6} + x_{i2} &\quad \le \quad 1,\ i = 1, \dots, 6,\ g = 2003, \dots, 2006 \\
s^{2006}_{i3} + x_{i3} &\quad \le \quad 1,\ i = 1, \dots, 6 \\
s^{g}_{i4} + x_{i4} &\quad \le \quad 1,\ i = 1, \dots, 6,\ g = 2002, \dots, 2006 \\
s^{2006}_{i3} + x_{i4} &\quad \le \quad 1,\ i = 1, \dots, 6 \\
s^{g}_{i5} + x_{i4} &\quad \le \quad 1,\ i = 1, \dots, 6,\ g = 2002, \dots, 2006 \\
s^{g}_{i5} + x_{i5} &\quad \le \quad 1,\ i = 1, \dots, 6,\ g = 2002, \dots, 2006 \\
s^{g}_{i4} + x_{i5} &\quad \le \quad 1,\ i = 1, \dots, 6,\ g = 2002, \dots, 2006 \\
s^{g}_{i2} + x_{i6} &\quad \le \quad 1,\ i = 1, \dots, 6,\ g = 2003, \dots, 2006 \\
s^{g}_{i6} + x_{i6} &\quad \le \quad 1,\ i = 1, \dots, 6,\ g = 2003, \dots, 2006 \\
x_{ij} &\quad \in \quad \{0,1\},\ i, j = 1, \dots, 6.
\end{aligned} \tag{4}$$

This problem belongs to class of the $0-1$ knapsack problem which is problem of combinatorial optimization. This class of problem was studied by George B. Dantzig. In 1957 he introduced *greedy algorithm* for solving this problem. About algorithms for solving this kind of problems one can read in [14], [15] and [17].

Solving problem *(4)* with mathematical software Matlab for different values of budget limit b, one gets results as in table *(VII)*. One can see that sugar beet, soybean and maize are most profitable crops.

TABLE VII
OPTIMAL CROP DISTRIBUTION FOR BINARY MODEL

	30% b	40% b	50%b	100%b
Plot 1	maize	sugar beet	soybean	sugar beet
Plot 2	sugar beet	sugar beet	sugar beet	sugar beet
Plot 3	soybean	–	soybean	maize
Plot 4	–	–	maize	maize
Plot 5	–	–	–	paparika
Plot 6	–	maize	maize	maize
Income	6705.02	8908.62	11077.80	14054.48

II. CONCLUSION

Comparing results of linear model of optimal crop distribution given in table *(VI)* and results of binary model of optimal crop distribution given in table *(VII)*, one can conclude that results are different. Income in linear model is higher than in binary model. One can explain this with binary constraint, because this constraint is rather restrictive. In linear model constraints of rotation cannot be implemented perfectly, because they take care of historical data of sowing plan just in hectares, so overlapping is possible. Advantage of this model is its simplicity, because it can be solved by simplex method. Also, it has less variable, then binary model. Linear model for farm in west Vojvodina has 6 variables, while binary model applied to the same farm has 36 variables. Binary model is problem of combinatorial optimization which is hard to solve. There are several algorithms like greedy algorithm which are rather complicated. Advantage of binary model is that it is more realistic than linear model because of the nature of agricultural production planning problem.

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