Primal Problem

$\min z = 2x_1 + 3x_2 + 4x_3$

s.t.

$x_1 - x_2 + x_3 \geq 10$

$x_1 - 2x_2 + 3x_3 \geq 6$

$3x_1 - 4x_2 + 5x_3 \geq 15$

$x_1, x_2, x_3 \geq 0$

$\Longrightarrow$

Dual Problem

$\max z_d = 10y_1 + 6y_2 + 15y_3$

$y_1 + y_2 + 3y_3 \leq 2$

$-y_1 - 2y_2 - 4y_3 \leq 3$

$y_1 + 3y_2 + 5y_3 \leq 4$

Slack form

$\max z_d = 10y_1 + 6y_2 + 15y_3$

s.t.

pivot y_3

$w_1 = 2 - y_1 - y_2 - 3y_3 \longrightarrow y_3 = \frac{2}{3}$; $y_3 = \frac{2}{3} - \frac{1}{3}y_1 - \frac{1}{3}y_2 - \frac{1}{3}w_1$

$w_2 = 3 + y_1 + 2y_2 + 4y_3 \longrightarrow y_3 = -\frac{3}{4}$

$w_3 = 4 - y_1 - 3y_2 - 5y_3 \longrightarrow y_3 = \frac{4}{5}$

substitute y_3 ;

$\max z_d = 10 + 5y_1 + y_2 - 5w_1$

s.t.

pivot y_1

$y_3 = \frac{2}{3} - \frac{1}{3}y_1 - \frac{1}{3}y_2 - \frac{1}{3}w_1 \longrightarrow y_1 = 2$; $y_1 = 2 - y_2 - w_1 - 3y_3$

$w_2 = \frac{17}{3} - \frac{1}{3}y_1 + \frac{2}{3}y_2 - \frac{4}{3}w_1 \longrightarrow y_1 = 17$

$w_3 = \frac{2}{3} + \frac{2}{3}y_1 - \frac{4}{3}y_2 + \frac{5}{3}w \longrightarrow y_1 = -1$

substitute y_1 ;

$\max z_d = 20 - 4y_2 - 15y_3 - 10w_1$

s.t.

$y_1 = 2 - y_2 - w_1 - 3y_3$

$w_2 = 5 + y_2 - w_1 + y_3$

$w_3 = 2 - 2y_2 + w_1 - 2y_3$

$\max z_d = 20$, $\min z = -20$

at $(y_1, y_2, y_3) = (2, 0, 0)$

It isn't appopriate to use the (Primal) Simplex Method because when we convert minimize problem to maximize problem, there's no $c_j > 0$ in the objective function. We also can't use the 2-Phase Method since the starting point isn't infeasible