Deep Learning for Visual Computing

Features, Parametric Models, Loss Functions

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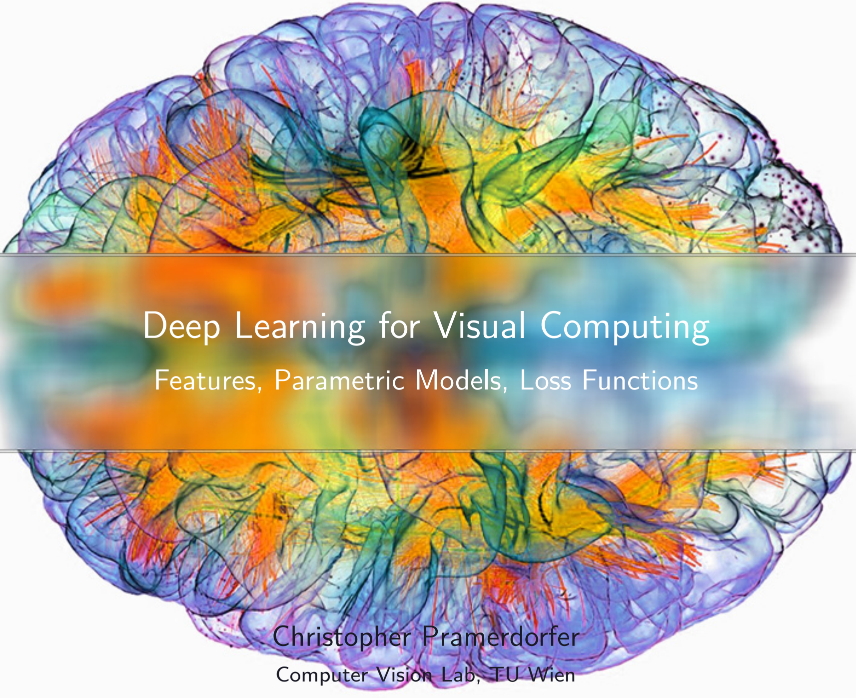


Image by Neuroscape Lab

Topics

Feature Extraction

Parametric models

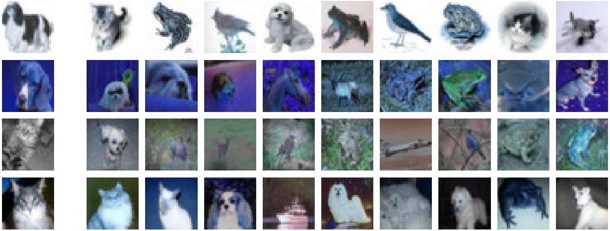
Linear models for classification

- Softmax classifiers

Loss functions

Our kNN classifier for cats and dogs performs poorly

- On CIFAR10 dataset, test accuracy is about 40%
- Humans achieve about 94%, similar to CNNs



Several reasons (later)

One is that we use raw images as feature vector $\mathbf{x}$

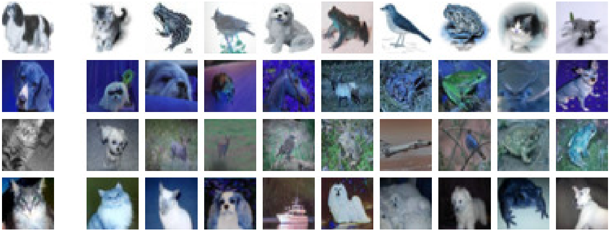
- But kNN has no understanding of images

Implications

- Input features $\mathbf{x}$ (pixel values) are indiscriminative
- D is large (curse of dimensionality)

Classification based on overall image similarity

- No concept of objects
- Background has strong effect (many pixels)



Feature Extraction
Motivation

Raw pixel values are poor features

- A feature is certain property of data

Goal of feature extraction

- Extract discriminative features from images
- Discriminative features help distinguish between classes

Steps

- Compute gradient magnitude and orientation
- Divide image into overlapping cells
- Compute histogram of quantized orientations in every cell
- Combine cells to blocks, concatenate histograms
- Normalize blocks, concatenate to single vector

See [1] for details

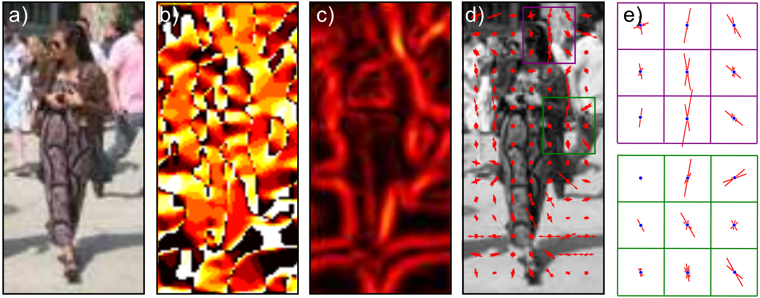


Image from [2]

HOG features improve performance by about 10%

- Still not close to human and CNN levels

Reasons

- kNN is a simple classifier
- HOG features are low-level

HOG features are low-level

- Capture directions of local brightness changes
- Generally useful but not task-specific

This applies to all manually designed feature extractors

- Impossible to design optimal task-specific extractors

We want task-specific high-level features

- Features that carry semantics
- E.g. presence of certain part of a human face

We cannot design them, so they must be learned

- CNNs are able to do this in a robust way
- Main reason why CNNs are so powerful

kNN classifier has several limitations

- Must keep all training data for testing
- Testing (predicting class labels) is slow
- Not particularly powerful

Parametric models overcome these limitations

Parametric Models
Definition

Let $\mathbf{x} \in \mathbb{R}^D$ be input and $\mathbf{w} \in \mathbb{R}^T$ be output

A model describes family of functions from $\mathbf{x}$ to $\mathbf{w}$

- Particular function $f : \mathbf{x} \mapsto \mathbf{w}$ learned during training

Model defines the hypothesis space of a ML algorithm

- Set of functions allowed as solution
- Extending family increases capacity

Parametric Models
Definition

In parametric models f depends on parameters $\boldsymbol{\theta}$

- We write $\mathbf{w} = f(\mathbf{x}; \boldsymbol{\theta})$
- Training entails finding good parameters
- Training set can be discarded after training

CNNs are parametric models

- Number of parameters can exceed 100 million (!)

The most basic example are linear models

Hypothesis space comprises linear functions from $\mathbb{R}^D$ to $\mathbb{R}^T$

- Formally $\mathbf{w} = f(\mathbf{x}; \boldsymbol{\theta}) = \mathbf{W}\mathbf{x} + \mathbf{b}$ with $\boldsymbol{\theta} = (\mathbf{W}, \mathbf{b})$

$\mathbf{W} \in \mathbb{R}^{T \times D}$ is called weight matrix

$\mathbf{b} \in \mathbb{R}^T$ is called bias vector

Used in almost all CNNs (linear layers)

- In the classifier stage
- For final mapping to class labels $\mathbf{w}$

We could directly estimate class label, $w \in \{1, \dots, T\}$

- Limited information, complicates training

Instead we let $\mathbf{w} = (w_1, \dots, w_T)^{\mathsf{T}}$

- Want $\arg\max(\mathbf{w}) = c$ if image $\mathbf{x}$ belongs to class c
- $\mathbf{w}$ encodes class scores (confidences)

A (unsuccessful) example prediction

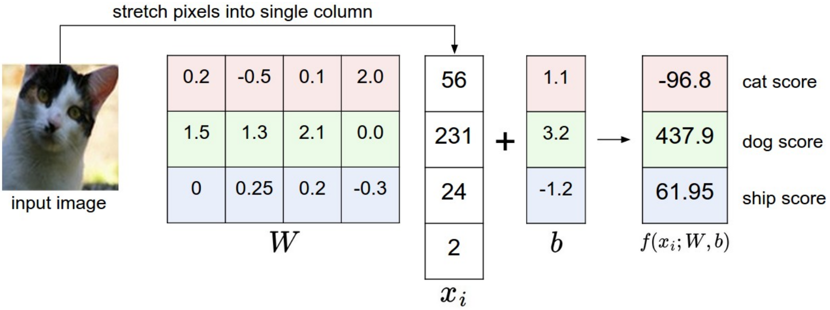
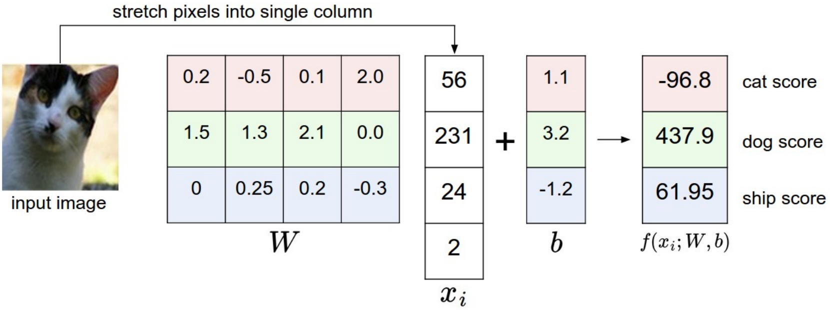


Image from cs231n.github.io

Linear Models for Classification
Example

Can think of T independent classifiers, one per class



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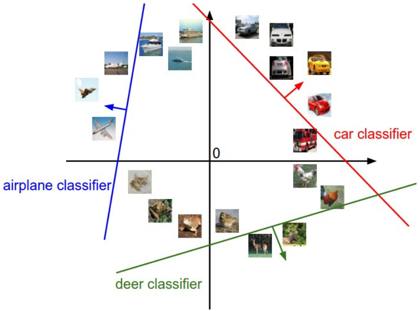
T hyperplanes as decision boundaries in $\mathbb{R}^D$

- Weights define orientation, bias defines offset from 0



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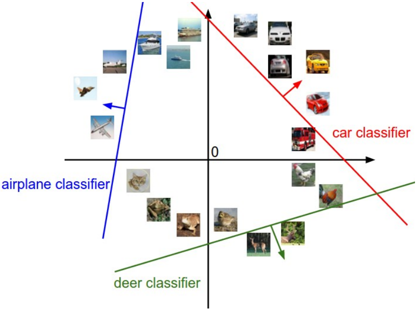
Linear Models for Classification

Geometric Interpretation

Hyperplane c should answer "is $\mathbf{x}$ of class c?"

- Want $\mathbf{x}$ on positive side of hyperplane c iff of class c



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Linear Models for Classification

Geometric Interpretation

$\mathbf{x}$ is on positive side if $\mathbf{w}_c\mathbf{x} + b_c \geq 0$

- $\mathbf{w}_c$ is row c of $\mathbf{W}$

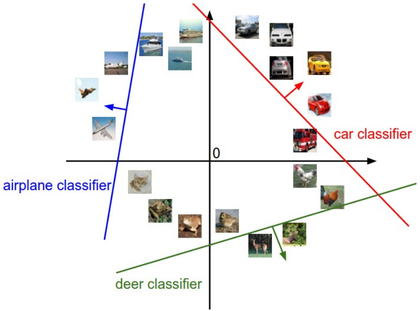
Left side equals score of class c, that is w_c

Signed distance to plane is $w_c/\|\mathbf{w}_c\|$

- Hence class score increases with distance/confidence

During training, make hyperplanes always answer correctly

- Only possible if classes are linearly separable



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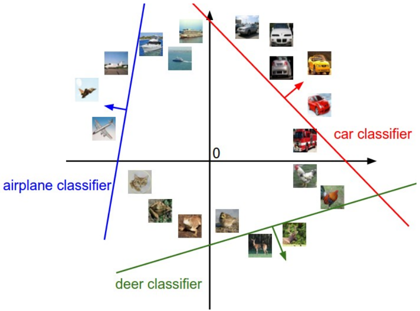
Hyperplanes do not work together

- Each hyperplane is an independent binary classifier



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Linear Models for Classification
Template-Matching Interpretation

T learned templates that are matched with input images

- Each $\mathbf{w}_c$ encodes a template
- Matching using inner product $\mathbf{w}_c\mathbf{x}$ (plus b_c)
- Class score increases with similarity of image to template

Image from cs231n.github.io

Linear Models for Classification
Template-Matching Interpretation

Most templates have clear interpretation

- Horse template shows something horse-like
- Most cars training data seem to be red
- Background is (again) very dominant (sky, grass, water)

Image from cs231n.github.io

Linear Models for Classification
Template-Matching Interpretation

Linear classifiers cannot properly model intraclass variation

- Templates merge modes of variation
- What about blue cars, planes on ground, gray horses?

CNNs solve this problem

Image from cs231n.github.io

Linear Models for Classification
Remarks

Much faster than kNN classifier

- Single matrix-vector multiplication

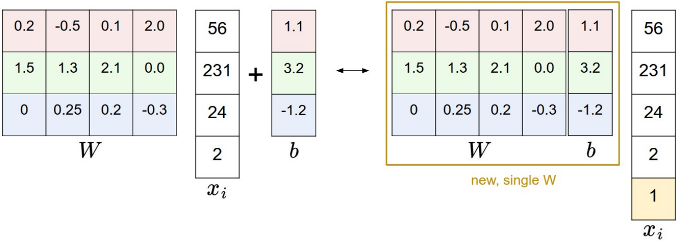
Number of parameters governed by D and T

- $\mathbf{W} \in \mathbb{R}^{T \times D}$ and $\mathbf{b} \in \mathbb{R}^{T}$
- CIFAR10 : $T = 10$ and $D = 3{,}072$, so $30{,}730$ parameters

Linear Models for Classification
Remarks

Can apply the bias trick to simplify f to $f(\mathbf{x}) = \mathbf{W}\mathbf{x}$

- Append $\mathbf{b}$ to $\mathbf{W}$, append 1 to $\mathbf{x}$



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Loss Functions

For training any parametric model, we need

- A loss function
- An optimization algorithm

Loss Functions

Recall that parametric models have form $f(\mathbf{x};\boldsymbol{\theta})$

A loss function $L(\boldsymbol{\theta})$ (or cost or objective function)

- Measures performance of $f(\cdot\,;\boldsymbol{\theta})$ (lower loss is better)
- On some dataset $\mathcal{D} = \{(\mathbf{x}_s, \mathbf{w}_s)\}_{s=1}^{S}$
- With respect to parameters $\boldsymbol{\theta}$

Choice of L depends on task

Goal during training is to change $\boldsymbol{\theta}$ to minimize $L(\boldsymbol{\theta})$

For classification, most popular choice is cross-entropy loss

- Standard loss for training CNNs

Dissimilarity between probability distributions p and q

- In terms of cross-entropy $H(p,q) = -\sum_x (p(x) \log q(x))$

p and q are discrete probability distributions

- So $p(x) \geq 0$ and $\sum_x p(x) = 1$ (same for q)

Thus $\log q(x)$ is always negative or 0, as are summands

- Cross-entropy is always positive
- Generally not 0 even if $p = q$ (but entropy of p)

In our case

- $p \sim \mathbf{w}_s$, encoding the true class distribution
- $q \sim \mathbf{w}$, the predicted class distribution

Use one-hot encoding to obtain $\mathbf{w}_s$ from single label w_s

- Vector of size T that is zero everywhere
- Except for element $c = 1$ if $w_s = c$
- $\mathbf{w}_s$ is valid probability distribution

Predictions $\mathbf{w}$ are not valid probability distributions

- Class-scores are unbounded
- $\mathbf{w}$ generally does not sum to 1

We solve this problem by

- Regarding $\mathbf{w}$ as unnormalized log probabilities
- And using the softmax function for normalization

$$\text{softmax}_k(\mathbf{w}) = \frac{\exp(w_k)}{\sum_{t=1}^{T} \exp(w_t)}$$

Softmax transforms any vector $\mathbf{w} \in \mathbb{R}^T$

- Such that $w_t \geq 0$ and $\sum_{t=1}^{T} w_t = 1$

$$\text{softmax}\left(\begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}\right) \approx \begin{pmatrix} 0.016 \\ 0.867 \\ 0.117 \end{pmatrix}$$

We calculate the cross-entropy loss on $\mathcal{D}$ as

$$L(\theta) = \frac{1}{S} \sum_{s=1}^{S} H(\mathbf{w}_s, \mathsf{softmax}(f(\mathbf{x}_s; \boldsymbol{\theta})))$$

Minimize the average cross-entropy over whole dataset

- Corresponds to maximum likelihood estimation (MLE)

MLE $\hat{\boldsymbol{\theta}}$ is optimal in terms of data

- Choice under which observed data $\mathcal{D}$ is most likely

Best we can do unless we have prior information about $\boldsymbol{\theta}$

- We'll talk about this later (regularization)

Loss Functions
Cross-Entropy Loss

Resulting classifier is called softmax classifier

- f still produces unnormalized log probabilities
- Must apply softmax to obtain probabilities

Loss Functions

Now that we have a good loss function, we can minimize it

- For this we need a suitable optimization algorithm
- Next lecture

Bibliography

[1] N. Dalal and B. Triggs, *Histograms of oriented gradients for human detection*, CVPR.

[2] Prince, *Computer Vision Models*. 2012.