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HARD ART OF THE UNIVERSE CREATION
(Stochastic approach to tunneling and baby universe formation)

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ABSTRACT

We develop a stochastic approach to the theory of tunneling with the baby universe formation. This method is applied also to the theory of creation of the universe in a laboratory.

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1 Introduction

Few years ago many people believed that the universe is topologically connected. Of course, it was possible to speculate about the universe consisting of many disconnected pieces, but such speculations did not seem to have any practical consequences since it was assumed that topologically disconnected pieces of the universe cannot influence each other. However, recently it was understood that topologically disconnected universes may have a non-local interaction with each other, which may help us to solve the cosmological constant problem [1]-[9]. The most popular approach to the cosmological constant problem is based on the possibility that even if one starts with one (topologically connected) universe, later on many new universes (so-called baby universes) are created from the original one due to quantum tunneling [10]-[13]. This idea is very interesting and it may prove to be very productive. However, the standard lore of the theory of tunneling with the universe production is based on the Euclidean approach to quantum cosmology [11]-[13]. In some cases this approach gives correct answers and can be justified with the help of more rigorous methods. In some other cases this approach also gives correct answers, but not to the questions which were asked originally. Being applied to the baby universe theory, this method contains many ambiguities which still remain unresolved [14]. One of the most difficult problems is related to the choice of sign of the Euclidean action in the expression for the wave function of the universe Ψ: Whether $\Psi \sim \exp(-S_E)$, like the Hartle-Hawking wave function [15] (or $\Psi \sim \exp(\exp(-S_E))$, with an account taken of baby universes [2]), or $\Psi \sim \exp(S_E)$, like the tunneling wave function [16] ? Or maybe (as we suspect) the true answer is much more complicated ?

The cosmological constant problem is not the only possible application of the theory of many topologically disconnected universes. It is not inconceivable that only in this context one can really understand the origin of our own universe. Indeed, the standard idea that our universe appeared from "nothing" (or singularity) is not much better (and conceptually is much less clear) than the idea that it appeared from another universe as a result of the baby universe creation. This brings us to another issue which was discussed recently: Is it possible to create the universe in a laboratory ?

In the context of the standard hot universe theory this question is not

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very interesting . In order to create a large universe which would be as long-living as ours, one would need to have at least as many particles as their total number in the observable part of our universe... In the inflationary universe scenario one can do the same job by producing a scalar field ϕ with the potential energy density $V(\phi)$ in a domain of space of a radius r bigger than the horizon, $r > H^{-1}(\phi) = \sqrt{\frac{3M_p^2}{8\pi V(\phi)}}$. Such a domain of the inflationary universe expands by its own laws, as a separate universe independent of what occurs outside it ("no-hair" theorem for de Sitter space). The total energy of the field ϕ in a bubble of a radius $r \sim H^{-1}$ is $E \sim \frac{M_p^3}{\sqrt{V(\phi)}}$. This amounts to about a hundred kilograms of matter for the new inflationary universe scenario in grand unified theories and to just a few Planck masses $M_p \sim 10^{-5}$ g for chaotic inflation, which may start at extremely large $V(\phi)$, up to $V(\phi) \sim M_p^4$ [14].

This does not mean that it is easy to create an inflationary universe in a laboratory, even with the help of chaotic inflation. An investigation of this problem shows that at the classical level this process is forbidden; one can only form small sub-critical bubbles with $r < H^{-1}$, which never expand up to the over-critical size $r > H^{-1}$ [19]. However, the over-critical domains can appear due to quantum effects.

In particular, one may consider a small (sub-critical) bubble filled with a scalar field with large $V(\phi)$ and investigate a possibility that such a bubble will transform into a large expanding bubble due to quantum tunneling. This possibility was considered in a series of very interesting papers [20, 21] by two different methods. The results of these two methods agree with each other and the authors conclude that one can actually create the universe in a laboratory in this way. However, there exist two problems with the methods used in [20, 21].

Investigation of quantum tunneling with an account taken of gravitational effects is extremely complicated. To simplify the problem, the authors of [20, 21] assumed that the bubble contains vacuum in a state $\phi = 0$, corresponding to a local minimum of the effective potential $V(\phi)$, and that the bubble wall is very thin. The thin wall approximation is valid in the old inflationary universe scenario, but old inflation does not lead to a good cosmology. This approximation is valid also in some inflationary models involving several

different scalar fields [22, 23], but the bubble formation in these models is much more complicated, and it was not studied in [20, 21]. Unfortunately, we are not aware of any situation where the thin wall approximation would work in the standard versions of new or chaotic inflation.

Another problem is related to the possibility to create the initial subcritical bubble surrounded by the ordinary Minkowski vacuum. In [20, 21] it was assumed that this is just a technical problem, which must have some solution. However, it is not quite clear whether such solution exists at all. Indeed, the only way of producing such bubbles which we know at present is to heat some part of the universe up to the critical temperature T_c, after which the phase transition to the false vacuum occurs in this domain. However, in all realistic models studied so far the thermal energy density $E \sim T^4$ at the time of the phase transition is much bigger than the vacuum energy density, $T_c^4 \gg V(0)$ [14]. Bubbles filled (and surrounded) by the gas of ultrarelativistic particles behave quite differently from the empty bubbles studied in [20, 21].

The problems discussed above are so complicated that it would be very desirable to find some simple intuitive approach which would help us to get at least partial understanding of what is going on. Few years ago, even before the notion of the baby universes was introduced, we suggested a stochastic approach to the inflationary universe formation due to quantum fluctuations in Minkowski space and estimated the probability of this process for a large class of models of the scalar field ϕ minimally coupled to gravity [10]. Later we used a simple generalization of this approach to describe the probability of the universe formation in a laboratory (at high temperature) [17, 18]. Stochastic approach to the universe formation has its own problems, to be discussed below. However, it is very simple, its results have a clear physical interpretation and, as we will show, it works very well being applied to the theory of tunneling in field theory and quantum statistics, both in Minkowski space and in de Sitter space. Therefore, this method may serve as a useful intermediate step towards a more complete and rigorous investigation of the baby universe formation. In this paper we will describe the stochastic approach to tunneling and apply it to the theory of the baby universe production, both in the empty Minkowski space and in a laboratory. In order to do it we must first say few words about Euclidean approach to

tunneling and to quantum cosmology.

2 Euclidean Methods in the Theory of Tunneling

One of the simplest and most elegant approaches to tunneling in the scalar field theory [24] is the Euclidean one [25]. The main idea of this approach is that tunneling is a motion with imaginary energy, which is equivalent to motion in imaginary time, i.e. in Euclidean space. The probability of tunneling is proportional to $\exp(-S_E)$, where S_E is the Euclidean action corresponding to the tunneling trajectory. In other words, S_E is the instanton action, where the instanton is the solution of the Euclidean field equations describing tunneling.

A most instructive example is tunneling in a theory with the effective potential

$$V = \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 + V(0) \ . \tag{1}$$

The tunneling trajectories (instantons) with the minimal action possess the O(4) symmetry of Euclidean space [25]. The Euclidean equation for O(4) symmetric tunneling is

$$d^2\phi/dr^2 + (3/r)d\phi/dr = V'(\phi) \ , \tag{2}$$

with the boundary conditions $\phi(r = \infty) = \phi_o$ and $d\phi/dr|_{r=0} = 0$. Here $r = \sqrt{x_i^2}$; the x_i are the Euclidean coordinates, i = 1,2,3,4.

In the theory (1) with $m = 0$ equation (2) has a class of solutions which are called Fubini instantons [26]:

$$\phi(r) = 2\sqrt{\frac{2}{\lambda}}\left(\frac{\rho}{r^2 + \rho^2}\right) \ , \tag{3}$$

where ρ is arbitrary. The corresponding Euclidean action does not depend on ρ,

$$S_E = 2\pi^2 \int r^3 \left(\frac{1}{2}(d\phi/dr)^2 + V(\phi)\right) dr = \frac{8\pi^2}{3\lambda} \ . \tag{4}$$

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The bubbles which appear after tunneling are also described by eq. (3) if one understands by r^2 its Minkowski counterpart $\mathbf{r}^2 - t^2$. The probability of the bubble formation per unit four-volume can be estimated by the expression

$$P \sim \rho^{-4} \exp\left(-\frac{8\pi^2}{3\lambda}\right) \tag{5}$$

for $m = 0$.

A somewhat surprising fact is that for any nonvanishing m there are *no* instanton solutions in the theory (1). This does not mean, however, that there is no tunneling in this theory. The point is that for any nonvanishing m the minimum of tunneling action corresponds to the $\rho \to 0$ limit of the field configurations (3). But one can easily check that the action of these configurations differs from its limiting value (4) by $\Delta S_E \ll 1$ for $\rho \ll \frac{\sqrt{\lambda}}{m}$, i.e. for $\phi \gg m/\lambda$ in the center of the instanton. This means that the Fubini instantons (3) with $\rho \ll \frac{\sqrt{\lambda}}{m}$ may play the role of instantons in the theory (1) with nonvanishing m as well.

A similar approach can be used to study tunneling at a finite temperature. Quantum statistics at finite T is equivalent to quantum field theory in the Euclidean space periodic in the time direction with the period $1/T$. In the limiting case $T \gg m$ one should look for O(3) symmetric instantons with the time-independent field ϕ and then multiply the three-dimensional action by $1/T$ [27]. The resulting four-dimensional action for the theory (1) is $S \sim \frac{19m}{\lambda T}$ [28] and the probability of tunneling is given by

$$P \sim \exp\left(-\frac{19m}{\lambda T}\right) . \tag{6}$$

To describe tunneling in the inflationary universe by Euclidean methods one should take into account that the Euclidean version of de Sitter space with the vacuum energy density $V(\phi)$ is the sphere S_4 with the radius $H^{-1}(\phi) = \sqrt{\frac{3M_p^2}{8\pi V(\phi)}}$. An instanton solution in this case is a sphere S_4 containing a constant scalar field ϕ corresponding to an extremum of the effective potential $V(\phi)$. The corresponding action is negative and is given by

$$S = -\frac{3M_p^4}{8V(\phi)} . \tag{7}$$

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Let us assume that the effective potential has a complicated form with a local minimum at $\phi = 0$, a local maximum at $\phi = \phi_1$, next local minimum at $\phi = \phi_2$, next local maximum at $\phi = \phi_3$, next local minimum at $\phi = \phi_4$, etc. Then, according to [29], the probability of tunneling from $\phi = 0$ to $\phi = \phi_2$ is given by

$$P = \exp\left(-S(\phi_1) + S(0)\right) = \exp\left(\frac{3M_p^4}{8}\left(V^{-1}(\phi_1) - V^{-1}(0)\right)\right) . \quad (8)$$

In particular, for the theory (1) with $m \ll \sqrt{V}/M_p$

$$P = \exp\left(-\frac{3M_p^4 m^4}{32\lambda V^2(0)}\right) . \quad (9)$$

Euclidean approach to tunneling in Minkowski space at $T = 0$ can be justified by the standard Hamiltonian methods. Euclidean methods at $T \neq 0$ give a correct expression for the exponential suppression of the probability of tunneling, but some care should be taken when one calculates sub-exponential terms. The situation with tunneling in de Sitter space is much more confusing:

i) From the derivation of eq. (8) given in [29] and from all subsequent "proofs" of this equation with the help of Euclidean methods by other authors it was not clear why one should write $S(\phi_1)$ in eq. (8) rather than, say, $S(\phi_2)$ or $S(\phi_3)$. Indeed, the instantons discussed in [29] describe de Sitter space filled by a homogeneous scalar field which may correspond to *any* extremum of $V(\phi)$.

ii) According to its derivation, eq. (8) should work equally well for $H > m$ and for $H < m$ since the instantons used for its derivation exist independently of the value of m. However, stochastic methods to be used in the next section show that eq. (8) is valid only if $V''(\phi) \gg H^2$ all the way from ϕ_0 to ϕ_1.

iii) The instanton solution obtained in [29] describes a homogeneous field ϕ rather than a bubble. Therefore eq. (8) was interpreted originally as an expression for the probability of tunneling which occurred simultaneously in the whole universe. This interpretation proved to be incorrect [30]. Another interpretation [31] was that eq. (8) described appearance of large bubbles with

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an interior which looked homogeneous on a scale $l > H^{-1}$. Unfortunately, it remained unclear how solutions with a constant field ϕ could describe the process of an inhomogeneous tunneling with bubble formation.

It took few years to prove that eq. (8) and its interpretation in [31] were actually correct, but only under some conditions which did not follow from its Euclidean derivation. The proof was given within the stochastic approach to tunneling [32, 30], which we will discuss now.

3   Stochastic Approach to Tunneling

The main idea of the stochastic approach can be illustrated on an example of tunneling in the theory (1) with $m = 0$ in Minkowski space. Equation of motion for the bubble in Minkowski space is

$$\ddot{\phi} = d^2\phi/dr^2 + (2/r)d\phi/dr - V'(\phi) . \qquad (10)$$

At the moment of its formation, the bubble wall does not move. Then it starts growing if $\ddot{\phi} > 0$, which requires that

$$|d^2\phi/dr^2 + (2/r)d\phi/dr| < -V'(\phi) . \qquad (11)$$

A bubble of a classical field is formed only if it contains a sufficiently big field ϕ (it should be over the barrier, so that $dV/d\phi < 0$) and if the bubble itself is sufficiently large. If the size of the bubble is too small, the gradient terms are bigger than the term $|V'(\phi)|$, and the field ϕ inside the bubble does not grow. Typically, the second term in (11) somewhat compensates the first one. To make a very rough estimate, one may write the condition (11) in the form

$$\frac{1}{2}r^{-2} \sim \frac{1}{2}k^2 < \frac{1}{2}k_{max}^2 \sim \phi^{-1}|V'| = \lambda\phi^2 = \frac{1}{3}m^2(\phi) \equiv \frac{1}{3}V''(\phi) . \qquad (12)$$

Let us estimate the probability of an event when vacuum fluctuations occasionally build up a configuration of the field satisfying this condition. In order to do it one should remember that the dispersion of quantum fluctuations of the field ϕ with $k < k_{max}$ is given by

$$< \phi^2 >_{k<k_{max}} = \frac{1}{4\pi^2} \int_0^{k_{max}} \frac{k^2 dk}{\sqrt{k^2 + m^2}} . \qquad (13)$$

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For the massless field $k_{max}^2 = 2C^2\lambda\phi^2$, where the factor $C^2 = O(1)$ reflects some uncertainty in our estimate of the value of k_{max}^2. This gives

$$< \phi^2 >_{k<k_{max}} = \frac{1}{4\pi^2} \int_0^{k_{max}} kdk = \frac{k_{max}^2}{8\pi^2} = \frac{C^2\lambda\phi^2}{8\pi^2} \ . \tag{14}$$

This is an estimate of the dispersion of perturbations which may sum up to produce a field ϕ which satisfies the condition (12). Of course, this estimate is rather crude. But let us nevertheless use eq. (14) to evaluate the probability that these fluctuations build up a bubble of the field ϕ of a radius $r > k_{max}^{-1}$. This can be done with the help of the Gaussian distribution:

$$P(\phi) \sim \exp(-\frac{\phi^2}{2 < \phi^2 >_{k<k_{max}}}) = \exp(-\frac{2\pi^2}{C^2\lambda}) \ . \tag{15}$$

Note that the factor in the exponent in (15) to within a factor of $C^2 = 3/4$ coincides with the Euclidean action S_E in eq. (4). Taking into account the very rough method we used to calculate the dispersion of the perturbations responsible for tunneling, the coincidence is rather impressive.

This method also helps to study tunneling in the theory (1) with $m \neq 0$, in which the instanton solutions do not exist. Indeed, a generalization of the previous calculation for this case gives the following result for the tunneling probability:

$$P(\phi) \sim= \exp\left(-\frac{2\pi^2}{C^2\lambda}\left(1 + O(\frac{m^2}{\lambda\phi^2})\right)\right) \ . \tag{16}$$

This means that tunneling is possible for $m \neq 0$. There is an additional exponential suppression of the probability of tunneling in this theory as compared with (15). However, this additional suppression disappears if tunneling occurs by formation of bubbles with $\phi > m/\lambda$. This is in an agreement with our arguments given in the previous section. For a discussion of other aspects of tunneling in the theory (1) within the stochastic approach see also [33, 34].

The agreement between the results of the method discussed above and a more complicated Euclidean approach becomes even more impressive if one remembers that most of the results obtained in the tunneling theory by Euclidean methods [14] can easily be reproduced (with an accuracy of the coefficients of $O(1)$ in the exponent) by this simple method.

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For example, let us consider the theory (1) at a temperature $T \gg m$. In this case

$$< \phi^2 >_{k<k_{max}} = \frac{1}{2\pi^2} \int_0^{k_{max}} \frac{k^2 dk}{\sqrt{k^2 + m^2} \left(\exp \frac{\sqrt{k^2 + m^2(\phi)}}{T} - 1 \right)} \sim \frac{T}{2\pi^2} \int_0^{k_{max}} \frac{k^2 dk}{k^2 + m^2} .$$

(17)

Note that the field inside the bubble should be somewhat bigger than $2m/\sqrt{\lambda}$ since otherwise $V(\phi) > V(0)$ and the phase transition is energetically impossible. At $\phi > 2m/\sqrt{\lambda}$ the main contribution to V' is given by $-\lambda\phi^3$ and the expression for k_{max} is the same as in the previous example, $k_{max}^2 = 2C^2\lambda\phi^2 > m^2$. This gives

$$P(\phi) \sim \exp \left(-\frac{\pi^2 \phi}{CT\sqrt{2\lambda}} \right) .$$

(18)

The probability increases with a decrease of ϕ, but ϕ should remain bigger than $2m/\sqrt{\lambda}$. To make a final estimate, let us take $\phi \sim 3m/\sqrt{\lambda}$. The result is

$$P(\phi) \sim \exp \left(-\frac{3\pi^2 m}{CT\sqrt{2\lambda}} \right) .$$

(19)

This coincides with the result of the Euclidean approach (6) for $C = \frac{3\pi^2}{19\sqrt{2}} \sim 1.1$.

Similar methods can be applied to tunneling in the inflationary universe. There are some differences though. First of all, the total energy of the bubble grows exponentially during inflation. In other words, the total energy of the scalar field inside the bubble is not conserved and there is no restriction $V(\phi) > V(0)$ mentioned above. Moreover, there is no need for the new phase to be energetically favorable for the exponentially large bubbles of the new phase to appear. It is sufficient for the radius of the bubble to be bigger than H^{-1} and for the *homogeneous* field ϕ inside it to be stable with respect to rolling back to $\phi = 0$. In other words, one should not worry about the bubble walls moving towards the center of the bubble until it disappears. Indeed, if the bubble walls originally are displaced at a distance $r > H^{-1}$ from the center of the bubble, this distance later grows with a speed greater than the speed of light due to the exponential expansion of the universe. Therefore, the local motion of the walls towards the center cannot lead to a disappearance of the bubble with an initial size $r > H^{-1}$. This means

that for tunneling to occur it is sufficient if the bubble of a radius $r > H^{-1}$ is formed containing any field $\phi > \phi_1$, where ϕ_1 corresponds to the nearby maximum of $V(\phi)$.

Calculation of $< \phi^2 >_{k<H}$ during inflation is rather complicated. Fortunately, for the theory with the effective potential $V(\phi) = \frac{m^2}{2}\phi^2 + V(0)$ the result is well known [35, 36],

$$< \phi^2 >_{k<H} = \frac{3H^4}{8\pi^2 m^2} , \tag{20}$$

where $H^2 = \frac{8\pi V(0)}{3M_p^2}$. According to (15), this gives the following estimate of the probability of formation of a bubble of a field $\phi \geq \phi_1 = m/\sqrt{\lambda}$ of a radius $r > H^{-1}$ (i.e. of the probability of tunneling):

$$P = \exp\left(-\frac{3M_p^4 m^4}{16\lambda V^2(0)}\right) . \tag{21}$$

This agrees, up to the coefficient 2 in the exponent, with the Euclidean result (9). The agreement becomes complete if one investigates the probability distribution in a more detailed way, without approximating $V(\phi)$ by its quadratic part.

The wave-lengths of all vacuum fluctuations of the scalar field ϕ grow exponentially in the expanding universe. When the wavelength of any particular fluctuation becomes greater than H^{-1}, this fluctuation stops propagating, and its amplitude freezes at some nonzero value $\delta\phi(x)$ because of the large friction term $3H\dot{\phi}$ in the equation of motion of the field ϕ. The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field $\delta\phi(x)$ that does not vanish after averaging over macroscopic intervals of space and time. The average amplitude of the frozen field generated during a typical time H^{-1} is given by $\delta\phi(x) = H/2\pi$ [36]. This field looks constant on a scale H^{-1}, but on a bigger scale it is inhomogeneous. If one is interested in the value of this field in each particular point, one should take into account that each new wave is frozen with a different phase. As a result, the field ϕ in each particular point moves as a Brownian particle. This makes it possible

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to write a diffusion equation for the probability distribution to find a field ϕ with the wavelength bigger than H^{-1} in a given point at a given time [32]:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi} \left(\frac{\partial (\mathcal{D}P)}{\partial \phi} + \frac{P}{3H} \frac{\partial V}{\partial \phi} \right) \ . \tag{22}$$

Here the coefficient of diffusion $\mathcal{D}$ is given by $H^3/8\pi^2$. If the probability of tunneling is sufficiently small, then the distribution P with a good accuracy is given by the stationary solution of eq. (22),

$$P \sim \exp \left(\frac{3M_p^4}{8V(\phi)} \right) \ , \tag{23}$$

To describe tunneling from the point $\phi = 0$ over the barrier at ϕ_1 one should consider the probability of formation of a bubble of a field $\phi \geq \phi_1$ of a radius $r > H^{-1}$,

$$P \sim \exp \left(\frac{3M_p^4}{8} \left(V^{-1}(\phi_1) - V^{-1}(0) \right) \right) \ , \tag{24}$$

where the last term is necessary for normalization of the probability distribution. This coincides with the Euclidean result (8). However, now we have a clear interpretation of this result. (For a more detailed discussion see [32, 30, 14].) We can also understand the limits of its validity. Namely, eq. (22) is valid only during inflation and only if $V'' \ll H^2$ in the interval from $\phi = 0$ to $\phi = \phi_1$. This condition does not follow at all from the derivation of eq. (8) in the Euclidean approach.

This comment proves to be very important in the context of quantum cosmology. Indeed, the stationary solution (23) looks as a square of the Hartle-Hawking wave function of the ground state of the universe [15] obtained by Euclidean methods. Therefore one could consider eq. (23) as a confirmation of the validity of the Hartle-Hawking wave function, which was criticized in [16]. However, in all realistic theories the condition $V'' \ll H^2$ of validity of eqs. (22), (23) is not satisfied near the absolute minimum of $V(\phi)$. As a result, all solutions of eq. (22) in realistic theories are non-stationary. The probability distribution (23) can be used at intermediate stages of inflation for an approximate description of tunneling, when the probability distribution is quasi-stationary, but it does not have any fundamental significance and cannot be used to describe the universe as a whole.

An important physical consequence of the Brownian motion of the field ϕ during inflation is the self-reproduction of the inflationary universe. This process is especially interesting in the context of chaotic inflation [37, 14]. It proves that at large ϕ the Brownian motion is much more rapid than the classical rolling of the field to the minimum of its potential energy density. As a result, many inflationary domains are formed which contain *growing* field ϕ. These domains expand much faster than the domains with small ϕ. This leads to a paradoxical situation where the main part of the physical volume of the universe (in the synchronous coordinate system) becomes occupied not by the field corresponding to the minimum of $V(\phi)$, but by the field fluctuating at a density close to the Planck energy density M_p^4 [37, 14]. It is quite clear that if the process of the baby universe formation can occur at all, inflationary domains with $V(\phi) \sim M_p^4$ are the best place for it. We will return to this question in the end of the paper.

4   Stochastic Approach to the Baby Universe Formation

Now that we demonstrated usefulness and reliability of the stochastic approach in many different situations where it can be confirmed by other methods, we will take a deep breath and try to apply it to the investigation of the possibility of creating an inflationary universe from Minkowski space [10, 17, 18]. The issue here is that quantum fluctuations in Minkowski space can bring into being an inflationary domain of a radius $r > H^{-1}(\phi)$, where ϕ is a scalar field produced by quantum fluctuations in this domain. The "no-hair" theorem for de Sitter space implies that such a domain inflates in an entirely self-contained manner, independent of what occurs in the surrounding space. We could then conceive of a ceaseless process of creation of inflationary mini-universes that could take place even at the very latest stages of development of the part of the universe that surrounds us.

Without pretending to provide a complete description of such a process, let us attempt to estimate its probability in theories with $V(\phi) = \frac{\lambda \phi^n}{n M_p^{\,n-4}}$ using the methods elaborated in the previous section. A domain formed with a large field ϕ can behave as a part of inflationary universe only if

$\phi \gtrsim M_p$ and the gradient and kinetic energy of this field $1/over2(\partial_\mu\phi)^2$ is smaller than $V(\phi)$ in its interior. The last condition implies that the size of the domain must exceed $r \sim \phi V^{-1/2}(\phi)$. Such a domain could arise through the build-up of quantum fluctuations $\delta\phi$ with a wavelength

$$r \sim k^{-1} \geq k_{max}^{-1} \sim \phi V^{-1/2}(\phi) \sim m^{-1}(\phi) . \qquad (25)$$

(Note that $m^{-1}(\phi) > H^{-1}(\phi)$ during inflation.) One can estimate the dispersion $< \phi^2 >_{k<m}$ of such fluctuations using the simple formula

$$< \phi^2 >_{k<m} \sim \frac{1}{4\pi^2} \int_0^{k_{max}} k \, dk \sim \frac{m^2}{\pi^2} \sim \frac{V(\phi)}{8\pi^2 \phi^2} , \qquad (26)$$

and for a Gaussian distribution $P(\phi)$ for the appearance of a field ϕ which is sufficiently homogeneous on a scale $r > m^{-1}(\phi)$, one has [10]

$$P(\phi) \sim \exp(-\frac{\phi^2}{2 < \phi^2 >_{k<m}}) \sim \exp(-C\frac{\pi^2\phi^4}{V(\phi)}) , \qquad (27)$$

where C = O(1). In particular, for a theory with $V(\phi) = \frac{\lambda}{4}\phi^4$,

$$P(\phi) \sim \exp(-C\frac{4\pi^2}{\lambda}) . \qquad (28)$$

Note, that eq. (28) does not show any suppression of the probability of the baby universe formation due to smallness of gravitational effects; $P(\phi)$ may be quite large in the theory with a large coupling constant λ.[3] Some suppression may appear at the sub-exponential level, since the "phase space" of all possible inflationary universes in this theory is constrained by the condition $\phi > M_p$.

The probability of the baby universe formation in the theory of a massive scalar field can be estimated by

$$P(\phi) \sim \exp(-C\frac{8\pi^2\phi^2}{m^2}) . \qquad (29)$$

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[3]One should remember that in a theory of several scalar fields with different coupling constants the probability of creation of an inflationary universe is given by the *largest* coupling constant, whereas the density perturbations $\delta\rho/\rho$ is determined by the *smallest* coupling constant λ [14].

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The leading contribution is given by the universes with the smallest ϕ compatible with inflation in this model, $\phi \sim M_p$:

$$P(\phi) \sim \exp(-C\frac{8\pi^2 M_p^2}{m^2}) \ . \tag{30}$$

This means that the baby universe formation should be very efficient in the theories in which the heaviest scalar particles have masses comparable with M_p.

The main objection to the possibility of quantum creation of an inflationary universe in Minkowski space is that the energy conservation forbids the production of an object with positive energy out of vacuum with vanishing energy density.[4] Within the scope of the classical field theory, in which the energy density is everywhere positive, such a process would therefore be impossible. But at the quantum level, the energy density of the vacuum is zero by virtue of the cancellation between the positive energy density of classical scalar fields, along with their quantum fluctuations, and the negative energy density associated with quantum fluctuations of fermions, or the bare negative energy of the vacuum. The creation of a positive energy-density domain through the build-up of long-wave fluctuations of the field ϕ is inevitably accompanied by formation of a region surrounding that domain in which the long-wave fluctuations of the field ϕ are suppressed, and the vacuum energy density is consequently negative. Here we are dealing with the familiar quantum fluctuations of the vacuum energy density about its zero point.

It is important that from the point of view of an external observer, the total energy of the inflationary region of the universe (and indeed the total energy of the closed inflationary universe) does not grow exponentially; the region that emerges forms a universe distinct from ours, to which it is joined only by a connecting throat (wormhole). The shortfall of long-wave fluctuations of the field ϕ surrounding the inflationary domain is quickly replenished by fluctuations arriving from neighboring regions, so the negative energy of the region near the throat can be be rapidly spread over a large volume around the inflationary domain. In such a scenario the total energy of the inflationary domain plus the energy of vacuum surrounding it will remain

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[4]This problem will not appear when we will study the process of the universe formation at large temperature.

zero, but after the negative energy of vacuum surrounding the inflationary domain will be distributed all over the rest of the universe an observer near the inflationary domain would see it as an evaporating black hole. To show that this is a viable possibility one can make a naive estimate of the Schwarzschild radius of the inflationary domain from the point of view of an external observer. The total energy of matter inside this domain is $E \sim V(\phi)m^{-3}(\phi)$. If one neglects the gravitational defect of mass (this is the place where the estimate is naive), then the Schwarzschild mass of this domain is equal to E and the Schwarzschild radius is $E/M_p^2 \sim m^{-1} \cdot H^2/m^2 \gg m^{-1}$. Thus, the Schwarzschild radius of the inflationary domain is much bigger than the size of the domain $r \sim m^{-1}$, so it really looks like a black hole. One can show also that the time of evaporation of such a black hole is microscopically small, but it is much bigger than $m^{-1}(\phi)$. For example, it can be shown that in the theory $\frac{\lambda}{4}\phi^4$ the time of the black hole evaporation is of the order

$$t \sim \frac{\phi^3}{\lambda^{3/2} M_p^4} > (\lambda m(\phi))^{-1} \ . \tag{31}$$

This means that for a very distant observer all what happens will look as a kind of an unusual long-living quantum fluctuation, an observer at a not too big distance from the inflationary domain will see it as an evaporating black hole surrounded by space with negative vacuum energy, whereas an observer inside the inflationary domain would believe that he lives inside an inflationary universe.

Since the universes created in the empty space do not carry any momentum, the process of their formation cannot be localized at any particular point. Their main role in the theory is the same as the role of the baby universes considered in [2] – [8]: They modify the properties of the vacuum state. However, the methods discussed in this section can be easily extended for the investigation of the process of formation of inflationary universes in a laboratory, where this process can be localized. For example, one can consider the process of production of inflationary bubbles inside a domain containing matter heated up to a temperature T. To this end one should just replace expression (26) for $< \phi^2 >_{k<k_{max}}$ in eq. (27) by its counterpart calculated at a temperature $T \gg m(\phi) \sim V^{1/2}/\phi$:

$$< \phi^2 >_{k<k_{max}} \sim \frac{TV^{1/2}(\phi)}{2\pi^2\phi} \ . \tag{32}$$

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Note, that this expression for $T \gg V^{1/2}/\phi$ is much larger than its empty space predecessor (26). This means (not unexpectedly) that heating leads to a more efficient creation of universes with a given ϕ, and this process is localized in the part of the universe with a large temperature (i.e. in a laboratory). The corresponding probability is given by

$$P_T(\phi) \sim \exp\left(-\frac{\phi^2}{2 < \phi^2 >_{k<m}}\right) \sim \exp\left(-C\frac{\pi^2\phi^3}{TV^{1/2}(\phi)}\right) . \tag{33}$$

For the theory $\frac{\lambda}{4}\phi^4$ the maximum contribution is given by $\phi \sim M_p$,

$$P_T \sim \exp\left(-C\frac{\pi^2 M_p}{T\sqrt{\lambda}}\right) , \tag{34}$$

whereas for the theory $\frac{m^2\phi^2}{2}$ the result is

$$P_T \sim \exp\left(-C\frac{\pi^2 M_p^2}{Tm}\right) . \tag{35}$$

These expressions remain exponentially small at $T \ll M_p$, for all values of λ and m. Moreover, even if one could rise the temperature up to M_p, one would not produce inflationary universes that way. Eqs. (33)-(35) are valid only if $T^4 \lesssim V(\phi)$, since otherwise the energy density of hot matter inside the bubble is bigger than the effective potential, and inflationary regime cannot be realized. (This is the same problem as the one discussed in the Introduction in relation to papers [20, 21].) Therefore, the probability of creation of inflationary universe due to high-temperature effects in the theory $\frac{\lambda}{4}\phi^4$ is bounded from above by

$$P_{max}(\phi) \sim \exp\left(-C\frac{\pi^2\phi^3}{V^{3/4}(\phi)}\right) \sim \exp\left(-C\frac{\pi^2}{\lambda^{3/4}}\right) . \tag{36}$$

This maximum is reached at $T \sim \lambda^{1/4}M_p$. The corresponding expression in the theory $\frac{m^2\phi^2}{2}$ is

$$P_{max}(\phi) \sim \exp\left(-C\pi^2\left(\frac{M_p}{m}\right)^{3/2}\right) , \tag{37}$$

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This maximum is achieved at $T \sim \sqrt{mM_p}$.

One could hope that it might be easier to create the universe in the new inflationary scenario, since inflation occurs there on a much smaller energy scale. Unfortunately, the result is quite opposite: The smaller is the energy density, the larger is H^{-1}, the larger is the size of the domain to be produced, the smaller is the probability of such event. As an example, we will consider here the theory with the effective potential often used in the new inflationary universe scenario,

$$V(\phi) = \lambda \phi^4 \left(\log \frac{\phi}{\phi_0} - \frac{1}{4} \right) + V(0) . \tag{38}$$

Here $V(0) = \lambda \phi_0^4/4$, ϕ_0 is the position of the minimum of $V(\phi)$, the mass of the field in this minimum is equal to $m = 2\sqrt{\lambda}\phi_0$. Inflationary domain should contain the field $\phi \ll \phi_0$. This means that the deviation from the minimum due to quantum fluctuations is given by ϕ_0. The size of the domain should exceed $H^{-1} = \frac{M_p}{\phi_0^2}\sqrt{\frac{3}{2\pi\lambda}}$. It should be even larger for tunneling to small ϕ, with $V'' \ll H^2$, but here we will consider the simplest case when V'' is just few times smaller than H^2. Note, that $m \gg H$ for $\phi_o \ll M_p$. In this case

$$< \phi^2 >_{k<H} \sim \frac{1}{4\pi^2} \int_0^H \frac{k^2 dk}{\sqrt{k^2 + m^2}} \sim \frac{H^3}{12\pi^2 m} . \tag{39}$$

This gives the following estimate of the probability of the baby universe creation in this theory:

$$P \sim \exp(-\frac{\phi_0^2}{2 < \phi^2 >_{k<H}}) \sim \exp\left(-C\frac{4\pi^2}{\lambda} \cdot \left(\frac{M^p}{\phi_0} \right)^3 \right) . \tag{40}$$

This is much smaller than the corresponding probability in the chaotic inflation scenario (28).

By a similar method one can get the following estimate of the probability to create the universe in a laboratory at $T \gg m$:

$$P_T \sim \exp\left(-C\frac{2\pi^2 \phi_0}{T\sqrt{\lambda}} \cdot \left(\frac{M_p}{\phi_0} \right)^3 \right) . \tag{41}$$

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From the condition $T^4 < V(0) = \lambda\phi_0^4$ it follows that

$$P_T < \exp\left(-C\frac{\pi^2}{\lambda^{3/4}} \cdot \left(\frac{M_p}{\phi_0}\right)^3\right) \ . \tag{42}$$

Again, for $\phi_0 \ll M_p$ this probability is much smaller than that in the chaotic inflation scenario, see eq. (36).

Thus, we see that the stochastic methods, which proved to be very efficient in the theory of tunneling, may help us to understand such issues as the baby universe formation in the vacuum and at a finite temperature. However, several problems are to be resolved before one gets too excited.

First of all, our estimates give us the probability of formation of a large domain with the energy density dominated by the potential energy density of the slowly changing field ϕ. Such a domain should behave as a part of de Sitter space. However, if it can be considered as a part of a *closed* de Sitter space, then in the beginning it may be rapidly contracting. Contraction may lead to appearance of large gradients of the field ϕ and to the collapse of the domain, instead of its exponential expansion.

Even though this problem is very complicated, we believe that it does not cause any difficulties in the case when the size of the bubble is much bigger than H^{-1}. Indeed, the acceleration of test particles in de Sitter space is always large and *positive*, $\ddot{r}/r = H^2$. One may expect that the bubble walls at the moment of its formation, being formed by overlapping of long-wave quantum fluctuations in Minkowski space, cannot move towards the center of the bubble with the speed bigger than the speed of light.[5] If this is true, then the bubble of de Sitter space of a size exceeding H^{-1} becomes exponentially expanding within a fraction of the Hubble time H^{-1} due to the acceleration $\ddot{r}/r = H^2$ mentioned above.

The second problem can be explained as follows. When we studied formation of an inflationary bubble, we did not take into account the backreaction of the changing metric inside the bubble on the spectrum of vacuum

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[5]In the standard tunneling theory in Minkowski space the initial speed of the bubble walls is equal to zero, whereas the bubbles formed in the inflationary universe may expand (but not contract) from the very beginning.

fluctuations during the process of the bubble formation. This is a good approximation for small bubbles with a radius $r < H^{-1}$, but for large bubbles the situation is not that simple. Let us try to visualize the process of growth of the field ϕ in Minkowski space due to overlapping of many long-wave fluctuations. The wavelength of each such wave should exceed $O(m^{-1}(\phi))$. Therefore one may expect that the total time necessary for the maxima of these waves to overlap should be also of the order of $m^{-1}(\phi) \gg H^{-1}$. If the bubble will start expanding exponentially at any stage of this process, then its expansion during the time $O(m^{-1}) \gg H^{-1}$ may push away all new incoming long-wave fluctuations of the scalar field which could lead to further growth of the field inside the bubble. This may make the process of direct formation of inflationary baby universes with $m^{-1}(\phi) \gg H^{-1}$ by the mechanism suggested above either strongly suppressed or even entirely forbidden. In the last case, the estimates we made could be only used to calculate the probability of formation of small, sub-critical bubbles with the energy density dominated by $V(\phi)$. (As we emphasized in the Introduction, this is an important problem by itself.) Then it will be necessary to consider further tunneling of a sub-critical bubble to a large exponentially expanding bubble along the lines of [20, 21], but without the use of the thin-wall approximation. One may expect that the probability of such a double tunneling will be even smaller than the probability of each of such events separately. In such case one may use our estimates as an upper bound on the probability of the baby universe formation.

However, this problem should not be exagerrated. First of all, this problem may not appear at all for the bubbles containing the field $\phi \sim M_p$ with $m^{-1}(\phi)$ being just few times larger than H^{-1}. This is quite sufficient for a production of the universe of our type with a size $l > 10^{28}$ cm [14]. Moreover, inflation of the interior of the bubble can push away incoming long-wave perturbations *only if inflation there already started.* But in this case inflation itself produces long-wave perturbations by stretching the short-wave ones, which always exist in the interior of the bubble. This is the same mechanism which leads to appearance of extremely large long-wave perturbations in the inflationary universe, eq. (20). Indeed, it is known that the total contribution to $< \phi^2 >$ in de Sitter space from perturbations with the wavelength smaller than H^{-1} is the same as in Minkowski space, and the contribution of long-wave perturbations with the wavelength larger than H^{-1} is even greater

than in Minkowski space, due to the process of stretching of the wavelengths mentioned above. This suggests that our estimate for the probability of bubble formation with $\phi \sim M_p$, $m^{-1}(\phi) \sim H^{-1}$ has a good chance to be correct, whereas the probability of the universe formation with $m^{-1}(\phi) \sim H^{-1}$ may be even somewhat higher than we expected.

5 Discussion

It is quite clear that the estimates which were obtained by the stochastic approach to tunneling cannot serve as a substitute of a complete investigation by more regular methods; a much more detailed investigation is needed to prove that the process of the baby universe formation can actually occur either in the empty Minkowski space or in a laboratory. However, the use of Euclidean methods in quantum cosmology never have been justified and often give ambiguous results, whereas the Hamiltonian approach usually is extremely complicated and may have its own conceptual limitations being applied to a system of many universes with different "times". Therefore, it would be very useful in the beginning to get at least partial understanding of the processes we are going to study. This was the main goal of the present investigation.

Our estimates indicate that the process of the baby universe formation in Minkowski space is actually possible. Moreover, in the theories with large coupling constants or with scalar particles with masses of the order of M_p this process may be even quite probable. This process by itself does not lead to any spectacular events like formation of a large hole in the ground. However, as it was argued in [2] – [8], baby universe formation may lead to important modifications of the properties of our vacuum state.

One should note also, that in the eternally existing self-reproducing inflationary universe this process may occur even at the present time in those domains of the universe which are now in the inflationary phase at a density close to the Planck density [37, 14]. Quantum jumps of the scalar field to the Planck density space-time foam, which regularly occur in this scenario, and subsequent jumps of the field back from the space-time foam may be interpreted as a process of creation of new inflationary universes which are not

attached to our universe by any regions of classical space-time. Of course, one may argue that the distance from us to the inflationary domains with the Planck density is exponentially large. Moreover, these regions may form huge black holes and become effectively disconnected from our part of the universe [38]. One should remember, however, that in the end we are going to investigate a *non-local* interaction of baby universes with our universe. It is important that *the main part* of the physical volume of a self-reproducing inflationary universe (in the synchronous coordinate system) is always occupied by the fluctuating scalar field with $V(\phi) \sim M_p^4$ [37, 14]. Moreover, it is clear that if the process of the baby universe formation may occur at all, it should be most efficient at the density close to the Planck density. This suggests (see also [39, 40]) that an investigation of the baby universe formation during inflation may be very important for the understanding of the properties of the gravitational vacuum.

The possibility that our own universe could appear as a result of decoupling from another universe may have other interesting implications, some of which may be experimentally testable. For example, if our universe was created "from nothing", then it typically appears in a state with a very large vacuum energy density, $V(\phi) \sim M_p^4$ [16], and later it enters an infinite process of self-reproduction [37]. In such case it does not seem possible to avoid the standard prediction of inflationary cosmology, $\Omega = 1$. However, if our universe is created due to decoupling from another universe, then from our estimates of $P(\phi)$ it follows that in some theories the universe should be typically created in a state with a small value of $V(\phi)$, which only leads to a very short stage of inflation, if any.[6] In such case our universe may be relatively small, and it may have $\Omega \neq 1$. We do not think that this model is natural, but it is better to know that such a possibility may exist. We mentioned it here, since it illustrates a novel way to solve the homogeneity and isotropy problem without much help of inflation: It is well known that only spherically symmetric bubbles are formed if the probability of their creation is strongly suppressed.

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[6]This issue is not very trivial, since the expectation value of the *volume* of the inflationary universe in the theories with $V(\phi) \sim \phi^n$ (without an account taken of its self-reproduction at very large ϕ) is proportional to $P(\phi) \exp\left(\frac{12\pi\phi^2}{nM_p^2}\right)$ [14], which may grow at large ϕ even if $P(\phi)$ decreases.

As for the possibility to create the universe in a laboratory, our estimates indicate that one would need a very good laboratory indeed. The probability to create the universe in a laboratory is not totally negligible only in the chaotic inflation scenario, only in the theories with large coupling constants or with scalar particles with masses of the order of M_p, and only if one can heat the system up to the temperature approaching M_p. However, the most ironical part of it is the question whether the new universe can be useful for us in any way. Of course, one may just consider the problem of the universe creation as an interesting theoretical problem to think about in a spare time, but if the universe creation is entirely useless, one may find other interesting problems to solve. Leaving aside the possibility to use the universe as a universal trash compactor, we were hardly able to find any good reason to spend our time and energy for its creation.

Indeed, one cannot "pump" energy from the new universe to ours, since this would contradict the energy conservation law. One cannot jump into the new universe, since at the moment of its creation it is microscopically small and extremely dense, and later it decouples from our universe. One even cannot send any information about himself to those people who will live in the new universe. If one tries, so to say, to write down something "on the surface of the universe", then, for the billions of billions years to come, the inhabitants of the new universe will live in a corner of one letter. This is a consequence of a general rule: All local properties of the universe after inflation do not depend on initial conditions at the moment of its formation. Very soon it becomes absolutely flat, homogeneous and isotropic, and any original message "imprinted" on the universe becomes unreadable.

We were able to find only one exception to this rule. As we already mentioned, if chaotic inflation starts at a sufficiently large energy density, then it goes forever, creating new and new inflationary domains. These domains contain matter in all possible "phase states" (or vacuum states), corresponding to all possible minima of the effective potential and all types of compactification compatible with inflation [37, 14]. However, if inflation starts at a sufficiently low energy density, as is often the case with the universes produced in a laboratory, then no such diversification occurs; inflation at a relatively small energy density does not change the symmetry breaking pattern of the theory and the way of compactification of space-time. There-

fore it seems that the only way to send a message to those who will live in the universe we are planning to create is to encrypt it into the properties of the vacuum state of the new universe, i.e. to the laws of the low-energy physics. Hopefully, one may achieve it by choosing a proper combination of temperature, pressure and external fields, which would lead to creation of the universe in a desirable phase state.

The corresponding message can be long and informative enough only if there are extremely many ways of symmetry breaking and/or patterns of compactification in the underlying theory. This is exactly the case, e.g., in the superstring theory, which was considered for a long time as one of the main problems of this theory. Another requirement to the informative message is that it should not be too simple. If, for example, masses of all particles would be equal to each other, all coupling constants would be given by 1, etc., the corresponding message would be too short. Perhaps, one may say quite a lot by creating a universe in a strange vacuum state with $m_p \sim 2000m_e$, $m_W \sim 100m_p$, $m_X \sim 10^{13}m_W$, $M_p \sim 10^4 m_X$. The stronger is the symmetry breaking, the more "unnatural" are relations between parameters of the theory after it, the more information the message may contain. Is it the reason why we must work so hard to understand strange features of our beautiful and imperfect world? Does this mean that our universe was created not by a divine design but by a physicist hacker? If it is true, then our results indicate that he did a very difficult job. Hopefully, he did not make too many mistakes...

6 Acknowledgements

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