SystemA
Advanced Control Systems Software

Iman Khademi

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1 Introduction

SystemA is an open source **C++** software for design and simulation of discrete-time control systems. The initial release contains the LQR and LQG control, and Kalman Filtering. The Model Predictive Control (MPC) techniques will be added in the future release.

2 LQR Control

The state-space model of the system is:

$$x_{k+1} = Ax_k + Bu_k,\ x(0) = x_0 \tag{1}$$
$$y_k = Cx_k \tag{2}$$

It will find the feedback gain K_k such that

$$u_k = L_k x_k \tag{3}$$

minimizes the following cost function:

$$J(u) := x_N^T Q_f x_N + \sum_{k=0}^{N-1} x_k^T Q x_k + u_k^T R u_k \tag{4}$$

where $Q \geq 0$, $Q_f \geq 0$, and $R > 0$. The feedback gain is given by:

$$L_k = -(R + B^T P_{k+1} B)^{-1} B^T P_{k+1} A \tag{5}$$

Matrix P_k is achieved by solving the following Ricatti equation backward in time:

$$P_{k-1} = Q + A^T P_k A - A^T P_k B(R + B^T P_k B)^{-1} B^T P_k A \tag{6}$$
$$P_N = Q_f \tag{7}$$

3 Kalman Filter

3.1 Process Model

The state-space model of the process is:

$$x_{k+1} = Ax_k + Bu_k + Gw_k, \tag{8}$$
$$y_k = Cx_k + v_k \tag{9}$$

where the initial condition of x is:

$$E\{x(0)\} = x_0 \tag{10}$$

3.2 Estimator

The optimal filtered estimate $\hat{x}(k+1|k+1)$ is given by[1]:

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)[y(k+1) - C\hat{x}(k+1|k)] \tag{11}$$

where

$$\hat{x}(k+1|k) = A\hat{x}(k|k) + Bu(k) \tag{12}$$

The Kalman gain is given by:

$$K(k+1) = P(k+1|k)C^T(k+1)[CP(k+1|k)C' + V_c]^{-1} \tag{13}$$
$$P(k+1|k) = AP(k|k)A^T + GW_cG^T \tag{14}$$
$$P(k+1|k+1) = [I - K(k+1)C]P(k+1|k) \tag{15}$$

where P is the covariance of the estimation error with known $P(0|0) = P(0)$. The covariance of the process noise, w, and the measurement noise, v, are given by W_c and V_c, respectively.

4 LQG Control

It takes advantages of the *Separation Principle* to plug in $\hat{x}$ instead of x into equation 3:

$$u_k = L_k\hat{x}_k \tag{16}$$

Therefore, it combines an LQR Controller with a Kalman Filter.

[1] J. S. Meditch, *Stochastic Optimal Linear Estimation and Control*, McGraw-HILL, 1969.