Chapter 9
Inference Based on Two Samples: Confidence Intervals and Tests of Hypotheses

9.1 Introduction

Chapter 9 introduces the reader to the inferences based on two samples. The two tools developed in the prior two chapters – confidence intervals and tests of hypothesis – are used to compare two population parameters. Three parameters - the population mean, proportion, and variance - are studied in this chapter. **XLSTAT** provides different techniques for comparing all three of these parameters.

Chapter 9 also introduces the reader to the concept of paired data. The paired difference comparison of two population means is presented as an additional technique for comparing two population means. **XLSTAT** offers both an independent and paired sampling comparison of two means.

We find all the confidence intervals and tests of hypothesis covered in Chapter 9 within the **Parametric Tests** menu located inside the **XLSTAT** tab.

We note that *A First Course in Statistics* does not include the topic of comparing two population variances. The *A First Course in Statistics* text covers comparing two population means in Chapter 7 and covers comparing two population proportions in Chapter 8 of that text.

The following examples from *Statistics* illustrate the confidence interval calculations that can be found using **XLSTAT** in this chapter:

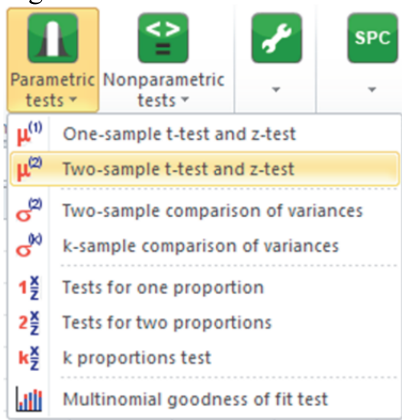
Excel Companion Exercise	**Page**	**Statistics Example**	**Excel File Name**
9.1	121	Example 9.1	DIETSTUDY
9.2	124	Example 9.5	GRADPAIRS
9.3	127	Example 9.6	
9.4	130	Example 9.10	MICEWTS

9.2 Comparing Two Population Means: Independent Sampling

There are two main sampling designs for comparing two population means. This section deals with samples collected randomly and independently from their respective populations. Sometimes, the data from the two samples collected is paired together. The paired difference analysis, covered in the next section, is appropriate for data collected in this manner.

Regardless of the sampling design utilized, **XLSTAT** uses the **Two-sample t-test and z-test** menu located within the **Parametric tests** option to compare two population means. Click on the **Parametric tests** option to access the **Two-sample t-test and z-test** menu (see Figure 9.1).

Figure 9.1



We illustrate how to use this technique with the following exercise:

Exercise 9.1: We use Example 9.1 found in the *Statistics* text.

Problem A dietician has developed a diet that is low in fats, carbohydrates, and cholesterol. Although the diet was initially intended to be used by people with heart disease, the dietitian wishes to examine the effect this diet has on the weights of obese people. Two random samples of 100 obese people each are selected, and one group of 100 is placed on the low-fat diet. The other 100 are placed on a diet that contains approximately the same quantity of food, but is not low in fats, carbohydrates, and cholesterol. For each person, the amount of weight lost (or gained) in a three-week period is recorded. The data is saved in the DIETSTUDY data file.

a. Form a 95% confidence interval for the difference between the population mean weight losses for the two diets.
b. Conduct a test of hypothesis to determine if the mean weight losses for the two diets differ. Use $\alpha = .05$.

Solution:

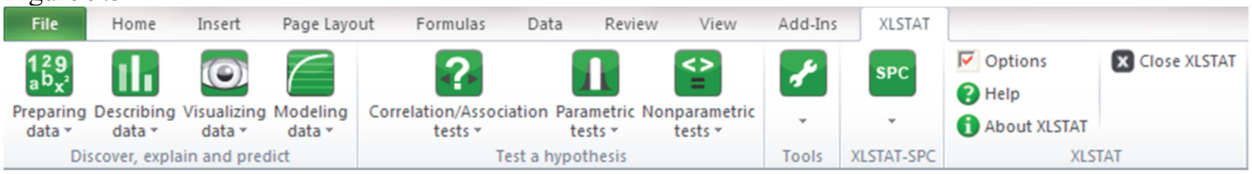
We solve Exercise 9.1 utilizing the **Two-sample t-test and z-test** menu presented in **XLSTAT**. **Open** the data file **DIETSTUDY** by following the directions found in the preface of this manual. If done correctly, the data should appear in a workbook similar to that shown in Figure 9.2.

Figure 9.2

	A	B
1	DIET	WTLOSS
2	LOWFAT	8
3	LOWFAT	10
4	LOWFAT	10
5	LOWFAT	12
6	LOWFAT	9
7	LOWFAT	3
8	LOWFAT	11
9	LOWFAT	7
10	LOWFAT	9
11	LOWFAT	2
12	LOWFAT	21
13	LOWFAT	8

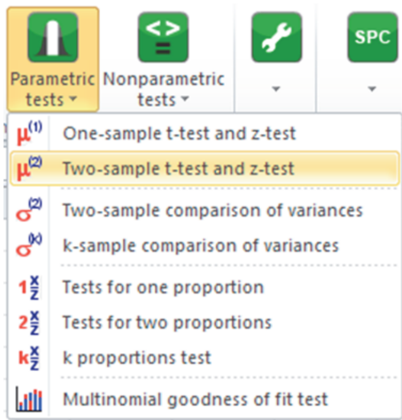
To compare the two population means, we click on the **XLSTAT** tab at the top of the **Excel** workbook to access the **XLSTAT** menus shown in Figure 9.3.

Figure 9.3



To conduct the desired test of hypothesis, we click on the **Parametric tests** menu and select the **Two-sample t-test and z-test** option shown in Figure 9.4.

Figure 9.4



This opens the **Two-sample t-test and z-test** menu shown in Figures 9.5-9.6. We need to first specify the location of the data that is to be analyzed. In our data set, the data is located in columns A and B, rows 2 – 201, with row 1 being the variable labels. We note that the data in column A represents the type of diet and the data in column B represent the corresponding weight loss. We specify the column B data in the **Data:** box and the column A data in the **Sample identifiers:** box, and check the **Column labels** box to indicate the first row of data represents the variable names, **DIET** and **WTLOSS**. We also check the **One column per variable** button to indicate the way our data appear in the data set. In order to utilize the z-distribution in the calculation of the test statistic, we need to check the **z test** box in this menu.

Click on the **Options** tab (shown in Figure 9.6) to specify the **alternative hypothesis**, the **hypothesized difference (D)**, the **significance level**, and to also indicate how we want **XLSTAT** to work with the population variance required in the test statistic calculations. If this value is known, we check the **User defined** button and enter the value in the **Variance** box. If not, we check the **Estimated using samples** button. In our example, we have entered **Mean 1 – Mean 2 ≠ D**, the value of **0** as the **Hypothesized difference (D):**, the value **5%** as the **Significance level,** and checked the **Estimated using samples** button. Click **OK** to conduct the test of hypothesis.

Figure 9.5

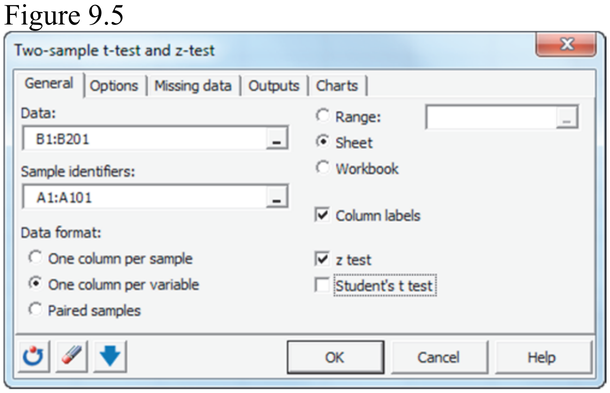
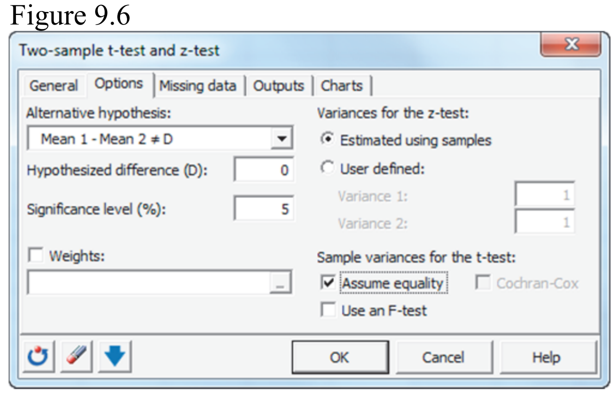


Figure 9.6



The XLSTAT output is shown in Figure 9.7.

Figure 9.7

z-test for two independent samples / Two-tailed test:

95% confidence interval on the difference between the means:
(0.7006, 3.1194)

Difference	1.9100
z (Observed value)	3.0954
|z| (Critical value)	1.9600
p-value (Two-tailed)	0.0020
alpha	0.05

Test interpretation:
H0: The difference between the means is equal to 0.
Ha: The difference between the means is different from 0.
As the computed p-value is lower than the significance level alpha=0.05,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 0.20%.

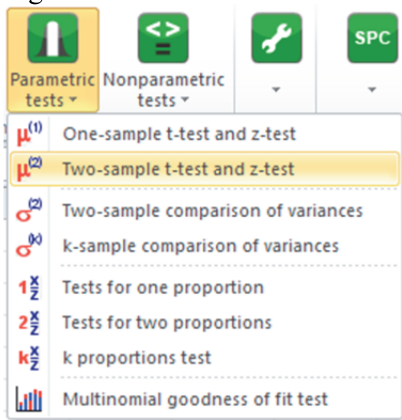
We see that the printout above contains both the confidence interval and test of hypothesis information that we desire. We compare this printout to the information shown in the text. We note that the test statistic shown in the printout (z = 3.0954) and the confidence interval given (0.7006, 3.1194) have been rounded in the text and differ slightly from the values shown on the printout.

We note that this example used two large samples in the calculations. The small-sample comparison would require us to check the Student's t test button in Figure 9.5 and specify how to estimate the sample variances in Figure 9.6. Otherwise the small-sample work is identical to the large-sample work.

9.3 Comparing Two Population Means: Paired Difference Experiments

This section deals with samples that are collected in pairs. The paired difference analysis of two population means is found in the same menu as the independent sampling analysis covered in the last section. **XLSTAT** uses the **Two-sample t-test and z-test** menu located within the **Parametric tests** option to compare two population means. Click on the **Parametric tests** option to access the **Two-sample t-test and z-test** menu (see Figure 9.8).

Figure 9.8



We illustrate how to use this technique with the following exercise:

Exercise 9.2: We use Example 9.5 found in the *Statistics* text.

Problem An experiment is conducted to compare the starting salaries of male and female college graduates who find jobs. Pairs are formed by choosing a male and a female with the same major and similar grade point averages (GPAs). Suppose a random sample of 10 pairs is formed in this manner and the starting annual salary of each person is recorded. The data is shown in Table 9.1 and found in the GRADPAIRS data file.

a. Compare the mean starting salary for males with the mean starting salary for females using a 95% confidence interval.
b. Conduct a test of hypothesis to determine if the means starting salary of males differs from the mean starting salary of females. Use $\alpha = .05$.

Table 9.1

PAIR	MALE	FEMALE	DIFFERENCE
1	29300	28800	500
2	41500	41600	-100
3	40400	39800	600
4	38500	38500	0
5	43500	42600	900
6	37800	38000	-200
7	69500	69200	300
8	41200	40100	1100
9	38400	38200	200
10	59200	58500	700

Solution:

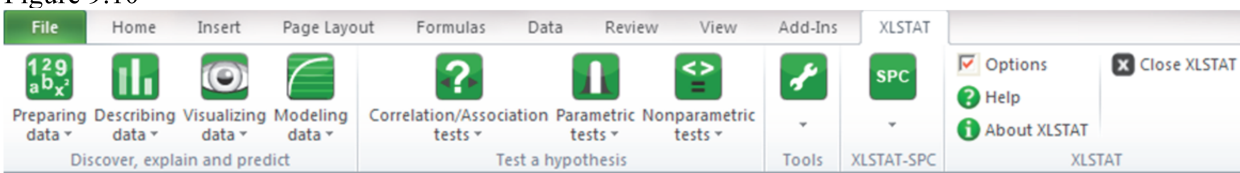
We solve Exercise 9.2 utilizing the **Two-sample t-test and z-test** menu presented in **XLSTAT**. **Open** the data file **GRADPAIRS** by following the directions found in the preface of this manual. If done correctly, the data should appear in a workbook similar to that shown in Figure 9.9.

Figure 9.9

	A	B	C
1	PAIR	MALE	FEMALE
2	1	29300	28800
3	2	41500	41600
4	3	40400	39800
5	4	38500	38500
6	5	43500	42600
7	6	37800	38000
8	7	69500	69200
9	8	41200	40100
10	9	38400	38200
11	10	59200	58500

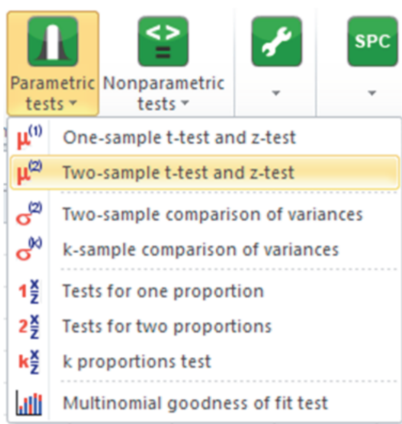
To compare the two population means, we click on the **XLSTAT** tab at the top of the **Excel** workbook to access the **XLSTAT** menus shown in Figure 9.10.

Figure 9.10



To conduct the desired test of hypothesis, we click on the **Parametric tests** menu and select the **Two-sample t-test and z-test** option shown in Figure 9.11.

Figure 9.11



This opens the **Two-sample t-test and z-test** menu shown in Figures 9.12-9.13. We need to first specify the location of the data that is to be analyzed. In our data set, the data is located in columns B and C, rows 2 – 11, with row 1 being the variable labels. We note that the data in column B represents the male starting salaries and the data in column C represent the female starting salaries. We specify the column B data in the **Sample 1:** box and the column C data in the **Sample 2:** box, and check the **Column labels** box to indicate the first row of data represents the variable names, **MALE** and **FEMALE**. We also check the **Paired samples** button to indicate the way our data appear in the data set. In order to utilize the t-distribution in the calculation of the test statistic, we need to check the **Student's t test** box in this menu.

Click on the **Options** tab (shown in Figure 9.13) to specify the **alternative hypothesis**, the **hypothesized difference (D)**, and the **significance level**. In our example, we have entered **Mean 1 – Mean 2 ≠ D**, the value of **0** as the **Hypothesized difference (D):**, and the value **5%** as the **Significance level**. Click **OK** to conduct the test of hypothesis.

Figure 9.12

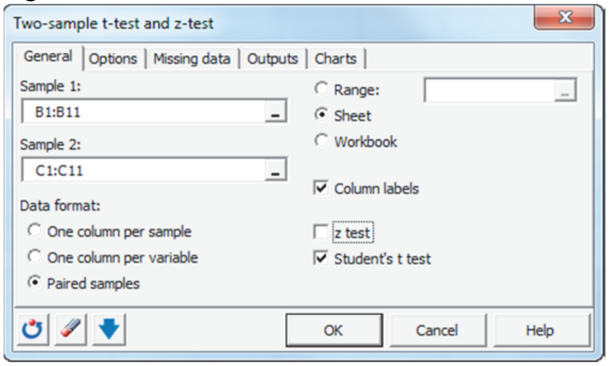
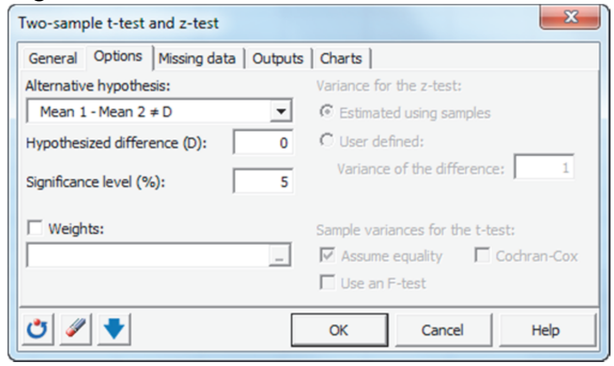


Figure 9.13



The XLSTAT output is shown in Figure 9.14.

Figure 9.14

t-test for two paired samples / Two-tailed test:

95% confidence interval on the difference between the means:

(89.0962, 710.9038)

Difference	400.0000
t (Observed value)	2.9104
|t| (Critical value)	2.2622
DF	9
p-value (Two-tailed)	0.0173
alpha	0.05

Test interpretation:

H0: The difference between the means is equal to 0.

Ha: The difference between the means is different from 0.

As the computed p-value is lower than the significance level alpha=0.05,

one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.

The risk to reject the null hypothesis H0 while it is true is lower than 1.73%.

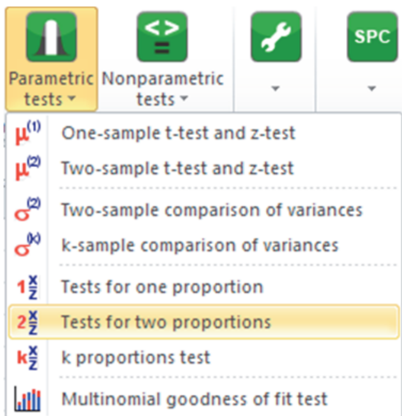
We see that the printout above contains both the confidence interval and test of hypothesis information that we desire. We compare this printout to the information shown in the text. We note that the test statistic shown in the printout ($z = 2.2622$) and the confidence interval given (89.0962, 710.9038) are identical to the values shown in the text.

We note that this example used two small samples in the calculations. The large-sample comparison would require us to check the z test button in Figure 9.12 and specify how to estimate the sample variances in Figure 9.13. Otherwise the large-sample work is identical to the small-sample work.

9.4 Comparing Two Population Proportions

As we saw in Chapters 7 and 8, we work with population proportions when the data we are collecting is qualitative in nature. Sometimes it is desirable to compare two populations. When random and independent samples are taken, the techniques that we use are similar to the confidence intervals and tests of hypotheses that we have previously looked at. **XLSTAT** uses the **Tests for two proportions** menu located within the **Parametric tests** option to compare two population means. Click on the **Parametric tests** option to access the **Tests for two proportions** menu (see Figure 9.15).

Figure 9.15



We illustrate how to use this technique with the following exercise:

Exercise 9.3: We use Example 9.6 found in the *Statistics* text.

Problem In the past decade, intensive antismoking campaigns have been sponsored by both federal and private agencies. Suppose the American Cancer Society randomly sampled 1,500 adults in 2000 and then sampled 1,750 adults in 2010 to determine whether there was evidence that the percentage of smokers had decreased. The results of the two surveys are shown in Table 9.2, where x_1 and x_2 represent the number of smokers in the 2000 and 2010 samples, respectively. Do these data indicate that the fraction of smokers decreased over this 10-year period? Use $\alpha = .05$.

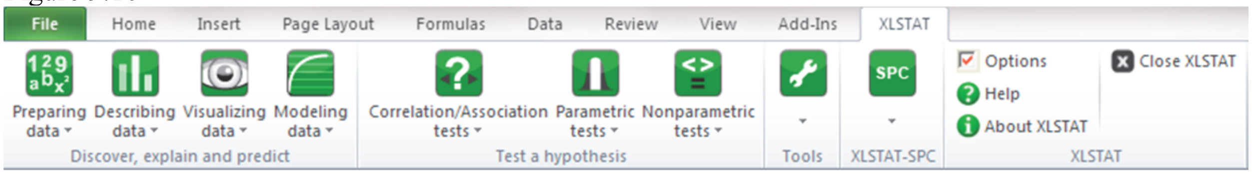
Table 9.2

2000	2010
$n_1 = 1,500$	$n_2 = 1750$
$x_1 = 555$	$x_2 = 578$

Solution:

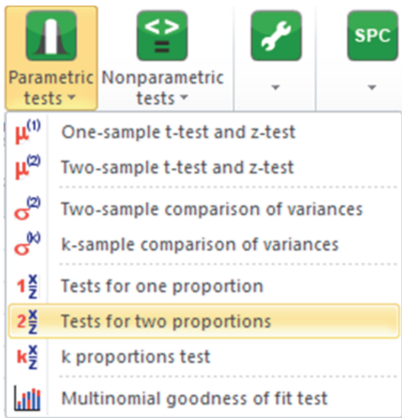
We solve Exercise 9.3 utilizing the **Tests for two proportions** menu presented in **XLSTAT**. To conduct the desired test of hypothesis, we click on the **XLSTAT** tab at the top of the **Excel** workbook to access the **XLSTAT** menus shown in Figure 9.16.

Figure 9.16



To conduct the desired test of hypothesis, we click on the **Parametric tests** menu and select the **Tests for two proportions** option shown in Figure 9.17.

Figure 9.17



This opens the **Tests for two proportions** menu shown in Figures 9.18-9.19. We need to first select how the data is presented in the problem. If the data is given in terms of the number of successes, we click on the **Frequency** button. If the data is given in terms of the proportion of successes, we would click on the **Proportion** button. In this problem, we are told that 555 of the 1,500 adults from the 2000 sample and 578 out of the 1,750 adults from the 2010 sample were smokers. We click on the **Frequency** button and enter a **Frequency 1:** of **555**, a **Sample size 1:** of **1500**, a **Frequency 2:** of **578**, and a **Sample size 2:** of **1750**.

Click on the **Options** tab (shown in Figure 9.19) to specify the **alternative hypothesis**, the **hypothesized difference (D)**, and the **significance level**. In our example, we have entered **Proportion 1 – Proportion 2 > D**, the value of **0** as the **Hypothesized difference (D):**, and the value **5%** as the **Significance level**. We also need to specify how the sample proportions will be estimated in the calculation of the test statistic and the confidence interval endpoints. When testing a hypothesized difference of 0, the second method shown in the menu is appropriate. For all other values of the hypothesized value, the first method shown in the menu would be appropriate. We click on the **pq(1/n1+1/n2)** button. Click **OK** to conduct the test of hypothesis.

Figure 9.18

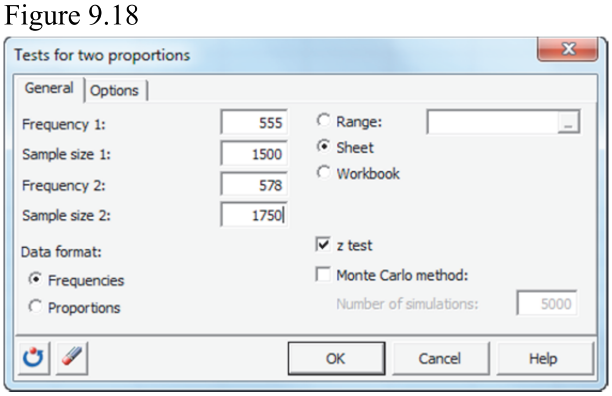
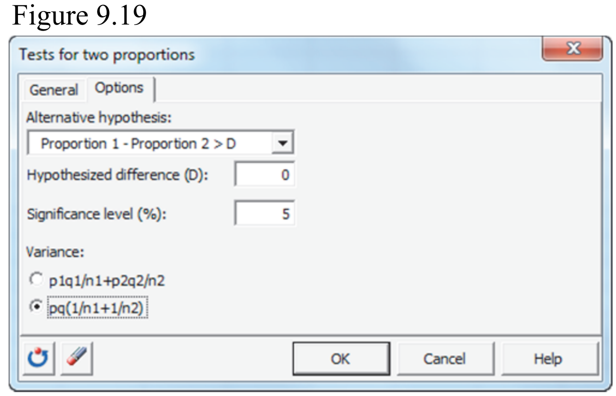


Figure 9.19



The XLSTAT output is shown in Figure 9.20.

Figure 9.20

z-test for two proportions / Upper-tailed test:

95% confidence interval on the difference between the proportions:

(0.0121, 1.0000)

Difference	0.0397
z (Observed value)	2.3685
z (Critical value)	1.6449
p-value (one-tailed)	0.0089
alpha	0.05

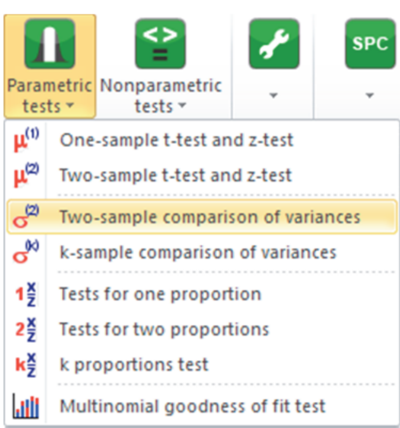
Test interpretation:
H0: The difference between the proportions is equal to 0.
Ha: The difference between the proportions is greater than 0.
As the computed p-value is lower than the significance level alpha=0.05,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 0.89%.

We see that the printout above contains both the confidence interval and test of hypothesis information that we desire. We compare this printout to the information shown in the text. We note that the test statistic and p-value shown in the printout are identical to the values shown on the MINITAB printout in the text.

9.5 Comparing Two Population Variances

When working with quantitative data, it is sometimes desired to compare the variation between two groups. The independent comparison of two population variances can be found in **XLSTAT** using the **Two-sample comparison of variances** menu located within the **Parametric tests** option to compare two population means. Click on the **Parametric tests** option to access the **Two-sample comparison of variances** menu (see Figure 9.21).

Figure 9.21



We illustrate how to use this technique with the following exercise:

Exercise 9.4: We use Example 9.10 found in the *Statistics* text.

Problem An Experimenter wants to compare the metabolic rates of white mice subjected to different drugs. The weights of the mice may affect their metabolic rates; thus, the experimenter wishes to obtain mice that are relatively homogeneous with respect to weight. Five hundred mice will be needed to complete the study. Currently, 13 mice from supplier 1 and another 18 mice from supplier 2 are available for comparison. The experimenter weights these mice and obtains the data shown in Table 9.3 (and the data file MICEWTS). Do these data provide sufficient evidence to indicate a difference in the variability of weights of mice obtained from the two suppliers? (Use $\alpha = .10$).

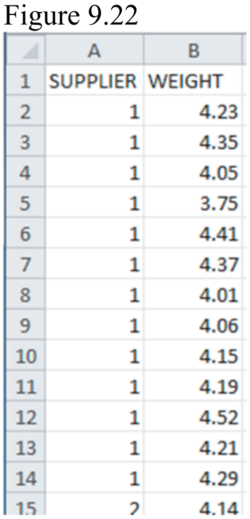
Table 9.3

Supplier 1					
4.23	4.35	4.05	3.75	4.41	4.37
4.01	4.06	4.15	4.19	4.52	4.21
4.29					
Supplier 2					
4.14	4.26	4.05	4.11	4.31	4.12
4.17	4.35	4.25	4.21	4.05	4.28
4.15	4.20	4.32	4.25	4.02	4.14

Solution:

We solve Exercise 9.4 utilizing the **Two-sample comparison of variances** menu presented in **XLSTAT**. **Open** the data file **MICEWTS** by following the directions found in the preface of this manual. If done correctly, the data should appear in a workbook similar to that shown in Figure 9.22.

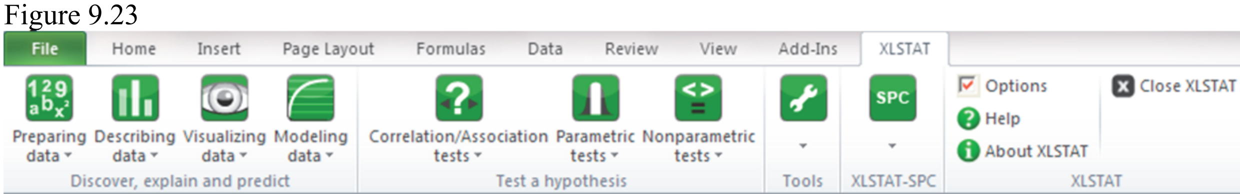
Figure 9.22



	A	B
1	SUPPLIER	WEIGHT
2	1	4.23
3	1	4.35
4	1	4.05
5	1	3.75
6	1	4.41
7	1	4.37
8	1	4.01
9	1	4.06
10	1	4.15
11	1	4.19
12	1	4.52
13	1	4.21
14	1	4.29
15	2	4.14

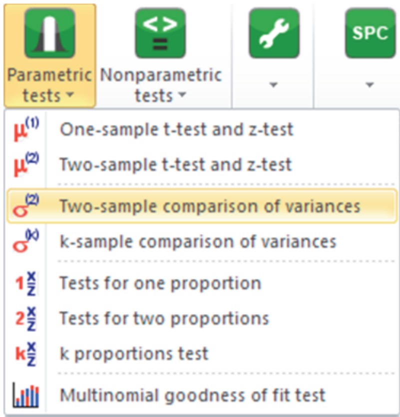
We solve Exercise 9.4 utilizing the **Two-sample comparison of variances** menu presented in **XLSTAT**. To conduct the desired test of hypothesis, we click on the **XLSTAT** tab at the top of the **Excel** workbook to access the **XLSTAT** menus shown in Figure 9.23.

Figure 9.23



To conduct the desired test of hypothesis, we click on the **Parametric tests** menu and select the **Two-sample comparison of variances** option shown in Figure 9.24.

Figure 9.24



This opens the **Tests for two proportions** menu shown in Figures 9.25-9.26. We need to first specify the location of the data that is to be analyzed. In our data set, the data is located in columns A and B, rows 2-32, with row 1 being the variable labels. We note that the data in column A represents the supplier and the data in column B represent the corresponding weight. We specify the column B data in the **Data:** box and the column A data in the **Sample identifiers:** box, and check the **Column labels** box to indicate the first row of data represents the variable names, **SUPPLIER** and **WEIGHT**. We also check the **One column per variable** button to indicate the way our data appear in the data set. To conduct the test we desire, we need to check the **Fisher's F-test** box in this menu.

Click on the **Options** tab (shown in Figure 9.26) to specify the **alternative hypothesis**, the **hypothesized ratio**, and the **significance level**. In our example, we have entered **Variance 1/Variance 2 ≠ R**, the value of **1** as the **Hypothesized ratio (R):**, and the value **10%** as the **Significance level**. Click **OK** to conduct the test of hypothesis.

Figure 9.25

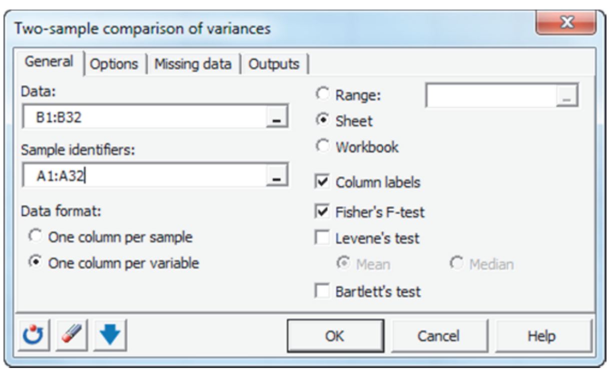
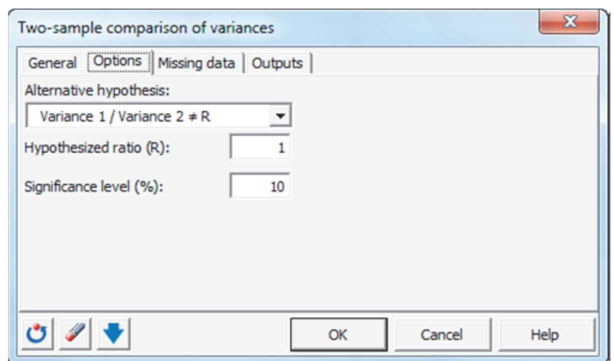


Figure 9.26



The XLSTAT output is shown in Figure 9.27.

Figure 9.27

Fisher's F-test / Two-tailed test:

90% confidence interval on the ratio of variances:
(1.7800, 10.9449)

Ratio	4.2375
F (Observed value)	4.2375
F (Critical value)	2.3807
DF1	12
DF2	17
p-value (Two-tailed)	0.0071
alpha	0.1

Test interpretation:
H0: The ratio between the variances is equal to 1.
Ha: The ratio between the variances is different from 1.
As the computed p-value is lower than the significance level alpha=0.1,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 0.71%.

We see that the printout above contains both the confidence interval and test of hypothesis information that we desire. We compare this printout to the information shown in the text. We note that the test statistic and p-value shown in the printout are identical to the values shown on the MINITAB printout in the text.

9.6 Technology Lab

The Technology Lab consists of problems for the student to practice the techniques presented in each lesson. Each problem is taken from the homework exercises within the *Statistics* text and includes an **Excel** data set (when applicable) that should be used to create the desired output. The completed output has been included with each problem so that the student can verify that he/she is generating the correct output.

1. **Lobster trap placement** Refer to the *Bulletin of Marine Science* (April, 2010) study of lobster trap placement. The variable of interest was the average distance separating traps – called *trap spacing* – deployed by teams of fishermen fishing for the red spiny lobster in Baja California Sur, Mexico. The trap spacing measurements (in meters) for a sample of 7 teams from the Bahia Tortugas (BT) fishing cooperative are repeated in the table (and in data file TRAPSPACE). In addition, trap spacing measurements for 8 teams from the Punta Abreojos (PA) fishing cooperative are listed.

    a. Find a 90% confidence interval for the difference in the mean trap spacing measurements of the two fishing cooperatives.

b. Test to determine if the variation in the mean trap spacing measurements of the two fishing cooperatives differs. Test using $\alpha = .10$

BT Cooperative:	93	99	105	94	82	70	86	
PA Cooperative	118	94	106	72	90	66	153	98

XLSTAT Output

90% confidence interval on the difference between the means:

(-29.5545 , 10.0188)

Difference	-9.7679
t (Observed value)	-0.8742
|t| (Critical value)	1.7709
DF	13
p-value (Two-tailed)	0.3979
alpha	0.1

Test interpretation:
H0: The difference between the means is equal to 0.
Ha: The difference between the means is different from 0.
As the computed p-value is greater than the significance level alpha=0.1,
one cannot reject the null hypothesis H0.
The risk to reject the null hypothesis H0 while it is true is 39.79%.

Fisher's F-test / Two-tailed test:
90% confidence interval on the ratio of variances:

(0.0466, 0.7583)

Ratio	0.1803
F (Observed value)	0.1803
F (Critical value)	3.8660
DF1	6
DF2	7
p-value (Two-tailed)	0.0532
alpha	0.1

Test interpretation:
H0: The ratio between the variances is equal to 1.
Ha: The ratio between the variances is different from 1.
As the computed p-value is lower than the significance level alpha=0.1,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 5.32%.

2. **Impact of red light cameras on car crashes** To combat red-light-running crashes – the phenomenon of a motorist entering an intersection after the traffic signal turns red and causing a crash – many states are adopting photo-red enforcement programs. In these programs, red light cameras installed at dangerous intersections photograph the license plates of vehicles that run the red light. How effective are photo-red enforcement programs ins reducing red-light-running crash incidents at intersections? The Virginia Department of Transportation (VDOT) conducted a comprehensive study of its newly adopted photo-red enforcement program and published the results in a June, 2007 report. In one portion of the study, the VDOT provided crash data both before and after installation of red light cameras at several intersections. The data (measured as the number of crashes caused by red light running per intersection per year) for 13 intersections in Fairfax County, VA are shown in the table (and in data file REDLIGHT). Conduct a test of hypothesis to determine if the average number of crashes at all photo-enforced intersections decreases with the installation of the cameras. Test at $\alpha = .05$.

Intersection	Before	After
1	3.6	1.36
2	0.27	0
3	0.29	0
4	4.55	1.79
5	2.6	2.04
6	2.29	3.14
7	2.4	2.72
8	0.73	0.24
9	3.15	1.57
10	3.21	0.43
11	0.88	0.28
12	1.35	1.09
13	7.35	4.92

XLSTAT Output

t-test for two paired samples / Upper-tailed test:

Difference	1.0069
t (Observed value)	3.0023
t (Critical value)	1.7823
DF	12
p-value (one-tailed)	0.0055
alpha	0.05

Test interpretation:
H0: The difference between the means is equal to 0.
Ha: The difference between the means is greater than 0.
As the computed p-value is lower than the significance level alpha=0.05,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 0.55%.

3. **Is steak your favorite barbeque food?** July is National Grilling Month in the United States. On July 1, 2008, the *Harris Poll #70* reported on a survey of Americans' grilling preferences. When asked about their favorite food prepared on a barbeque, 662 of 1,250 randomly sampled Democrats preferred steak, as compared to 586 of 930 randomly sampled Republicans.

    a. Construct a 95% confidence interval for the difference between the proportions of all Democrats and all Republicans who prefer steak as their favorite barbeque food.
    b. Test to determine if a difference exists between the proportions of all Democrats and all Republicans who prefer steak as their favorite barbeque food. Test at $\alpha = .05$.

XLSTAT Output

95% confidence interval on the difference between the proportions:

(-0.1425 , -0.0585)

Difference	-0.1005
z (Observed value)	-4.6915
z (Critical value)	1.9600
p-value (Two-tailed)	< 0.0001
alpha	0.05

Test interpretation:
H0: The difference between the proportions is equal to 0.
Ha: The difference between the proportions is different from 0.
As the computed p-value is lower than the significance level alpha=0.05,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 0.01%.