Chapter 14
Nonparametric Statistics

14.1 Introduction

Chapter 14 introduces the reader to the topic of nonparametric statistics and gives the reader several different examples of methods to analyze data using the nonparametric techniques. Many of these techniques are available within the **XLSTAT** software program. We note that the *A First Course in Statistics* text covers this material in chapters 6 and 7 of that text.

We illustrate the nonparametric procedures available in **XLSTAT** using the following examples from the text:

Excel Companion Exercise	**Page**	**Statistics Example**	**Excel File Name**
14.1	193	Example 14.2	DRUGS
14.2	196	Example 14.3	CRIMEPLAN
14.3	199	Example 14.4	HOSPBEDS
14.4	201	Example 14.5	REACTION2

14.2 The Wilcoxon Rank Sum Test for Comparing Two Independent Samples

The *Statistics* text offers the Wilcoxon Rank Sum technique for comparing two population means with independent samples. **XLSTAT** provides this test and the corresponding p-value that can be used to make conclusions without the need for the tables that are shown in the text. **XLSTAT** allows the user to perform this test of hypothesis when the data has been collected using two random, independent samples. We illustrate how to use the Wilcoxon Rank Sum test below.

Exercise 14.1: We use Example 14.2 found in the *Statistics* text.

Problem: Consider an experimental psychologist who wants to compare reaction times for adult males under the influence of drug A with reaction times for those under the influence of drug B. Experience has shown that the populations of reaction time measurements often possess probability distributions that are skewed to the right. Consequently, a *t*-test should not be used to compare the mean reaction times for the two drugs, because the normality assumption that is required for the *t*-test may not be valid.

Suppose the psychologist randomly assigns seven subjects to each of two groups, one group to receive drug A and the other to receive drug B. The reaction time for each subject is measured at the completion of the experiment. These data (with the exception of the measurement for one subject in group A who was eliminated from the experiment for personal reasons) are shown in Table 14.1 and are saved in the DRUGS data file.

Do the data provide sufficient evidence to indicate a shift in the probability distributions for drugs A and B – that is, that the probability distribution corresponding to drug A lies either to the right or to the left of the probability distribution corresponding to drug B? Test at the .05 level of significance.

Table 14.1

Drug A	Drug B
1.96	2.11
2.24	2.43
1.71	2.07
2.41	2.71
1.62	2.50
1.93	2.84
	2.88

Solution:

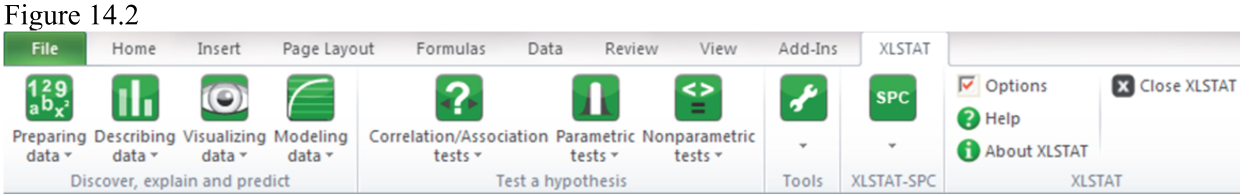
We solve Exercise 14.1 utilizing the **Comparison of two samples (Wilcoxon, Mann-Whitney,…)** option presented in the **Nonparametric tests** tab in **XLSTAT**. **Open** the data file **DRUGS** by following the directions found in the preface of this manual. If done correctly, the data should appear in a workbook similar to that shown in Figure 14.1.

Figure 14.1

	A	B
1	DRUG	REACTIME
2	A	1.96
3	A	2.24
4	A	1.71
5	A	2.41
6	A	1.62
7	A	1.93
8	B	2.11
9	B	2.43
10	B	2.07
11	B	2.71
12	B	2.5
13	B	2.84
14	B	2.88

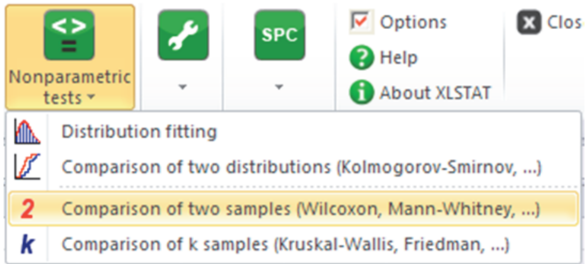
To conduct the desired analysis, we click on the **XLSTAT** tab at the top of the **Excel** workbook to access the **XLSTAT** menus shown in Figure 14.2.

Figure 14.2



To conduct the desired test, we click on the **Nonparametric tests** menu and select the **Comparison of two samples (Wilcoxon, Mann-Whitney,…)** option shown in Figure 14.3.

Figure 14.3



This opens the **Comparison of two samples (Wilcoxon, Mann-Whitney,…)** menu shown in Figures 14.4 and 14.5. We need to first specify the location of the data that is to be analyzed. In our data set, the reaction times are located in column B, rows 2 – 14, with row 1 being the variable label. The drug types are located in column A, rows 2-14. We specify the column **A** data in the **Sample identifiers** box and the column **B** data in the **Data** box. We make sure that the **One column per variable** button is selected and that the **Column labels** box is checked and we check the **Mann-Whitney test** box to conduct the desired test.

We next click on the **Options** tab to specify the type of test we want to conduct. We specify the two-direction test by selecting **Sample 1 – Sample 2 ≠ D** and by entering the value of **D = 0.** We enter the **Significance Level (%)** of **5**% and click **OK** to create the desired output.

Figure 14.4

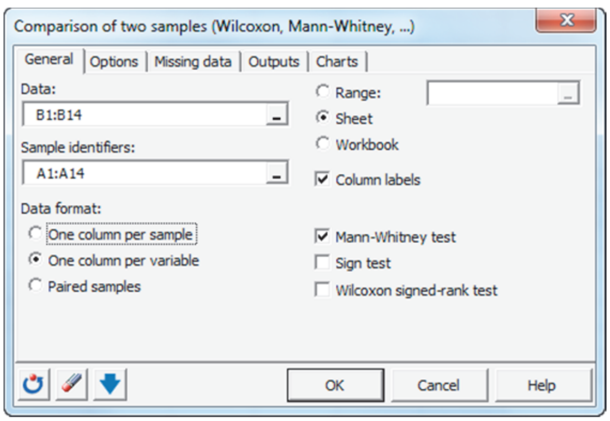


Figure 14.5

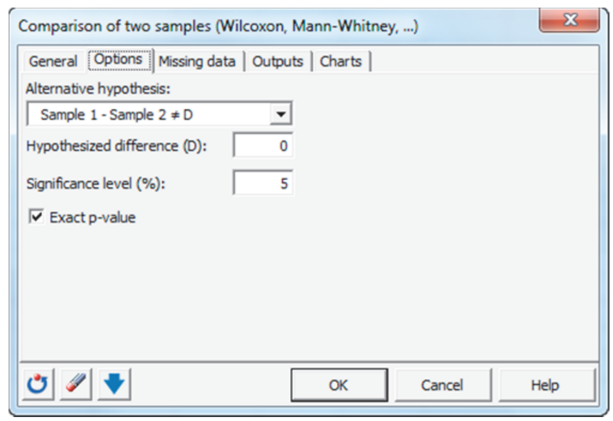


Figure 14.6

Mann-Whitney test / Two-tailed test:

U	4.0000
Expected value	21.0000
Variance (U)	49.0000
p-value (Two-tailed)	0.0140
alpha	0.05

The p-value is computed using an exact method.

Test interpretation:
H0: The difference of location between the samples is equal to 0.
Ha: The difference of location between the samples is different from 0.
As the computed p-value is lower than the significance level alpha=0.05,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 1.40%.

We note that the output given in **XLSTAT** contains the two-tailed p-value of p = .0140. We compare this to the one-tailed p-value shown in the text (p = 0.007) to see that the results are identical.

14.3 The Wilcoxon Signed Rank Test for Dependent Sampling

The *Statistics* text offers the Wilcoxon Signed Rank Test for comparing two population means with dependent samples. **XLSTAT** allows the user to perform this test of hypothesis when the data has been collected using a paired difference sampling design. We illustrate how to use the Wilcoxon Signed Rank Test below.

Exercise 14.2: We use Example 14.3 found in the *Statistics* text.

Problem: Suppose the police commissioner in a small community must choose between two plans for patrolling the town's streets. Plan A, the less expensive plan, uses voluntary citizen groups to patrol certain high-risk neighborhoods. In contrast, plan B would utilize police patrols. As an aid in reaching a decision, both plans are examined by 10 trained criminologists, each of whom is asked to rate the plans on a scale from 1 to 10. (High ratings imply a more effective crime prevention plan.) The city will adopt plan B (and hire extra police) only if the data provide sufficient evidence that criminologists tend to rate plan B more effective than plan A. The results of the survey are shown in Table 14.2 (and saved in the data file CRIMEPLAN). Do the data provide evidence at the $\alpha = .05$ level that the distribution of ratings for plan B lies above that for plan A? Use the Wilcoxon signed rand test to answer the question.

Table 14.2

EXPERT	A	B
1	9	7
2	5	4
3	8	8
4	8	9
5	6	3
6	10	6
7	9	8
8	8	10
9	4	9
10	9	5

Solution:

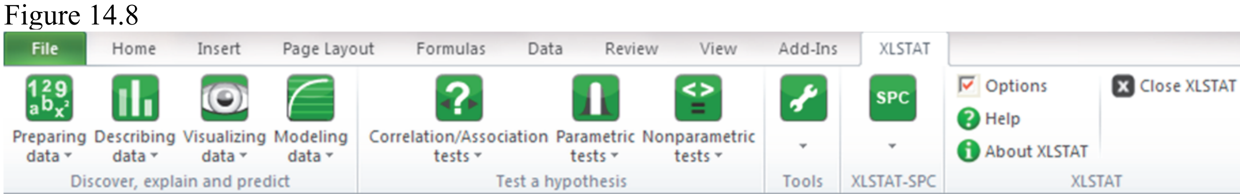
We solve Exercise 14.2 utilizing the **Comparison of two samples (Wilcoxon, Mann-Whitney,…)** option presented in the **Nonparametric tests** tab in **XLSTAT**. **Open** the data file **CRIMEPLAN** by following the directions found in the preface of this manual. If done correctly, the data should appear in a workbook similar to that shown in Figure 14.7.

Figure 14.7

	A	B	C
1	EXPERT	A	B
2	1	9	7
3	2	5	4
4	3	8	8
5	4	8	9
6	5	6	3
7	6	10	6
8	7	9	8
9	8	8	10
10	9	4	9
11	10	9	5

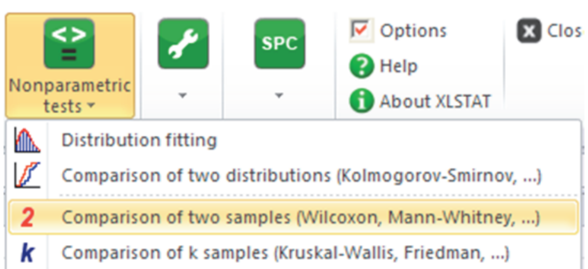
To conduct the desired analysis, we click on the **XLSTAT** tab at the top of the **Excel** workbook to access the **XLSTAT** menus shown in Figure 14.8.

Figure 14.8



To conduct the desired test, we click on the **Nonparametric tests** menu and select the **Comparison of two samples (Wilcoxon, Mann-Whitney,…)** option shown in Figure 14.9.

Figure 14.9



This opens the **Comparison of two samples (Wilcoxon, Mann-Whitney,…)** menu shown in Figures 14.10 and 14.11. We need to first specify that the data is formatted in paired samples by selecting the **Paired samples** button. We then select the location of the data that is to be analyzed. In our data set, **Sample 1** ratings are located in column B, rows 2 – 11, with row 1 being the variable label. **Sample 2** ratings are located in column C, rows 2 – 11, with row 1 being the variable label. We make sure that the **Column labels** box is checked and we check the **Wilcoxon signed-rank test** box to conduct the desired test.

We next click on the **Options** tab to specify the type of test we want to conduct. We specify the one-direction test by selecting **Sample 1 – Sample 2 > D** and by entering the value of **D = 0.** We enter the **Significance Level (%)** of **5**% and check the **Hollander & Wolfe** button as our method of handling tied rankings. Click **OK** to create the desired output.

Figure 14.10

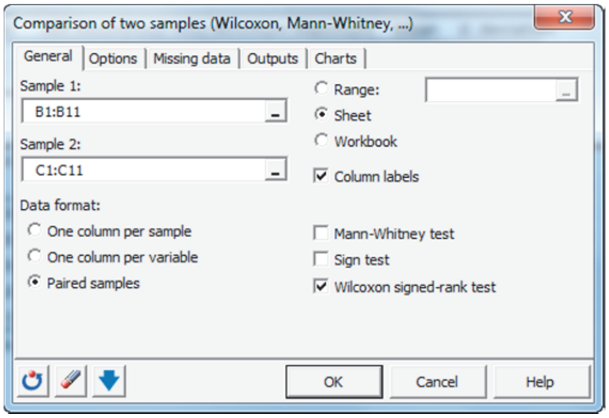


Figure 14.11

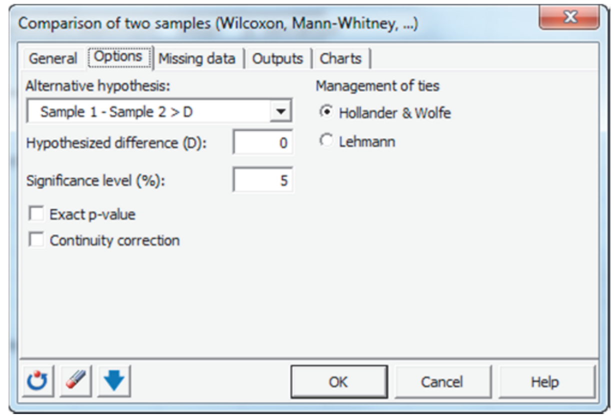


Figure 14.12

Wilcoxon signed-rank test / Upper-tailed test:

V	29.5000
Expected value	22.5000
Variance (V)	70.5000
p-value (one-tailed)	0.2022
alpha	0.05

An approximation has been used to compute the p-value.

Test interpretation:
H0: The two samples follow the same distribution.
Ha: The distribution of the first sample is shifted to the right of the distribution of the second sample.
As the computed p-value is greater than the significance level alpha=0.05,
one cannot reject the null hypothesis H0.
The risk to reject the null hypothesis H0 while it is true is 20.22%.

Ties have been detected in the data and the appropriate corrections have been applied.

We note that the output given in XLSTAT contains the one-tailed p-value of p = .2022. We compare this to the one-tailed p-value shown in the text (p = 0.202) to see that the results are identical.

Section 14.4: The Kruskal-Wallis H-Test for a Completely Randomized Design

The *Statistics* text offers the Kruskal-Wallis H-Test for comparing three or means collected using a completely randomized sampling design. XLSTAT allows the user to perform this test of hypothesis utilizing the **Kruskal Wallis** option form the **Nonparametric Tests** menu. We illustrate how to use the Kruskal-Wallis Test in the next example.

Exercise 14.3: We use Example 14.4 found in the *Statistics* text.

Problem: Consider the data in Table 14.3 (and saved in the data file HOSPBEDS). Recall that a health administrator wants to compare the unoccupied bed space of the three hospitals. Apply the Kruskal-Wallis H-test to the data. What conclusions can you draw? Test using $\alpha = .05$.

Table 14.3

Hospital 1	Hospital 2	Hospital 3
6	34	13
38	28	35
3	42	19
17	13	4
11	40	29
30	31	0
15	9	7
16	32	33
25	39	18
5	27	24

Solution:

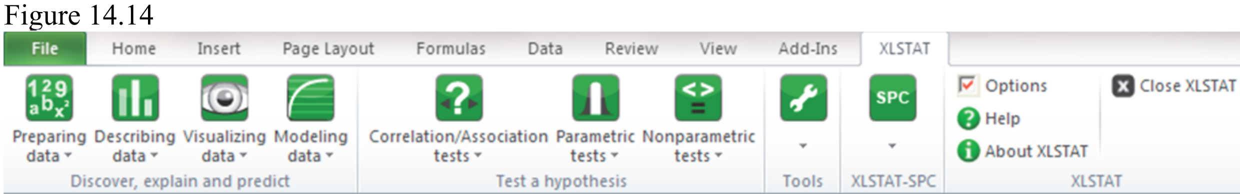
We solve Exercise 14.3 utilizing the **Comparison of k samples (Kruskal-Wallis, Friedman,…)** option presented in the **Nonparametric tests** tab in **XLSTAT**. **Open** the data file **HOSPBEDS** by following the directions found in the preface of this manual. If done correctly, the data should appear in a workbook similar to that shown in Figure 14.13.

Figure 14.13

	A	B
1	Beds	Hospital
2	6	1
3	38	1
4	3	1
5	17	1
6	11	1
7	30	1
8	15	1
9	16	1
10	25	1
11	5	1
12	34	2
13	28	2
14	42	2

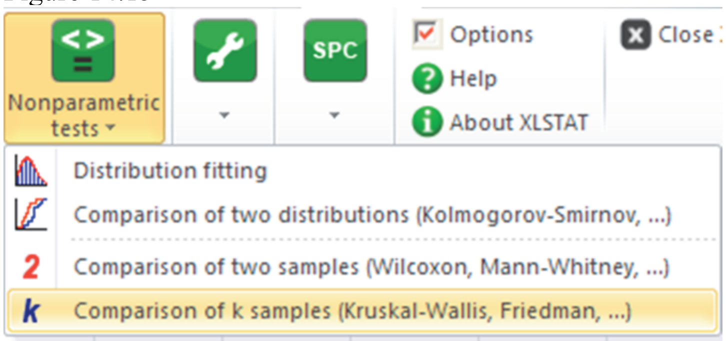
To conduct the desired analysis, we click on the **XLSTAT** tab at the top of the **Excel** workbook to access the **XLSTAT** menus shown in Figure 14.14.

Figure 14.14



To conduct the desired test, we click on the **Nonparametric tests** menu and select the **Comparison of k samples (Kruskal-Wallis, Friedman,…)** option shown in Figure 14.15.

Figure 14.15



This opens the **Comparison of k samples (Kruskal-Wallis, Friedman,…)** menu shown in Figures 14.16 and 14.17. We need to first specify the location of the data that is to be analyzed. In our data set, the number of unoccupied beds data is located in column B, rows 2 – 31, with row 1 being the variable label. The Hospital identifier is located in column A, rows 2-31. We specify the column **A** data in the **Sample identifiers** box and the column **B** data in the **Data** box. We make sure that the **One column per variable** button is selected and that the **Column labels** box is checked and we check the **Kruskal-Wallis test** box to conduct the desired test.

We next click on the **Options** tab to specify the type of test we want to conduct. We specify the **Significance Level (%)** of **5**%, check the **Asymptotic p-value** box and click **OK** to create the desired output.

Figure 14.16

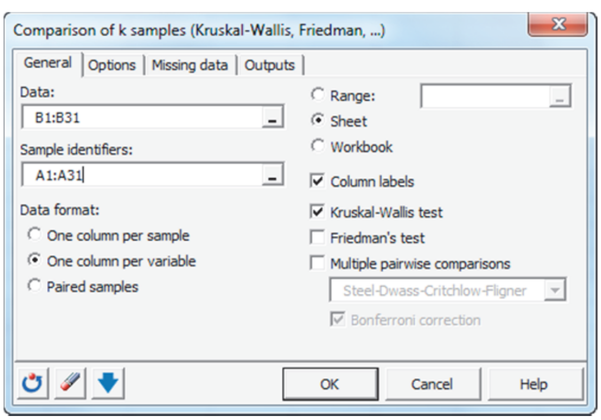


Figure 14.17

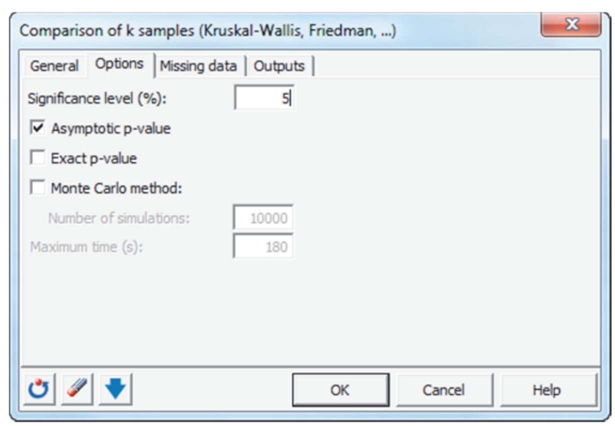


Figure 14.18

Kruskal-Wallis test (BEDS):

K (Observed value)	6.0988
K (Critical value)	5.9915
DF	2
p-value (Two-tailed)	0.0474
alpha	0.05

An approximation has been used to compute the p-value.

Test interpretation:
H0: The samples come from the same population.
Ha: The samples do not come from the same population.
As the computed p-value is lower than the significance level alpha=0.05,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 4.74%.

Ties have been detected in the data and the appropriate corrections have been applied.

We note that the output given in **XLSTAT** contains the two-tailed p-value of p = .0474. We compare this to the p-value shown in the text (p = 0.047) to see that the results are identical.

Section 14.5: The Friedman F_r-Test for a Randomized Block Design

The *Statistics* text offers the Friedman F_r-Test for comparing three or means collected using a randomized block sampling design. XLSTAT allows the user to perform this test of hypothesis utilizing the Friedman test option form within the **Nonparametric tests** menu. We illustrate how to use the Friedman F_r-Test below.

Exercise 14.4: We use Example 14.5 found in the *Statistics* text.

Problem: Consider the data in Table 14.4 and saved in the data file REACTION2. Recall that a pharmaceutical firm wants to compare the reaction times of subjects under the influence of three different drugs that it produces. Apply the Friedman F_r-test to the data. What conclusions can you draw? Test using $\alpha = .05$.

Table 14.4

Subject	Drug A	Drug B	Drug C
1	1.21	1.48	1.56
2	1.63	1.85	2.01
3	1.42	2.06	1.70
4	2.43	1.98	2.64
5	1.16	1.27	1.48
6	1.94	2.44	2.81

Solution:

We solve Exercise 14.4 utilizing the **Comparison of k samples (Kruskal-Wallis, Friedman,…)** option presented in the **Nonparametric tests** tab in **XLSTAT**. **Open** the data file **REACTION2** by following the directions found in the preface of this manual. If done correctly, the data should appear in a workbook similar to that shown in Figure 14.19. To more easily conduct the analysis desired, we reformat the data set to look like the data shown in Figure 4.20.

Figure 14.19

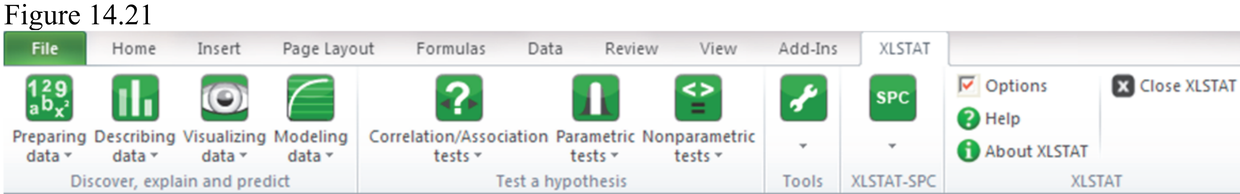
	A	B	C
1	SUBJECT	DRUG	TIME
2	1	A	1.21
3	2	A	1.63
4	3	A	1.42
5	4	A	2.43
6	5	A	1.16
7	6	A	1.94
8	1	B	1.48
9	2	B	1.85
10	3	B	2.06
11	4	B	1.98
12	5	B	1.27
13	6	B	2.44
14	1	C	1.56
15	2	C	2.01
16	3	C	1.7
17	4	C	2.64
18	5	C	1.48
19	6	C	2.81

Figure 4.20

	A	B	C	D
1	SUBJECT	A	B	C
2	1	1.21	1.48	1.56
3	2	1.63	1.85	2.01
4	3	1.42	2.06	1.7
5	4	2.43	1.98	2.64
6	5	1.16	1.27	1.48
7	6	1.94	2.44	2.81

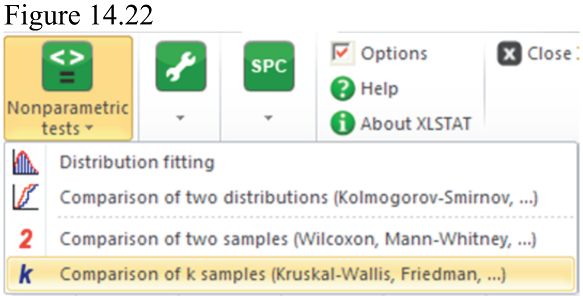
To conduct the desired analysis, we click on the **XLSTAT** tab at the top of the **Excel** workbook to access the **XLSTAT** menus shown in Figure 14.21.

Figure 14.21



To conduct the desired test, we click on the **Nonparametric tests** menu and select the **Comparison of k samples (Kruskal-Wallis, Friedman,…)** option shown in Figure 14.22.

Figure 14.22



This opens the **Comparison of k samples (Kruskal-Wallis, Friedman,…)** menu shown in Figures 14.23 and 14.24. We need to first specify that the data are collected using a randomized block design by checking the **Paired samples** box.. In our data set, the reaction times are located in columns B - D, rows 2 – 7, with row 1 being the variable label. We specify the data in the **Samples** box and make sure that the **Column labels** box is checked. We check the **Friedman's test** box to conduct the desired test.

We next click on the **Options** tab to specify the type of test we want to conduct. We specify the **Significance Level (%)** of **5**% and click **OK** to create the desired output.

Figure 14.23

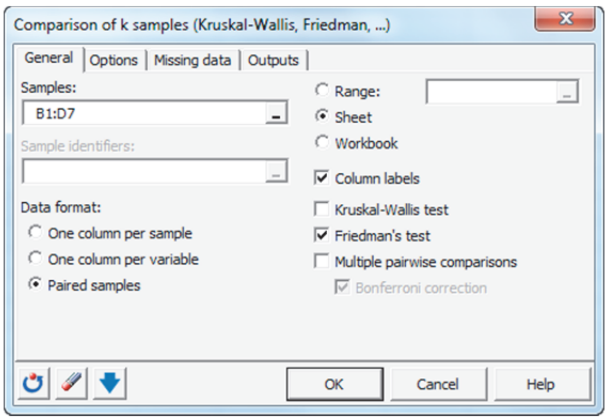


Figure 14.24

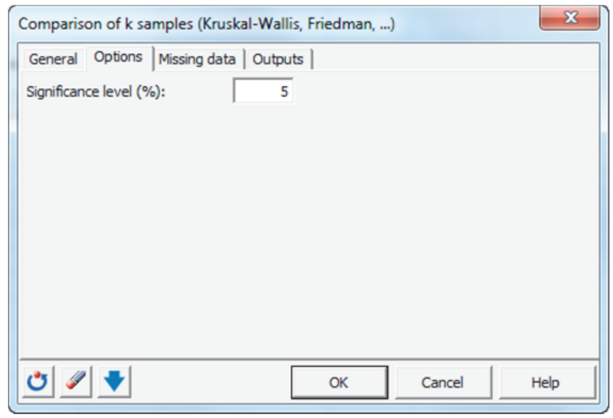


Figure 14.25

Friedman's test:

Q (Observed value)	8.3333
Q (Critical value)	5.9915
DF	2
p-value (Two-tailed)	0.0155
alpha	0.05

Test interpretation:
H0: The samples come from the same population.
Ha: The samples do not come from the same population.
As the computed p-value is lower than the significance level alpha=0.05,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 1.55%.

We note that the output given in XLSTAT contains the two-tailed p-value of p = .0155. We compare this to the p-value shown in the text (p = 0.016) to see that the results are identical.

14.6 Technology Lab

The Technology Lab consists of problems for the student to practice the techniques presented in each lesson. Each problem is taken from the homework exercises within the *Statistics* text and includes an **Excel** data set (when applicable) that should be used to create the desired output. The completed output has been included with each problem so that the student can verify that he/she is generating the correct output.

1. **Is honey a cough remedy? Refer to the Archives of Pediatrics and Adolescent Medicine** (Dec., 2007) study of honey as a children's cough remedy. Recall that 70 children who were ill with an upper respiratory tract were given either a dosage of dextromethorphan (DM) – an over-the-counter cough medicine – or a similar dose of honey. Parents then rated their children's cough symptoms and the improvement in total cough symptoms score was determined for each child. The data (improvement scores) are reproduced in the accompanying table and saved in the HONEYCOUGH data file. The researchers concluded that "honey may be a preferable treatment for the cough and sleep difficulty associated with childhood upper respiratory tract infection." Use the nonparametric method presented in this chapter to analyze the data (use α = .05). Do you agree with the researchers?

   Honey Dosage: 12 11 15 11 10 13 10 4 15 16 9 14 10 6 10 8 11 12 12 8 12 9 11 15 10 15 9 13 8 12 10 8 9 5 12
   DM Dosage: 4 6 9 4 7 7 7 9 12 10 11 6 3 4 9 12 7 6 8 12 12 4 12 13 7 10 13 9 4 4 10 15 9

   XLSTAT Output

   Mann-Whitney test / Upper-tailed test:

U	810.5000
Expected value	577.5000
Variance (U)	6563.1683
p-value (one-tailed)	0.0020
alpha	0.05

   An approximation has been used to compute the p-value.

   Test interpretation:
   H0: The difference of location between the samples is equal to 0.
   Ha: The difference of location between the samples is greater than 0.
   As the computed p-value is lower than the significance level alpha=0.05,
   one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
   The risk to reject the null hypothesis H0 while it is true is lower than 0.20%.

2. Agent Orange – the code name for a herbicide developed for the U.S. Armed Forces in the 1960's – was found to be extremely contaminated with TCDD, or dioxin. During the Vietnam War, and estimated 19 million gallons of Agent Orange was used to destroy the dense plant and tree cover of the Asian jungle. As a result of this exposure, many Vietnam veterans have dangerously high levels of TCDD in their blood and adipose (fatty) tissue. A study published in *Chemosphere* (Vol. 20, 1990) reported on the TCDD levels of 20 Massachusetts Vietnam vets who were possibly exposed to Agent Orange. The TCDD amounts (measured in parts per trillion) in both plasma and fat tissue of the 20 vets are listed in the accompanying table and saved in the TCDD data file.

   Medical researchers also are interested in comparing the TCDD levels in fat tissue and plasma for Vietnam veterans. Specifically, they want to determine if the distribution of TCDD levels in fat is shifted above or below the distribution of TCDD levels in plasma. Conduct this analysis (at $\alpha = .05$) and make the appropriate inference.

Vet	Fat	Plasma
1	4.9	2.5
2	6.9	3.5
3	10.0	6.8
4	4.4	4.7
5	4.6	4.6
6	1.1	1.8
7	2.3	2.5
8	5.9	3.1
9	7.0	3.1
10	5.5	3.0
11	7.0	6.9
12	1.4	1.6
13	11.0	20.0
14	2.5	4.1
15	4.4	2.1
16	4.2	1.8
17	41.0	36.0
18	2.9	3.3
19	7.7	7.2
20	2.5	2.0

XLSTAT Output

Wilcoxon signed-rank test / Two-tailed test:

V	140.0000
Expected value	95.0000
Variance (V)	617.1250
p-value (Two-tailed)	0.0701
alpha	0.05

An approximation has been used to compute the p-value.

Test interpretation:
H0: The two samples follow the same distribution.
Ha: The distributions of the two samples are different.
As the computed p-value is greater than the significance level alpha=0.05, one cannot reject the null hypothesis HO.
The risk to reject the null hypothesis H0 while it is true is 7.01%.

3. **Commercial eggs produced from different housing systems.** Refer to the Food Chemistry (Vol. 106, 2008) study of commercial eggs produced from different housing systems for chickens. Recall that the four housing systems investigated were (1) cage, (2) barn, (3) free range, and (4) organic. Twenty-eight commercial grade A eggs were randomly selected from supermarkets – 10 of which were produced in cages, 6 in barns, 6 with free range, and 6 organic. A number of quantitative characteristics were measured for each egg, including penetration strength (Newtons). The data (simulated from summary statistics provided in the journal article are shown below and are saved in the EGGS data file.

Cage	36.9	39.2	40.2	33.0	39.0	36.6	37.5	38.1	37.8	34.9
Free	31.5	39.7	37.8	33.5	39.9	40.6				
Barn	40.0	37.6	39.6	40.3	38.3	40.2				
Organic	34.5	36.8	32.6	38.5	40.2	33.2				

Is there sufficient evidence to infer that the strength distributions for the four housing systems differ? Use $\alpha = .05$.

XLSTAT Output

Kruskal-Wallis test (STRENGTH):

K (Observed value)	5.0029
K (Critical value)	7.8147
DF	3
p-value (Two-tailed)	0.1716
alpha	0.05

An approximation has been used to compute the p-value.

Test interpretation:
H0: The samples come from the same population.
Ha: The samples do not come from the same population.
As the computed p-value is greater than the significance level alpha=0.05,
one cannot reject the null hypothesis H0.
The risk to reject the null hypothesis H0 while it is true is 17.16%.

4. **Stress in cows prior to slaughter.** Refer to the *Applied Animal Behaviour Science* (June, 2010) study of stress in cows prior to slaughter. In the experiment, recall that the heart rate (beats per minute) of a cow was measured at four different pre-slaughter phases – (1) first phase of visual contact with pen mates, (2) initial isolation from pen mates for prepping, (3) restoration of visual contact with pen mates, and (4) first contact with human prior to slaughter. Thus, a randomized block design was employed. The simulated data for eight cows are reproduced on the next page and saved in the COWSTRESS data file. Consider applying the nonparametric Friedman test to determine whether the heart rate distributions differ for cows in the four pre-slaughter phases. Test at $\alpha = .05$.

	Phase			
COW	1	2	3	4
1	124	124	109	107
2	100	98	98	99
3	103	98	100	106
4	94	91	98	95
5	122	109	114	115
6	103	92	100	106
7	98	80	99	103
8	120	84	107	110

XLSTAT Output

Friedman's test:

Q (Observed value)	10.6538
Q (Critical value)	7.8147
DF	3
p-value (Two-tailed)	0.0138
alpha	0.05

Test interpretation:
H0: The samples come from the same population.
Ha: The samples do not come from the same population.
As the computed p-value is lower than the significance level alpha=0.05,
one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.
The risk to reject the null hypothesis H0 while it is true is lower than 1.38%.