

Choose independently two numbers  $B$  and  $C$  at random from the interval  $[0, 1]$  with uniform density. Prove that  $B$  and  $C$  are proper probability distributions. Note that the point  $(B, C)$  is then chosen at random in the unit square.

Find the probability that

(a)  $B + C < 1/2$ .

(b)  $BC < 1/2$ .

(c)  $|B - C| < 1/2$ .

(d)  $\max\{B, C\} < 1/2$ .

(e)  $\min\{B, C\} < 1/2$ .