Chapter 6
Sampling Distributions

6.1 Introduction

Chapter 6 introduces the reader to the concept of sampling distributions. A general discussion of what sampling distributions are is given in the first section of the chapter. Section two discusses some properties common to all sampling distributions. Finally, the last section of the chapter deals specifically with the sampling distribution of the sample mean. This sampling distribution, in certain situations, can be modeled using the normal distribution discussed in Chapter 5. The **Excel** normal distribution functions discussed in Chapter 5 can be used when working with the sampling distribution of the sample mean found in Chapter 6. We demonstrate how to use these functions below.

We note here that the sampling distribution material covered in *A First Course in Statistics* appears in Chapter 4 of that text.

The following example from *Statistics* is solved with **Excel** in this chapter:

Excel Companion Exercise	**Page**	**Statistics Text**
6.1	91	Example 6.7

6.2 Calculating Probabilities Using the Sampling Distribution of $\bar{x}$

The Central Limit Theorem guarantees that for large n, the sampling distribution of the sample mean possesses an approximate normal sampling distribution. In order to calculate probabilities for these sampling distributions, we must utilize the normal probability distribution functions that we discussed in Section 5.2 in the preceding chapter. We illustrate with the following example.

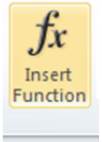
Exercise 6.1: We use Example 6.7 from the *Statistics* text.

Problem Suppose we have selected a random sample of n = 36 observations from a population with mean equal to 80 and standard deviation equal to 6. It is known that the population is not extremely skewed. Find the probability that the sample mean will be larger than 82.

Solution:

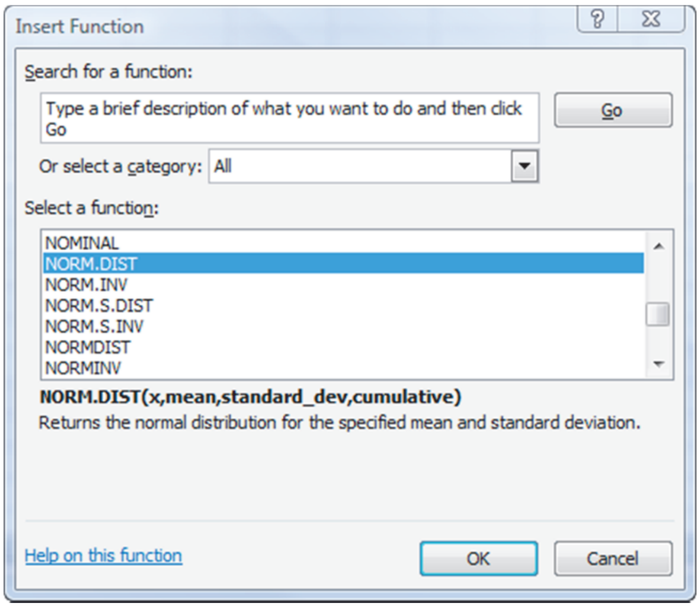
We first identify that we are working with the normal distribution with a mean of $\mu_{\bar{x}} = 80$ and a standard deviation of $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 6/\sqrt{36} = 1$. We are asked to determine the probability of selecting a value from this normal distribution that falls larger than the value 82. The **NORM.DIST** function is appropriate to use for this problem. To use this function within **Excel**, we begin by opening up **Excel** and placing the cursor on any cell in the blank worksheet. We click on the **Formulas** tab and then select the **Insert Function** icon shown in Figure 6.1.

Figure 6.1



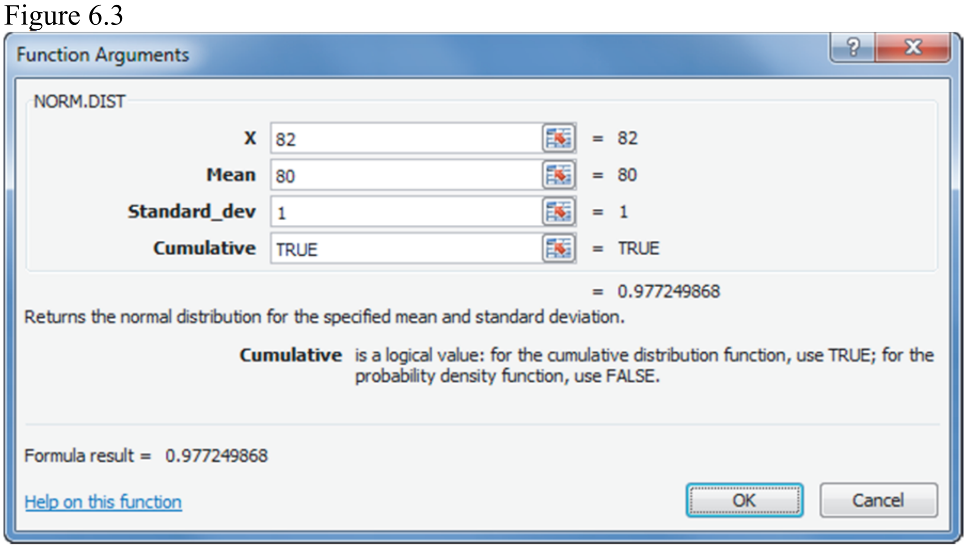
This opens the **Insert Function** menu shown in Figure 6.2. Click on the arrow to select that **All** categories are being used and scroll down until you reach the **NORM.DIST** function. **Highlight** the **NORM.DIST** option and click **OK**.

Figure 6.2



The **NORMDIST** function requires the user to identify values of the mean (**Mean=80**) and standard deviation (**Standard_dev=1**) of the normal distribution that we are working with. We also need to identify the value (**X=82**) in the distribution that we want probabilities for. Lastly, we need to specify that we want to work with cumulative probabilities by entering **TRUE** in the **Cumulative** box in the menu. Figure 6.3 shows the completed function.

Figure 6.3



We note that the probability is shown in Figure 6.3 and then placed in our worksheet cell once we click OK. That probability is given as 0.97725. This cumulative probability can be used to find the probability we desire as follows:

$$P(X > 82) = 1 - P(X \leq 82) = 1 - 0.97725 = .02275$$

Compare this probability to the one found in the text.

6.3 Technology Lab

The Technology Lab consists of problems for the student to practice the techniques presented in each lesson. Each problem is taken from the homework exercises within the *Statistics* text and includes an **Excel** data set (when applicable) that should be used to create the desired output. The completed output has been included with each problem so that the student can verify that he/she is generating the correct output.

1. **Shell lengths of sea turtles**. Refer to the *Aquatic Biology* (Vol. 9, 2010) study of green sea turtles inhabiting the Grand Cayman South Sound lagoon. Research shows that the curved carapace (shell) lengths of these turtles has a distribution with mean μ = 50 cm and standard deviation σ = 10 cm. In the study, n = 76 green sea turtles were captured from the lagoon; the mean shell length for the sample was $\bar{x}$ = 55.5 cm. How likely is it to observe a sample mean of 55.5 cm or larger?

   Excel Output

	A	B
1	Question	Probability
2	1	0.00000081423

2. **Tomato as a taste modifier.** Miraculin is a protein naturally produced in a rare tropical fruit that can convert a sour taste into a sweet taste. Refer to the *Plant Science* (May, 2010) investigation of the ability of a hybrid tomato plant to produce miraculin. Recall that the amount x of miraculin produced in the plant had a mean of 105.3 micrograms per gram of fresh weight with a standard deviation of 8.0. Consider a random sample of n = 64 hybrid tomato plants and let $\bar{x}$ represent the sample mean amount of miraculin produced. Would you expect to observe a value of $\bar{x}$ less than 103 micro-grams per gram of fresh weight?

   Excel Output

	A	B
1	Question	Probability
2	2	0.01072492425