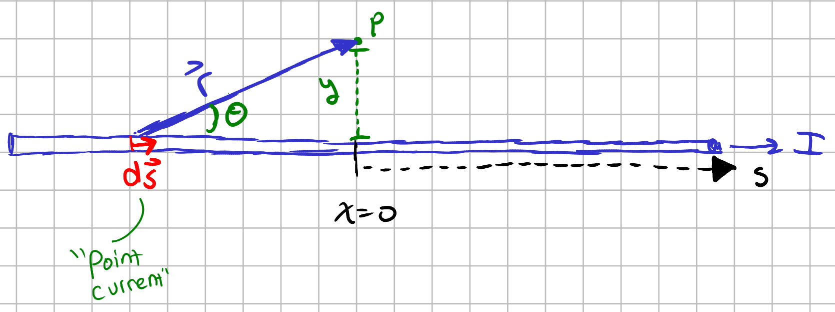
1. Use the Biot-savart law to find the B field a distance y from a long straight wire



$$dB = \frac{\mu_0}{4\pi}\frac{I\, d\vec{s}\times\hat{r}}{r^2}$$

$d\vec{s}$ Points in x-direction, So changing to dx

$$B = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{d\vec{x}\times\hat{r}}{r^2}$$

integrate over infinitely long wire

- due to Right Hand Rule, $\vec{B}$ is Pointing out of Page
- $d\vec{x}\times\hat{r} = |dx||\hat{r}|\sin\theta = dx\sin\theta$

$$= \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{dx\sin\theta}{r^2}$$

- from SOH CAH TOA: $\sin\theta = \frac{opp}{hyp} = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$

$$= \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{dx}{r^2}\cdot\frac{y}{r}$$

- y is constant

$$= \frac{\mu_0 I y}{4\pi}\int_{-\infty}^{\infty}\frac{dx}{(x^2+y^2)^{3/2}}$$

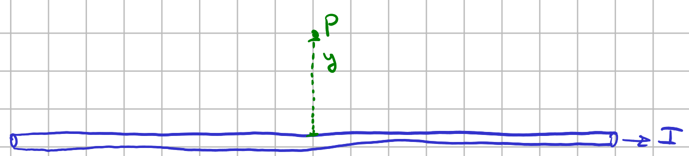
- Physics is done, now we just integrate
- use a table: $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$

$$= \frac{\mu_0 I y}{4\pi}\left[\frac{x}{y^2\sqrt{x^2+y^2}}\right]\Bigg|_{-\infty}^{\infty}$$

- When $x >> y$, $\frac{x}{y^2\sqrt{x^2+y^2}} \Rightarrow \frac{x}{y^2\sqrt{x^2}} = \frac{1}{y^2}$

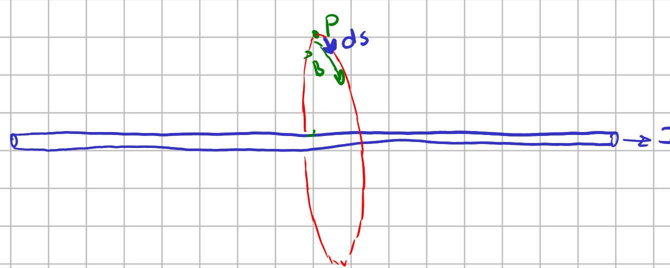
$$= \frac{\mu_0 I y}{4\pi}\left[\frac{1}{y^2} - \left(-\frac{1}{y^2}\right)\right] = \frac{\mu_0 I y}{4\pi}\left(\frac{2}{y^2}\right) \Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{2\pi y}\hat{z}}$$

2. Use Ampere's law to find the B field a distance y from a long straight wire



use $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$

- $\oint$ is over a Path we define
- Pick a Path such that $\vec{B}$ is constant (ring)



$$\oint \vec{B}\cdot ds = B\oint ds$$

- $\vec{B}$ is constant

$$= B\cdot s$$

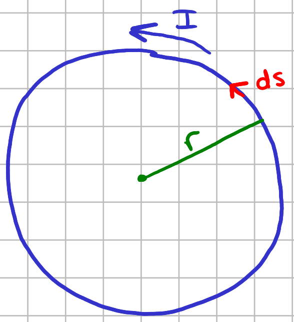
- S is length of the Path
- aka Circumference of circle

$$= B\cdot 2\pi y$$

$$B\cdot 2\pi y = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi y}}$$

3. Use the Biot-savart law to find the B field at the center of a loop of current



$$dB = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

- r is constant
- I is constant
- from Right Hand Rule: B is in $\hat{z}$ direction

$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int ds$$

- $\int ds$ is over a circle, so it evaluates to the circumference

$$B = \frac{\mu_0}{\cancel{4\pi}_{2}} \frac{I}{r^{\cancel{2}}} \cdot \cancel{2\pi r}$$

$$\boxed{B = \frac{\mu_0 I}{2r}}$$

- Trying this with Ampere's Law wouldn't make sense because $\vec{B}$ is only constant around a Path that includes the center.