Image formation

2.1: Geometric Primitives & transforms

(i.e.: Points, lines, & Planes)

Points: $\vec{x} = (x,y) \in \mathbb{R}^2$, or $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

homogeneous coordinates: $\tilde{x} = (\tilde{x}, \tilde{y}, \tilde{w}) \in P^2$

where vectors that differ only by scale are equivalent to $P^2 = R^3 - (0,0,0)$ - 2D Projective space

homogeneous to inhomogeneous: $\tilde{x} = (\tilde{x}, \tilde{y}, \tilde{w}) = \tilde{w}\underbrace{(x, y, 1)}_{\text{augmented vector}} = \tilde{w}\bar{x}$

for $\tilde{w} = 0 \rightarrow$ ideal points / points @ infinity

augmented vector $= \bar{x}$

no inhomogeneous representation

2D lines rep. using homogeneous coordinates $\tilde{l} = (a, b, c)$

line equation: $\bar{x} \cdot \tilde{l} = ax + by + c = 0$

normalize: $l = (\hat{n}_x, \hat{n}_y, d) = (\hat{n}, d)$, $\|\hat{n}\| = 1$

$\hat{n}$ = normal vector, d = distance to origin

line @ infinity: $\tilde{l} = (0, 0, 1)$

Can also represent w/ Polar: $\hat{n} = (\hat{n}_x, \hat{n}_y) = (\cos\theta, \sin\theta)$

this representation is commonly used in hough-transforms

intersection of 2 lines: $\tilde{x} = \tilde{l}_1 \times \tilde{l}_2$ (cross product)

line joining 2 points: $\tilde{l} = \tilde{x}_1 \times \tilde{x}_2$

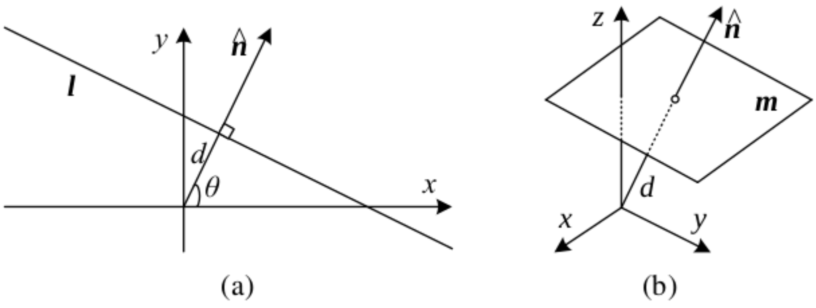

Figure 2.2 *(a) 2D line equation and (b) 3D plane equation, expressed in terms of the normal* $\hat{\mathbf{n}}$ *and distance to the origin* d.

2D conics: $\tilde{x}^T Q \tilde{x} = 0$

3D planes: $\tilde{m} = (a, b, c, d)$, $\bar{x} \cdot \tilde{m} = ax + by + cz + d = 0$

$m = (\hat{n}_x, \hat{n}_y, \hat{n}_z, d) = (\hat{n}, d)$, $\|\hat{n}\| = 1$

plane @ ∞: $\tilde{m} = (0, 0, 0, 1)$

$\hat{n} = (\cos\theta \cos\phi, \sin\theta \cos\phi, \sin\phi)$ — usual mess w/ spherical coordinates

3D lines: - any point on a line as a linear combo of 2 points: $\vec{r} = (1-\lambda)\vec{p} + \lambda \vec{q}$

homogeneous coords: $\tilde{r} = \mu \tilde{p} + \lambda \tilde{q}$

these have 6 dof, instead of 4 (which is how many a 3D line actually has)
can be fixed by fixing the two points to lie in specific planes

Used in light field & lumigraph based rendering

Plücker coordinates - represents all possible lines w/ out any bias towards orientation

$L = \tilde{p}\tilde{q}^T - \tilde{q}\tilde{p}^T$