Circuits in a nutshell

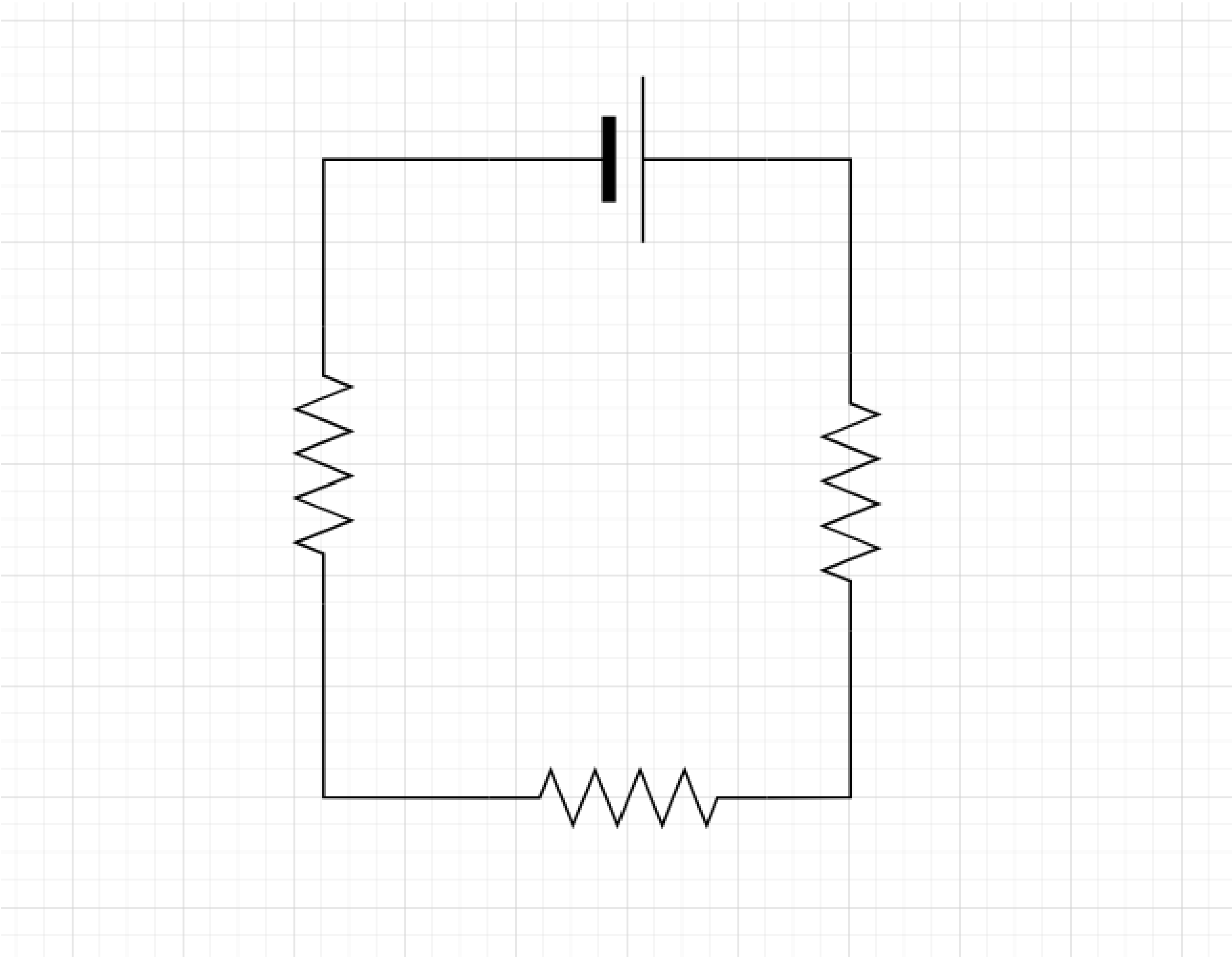
Important quantities

- current - amount of charge flowing through a component or wire (unit = amps = A = coulombs per second = $\frac{C}{s}$)
- resistance - ability of a material to dissipate charge (unit = ohms = Ω (greek letter omega))
- voltage - potential difference between two points (this is electric potential - $k\frac{q}{r}$) (unit = voltage = V)
- capacitance - ability of a material to hold charge (unit = Farad = F = seconds per ohm = $\frac{s}{\Omega}$, base SI untis = $\frac{s^4A^2}{kgm^2}$)

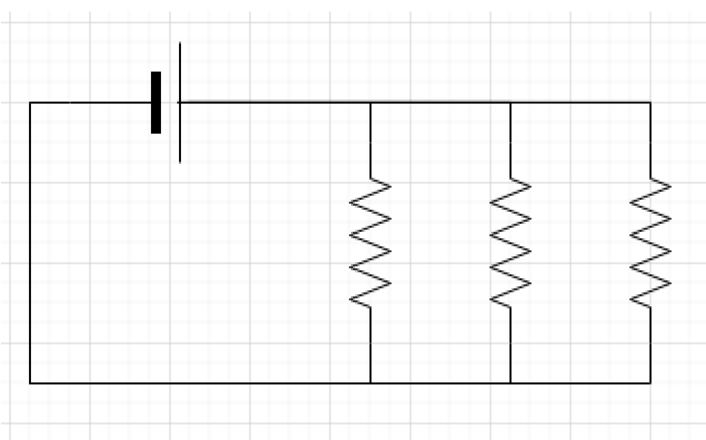
Parallel vs. Series circuits

Series circuits have elements in a chain. **Parallel** circuits have branching elements. For more complex circuits, it is not uncommon for some elements to be in series and others to be in parallel

series circuit



Parallel circuit

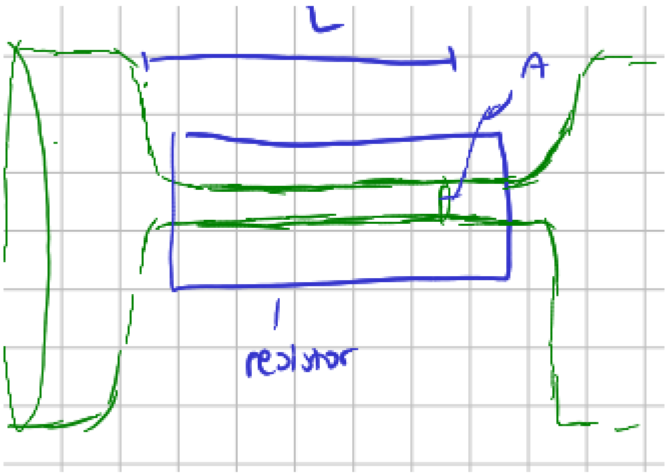


When calculating equivalent capacitance or resistance, you will frequently encounter circuits with elements in both series and parallel. In this case, identify groupings of elements that are either all in series or all in parallel and combine elements accordingly see ['Exercises - circuits.pdf'] for some examples.

Resistance and Capacitance at the microscopic level

Resistors

Think about electricity as water flowing through a pipe. In this analogy, a resistor will be a constriction in the pipe (i.e. it keeps water / electricity from flowing).



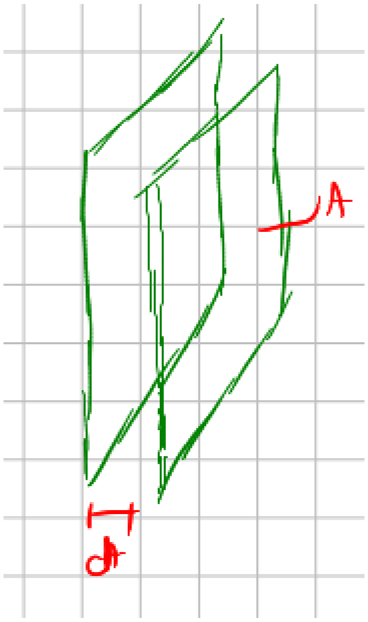
In this example, the resistor is a small tube with cross sectional area *A* and length *L*. The resistance of this resistor is modeled by the equation

$$R = \rho \frac{L}{A}$$

where $L \text{ and } A$ are the quantities mentioned above and ρ is resistivity, which is a property of the material that is normally given if needed

Capacitors

Think of a parallel plate capacitor. The capacitance, or the ability of the component to store an electric charge, like the resistor, is dictated by the geometry of the component.



Here we have a parallel plat capacitor where the two plates have an area A and are separated by a distance d. The capacitance is defined by:

$$C = \epsilon \frac{A}{d}$$

where ϵ is the permitivity of the material between the two plates (this is also called the *dielectric constant*, denoted by κ (greek letter kappa). If there is nothing between the two plates (e.g. vacuum or air), then use $\epsilon = \epsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{F}}{\mathrm{m}}$. If there is a material between the two plates (called a dielectric), then the permitivity is given by $\epsilon = \epsilon_0 \epsilon_r$, where ϵ_r is the relative permitivity and is a property of the dielectric.

Important equations

Ohm's law

$$V = \frac{I}{R}$$

Power dissipated

$$P = IV = \frac{V^2}{R} = I^2 R$$

Voltage, current, resistance, and capacitance in series vs. parallel

Series	parallel
$V_{\text{tot}} = V_1 + V_2 + \ldots$	$V_{\text{tot}} = V_1 = V_2 = \ldots$
$I_{\text{tot}} = I_1 = I_2 = \ldots$	$I_{\text{tot}} = I_1 + I_2 + \ldots$

Series	parallel
$R_{\text{tot}} = R_1 + R_2 + ...$	$\frac{1}{R_{\text{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$
$\frac{1}{C_{\text{tot}}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$	$C_{\text{tot}} = C_1 + C_2 + ...$

Capacitors

charge stored in a capacitor:

$$Q = CV$$

C = capacitance, V = voltage across capacitor

energy stored in a capacitor:

$$E = \frac{1}{2}CV^2$$

this equation is similar in structure to the formulas for kinetic energy ($\frac{1}{2}mv^2$) and spring potential energy ($\frac{1}{2}kx^2$)

Charging and discharging capacitors

value	charging
time constant (τ (tau))	$\tau_c = R_C \times C$
potential across capacitor	$V_C = V(1 - e^{-t/\tau_C})$
current	$I_C = V/R_C e^{-t/\tau_C}$

Conceptualizing equivalent capacitance and equivalent resistance.

I've frequently found myself in situations where I need to find the equivalent capacitance of a group of components but I can't remember which equation to use, so I've come to use this line of reasoning to figure out which equation goes with which kind of circuit:

Resistors

recall the microscopic equation for resistance:

$$R = \rho\frac{L}{A}$$

- series: resistors in series make one long resistor, so L gets bigger in the above equation. If L gets bigger, then R should get bigger, so we should use the equation that results in a larger equivalent resistance: $R_{eq} = R_1 + R_2 + ...$
- parallel: when combining resistors in parallel, imagine sticking all the resistors together side-by-side - this effectively makes one resistor that is wider than all the others, so when combining resistors in parallel we're increasing A. If A gets bigger, then R must get smaller, so we use the equation that decreases equivalent resistance: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$

Capacitors

recall the microscopic equation for capacitance:

$$C = \epsilon \frac{A}{d}$$

- series: capacitors in series combine to form a capacitor with a larget gap. So if d gets bigger in the above equation, then capacitance gets smaller, therefore we use the equation that decreases equivalent capacitance: $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$
- parallel: when combining capacitors in parallel, imagine sticking several capacitors together side-by-side. This has the effect of making one capacitor with a larger area, thus increasing A. Increasing A in the above equation will increase capacitance, so use the equation that results in a larger equivalent capacitance: $C_{\text{eq}} = C_1 + C_2 + ...$