

Separation of variables

ex 1: $y' = x(y-1)$

$$\frac{dy}{dx} = x(y-1)$$

$$\int \frac{dy}{y-1} = \int x dx$$

$$\int \frac{dy}{y-1} = \int \frac{du}{u} = \ln(u) + c = \ln(y-1) + c$$

$$u = y-1$$

$$du = dy$$

$$\ln(y-1) + c_1 = \frac{1}{2}x^2 + c_2$$

$$\ln(y-1) = \frac{1}{2}x^2 + c_3$$

$$y-1 = e^{\frac{1}{2}x^2 + c_3} = C_4 e^{\frac{1}{2}x^2}$$

$$\boxed{y = 1 + C e^{\frac{1}{2}x^2}}$$

② $y' = 2x(1-y)^2$ $\int \frac{dy}{(1-y)^2} = - \int \frac{du}{u^2} = - \frac{-1}{u} = \frac{1}{u} + c$

$$\frac{dy}{dx} = 2x(1-y)^2 \quad \begin{array}{l} u = 1-y \\ -du = dy \end{array} \quad = \frac{1}{1-y} + c$$

$$\int 2x dx = x^2 + c_2$$

$$\frac{dy}{(1-y)^2} = 2x dx$$

$$\frac{1}{1-y} + c_1 = x^2 + c_2$$

$$\frac{1}{1-y} = x^2 + c^3$$

$$\frac{1}{1-y} = x^2 + C$$

$$\frac{1}{x^2 + C} = 1 - y$$

$$y = 1 - \frac{1}{x^2 + C}$$

Note: $y(x) = 1$ is not in the parametrized family -

this is a **lost solution**

(lost by separation of variables)

lost when we get to $\frac{dy}{(1-y)^2}$,

only valid if $y \neq 1$

in general, for $y' = f(x)g(y)$, all roots of g give lost (constant) solutions

③ find all lost solutions for $y' = (x+1)e^x(y^2 - 8y + 7)$

roots of $y^2 - 8y + 7 \rightarrow (y-7)(y-1) = 0$,

So $y = 7, 1$

$$y(x) = 1, \quad y(x) = 7$$

④ (growth/decay) $\dot{y} = ky$

$$\frac{dy}{dx} = ky \rightarrow \frac{dy}{y} = k dx$$

$$\int \frac{dy}{y} = \ln(y) + C_1, \quad \int k dx = kx + C_2$$

$$\ln(y) + C_1 = kx + C_2$$

$$\ln(y) = kx + C_3$$

$$y = e^{kx + C_3}$$

$$y = Ce^{kx}$$

Note: had a lost solution with $y(x) = 0$, but found it again with $C = 0$ (got lucky by being a little sloppy)