Solutions that blow up: the domain of a solution

ex: Solve the IVP $y' = y^2$, $y(0) = 1$

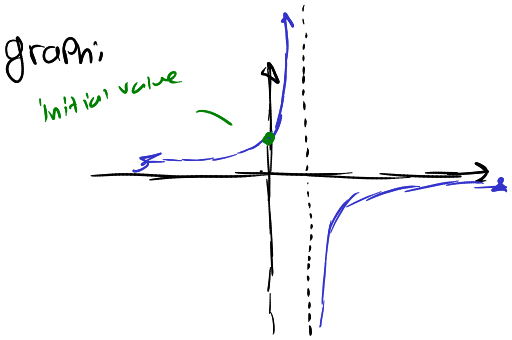
$$\frac{dy}{dx} = y^2 \rightarrow \int \frac{dy}{y^2} = \int dx \rightarrow -\frac{1}{y} + C_1 = x + C_2 \rightarrow -\frac{1}{y} = x + C_3$$

$$-y = \frac{1}{x+C} \rightarrow \boxed{y = \frac{-1}{x+C}}$$

$$y(0) = \frac{-1}{0+C} = \frac{-1}{C} = 1 \rightarrow C = -1$$

$$\boxed{y = \frac{-1}{x-1} = \frac{1}{1-x}}$$

graph:



To be precise, say there are 2 solutions:

$$y = \frac{1}{1-x}, \quad x \in (-\infty, 1)$$

$$y = \frac{1}{1-x}, \quad x \in (1, \infty)$$

Solution to IVP is $y = \frac{1}{1-x}$, $x \in (-\infty, 1)$

Solutions to ODEs have a domain constraint of a single variable

Why? Starting @ (0,1) (IV), then is no way to follow the solution continuously to the other branch