sources:

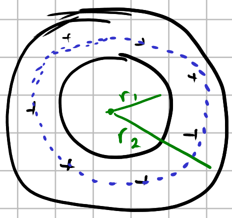
https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Book%3A_University_Physics_II_-_Thermodynamics_Electricity_and_Magnetism_(OpenStax)/06%3A_Gauss's_Law/6.0E%3A_6.E%3A_Gauss's_Law_(Exercises)

46. A total charge **Q** is distributed uniformly throughout a spherical shell of inner and outer radii r_1 and r_2, respectively. Show that the electric field due to the charge is

$$\vec{E} = \vec{0} \ (r \leq r_1);$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right)\hat{r} \ (r_1 \leq r \leq r_2);$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \ (r \geq r_2).$$



$r < r_1$: $q_{enc} = 0$, so $E = \vec{0}$

$r_1 \leq r \leq r_2$: $\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0}$

(E is constant over surface)

$E \cdot 4\pi r^2 = \frac{q_{enc}}{\epsilon_0}$

$\rho = \frac{Q_{tot}}{V}$ $\quad V = \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3$

$= \frac{4}{3}\pi(r_2^3 - r_1^3)$

$q_{enc} = \rho V_{enc} = \rho \frac{4}{3}\pi(r^3 - r_1^3)$

$q_{enc} = \frac{Q_{tot}}{\frac{4}{3}\pi(r_2^3 - r_1^3)}\left(\frac{4}{3}\pi(r^3 - r_1^3)\right)$

$q_{enc} = Q_{tot}\frac{r^3 - r_1^3}{r_2^3 - r_1^3}$

$E = \frac{1}{4\pi r^2}\frac{q_{enc}}{\epsilon_0} = \boxed{\frac{Q}{4\pi\epsilon_0 r^2}\frac{r^3 - r_1^3}{r_2^3 - r_1^3}}$

$r \geq r_2$: Same as point charge