Vertical asymptotes:

- factor numerator & denominator
- cancel common terms
- vertical asymptotes where denominator = 0

ex:

$$g(x) = \frac{x-2}{x^2-4x+3} = \frac{(x-2)}{(x-3)(x-1)}, \quad \text{VA @ } x = 3, 1$$

$$h(x) = \frac{x^2-4}{x^2+x-2} = \frac{(x-2)(x+2)}{(x-1)(x+2)} = \frac{(x-2)}{(x-1)} \rightarrow \text{VA @ } x=1$$

$x = -2$ is a hole (removable discontinuity)

Horizontal asymptotes

N = degree of numerator
D = degree of denominator

- if $N < D$, horizontal asymptote = 0 — ex: $\frac{2x}{3x^2+1}$ (HA = 0)
- if $N = D$, horizontal asymptote is ratio of leading coefficients — ex: $y = \frac{2x^2}{3x^2+1}$ (HA = $\frac{2}{3}$)
- if $N > D$, no horizontal asymptotes — ex: $y = \frac{2x^4}{3x+1}$

Slant asymptotes

$N = D+1 \rightarrow$ divide the fraction & ignore the remainder

ex: $y = \frac{2x^2}{3x+1}$ So slant asymptote is $\frac{2}{3}x - \frac{2}{9}$

$$\begin{array}{r} \frac{2}{3}x - \frac{2}{9} \\ 3x+1 \overline{)\,2x^2+0x+0} \\ \underline{2x^2+\frac{2}{3}x+0} \\ -\frac{2}{3}x+0 \\ \underline{-\frac{2}{3}x-\frac{2}{9}} \\ \frac{2}{9} \end{array}$$

$-\frac{2x}{3} \cdot \frac{1}{3x} = -\frac{2}{9}$

ref: https://www.andrews.edu/~rwright/Precalculus-RLW/Text/02-07.html