

# Linear equations, systems of linear equations

Tuesday, May 4, 2021 4:45 PM

13. The equations  $8x + 4y = 28$  and  $-9x + 5y = 36$  represents the amount of money collected from the Stone Creek Movie Theater. If "y" represents the cost of an adult ticket to get into the movie and "x" represents the cost of a child ticket to get into a movie then what is the cost of each adult ticket? Use any method! ☺

$$\begin{aligned} 8x + 4y &= 28 \\ -9x + 5y &= 36 \end{aligned} \quad \begin{aligned} 4x + 2y &= 14 \\ -x + y &= 4 \rightarrow y = 4 + x \end{aligned}$$

$$3x + 3y = 12$$

$$3(y = 4 + x) + 3y = 12$$

$$3y + 12 + 3y = 12$$

$$6y = 0$$

$$y = 0$$

$$x = 1$$

$$y = |x - 3|$$

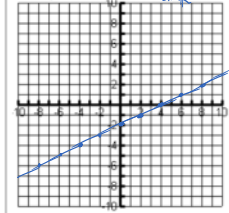
$$y = \frac{-2}{5}x + \frac{27}{5}$$

Solutions:

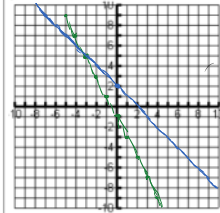
$$\begin{aligned} y &= |x - 3|, & y &= -\frac{2}{5}x + \frac{27}{5} \\ y &\geq 0, & 5y &= -2x + 27 \\ y &= x - 3, & 5(x - 3) &= -2x + 27 \\ & & 5x - 15 &= -2x + 27 \\ & & 7x &= 42 \\ & & x &= 6 \\ 16 - 11 &= 5 \checkmark \\ -\frac{2}{5}(6) + \frac{27}{5} &= -\frac{12}{5} + \frac{27}{5} = \frac{15}{5} = 3 \checkmark \end{aligned}$$

## Solve by Graphing.

$$\begin{aligned} 3x - 6y &= 12 \rightarrow x - 2y = 4 \rightarrow y = -\frac{1}{2}x - 2 \\ 2x - 4y &= 8 \rightarrow x - 2y = 4 \end{aligned}$$



$$\begin{aligned} x + y &= 2 \\ y &= -2x - 1 \end{aligned}$$



2. Suppose you have \$200 in your account and you save \$10 dollars each week. Your friend has \$110 in their account and starts saving \$15 each week. When will your account balances be the same?

$$\begin{aligned} y &= 200 + 10(w) = y \\ y &= 110 + 15(w) = y \end{aligned} \quad \text{When equal?}$$

$$200 + 10w = 110 + 15w$$

$$90 = 5w$$

$$w = 18 \text{ weeks}$$

7. Joey has \$5.75 made up of all dimes and quarters. If Joey has 38 coins, how many of each coin does he have?

$$\begin{aligned} d(0.10) + q(0.25) &= 5.75 \\ d + q &= 38 \\ d &= 38 - q \\ (38 - q)(0.10) + q(0.25) &= 5.75 \\ 3.8 - 0.10q + 0.25q &= 5.75 \\ 0.15q &= 1.95 \\ q &= 13 \\ d &= 25 \end{aligned}$$

## Solve by Substitution.

$$\begin{aligned} 2x - 3y &= -1 \\ y &= x - 1 \end{aligned}$$

$$\begin{aligned} 2x - 3(x - 1) &= -1 \\ 2x - 3x + 3 &= -1 \\ -x + 3 &= -1 \\ -x &= -4 \\ x &= 4 \\ y = 4 - 1 &= 3 \end{aligned}$$

$$\begin{aligned} y &= -3x + 5 \\ 5x - 4y &= -3 \\ 5x - 4(-3x + 5) &= -3 \\ 5x + 12x - 20 &= -3 \\ 17x &= 17 \\ x &= 1 \\ y = -3(1) + 5 &= 2 \end{aligned}$$

## Solve by Elimination.

$$\begin{aligned} (5x + y = 9) \times -2 \\ 10x - 7y &= -18 \\ 5x + 4y &= 9 \\ 5x &= 5 \\ x &= 1 \\ -10x - 7y &= -19 \\ 10x - 7y &= -19 \\ -7y &= -36 \\ y &= 4 \end{aligned}$$

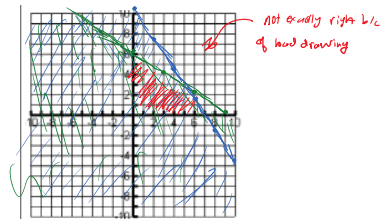
$$\begin{aligned} (-3x + 7y = -16) \times -3 \\ -9x + 5y &= 16 \\ -9x + 5(-4) &= 16 \\ -9x - 20 &= 16 \\ -9x &= 36 \\ x &= -4 \\ 0 + -14 &= 6y \\ y &= -4 \end{aligned}$$

## Systems of inequalities:

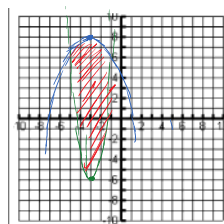
A sundae requires 3 ice-cream scoops and 4 strawberries, and a milkshake requires 2 ice-cream scoops and 6 strawberries. Ramses wants to make sundaes and milkshakes with at most 25 ice-cream scoops and 37 strawberries. Let's form a system of inequalities to represent his conditions. Let x denote the number of sundaes he makes and y the number of milkshakes he makes. Graph your solution on the following graph.

Sundae: 3 scoops, 4 strawberries  
Milkshake: 2 scoops, 6 strawberries  
Total: 25 scoops, 37 strawberries  
x = # sundaes  
y = # milkshakes

$$\begin{aligned} 3x + 2y &\leq 25 \\ 4x + 6y &\leq 37 \\ 2y &\leq 25 - 3x \\ y &\leq \frac{25}{2} - \frac{3}{2}x \\ 6y &\leq 37 - 4x \\ y &\leq \frac{37}{6} - \frac{2}{3}x \end{aligned}$$



$$\begin{aligned} f(x) &\leq -(x + 3)^2 + 8 \\ f(x) &\geq 2(x + 3)^2 - 6 \end{aligned}$$



Crash course from SAT Prep

# Systems of equations

$n$  unknowns  $\rightarrow n$  equations

## 3 methods

- 1) graphing
- 2) substitution
- 3) elimination

## 3 possible results

- 1) one solution  $x=5, y=2$
- 2) no solution  $3=2$
- 3) infinitely many solutions  $2=2$

$$\begin{aligned} x+y &= 3 \\ x-y &= 1 \end{aligned} \rightarrow \begin{aligned} x+y &= 3 \\ y &= -x+3 \end{aligned}$$

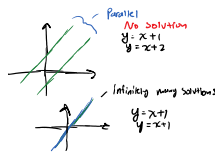
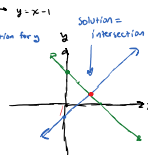
graphing: solve each equation for  $y$

$$y = -x + 3 \quad (1)$$

$$y = x - 1 \quad (2)$$

slope  $\downarrow$  y-intercept

$$\boxed{x=2, y=1}$$



## elimination

$$\begin{aligned} x+y &= 3 \\ x-y &= 1 \end{aligned}$$

$$+ \quad (x-y=1)$$

$$2x = 4$$

$$x = 2$$

$$2 - y = 1$$

$$-y = -1$$

$$y = 1$$

$$x+y=3$$

$$2(x-y=1)$$

$$x+y=3$$

$$2x-2y=2$$

## substitution

$$x+y=3$$

$$x-y=1 \rightarrow x=y+1$$

$$y+1+y=3$$

$$2y=2$$

$$y=1$$

$$x=1+1=2$$

12

$$\begin{aligned} 3x+4y &= -23 \\ 2y-x &= -19 \end{aligned}$$

What is the solution  $(x, y)$  to the system of equations above?

- A)  $(-5, -2)$
- B)  $(1, -8)$
- C)  $(4, -6)$
- D)  $(8, -6)$

$$\begin{aligned} 3x+4y &= -23 \\ 2x+10y &= -40 \end{aligned}$$

$$y = -9$$

get rid of  $x$

$$3(-x+2y) = -19$$

$$-3x+6y = -19$$

$$-3x+6y = -57$$

$$2x-4y = 32$$

$$2y-x = -19$$

$$2y+x = 19$$

$$4y = 38$$

$$y = 9.5$$

$$2(9.5) + 19 = 38 + 19 = 57$$

$$10y + 57 = 57 + 57 = 114$$

$$10y + 57 = 114$$

$$10y = 57$$

$$y = 5.7$$