Separation of variables

ex1 : $y' = x(y-1)$

$$\frac{dy}{dx} = x(y-1)$$

$$\int \frac{dy}{y-1} = \int x\,dx$$

$$\int \frac{dy}{y-1} = \int \frac{du}{u} = \ln(u) + c = \ln(y-1) + c$$

$u = y-1$

$du = dy$

$$\ln(y-1) + c_1 = \frac{1}{2}x^2 + c_2$$

$$\ln(y-1) = \frac{1}{2}x^2 + c_3$$

$$y-1 = e^{\frac{1}{2}x^2 + c_3} = c_4 e^{\frac{1}{2}x^2}$$

$$\boxed{y = 1 + ce^{\frac{1}{2}x^2}}$$

(2) $y' = 2x(1-y)^2$

$$\frac{dy}{dx} = 2x(1-y)^2$$

$$\frac{dy}{(1-y)^2} = 2x\,dx$$

$$\int \frac{dy}{(1-y)^2} = -\int \frac{du}{u^2} = -\frac{-1}{u} + c = \frac{1}{u} + c$$

$$= \frac{1}{1-y} + c$$

$u = 1-y$

$-du = dy$

$$\int 2x\,dx = x^2 + c_2$$

$$\frac{1}{1-y} + c_1 = x^2 + c_2$$

$$\frac{1}{1-y} = x^2 + c^3$$

$$\frac{1}{1-y} = x^2 + C$$

$$\frac{1}{x^2+C} = 1-y$$

$$y = 1 - \frac{1}{x^2+C}$$

Note: $y(x)=1$ is not in the parameterized family.

this is a lost solution

(lost by separation of variables

lost when we get to $\frac{dy}{(1-y)^2}$,

only valid if $y \neq 1$

In general, for $y' = f(x)g(y)$, all roots of y give lost (constant) solutions

(3) find all lost solutions for $y' = (x+1)e^x(y^2 - 8y + 7)$

roots of $y^2 - 8y + 7 \to (y-7)(y-1)=0$,

so $y = 7, 1$

$$\boxed{y(x) = 1, \quad y(x) = 7}$$

(4) (growth/decay) $\dot{y} = ky$

$$\frac{dy}{dx} = ky \to \frac{dy}{y} = k\,dx$$

$$\int \frac{dy}{y} = \ln(y) + C_1 \qquad \int k\,dx = kx + C_2$$

$$\ln(y) + C_1 = kx + C_2$$

$$\ln(y) = kx + C_3$$

$$y = e^{kx + C_3}$$

$$\boxed{y = Ce^{kx}}$$

Note: had a lost solution with $y(x)=0$, but found it again with $C=0$

(got lucky by being a little sloppy)