Sources:
https://openstax.org/books/physics/pages/8-multiple-choice
3000 solved problems in physics, chapters 9, 12.3

9.1 A mass m undergoes free fall. What is its linear momentum after it has fallen a distance h?

$F = mg$

$\Delta P = F\Delta t = m\Delta v$

$P = Ft = mgt$

$P = mg\sqrt{\frac{2h}{g}} = m\sqrt{2gh}$

time to fall distance h?

$h = \frac{1}{2}gt^2$

$\sqrt{\frac{2h}{g}} = t$

9.4 While waiting in his car at a stoplight, an 80-kg man and his car are suddenly accelerated to a speed of 5 m/s as the result of a rear-end collision. Assuming the time taken to be 0.3 s, find (*a*) the impulse on the man and (*b*) the average force exerted on him by the back of the seat of his car.

a) $F\Delta t = m\frac{\Delta v}{\Delta t}\Delta t = m\Delta v = (80\,kg)(5.0\,m/s) = 400\,N\cdot s$

b) $F = \frac{\Delta(mv)}{\Delta t} = \frac{400\,N\cdot s}{0.3} = 1330\,N$

9.18 A ball of 0.4-kg mass and a speed of 3 m/s has a head-on, completely elastic collision with a 0.6-kg mass initially at rest. Find the speeds of both bodies after the collision.

KE is conserved

$m_1 = 0.4\,kg$
$v_{1i} = 3\,m/s$

$m_2 = 0.6\,kg$
$v_{2i} = 0\,m/s$

$m_1v_{1i} + \cancel{m_2v_{2i}} = m_1v_{1f} + m_2v_{2f}$

$v_{1i} = v_{1f} + \frac{m_2}{m_1}v_{2f}$

$v_{1f} = \frac{m_2}{m_1}v_{2f} - v_{1i}$

$\cancel{\frac{1}{2}}m_1v_{1i}^2 + \cancel{\frac{1}{2}m_2v_{2i}^2} = \cancel{\frac{1}{2}}m_1v_{1f}^2 + \cancel{\frac{1}{2}}m_2v_{2i}^2$

$m_1(v_{1i}^2 - v_{1f}^2) = m_2v_{2i}^2$

$\cancel{m_1(v_{1i}^2 - (\frac{m_1}{m_2}v_{2f} - v_{1i})^2) = m_2v_{2i}^2}$

$\cancel{m_1(v_{1i}^2 - [\frac{m_1^2}{m_2^2}v_{2f}^2 - 2\frac{m_1}{m_2}v_{2f}v_{1i} + v_{1i}^2]) = m_2v_{2i}^2}$

$\cancel{m_1[-\frac{m_1^2}{m_2^2}v_{2f}^2 + 2\frac{m_1}{m_2}v_{2f}v_{1i}] = m_2v_{2i}^2}$

$\cancel{-\frac{m_1^3}{m_2^2}v_{2f}^2 + 2\frac{m_1^2}{m_2}v_{2f}v_{1i} = m_2v_{2i}^2}$

Note: units wont

$-\frac{m_1^3}{m_2^2}v_{2f}^2 + 2\frac{m_1^2}{m_2}v_{2f}v_{1i} - m_2v_{2i}^2$

$\cancel{a = -\frac{m_1^3}{m_2^2} = -0.18}$

$\cancel{b = 2\frac{m_1^2}{m_2}v_{1i} = 2.6}$

$c = -0.6$

$v_{2f} = \cancel{\frac{-2.6 \pm \sqrt{2.6^2 - 4(-0.18)(-0.6)}}{2(-0.18)}} = 0.23,\ 14.21$

using velocity of separation = velocity of approach: $-v_{1f} + v_{2f} = 3 \rightarrow v_{2f} = 3 + v_{1f}$

$0.4 \times 3 + 0 = 0.4v_{1f} + 0.6v_{2f}$

$1.2 = 0.4v_{1f} + 0.6(3 + v_{1f})$

$1.2 = 0.4v_{1f} + 1.8 + 0.6v_{1f} = v_{1f} + 1.8$

$\boxed{v_{1f} = -0.6\,m/s}$

$v_{2f} = 3 + (-0.6) = \boxed{2.4\,m/s}$

9.19 A proton of mass 1.66×10^{-27} kg collides head-on with a helium atom at rest. The helium atom has a mass of 6.64×10^{-27} kg and recoils with a speed of 5×10^5 m/s. If the collision is elastic, what are the initial and final speeds of the proton and the fraction of its initial energy transferred to the helium atom?

$m_1 = 1.66\times10^{-27}\,kg$
$V_{1i} = ?$
$V_{1f} = ?$

$m_2 = 6.64\times10^{-27}\,kg$
$V_{2i} = 0\,m/s$
$V_{2f} = 5\times10^5\,m/s$

$$m_1V_{1i} + \underbrace{\cancel{m_2V_{2i}}}_{0} = m_1V_{1f} + m_2V_{2f} \qquad m_1V_{1i}^2 + \underbrace{\cancel{m_2V_{2i}^2}}_{0} = m_1V_{1f}^2 + m_2V_{2f}^2$$

Solve for V_{1i} V_{1f}

$$m_1V_{1i} = m_1V_{1f} + m_2V_{2f} \qquad m_1V_{1i}^2 = m_1V_{1f}^2 + m_2V_{2f}^2$$

$$V_{1i} = V_{1f} + \frac{m_2}{m_1}V_{2f} \qquad V_{1i}^2 = V_{1f}^2 + \frac{m_2}{m_1}V_{2f}^2$$

$$\cancel{V_{1f}^2} + 2\frac{\cancel{m_2}}{\cancel{m_1}}V_{1f}V_{2f} + \frac{m_2^2}{m_1^2}V_{2f}^2 = \cancel{V_{1f}^2} + \frac{\cancel{m_2}}{\cancel{m_1}}V_{2f}^2$$

$$2V_{1f}V_{2f} + \frac{m_2}{m_1}V_{2f}^2 = V_{2f}^2$$

$$2V_{1f}V_{2f} + \frac{m_2}{m_1}V_{2f}^2 - V_{2f}^2 = 0$$

$$2V_{1f}V_{2f} + V_{2f}^2\left(\frac{m_2}{m_1} - 1\right) = 0$$

$$2V_{1f} + V_{2f}\left(\frac{m_2}{m_1} - 1\right) = 0$$

$$2V_{1f} = -V_{2f}\left(\frac{m_2}{m_1} - 1\right)$$

$$V_{1f} = -\frac{1}{2}V_{2f}\left(\frac{m_2}{m_1} - 1\right) = -\frac{1}{2}\left(5\times10^5\,\frac{m}{s}\right)\left(\frac{6.64\times10^{-27}\,kg}{1.66\times10^{-27}\,kg} - 1\right) = \boxed{-7.5\times10^5\,\frac{m}{s}}$$

$$V_{1i} = \left(-7.5\times10^5\,\frac{m}{s}\right) + 4\left(5\times10^5\,\frac{m}{s}\right) = \boxed{1.25\times10^6\,\frac{m}{s}}$$

energy fraction:
$$\frac{KE\ of\ HE}{KE_i} = \frac{m_2V_{2f}^2}{\underbrace{m_1V_{1i}^2}_{\text{all of the initial KE is in here}}} = \frac{4\cancel{m_1}V_{2f}^2}{\cancel{m_1}V_{1i}^2} = 4\frac{\left(5\times10^5\,\frac{m}{s}\right)^2}{\left(1.25\times10^6\,\frac{m}{s}\right)^2} = \boxed{0.64}$$

9.31 An 8-g bullet is fired horizontally into a 9-kg block of wood and sticks in it. The block, which is free to move, has a velocity of 40 cm/s after impact. Find the initial velocity of the bullet.

Perfectly inelastic collision

$m_1 = 8\times10^{-3}\,kg$
$V_{1i} = ?$
$V_{2i} = 40\times10^{-2}\,m/s$

$m_2 = 9\,kg$
$V_{2i} = 0$
$V_{2f} = 40\times10^{-2}\,m/s$

let $V_f = V_{1f} = V_{2f}$

$$m_1V_{1i} + \underbrace{\cancel{m_2V_{2i}}}_{0} = m_1V_{1f} + m_2V_{2f}$$

$$m_1V_{1i} = (m_1+m_2)V_f$$

$$V_{1i} = \frac{(m_1+m_2)}{m_1}V_f = \frac{(8\times10^{-3} + 9)\,kg}{8\times10^{-3}\,kg} \times 40\times10^{-2}\,m/s$$

$$= \boxed{450\,m/s}$$

9.70 An object at rest explodes into three pieces of equal mass. One moves east at 20 m/s; a second moves southeast at 30 m/s. What is the velocity of the third piece?

9.73 A 7500-kg truck traveling at 5 m/s east collides with a 1500-kg car moving at 20 m/s in a direction 30° south of west. After collision, the two vehicles remain tangled together. With what speed and in what direction does the wreckage begin to move?

9.81 A particle of mass m traveling with speed v_0 along the x axis suddenly shoots out one-third its mass with speed $2v_0$ parallel to the y axis. Express the velocity of the remainder of the particle in **i**, **j**, **k** notation.

Angular momentum Problems

12.47 Find the rotational energy and angular momentum due to the daily rotation of the earth about its axis. Data: $M_e = 6 \times 10^{24}$ kg, $R_e = 6.4 \times 10^6$ m, $\omega = 1/86\,400$ rev/s. Assume the earth to be a uniform sphere.

$KE = \frac{1}{2} I \omega^2,$ $\qquad I_{sphere} = \frac{2}{5} M R^3$

$KE = \frac{1}{2} \cdot \frac{2}{5} M R^3 \omega^2 = \frac{1}{5} M R^2 \omega^2 = 2.6 \times 10^{29}\,J$ $\qquad \frac{1}{86400} \frac{rev}{s} = \frac{2\pi}{86400} \frac{rad}{s}$

12.58 A boy stands on a freely rotating platform. With his arms extended, his rotation speed is 0.25 rev/s. But when he draws them in, his speed is 0.80 rev/s. Find the ratio of his moment of inertia in the first case to that in the second.

$I_0 \omega_0 = I_f \omega_f$

$\frac{I_0}{I_f} = \frac{\omega_f}{\omega_0} = \frac{0.8\ rev/s}{0.25\ rev/s} = \boxed{3.2}$