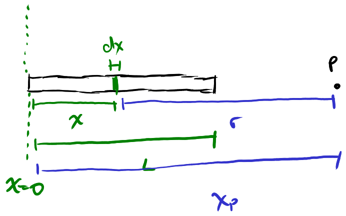
Electric fields of Some Continuous Charge distributions

ref: http://web.phys.ntnu.no/~stovneng/TFY4155/TiplerCH22.pdf

① $\vec{E}$ on the axis of a finite line of charge



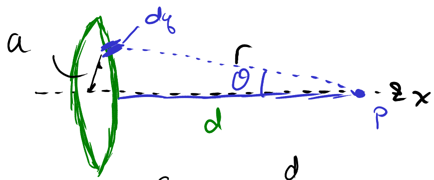
$$E = \int \frac{k\,dq}{r^2} \qquad \lambda = \frac{Q}{L} \;\&\; \lambda = \frac{dq}{dx} \to dq = \lambda\,dx \qquad r = x_p - x$$

$$dE = \frac{k\lambda\,dx}{(x_p - x)^2} \to E = k\lambda \int_0^L \frac{dx}{(x_p-x)^2} = -k\lambda \int_{x_p}^{x_p-L} \frac{dx}{u^2} = k\lambda \int_{x_p-L}^{x_p} \frac{dx}{u^2} = k\lambda \left[\frac{-1}{u}\right]_{x_p-L}^{x_p}$$

$$u = x_p - x \qquad u(0) = x_p$$
$$du = -dx \qquad u(L) = x_p - L$$

$$= k\lambda\left[\frac{-1}{x_p} + \frac{1}{x_p - L}\right] = k\lambda\left[\frac{-x_p + L + x_p}{x_p(x_p-L)}\right] = k\lambda\left[\frac{L}{(x_p-L)^2}\right] = \frac{k\lambda L}{\underbrace{(x_p-L)}_{\text{distance from end of line}}{}^2} = \boxed{\frac{kQ}{(x_p-L)^2}}$$

② $\vec{E}$ on the axis of a ring of charge



y-components cancel out due to symmetry, so $\vec{E} = E_x$

$$dE_x = |dE|\cos\theta = k\frac{dq}{r^2}\cos\theta = k\frac{dq}{r^2}\cdot\frac{d}{r} = k\frac{d\,dq}{(d^2+a^2)^{3/2}}$$

$$\cos(\theta) = \frac{d}{r} \qquad r = \sqrt{d^2+a^2} \qquad E = \int k\frac{d\,dq}{(d^2+a^2)^{3/2}} = k\frac{d}{(d^2+a^2)^{3/2}}\int dq = \boxed{k\frac{Qd}{(d^2+a^2)^{3/2}}}$$