Covariance matrices

Covariance matrices

sources:

- https://datascienceplus.com/understanding-the-covariance-matrix/
- https://towardsdatascience.com/5-things-you-should-know-about-covariance-26b12a0516f1
- https://builtin.com/data-science/covariance-vs-correlation
- https://www.simplilearn.com/covariance-vs-correlation-article
- https://medium.com/swlh/covariance-correlation-r-sqaured-5cbefc5cbe1c
- https://www.mygreatlearning.com/blog/covariance-vs-correlation/ (has a useful table at the bottom comparing covariance and correlation)

other related topics:

- linear algebra - matrix basics
- statistics - variance and standard deviation
- statistics - pearson correlation coefficient

linked ideas:
Principal Component Analysis.pdf

Definition of variance

$$\sigma^2_{x_i} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

definition of covariance

$$\sigma(x,y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

The covariance matrix is a matrix whose entries are the covariances of each combined row. For example - in the case of a 2-dimensional dataset of (x,y) pairs, the covariance matrix would be

$$C = \begin{bmatrix} \sigma(x,x) & \sigma(x,y) \\ \sigma(y,x) & \sigma(y,y) \end{bmatrix}$$

diagonal entries are the variances, off-diagonal are covariances

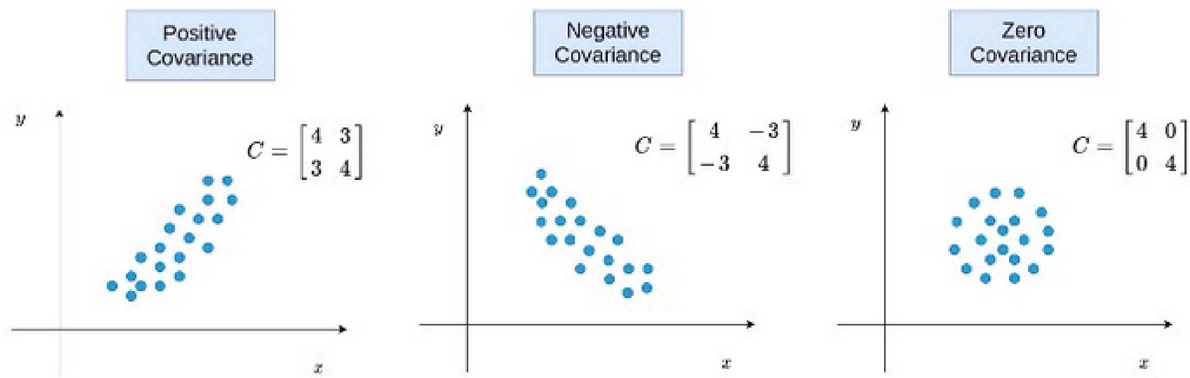
matrix calculation of covariance:

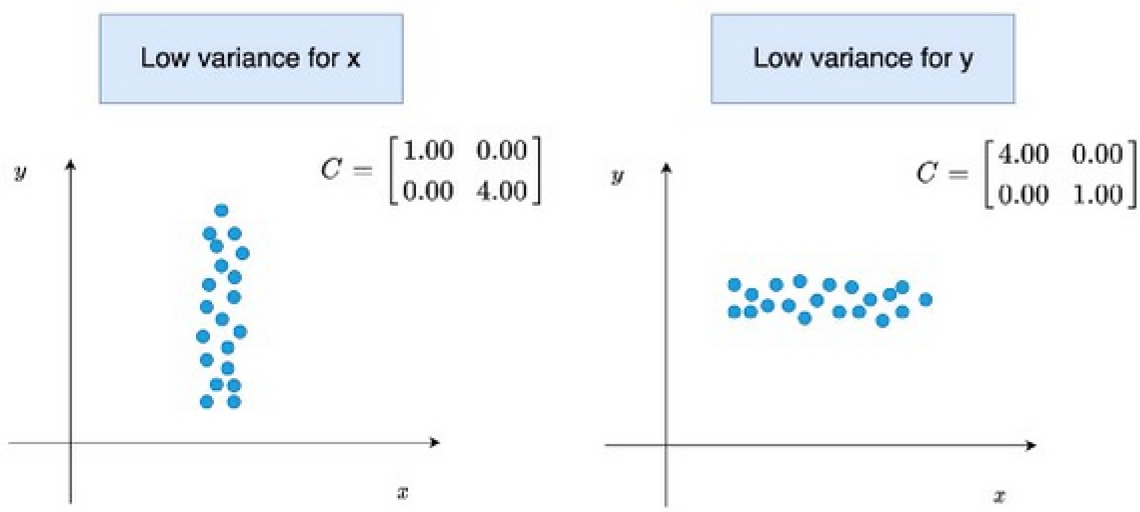
$$C = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})(X_i - \bar{X})^T$$

if the matrix X has zero mean, then:

$$C = \frac{XX^T}{n-1}$$

Interpretation

- describes how two variables vary together
- 

- 

- What's important are the relative values in the covariance matrix, not absolute values.

connection to correlation coefficient

covariance is unstandardized, so dividing by the product of the standard deviation of each variable will give us a covariance within the range -1, 1 → which is exactly what the correlation coefficient is

$$R = corr(x, y) = \frac{cov(x, y)}{\sigma_x \sigma_y}$$

so pearson correlation coefficient is related to covariance by:

$$R^2 = (\frac{cov(x, y)}{\sigma_x \sigma_y})^2$$