

# Quantities

**Impedance** - analogue of resistance in DC circuits. Associated with a phases angle -> because peaks of current and voltage do not occur at the same time

**Average Power** - accounts for phase difference in AC circuits

**angular frequency ( $\omega$ )** - relates to angular frequency of sin wave representing emf

# Formulas

Ohm's law for AC circuits:

$$I = \frac{V}{Z}$$

$I$  and  $V$  correspond to rms / effective values

**Average power:**

$$P_{\text{avg}} = VI \cos \phi = V_{\text{rms}} I_{\text{rms}}$$

Where  $\phi$  is the phase angle

**Peak voltage and current**

$$V = \frac{V_m}{\sqrt{2}}$$
$$I = \frac{I_m}{\sqrt{2}}$$

# Impedance of various components

**Resistor**

- same as DC resistance (use rms values for V and I)

**Capacitor**

- called capacitive reactance
- complex contribution to impedance ( $\frac{-j}{\omega C}$ )

$$X_c = \frac{1}{\omega C}$$

- pure C circuit will cause voltage to LAG current by 90 deg

## Inductor

- called inductive reactance
- complex contribution to impedance:  $j\omega L$

$$X_L = \omega L$$

- pure L circuit will cause voltage to LEAD current by 90 deg

## RC circuit

- voltage LAGS current, but by less than 90 deg

$$\begin{aligned}\text{contribution to complex impedance} &= R - \frac{j}{\omega C} \\ Z &= \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \\ \text{phase angle} = \phi &= \tan^{-1} \frac{-1/\omega C}{R}\end{aligned}$$

(note: formula for phase angle aligns with opp / adj = (impedance from capacitor) / (impedance from resistor))

## RL circuit

- voltage LEADS current, but by less than 90 deg

$$\begin{aligned}\text{contribution to complex impedance} &= R + j\omega L \\ Z &= \sqrt{R^2 + \omega^2 L^2} \\ \text{phase angle} = \phi &= \tan^{-1} \frac{\omega L}{R}\end{aligned}$$

## RLC Series Circuit

example of a resonant circuit. Minimum impedance of  $Z = R$  at resonant frequency

Resonant condition:

$$\begin{aligned}Z &= R \\ \omega &= \frac{1}{\sqrt{LC}} \\ X_C &= X_L \\ \text{Phase} = \phi &= 0\end{aligned}$$

Normal conditions:

$$\begin{aligned}
 X_C &= \frac{1}{\omega C} \\
 X_L &= \omega L \\
 Z &= \sqrt{R^2 + (X_L - X_C)^2} \\
 \text{phase} = \phi &= \tan^{-1} \frac{X_L - X_C}{R}
 \end{aligned}$$

### RLC Parallel (and combination)

Dealing with parallel circuits can be more complex and tedious because each branch can have a different phase angle that combines differently.

In general, impedance combines like resistance:

$$\begin{aligned}
 Z_{\text{parallel}} &= \left( \frac{1}{Z_1} + \frac{1}{Z_2} + \dots \right)^{-1} \\
 Z_{\text{series}} &= Z_1 + Z_2 + \dots
 \end{aligned}$$

remember that we're doing these calculations with complex numbers, so the denominators must be rationalized.

See [this](#) for a more detailed overview of the forms we can use for dealing with complex impedance