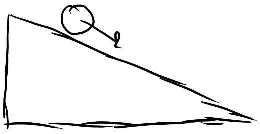
Sources:
https://courses.lumenlearning.com/suny-osuniversityphysics/chapter/11-1-rolling-motion/
3000 solved problems in physics section 6.1

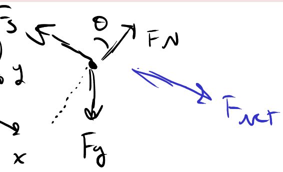
Rolling Down an Inclined Plane

A solid cylinder rolls down an inclined plane without slipping, starting from rest. It has mass m and radius r. (a) What is its acceleration? (b) What condition must the coefficient of static friction μ_S satisfy so the cylinder does not slip?



$V_0 = 0$, $a = ?$

a)



$\sum F_x = F_g \sin\theta - \mu_s N = ma$

$\sum F_y = F_N - F_g \cos\theta = 0$

$F_N = F_g \cos\theta$

$F_g \sin\theta - \mu_s mg\cos\theta = ma$

$g\sin\theta - \mu_s g\cos\theta = a$

$\boxed{a = g(\sin\theta - \mu_s \cos(\theta))}$

b) $\boxed{\mu_s = \sin\theta}$

6.2 A small ball is fastened to a string 24 cm long and suspended from a fixed point P to make a conical pendulum, as shown in Fig. 6-1. The ball describes a horizontal circle about a center vertically under point P, and the string makes an angle of 15° with the vertical. Find the speed of the ball.

$L = 24\text{cm}, \quad \theta = 15°$

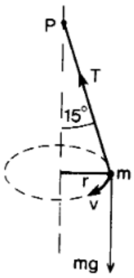
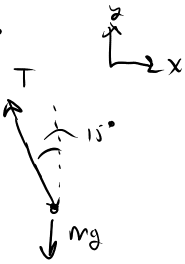


Fig. 6-1



net force $= F_c = m\frac{v^2}{r}$

$mg = T\cos\theta \qquad m\frac{v^2}{r} = T\sin\theta$

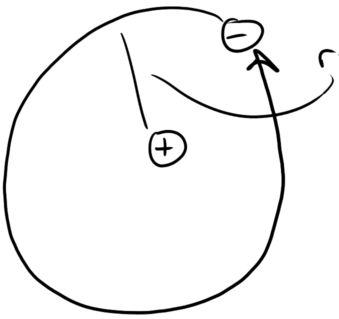
$T = \frac{mg}{\cos\theta}$

$m\frac{v^2}{r} = \frac{mg}{\cos\theta}\sin\theta \rightarrow \frac{v^2}{r} = g\tan\theta$

$v = \sqrt{gr\tan\theta} \qquad L\sin\theta = r$

$v = \sqrt{gL\sin\theta\tan\theta} = \sqrt{(9.81\frac{m}{s^2})(24\times10^{-2}m)\sin(15°)\tan(15°)} = \boxed{0.40\frac{m}{s} = 40.4 \text{cm/s}}$

6.3 In the Bohr model of the hydrogen atom an electron is pictured rotating in a circle (with a radius of 0.5×10^{-10} m) about the positive nucleus of the atom. The centripetal force is furnished by the electric attraction of the positive nucleus for the negative electron. How large is this force if the electron is moving with a speed of 2.3×10^6 m/s? (The mass of an electron is 9×10^{-31} kg.)



$r = 0.5\times10^{-10}m$

$V_e = 2.3\times10^6 \frac{m}{s}$

$m_e = 9\times10^{-31} kg$

$F_c = m\frac{v^2}{r} = (9\times10^{-31}kg)\frac{(2.3\times10^6\frac{m}{s})^2}{0.5\times10^{-10}m} = \boxed{9.5\times10^{-8} N}$

A formula 1 car is initially at top speed and slams on the brakes to take a tight turn. Before the driver hits the brakes, the wheels are rotating at 408 rad/s. When the driver releases the brakes to turn into the corner, the wheels are rotating at 97 rad/s. The brakes provide an angular acceleration of 180 rad/s^2. Assume the wheels have a diameter of 18".

a) what is the initial and final speed of the car in m/s and mph?

b) in what distance did the car slow down?

c) how many g's did the driver pull when slamming on the brakes?

d) How does this compare to a road car? (assume an average road car can come to a stop from 60 mph in 200 feet.

a) $r = 9'' \times \frac{0.0254\,m}{1''} = 0.23\,m$

$V = \omega r$: $V_i = \omega_i r = 408\,\frac{rad}{s} \times 0.23\,m = 93.84\,\frac{m}{s} \times \frac{1\,km}{1000\,m} \times \frac{0.62\,mi}{1\,km} \times \frac{3600\,s}{1\,hr} = 209\,mph$

$V_f = \omega_f r = 97\,\frac{rad}{s} \times 0.23\,m = 22.31\,\frac{m}{s} \times \frac{1\,km}{1000\,m} \times \frac{0.62\,mi}{1\,km} \times \frac{3600\,s}{1\,hr} = 50\,mph$

$\boxed{V_i = 93.8\,\frac{m}{s} = 209\,mph, \quad V_f = 22.3\,m/s = 50\,mph}$

b) $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta \rightarrow \Delta\theta = \frac{\omega_f^2 - \omega_0^2}{2\alpha} = \frac{(97\,\frac{rad}{s})^2 - (408\,\frac{rad}{s})^2}{2(-180\,\frac{rad}{s^2})} = 436\,rad$

ω initially pos car slowing down, so $\omega \downarrow$, which means $\alpha < 0$

distance travelled = arc length = $S = r\theta = (0.23\,m)(436\,rad) = \boxed{100\,m}$

c) calculate linear accel : $V_f^2 = V_0^2 + 2a\Delta x \rightarrow$

$a = \frac{V_f^2 - V_0^2}{2\Delta x} = \frac{(93.8\,\frac{m}{s})^2 - (22.3\,\frac{m}{s})^2}{2(100\,m)} = 41.5\,m/s^2$

$a = \#\text{ of g's} \times g \rightarrow \#\text{-g's} = \frac{41.5\,m/s^2}{9.81\,m/s^2} = \boxed{4.2g_s}$

d) road car: 60 mph – 0 in 200 ft

$\frac{60\,mi}{hr} \times \frac{1\,km}{0.62\,mi} \times \frac{1000\,m}{1\,km} \times \frac{1\,hr}{3600\,s} = 26.8\,m/s$

$200\,ft \times \frac{0.3048\,m}{1\,ft} = 60.96\,m$

$\cancel{V_f^2} = V_0^2 + 2a\Delta x$
0

$\frac{-V_0^2}{2\Delta x} = a = -\frac{(26.8\,m/s)^2}{2(60.96\,m)} = \boxed{-5.89\,m/s^2}$

$\frac{41.5}{5.89} = 7 \rightarrow$ $\boxed{7\times \text{ more intense accel than a normal car}}$