**Problem 1:** [Natural growth, separable equations] In recitation a population model was studied in which the natural growth rate of the population of oryx was a constant $k > 0$, so that for small time intervals $\Delta t$ the population change $x(t + \Delta t) - x(t)$ is well approximated by $kx(t)\Delta t$. (You also studied the effect of hunting them, but in this problem we will leave that aside.) Measure time in years and the population in kilo-oryx (ko).

A mysterious virus infects the oryxes of the Tana River area in Kenya, which causes the growth rate to decrease as time goes on according to the formula $k(t) = k_0/(a + t)^2$ for $t \geq 0$, where $a$ and $k_0$ are certain positive constants.

**(a)** What are the units of the constant $a$ in "$a + t$," and of the constant $k_0$?

**(b)** Write down the differential equation modeling this situation.

**(c)** Write down the general solution to your differential equation. Don't restrict yourself to the values of $t$ and of $x$ that are relevant to the oryx problem; take care of all values of these variables. Points to be careful about: use absolute values in $\int \frac{dx}{x} = \ln|x| + c$ correctly, and don't forget about any "lost" solutions.

**(d)** Now suppose that at $t = 0$ there is a positive population $x_0$ of oryx. Does the progressive decline in growth rate cause the population stabilize for large time, or does it grow without bound? If it does stabilize, what is the limiting population as $t \to \infty$?

- growth rate $k > 0$
- Pop change approximated by $k\,x(t)\,\Delta t$
- growth rate decreases w/ $k(t) = \frac{k_0}{(a+t)^2}$

a) $k(t)$ must have units of years
Since we want $k(t)\,x(t)\Delta t$ to have the same units as $x(t)$

$a+t$ = units of years, $k_0$ = units of years

b) $x(t+\Delta t) - x(t) \approx k(t)\,x(t)\,\Delta t \rightarrow \quad \Delta x \approx k(t)\,x\,dt \quad \frac{dx}{dt} = \frac{k_0 x}{(a+t)^2}$

c) $\int \frac{dx}{x} = \int \frac{dt\, k_0}{(a+t)^2} \rightarrow \quad \ln|x| + C_1 = \frac{-k_0}{a+t} + C_2$

lost solution: $x = 0$

$\ln|x| = \frac{-k_0}{a+t} + C_3$

$|x| = e^{\frac{-k_0}{a+t} + C_3} = Ce^{\frac{-k_0}{a+t}} \rightarrow \boxed{x = Ce^{\frac{-k_0}{a+t}}}$

d) @ $t=0$, $x_0 > 0$

$A = Ce^{\frac{-k_0}{a}} > 0$

$\lim_{t\to\infty} Ce^{\frac{-k_0}{a+t}} = Ce^{\frac{-k_0}{a + \text{big \#}}} = Ce^{\frac{-k_0}{\text{big \#}}} = Ce^0 = C$

$\boxed{\text{Population stabilizes to } C}$