Electromagnetic Torquing in Seeker Head Gyroscopes[a]

Anurag Pallaprolu,[b] Animesh Agarwal,[c] and Harshit Sharma[d]

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I. INTRODUCTION

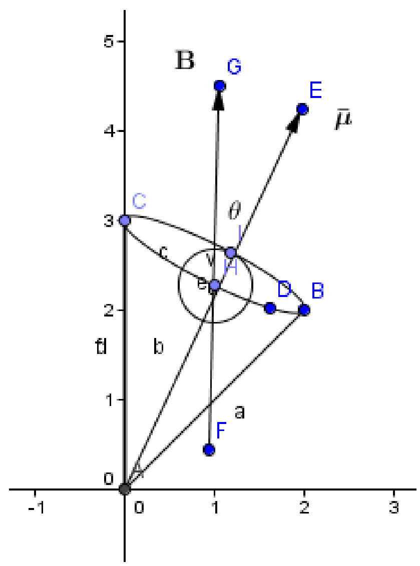

FIG. 1. The electromagnetic torquing problem: Schematic

In any standard anti-tank missile system which is guided by a Command Line-of-sight system, there is a mechanical gyro installed in the seeker head which plays an integral role in the inertial navigation process. The gyroscope needs to be kept running for the guidance system to determine the direction of motion of the missile. In real life, as opposed to theoretical situations, friction leads to systems damping out. In order to counter this, the gyro is circumvented with a reasonably constant current so that it behaves like a current carrying coil around an armature(which is the body of the gyroscope). When placed in an external magnetic field, based on the operational principle of electromagnetic motors, a torque is generated on the coil which drives the armature to precess. The complexities arise when this torquing process happens with the gyro precessing and nutating at the same time. The angle of the plane of the current carrying loop with the magnetic field will indeed change in a pretty complex way.

II. THE PAPER

We will first look at the standard torque-magnetic moment relationship for simple closed loops. If $\bar{\mu}$ is the magnetic moment of the gyroscope (which is $I_{gyro} \times Area_{cross\ section}$ but for simplicity let's just call it that), then the torque it would get from the solenoid field $\vec{\mathrm{B}}$ would be

$$\tau = \bar{\mu} \times \vec{\mathrm{B}}$$

Now, we know that for a solenoid, neglecting the bend in the field lines at the circumference, and also assuming that the dimensions of the gyroscope are small when compared to the diameter of the solenoid, the field is almost constant, $\vec{\mathrm{B}} = \mu_0 N_{turns} I_{soleniod}$, in which we shall neglect the subscripts for there is no ambiguity. Plugging this into the torque equation, and noting that $\tau = \mathcal{I}\bar{\alpha}$, we get

$$\bar{\mu}\mu_0 NI \sin\theta = \tau$$

$$\implies \mu\mu_0 NI \sin\theta = \mathcal{I}\bar{\alpha}$$

Noticing that staying here, we encounter a dead end, we can average over θ, the nutation angle(!) over one *precession* time period T_p, and so, we must average on the RHS too, leading us to,

$$\implies \mu\mu_0 NI \leq \sin\theta \geq = \mathcal{I} \leq \bar{\alpha} \geq$$

$$\implies \mu\mu_0 NI \frac{\int_0^{T_p} \sin\theta \mathrm{dt}}{T_p} = \mathcal{I}\frac{\int_0^{T_p} \bar{\alpha}\mathrm{dt}}{T_p}$$

Now, we know that the antiderivative of the angular acceleration is the frequency, in this case the frequency is in the precessing direction, since the torque is clearly in the direction of precession (And can be verified by the cross product rule). Changing a few things, we get

$$\implies \mu\mu_0 NI \int_0^{T_p} \sin\theta \mathrm{dt} = \mathcal{I}\bar{\omega}_p$$

$$N = \frac{\mathcal{I}\bar{\omega}}{\bar{\mu}\mu_0 I \int_0^{T_p} \sin\theta \mathrm{dt}}$$

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[b] Physics Department, BITS Pilani, Pilani.; Electronic mail: f2012405@pilani.bits-pilani.ac.in

[c] Electrical and Electronics Engineering, BITS Pilani, Hyderabad.; Electronic mail: coolanimesh50@gmail.com

[d] Mechanical Engineering, BITS Pilani, Pilani.; Electronic mail: f2012814@pilani.bits-pilani.ac.in

Hence, theoretically speaking, we can solve the conservation equations for the heavy top (need not necessarily be a symmetric case, in most scenarios, seeker head gyros tend to be rather disk like objects) and derive the nutation angle as a function of time which comes out as one of the solutions, but a simplistic assumption of low and constant nutation angles at high precession speeds can be made (*This assumption is actually proved in the book by Goldstein. As the precession and main axial rotation rates increase, the nutation is observed to decrease as is confirmed by obvious experiments with the childhood top*). Assuming that $\bar{\omega}_p$ is large enough to let $\theta \to \theta_N$, we get the following relations,

$$\bar{\mu}\mu_0 NI \sin\theta_N T_p = \mathcal{I}\bar{\omega}_p$$

$$\implies \bar{\mu}\mu_0 NI \sin\theta_N = \frac{\mathcal{I}\bar{\omega}_p^2}{2\pi} \tag{1}$$

We shall use a result from Goldstein, which is proved in the book. The idea is to relate the nutation angle to the constant frequency of rotation. The relation is given as

$$\Delta\cos\theta = \frac{\lambda\sin^2\theta_0}{\bar{\omega}_r^2}$$

where λ is a constant which will be introduced later, but now it is labeled so for clarity. If we leave the top from some angle say θ_0 then we can say

$$\sin\theta_N = \sqrt{1 - (\cos\theta_0 - \frac{\lambda\sin^2\theta_0}{\bar{\omega}_r^2})^2}$$

Plugging this into where we left off (1), we get

$$\bar{\mu}\mu_0 NI\sqrt{1 - (\cos\theta_0 - \frac{\lambda\sin^2\theta_0}{\bar{\omega}_r^2})^2} = \frac{\mathcal{I}\bar{\omega}_p^2}{2\pi} \tag{2}$$

Now, the precession frequency as a function of time is given by the following formula which is also proved in Goldstein and we adapt it directly from the book.

$$\omega_p = \frac{\beta}{2a}(1 - \cos at)$$

$$\omega_p^2 = \frac{\beta^2}{4a^2}(1 + \cos^2 at - 2\cos at)$$

$$\leq \omega_p^2 \geq = \frac{\beta^2}{4a^2}(1 + \frac{1}{2})$$

$$\leq \omega_p^2 \geq = \frac{3\beta^2}{8a^2}$$

Assuming the precession is fast, the average value can be taken as approximately the instantaneous value. Plugging this facet too in the previous equation (2)

$$\bar{\mu}\mu_0 NI\sqrt{1 - (\cos\theta_0 - \frac{\lambda\sin^2\theta_0}{\bar{\omega}_r^2})^2} = \frac{3\beta^2\mathcal{I}}{16\pi a^2}$$

Doing a binomial approximation on the square root, or also known as Bernoulli's approximation or Bernoulli's inequality, we can simplify the expression a bit more,

$$\bar{\mu}\mu_0 NI(1 - \frac{1}{2}(\cos\theta_0 - \frac{\lambda\sin^2\theta_0}{\bar{\omega}_r^2})^2) = \frac{3\beta^2\mathcal{I}}{16\pi a^2}$$

$$\implies \bar{\mu}\mu_0 NI(1 - \frac{1}{2}(\cos\theta_0) + \frac{\lambda\cos\theta_0\sin^2\theta_0}{\bar{\omega}_r^2})^2 = \frac{3\beta^2\mathcal{I}}{16\pi a^2} \tag{3}$$

Now, we introduce the value of λ to be $\frac{2\mathcal{I}_1 mgh}{\mathcal{I}_3^2\bar{\omega}_r^2}$ (*The value of λ is derived in Goldstein, it is actually used on the way to the derivation of the relation between the precession frequency and the rotation rate*) and plugging it in (3) and simplifying we get

$$\bar{\mu}\mu_0 NI(\frac{2\mathcal{I}_1 mgh\cos\theta_0\sin^2\theta_0}{\mathcal{I}_3^2\bar{\omega}_r^4} - \frac{1}{2}\cos^2\theta + 1) = \frac{3\beta^2\mathcal{I}}{16\pi a^2}$$

In this expression, $\mathcal{I}_1, \mathcal{I}_3$ are both moments of inertia across the auxiliary prinicipal axes, that is, the moments of inertia about the axes other than the Euler axis which is the axis of symmetry of the gyroscope. Making N the subject, we obtain our final glorious result

$$N = \frac{3\beta^2\mathcal{I}}{16\pi a^2\bar{\mu}\mu_0 I}(1 - \frac{1}{2}\cos^2\theta + \frac{2\mathcal{I}_1 mgh\cos\theta_0\sin^2\theta_0}{\mathcal{I}_3^2\bar{\omega}_r^4})^{-1}$$

The result of the exercise was worth the effort, as clearly, to increase the rotation speed even with deferring processes like precession and nutation, the only way out would be to increase the number of coils in the solenoid or indirectly, making the magnetic field stronger. Notice that the derivation was highly approximated, but the result is useful in not only calibrating the configuration but is also useful in studying the properties of the system for long time periods(relatively). Another interesting result which can be derived from the final equation is by taking further approximation of the initial nutation angle θ_0 to be small, in which case,

$$N = \frac{3\beta^2\mathcal{I}}{16\pi a^2\bar{\mu}\mu_0 I}(1 - \frac{1}{2}\cos^2\theta + \frac{2\mathcal{I}_1 mgh\cos\theta_0\sin^2\theta_0}{\mathcal{I}_3^2\bar{\omega}_r^4})^{-1}$$

$$\implies N = \frac{1}{\mu}(\lambda_1 - \frac{\lambda_2\theta_0^2}{\mathcal{I}\omega_r^4})$$

again, where λ_1 and λ_2 are suitable constants which can be correspondingly found out from the previous equations. Now, taking the circumferential constant current in the gyroscope frame to be I_g and the average radius of the gyro to be r, we get, the following

$$\implies N = \frac{1}{I_g\pi r^2}(\lambda_1 - \frac{\lambda_2\theta_0^2}{\mathcal{I}\omega_r^4})$$

$$\implies NI_g\pi r^2 = \lambda_1 - \frac{\lambda_2\theta_0^2}{\mathcal{I}\omega_r^4}$$

$$\implies \frac{\lambda_2\theta_0^2}{\mathcal{I}\omega_r^4} = \lambda_1 + NI_g\pi\mathrm{r}^2$$

$$\implies \omega_r = (\frac{\lambda_2\theta_0^2}{\mathcal{I}(\lambda_1 - I_gN\pi\mathrm{r}^2)})^{\frac{1}{4}}$$

Doing another round of Bernoulli approximations all around the expression, we get the following result

$$\bar{\omega}_r = (\frac{\lambda_2}{\mathcal{I}\lambda_1})^{1/4}\sqrt{\theta_0}(1 + \frac{1}{4}I_{gyro}N\pi r^2)$$

where the constants are dependent again on the gyrating parameters.

III. BIBLIOGRAPHY

Most of the calculations were made originally and useful theorems and lemmas were picked up mercilessly from Herbert Goldstein's omnipresent book *Classical Mechanics*. Graphics was done using the *Geogebra* graphing package.