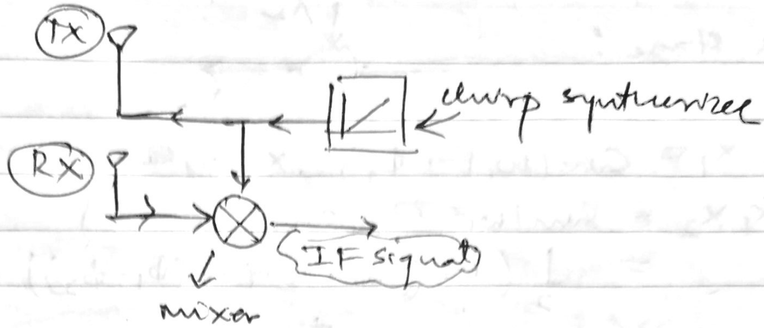
FMCW Radars:

* We have consistent phase information because TX and RX reside on the same ~~chip~~ radar board, so it is a classic monostatic radar.

IF Signal concept



As the mixer is located on the same SoC there is no "sampling time offset", however I am not sure about CFO and other PLL based issues

→ Quadrature mixer and complex baseband architecture in place of a traditional real mixer and real baseband architecture

TODO: what is the difference b/w a "real valued" mixer and baseband vs complex baseband architecture? How is this a novelty?
(Answer on the backside)

L-FMCW: An FM signal whose TX frequency follows a sawtooth pattern periodic with T_{chirp} period, whose $f_T(t)$ is given as

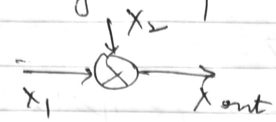
$$f_T(t) = f_c + \left(\frac{BW}{T_{chirp}}\right)t, \quad \Phi_T(t) = 2\pi f_c t + \pi\left(\frac{BW}{T_{chirp}}\right)t^2$$

Do recall that these RADAR boards are not used for any communications based use cases so they do not have any constellation based I/Q encoder and DAC unit.

"For moving objects the IF signal also has a Doppler component that depends on the relative velocity between the RADAR and the target. You can estimate this Doppler component, and hence the relative velocity, by performing a second FFT across chirps and looking at the phase shift in the IF signal from one chirp block to the next"

The "mixer" block ("Real" mixer)

consists of an analog multiplier followed by a low p[ass]
filter stage:



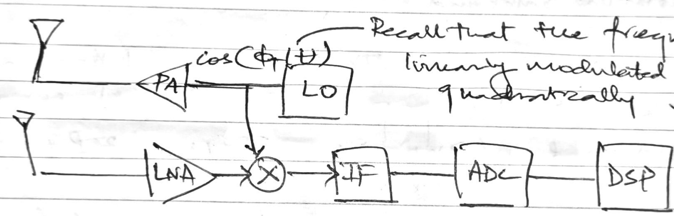
If $X_1 = \sin(\omega_1 t + \phi_1)$, $X_2 = \sin(\omega_2 t + \phi_2)$

$$X_1 X_2 = \sin(\omega_1 t + \phi_1)\sin(\omega_2 t + \phi_2)$$
$$= \frac{1}{2}\Big(\cos\big((\omega_1 - \omega_2)t + (\phi_1 - \phi_2)\big) + \cos\big((\omega_1 + \omega_2)t + (\phi_1 + \phi_2)\big)\Big)$$

$$LPF\{X_1 X_2\} = \frac{1}{2}\cos\big((\omega_1 - \omega_2)t + (\phi_1 - \phi_2)\big)$$

So the output of the mixer is a sinusoid whose frequency is the difference of the input frequencies.

Real baseband FMCW Receiver



Recall that the frequency is linearly modulated so phase is quadratically modulated

The output after mixing is a baseband signal that is real valued, the problem being the downconversion will also fold any "in-band" noise along with the IF signal content.

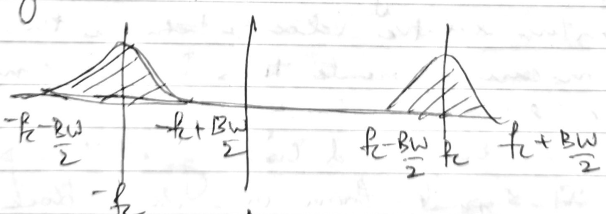
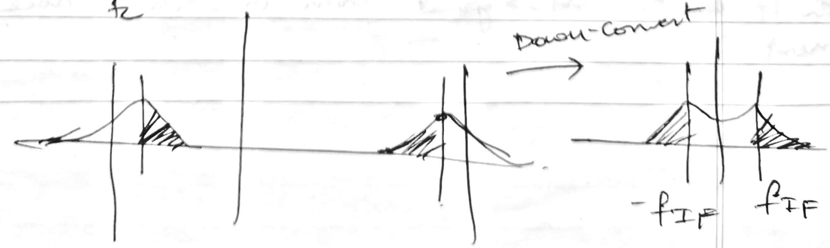
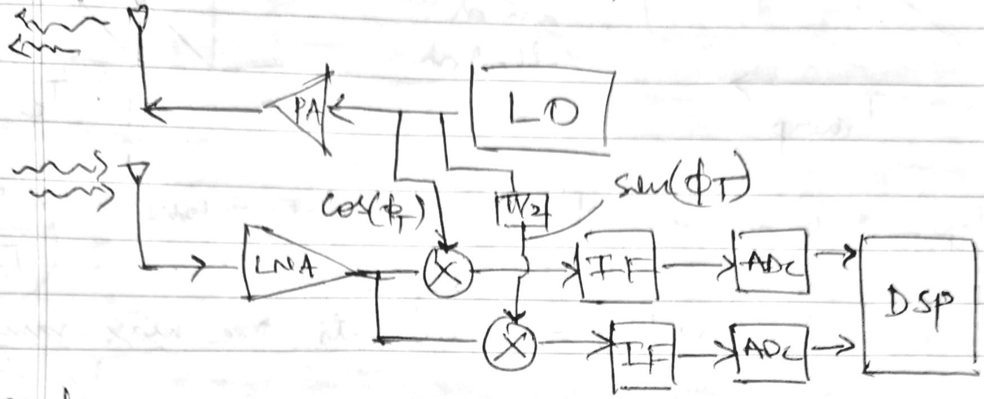


Image Band Noise Foldback



...band FMCW Receiver :

... some additional hardware we can effectively restrict IF filtering / "mixing" stage spectrum to a single side



Now, we have

$$= \sin(\omega_1 t + \phi_1)\cos(\omega_2 t + \phi_2) + j\sin(\omega_1 t + \phi_1)\sin(\omega_2 t + \phi_2)$$

$$= \frac{1}{2}\Big[\sin\big((\omega_1+\omega_2)t + (\phi_1+\phi_2)\big) + \sin\big((\omega_1-\omega_2)t + (\phi_1-\phi_2)\big) + j\cos\big((\omega_1+\omega_2)t + (\phi_1+\phi_2)\big) + j\cos\big((\omega_1-\omega_2)t + \phi_1 - \phi_2\big)\Big]$$

Applying LPF + LNA

$$= \sin\big((\omega_1-\omega_2)t + \phi_1-\phi_2\big) + j\cos\big((\omega_1-\omega_2)t + \phi_1-\phi_2\big)$$

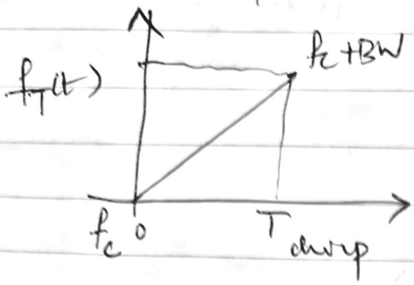
The FT is $= \frac{1}{2j}\big(\delta(\omega-\omega_{IF}) - \delta(\omega+\omega_{IF})\big)$

$$+ j\left(\frac{1}{2}\big(\delta(\omega-\omega_{IF}) + \delta(\omega+\omega_{IF})\big)\right)$$

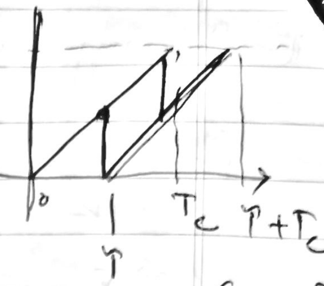
upon simplification it can be seen that only a single sideband at $\omega_2 + \omega_{IF}$ remains thus eliminating noise folding. In reality, this is just two real baseband RF chains in quadrature, and all the processing incurs the image band noise folding along indep. paths.

However, when the two bands are combined, the redundancy in the scenario allows for effective SNR improvement (as FT is linear, complex summation is linear, $\mathbb{C} \cong \mathbb{R}^2$)

IF chirp Signal:



upon transmission and reflection

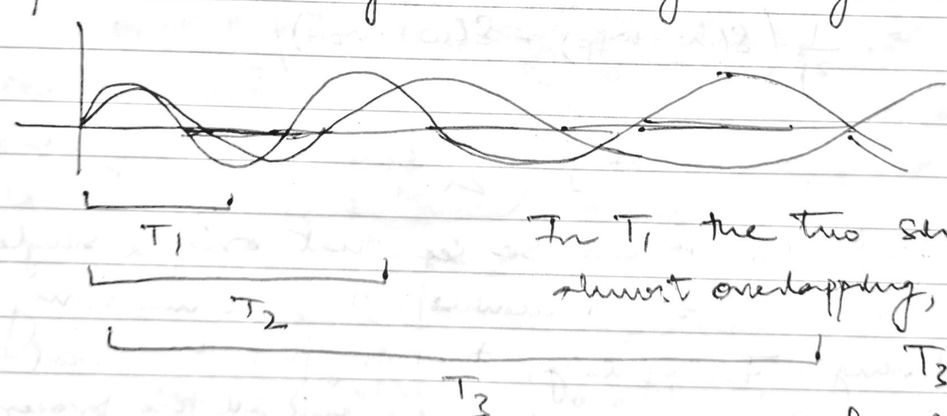


Obviously, it is assumed that $\tau \ll T_{chirp}$ (or else the IF signal would be null) — This sets the max range bound.

$\omega_2 - \omega_1 = \text{constant} = \left(\frac{BW}{T_{chirp}}\right)\tau$

The relationship between range resolution and bandwidth can be understood either through physical arguments based on total turnaround time, or, by noting the fact that two objects that are very close to each other will produce IF outputs at frequencies (or rather, the differences in the frequencies) that are very close to one another.

Observe that if your observation time horizon is minimal you cannot fully separate two sinusoids at very close frequencies as they take longer to go out of phase



In T_1 the two sinusoids are almost overlapping, whereas in T_3 they are clearly distinguishable

$\Rightarrow$ longer the T_{chirp}, better the range resolution

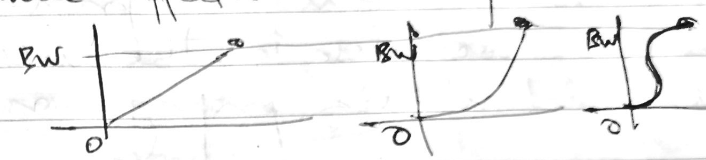
$\boxed{\text{We can have a longer } T_{chirp} \text{ with L-FMCW only by increasing the BW}}$

...that the maximal temporal difference between [two] "ping-back" echos from a chirp of T_{chirp} width is T_{chirp} itself ie, $\Delta T \leq T_{chirp}$

$$\Rightarrow \left(\Delta f_{IF}\right) \geq \frac{1}{T_{chirp}} \quad . \text{ But } \Delta f_{IF} = \frac{BW}{T_{chirp}}\left(\frac{2\Delta d}{c}\right)$$

$$\Rightarrow \left(\frac{BW}{\cancel{T_{chirp}}}\right)\left(\frac{2\Delta d}{c}\right) \geq \frac{1}{\cancel{T_{chirp}}} \Rightarrow \boxed{\Delta d \geq \frac{c}{2BW}}$$

So two chirps with same effective BW span have same range resolution



<u>How does the chirp slope impact range resolution?</u>

It does not impact the range resolution, but it surely impacts the max range. As the IF signal is sent through an ADC for further processing there is an ~~[illegible]~~ LPF to prevent aliasing artifacts during ADC quantization (making the IF signal bandlimited)

Thus, the max f_{IF} is given by the sampling rate of the ADC, given by the Nyquist window size

$$f_s \geq \left(\frac{BW}{T_{chirp}}\right)\left(\frac{2 d}{c}\right)$$

$$\Rightarrow \boxed{d_{max} = \frac{c F_s T_{chirp}}{2BW}}$$

So a higher slope for a fixed ADC implies a smaller max ranging, although this is strictly not a physical artifact but rather an issue with analog RF quantization

In summary, if you perform FFT on ADC samples your frequency axis is just a scaled multiple of the range = $f_{ADC} \Leftrightarrow 2\left(\frac{BW}{T_{chirp}}\right)\left(\frac{range}{c}\right)$. Thus, by appropriately back-scaling ADC FFT you get the so called "range-FFT".

If you sample ADC at higher rate you can grow the Nyquist window and consequently uncover objects at larger distances, provided the ping-back chirp is not too close to the noise floor (due to signal degradation via propagation loss.

→ ~~Bu~~ All objects on a circle around the TRx have an overlapping FFT peak. You'd need velocity FFT resolution to further distinguish the peaks.

IF Phase Variation

We have confirmed that ping-back time

$\cancel{\left(\frac{BW}{T_{chirp}}\right)}$ $\tau = \frac{2d}{c}$ leads to $\Delta f = \frac{2\Delta d}{c}\left(\frac{BW}{T_{ch}}\right)$

If the object moves $\Delta d << d$ then this would lead to a very small $\Delta f' \neq \Delta f$ but a noticeable phase shift

$$\Delta\Phi = 2\pi f_{IF} \Delta\tau = \frac{4\pi f_{IF} \Delta d}{c}$$

Notice that $f \approx f_{IF}$

$$\boxed{\Delta\Phi = \frac{4\pi \Delta d}{\lambda}}$$

$\boxed{\lambda = \text{wavelength of IF signal}}$ * *

ple If an object shifts by $\Delta d = \lambda/4$ we have $\Delta\Phi = \pi$, and $\Delta f = \cancel{\frac{(BW)}{2}} \; \frac{2(\lambda/4)}{f_c \lambda} \cdot \frac{BW}{T_{ch}}$

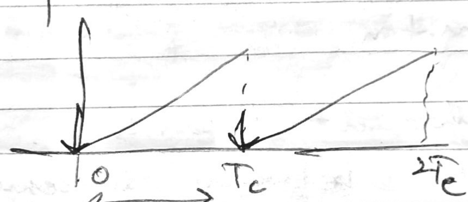
$$= \frac{BW}{2 f_c T_{ch}}$$

which is kind of small if f_c is large!

How to measure velocity of an object?

Send two consecutive chirps! Even though the range-FFT peaks are near each other, the phase at the peak would show a shift that can estimate Δd.

If two chirps were sent with a space of T_c



And if the distance traveled in that duration is Δd then $V_{avg} = \frac{\Delta d}{T_c}$. Now we know,

$$\Delta\Phi = \frac{4\pi \Delta d}{\lambda} \Rightarrow \boxed{\Delta\Phi = \frac{4\pi V_{avg} T_c}{\lambda}}$$

$$\Rightarrow V_{avg} = \frac{\lambda \Delta\Phi}{4\pi T_c}$$

phases and phase shifts need consistent PLLs on both sides of TX/RX or a single TRx

In this aspect, mmWave transceivers are perfect for heart rate / chest cavity velocity / frequency measurements continuing the chirp sequence, if we collect Φ we can basically study the velocity/small displacement profile over units of T_{chirp}.

That is, a single chirp can be used to estimate a zero velocity or a static object, and all subsequent chirp blocks will be redundant measurements. However if an object moves by $\Delta d << d$ such that $f_T(t+T)$ does not significantly change or deviate from $f_T(t)$ then the $\Delta\Phi$ of the IF signal can yield valuable information regarding the instantaneous velocity of the object.

Note that $\Delta\Phi$ is computed across successive chirps so we will need atleast two chirps for velocity estimation.

+ chirp blocks.

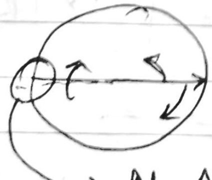
But we can estimate range with just a single CB.

<u>Direction of motion estimation</u>

How can we detect movement towards or away from the RADAR? The sign of $\Delta\Phi$ between successive chirp blocks gives us the direction of the velocity vector. (why?)
Because $\Delta\Phi$ is proportional to Δd and Δd is a vector quantity / displacement around the mean (d_0)

Physically, the second chirp would measure a lesser distance or range if the object moves closer to the radar so it would have a smaller net phase than the mean position $\Longrightarrow \Delta\Phi < 0$ when object moves towards the RADAR

$$\Longrightarrow |\Delta\Phi|_{max} = \pi$$



$$\Longrightarrow |V_{max}| = \frac{\lambda}{4T_{chirp}}$$

At $\Delta\Phi = \pi$ we are unable to unambiguously estimate the direction of motion.

~~FFT : If you take the FFT of $\Delta\Phi$ of each chirp~~

So to recap, we compute the phase spectrum of the FFT of the IF signal of each chirp block and then calculate phase shifts relative to the original position to compute the small scale motion parameters.

~~In other words $\Delta\Phi(t)$~~

Recall that FFT or FT of a complex exponential (or) a phasor $\mathcal{F}\{e^{j\omega_0 t}\} = \delta(\omega - \omega_0)$

Proof: $\mathcal{F}^{-1}(\delta(\omega-\omega_0)) = \int_{-\infty}^{\infty} \delta(\omega-\omega_0) e^{j\omega t} d\omega = e^{j\omega_0 t}$ (QED).

Forward proof involves approximating Fourier integral as Riemann partial sums and showing that the limiting case is a summation of complex exp. discrete i.e. Dirichlet Kernel $\Rightarrow$ sinc of 0 BW $\Rightarrow$ Dirac delta

Now consider > 1 chirp block with two objects moving at V_1 and V_2. They would lead to two interfering "phasors" $A_1 e^{j\Delta\phi_1(t)} + A_2 e^{j\Delta\phi_2(t)}$. where

$$\Delta\phi_1 = \frac{4\pi V_1 T_c}{\lambda_{IF}}, \quad \Delta\phi_2 = \frac{4\pi V_2 T_c}{\lambda_{IF}}$$

So if we take the Fourier Transform of this we should obtain $\delta(\omega - \Delta\phi_1) + \delta(\omega - \Delta\phi_2)$ or some sinc-style analogue. However, our samples over which FT is being done are discrete chirp blocks, unlike range estimation where it was a continuous IF wave in a single chirp block.

This implies that velocity resolution will now depend on the # of chirp blocks observed instead of the total duration of the chirp itself

Theorem: Prove that $A_1 e^{j\phi_1} + A_2 e^{j\phi_2}$ observed over N points for FFT has the resolution bound

$$\boxed{\phi_2 - \phi_1 \geq \frac{2\pi}{N}}$$

Proof is left as an exercise for now.....

So if $\Delta\phi_1 - \Delta\phi_2 \geq \frac{2\pi}{N} \Rightarrow \frac{4\pi(V_1 - V_2)T_{chirp}}{\lambda_{IF}} \geq \frac{2\pi}{N}$

$\Rightarrow \Delta V \geq \frac{\lambda_{IF}}{2(NT_{chirp})}$ Note that $NT_{chirp} = T_{total} = T_f$

$\Rightarrow \boxed{\Delta V \geq \frac{\lambda}{2T_f}}$ Velocity resolution is inversely proportional to the total # of chirp blocks observed.

Range - Doppler FFT :

So if we have a 2D matrix of chirps and chirp blocks

$$
\begin{array}{c} \\ \text{ADC 1} \\ \text{ADC 2} \\ \vdots \\ \text{ADC M} \end{array}
\begin{array}{c}
\begin{array}{ccc} \text{Chirp 1} & \text{Chirp 2} \; \ldots & \text{Chirp N} \end{array} \\
\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{M1} & \cdots & \cdots & a_{MN} \end{bmatrix}
\end{array}
$$

this gives us the range and doppler FT with one single 2D FFT operation.

→ V_{max} specifies chirp duration T_{chirp}.

→ ~~xxx~~ Δd resolution determines BW

→ ΔV_{res} determines T_f or T_{total}

→ d_{max} is fully specified if above are fixed.

→ ~~xxx~~ d_{max} determ specifies ADC sampling ~~xxx~~ rate for Nyquist sampling and anti-aliasing filter design

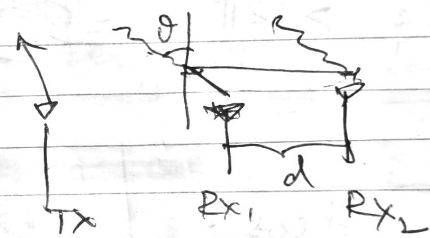
Angle Estimation: The Third Dimension

To separate two objects on a circle we used the velocity dimension under the assumption that there is separability on the Doppler-FFT when range information is redundant.

But if the objects on a circle have the same speed and ~~orientation~~ directionality then they will appear as the same point in Range-Doppler 2D FFT. In such situations we need to rely on the AoA/AoD/ Bearing angle dimension.

This requires spatial redundancy / diversity i.e., multiple RX antennas / an antenna array!

It is easy to group this as the spatial analogue of the chirp block based Doppler FFT, which considers phase shifts temporally.



Consider a 2RX case operating in a bi-static mode i.e the TX and RX are not in the same spatial vicinity. Also we assume far field arrivals at RXes.

If we compute the 2D range-doppler FFTs across RX_1 and RX_2 we will see that their phase difference yields us the path difference compensation for the incoming plane waves

$$\Delta\Phi_{12} = \left(\frac{2\pi}{\lambda}\right)(d\sin\theta) \implies \theta = \arcsin\left(\frac{\lambda\,\Delta\Phi_{12}}{2\pi d}\right)$$

And once again $|\Delta\Phi_{12}|_{max} = \pi$ and therefore

$$\boxed{\theta_{max} = \arcsin\left(\frac{\lambda}{2d}\right)}$$ Far field bearing limit angle

Similarly, if we have N RX antennas, we can compute $\Delta\Phi_i$ for each of $i = 2, \ldots, N$ ~~[illegible]~~ to get bearing angle estimation for multiple objects

The theory is similar to that of Doppler-FFT except it degenerates to the fourier spectrum only for (a) Far Field condition / zero wavefront curvature / Plane Wave Arrival (b) Uniform Linear Arrays.

For uniform circular arrays it degenerates to the Fourier-Bessel Transform or Hankel Transform

$$\hat{f}(q) = 2\pi \int_0^\infty f(r) J_0(2\pi q r)\, r\, dr.$$

Consequently, ~~[illegible]~~

$$\Delta\Phi_i > \frac{2\pi}{N} \Rightarrow \frac{2\pi d}{\lambda}\left(\sin(\theta + \Delta\theta) - \sin\theta\right) > \frac{2\pi}{N}$$

$$\Rightarrow \boxed{\Delta\theta > \frac{\lambda}{N d \cos\theta}}$$

$$N d = l_{array}$$

$$\Rightarrow \Delta\theta_{res} = \frac{\lambda}{l_{array}\cos\theta}$$

$$\boxed{(\Delta\theta_{res})_{max} = \frac{\lambda}{l_{array}}},$$

The Abbe Diffraction Limit