# Bayesian Hierarchical Model for Characterizing Electric Vehicle Charging Station Arrivals and Departures

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https://github.com/apalom/bayesian\_countModel

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### 1 Introduction

Electric vehicle (EV) charging load presents a new power demand to power systems all over the world. EV charging load poses overloading risks for distribution systems operation [1], while providing opportunities for distribution system operators (DSOs) to take advantage of EV charging flexibility for enhancing grid operation [2, 3]. Energy flexibility is inherent to loads in which energy demand can be stored, deferred or both. Such flexibility offers a demand-side approach to real-time load balancing. The approach in [2] harnesses EV flexibility to serve real-time fast ramping needs resulting in significant cost savings. The temporal uncertainty presented by EV charging load, however, pose challenges to the utilization of EV charging load as a flexibility asset in power systems [6].

Control strategies presented to optimally harness energy flexibility rely on the premise that demand is known or can be assumed [2, 3]. For EV charging load, the prospect of modeling EV arrivals to and departures from publicly sited EV charging stations is inhibited by a dearth of available data. EV charging load presents a unique spatial-temporal uncertainty to energy flexibility utilization as publicly sited EV chargers are dispersed across the power distribution system to serve the uncertain charging needs of an unknown number of drivers. In the United States, EV adoption is expected to grow from 500,000 in 2016 to up to 21 million in 2030 and additional charging infrastructure is needed to serve the expansion charging demands [7]. Residential participation in demand-side management and energy trading is formulated as a Bayesian game where connected EVs act as trading nodes [10]. The work in [2, 10, 9] relies on assumed EV charging characterization. The reliance on such deterministic approaches to EV charging load characterization diminishes the benefit that EV charging load flexibility can yield to the power system in practice.

# 2 The Proposed Bayesian Hierarchical Model

Bayesian hierarchical modeling is a Bayesian approach to stochastic process modeling which recognizes a hierarchy of latent random variables that influence the realization of the representative probability distributions.

### 2.1 Bayesian Inference

A Bayesian model is defined by a likelihood function  $\pi(X|\Theta)$  and a prior  $\pi(\Theta)$  with data observations X and distribution parameters  $\Theta$  [12]. The likelihood function  $\pi(X|\Theta)$  is a probability distribution function which maps the probability of a realization to parameters (1). Maximization of the likelihood function results in a set of parameters  $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$  which best define the target distribution to model the data generating process. Such a maximization results in  $\Theta_* = argmax_{\Theta} \pi(\Theta|X)$  where  $\Theta_*$  are the parameters which maximize the likelihood function.

$$\pi(\Theta|X) = \prod_{n=1}^{N} \pi(x_i|\Theta) \tag{1}$$

The likelihood  $\pi(\Theta|X)$  function, generally defined in (1), expresses the likelihood that a given set of data X are realized from the target distribution parameterized by  $\Theta$ . Importantly then, is the selection of the target distribution, the likelihood, for which to stochastically characterize the data generating phenomenon under study. The prior distribution  $\pi(\Theta)$  defines the distribution of parameters before any data is observed. The posterior  $\pi(\Theta|X)$ , proportional to the product of the likelihood and prior as shown in 2, is then a probability distribution of parameters conditioned upon input data that captures everything known and unknown about the phenomenon under study.

$$\pi(\Theta|X) = \frac{\pi(X|\Theta) \cdot \pi(\Theta)}{\int \pi(X|\Theta) \cdot \pi(\Theta)d\Theta}$$
 (2)

The posterior predictive distribution  $\pi(x^*, \Theta|X)$ , in (3), provides the probability distribution of a new data observation  $x^*$  marginalized over the posterior distribution.

$$\pi(x^*|X) = \int_{\Theta} p(x^*|\Theta) \cdot p(\Theta|X) \cdot d\Theta$$
 (3)

In this manner Bayesian inference allows predictive distributions to be updated as additional evidence becomes available.

## 2.2 Hierarchical Model Pooling

Bayesian hierarchical modeling provides a framework which results in posterior function definition subject to a set of explanatory parameters. In the proposed model, we observe that a relationship exists between the number of EV arrivals and the hour of the day and thus, we cluster the data to the nearest whole hour of the charging session initiation. Within this hierarchical framework we introduce the concept of partial pooling. The total number of charging sessions initiated in each hour of the data is illustrated in Figure 1. A relatively large number of data observations, EV arrivals, are available in the study dataset to characterize arrivals around "data-rich" periods around 08:00 and 13:00. On the other hand, there exist very few data observations for early morning EV arrival characterization. This temporal disparity in data availability can lead to under/overfitting EV arrivals across hours when modeling each hour in aggregate, complete pooling, or as completely independent, no pooling. Partial pooling permits the sharing of information, via commonly defined hyper-parameters,

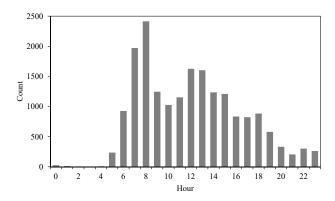


Figure 1: Aggregate hourly EV arrival frequency.

across nearby data clusters and implies that the data observed is the result of the same data generating process. As such, EV arrivals in any hour inform the stochastic characterization of arrivals in nearby hours leading to more robust learning across real-world data.

### 2.3 Hierarchical Model Construction

The proposed hierarchical model, illustrated in Figure 2, is abstracted into a hierarchy of distributions from the hyper-parameters, informed by the input data, a Gamma prior, the conjugate for the Poisson likelihood, and finally the predicted values  $y_{t,i}$ . Twenty-four hourly distributions are anticipated by the prior to be updated by the observations informing the likelihood from the training dataset. We can describe the model top-down from evidence, or

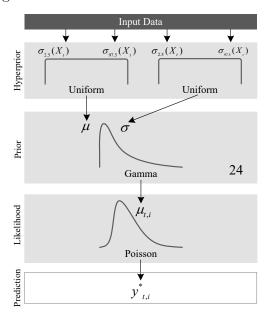


Figure 2: Bayesian hierarchical model for EV arrival and departure count modeling.

input training data, to prediction. The proposed model comprises two uniform hyper-prior characterizing the parameters for the Gamma prior. Realizations of the hyper-parameter distributions (5) and (6) define the Gamma prior distribution (7). Realizations of the Gamma distribution then provide the  $\mu_{t,i}$  parameterization for the Poisson likelihood (8). The Poisson

distribution is a discrete, non-negative distribution of the number of events that can be expected to occur in a given period of time according to the average rate of occurrence. The Poisson distribution is often employed in literature to model EV arrival counts to charging stations [14, 15, 16] and is employed in this work to model arrival and departure count data.

$$\mathbf{E}_{\pi}[f] = \int_{\Theta} f(\Theta) \cdot f(\Theta|X) \cdot d\Theta \approx \frac{1}{N} \sum_{i=1}^{N} f(\Theta_i)$$
 (4)

The proposed model with a Poisson likelihood and Gamma prior can be written compactly as in (5)-(8).

$$\hat{\mu}_i = Uniform\left(\underline{X}_{t_{2.75}}, \overline{X}_{t_{97.25}}\right) \tag{5}$$

$$\hat{\sigma}_i = Uniform\left(\underline{X}_{t_{2.75}}, \overline{X}_{t_{97.25}}\right) \tag{6}$$

$$\mu_{t,i} = Gamma\left(\hat{\mu}_i, \hat{\sigma}_i\right) \tag{7}$$

$$y_{t,i}^* = Poisson\left(\mu_{t,i}\right) \tag{8}$$

Finally, the posterior distribution is simulated from the prior and likelihood to yield predictions  $y_{t_i}^*$  using Markov-Chain Monte Carlo (MCMC) sampling. The construction of the Bayesian model with a Gamma prior 9 and Poisson likelihood 10 results in the following posterior 11.

$$p(\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha - 1} e^{-\beta \lambda}, \quad \lambda > 0$$
 (9)

$$L(\lambda|\mathbf{x}) = \prod_{i=1}^{n} \frac{e^{-\lambda} \lambda^{x_i}}{x_i!} = \frac{e^{-n\lambda} \lambda \sum x_i}{\prod_{i=1}^{n} (x_i!)}$$
(10)

$$\pi(\lambda|\mathbf{x}) \propto \lambda^{\sum x_i + \alpha - 1} e^{-(n+\beta)\lambda}, \quad \lambda > 0$$
 (11)

The posterior, in 11, is a Gamma distribution with parameters  $Gamma(\sum x_i + \alpha, n + \beta)$ .

# 3 Numerical Analysis

The Bayesian hierarchical model discussed in 2.2 is implemented in Python as a PyMC3 model [19]. One year of data is taken from the EV charging network in Salt Lake City, Utah to train and test the proposed model. Ultimately, the proposed model seeks to accurately predict the arrival of EVs to charging stations in Salt Lake City. Predictions are made by MCMC sampling of the posterior distribution and then compared to the testing dataset.

The data employed in this work is collected from publicly sited EV charging stations in Salt Lake City, Utah. In the 2018 dataset used, there were 46,000 EV charging sessions across 64 publicly sited chargers. Of concern in this data, is the aggregate arrival and departure of EVs to charging stations per hour. Thus, individual charging sessions are aggregated per hour per day to arrive at the number of EV arrivals to chargers.

### 3.1 Model Implementation

The hierarchical model is implemented in python as a PyMC3 model using the No-U Turn sampling algorithm. Each of 4 sampling chains are tuned with 4,000 samples. Then an additional 1,000 samples per chain are taken to define the posterior. A trace which moves wildly or gets stuck indicates that sampling failed to cover the parameter space adequately. A key strength of Bayesian modeling is its application of probability distributions for all parameters as opposed to point-value results as would be expected by frequentist techniques. Thus, a trace which does not sample comprehensively will fail to yield reliable results.

The resulting trace and diagnostics indicate robust parameter space exploration. A subset of  $\mu_t$  trace plots for the hours of 04:00, 12:00 and 20:00 are shown in figure 3. Quantitatively,

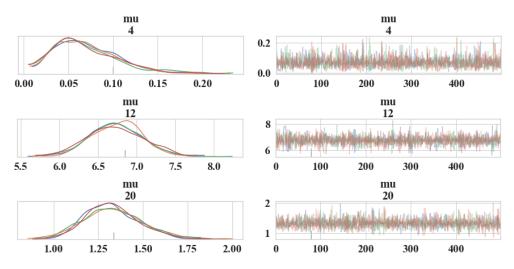


Figure 3: Trace sampling exploration of credible parameter space.

the Gelman-Rubin  $\hat{R}$  statistic can evaluate model trace convergence [21]. The Gelman-Rubin diagnostic analyzes the sampling difference between Markov chains for each model parameter where a value close to 1 indicates that the value observations resulting from sampling has converged to the target distribution. The values shown in 1 illustrate the nearness of the Markov chain sample distributions to the target distributions. Across all hours of model training, we see  $\hat{R}$  values very close to 1.0, indicating model convergence.

Table 1: Gelman-Rubin diagnostic results for the EV arrival expectation,  $\mu_t$ , parameter.

Hour	Ŕ	Hour	Ŕ	Hour	Ŕ	Hour	Ŕ
0	1.000223	6	0.999920	12	0.999910	18	0.999910
1	0.999973	7	0.999901	13	0.999981	19	0.999904
2	1.000602	8	1.000216	14	0.999995	20	1.000499
3	1.000295	9	0.999919	15	1.000011	21	0.999913
4	0.999928	10	1.000547	16	0.999974	22	0.999960
5	1.000073	11	0.999927	17	0.999900	23	1.000230

### 3.2 EV Arrival Prediction

The prediction of EV arrivals per hour is realized from the expected value of the stochastic process resulting from model training. The expectation of EV arrivals from the posterior distributions is overlapped with the expectation from the testing data in Figure 4. Comparison shows nearly identical EV arrival expectation and standard deviation, shown as shaded bounds, from both the Bayesian hierarchical model posterior predictive counts (PPC) and testing data. Finally, we can address the total prediction error across the test data set by

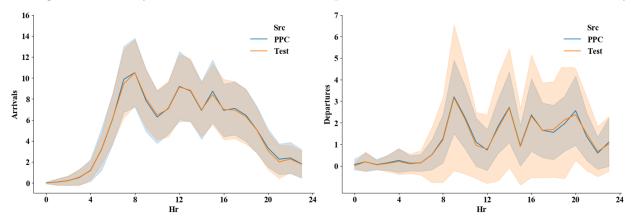


Figure 4: Hourly expectations distribution resulting from model prediction, PPC, and testing data.

measuring symmetric mean absolute percentage error (12) is employed to quantify the actual  $A_t$  to predicted  $P_t$  error.

$$SMAPE = \frac{100\%}{n} \sum_{t=1}^{n} \frac{|P_t - A_t|}{(|A_t| + |P_t|)/2}$$
 (12)

Overall, arrival model prediction is 10.70% SMAPE and departure model prediction is of 15.35% where the departure test data exhibits greater variance than predicted.

# 4 Concluding Remarks

In conclusion, the hierarchical model proposed handles inter-hourly heterogeneity in data availability and count expectation successfully. Partial-pooling recognizes that the parameters defining the hourly EV arrival process are realizations from a population distribution of parameters and are neither completely dependent, pooled, or completely independent, unpooled. This work demonstrates the efficacy of Bayesian techniques to stochastically characterize the availability of EV charging load. Utilization of EV flexibility demands such understanding to inform real-time operational decision making and market innovation.

### 4.1 Future Work

Future work seeks to quantify the energy intensity of charging sessions (a critical aspect to EV charging flexibility). A similar Bayesian approach requires learning a likelihood function that accurately captures the phenomenon under study.

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