

FOCS Homework 16

You may edit your answers into this file, or add a separate file in the same directory.

If you add a separate file, please include the following at the top:

Student Name: Frankly Olin [change to your name]

Check one:

☒ I completed this assignment without assistance or external resources.

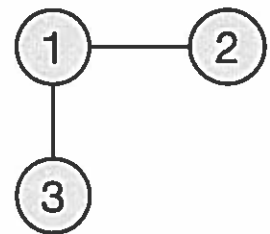
☐ I completed this assignment with assistance from ____
and/or using these external resources: ____

I. (Undirected) Graphs

A **graph** is a set of points (called **nodes** or **vertices**), connected pair-wise by lines (called **edges**).

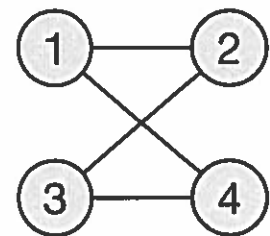
Formally, a graph is an ordered pair $G = (V, E)$: V is a set of vertices; E is a set of edges; and an edge is a set of the nodes that are its ends.

Example:

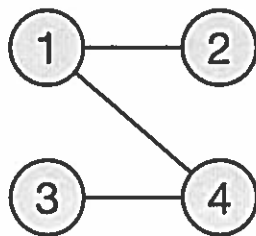


$G = (\{1, 2, 3\}, \{\{1, 2\}, \{1, 3\}\})$

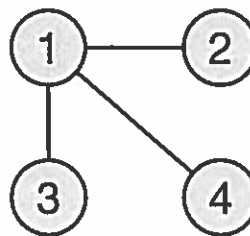
1. Match the diagrams to the graph structures.



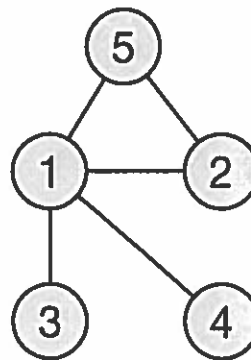
(a)



(b)



(c)



(d)

- (i) $G = (\{1, 2, 3, 4\}, \{\{1, 2\}, \{1, 4\}, \{3, 4\}\})$ *b*
- (ii) $G = (\{1, 2, 3, 4\}, \{\{1, 2\}, \{1, 4\}, \{2, 3\}, \{3, 4\}\})$ *a*
- (iii) $G = (\{1, 2, 3, 4, 5\}, \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 5\}\})$ *d*
- (iv) $G = (\{1, 2, 3, 4\}, \{\{1, 2\}, \{1, 4\}, \{1, 3\}\})$ *c*

2. Cycles

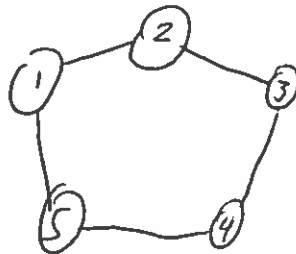
Two vertices are **adjacent** if there's an edge between them.

A **cycle** is a sequence of adjacent vertices that begin and end with the same vertex.

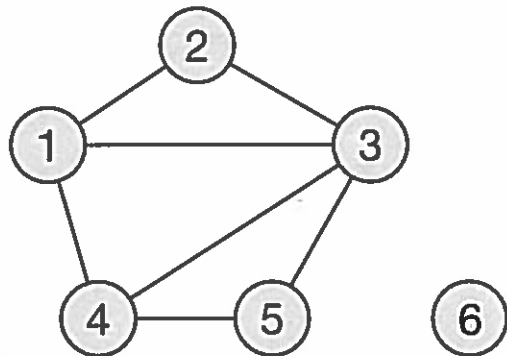
Which of (1a-d) contains a cycle? For those graphs that contain a cycle, what is the cycle?

3. Draw a diagram for this graph.

$$G = (\{1, 2, 3, 4, 5\}, \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{1, 5\}\})$$



4. Write the graph (V, E) for this diagram.



This is a single graph. It is not a **connected graph**.

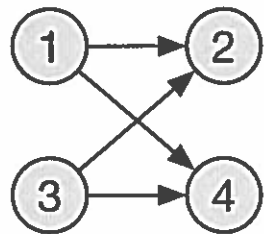
$$G = (\{1, 2, 3, 4, 5\}, \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{1, 5\}, \{1, 3\}, \{2, 4\}, \{3, 5\}, \{4, 5\}\})$$

II. Directed Graphs

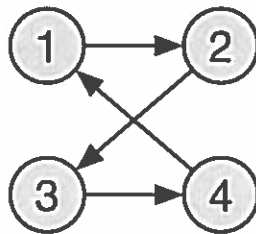
A **directed graph** ("digraph") is a set of vertices, together with arrows that begin and end on a vertex.

Formally, $G = (V, A)$, where A is a set of tuples (V_i, V_j) .

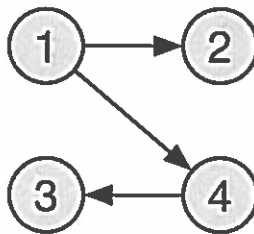
5. Match the diagrams to the graphs.



(a)



(b)



(c)

(i) $G = (\{1, 2, 3, 4\}, \{(1, 2), (1, 4), (3, 2), (3, 4)\})$ *a*

(ii) $G = (\{1, 2, 3, 4\}, \{(1, 2), (1, 4), (4, 3)\})$ *c*

(iii) $G = (\{1, 2, 3, 4\}, \{(1, 2), (4, 1), (2, 3), (3, 4)\})$ *b*

6. Directed acyclic graphs

A cycle in a directed graph is a sequence of vertices, that starts and ends with the same vertex, where for each pair of consecutive vertices in the sequence there's an arrow from the first to the second.

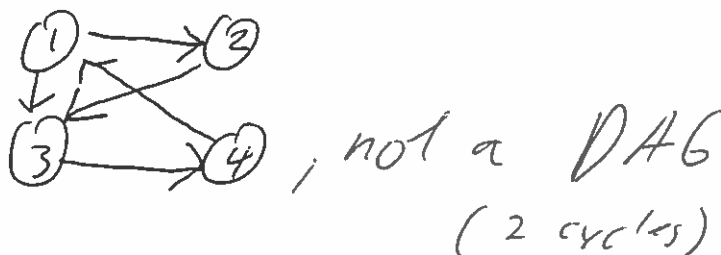
A **directed acycle graph** (DAG) is a digraph that doesn't contain cycles.

Which of (5a-c) are DAGs? *a and c*

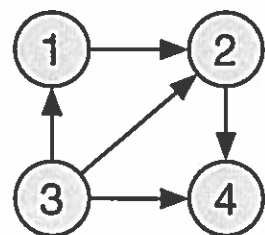
7. Draw a diagram for this graph.

$G = (\{1, 2, 3, 4\}, \{(1, 2), (2, 3), (3, 4), (4, 1), (1, 3)\})$

Is this graph a DAG?



6. Write the graph (V, A) for this diagram.



$G = (\{1, 2, 3, 4\}, \{(1, 2), (2, 4), (3, 1), (3, 2), (3, 4)\})$

7. Adjacency matrices

An **adjacency matrix** is a square matrix A such that $A_{i,j} = 1$ iff there is an arrow from vertex i to vertex j .

Write the adjacency matrix for the graph in (5a).

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

III. (Optional) Reading

One of:

- Cormen *et al.* Section 6 "Graph Algorithms", Chapters 22-24.
- Equivalent material in your favorite data structures text: graphs, Hamiltonian paths, algorithms to compute spanning trees, shortest paths.
- Wikipedia: [Graph](#), [graph data type](#), [directed graph](#), [adjacency matrix](#), [Hamiltonian path](#)

