Andrew

1\. 1. $p \to q$

2\. $p \to r$

prove $p \to (q \land r)$

3\. $(p \to q) \land (p \to r)$, 1, 2,

4\. $(\sim p \lor q)$, 1, conditional

5\. $(\sim p \lor r)$, 2, cond

6\. $(\sim p \lor q) \land (\sim p \lor r)$, 4, 5,

~~7. $((\sim p \lor q) \land \sim p) \lor ((\sim p \lor q) \land r)$~~

~~8. $((\sim p \land \sim p) \lor (\sim p \land q)) \lor ((\sim p \land r) \lor (q \land$~~

~~9. $(\sim p \lor (\sim p \land q)) \lor ((\sim p \land r) \lor (q \land r))$~~

~~10. $(\sim p \lor (q \land r)) \lor ((\sim p \land q) \lor (q \land r)) \lor ((\sim p \land q) \lor ($~~

~~$\lor (\sim p \lor (\sim p \land r))$, 1, distr~~

7\. $\sim p \lor (q \land r)$, 6, distributive

8\. $p \to (q \land r)$, 7, conditional

1\. 1. $p \to (q \lor r)$

2\. $p \to (q \lor \sim r)$

prove $p \to q$

3\. $(p \to (q \lor r)) \land (p \to (q \lor \sim r))$, 1, 2, conjunc

4\. $(\sim p \lor (q \lor r))$, 1, conditional

5\. $(\sim p \lor (q \lor \sim r))$, 2, conditional

6\. $(\sim p \lor (q \lor r)) \land (\sim p \lor (q \lor \sim r))$, 4, 5, conjunc

7\. $((\sim p \lor q) \lor r) \land ((\sim p \lor q) \lor \sim r)$, 6, associ

8\. $(\sim p \lor q) \lor (r \land \sim r)$, 7, distributive

~~9. $(\sim p \lor q) \lor F$, 8, idempotent~~

9\. ~~$\sim\sim r$, 8, negation~~ $(\sim p \lor q) \lor F$, 8, negation

10\. $\sim p \lor q$, 9, elimination

11\. $p \to q$, conditional

2\. 1.

p	q	r	$p \to q$	$p \to r$	$q \land r$	$p \to q$
0	0	0	1	1	0	1
0	1	0	1	1	0 0	1
1	0	0	0	0	0	0
1	1	0	1	0	0	0
0	0	1	1	1	1	1
0	1	1	1		0	1
1	0	1	0			1

2. 2.

p	q	r	$\sim r$	$q \vee r$	$p \to (q \vee r)$	$q \vee \sim r$	$p \to (q \vee \sim r)$
0	0	0	1	0	1	1	1
0	1	0	1	1	1	1	1
1	0	0	1	0	0	1	1
1	1	0	1	1	1	1	1
0	0	1	0	1	1	0	1
0	1	1	0	1	1	1	1
1	0	1	0	1	1	0	0
1	1	1	0	1	1	1	1

$p \to q$
1
1
0
1
1
1
0
1

3. 1.

p	q	r	$p \wedge q \wedge r$	$p \vee q$	$(p \wedge q \wedge r) \Rightarrow (p \vee q$
0	0	0	0	0	1
0	1	0	0	1	1
1	0	0	0	1	1
1	1	0	0	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	0	1	1
1	1	1	1	1	1

satisfiable tautology

~~(p→q~~
((p→q

3. 2.

p	q	r	p→q	q→r	(p→r)	(p→q)∧(q
0	0	0	1	1	1	1
0	1	0	1	0	1	0
1	0	0	0	1	0	0
1	1	0	1	0	0	0
0	0	1	1	1	1	1
0	1	1	1	1	1	1
1	0	1	0	1	1	0
1	1	1	1	1	1	1

satisfiable tautology

3. 3.

p	q	p→q	(p→q)→p
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	1

Not a tautology

3. 4.

p	q	r	p∨q∨r	¬p	¬q	¬r	¬p∨¬q∨¬r	p∨¬q	
0	0	0	0	1	1	1	1	1	
0	1	0	1	1	0	1	1	0	
1	0	0	1	0	1	1	1	1	
1	1	0	1	0	0	1	1	1	
0	0	1	1	1	1	0	1	1	[illegible]
0	1	1	1	1	0	0	1	0	1
1	0	1	1	0	1	0	1	1	0
1	1	1	1	0	0	0	0	1	1

r∨¬p	~~(q∨¬r~~ (p∨q∨r)∧(¬p∨¬q∨¬r)∧(p∨¬q)∧(q∨¬r)∧
1	0
1	0
0	0
0	0
1	0
1	0

Unsatisfiable

4. $p \vee (q \wedge \sim(r \wedge (s \rightarrow t)))$

$p \vee (q \wedge \sim(r \wedge (\sim s \vee t)))$	conditional
$p \vee (q \wedge \sim((r \wedge \sim s) \vee (r \wedge t)))$	distributive
$p \vee (q \wedge (\sim(r \wedge \sim s) \wedge \sim(r \wedge t)))$	de Morgan's law
$p \vee (q \wedge (\sim r \vee s) \wedge (\sim r \vee \sim t)))$	~~distri~~ de Morgan's, double neg.
$(p \vee q) \wedge (p \vee (\sim r \vee s)) \wedge (p \vee (\sim r \vee \sim t))$	distributive
$(p \vee q) \wedge (p \vee \sim r \vee s) \wedge (p \vee \sim r \vee \sim t)$	associative

5.

p	q	r	s	t	$\sim r$	$\sim t$	$p \vee \sim r \vee s$	$p \vee q$	$p \vee \sim r \vee \sim t$
0	0	0	0	0	1	1	1	0	1
0	1	1	0	0	0	1	0	1	1
1	0	0	0	0	1	1	1	1	1
1	1	1	0	0	0	1	1	1	1
0	0	0	1	0	1	1	1	0	1
0	1	1	1	0	0	1	1	1	1
1	0	0	1	0	1	1	1	1	1
1	1	1	1	0	0	1	1	1	1
0	0	0	0	1	1	0	1	0	1
0	1	1	0	1	0	0	0	1	0
1	0	0	0	1	1	0	1	1	1
1	1	1	0	1	0	0	1	1	1
0	0	0	1	1	1	0	1	0	1
0	1	1	1	1	0	0	1	1	0
1	0	0	1	1	1	0	1	1	1
1	1	1	1	1	0	0	1	1	1

I used exhaustive enumeration to prove that this formula is satisfiable, because the conjunctive normal form involved the ANDs of multiple statements, requiring all substatements to be true for the formula to be true. I wrote q and r the same because q is only in