Particle Filtering

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Hidden Markov Models (HMM)

$$X_1 \sim \mu(x_1) \text{ and } X_n|(X_{n-1} = x_{n-1}) \sim f(x_n|x_{n-1})$$
$$Y_n|(X_n = x_n) \sim g(y_n|x_n)$$

Problem Setup

Goal:

$$p(x_n|y_{1:n})$$

Have:

From Markov Property:

$$p(x_{1:n}) = \mu(x_1) \prod_{i=2}^{n} f(x_i|x_{i-1})$$

$$p(y_{1:n}|x_{1:n}) = \prod_{i=1}^{n} g(y_i|x_i)$$

Deriving our Goal $p(x_n|y_{1:n})$ from our Markovian Assumptions

$$
\begin{aligned}
p(x_{1:n}|y_{1:n}) &= \frac{p(x_{1:n}, y_{1:n})}{p(y_{1:n})} \\
&= \frac{p(y_{1:n}|x_{1:n})p(x_{1:n})}{p(y_{1:n})}
\end{aligned}
$$

Notice that $p(x_n|y_{1:n})$ is just a marginal of the above equation.

$$
p(x_n|y_{1:n}) = \int \cdots \int p(x_{1:n}|y_{1:n})\, dx_{1:n-1}
$$

Not always integrable e.g. non-linear, non-gaussian distributions

. . . At least we still have this . . .

$$\begin{aligned}
p(x_{1:n}|y_{1:n}) &= \frac{p(x_{1:n}, y_{1:n})}{p(y_{1:n})} \\
&= \frac{p(y_{1:n}|x_{1:n})p(x_{1:n})}{p(y_{1:n})} \\
&= \frac{\left(\prod_{i=1}^{n} g(y_i|x_i)\right)\left(\mu(x_1)\prod_{i=2}^{n} f(x_i|x_{i-1})\right)}{p(y_{1:n})} \\
&= p(x_{1:n-1}|y_{1:n-1})\frac{g(y_n|x_n))f(x_n|x_{n-1})}{p(y_n|y_{1:n-1})} \\
&= \underbrace{p(x_{1:n-1}|y_{1:n-1})}_{\text{recursion}}\underbrace{\frac{g(y_n|x_n)}{p(y_n|y_{1:n-1})}}_{\text{update/weight}}\underbrace{f(x_n|x_{n-1})}_{\text{transition probability}}
\end{aligned}$$