1 HMM

$$X_1 \sim \mu(x_1) \text{ and } X_n|(X_{n-1} = x_{n-1}) \sim f(x_n|x_{n-1}) \tag{1}$$

$$Y_n|(X_n = x_n) \sim g(y_n|x_n) \tag{2}$$

2 Particle Filters

$$\text{Prior: } p(x_{1:n}) = \mu(x_1)\prod_{i=2}^{n} f(x_1|x_{i-1}) \tag{3}$$

$$\text{Conditional of Y: } p(y_{1:n}|x_{1:n}) = \prod_{i=1}^{n} g(y_i|x_i) \tag{4}$$

$$\text{Conditional of X: } p(x_{1:n}|y_{1:n}) = \frac{p(x_{1:n}, y_{1:n})}{p(y_{1:n})} \tag{5}$$

$$\text{Joint: } p(x_{1:n}, y_{1:n}) = p(x_{1:n})p(y_{1:n}|x_{1:n}) \tag{6}$$

$$\text{Marginal: } p(y_{1:n}) = \int p(x_{1:n}, y_{1:n}) \tag{7}$$

$$\text{We have: } \{p(x_{1:n}|y_{1:n})\}_{n>0} \text{ and } \{p(y_{1:n})\}_{n>0} \tag{8}$$

$$\text{Want to find: } \{p(x_n|y_{1:n})\}_{n>0} \tag{9}$$

$$\text{Reformulate the joint: } p(x_{1:n}, y_{1:n}) = p(x_{1:n-1}, y_{1:n-1})f(x_n|x_{n-1})g(y_n|x_n) \tag{10}$$

$$\begin{aligned}
p(x_{1:n}, y_{1:n}) &= p(x_{1:n})p(y_{1:n}|x_{1:n}) \\
p(x_{1:n}) &= \mu(x_1)\prod_{i=2}^{n} f(x_1|x_{i-1}) = (\mu(x_1)\prod_{i=2}^{n-1} f(x_1|x_{i-1}))f(x_n|x_{n-1}) = p(x_{1:n-1}(x_n|x_{n-1})) \\
p(y_{1:n}|x_{1:n}) &= \prod_{i=1}^{n} g(y_i|x_i) = (\prod_{i=1}^{n-1} g(y_i|x_i))g(y_n|x_n) = p(y_{1:n-1}|x_{1:n-1})g(y_n|x_n) \\
p(y_{1:n-1}|x_{1:n-1}) &= \frac{p(x_{1:n-1}, y_{1:n-1})}{p(y_{1:n-1})} \\
p(x_{1:n}, y_{1:n}) &= p(x_{1:n-1}(x_n|x_{n-1}))\frac{p(x_{1:n-1}, y_{1:n-1})}{p(y_{1:n-1})}g(y_n|x_n) = p(x_{1:n-1}, y_{1:n-1})f(x_n|x_{n-1})g(y_n|x_n)
\end{aligned} \tag{11}$$

$$\text{Reformulate the conditional of X: } p(x_{1:n}|y_{1:n}) = p(x_{1:n-1}|y_{1:n-1})\frac{f(x_n|x_{n-1})g(y_n|x_n)}{p(y_n|y_{1:n-1})} \tag{12}$$

$$\begin{aligned}
p(x_{1:n}|y_{1:n}) &= \frac{p(x_{1:n}, y_{1:n})}{p(y_{1:n})} \\
p(x_{1:n}, y_{1:n}) &= p(x_{1:n-1}, y_{1:n-1}) f(x_n|x_{n-1}) g(y_n|x_n) \\
p(x_{1:n-1}, y_{1:n-1}) &= p(y_{1:n-1}) p(x_{1:n-1}|y_{1:n-1}) \\
p(y_{1:n-1}) &= \frac{p(y_{1:n-1}, y_n)}{p(y_n|y_{1:n-1})} = \frac{p(y_{1:n})}{p(y_n|y_{1:n-1})} \\
p(x_{1:n}|y_{1:n}) &= \frac{1}{p(y_{1:n})} \frac{p(y_{1:n})}{p(y_n|y_{1:n-1})} p(x_{1:n-1}|y_{1:n-1}) f(x_n|x_{n-1}) g(y_n|x_n) \\
&= p(x_{1:n-1}|y_{1:n-1}) \frac{f(x_n|x_{n-1}) g(y_n|x_n)}{p(y_n|y_{1:n-1})}
\end{aligned} \tag{13}$$