A detailed example of how causality is violated when information travels faster than speed of light in vaccum

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Abstract

In certain reference frames, a signal travelling faster than light can reach the target before it was sent. This puzzling phenomenon is illustrated in the this article.

1 Two people throwing tachyons[1] at each other

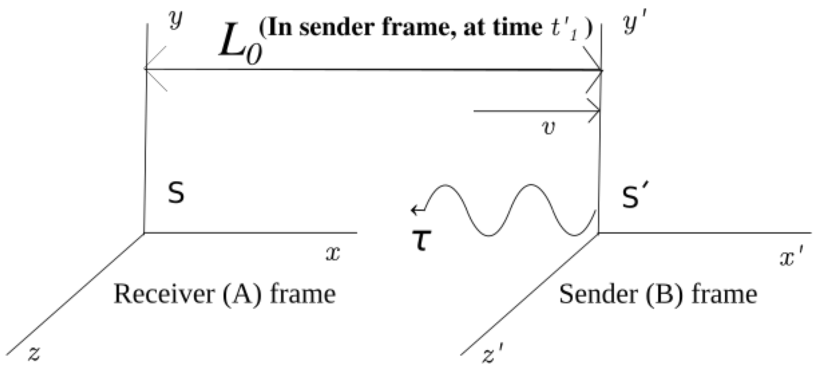

Figure 1: The sender $\mathcal{B}$ sends a particle with speed τ towards left

Suppose there are two persons $\mathcal{A}$ and $\mathcal{B}$. In $\mathcal{A}$'s reference frame, $\mathcal{B}$ is moving towards $+x$ axis with constant speed $v(< c)$.
Let us denote the reference frame of $\mathcal{A}$ by S, and that of $\mathcal{B}$ by S′. When $\mathcal{A}$ and $\mathcal{B}$ were at the same place, (origin of S, in S frame), their clocks were synchronized.
Suppose, in S′ frame, when time is t'_1, $\mathcal{B}$ sends a particle to $\mathcal{A}$, with speed τ.
As we proceed, we will determine the appropriate conditions τ must satisfy, so that the particle is received before it was sent (as observed in $\mathcal{A}$'s reference frame).

Note: $\tau > v$, otherwise $\mathcal{A}$ will never receive the particle.
When the particle was released, the distance between $\mathcal{A}$ and $\mathcal{B}$ in S′ frame is $vt'_1 = L_0$ (say).
Suppose, in $\mathcal{B}$'s frame, $\mathcal{A}$ receives the particle at time t'_2.
In this time interval, $\mathcal{B}$ will observe the particle to move distance $\tau(t'_2 - t'_1)$, and $\mathcal{A}$ to move a distance $v(t'_2 - t'_1)$.
Therefore, $\tau(t'_2 - t'_1) = v(t'_2 - t'_1) + L_0$
$\implies t'_2 = t'_1 + \frac{L_0}{\tau - v}$

In S′ frame, the particle is sent from coordinate $x'_1 = 0$ and $\mathcal{B}$ observes that $\mathcal{A}$ received the particle at $x'_2 = -vt'_2 = -v(t'_1 + \frac{L_0}{\tau - v}) = -\frac{\tau L_0}{\tau - v}$.

Suppose, in reference frame S, it is observed that the particle is thrown when $\mathcal{B}$ was at position x_1, at time t_1, and it was received at origin of S, at time t_2. Let $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. So, $x'_1 = 0$, $x'_2 = -\frac{\tau L_0}{\tau - v}$ Applying Lorentz transformations,

$$x_1 = \gamma(x'_1 + vt'_1) = \gamma vt'_1 = \gamma L_0$$
$$t_1 = \gamma(t'_1 + \frac{vx'_1}{c^2}) = \gamma t'_1 = \frac{\gamma L_0}{v}$$
$$t_2 = \gamma(t'_2 + \frac{vx'_2}{c^2}) = \gamma(t'_1 + \frac{L_0}{\tau - v} - \frac{v\tau L_0}{(\tau - v)c^2})$$
$$= t_1 + \gamma(\frac{L_0}{\tau - v} - \frac{v\tau L_0}{(\tau - v)c^2}) = t_1 + \gamma L_0 \frac{v}{c^2(\tau - v)}(\frac{c^2}{v} - \tau).$$

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[1] In this article, any particle travelling faster than light is called a tachyon.

Thus $t_2 - t_1 = \gamma L_0 \frac{v}{c^2(\tau - v)}(\frac{c^2}{v} - \tau)$.

Therefore, if $\tau > \frac{c^2}{v}$, then $t_1 > t_2$, i.e., $\mathcal{A}$ will **observe that the particle is received before it was sent**!

Now, $\frac{c^2}{v} > c$, so τ must be greater than c. So, this can happen only if the particle is a tachyon.

1.1 Explanation from another point of view

In the reference frame of the $\mathcal{A}$, the $\mathcal{B}$ is moving away with velocity $v\hat{x}$. In the reference frame of the $\mathcal{B}$, the tachyon is moving with velocity $-\tau\hat{x}$. Thus, the velocity of the tachyon w.r.t. $\mathcal{A}$ will be, $\frac{v-\tau}{1-\frac{\tau v}{c^2}}\hat{x} = \frac{c^2}{v} \cdot \frac{\tau - v}{\tau - \frac{c^2}{v}}\hat{x}$, which is positive for $\tau > \frac{c^2}{v}$.

Since the tachyon has positive velocity in the S frame, it will pass through the origin of S before it passes through the origin of S′, which is further away towards the +ve x axis. Naturally, the $\mathcal{A}$ will receive it before the $\mathcal{B}$ sends it!

1.2 Can $\mathcal{A}$ disable the tachyon producing machine of $\mathcal{B}$?

Suppose, $\mathcal{A}$ can send a special signal (with speed θ), which if received, will disable the tachyon sending machine of $\mathcal{B}$. A curious question - Now that $\mathcal{A}$ has received the particle (a tachyon) when $\mathcal{B}$ is yet to send it, can $\mathcal{A}$ stop $\mathcal{B}$ from sending the tachyon altogether? Suppose $\mathcal{A}$ sends this special signal as soon as receiving the tachyon (at time t_2, in S frame).

Suppose $\mathcal{A}$ observes that this signal is received at time t_3. By similar arguments as before,
$t_3 = t_2 + \frac{vt_2}{\theta - v} = \frac{\theta t_2}{\theta - v}$, and $x_3 = vt_3$.

In S frame, $\mathcal{B}$ receives this tachyon at $t'_3 = \gamma(t_3 - \frac{vx_3}{c^2})$

$= \gamma t_3(1 - \frac{v^2}{c^2})$

$= \frac{t_3}{\gamma}$

$= \frac{\theta t_2}{\gamma(\theta - v)}$

$= \frac{\theta}{(\theta - v)} \cdot L_0 \cdot [\frac{1}{v} + \frac{c^2 - v\tau}{c^2(\tau - v)}]$

$= \frac{\theta}{(\theta - v)} \cdot t'_1 \cdot [1 + \frac{c^2 - v\tau}{c^2(\tau - v)}v]$

So, $t'_3 - t'_1 = [\frac{\theta}{(\theta - v)}\{1 + \frac{c^2 - v\tau}{c^2(\tau - v)}v\} - 1]t'_1$

$= [\frac{\theta}{(\theta - v)} \cdot \frac{\tau}{(\tau - v)} \cdot \frac{c^2 - v^2}{c^2} - 1]t'_1$.

When this quantity is positive, the signal from $\mathcal{A}$ reaches $\mathcal{B}$, before $\mathcal{B}$ releases the particle. When this is negative, the tachyon sending machine of $\mathcal{B}$ is turned off before it fires, so no tachyon should reach $\mathcal{A}$. For this to be negative,

$\theta\tau(c^2 - v^2) < (\theta - v)(\tau - v)c^2$

$\implies vc^2 - \tau c^2 - \theta c^2 + \theta\tau v > 0$

$\implies \theta(\tau - \frac{c^2}{v}) > c^2(\frac{\tau}{v} - 1)$ ——————(*)

$\implies \theta > c^2 \frac{\frac{\tau}{v} - 1}{\tau - \frac{c^2}{v}}$ (since $\tau > \frac{c^2}{v}$)

When this condition is satisfied, $\mathcal{B}$ cannot send the tachyon anymore, because the machine would not work. So, $\mathcal{A}$ should not receive any tachyon. However, $\mathcal{A}$ released the signal to disable the machine of $\mathcal{B}$ only after receiving the tachyon. Thus, $\mathcal{A}$ ***has received a tachyon implies $\mathcal{A}$ must not have received any tachyon!***

Note: When $\tau < \frac{c^2}{v}$, $\theta(\tau - \frac{c^2}{v}) < 0$, and $c^2(\frac{\tau}{v} - 1) > 0$, then $(*)$ is wrong, and $t'_3 > t'_1$. So, this absurd situation can only arise when the other absurd situation has taken place - when the $\mathcal{A}$ receives the tachyon before it was sent (in the S frame).

Previously, we showed that $\tau > c$. Now, we claim that $\theta \nleq c$. Because, if the contrary is true, $c > c^2 \frac{\frac{\tau}{v} - 1}{\tau - \frac{c^2}{v}} (= \theta_{min})$

$\implies \tau - \frac{c^2}{v} > \tau\frac{c}{v} - c$

$\implies \underbrace{\tau(\frac{c}{v} - 1)}_{positive} < \underbrace{c - \frac{c^2}{v}}_{negative}$, which is absurd.

2 Conclusion

We see that, when the speed of the tachyon is high enough ($\tau > \frac{c^2}{v} > c$), it is received before it was sent (in the receiver frame). Just after receiving, if the receiver sends another tachyon to the sender with high enough speed ($\theta > c^2 \frac{\frac{\tau}{v} - 1}{\tau - \frac{c^2}{v}} > c$), it will be received before the first tachyon was sent. Even if it was apriory decided that no tachyon will be sent after this (second tachyon) is received, the sender must have sent the first tachyon after receiving the second one, despite being instructed not to do so, or, despite not being in a state to send the first tachyon!

All these self-contradictory statements can only arise when the speed of the signals are greater than the speed of light.