# Some consequences of the *Special Theory of Relativity* - Doppler effect of electromagnetic waves in refractive medium and violation of causality due to Tacyons

Archisman Panigrahi[1]
UG $1^{st}$ year, Indian Institute of Science, Bangalore
SR No. - 11-01-00-10-91-17-1-14503
KVPY SA Fellow, 2015
KVPY Registration No. - SA-1510017

2018

[1]archismanp@iisc.ac.in

**Acknowledgement**

I am thankful to Prof. Subroto Mukerjee[1] for guiding me in this project.
I am also thankful to my classmates Agrim Sharma and Vinay Vikramaditya.

[1]Dept. of Physics, Indian Institute of Science

# Contents

# Chapter 1

# Doppler effect of light travelling in refractive medium and relativistic Doppler effect of any other wave

## 1.1 Introduction

When a moving train whistles by a stationary observer, the pitch of sound suddenly decreases as the train moves away. When the source of sound and receiver are moving relative to the medium (in this case, air), the frequency of the sound perceived by the receiver will not be same as the emitted frequency, when the relative velocity between the source and the observer is non-zero. Suppose, the source and the receiver are moving w.r.t. the medium at speed $v_s$ and $v_r$, (Fig. 1) respectively, and both of their motion is along the line joining them. Suppose, the frequency emitted by the source is $f$ and the speed of sound in the medium is $s$ $(v_s, v_r < s)$.


Figure 1.1: The sound source and the receiver are moving through the medium

Then the frequency observed by the receiver is given by the classical (non-relativistic) formula —[1, Section 16.3]

$$f' = f \cdot \frac{s - v_r}{s - v_s} \tag{1.1}$$

In case of the train and the observer, the apparent frequency of the approaching train was $f'_1 = f \cdot \frac{s}{s - v_{train}}$, and as the train passed, it became $f'_2 = f \cdot \frac{s}{s + v_{train}}$. The observer noticed the sudden change in frequency.

## 1.2 Doppler effect of Electromagnetic waves in vaccum

Doppler effect is also observed in other waves like electromagnetic waves. To analyze it, we must consider relativistic effects.

Consider two inertial frames S and S′ (Fig. 2). Suppose, according to the source (at the origin of S) the receiver (at the origin of S′) is at point $(x_0, 0, 0)$ at $t = 0$, and moving away along $x$ axis with constant speed $v$. Consider an observer O at $(x_0, 0, 0)$, stationary w.r.t. S. O has a clock,

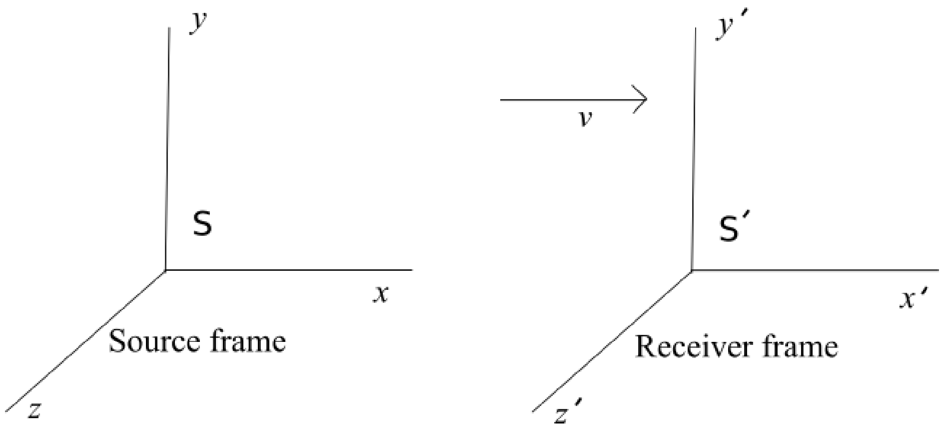


Figure 1.2: The light source is at origin of S. The receiver is at origin of S$'$.

synchronized with the source (at origin of S). If the $x$ coordinate of a point measured by O is $\bar{x}$, then the $x$ coordinate of that point in S frame will be $x = \bar{x} + x_0$.

The measurements made in the receiver frame are related to the measurements made in the source frame by the Lorentz transformations — [2, Section 12.1.3]

$$x' = \frac{\bar{x} - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{(x - x_0) - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.2}$$

$$y' = y \tag{1.3}$$

$$z' = z \tag{1.4}$$

and,

$$t' = \frac{t - \frac{v\bar{x}}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t - \frac{v(x - x_0)}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.5}$$

The inverse for $x$ and $t$ are,

$$x - x_0 = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.6}$$

and,

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.7}$$

### 1.2.1 Analysis from the source frame (S)

Suppose, the source keeps sending light waves of very low intensity through vaccum, so that only one photon is emitted at a time. So, the source sends two light pulses of the phase difference $2\pi$ at $t = 0$ and $t = \frac{1}{f}$ (where $f$ is the frequency of the light pulse), and it is observed in the source frame that they are received at $t = t_1$ and $t = t_2$, respectively. To find the frequency of the light detected by the receiver, we have to find the difference between time at which the light pulses arrive to the receiver in its frame. The frequency will be inverse of this time period.

Within time $t_1$, the receiver moves distance $vt_1$, and in that time, the light must cover a distance $x_0 + vt_1$. So, $ct_1 = x_0 + vt_1, \implies t_1 = \frac{x_0}{c-v}$.
Similarly, $t_2 = \frac{1}{f} + \frac{x_0 + \frac{v}{f}}{c-v}$.
Thus, $\Delta t = t_2 - t_1 = \frac{1}{f}(1 + \frac{v}{c-v}) = \frac{1}{f(1-\frac{v}{c})}$

Suppose, in the receiver's frame, the signals are received at $t'_1$ and $t'_2$, respectively.

From (7), $t_1 = \frac{t'_1}{\sqrt{1-\frac{v^2}{c^2}}}$ ($x'_1 = 0$, as the signal is received at origin of S$'$)
Similarly, $t_2 = \frac{t'_2}{\sqrt{1-\frac{v^2}{c^2}}}$ ($x'_2 = 0$)

So,

$$t'_2 - t'_1 = (t_2 - t_1)\sqrt{1 - \frac{v^2}{c^2}} \tag{1.8}$$

$$= \frac{\frac{1}{f}\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}} = \frac{1}{f}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

Now, $t'_2 - t'_1$ is the interval between the two signals received by the receiver in its frame, and its reciprocal will be the frequency perceived by the receiver.
Thus, $\boxed{f' = f\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}}$
**Note**: If the time interval between two events occuring at the same place in a certain inertial reference frame is $\Delta\tau$ and in another reference frame moving at speed $v$ w.r.t. the previous frame, the observed time interval is $\Delta t$, then

$$\Delta\tau = \frac{\Delta t}{\gamma} \tag{1.9}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is known as the Lorentz factor [2, Section 12.2.1]. This is also known as the time dilation formula and $\Delta\tau$ is known as the 'proper time interval'. For example, in S$'$, the signals were received at the same place (the origin), and in (8), we got $\Delta\tau = t'_2 - t'_1 = \frac{t_2-t_1}{\gamma} = \frac{\Delta t}{\gamma}$.

### 1.2.2 Analysis from the receiver frame (S$'$)

Let us analyze the situation from the receiver's frame.
In the receiver's frame, the first pulse is sent from $x'_1 = \frac{(0-x_0)-v\cdot 0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{-x_0}{\sqrt{1-\frac{v^2}{c^2}}}$ (from equation (2)), at time $t'_1 = \frac{0-\frac{v\cdot(0-x_0)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{vx_0}{c^2\sqrt{1-\frac{v^2}{c^2}}}$.
Since the frame of the observer is an inertial frame, the speed of light in space w.r.t. the receiver is $c$ (this is one of the two postulates of Special Theory of relativity).
The first pulse will be received at time

$$\tau_1 = t'_1 + \frac{|x_1|}{c}$$

In the receiver's frame, the second pulse is sent from $x'_2 = \frac{(0-x_0)-v\cdot\frac{1}{f}}{\sqrt{1-\frac{v^2}{c^2}}} = -\frac{x_0+\frac{v}{f}}{\sqrt{1-\frac{v^2}{c^2}}}$, at time $t'_2 = \frac{\frac{1}{f}+\frac{vx_0}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$.
The second pulse will be received at

$$\tau_2 = t'_2 + \frac{|x'_2|}{c}$$

The time interval of receiving the two pulses from the receiver's frame is, $\tau_2-\tau_1 = (t'_2-t'_1)+\frac{|x'_2|-|x'_1|}{c} = \frac{1}{f}\cdot\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{v}{c}\cdot\frac{1}{f}\cdot\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{f}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$. Thus, $\boxed{f' = \frac{1}{\tau_2-\tau_1} = f\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}}$

### 1.2.3 Analysis using energy-momentum four vector

There is anotherway to obtain this formula. Consider a photon of frequency $f$ (in the frame S of source) travelling in space along +ve $x$ axis. Now, energy and momentum form a four vector $(\frac{E}{c}, p_x, p_y, p_z)$. The energy and momentum of the photon in the S′ frame (of the receiver, travelling at speed $v$ along $+x$ axis) can be obtained by applying the Lorentz transformations. [3, Eq. 17.12]

$$\begin{cases} p'_x = \frac{p_x - \frac{vE}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \\ p'_y = p_y \\ p'_z = p_z \\ E' = \frac{E - vp_x}{\sqrt{1-\frac{v^2}{c^2}}} \end{cases} \tag{1.10}$$

Then, applying the relation between energy and frequency of photon, we can get its frequency in the S′ frame. [3, Section 17-5]
Consider a photon of frequency $f$ (in S frame), travelling in $+x$ direction. For this photon, $E = hf$, and $p_x = \frac{hc}{\lambda}$.
So, $E' = \frac{hf - \frac{vhf}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = hf'$
Or, $\boxed{f' = f\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}}$, which is the same relation we obtained previously.

## 1.3 Doppler effect of Electromagnetic waves in refractive medium

Suppose, a medium with refractive index $n$ is at rest in a certain inertial frame. In this frame, the source is moving with constant velocity $v_s\hat{x}$ and the receiver is moving with constant velocity $v_r\hat{x}$ (Fig. 3). Speed of light w.r.t the medium is $\frac{c}{n}$. Let us assume $v_s, v_r < \frac{c}{n}$. The speed of light w.r.t.

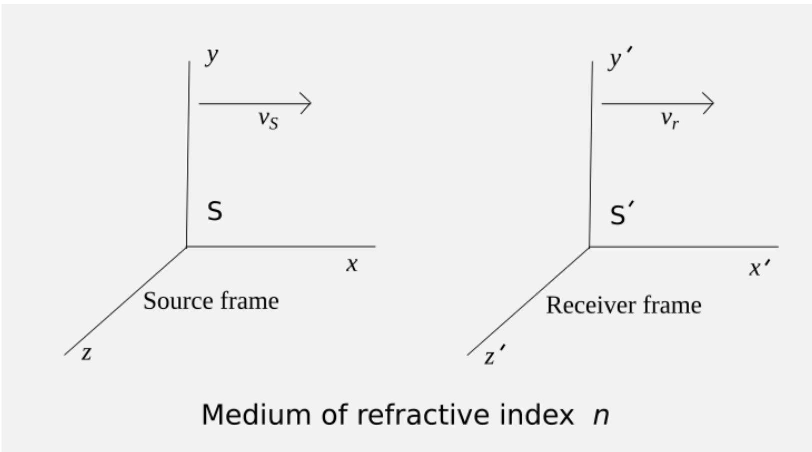


Figure 1.3: The source and receiver are moving in an inertial refractive medium

the source is $v_c = \frac{\frac{c}{n} - v_s}{1 - \frac{v_s \cdot \frac{c}{n}}{c^2}} = \frac{\frac{c}{n} - v_s}{1 - \frac{v_s}{nc}}$ (according to the velocity addition formula [4, Eq. 5-2]).
As in the previous situation, the receiver (at the origin of S′ frame) is at distance $x_0$ from the source (at the origin of S frame) at $t = 0$ (measured in S frame). The source emits two light pulses of phase difference $2\pi$, at $t = 0$ and $t = \frac{1}{f}$.
The speed of the receiver w.r.t. the source is $v_{rel} = \frac{v_r - v_s}{1 - \frac{v_r \cdot v_s}{c^2}}$.
According to the source, the first signal is received at $t_1 = \frac{x_0}{v_c - v_{rel}}$.
And the second signal is received at $t_2 = \frac{1}{f} + \frac{x_0 + \frac{v_{rel}}{f}}{v_c - v_{rel}}$.

Thus, according to the sourse in S frame, the time interval between the receiver receiving the two signals is $\Delta t = t_2 - t_1 = \frac{1}{f} \cdot [1 + \frac{v_{rel}}{v_c - v_{rel}}] = \frac{1}{f} \cdot [\frac{v_c}{v_c - v_{rel}}]$
Thus, $\frac{1}{\Delta t} = f \cdot [1 - \frac{v_{rel}}{v_c}]$
$= f \cdot [1 - \frac{v_{rel}(1 - \frac{v_s}{nc})}{\frac{c}{n} - v_s}]$

To get the proper time interval, we have to divide $\Delta t$ by the appropriate Lorentz factor, which is,

$$\gamma_{v_{rel}} = (1 - \frac{v_{rel}^2}{c^2})^{-\frac{1}{2}}$$
$$= [1 - \frac{1}{c^2} \cdot (\frac{v_r - v_s}{1 - \frac{v_r v_s}{c^2}})]^{-\frac{1}{2}}$$
$$= \frac{1 - \frac{v_r v_s}{c^2}}{\sqrt{(1 - \frac{v_r v_s}{c^2})^2 - (\frac{v_r - v_s}{c})^2}}$$
$$= \frac{1 - \frac{v_r v_s}{c^2}}{\sqrt{(1 - \frac{v_r^2}{c^2})(1 - \frac{v_s^2}{c^2})}}$$

So, the proper time interval is $\Delta \tau = \frac{\Delta t}{\gamma_{v_{rel}}}$
The frequency observed by the receiver will be $f' = \frac{1}{\Delta \tau} = \frac{\gamma_{v_{rel}}}{\Delta t}$. Therefore,

$$\boxed{f' = f \frac{1 - \frac{v_r v_s}{c^2}}{\sqrt{(1 - \frac{v_r^2}{c^2})(1 - \frac{v_s^2}{c^2})}} [1 - \frac{v_{rel}(1 - \frac{v_s}{nc})}{\frac{c}{n} - v_s}]} \tag{1.11}$$

### 1.3.1 Special cases

1) When $v_r = v_s = v$, then $v_{rel} = 0$.

Then, $f' = f \frac{1 - \frac{v^2}{c^2}}{\sqrt{(1 - \frac{v^2}{c^2})(1 - \frac{v^2}{c^2})}} [1 - 0] = f$.
So, when there is **no relative velocity**, there is **no change in observed frequency**.

2) When the **receiver is moving away** at speed $v$, and the source is fixed w.r.t. the medium, $v_r = v$, and $v_s = 0$, so $v_{rel} = v$.

Then, $f' = \frac{f}{\sqrt{1 - \frac{v^2}{c^2}}} [1 - \frac{v}{\frac{c}{n}}] = \frac{f}{\sqrt{1 - \frac{v^2}{c^2}}} [1 - \frac{nv}{c}]$

3) When the **source is moving away** with speed $v$ (along -ve $x$ axis) and the receiver is fixed w.r.t. the medium, $v_r = 0$, and $v_s = -v$, so $v_{rel} = v$.
Then, $f' = \frac{f}{\sqrt{1 - \frac{v^2}{c^2}}} [1 - \frac{v(1 + \frac{v}{nc})}{\frac{c}{n} + v}]$

$$= \frac{f}{\sqrt{1 - \frac{v^2}{c^2}}} [\frac{c - \frac{v^2}{c}}{c + nv}]$$
$$= \frac{f}{1 + \frac{nv}{c}} \sqrt{1 - \frac{v^2}{c^2}}$$

Thus, the **observed frequencies are different** when the source is moving away with respect to the receiver (fixed in the medium) and when the receiver is moving away with respect to the source (fixed in the medium). However, if we substitute $n = 1$, these two become identical to the previous formula $f' = f\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$ When source and receivers are approaching each other in special cases 2) or 3), we have to replace $v$ with $-v$ to get the required formula.

## 1.4 Relativistic Doppler effect of sound waves or any other wave

Suppose, the speed of sound in a medium is $s$, and the medium is fixed in an inertial frame. In the previous section, the speed of the signal sent was $\frac{c}{n}$. We can work out the formula for Doppler effect

of sound by choosing $n$ such that, $\frac{c}{n} = s$, or, $n = \frac{c}{s}$.
Substituting this value of $n$ in (11) we get,

$$\boxed{f' = f \cdot \frac{1-\frac{v_r v_s}{c^2}}{\sqrt{(1-\frac{v_r^2}{c^2})(1-\frac{v_s^2}{c^2})}} \cdot [1 - \frac{v_{rel}(1-\frac{s v_s}{c^2})}{s-v_s}]}, \tag{1.12}$$

where $v_{rel} = \frac{v_r - v_s}{1-\frac{v_r v_s}{c^2}}$
In low speed limits, $v_s \ll c^2, v_r \ll c^2$.
Then, $v_{rel} \approx v_r - v_s$
Finally we get, $f' \approx [1 - \frac{v_r - v_s}{s-v_s}] = \frac{s-v_r}{s-v_s}$, which is the fimiliar classical formula (Eq. (1)).

## 1.5 Conclusion

Thus, we have verified that the relativistic formula for Doppler effect of Sound waves turns into the classical formula under low speed limit. The same formula will apply for mechanical waves in a material, or any other kind of signal. The observed frequency of electromagnetic waves through a refractive medium and observed frequency of sound waves depends not only on the relative velocity of the source and the observer, but also on the velocities of the source and observer with respect to the medium.

# Chapter 2

# Tachyons and violation of causality

## 2.1 Introduction

Travelling faster than light is not allowed by the special theory of relativity, as, then, the energy and momentum of the body (having a positive mass) travelling faster than light will become imaginary quantities. A tachyon is a (hypothetical) particle travelling faster than light. If a signal travelling faster than light is sent, in certain reference frames it can reach the target before it was sent.

This puzzling phenomenon is illustrated in the next section.

## 2.2 Two persons throwing tachyons at each other

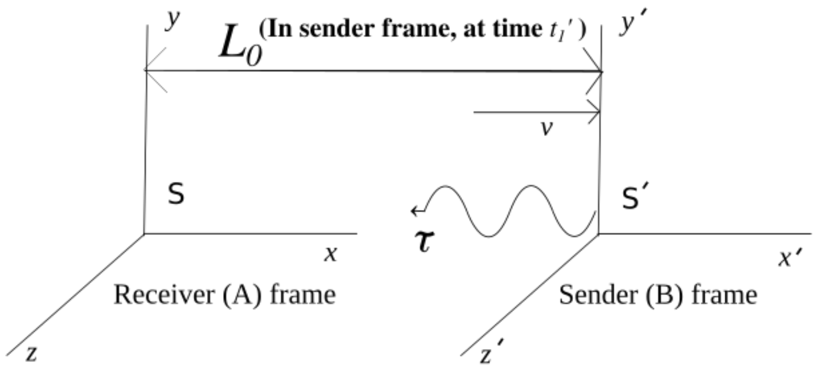


Figure 2.1: The sender $\mathcal{B}$ sends a particle with speed $\tau$ towards left

Suppose there are two persons $\mathcal{A}$ and $\mathcal{B}$. In $\mathcal{A}$'s reference frame, $\mathcal{B}$ is moving towards $+x$ axis with constant speed $v(< c)$.
Let us denote the reference frame of $\mathcal{A}$ by S, and that of $\mathcal{B}$ by S$'$. When $\mathcal{A}$ and $\mathcal{B}$ were at the same place, (origin of S, in S frame), their clocks were synchronized.
Suppose, in S$'$ frame, when time is $t_1'$, $\mathcal{B}$ sends a particle to $\mathcal{A}$, with speed $\tau$.
As we proceed, we will determine the appropriate conditions $\tau$ must satisfy, so that the particle is received before it was sent (as observed in $\mathcal{A}$'s reference frame).

**Note**: $\tau > v$, otherwise $\mathcal{A}$ will never receive the particle.
When the particle was released, the distance between $\mathcal{A}$ and $\mathcal{B}$ in S$'$ frame is $vt_1' = L_0$ (say).
Suppose, in $\mathcal{B}$'s frame, $\mathcal{A}$ receives the particle at time $t_2'$.
In this time interval, $\mathcal{B}$ will observe the particle to move distance $\tau(t_2' - t_1')$, and $\mathcal{A}$ to move a distance $v(t_2' - t_1')$.
Therefore, $\tau(t_2' - t_1') = v(t_2' - t_1') + L_0$
$\implies t_2' = t_1' + \frac{L_0}{\tau - v}$

In S$'$ frame, the particle is sent from coordinate $x_1' = 0$ and $\mathcal{B}$ observes that $\mathcal{A}$ received the particle at $x_2' = -vt_2' = -v(t_1' + \frac{L_0}{\tau - v}) = -\frac{\tau L_0}{\tau - v}$.

Suppose, in reference frame S, it is observed that the particle is thrown when $\mathcal{B}$ was at position $x_1$, at time $t_1$, and it was received at origin of S, at time $t_2$. Let $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. So, $x'_1 = 0$, $x'_2 = -\frac{\tau L_0}{\tau - v}$

Applying Lorentz transformations,
$x_1 = \gamma(x'_1 + vt'_1) = \gamma v t'_1 = \gamma L_0$
$t_1 = \gamma(t'_1 + \frac{vx'_1}{c^2}) = \gamma t'_1 = \frac{\gamma L_0}{v}$
$t_2 = \gamma(t'_2 + \frac{vx'_2}{c^2}) = \gamma(t'_1 + \frac{L_0}{\tau - v} - \frac{v\tau L_0}{(\tau - v)c^2})$
$= t_1 + \gamma(\frac{L_0}{\tau - v} - \frac{v\tau L_0}{(\tau - v)c^2}) = t_1 + \gamma L_0 \frac{v}{c^2(\tau - v)}(\frac{c^2}{v} - \tau)$.
Thus $t_2 - t_1 = \gamma L_0 \frac{v}{c^2(\tau - v)}(\frac{c^2}{v} - \tau)$.
Therefore, if $\tau > \frac{c^2}{v}$, then $t_1 > t_2$, i.e., $\mathcal{A}$ will **observe that the particle is received before it was sent**!

Now, $\frac{c^2}{v} > c$, so $\tau$ must be greater than $c$. So, this can happen only if the particle is a tachyon.

### 2.2.1 Explanation from another point of view

In the reference frame of the $\mathcal{A}$, the $\mathcal{B}$ is moving away with velocity $v\hat{x}$. In the reference frame of the $\mathcal{B}$, the tachyon is moving with velocity $-\tau\hat{x}$. Thus, the velocity of the tachyon w.r.t. $\mathcal{A}$ will be, $\frac{v-\tau}{1-\frac{\tau v}{c^2}}\hat{x} = \frac{c^2}{v} \cdot \frac{\tau - v}{\tau - \frac{c^2}{v}}\hat{x}$, which is positive for $\tau > \frac{c^2}{v}$.

Since the tachyon has positive velocity in the S frame, it will pass through the origin of S before it passes through the origin of S′, which is further away towards the +ve $x$ axis. Naturally, the $\mathcal{A}$ will receive it before the $\mathcal{B}$ sends it!

### 2.2.2 Can $\mathcal{A}$ disable the tachyon producing machine of $\mathcal{B}$?

Suppose, $\mathcal{A}$ can send a special signal (with speed $\theta$), which if received, will disable the tachyon sending machine of $\mathcal{B}$. A curious question - Now that $\mathcal{A}$ has received the particle (a tachyon) when $\mathcal{B}$ is yet to send it, can $\mathcal{A}$ stop $\mathcal{B}$ from sending the tachyon altogether? Suppose $\mathcal{A}$ sends this special signal as soon as receiving the tachyon (at time $t_2$, in S frame).

Suppose $\mathcal{A}$ observes that this signal is received at time $t_3$. By similar arguments as before,
$t_3 = t_2 + \frac{vt_2}{\theta - v} = \frac{\theta t_2}{\theta - v}$, and $x_3 = vt_3$.

In S frame, $\mathcal{B}$ receives this tachyon at $t'_3 = \gamma(t_3 - \frac{vx_3}{c^2})$
$= \gamma t_3(1 - \frac{v^2}{c^2})$
$= \frac{t_3}{\gamma}$
$= \frac{\theta t_2}{\gamma(\theta - v)}$
$= \frac{\theta}{(\theta - v)} \cdot L_0 \cdot [\frac{1}{v} + \frac{c^2 - v\tau}{c^2(\tau - v)}]$
$= \frac{\theta}{(\theta - v)} \cdot t'_1 \cdot [1 + \frac{c^2 - v\tau}{c^2(\tau - v)}v]$
So, $t'_3 - t'_1 = [\frac{\theta}{(\theta - v)}\{1 + \frac{c^2 - v\tau}{c^2(\tau - v)}v\} - 1]t'_1$
$= [\frac{\theta}{(\theta - v)} \cdot \frac{\tau}{(\tau - v)} \cdot \frac{c^2 - v^2}{c^2} - 1]t'_1$.

When this quantity is positive, the signal from $\mathcal{A}$ reaches $\mathcal{B}$, before $\mathcal{B}$ releases the particle. When this is negative, the tachyon sending machine of $\mathcal{B}$ is turned off before it fires, so no tachyon should reach $\mathcal{A}$. For this to be negative,

$\theta\tau(c^2 - v^2) < (\theta - v)(\tau - v)c^2$
$\implies vc^2 - \tau c^2 - \theta c^2 + \theta\tau v > 0$
$\implies \theta(\tau - \frac{c^2}{v}) > c^2(\frac{\tau}{v} - 1)$ ——————(*)
$\implies \theta > c^2\frac{\frac{\tau}{v} - 1}{\tau - \frac{c^2}{v}}$ (since $\tau > \frac{c^2}{v}$)

*When this condition is satisfied, $\mathcal{B}$ cannot send the tachyon anymore, because the machine would not work. So, $\mathcal{A}$ should not receive any tachyon. However, $\mathcal{A}$ released the signal to disable the machine of $\mathcal{B}$ only after receiving the tachyon. Thus, $\mathcal{A}$* ***has received a tachyon implies*** *$\mathcal{A}$* ***must not***

***have received any tachyon!***
**Note**: When $\tau < \frac{c^2}{v}$, $\theta(\tau - \frac{c^2}{v}) < 0$, and $c^2(\frac{\tau}{v} - 1) > 0$, then $(*)$ is wrong, and $t_3' > t_1'$. So, this absurd situation can only arise when the other absurd situation has taken place - when the $\mathcal{A}$ receives the tachyon before it was sent (in the S frame).
Previously, we showed that $\tau > c$. Now, we claim that $\theta \nleq c$. Because, if the contrary is true,
$c > c^2 \frac{\frac{\tau}{v} - 1}{\tau - \frac{c^2}{v}} (= \theta_{min})$
$\implies \tau - \frac{c^2}{v} > \tau \frac{c}{v} - c$
$\implies \underbrace{\tau(\frac{c}{v} - 1)}_{positive} < \underbrace{c - \frac{c^2}{v}}_{negative}$, which is absurd.

## 2.3 Conclusion

We see that, when the speed of the tachyon is high enough ($\tau > \frac{c^2}{v} > c$), it is received before it was sent (in the receiver frame). Just after receiving, if the receiver sends another tachyon to the sender with high enough speed ($\theta > c^2 \frac{\frac{\tau}{v} - 1}{\tau - \frac{c^2}{v}} > c$), it will be received before the first tachyon was sent. Even if it was apriory decided that no tachyon will be sent after this (second tachyon) is received, the sender must have sent the first tachyon after receiving the second one, despite being instructed not to do so, or, despite not being in a state to send the first tachyon!

All these self-contradictory statements can only arise when the speed of the signals are greater than the speed of light.

# Bibliography

[1] H. C. Verma, *Concepts of Physics*, vol. I. Bharati Bhawan P&D, New Delhi, 2013.

[2] D. J. Griffiths, *Introduction to Electrodynamics*. Pearson, Boston, $4^{th}$ ed., 2013.

[3] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics - Volume-I*. Pearson, India, 2016.

[4] A. P. French, *Special Relativity: The MIT Introductory Physics Series*. W. W. Norton & Company Inc., New York, 1968.