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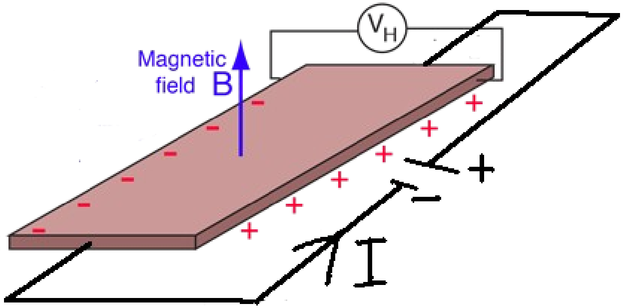

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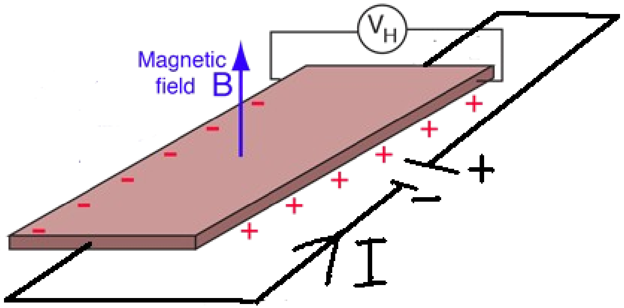

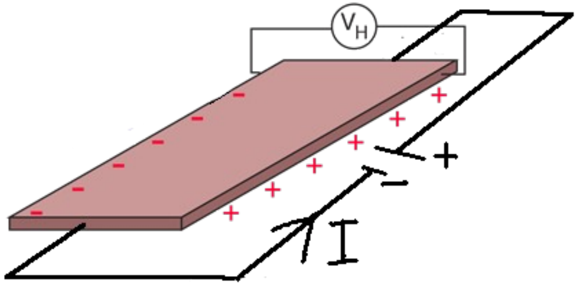

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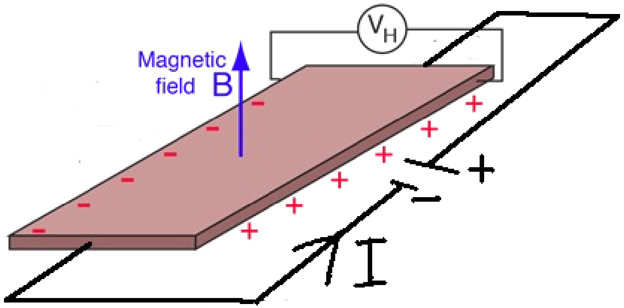

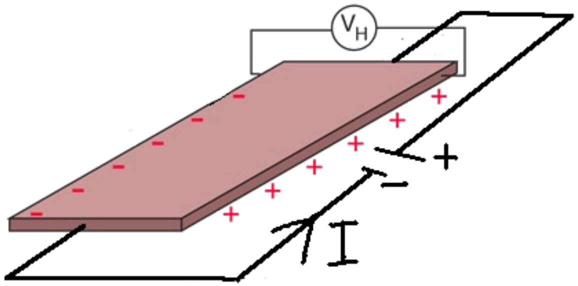

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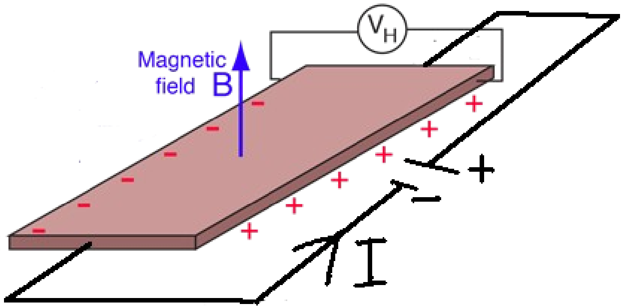

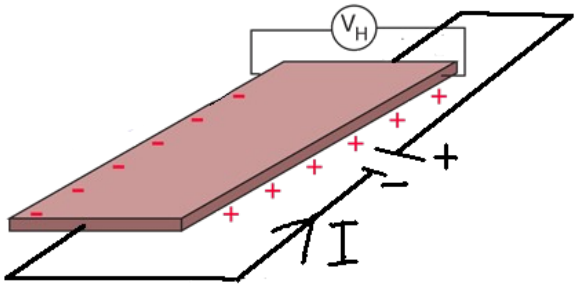

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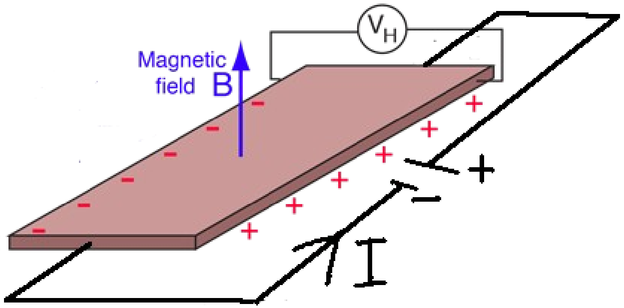

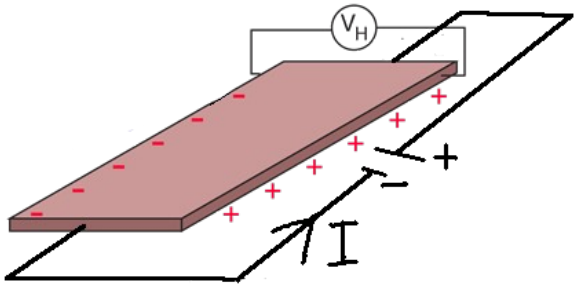

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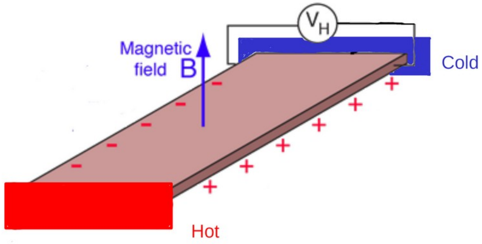

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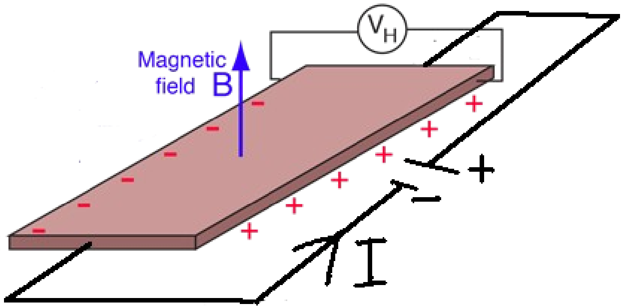

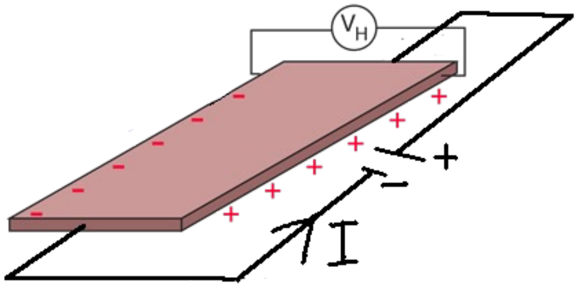

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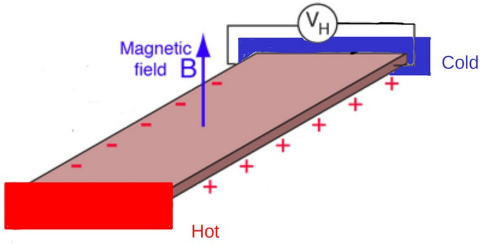

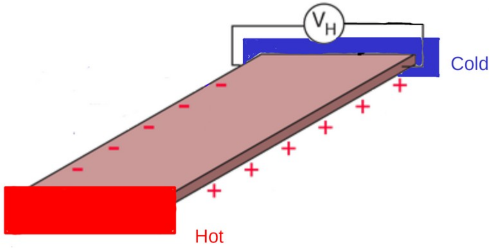

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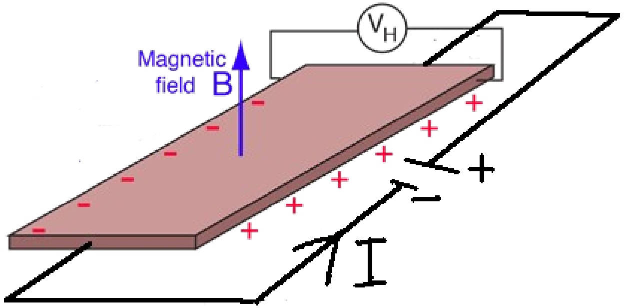

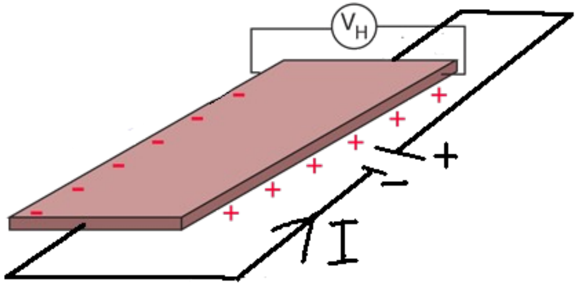

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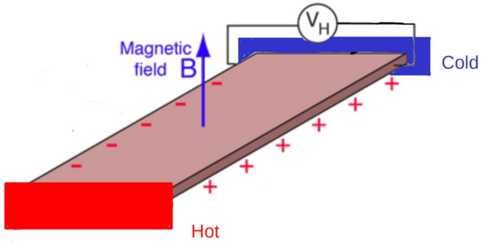
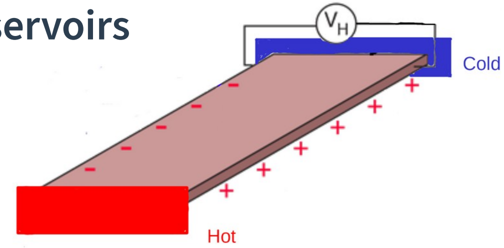


Electrically insulating temperature reservoirs



Thermoelectric transport: Einstein and Onsager relations

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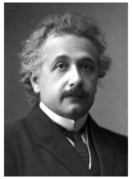

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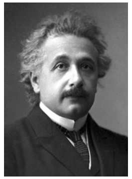

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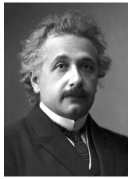

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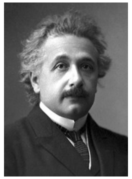

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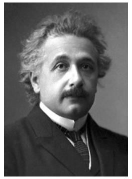

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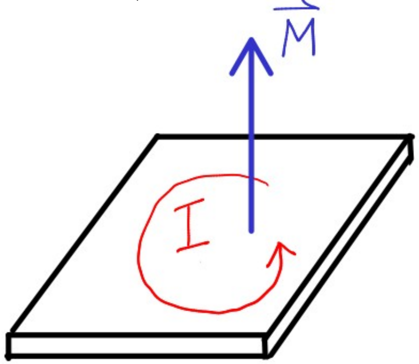

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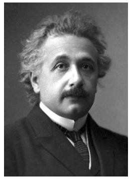

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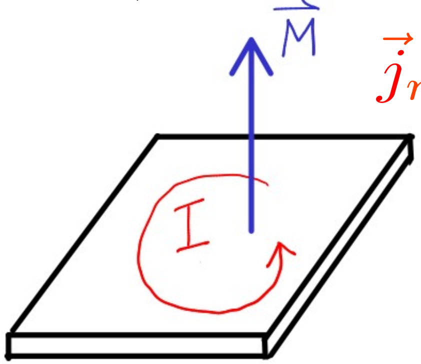

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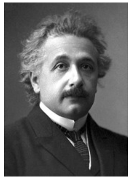

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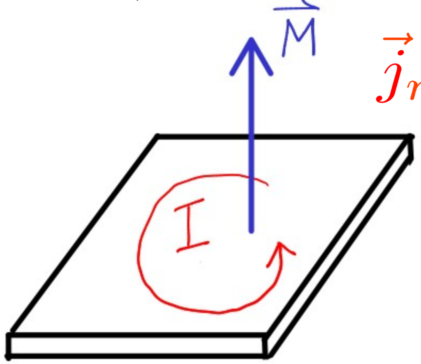

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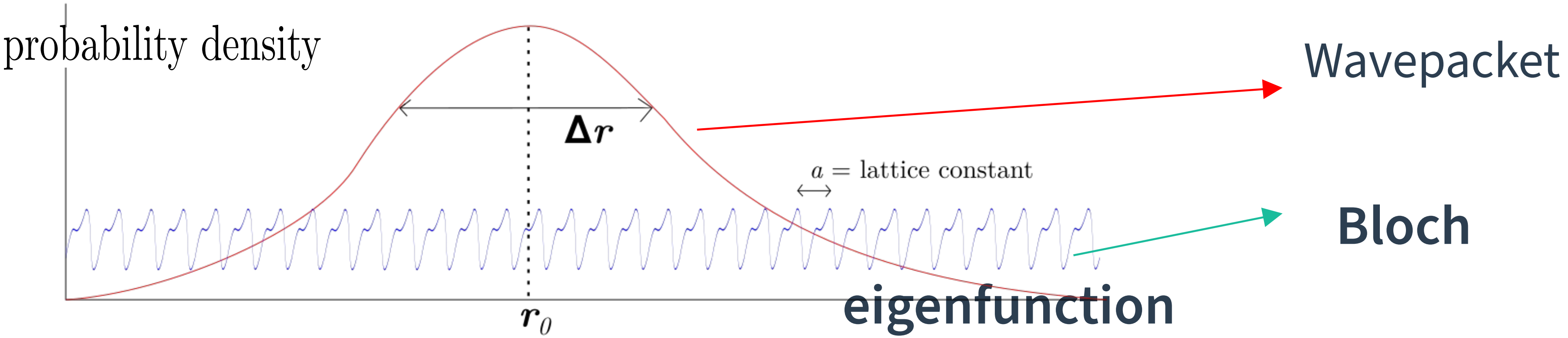
N. R. Cooper, B. I. Halperin, and I. M. Ruzin. PRB **55** 2344 (1997)

Geometric phase and Berry Curvature

Berry Curvature in reciprocal space

Construction of Bloch wavepacket, and its evolution



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(2D)

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(2D)

Effects of Berry curvature

- The semiclassical equation of velocity is modified

$$\dot{\vec{r}} = \frac{1}{\hbar}\nabla_k \varepsilon_k - \dot{\vec{k}} \times \boldsymbol{\Omega}(\boldsymbol{k})$$

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Anomalous velocity

ecoupling

(2D)

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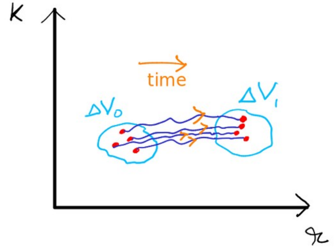

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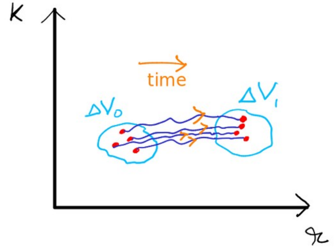

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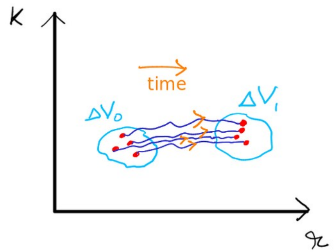

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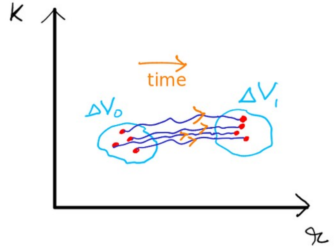

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$$\langle\mathcal{O}\rangle\,(\vec{B}=0) = 2\int \frac{d\vec{k}}{(2\pi)^d}\,\langle\mathcal{O}\rangle_{\vec{k}}\,\tilde{g}_{\vec{k}}$$

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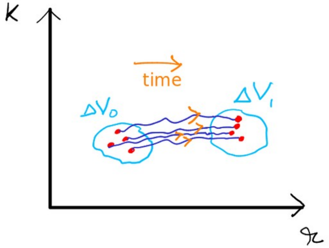

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Boltzmann Transport framework

Classical charge and energy currents

$$\vec{j}_e = -nev, \vec{j}_\epsilon = n\epsilon v$$

Boltzmann Transport framework

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$$\left\langle \hat{\vec{j}}_e \right\rangle =$$

Boltzmann Transport framework

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Semiclassical framework

$$\left\langle \hat{\vec{j}}_e \right\rangle = \qquad (-e\dot{\vec{r}})_{\vec{k}}\, \tilde{g}_{\vec{k}}$$

Boltzmann Transport framework

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$$\left\langle \hat{\vec{\dot{j}}}_Q \right\rangle = 2 \int \frac{d\vec{k}}{(2\pi)^d} \left(1 + \frac{e}{\hbar}\vec{B}\cdot\vec{\Omega}\left(\vec{k}\right)\right) \left(\varepsilon_{\vec{k}} - \mu\right) \dot{\vec{r}}_{\vec{k}} \tilde{g}_{\vec{k}}$$

Boltzmann Transport Equation

- Validity of Perturbation Theory

New results