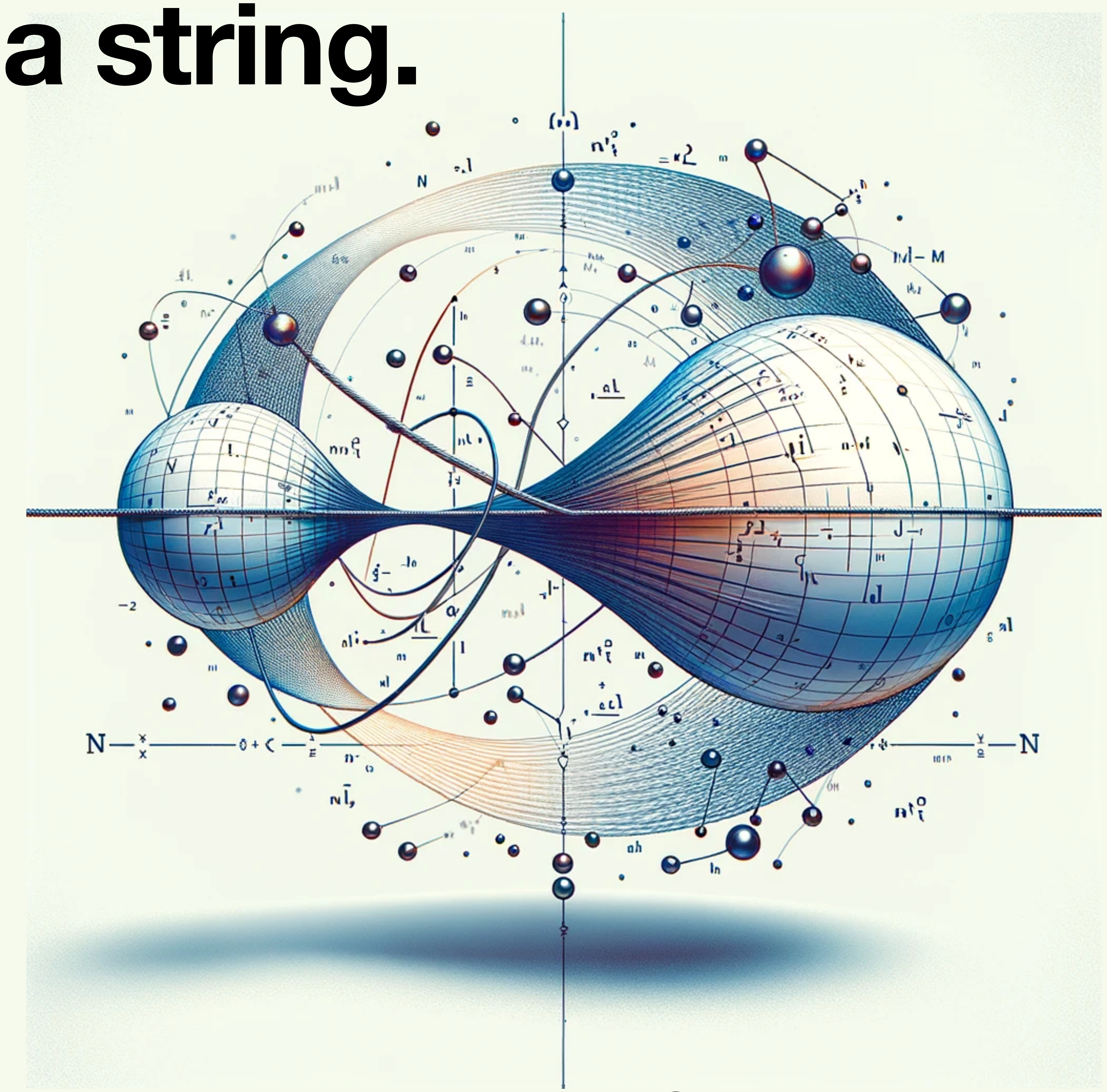
Computational Physics I
Exercise 6.2: N-Dimensional Newton-Raphson: Two masses on a string.

PHYS 3500K



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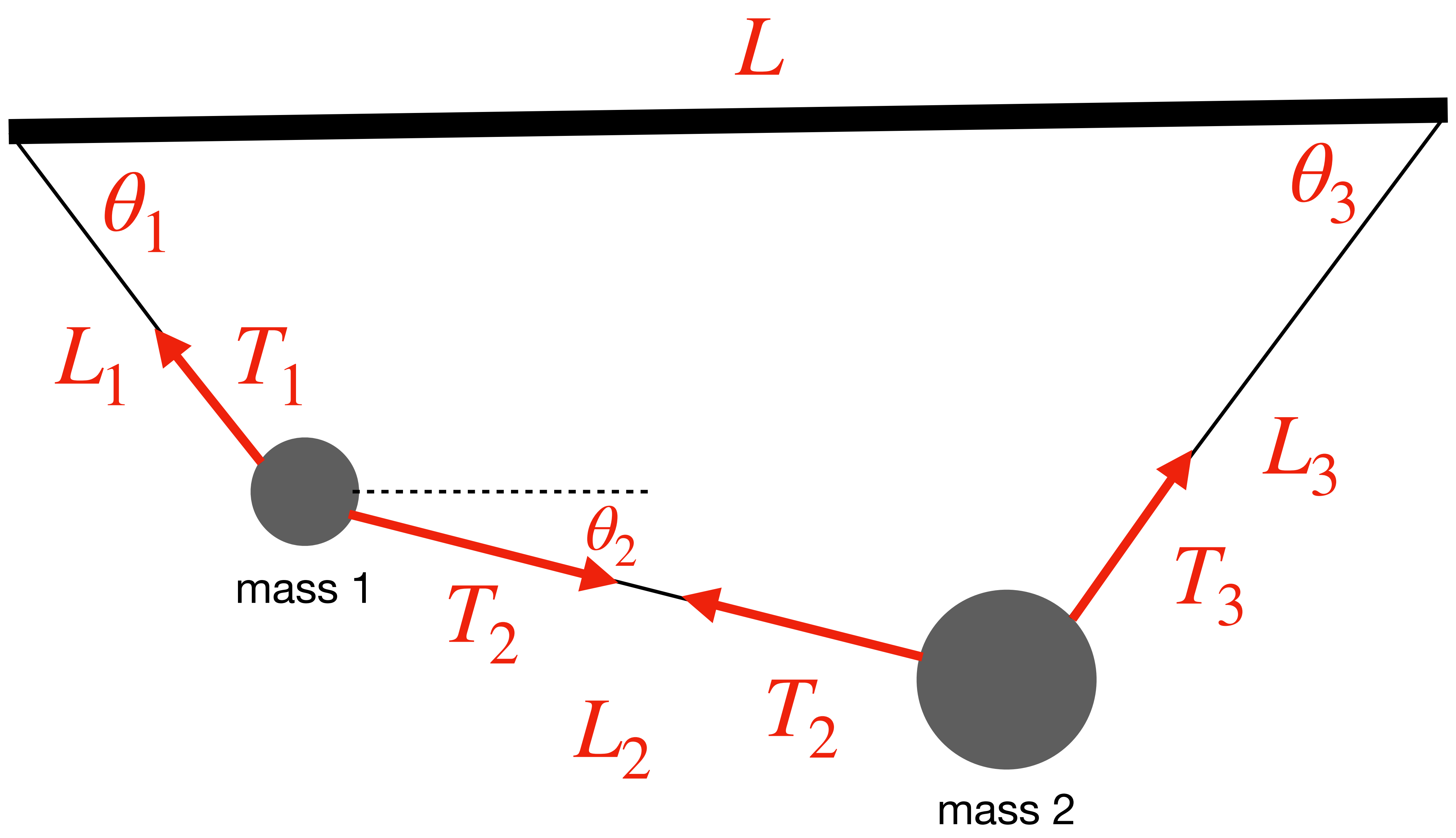
Problem Setup

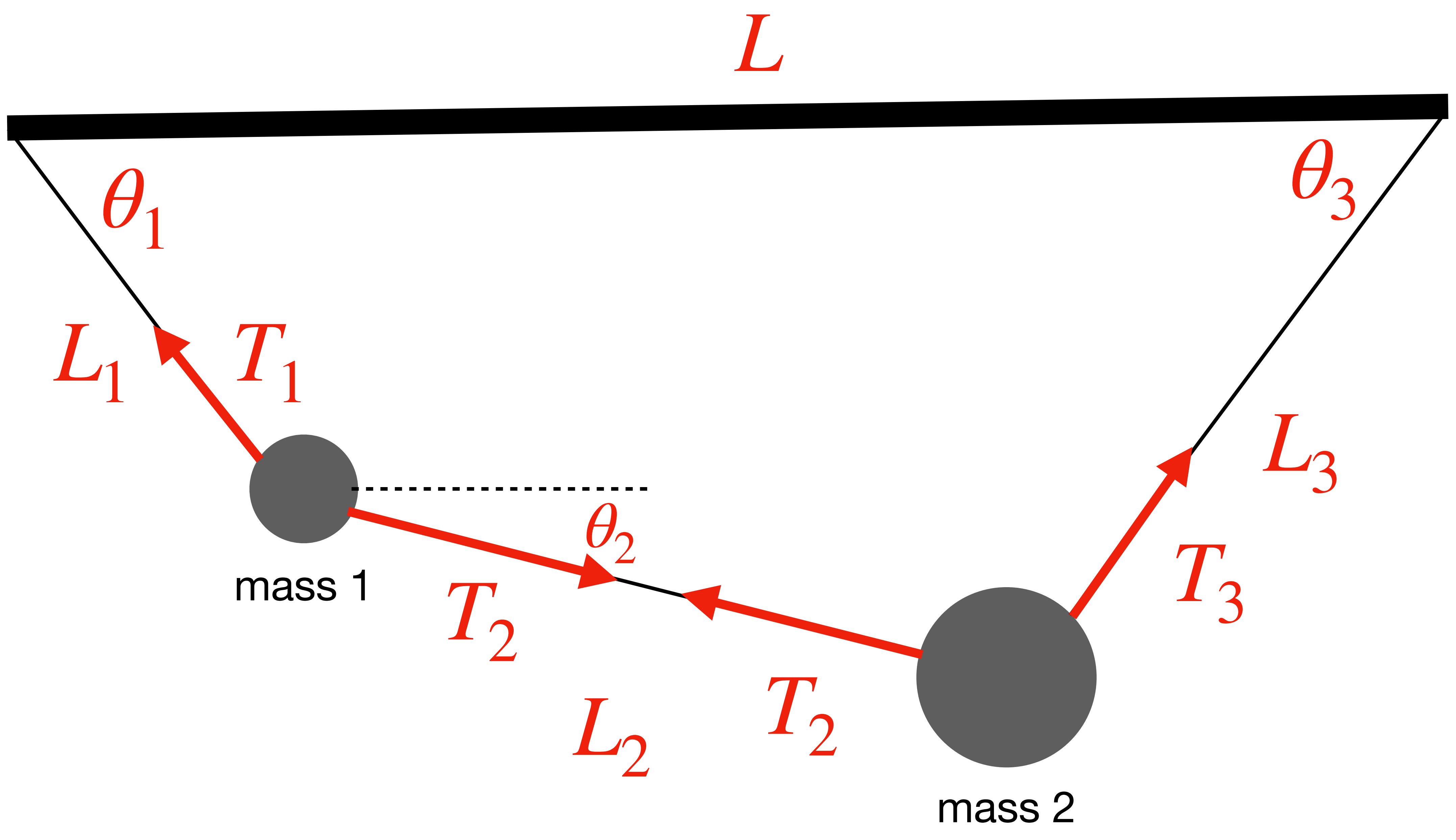
- Two masses 1 and 2, of weights W_1, W_2, respectively, are hung from three pieces of string with lengths L_1, L_2, L_3 and a horizontal bar of length L.

- Using N-dimensional Newton-Raphson searching, find the angles θ_1, θ_2 and θ_3, and the tensions exerted by the strings T_1, T_2, T_3.



Statics

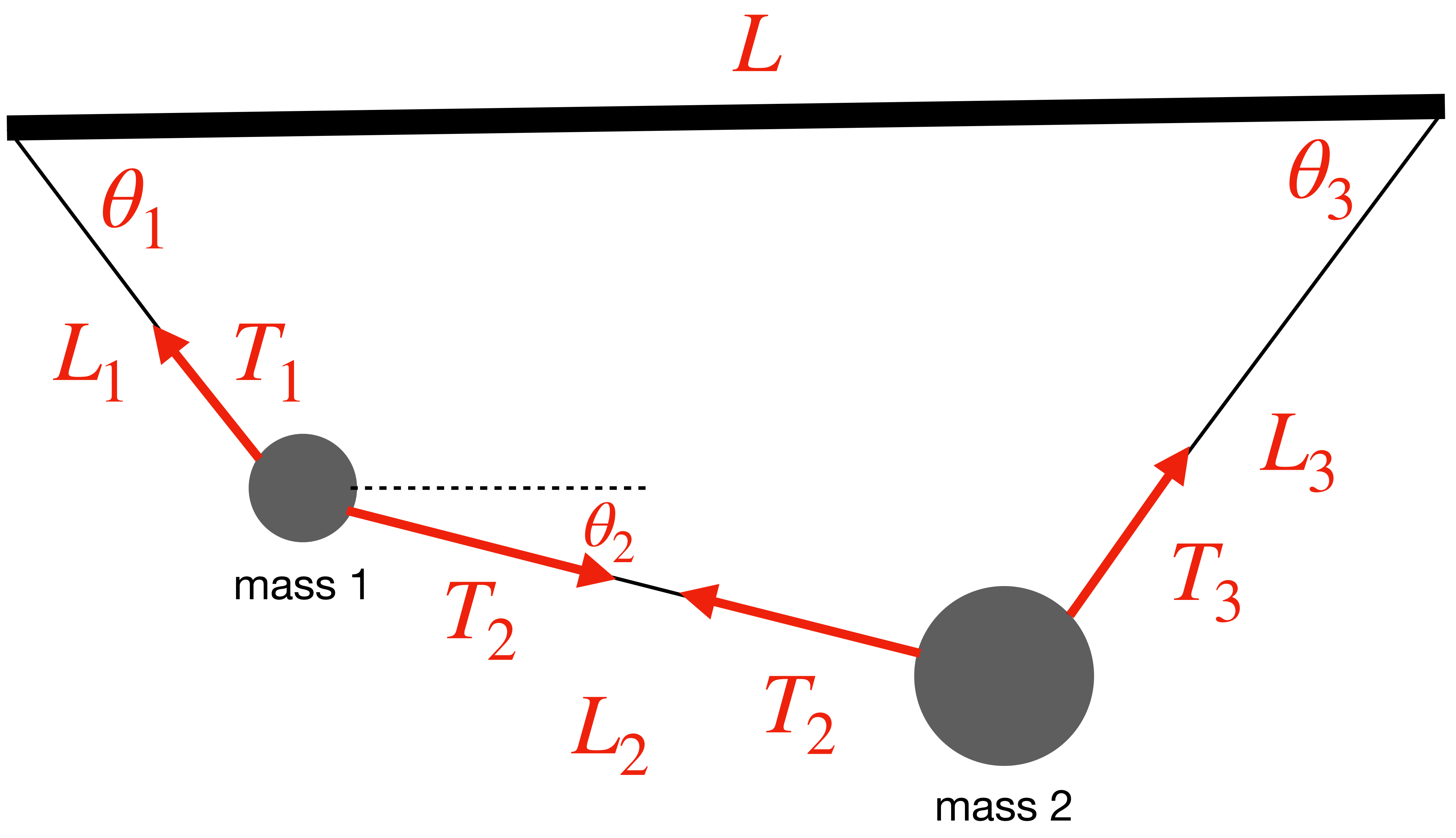
- Since the system is in equilibrium, the sum of all forces forces acting on the masses should be zero.



Other Constraints and Identities

- Other constraints also need to be satisfied, related to the lengths of the strings.

- We also use the trigonometric identities as independent non-linear equations (not necessary, but more straightforward to implement algorithm).



Full System of Equations:

- For the sake of simplicity of the algorithm implementation, we are treating the sines and cosines of the angles as independent variables. Hence, in our problem, there are $N = 9$ unknowns: $(x_1, x_2, \ldots, x_9) = (\sin\theta_1, \sin\theta_2, \sin\theta_3, \cos\theta_1, \cos\theta_2, \cos\theta_3, T_1, T_2, T_3)$.

- And there will be 9 nonlinear equations of the form: $f_i(x_1, x_2, \ldots, x_N) = 0$, for $i = 1,...,9$:

$$T_1 \sin\theta_1 - T_2 \sin\theta_2 - W_1 = 0$$

$$T_1 \cos\theta_1 - T_2 \cos\theta_2 = 0$$

$$T_2 \sin\theta_2 + T_3 \sin\theta_3 - W_2 = 0$$

$$T_2 \cos\theta_2 - T_3 \cos\theta_3 = 0$$

$$L_1 \cos\theta_1 + L_2 \cos\theta_2 + L_3 \cos\theta_3 - L = 0$$

$$L_1 \sin\theta_1 + L_2 \sin\theta_2 - L_3 \sin\theta_3 = 0$$

$$\sin^2\theta_1 + \cos^2\theta_1 - 1 = 0$$

$$\sin^2\theta_2 + \cos^2\theta_2 - 1 = 0$$

$$\sin^2\theta_3 + \cos^2\theta_3 - 1 = 0$$

Newton-Raphson Algorithm Summarized

- We begin the searching algorithm with initial guesses for the 9 unknowns $x_1, x_2, \ldots, x_N$, and then find the next guess $x_1 + \Delta x_1, x_2 + \Delta x_2, \ldots, x_N + \Delta x_N$ by solving the matrix equation:

$$\mathbf{J}\Delta\vec{x} = -\vec{f}$$

where: $\vec{f} = \begin{pmatrix} f_1 \\ f_2 \\ \ddots \\ f_N \end{pmatrix}$ and $\mathbf{J} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_N} \\ \ddots & \ddots & \ddots & \ddots \\ \frac{\partial f_N}{\partial x_1} & \frac{\partial f_N}{\partial x_2} & \cdots & \frac{\partial f_N}{\partial x_N} \end{pmatrix}$, both evaluated at the initial guess $x_1, x_2, \ldots, x_N$.

The derivatives can be evaluated using either a forward- or central-difference approximation.

On to Exercise 6.2!