Bezier Curve

Asha (1)

Construct Bezier curve for the control points:

(4, 2) (8, 8) (16, 4)

$$Q(u) = \sum_{i=0}^{2} P_i B_{i,2}(u), \quad 0 \le u \le 1$$

$$Q(u) = P_0 B_{0,2}(u) + P_1 B_{1,2}(u) + P_2 B_{2,2}(u)$$

$$\Rightarrow x(u) = x_0 B_{0,2}(u) + x_1 B_{1,2}(u) + x_2 B_{2,2}(u)$$

$$y(u) = y_0 B_{0,2}(u) + y_1 B_{1,2}(u) + y_2 B_{2,2}(u)$$

$$B_{i,n}(u) = {}^{n}C_i \, u^i (1-u)^{n-i}$$

$${}^{n}C_i = \frac{n!}{i!(n-i)!}$$

$$B_{0,2}(u) = (1-u)^2$$

$$B_{1,2}(u) = {}^{2}C_1 \, u^1 (1-u)^{2-1}$$

$$= \frac{2!}{1!(2-1)!} u(1-u)$$

$$= \frac{2}{1 \cdot 1} u(1-u)$$

$$= 2u(1-u)$$

$$B_{2,2}(u) = {}^{2}C_2 \, u^2 (1-u)^{2-2}$$

$$= u^2$$

$$x(u) = x_0(1-u)^2 + x_1 \cdot 2u(1-u) + x_2u^2$$
$$y(u) = y_0(1-u)^2 + y_1 \cdot 2u(1-u) + x_2u^2$$

For: $P_0(4,2)$ $P_1(8,8)$ $P_2(16,4)$

$$x(u) = 4(1-u)^2 + 8 \cdot 2u(1-u) + 16u^2$$
$$y(u) = 2(1-u)^2 + 8\,2u(1-u) + 4u^2$$

$$x(u) = 4(1-u)^2 + 16u - 16u^2 + 16u^2$$
$$= 4(1+u^2-2u) + 16u$$
$$= 4 + 4u^2 - 8u + 16u$$

$$\boxed{x(u) = 4 + 4u^2 + 8u}$$

$$y(u) = 2(1-u)^2 + 16u - 16u^2 + 4u^2$$
$$= 2(1+u^2-2u) + 16u - 12u^2$$
$$= 2 + 2u^2 - 4u + 16u - 12u^2$$

$$\boxed{y(u) = 2 - 10u^2 + 12u}$$

$x(u) = 4 + 4u^2 + 8u$

$y(u) = 2 - 10u^2 + 12u$

$u = 0$:

$x(0) = 4$

$y(0) = 2$

$u = 0.2$:

$x(0.2) = 4 + 4(0.2)^2 + 8(0.2)$

$= 4 + 0.16 + 1.6$

$= 5.76$

$y(0.2) = 2 - 10(0.2)^2 + 12(0.2)$

$= 2 - 0.4 + 2.4$

$= 4$

$u = 0.4$:

$x(0.4) = 4 + 4(0.4)^2 + 8(0.4)$

$= 4 + 0.64 + 3.2$

$= 7.84$

$y(0.4) = 2 - 10(0.4)^2 + 12(0.4)$

$= 2 - 1.6 + 4.8$

$= 5.2$

$u = 0.6$:

$x(0.6) = 4 + 4(0.6)^2 + 8(0.6)$

$= 4 + 1.44 + 4.8$

$= 10.24$

$y(0.6) = 2 - 10(0.6)^2 + 12(0.6)$

$= 2 - 3.6 + 7.2$

$= 5.6$

$u = 0.8$:-

$x(0.8) = 4 + 4(0.8)^2 + 8(0.8)$

$= 4 + 2.56 + 6.4$

$= 12.96$

$y(0.8) = 2 - 10(0.8)^2 + 12(0.8)$

$= 2 - 6.4 + 9.6$

$= 5.2$

$u = 1$:-

$x(1) = 4 + 4(1)^2 + 8(1)$

$= 4 + 4 + 8$

$= 16$

$y(1) = 2 - 10(1)^2 + 12(1)$

$= 2 - 10 + 12$

$= 4$

u	$x(u)$	$y(u)$
0	4	2
0.2	5.76	4
0.4	7.84	5.2
0.6	10.24	5.6
0.8	12.96	5.2
1	16	4

Bezier Curve

Asha (5)

P_0 : (4,2)

P_1 : (8,8)

P_2 : (16,4)