P. Asha ①

Liang Barsky Line clipping Algorithm

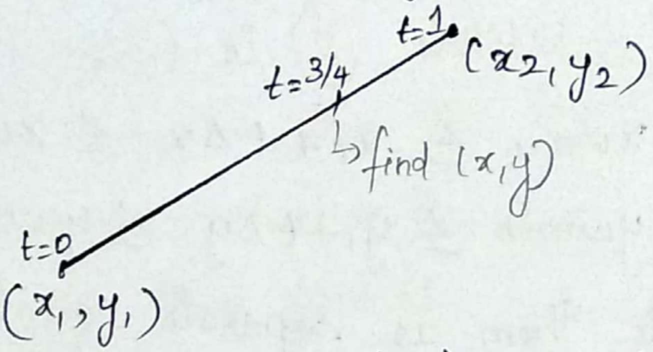
→ Use Parametric equation of the line

→ Consider line (x_1, y_1) to (x_2, y_2)

Consider t (time) : range from 0 to 1.

At start of the line (x_1, y_1), $t = 0$

At end of the line (x_2, y_2), $t = 1$.



Consider the line is at 3/4 between 0 and 1.

At $3/4^{th}$ of line path, $t = 3/4$.

The location is,

$$x = \tfrac{1}{4} x_1 + \tfrac{3}{4} x_2$$
$$y = \tfrac{1}{4} y_1 + \tfrac{3}{4} y_2$$

still we want to travel 1/4 from x_1 & have crossed $3/4^{th}$ of x_2

∴ In general,

If time is t,

$$\boxed{\begin{aligned} x &= (1-t)\,x_1 + t\,x_2 \\ y &= (1-t)\,y_1 + t\,y_2 \end{aligned}} \rightarrow ①$$

From ①,

$$x = x_1 - t x_1 + t x_2$$
$$= x_1 + t(x_2 - x_1)$$

$\boxed{x = x_1 + t\,\Delta x}$ → location of x at time t, inbetween (x_1, y_1) to (x_2, y_2)

Similarly, $y = y_1 - t y_1 + t y_2$

$\boxed{y = y_1 + t\,\Delta y}$

$\therefore$ Parametric line equation is,

$$\boxed{\begin{aligned} x &= x_1 + t\Delta x \\ y &= y_1 + t\Delta y \end{aligned}} \rightarrow (2)$$

Algorithm derivation:-

$$\left.\begin{aligned} xw_{min} &\le x \le xw_{max} \\ yw_{min} &\le y \le yw_{max} \end{aligned}\right\} \rightarrow (3)$$

Substitute (2) in (3),

$$xw_{min} \le x_1 + t\Delta x \le xw_{max}$$

$$yw_{min} \le y_1 + t\Delta y \le yw_{max}$$

Write them as separate equation,

$xw_{min} \le x_1 + t\Delta x$ $\Rightarrow -t\Delta x \le x_1 - xw_{min}$	Illy, $yw_{min} \le y_1 + t\Delta y$ $-t\Delta y \le y_1 - yw_{min}$
$x_1 + t\Delta x \le xw_{max}$ $\Rightarrow t\Delta x \le xw_{max} - x_1$	Illy, $y_1 + t\Delta y \le yw_{max}$ $\Rightarrow t\Delta y \le yw_{max} - y_1$

Similarly, generally, we can write as,

$$tP_k \le q_k \quad [k = 1, 2, 3, 4] \rightarrow$$ 4 different equations

$$\left.\begin{aligned} P_1 &= -\Delta x & q_1 &= x_1 - xw_{min} \\ P_2 &= \Delta x & q_2 &= xw_{max} - x_1 \\ P_3 &= -\Delta y & q_3 &= y_1 - yw_{min} \\ P_4 &= \Delta y & q_4 &= yw_{max} - y_1 \end{aligned}\right\}$$

Now we have to identify whether the line is inside/outside (or) partially inside/outside

Algorithm:

Step 1 : Get the endpoints of line (x_1, y_1) to (x_2, y_2)

Step 2 : Find Δx, Δy, $P1, P2, P3, P4, q_1, q_2, q_3, q_4$.

Step 3 : Assign $t1 = 0$, $t2 = 1$

(i) if $P_k = 0$ $(k = 1,2,3,4)$, then the line is llel to the window.

if $q_k < 0$ $(k = 1,2,3,4)$, then the line is outside the window.

(ii) if $P_k = 0$ & $q_k \geq 0$, then the line is inside the window.

(iii) For non-zero value of P_k, (line is Partially inside & Partially outside)

if $P_k < 0$ then find $t1$

(line proceeds from outside to inside)

$$t_1 = Max(0, q_k / P_k)$$

else $P_k > 0$ then find $t2$ (line proceeds from inside to outside)

$$t_2 = Min(1, q_k / P_k)$$

if $t1 > t2$ then line is completely outside - reject

else

find new set of (x, y) if t_1, t_2 is changed.

$$\boxed{\begin{aligned} x &= x_1 + t \Delta x \\ y &= y_1 + t \Delta y \end{aligned}}$$

Example:

Consider the window size from 5 to 9.
clip the following line:

(4,12) and (8,8)
(6,6) and (8,6)
(3,17) and (4,1)

Soln:

Given $x_{wmin} = y_{wmin} = 5$,
$x_{wmax} = y_{wmax} = 9$.

① (4,12) and (8,8):

$\Delta x = x_2 - x_1 = 8 - 4 = 4$

$\Delta y = y_2 - y_1 = 8 - 12 = -4$

$P_1 = -\Delta x = -4$

$P_2 = \Delta x = 4$

$P_3 = -\Delta y = 4$

$P_4 = \Delta y = -4$

$q_1 = 4 - 5 = -1$

$q_2 = 9 - 4 = 5$

$q_3 = 12 - 5 = 7$

$q_4 = 9 - 12 = -3$

Initial value of $t_1 = 0$, $t_2 = 1$.

P_1 & $P_4 < 0$ (choose corresponding q_k)
(here q_1 & q_4)

$\therefore t_1 = Max(0, q_k/p_k)$
$= Max(0, -1/-4, -3/-4) = \underbrace{max(0, 1/4, 3/4)}_{3/4}$

$\therefore \boxed{t_1 = 3/4}$

$\boxed{\therefore\ t_1 = 3/4}$

⇒ The line starting point is outside the window, at 3/4 of time, it crosses the window.

$P_2, P_3 > 0$, (choose corresponding q_2 & q_3)

$t_2 = \min(1, q_2/p_2, q_3/p_3)$

$= \min(1, 5/4, 7/4)$

$\boxed{\therefore\ t_2 = 1}$ ⇒ The line ending pt is inside the window.

t_1 value alone is changed,

$x = x_1 + t_1 \Delta x$

$= 4 + 3/4 \cdot 4$

$\boxed{x = 7}$

$y = y_1 + t_1 \Delta y$

$= 12 + 3/4(-4)$

$\boxed{y = 9}$

∴ New point is $\boxed{(7,9) \text{ to } (8,8)}$

If t_2 is changed, then use,

$x = x_1 + t_2 \Delta x$ // x or x_2

$y = y_1 + t_2 \Delta y$ // y or y_2

Line L_2:

(6,6) and (8,6)

$x_1\ y_1$ $\quad$ $x_2\ y_2$

$\boxed{\Delta x = 2 \quad \Delta y = 0}$

$P_1 = -2 \quad q_1 = 6-5 = 1$

$P_2 = 2 \quad q_2 = 9-6 = 3$

$P_3 = -0 \quad q_3 = 6-5 = 1$

$P_4 = 0 \quad q_4 = 9-6 = 3$

$\boxed{P=0}$ & $\boxed{q \geq 0}$

∴ Line is ||el & the line is inside the clipping window.