DTFT

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega \quad X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Symmetry Properties

$x[n]$	$X(e^{j\omega})$
$x^*[n]$	$X^*(e^{-j\omega})$
$x^*[-n]$	$X^*(e^{j\omega})$
Conjugate Symmetric signal	Real Fourier Transform
Conjugate Antisymmetric signal	Imaginary Fourier
$x[n] = x_e[n] + x_o[n]$	$Re\{X(e^{j\omega})\} + jIm\{X(e^{j\omega})\}$
Real x[n]	Conjugate Symmetric Real part is even Imaginary part is odd Magnitude is even Phase is odd

Theorems

$x[n]$	$X(e^{j\omega})$
$\alpha x[n] + \beta y[n]$	$\alpha X(e^{j\omega}) + \beta Y(e^{j\omega})$
$x[n-n_d]$	$e^{-j\omega n_d}X(e^{j\omega})$
$x[-n]$	$X(e^{-j\omega})$
$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
$x[n]y[n]$	$X(e^{j\omega}) * Y(e^{j\omega})$
$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$	
$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$	

Transform Pairs

Signal	Fourier Transform
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] (\alpha < 1)$	$\frac{1}{1-\alpha e^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n]$	$\frac{1}{(1-\alpha e^{-j\omega})^2}$
$\frac{\sin \omega_c n}{\pi n}$	$\left\{ \begin{array}{ll} 1 & \lvert\omega\rvert < \omega_c, \\ 0 & \omega_c < \lvert\omega\rvert \leq \pi \end{array} \right\}$
$\left\{ \begin{array}{ll} 1 & \lvert\omega\rvert < \omega_c, \\ 0 & \omega_c < \lvert\omega\rvert \leq \pi \end{array} \right\}$	$\frac{sin(\omega(M+1)/2)}{sin[\omega]/2}e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
$cos(\omega_0 n + \phi)$	$\sum_{k=-\infty}^{\infty} \pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)$

Z - Transform

Theorems

Sequence	Transform	ROC
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$R_1 \bigcap R_2$
$x[n-n_0]$	$z^{-n_0}X(z)$	R
$z_0^n x[n]$	$X(\frac{z}{z_0})$	$\|z_0\|R$
$nx[n]$	$-z\frac{dX(z)}{dz}$	R
$x^*[n]$	$X^*(z^*)$	R
$Re\{x[n]\}$	$\frac{1}{2}\left[X(z) + X^*(z^*)\right]$	Contains R
$Im\{x[n]\}$	$\frac{1}{2j}\left[X(z) - X^*(z^*)\right]$	Contains R
$x^*[-n]$	$X^*\left(\frac{1}{z^*}\right)$	$\frac{1}{R}$
$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$R_1 \bigcap R_2$

Transforms

Sequence	Transform	ROC
$\delta[n]$	1	$\mathbb{C}$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	$\mathbb{C}$ except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
$r^n cos(\omega_0 n)u[n]$	$\frac{1-rcos(\omega_0)z^{-1}}{1-2rcos(\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$r^n sin(\omega_0 n)u[n]$	$\frac{rsin(\omega_0)z^{-1}}{1-2rcos(\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$\left\{ \begin{array}{ll} a^n & 0 \le n \le N-1 \\ 0 & \end{array} \right\}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\|z\| > 0$

ROC properties

1. ROC is either $|z| \ge r$, $|z| \le r$, or $r_1 \le |z| \le r_2$
2. Fourier transform exists iff ROC includes unit circle
3. ROC contains no poles
4. If $x[n]$ is finite, ROC is entire plane
5. If $x[n]$ is right sided, ROC extends from outermost pole to ∞
6. If $x[n]$ is left sided, ROC extends from innermost pole to 0
7. If $x[n]$ is two sided, then ROC is an annulus
8. ROC is connected region
   For LTI Systems
9. Stable $\implies$ Causal if and only if right-sided ROC
10. Causal $\implies$ Stable if and only if poles inside unit circle