

# System Properties

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**Causal:**  $x_1(t) = x_2(t) \forall t \leq \tau \implies S\{x_1(t)\} = S\{x_2(t)\} \forall t \leq \tau$

**Strictly Causal:**  $x_1(t) = x_2(t) \forall t < \tau \implies S\{x_1(t)\} = S\{x_2(t)\} \forall t \leq \tau$

**Memoryless:**  $S\{x(t)\} = f(x(t))$

**Linearity**  $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha S\{x_1(t)\} + \beta S\{x_2(t)\}$

**Time Invariance:**  $\forall \tau \in \mathbb{R}, S\{x_1(t - \tau)\} = S\{x(t)\}(t - \tau)$

**Stability**  $\exists A < \infty$  such that  $|x(t)| \leq A \forall t \implies \exists B < \infty$  such that  $|S\{x(t)\}| \leq B \forall t$

## Discrete Dynamics

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**Discrete Signal:** A signal  $e$  is discrete if  $\exists f : T \rightarrow \mathbb{N}$  which is order-preserving. In other words, we can count the number of times  $e$  is present

## State Machines

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**States::** Finite set ( $S$ )

**Inputs::** Finite set of inputs ( $I$ )

**Outputs::** Finite set of outputs ( $O$ )

**update::**  $S \times I \rightarrow S \times O$

$$(s(n+1), y(n)) = \text{update}(s(n), x(n))$$

(If the machine is non-deterministic, then  $\text{update} : S \times I \rightarrow 2^{S \times O}$ )

**Initial State:** The beginning state of the state machine

**Behavior:** An assignment of a signal such that the output signals are the output produced for the given inputs

**Observable Trace:**  $((x_0, y_0), (x_1, y_1), (x_2, y_2), \dots)$  where  $x_i$  are inputs and  $y_i$  are outputs

**Execution Trace:**  $((x_0, s_0, y_0), (x_1, s_1, y_1), \dots)$  where  $x_i$  are inputs,  $s_0$  is the state the machine is leaving, and  $y_i$  are outputs

## Components

**Self-Transition:** A transition starting and ending at the same state

**Stuttering Transition:** A transition where all inputs and outputs are absent and the machine does not change state

**Default Transition:** A transition enabled if no non-default transition is enabled and if the guard evaluates to true.

**Set Action:** Specifies assignment to a variable after the guard is evaluated and the output is produced.

**State Space:** All possible settings of modes + variables.  $|States| = np^m$  where  $n$  is the number of modes, and there are  $m$  variables taking on  $p$  values each.

**Pre-emptive Transition:** The guard of a pre-emptive transition is evaluated before the refinement, and if it is true, the refinement does not act

**Reset Transition:** The destination refinement of the transition is set to its initial state

**History Transition:** The destination refinement of the transition resumes in the state where it was last.

## Composition

**Synchronous Composition:** Two or more state machines react simultaneously.

**Asynchronous Composition:** When two or more state machines react independently of each other.

**Side by Side Synchronous Composition:** One react of the overall machine is the simultaneous react of the sub-machines

**Side by Side Asynchronous Composition with Interleaving Semantics:** A react of the overall state machine is the react of one of the sub-machines where the choice is non-deterministic.

**Cascade Composition:** When the output port of state machine  $A$  feeds into the input of state machine  $B$ . The reactions are simultaneous and instantaneous but  $A$  reacts first to produce the input to  $B$  (if any).