EE128 Course Notes

Anmol Parande

Fall 2020 - Professor Ronald Fearing

Disclaimer: These notes reflect 128 when I took the course (Fall 2020). They may not accurately reflect current course content, so use at your own risk. If you find any typos, errors, etc, please raise an issue on the GitHub repository.

Contents

1 Introduction to Control

The general goal of control is to get some physical system to respond to a reference input in the way we would like.

Definition 1 *The plant is the physical system which we would like to control*

In general, there are two different types of control.

Definition 2 *Open-Loop control is where we pass a reference directly to the actuator to control the plant (see fig. 1.*

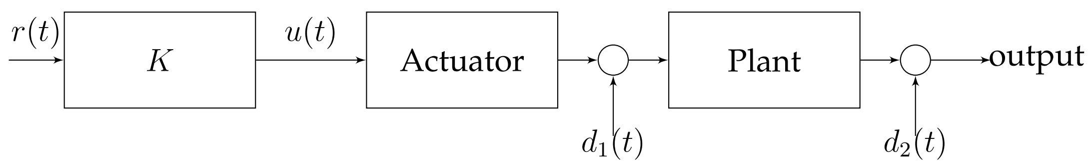


Figure 1: Open-Loop Control

Open-Loop control is generally difficult because the disturbances make it difficult to copy the reference exactly.

Definition 3 *Closed loop control is using the output of our system and comparing it to the reference in order to generate the control signal (see fig. 2).*

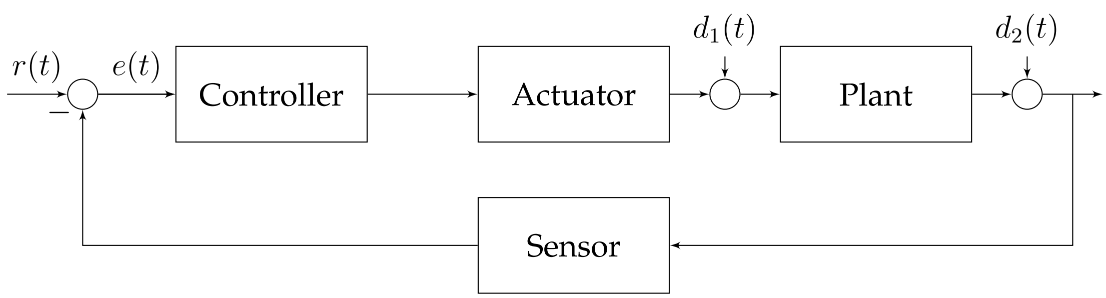


Figure 2: Closed-Loop Control

Notice how the output signal is subtracted from a reference signal, and we use the difference (a.k.a the error) to determine what input we pass into the plant. Looking at the overall transfer function of the system, we see that

$$\begin{aligned} Y(s) &= (R(s) - Y(s))H_c(s)H_p(s) \\ (1 + H_c(s)H_p(s))Y(s) &= H_c(s)H_p(s)R(s) \\ H(s) = \frac{Y(s)}{R(s)} &= \frac{H_c(s)H_p(s)}{1 + H_c(s)H_p(s)} \end{aligned}$$

Depending on what control we use for $H_c(s)$, we can shape this transfer function to be what we want.

1.1 Types of Control

1.1.1 Constant Gain Control

When $H_c(s) = K_0$, this is known as constant gain control.

$$H(s) = \frac{K_0 H_p(s)}{1 + K_0 H_p(s)}$$

The poles of this system are clearly when $1 + K_0 H_p(s) = 0$.

1.1.2 Lead Control

Lead controllers are of the form

$$H_c(s) = K_0 \frac{s - \beta}{s - \alpha}$$

Their poles are when

$$1 + K_0 \frac{s - \beta}{s - \alpha} H_p(s) = 0$$

1.1.3 Integral Control

Integral controller are of the form

$$H_c(s) = \frac{K_0}{s}$$

Their poles are when

$$1 + \frac{K_0}{s} H_p(s) = 0$$

1.2 Root Locus Analysis

For all forms of control, we need to choose a constant which places our poles where we want. Root Locus Analysis is the technique which helps us determine how our poles will move as $K_0 \to \infty$. Assuming we only have a single gain to choose, we the poles of the new transfer function will be the roots of

$$1 + K_0 H(s)$$

where $H(s)$ is some transfer function that results in the denominator (For example, in constant gain control, $H(s) = H_p(s)$ but for lead control, $H_s = \frac{s-\beta}{s-\alpha} H_p(s)$).

Definition 4 *The root locus is the set of all points $s_0 \in \mathbb{C}$ such that $\exists K_0 > 0$ such that $1 + K_0 H(s) = 0$.*

This definition implies that $H(s_0) = \frac{1}{K_0}$ for some K_0, meaning the root locus is all points such that $\angle H(s_0) = -180°$. The first step of RLA is to factor the numerator and denominator of $H(s)$

$$H(s) = \frac{\prod_{i=0}^{m}(s - \beta_i)}{\prod_{i=0}^{n}(s - \alpha_i)}$$

As $k \to 0, H(s_0) = -\frac{1}{K_0} \to \infty$, so the root locus begins at the poles. As $k \to \infty, H(s_0) = -\frac{1}{K_0} \to 0$, so the root locus will end at the open loop zeros. However, if $m < n$, (i.e there are more poles than 0's), not all of the poles can converge to a zero. Instead $n - m$ branches will approach ∞ witth asymptotes at

$$\frac{\sum_i^n \alpha_i - \sum_i^m \beta_i}{n - m}$$

and angles of

$$\frac{180 + (i - 1) * 360}{n - m}$$

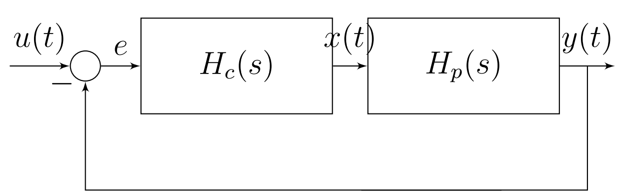
The final rule of RLA is that parts of the real line left of an odd number of real poles and zeros are on the root locus. RLA tells us that we have to be careful when choosing our gain K_0 because we could by mistake cause instability. In particular, high gain will cause instability if

- $H(s)$ has zeros in the right half of the plane
- if $n - m \geq 3$, then the asympototes will cross the imaginary axis.

1.3 Feedback Controller Design

When we design systems to use in feedback control, there are certain properties we want besides just basic ones like stability. Because signals can be thought of as a series of step signals, when analyzing these properties, we will assume $r(t) = u(t)$

1.3.1 Steady State Tracking Accuracy



Definition 5 *A system has steady state tracking accurracy if the different between the reference and the output signals tends to 0 as* $t \to \infty$

$$e_{ss} := \lim_{t\to\infty} e(t) = 0$$

A useful theorem which can help us evaluate this limit is the final value theorem.

Theorem 1 (Final Value Theorem)

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

As long as a limit exists and $e(t) = 0$ *for* $t < 0$

Looking at the relationship between $E(s)$ and $R(s)$, we see that

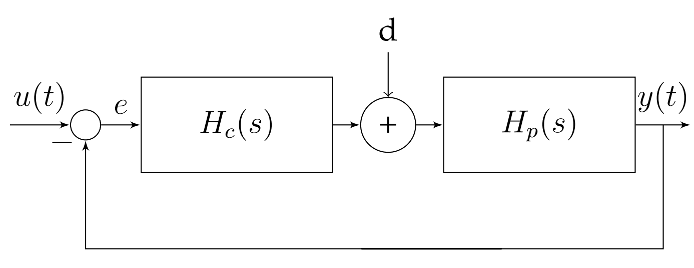
$$\frac{E(s)}{R(s)} = \frac{1}{1 + H_c(s)H_p(s)} \implies E(s) = \frac{\frac{1}{s}}{1 + H_c(s)H_p(s)}$$

Thus

$$e_{ss} = \lim_{s\to 0} \frac{1}{1 + H_c(s)H_p(s)}$$

Thus as long as $H_c(s)H_p(s)$ has at least one pole at $s = 0$, then $e_{ss} = 0$. Notice that integral control gives us a pole at s_0, so it is guaranteed that an integral controller will be steady-state accurate.

1.3.2 Disturbance Rejection



Sometimes the output of our controller can be disturbed before it goes into the plant. Ideally, our system should be robust to these disturbances.

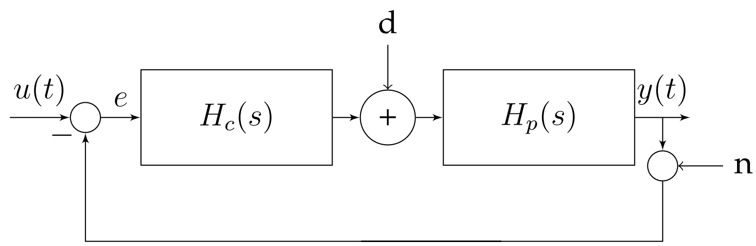
$$Y(s) = H_p(s)\left[H_c(s)(R_s - Y_s) + D(s)\right]$$
$$Y(s) = \frac{H_c(s)H_p(s)}{1 + H_c(s)H_p(s)}R(s) + \frac{H_p(s)}{1 + H_c(s)H_p(s)}D(s)$$

The system will reject disturbances if the $\frac{H_p(s)}{1+H_c(s)H_p(s)}D(s)$ is close to 0 in the steady state. Assuming that $d(t) = u(t)$, we see that

$$\delta_{ss} = \lim_{s\to 0} s\frac{H_p(s)}{1+H_c(s)H_p(s)}\frac{1}{s} = \lim_{s\to 0}\frac{H_p(s)}{1+H_c(s)H_p(s)}$$

Thus as long as H_c has a pole at 0, then the system will reject disturbances. Notice that integral control guarantees disturbance rejection as well.

1.3.3 Noise Insensitivity



In real systems, our measurement of the output $y(t)$ is not always 100% accurate. One way to model this is to add a noise term to the output. Looking at the relationship between the noise and the output signal, we see

$$H(s) = \frac{-H_c(s)H_p(s)}{1+H_c(s)H_p(s)}$$

In order to reject this noise, we want this term to be close to 0, so ideally $H_c(s)H_p(s) << 1$ as $s \to 0$. However, this conflicts with our desire for $H_c(s)H_p(s)$ to have a pole at 0 to guarantee steady state tracking. Thus it is difficult to make a controler that is both accurate and robust to noise. However, because noise is usually a high frequency signal and the reference is a low frequency signal, we can mitigate this by choosing $H_c(s)H_p(s)$ to be a low-pass filter.