

# Unilateral Laplace Transform

$$X(s)=\int_{0-}^{\infty}x(t)e^{-st}dt$$

## Theorems

| $x(t)$              | $X(s)$   | ROC              |
|---------------------|--|------------------|
| $x(t-t_0)$          | $e^{-st_0}X(s)$  | $R$              |
| $e^{s_0t}x(t)$      | $X(s-s_0)$   | $R+Re(s_0)$      |
| $x(at)$             | $\frac{1}{ a }X\left(\frac{s}{a}\right)$                     | $aR$             |
| $x^*(t)$            | $X(s^*)^*$   | $R$              |
| $(x_1*x_2)(t)$      | $X_1(s)X_2(s)$   | $R_1\bigcap R_2$ |
| $-tx(t)$            | $\frac{dx}{ds}$  | $R$              |
| $\frac{d^nx}{dt^n}$ | $s^nX(s)-\sum_{i=0}^{n-1}s^{n-i-1}\frac{d^ix}{dt^i} _{t=0-}$ | $R$              |

## Transforms

| Signal                              | Transform                             | ROC          |
|-------------------------------------|---------------------------------------|--------------|
| $\delta(t-T)$                       | $e^{-sT}$                             | $\mathbb{C}$ |
| $\frac{t^{n-1}}{(n-1)!}u(t)$        | $\frac{1}{s^n}$                       | $Re(s)>0$    |
| $\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$ | $\frac{1}{(s+a)^n}$                   | $Re(s)>a$    |
| $e^{-at}\cos(\omega_0t)u(t)$        | $\frac{s+a}{(s+a)^2+\omega_0^2}$      | $Re(s)>a$    |
| $e^{-at}\sin(\omega_0t)u(t)$        | $\frac{\omega_0}{(s+a)^2+\omega_0^2}$ | $Re(s)>a$    |