

# Discrete Distributions

## Probabilities

Distribution	PMF	CDF
Bernoulli	$p^k(1-p)^{1-k}, \quad k \in \{0,1\}$	$\begin{cases} 0 & k < 0 \\ 1-p & 0 \leq k < 1 \\ 1 & k \geq 1 \end{cases}$
Binomial	$\binom{n}{k}p^k(1-p)^{n-k}$	-
Geometric	$p(1-p)^{k-1}$	$1-(1-p)^k$
Poisson	$\frac{\lambda^k e^{-\lambda}}{k!}$	-

## Moments

Distribution	Expectation	Variance	MGF
Bernoulli	$p$	$p(1-p)$	$(1-p) + pe^t$
Binomial	$np$	$np(1-p)$	$(1-p + pe^t)^n$
Geometric	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Poisson	$\lambda$	$\lambda$	$e^{\lambda(e^t-1)}$

# Continuous Distributions

## Probabilities

Distribution	PDF	CDF
Uniform	$\frac{1}{b-a}$	$\frac{x-a}{b-a}$
Exponential	$\lambda e^{-\lambda x}$	$1 - e^{-\lambda x}$
Normal	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\Phi(x)$
Erlang	$\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$	$1 - \sum_{n=0}^{k-1} \frac{1}{n!} e^{-\lambda x} (\lambda x)^n$

## Moments

Distribution	Expectation	Variance	MGF
Uniform	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Exponential	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}$
Normal	$\mu$	$\sigma^2$	$e^{\mu t + \frac{\sigma^2 t^2}{2}}$
Erlang	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$	$(1 - \frac{t}{\lambda})^{-k}$

## Derived Distributions

1. If  $X_1, X_2, \dots, X_N \sim \text{Exp}(\lambda_i)$  are independent, then  $\min X_i \sim \text{Exp}(\sum \lambda_i)$ .
2. If  $X_1, X_2, \dots, X_N \sim \text{Exp}(\lambda_i)$  are independent, then  $P(X_i = \min X_k) = \frac{\lambda_i}{\sum \lambda_k}$ .

## Assorted Facts

1. If  $X$  is a random variable, then  $P(X = 0) \leq \frac{\text{Var}(X)}{\mathbb{E}[X]^2}$
2. If  $(N_t)_{t \geq 0}$  is a Poisson Process with rate  $\lambda$ , then the time from the most recent arrival to the current “random” time is  $t_0 - \bar{T}_i \sim \text{Exp}(\lambda)$