Unilateral Laplace Transform

$$X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$$

Theorems

$x(t)$	$X(s)$	ROC
$x(t-t_0)$	$e^{-st_0}X(s)$	R
$e^{s_0t}x(t)$	$X(s-s_0)$	$R+Re(s_0)$
$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	aR
$x^*(t)$	$X(s^*)^*$	R
$(x_1 * x_2)(t)$	$X_1(s)X_2(s)$	$R_1 \bigcap R_2$
$-tx(t)$	$\frac{\mathrm{d}X}{\mathrm{d}s}$	R
$\frac{\mathrm{d}^n x}{\mathrm{d}t^n}$	$s^nX(s) - \sum_{i=0}^{n-1} s^{n-i-1}\frac{\mathrm{d}^i x}{\mathrm{d}t^i}\rvert_{t=0^-}$	R

Transforms

Signal	Transform	ROC
$\delta(t-T)$	e^{-sT}	$\mathbb{C}$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$Re(s) > 0$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(s+a)^n}$	$Re(s) > a$
$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$Re(s) > a$
$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0}$	$Re(s) > a$

Electro-Mechanical Equivalence

Equivalent Quantities

Translational Mechanical System	Rotational Mechanical System	Electrical System
Force (F)	Torque	Voltage (V)
Mass (M)	Moment of Inertia (J)	Inductance (L)
Damping Coefficient (B)	Rotational Damping Coefficient (B)	Resistance (R)
Spring Constant (K)	Torsional Spring Constant (K)	Reciprocal of Capacitance $\left(\frac{1}{C}\right)$
Displacement (x)	Angular Displacement (θ)	Charge (Q)
Velocity (v)	Angular Velocity (ω)	Current (I)

Equation Equivalence

Translational Mechanical System	Rotational Mechanical System	Electrical System
$Ms^2X(s)$	$Js^2\Theta(s)$	$LsI(s)$
$BsX(s)$	$Bs\Theta(s)$	$RI(s)$
$KX(s)$	$K\Theta(s)$	$\frac{1}{Cs}I(s)$
-	$\frac{T_2(s)}{T_1(s)} = \frac{\Theta_1(s)}{\Theta_2(s)} = \frac{N_2}{N_1}$	$\frac{N_p}{N_s} = \frac{V_p(s)}{V_s(s)} = \frac{I_s(s)}{I_p(s)}$

Conversion Rules

1. The force at two ends of a damper (or spring) must be equal $\Leftrightarrow$ the voltage across the resistor (or capacitor) must be equal
2. Parallel in one domain $\implies$ Series in the other domain
3. $\sum F = 0$ at a massless node $\Leftrightarrow \sum V = 0$ at an electrical node
4. Rotational impedances are reflected through gear trains by multiplying by $\left(\frac{N_{dest}^2}{N_{source}^2}\right)$

Conversion Procedure

Electrical to Mechanical

1. Label all currents such that only one current flows through inductors
2. Write loop equations for each loop
3. Re-write equations using the analogous quantities. Each loop is replaced by a position
4. Draw mechanical system corresponding to equations

Mechanical to Electrical

1. Write force equations for each position
2. Re-write equations using analogous quantities. Each equation becomes a loop
3. Draw loops such that only one current flows through each inductor