CS70 Course Notes

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Fall 2019 - Professors Alistair Sinclair and Yun Song

Disclaimer: These notes reflect CS60 when I took the course (Fall 2019). They may not accurately reflect current course content, so use at your own risk. If you find any typos, errors, etc, please report them on the GitHub repository.

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1 Propositional Logic

Definition 1 *A proposition P is a statement that is either true or false*

Propositions can depend on one or more variables. This is denoted by $P(x, y, ...)$

1.1 Connectives

Definition 2 *A connective is an operator which joins two or more propositions together in some way*

Connectives are fully defined by a **Truth Table** which enumerates the values of the connective given all possible combination of inputs.

Definition 3 *The base connectives are those which can be combined to create any other connective.*

- $\wedge$*: P AND Q*
- $\vee$*: P OR Q*
- $\neg$*: NOT P*

One important connective is the **implies** connective.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

Notice that $P \implies Q$ has the same truth table as $\neg P \vee Q$. This means they are equivalent.

Definition 4 *The contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$*

Notice: The contrapositive is logically equivalent to the original statement

Definition 5 *The converse of $P \implies Q$ is $Q \implies P$*

Notice: This is not always equivalent to the original statement The **if and only if** connective $P \iff Q$ is equivalent to $(P \implies Q) \wedge (Q \implies P)$

P	Q	$P \iff Q$
T	T	T
T	F	F
F	T	F
F	F	T

1.2 Quantifiers

Quantifiers help introduce variables into our propositions.

- $\forall$: For all
- $\exists$: There exists

Important: The order of quantifiers matters.

1.3 DeMorgan's Laws

- $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
- $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
- $\neg(\forall x P(x)) \equiv \exists x(\neg P(x))$
- $\neg(\exists x P(x)) \equiv \forall x(\neg P(x))$

2 Proofs

Definition 6 *A proof is a sequency of statements, each of which follows from the preceding ones by a valid law of reasoning.*

Definition 7 *An Axiom is a basic fact which can be assumed without proof*

2.1 Direct Proof

Goal: Prove $P \implies Q$
Approach:

1. Assume P
2. Deduce Q through logical steps

2.2 Proof by Contraposition

Since the contrapositive of a statement is logically equivalent to the original statement, we can prove the original statement by proving the contrapositive.
Goal: To prove $P \implies Q$
Approach:

1. Assume $\neg Q$
2. Deduce $\neg P$ through logical steps

Theorem 1 *The pigeonhole principle says that if n objects are placed in k boxes and $n > k$, then some box contains ≥ 1 object.*

2.3 Proof by Contradiction

Goal: Prove $P \implies Q$ **Approach:**

1. Assume $\neg P$
2. Deduce $\neg P \implies (R \neg R)$

R and $\neg R$ are two facts which can be deduced from assumign $\neg P$

2.4 Proof by Cases

Goal: Prove $P \implies Q$
Approach:

1. Break P into cases $1...n$
2. Prove P holds in all cases

2.5 Proof by Induction

Goal: Prove $\forall n, P(n)$ **Approach:**

1. Prove a based case
2. Assume P(k) (Induction hypothesis)
3. Prove $P(k) \implies P(k+1)$ (Inductive Step)

2.5.1 Strong Induction

Because induction requires that $P(k)$ be true to prove $P(k+1)$, it might be easier to just assume $P(0) \wedge P(1) \wedge ... \wedge P(k)$ when proving $P(k+1)$.