Definitions

Causality: $y[n]$ only depends on $x[n]$

Memoryless: $y[n]$ depends only on $x[n]$

Time Invariance: $y[n] = T\{x[n]\} \Rightarrow y[n-n_0] = T\{x[n-n_0]\}$

Bibo Stability: $|x[n]| \le B_x < \infty \ \forall n \Rightarrow |y[n]| \le B_y < \infty \ \forall n$

Stable Sequence: $\sum_{n=-\infty}^{\infty} |x[n]| < \infty$

LTI Systems

- Causal: $h[n<0] = 0$
- Stable: $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$
- Stable $\Rightarrow$ Causal $\Leftrightarrow$ ROC of $H(z)$ right sided
- Causal $\Rightarrow$ Stable $\Leftrightarrow$ poles of $H(z)$ inside unit circle

Linear Convolution via DFT

$x[n]$ $0 \le n \le L-1$ & $h[n]$ $0 \le n \le P-1$

1. Pad vectors to length $N \ge L+P-1$
2. Compute $Y[k] = H[k]X[k]$
3. Take the inverse DFT

Overlap-Add Linear Convolution

1. $x_r[n] = \begin{cases} x[n] & rL \le n < (r+1)L \\ 0 & \text{else} \end{cases}$
2. Zero pad $x_r[n]$ & $h[n]$ to $N \ge L+P-1$
3. Linearly convolve $x_r[n]$ & $h[n]$
4. $y[n] = \sum_r x_r[n] * h[n]$

* Neighboring outputs share $P-1$ points

Overlap-Save Linear Convolution

1. $x_r[n] = x[n + r(L-P+1) - P + 1]$ $(0 \le n \le L-1)$
2. $y_r[n] = \begin{cases} x_r[n] * h[n] & P-1 \le n \le L-1 \\ 0 & \text{else} \end{cases}$
3. $y[n] = \sum_r y_r[n - r(L-P+1) + P - 1]$

* First $P-1$ samples of each y_r are bad

FFT

$W_N = e^{-j\frac{2\pi}{N}}$

$W_N^{k(N-n)} = W_N^{-n} = (W_N^{kn})^*$

$W_N^{kn} = W_N^{k(n+N)}$

$W_N^2 = W_{N/2}$

Decimation in Time

$X[k] = \sum x[2r] W_N^{2rk} + \sum x[2r+1] W_N^{(2r+1)k}$

$\hookrightarrow X[k] = G[k] + W_N^k H[k]$

$X[k+\frac{N}{2}] = G[k] - W_N^k H[k]$

Decimation in Frequency

$X[2r] = \sum x[n] W_N^{2rn} + \sum x[n+\frac{N}{2}] W_N^{2r(n+\frac{N}{2})}$

$= \sum (x[n] + x[n+\frac{N}{2}]) W_{N/2}^{rn}$

$X[2r+1] = \sum (x[n] - x[n+\frac{N}{2}]) W_{N/2}^{rn}$

Windowing

- Larger window $\rightarrow$ smaller main lobe $\rightarrow$ better frequency resolution
- Smaller main lobe $\rightarrow$ larger side lobes
- More zero-padding $\rightarrow$ DFT better samples DTFT
- Smaller window $\rightarrow$ larger main lobe $\rightarrow$ better time resolution

STFT & DTDFT

STFT: $X[n,w] = \sum_m x[n+m]w[m]e^{-jwm}$

DTDFT: $X_r[k] = \sum_{m=0}^{L-1} x[rR+m]w[m]e^{-j\frac{2\pi}{N}km}$

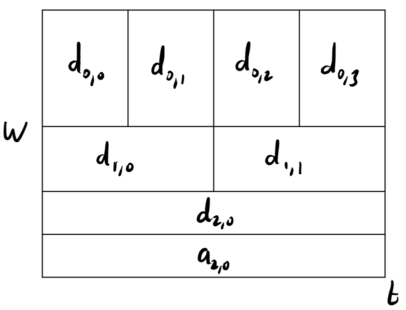
L: window length, R: Sample jump, $N \ge L$: DFT

In time frequency tiling, $\Delta w = \frac{2\pi}{L}$, $\Delta t = L$

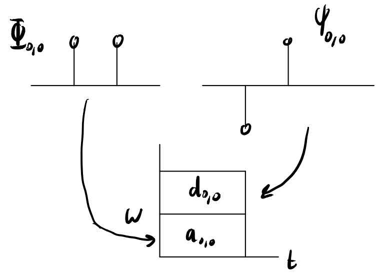
R, zero pad blends tiling together

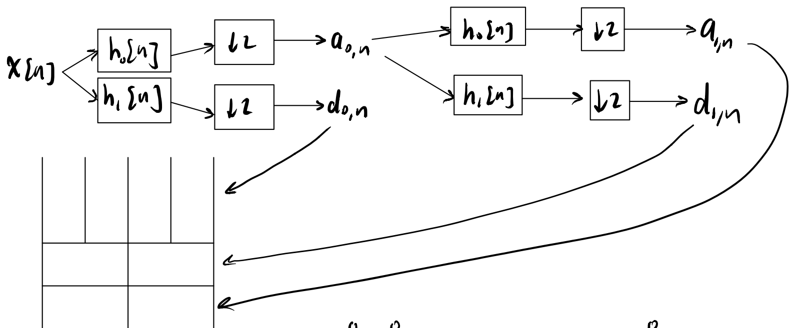
Discrete Wavelet Transform

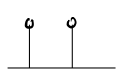
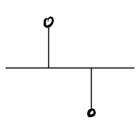
$d_{s,u} = \sum_{n=0}^{N-1} x[n]\psi_{s,u}[n]$, $a_{s,u} = \sum_{n=0}^{N-1} x[n]\phi_{s,u}[n]$



Haar For $n=2$





Normalize! $h_0[n] =$  $h_1[n]$ 

Wavelet Transform

$$Wf(u,s) = \int_{-\infty}^{\infty} f(t) \cdot \frac{1}{\sqrt{s}} \psi^*\left(\frac{t-u}{s}\right) dt = \{f(t) * \bar{\psi}_s(t)\}(u)$$

$\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1$ $\quad$ $\int_{-\infty}^{\infty} \psi(t) dt = 0$

Haar Mother: $\psi(t) = \begin{cases} -1 & 0 \le t < \frac{1}{2} \\ 1 & \frac{1}{2} \le t < 1 \end{cases}$

Haar Father: $\Phi(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{else} \end{cases}$