

Assignment 4

Part II: Semi-supervised Low-level Segmentation

NOTE: Your code should "recreate" representative results shown for graph cuts in Topic 9, but they do not have to be identical.

Problem 1

Implement interactive seed-based segmentation using s/t graph cut.

A basic seed-interactive GUI "GraphCutsPresenter" is available (implemented in "asg1.py"). The starter code below uses it. Presenter allows to add seeds (left and right mouse clicks for object and background seeds) and displays these seeds over the image. However, instead of proper graph cut segmentation the provided code displays some fixed binary mask (just as an example of a mask). This "fixed" mask should be replaced by the output of an interactive image segmentation method based on minimum s/t graph cut respecting the hard constraints marked by the user-seeds. You can use an existing python library for computing minimum s/t cuts on arbitrary graphs (run "pip install PyMaxFlow" in Anaconda Prompt , see [documentation \(<http://pmneila.github.io/PyMaxflow/maxflow.html>\)](http://pmneila.github.io/PyMaxflow/maxflow.html)). You can use this library to build a weighted graph based on selected image and user-entered seeds.

As a **first milestone** , you should implement graph cut algorithm for minimizing the loss (see slide 15, topic 9B)

$$-\sum_{p \in \Omega_L} \log S_p^{y_p} + \lambda \sum_{pq \in N} w_{pq} [S_p \neq S_q]$$

combining the seed loss (hard constraints for user-defined labels) and basic pair-wise regularization loss with "contrast-weights" $w_{pq} = \exp \frac{-\|I_p - I_q\|^2}{\sigma^2}$ for 4-connected grid neighborhood. Note that the scalar (hyper-parameter) λ controlling the regularization strength can be integrated into edge weights $\tilde{w}_{pq} = \lambda \exp \frac{-\|I_p - I_q\|^2}{\sigma^2}$ (parameter λ is primarily needed for the second milestone below). Terminal "t-links" for seed-points should make sure that such graph nodes can not be severed from the corresponding terminals. You have to use "large-enough" finite cost t-links to enforce hard-constraints (user seeds) since "infinity" weight is not possible to implement. One can argue that $N \cdot \max\{\tilde{w}_{pq}\} \equiv N\lambda$ (number of neighbors at each point times the largest n-link weight) is sufficient.

Once the first version of your interactive segmentation above is implemented and tested, your **second milestone** is to add color-based likelihood term (slide 55, topic 9B) into the total loss

$$-\sum_p \log Pr(I_p | \theta_{S_p}) - \sum_{p \in \Omega_C} \log S_p^{y_p} + \lambda \sum_{pq \in N} w_{pq} [S_p \neq S_q]$$

where two distinct color distributions $\Pr(I|\theta_1)$ and $\Pr(I|\theta_0)$ for two segments (object and background) should be estimated by GMMs, as follows. As discussed in topic 9A (slides 60-61), GMM is a standard way to efficiently estimate arbitrary (probability) density functions using relatively few parameters. Note that this assignment is not using GMMs with the goal of clustering seeds. We estimate one set of parameters θ_1 of GMM probability density function $\Pr(I|\theta_1)$ for RGB colors at pixels with "red" seeds (treated as a sample of object features), and the second set of parameters θ_0 of GMM probability density function $\Pr(I|\theta_0)$ for RGB colors at pixels with "blue" seeds (sample of background features). You do not need to implement EM algorithm (slide 67, topic 9A) for estimating θ_1 and θ_0 , just use "GaussianMixture" function from the standard "mixture" library in "sklearn"; mixtures of six Gaussian modes ($K = 6$) should suffice for each of the two GMMs. Note that once you estimate two color distributions/densities (from seeds), then you can evaluate two likelihoods $\Pr(I_p|\theta_1)$ and $\Pr(I_p|\theta_0)$ at any pixel p , as required for the color likelihoods term in the total loss above.

NOTE 1: max-flow/min-cut libraries are typically more efficient when using integer edge weights in a relatively small range. You can use integer-weighted graph where edge weights are discretized/truncated values of your edge-weighting function.

NOTE 2: Test different values of "regularization parameter" λ controlling relative weight of the regularization term versus the likelihoods terms.

NOTE 3: Play with parameter σ for exponential n-link weighting function in $w_{pq} \propto \exp \frac{-\|I_p - I_q\|^2}{\sigma^2}$ using intensity differences/contrast between two pixels. Test different values of σ . Show 2-3 representative results (in different cells). Use markdown cell to discuss your observations, if any. If you can suggest some specific way of selecting some σ adaptively to each image, provide a brief technical motivation for it.

NOTE 4: You can use either 4 or 8 connected grid.

NOTE 5: One numerical issue you may face are intensities/colors I such that $\Pr(I|\theta_i) \approx 0$ where pixels with I may still be present in segment i , but this intensity was not represented in the sample (seeds) used to estimate the distribution for segment i . This implies near-infinity log-likelihoods forbidding any pixel of color I from segment i . Another issue could be that seeds for category i may mistakenly include some pixels from the other category (e.g. if the user made errors when placing seeds). The core issue here is robustness of the log-likelihood loss with respect to imperfect

distributions/densities. There is a standard robustification trick (also used for log-likelihood losses in supervised network training to address labels with errors). The general idea is to modify (both segments) likelihoods as follows:

$$P'(I|\theta_i) = \gamma P(I|\theta_i) + \frac{1-\gamma}{\|RGB\|}$$

which can be derived as a mixture model for two distributions: P and the uniform distribution over color space where $\|RGB\|$ is its cardinality. Essentially, robust modification of the log-likelihood loss boils down to

$$-\log P(I|\theta_i) \rightarrow -\log(P(I|\theta_i) + \epsilon)$$

where ϵ is an additional "robustness" parameter that bounds/truncates near-infinite log-likelihood penalties.

NOTE 6: (Creating n-links) Here is an illustration of one way of creating (undirected) n-links for the simplest 4-connected grid used in all examples of topic 9. First, you should compute two 2D arrays `n_right` and `n_below` (same shape as your image) where each element corresponding to pixel p is a weight w_{pq} of n-link from this pixel to its neighbor q on the right or below (respectively). The weights should be computed according to the provided formula. You can use `np.roll`, `np.exp`, and other numpy functions to compute such arrays avoiding double-for-loops. Then, you can add the corresponding n-links to your graph as follows:

```
g = maxflow.GraphFloat()

nodeids = g.add_grid_nodes((num_rows, num_cols))

structure_x = np.array([[0, 0, 0], [0, 0, 1], [0, 0, 0]])

structure_y = np.array([[0, 0, 0], [0, 0, 0], [0, 1, 0]])

g.add_grid_edges(nodeids, weights=n_right, structure=structure_x, symmetric=True)

g.add_grid_edges(nodeids, weights=n_below, structure=structure_y, symmetric=True)
```

NOTE 7: (Creating t-links) Assume you already computed two 2D arrays `t_sink` and `t_source` (same shape as your image) where each element corresponding to pixel p is a weight of t-link from this pixel to the source or sink terminals (respectively). When computing such arrays, you should avoid double for-loops. Then, t-links can be created as below.

```
g.add_grid_tedges( nodeids, t_source, t_sink )
```

This function increments the weight of existing t-links, which are automatically created with each node (initialized to zero). You can call this function for the same nodes many times before or after the max-flow is computed. To update the solution after each modification of edge weights, you should call max-flow again. HINT: if you need to change t-links to be equal to

some particular "new_weights", you may need to create additional arrays storing current values of t-links and update t-links as follows:

```
tlinks_source = np.zeros((num_rows,num_cols))

tlinks_sink = np.zeros((num_rows,num_cols))

...
...

g.add_grid_tedges( nodeids, new_weights_source - tlinks_source, new_weights_sink - tlinks_sink)

tlinks_source = new_weights_source

tlinks sink = new weights sink
```

In [1]: ┌ %matplotlib widget

```
# Loading standard modules
import numpy as np
import matplotlib.pyplot as plt
import maxflow
from skimage import img_as_ubyte
from sklearn import mixture
from skimage.color import rgb2gray
# Loading custom module (requires file asg1.py in the same directory as the notebook file)
from asg1_error_handling import Figure, GraphCutsPresenter
```

```
In [2]: ┌ class MyGraphCuts:  
  bgr_value = 0  
  obj_value = 1  
  none_value = 2  
  
  def __init__(self, img, sigma:float = 0.5, lam :float =1, gamma: float =0.5):  
    self.fig = Figure()  
    self.pres = GraphCutsPresenter(img, self)  
    self.pres.connect_figure(self.fig)  
  
    self.img = img  
  
    self.num_rows = img.shape[0]  
    self.num_cols = img.shape[1]  
  
    self.sigma = sigma  
    self.lam = lam  
    self.gamma = gamma  
  
  def run(self):  
    self.fig.show()  
  
  def compute_labels(self, seed_mask):  
    num_rows = self.num_rows  
    num_cols = self.num_cols  
  
    # +-----+-----+  
    # |       |       |  
    # |   bgr   |   none  |  
    # |       |       |  
    # +-----+-----+  
    # |       |       |  
    # |   none   |   obj   |  
    # |       |       |  
    # +-----+-----+  
    label_mask = np.full((num_rows, num_cols), self.none_value, dtype='uint8')  
    label_mask[:num_rows // 2, :num_cols // 2] = self.bgr_value  
    label_mask[num_rows // 2:, num_cols // 2:] = self.obj_value  
  
  return self.milestone_2(seed_mask) #Label_mask
```

```
def milestone_1(self, seed_mask):
    sigma = self.sigma
    lam = self.lam

    intensities = np.linalg.norm(x=self.img, axis=2)

    right = np.square(intensities - np.roll(a=intensities, shift=-1, axis=0)) / (-2 * sigma ** 2)
    below = np.square(intensities - np.roll(a=intensities, shift=-1, axis=1)) / (-2 * sigma ** 2)

    # mask_right = np.where(seed_mask == np.roll(a=seed_mask, shift=-1, axis=0), 0, 1)
    # mask_below = np.where(seed_mask == np.roll(a=seed_mask, shift=-1, axis=1), 0, 1)

    # n_right = np.multiply(np.exp(right), mask_right)
    # n_below = np.multiply(np.exp(below), mask_below)
    n_right = np.exp(right) * lam
    n_below = np.exp(below) * lam

    t_source = np.where(seed_mask == self.bgr_value, np.inf, 0)
    t_sink = np.where(seed_mask == self.obj_value, np.inf, 0)

    g = maxflow.GraphFloat()
    nodeids = g.add_grid_nodes((self.num_rows, self.num_cols))

    structure_x = np.reshape(a=[0,0,0,0,0,1,0,0,0], newshape=(3,3))
    structure_y = np.reshape(a=[0,0,0,0,0,0,0,1,0], newshape=(3,3))

    g.add_grid_edges(nodeids, weights=n_right, structure=structure_x, symmetric=True)
    g.add_grid_edges(nodeids, weights=n_below, structure=structure_y, symmetric=True)

    g.add_grid_tedges(nodeids, t_source, t_sink)

    g.maxflow()

    seg = g.get_grid_segments(nodeids)

    return np.array(seg, dtype=int)

def milestone_2(self, seed_mask):
    sigma = self.sigma
    lam = self.lam
    gamma = self.gamma
```

```
intensities = np.linalg.norm(x=self.img, axis=2)

right = np.square(intensities - np.roll(a=intensities, shift=-1, axis=0)) / (-2 * sigma ** 2)
below = np.square(intensities - np.roll(a=intensities, shift=-1, axis=1)) / (-2 * sigma ** 2)

n_right = np.exp(right) * lam
n_below = np.exp(below) * lam

t_source = np.where(seed_mask == self.bgr_value, np.infty, 0) # bgr=0
t_sink = np.where(seed_mask == self.obj_value, np.infty, 0)

g = maxflow.GraphFloat()
nodeids = g.add_grid_nodes((self.num_rows, self.num_cols))

structure_x = np.reshape(a=[0,0,0,0,0,1,0,0,0], newshape=(3,3))
structure_y = np.reshape(a=[0,0,0,0,0,0,0,1,0], newshape=(3,3))

g.add_grid_edges(nodeids, weights=n_right, structure=structure_x, symmetric=True)
g.add_grid_edges(nodeids, weights=n_below, structure=structure_y, symmetric=True)

## T-Links
g.add_grid_tedges(nodeids, t_source, t_sink)

# Method 2
bgr_gmm = mixture.GaussianMixture(n_components=6)
obj_gmm = mixture.GaussianMixture(n_components=6)

bgr_mask = np.array(seed_mask == self.obj_value)
obj_mask = np.array(seed_mask == self.bgr_value)

if bgr_mask.sum() > 0 and obj_mask.sum() > 0:
    bgr_samples = np.reshape(self.img[bgr_mask], newshape=(bgr_mask.sum(), self.img.shape[-1]))
    obj_samples = np.reshape(self.img[obj_mask], newshape=(obj_mask.sum(), self.img.shape[-1]))

    bgr_gmm.fit(bgr_samples)
    obj_gmm.fit(obj_samples)

    im_2d = np.reshape(self.img, newshape=(self.num_cols*self.num_rows, self.img.shape[-1]))

    bgr_lhood = np.exp(bgr_gmm.score_samples(im_2d))
    obj_lhood = np.exp(obj_gmm.score_samples(im_2d))
```

```
bgr_llhood = -np.log(gamma*bgr_llhood + (1-gamma)/256)
obj_llhood = -np.log(gamma*obj_llhood + (1-gamma)/256)

bgr_llhood_im = np.reshape(bgr_llhood, newshape=(self.num_rows, self.num_cols))
obj_llhood_im = np.reshape(obj_llhood, newshape=(self.num_rows, self.num_cols))

g.add_grid_tedges(nodeids, bgr_llhood_im, obj_llhood_im)

g.maxflow()

seg = g.get_grid_segments(nodeids)

return np.array(seg, dtype=int)
```

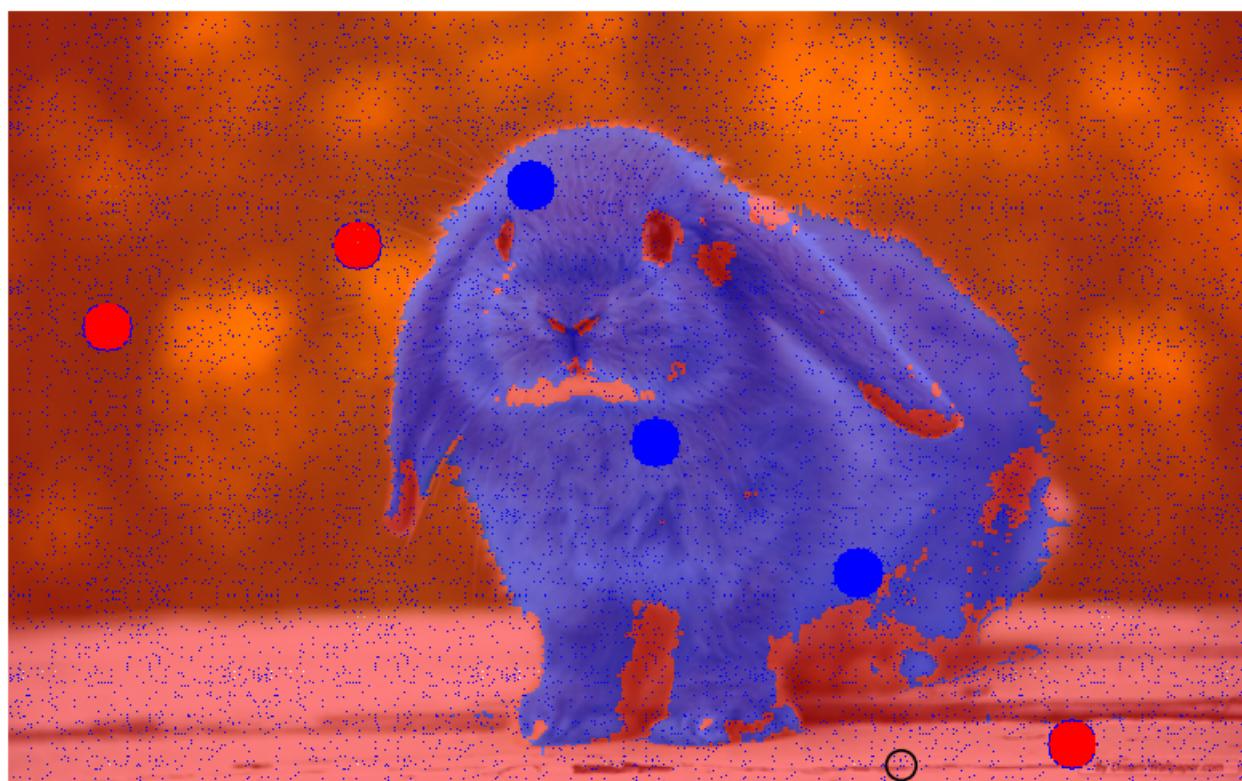
Notes about the basic graph cut interface:

1. To provide the regional hard constraints (seeds) for object and background segments use left and right mouse clicks (mouse dragging works somewhat too). Use mouse wheel to change the brush size.
2. The seed mask is built by the "GraphCutsPresenter". Each mouse release activates "on_mouse_up" function of the presenter, which asks the linked MyGraphCuts object to "compute_labels" for all pixels based on the provided seed mask.
3. You should use "PyMaxflow" library (already imported into this notebook if you ran the second cell) to build a weighted graph and to compute a minimum s/t cut defining all pixel labels from the seeds as explain in topic 5.

```
In [3]: ┌─ img1 = plt.imread('images/bunny.bmp')
      app = MyGraphCuts(img1)
      app.run()
```

Figure 1

Graph Cuts



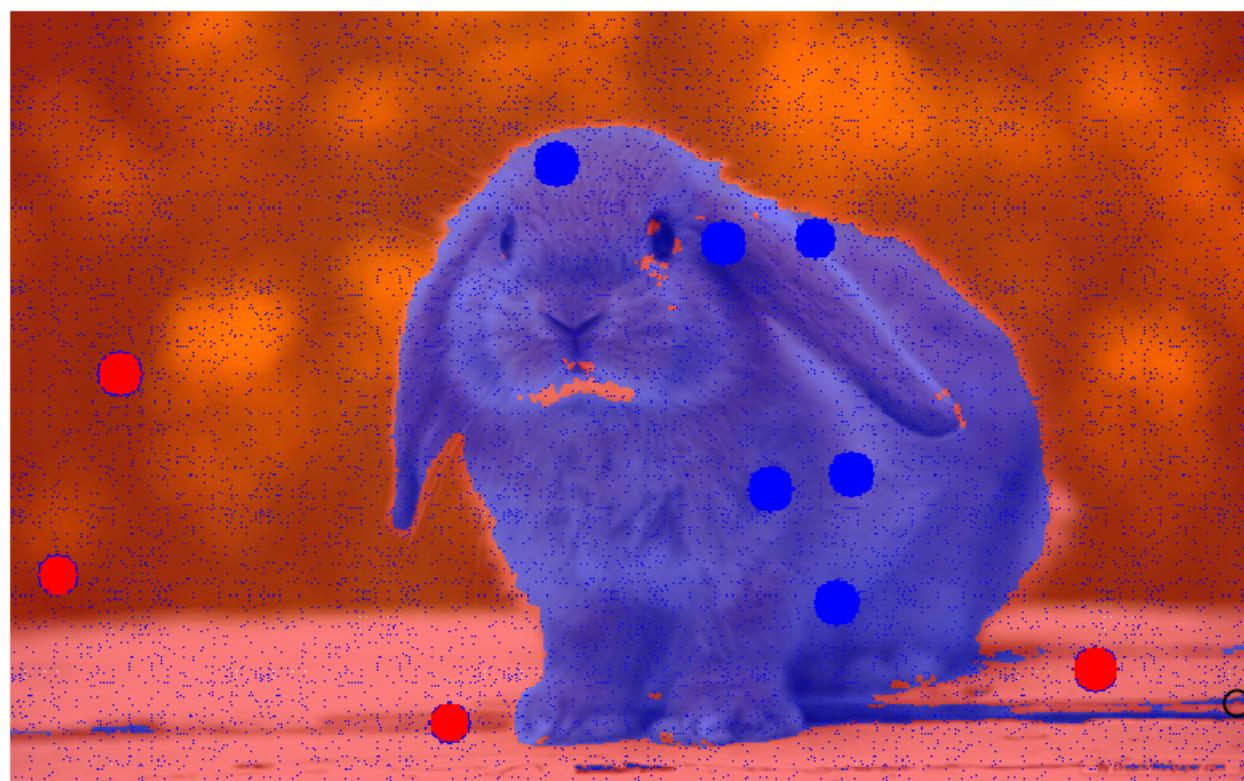


x=417.2 y=220.6 [2.]

```
In [4]: ┌─ img1_1 = plt.imread('images/bunny.bmp')
      app = MyGraphCuts(img1_1, sigma=0.05)
      app.run()
```

Figure 2

Graph Cuts



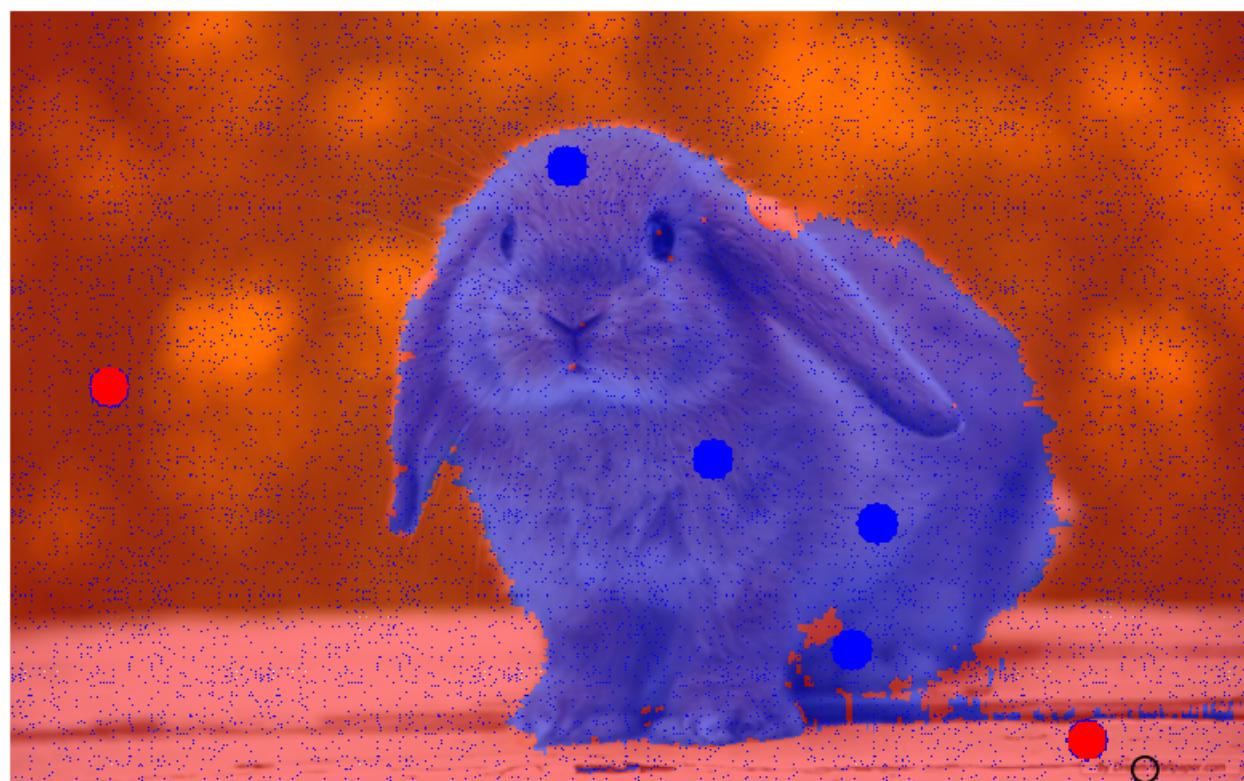


x=573.5 y=191.5 [2.]

```
In [5]: ┌─ img1_2 = plt.imread('images/bunny.bmp')
      app = MyGraphCuts(img1_2, sigma=0.8)
      app.run()
```

Figure 3

Graph Cuts



In []:

Comparing the cases $\sigma=0.5, 0.05, 1$, I found that the second case required a lot more seeds compared to the other two for segmentation. It also seems to follow the boundary between the object and background much better. One thing interesting about the second and third case is that the eyes are also part of the bunny unlike the other first one. This may have just been due to the seeds inputted.

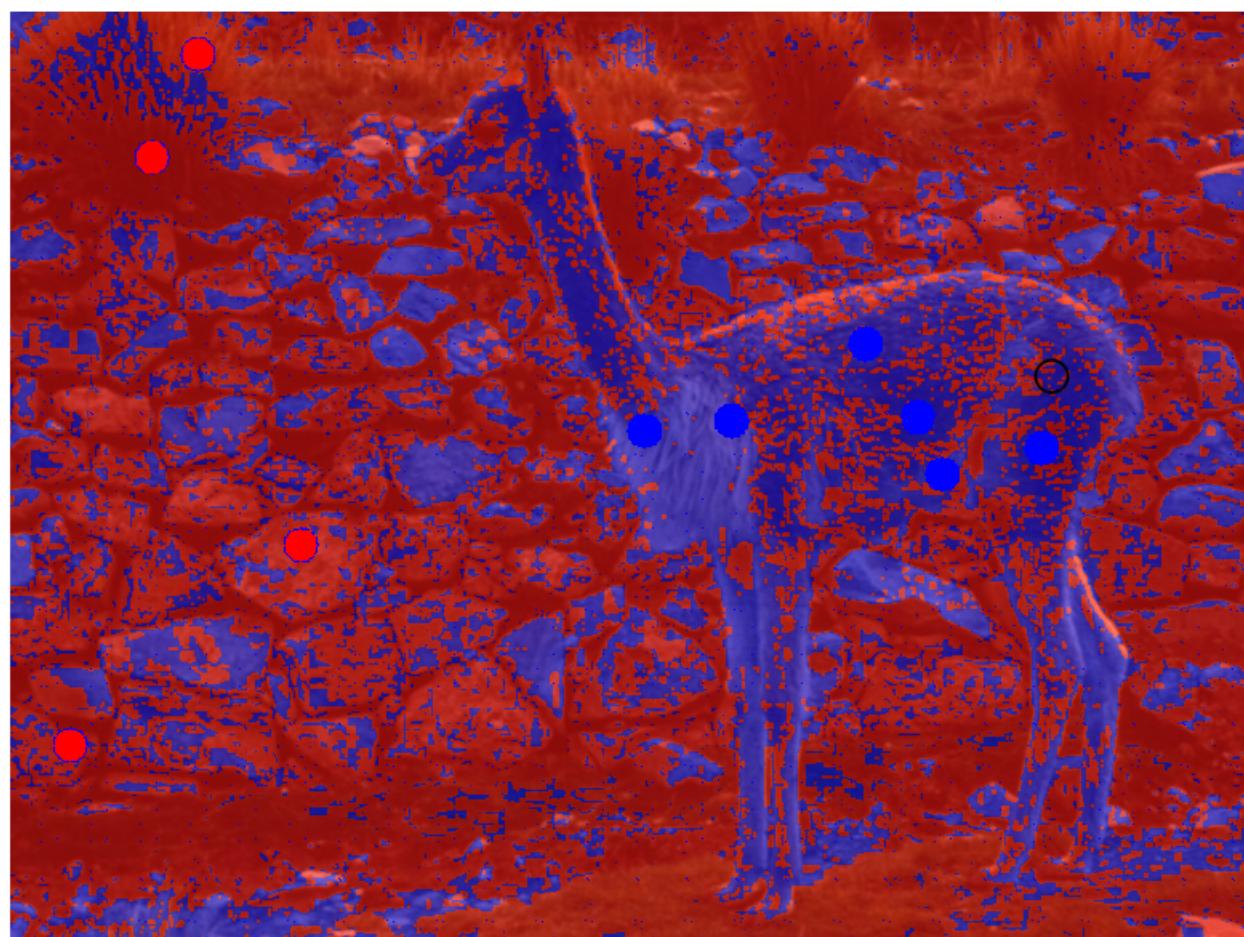
x=24.3 y=360.2 [2.]

Show how your interactive segmenter works on more challenging images where there is some overlap between the color-models in the object and background (as in the "lama" image). Compare the results for $\lambda = 0$ (no regularization) and for some $\lambda > 0$. For convenience, you might want to include λ in the list of parameters for the function "MyGraphcuts".

```
In [6]: ┌─ img2 = plt.imread('images/lama.jpg')
      app = MyGraphCuts(img2, lam=0)
      app.run()
```

Figure 4

Graph Cuts

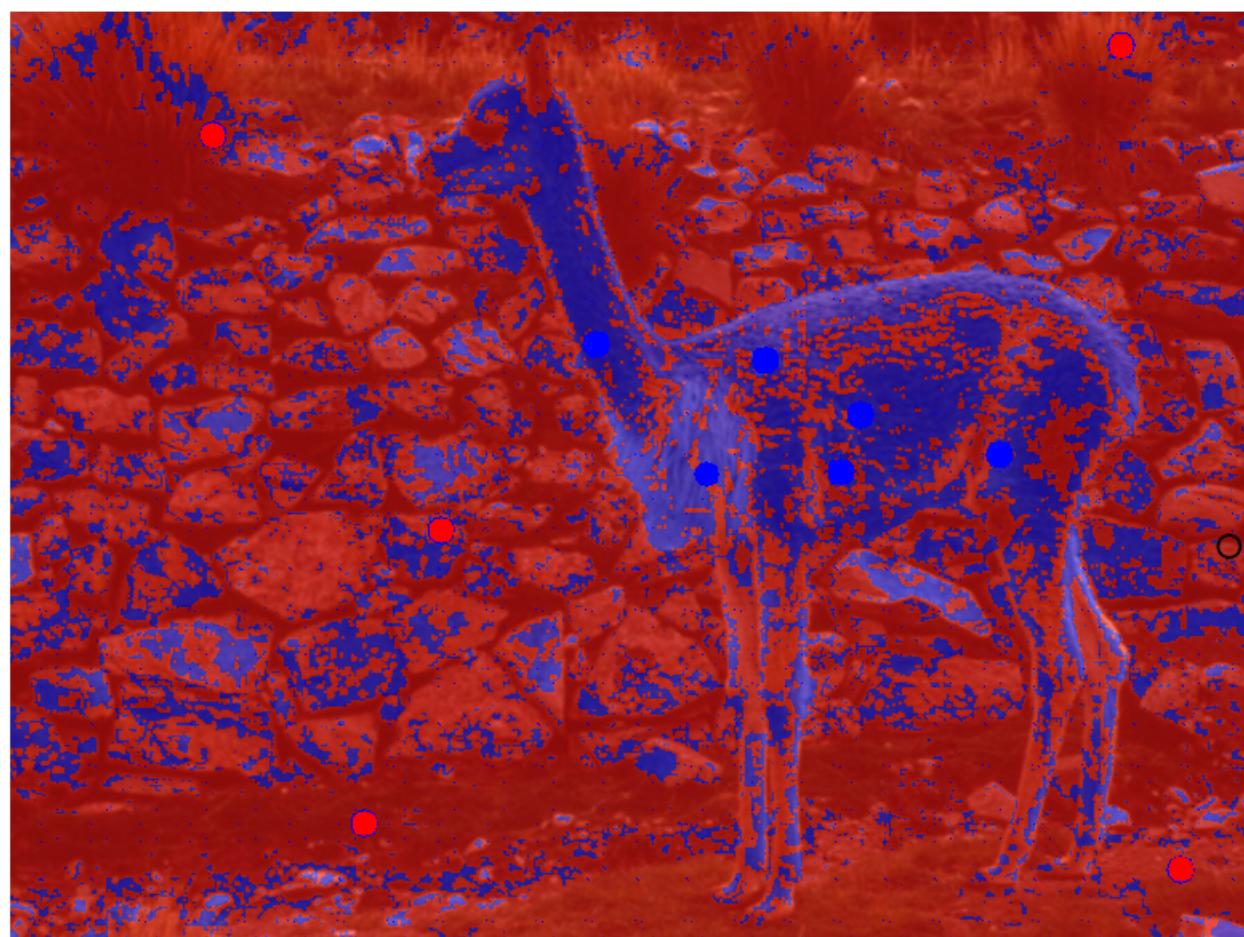




```
In [7]: ┌─ img2_1 = plt.imread('images/lama.jpg')
      app = MyGraphCuts(img2_1, lam=0.5)
      app.run()
```

Figure 5

Graph Cuts

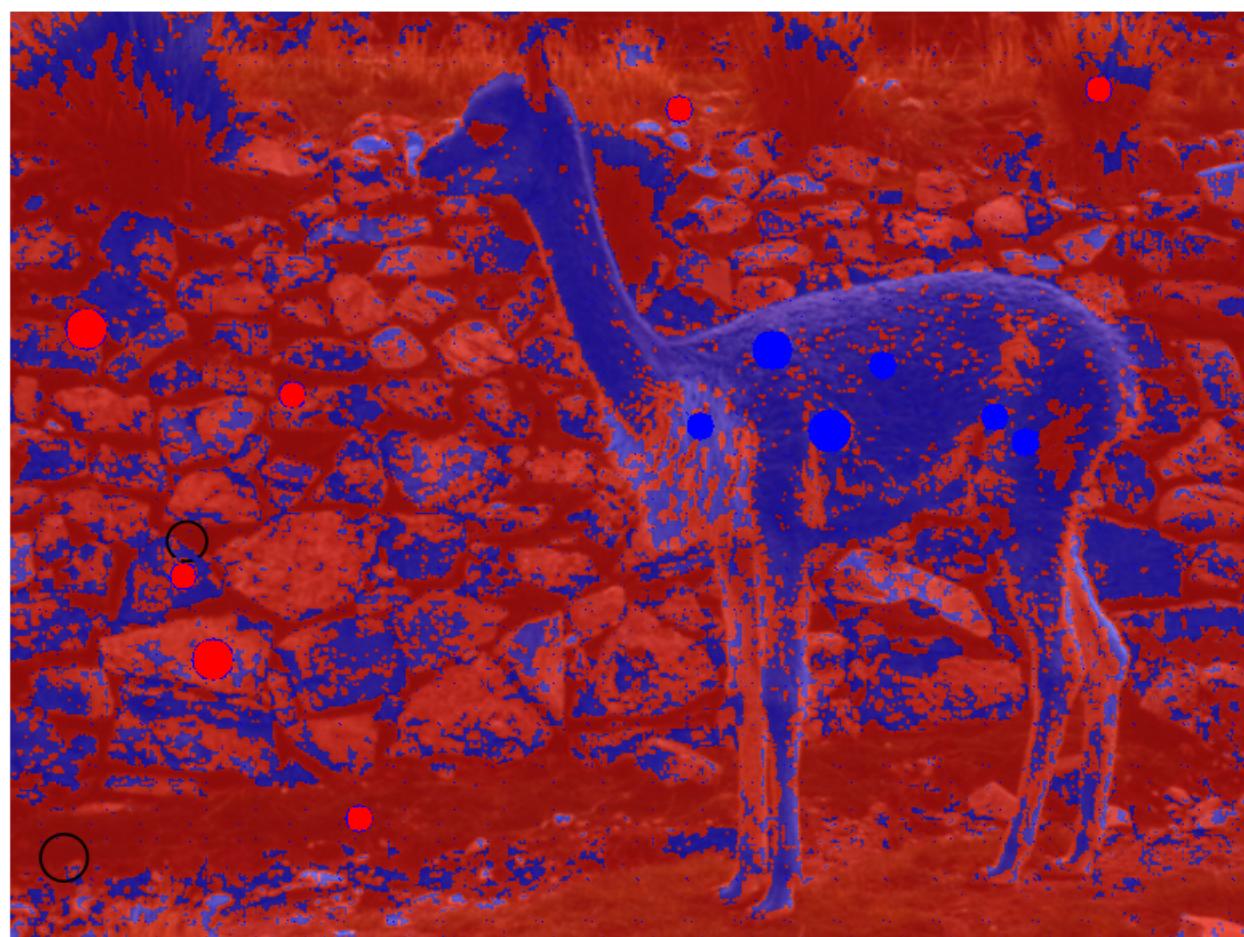




```
In [8]: ┌─ img2_2 = plt.imread('images/lama.jpg')
      app = MyGraphCuts(img2_2, lam=1)
      app.run()
```

Figure 6

Graph Cuts



The addition of lamda improved the results by not looking at the background as much. However as it was increased, any blue in the background area became larger.

In []: