

a)

$$P(x) = \frac{w(x)}{\int_a^b w(x) dx}$$

$$I_w = \int_a^b w(x) dx = \int_a^b x^{-1/2} dx = \frac{x^{1/2}}{2} \Big|_a^b = \frac{1}{2} (b^{1/2} - a^{1/2})$$

$$\int_{x_0}^{x(z)} P(x') dx' = \int_0^z dz' = z$$

$$w(x) = x^{-1/2}$$

$$\frac{1}{I_w} \int_{x_0}^{x(z)} x^{-1/2} dx = z = \frac{2}{I_w} x^{1/2} \Big|_{x_0}^{x(z)}$$

$$x(z) - \cancel{x_0} = \left( \frac{I_w z}{2} \right)^2 \quad \begin{matrix} x_0 = 0 \\ I_w = \frac{1}{2} \end{matrix}$$

$$x(z) = z^2$$

b)

$$\int_0^x \frac{1}{2\sqrt{x}} dx = \frac{1}{2} \int_0^x x^{-1/2} dx$$

$$= \frac{1}{2} [2x^{1/2}] \Big|_0^x = \frac{1}{2} (2\sqrt{x} - 0) = \sqrt{x}$$