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Flight Revenue Management Optimization
Optimization Analysis

Optimization Project 2 - Group 13

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Table of Contents

1. Introduction

The fundamental approach for an airline company is straightforward: fill as many seats as possible on a flight to increase revenue. Yet, they also have to fine-tune airfares to attract the highest number of passengers. Moreover, airlines can boost profits by overbooking flights, selling more tickets than there are seats available, banking on the likelihood of no-shows or empty seats in first class on travel day. Therefore, it's crucial for an airline to implement distinct fare policies for first class and coach, and to adjust these fares daily based on demand and supply. Pricing decisions hinge on two main factors: the allowable number of overbooked seats and the anticipated total profit. We'll delve into two distinct pricing strategies. The first employs a strict limit on overbookings, continuing ticket sales up to the flight date or until sold out. The second strategy involves halting ticket sales preemptively to strategically manage overbookings. Both strategies will be analyzed through dynamic programming, determining the optimal daily price for first class and coach tickets to maximize profit.

In the dynamic landscape of the airline industry, optimizing revenue through effective utilization of flight seating capacity is paramount. However, the persistent challenge of passenger no-shows necessitates airlines to employ overbooking strategies to maximize revenue opportunities. This report delves into the strategic analysis of overbooking practices within the airline industry, aiming to strike the delicate balance between increased ticket sales and the potential costs incurred from accommodating passengers when flights exceed seating capacity. While overbooking offers financial benefits, it also carries risks, including passenger dissatisfaction and reputational damage. Utilizing dynamic programming, the analysis meticulously evaluates pricing and overbooking policies, accounting for varied demand probabilities across different ticket pricing strategies within a dual-class seating system.

Problem Statement

For our report on the airline company's flight booking strategy for a service scheduled to depart 365 days from now, we examine a flight offering two distinct classes: first-class and coach. The sale dynamics for these two classes operate independently, meaning the demand for one class does not affect the other.

Initially, our programming explores ticket sales and overbooking management for a flight one year prior to departure, assessing expected discounted profits. This examination includes show-up probabilities for both coach (95%) and first-class (97%) passengers, diverse ticket pricing for each class, and a 3% increase in sales chances when first-class is fully booked. Moreover, the analysis incorporates overbooking costs for coaches, including specific penalties for bumping passengers to first-class or off the flight. Financial impacts are quantified using the binomial distribution to estimate costs on the departure day, factoring in a daily discount rate of 17%. Consequently, the daily discount factor amounts to 1/(1+0.17/365) over the one-year timeframe.

Sale Probability: There is an inverse relationship between ticket price and the likelihood of a sale. Notably, the sale probability for coach tickets priced at $300 surpasses that of tickets priced at $350, indicating significant price sensitivity in this segment. This sensitivity diminishes as we move into the first-class ticket segment, where the variance in sale probability between $425 and $500 tickets is less pronounced. This suggests that first-class passengers may exhibit lower price sensitivity, a factor to consider when devising overbooking strategies for different classes.

Show Up Probability: The graph also reveals a subtle increase in the probability of passengers showing up as ticket prices rise. While the show-up probability for coach tickets is already high at 95%, it slightly increases for first-class passengers, peaking at 97% for the $500 ticket price. This disparity underscores the greater reliability of first-class passengers in attending their flights, likely due to the higher investment associated with their ticket costs.

Our decision-making revolves around setting the daily prices for both classes, with four possible pricing strategies:

Low price for Coach and First Class
High price for Coach, Low price for First Class
Low price for Coach, High price for First Class
High price for both Coach and First Class

Furthermore, each day presents nine potential scenarios based on ticket sales and pricing strategies for both classes, including scenarios where tickets in either class have not been sold.

We aim to explore two distinct strategies. The first involves establishing a fixed range for overbooked coach tickets, between 5 to 15 tickets, to identify the policy yielding the highest expected profit through a series of simulations. The second strategy employs a dynamic programming approach to ascertain the optimal overbooking limit, considering expected costs, with a flexible stopping point for coach ticket sales. This introduces an upper limit of 20 overbooked tickets and an additional decision-making point where coach tickets may not be sold on a given day, enhancing our pricing strategy options to include:

- Not selling Coach tickets while setting First Class ticket prices as low or high.

These strategies maintain the same nine daily scenarios as previously outlined, with the added dimension of potentially halting coach ticket sales represented in the "Not Sold Coach" states, which allow the demand for Coach class ticket to be 0. This comprehensive approach aims to maximize profitability through strategic pricing and overbooking policies, adjusted daily to respond to evolving market conditions and booking patterns.

In the initial step, evaluate the anticipated discounted revenue resulting from permitting an overbooking of 5 seats in the coach section. First we need to define the cost for better financial implications of overbooking coach seats on an airline's revenue, specifically when exceeding the seat capacity by up to 5 seats. The terminal function will be the cost incurred, therefore, it calculates the potential costs incurred from having to either upgrade overbooked passengers to first-class or compensate them if bumped off the flight, factoring in the probabilities of passengers showing up. We then iterate over each day leading up to a flight's departure, analyzing various pricing strategies for coach and first-class tickets. It calculates the expected revenue for each pricing combination, considering the probabilities of tickets being sold at each price point. The decision to sell at high or low prices in each class is made based on the maximization of expected revenue, factored by the probability of seat occupancy and adjusted for potential overbooking costs. The ultimate goal is to identify the pricing strategy that yields the highest expected discounted profit, considering the dynamic nature of ticket sales and customer behavior.

2. Strategy-1

Mathematical Formulation: Dynamic pricing model

In the report, we outline a dynamic pricing model used to maximize revenue from seats sales for a flight. The model incorporates the following elements:

Objective:
The model aims to maximize expected discounted profit by determining the optimal number of tickets to sell (including overbooking) and the most effective ticket pricing strategy for each day leading up to a flight's departure.

State Variables:
These variables encompass the number of days until departure, the quantity of tickets sold in each class (coach and first-class), and the current overbooking level.

- Time (t): representing the number of days until the flight takes off.
- Coach seats Sold (Sc): indicating the number of seats sold in coach class.
- First Class seats Sold (Sf): indicating the number of seats sold in first class.

Decision Variables/Choice Variables:

For each day and given state, the model decides on the ticket price for coach and first-class seats and whether to sell additional tickets beyond the plane's seating capacity.

- Pricing Strategy: High or Low pricing options for both first class (H or L) and coach class (H or L).

Constraints:

The model accounts for physical limitations, such as the total number of seats in each class and the maximum allowable overbooking limit. Additionally, it accommodates the probabilistic nature of ticket sales and passenger show-ups.

Dynamics:

To transition from one state to the next, we must determine the number of tickets sold on a given day as well as the cumulative number of tickets already sold for both coach and first-class. As time progresses, each state naturally advances to the following day. The potential subsequent states are characterized by:

- (Sc, t+1, Sf) - unsold coach and first-class ticket today

- (Sc-1, t+1, Sf) - sold coach ticket, but not first-class ticket today
- (Sc-1, t+1, Sf-1) - sold both coach and first-class ticket
- (Sc, t+1, Sf-1) - sold first-class ticket, but not coach

Value Function:

$$V(Sc, t, Sf) = \max\left(E\left(\sum_{i=0}^{T-t} (profits@t + 1)\gamma i\right)\right.$$

Bellman Equation:
Prior formulating the Bellman equation, we can streamline our approach by reworking the variables to simplify the equation.

Simplifying the Price and Probability Equation

- The expected revenue and price, from selling tickets can be broken down into four scenarios based on the probabilities of selling coach and first-class tickets. Let's denote A as the price of a coach ticket and **B** as the price of a first-class ticket.

- Price: We get A if the ticket sold in a coach and 0 if the ticket doesnot.
- For the above scenario i.e the coach Probability will be x and x-1 respectively.
- Price: We get B if the ticket sold in a first class and 0 if the ticket doesnot.
- For the above scenario i.e. the first-class Probability will be y and y-1 respectively.

4 Scenarios:
- Both coach and first-class tickets are sold: expected value- xy(A+B)
- Only the coach ticket is sold: expected value- x(1-y)A
- Only the first-class ticket is sold: expected value- (1-x)yB
- Neither ticket is sold: expected value- (1-x)(1-y)0

Bellman equation by also discounting future possible dates as well:

$$E[Price] = xy(A+B) + x(1-y)A + (1-x)yB + (1-x)(1-y)0$$
$$= xA + yB$$

$$V(Sc, t, Sf) = xA + xB + \gamma(..)$$

Coach and First-Class tickets are Available:

Value when set both coach and first-class ticket prices low

LL = prob_coach_sale_low ∗ price_coach_low + prob_first_sale_low ∗ price_first_low + γ∗(prob_coach_no_sale_low ∗prob_first_no_sale_low ∗ V(Sc,t+1,Sf) +prob_coach_sale_low ∗ prob_first_no_sale_low ∗ V(Sc − 1, t + 1, Sf) +prob_coach_sale_low ∗ prob_first_sale_low ∗ V(Sc − 1, t + 1, Sf − 1) +prob_coach_no_sale_low ∗ prob_first_sale_low ∗ V(Sc, t + 1, Sf − 1))

Value when set coach ticket price low and first-class ticket price high

LH = prob_coach_sale_low ∗ price_coach_low + prob_first_sale_high ∗ price_first_high + γ∗(prob_coach_no_sale_low ∗prob_first_no_sale_high ∗ V(Sc,t+1,Sf) +prob_coach_sale_low ∗ prob_first_no_sale_high ∗ V(Sc − 1, t + 1, Sf)+prob_coach_sale_low ∗ prob_first_sale_high ∗ V(Sc − 1, t + 1, Sf − 1) +prob_coach_no_sale_low ∗ prob_first_sale_high ∗ V(Sc, t + 1, Sf − 1))

Value when set coach ticket price high and first-class ticket price low

HL = prob_coach_sale_high ∗ price_coach_high + prob_first_sale_low ∗ price_first_low + γ∗(prob_coach_no_sale_high ∗prob_first_no_sale_low ∗ V(Sc,t+1,Sf) +prob_coach_sale_high ∗ prob_first_no_sale_low ∗ V(Sc − 1, t + 1, Sf) +prob_coach_sale_high ∗ prob_first_sale_low ∗ V(Sc − 1, t + 1, Sf − 1) +prob_coach_no_sale_high ∗ prob_first_sale_low ∗ V(Sc, t + 1, Sf − 1))

Value when set both coach and first-class ticket prices high

HH = prob_coach_sale_high ∗ price_coach_high + prob_first_sale_high ∗ price_first_high
+γ∗(prob_coach_no_sale_high ∗prob_first_no_sale_high ∗V(Sc, t + 1, Sf)
+prob_coach_sale_high ∗ prob_first_no_sale_high ∗ V(Sc − 1, t + 1, Sf)
+prob_coach_sale_high ∗ prob_first_sale_high ∗ V(Sc − 1, t + 1, Sf − 1)
+prob_coach_no_sale_high ∗ prob_first_sale_high ∗ V(Sc, t + 1, Sf − 1))

V(Sc,t,Sf) = max(LL,LH,HL,HH)

Code:

Ticket Price Optimizer Function in Code:
This function(ticket_price_optimizer) utilizes the previously defined variables to generate Value and Decision matrices. These matrices are instrumental in calculating the expected profit and determining the optimal decisions for various situations.

```
            else:
                # Value when set both coach and first-class ticket prices low
                value_LL = prob_Lc[1]*priceLc + prob_Lf[1]*priceLf + delta* (prob_Lf[0]*prob_Lc[0]*V[Sc,t+1,Sf]
                                                                     +prob_Lf[0]*prob_Lc[1]*V[Sc-1,t+1,Sf]
                                                                     +prob_Lf[1]*prob_Lc[1]*V[Sc-1,t+1,Sf-1]
                                                                     +prob_Lf[1]*prob_Lc[0]*V[Sc,t+1,Sf-1])

                # value when set coach ticket price low and first-class ticket price high
                value_LH = prob_Lc[1]*priceLc + prob_Hf[1]*priceHf + delta* (prob_Hf[0]*prob_Lc[0]*V[Sc,t+1,Sf]
                                                                     +prob_Hf[0]*prob_Lc[1]*V[Sc-1,t+1,Sf]
                                                                     +prob_Hf[1]*prob_Lc[1]*V[Sc-1,t+1,Sf-1]
                                                                     +prob_Hf[1]*prob_Lc[0]*V[Sc,t+1,Sf-1])

                # value when set coach ticket price high and first-class ticket price low
                value_HL = prob_Hc[1]*priceHc + prob_Lf[1]*priceLf + delta* (prob_Lf[0]*prob_Hc[0]*V[Sc,t+1,Sf]
                                                                     +prob_Lf[0]*prob_Hc[1]*V[Sc-1,t+1,Sf]
                                                                     +prob_Lf[1]*prob_Hc[1]*V[Sc-1,t+1,Sf-1]
                                                                     +prob_Lf[1]*prob_Hc[0]*V[Sc,t+1,Sf-1])

                # value when set both coach and first-class ticket prices high
                value_HH = prob_Hc[1]*priceHc + prob_Hf[1]*priceHf + delta* (prob_Hf[0]*prob_Hc[0]*V[Sc,t+1,Sf]
                                                                     +prob_Hf[0]*prob_Hc[1]*V[Sc-1,t+1,Sf]
                                                                     +prob_Hf[1]*prob_Hc[1]*V[Sc-1,t+1,Sf-1]
                                                                     +prob_Hf[1]*prob_Hc[0]*V[Sc,t+1,Sf-1])


                V[Sc,t,Sf]=max(value_LL,value_LH,value_HL,value_HH) # value funciton maximizes expected profit

                U[Sc,t,Sf]=(int(np.argmax([value_LL,value_LH,value_HL,value_HH])/2)+1)*10\
                +int((np.argmax([value_LL,value_LH,value_HL,value_HH])%2)!=0)+1
    return U,V
```

<u>First-class tickets are available but not coach:</u>

*Lf = prob_first_sale_low ∗ price_first_low + γ ∗ (prob_first_no_sale_low)*V(Sc,t + 1,Sf) + prob_first_no_sale_low ∗ V(Sc, t + 1, Sf − 1))*

*Hf = prob_first_sale_high ∗ price_first_high + γ ∗ (prob_first_no_sale_low)*V(Sc,t + 1,Sf) + prob_first_no_sale_low ∗ V(Sc, t + 1, Sf − 1))*

$$v(sc,t,sf) = max(Lf,Hf)$$

Code:

```
    elif Sc !=0 and Sf==0: # There are coach tickets available but not first-class. This increases the
        # probability of sale coach tickets by 0.03.

        # Value when set coach ticket price low
        value_Lc = (prob_Lc[1]+0.03)*priceLc + delta* ((prob_Lc[0]-0.03)*V[Sc,t+1,Sf] + (prob_Lc[1]+0.03)*V[Sc-1,t

        # Value when set coach ticket price high
        value_Hc = (prob_Hc[1]+0.03)*priceHc + delta* ((prob_Hc[0]-0.03)*V[Sc,t+1,Sf] + (prob_Hc[1]+0.03)*V[Sc-1,t

        V[Sc,t,Sf]=max(value_Lc,value_Hc) # value function maximizes expected profit
```

No seats are Available:

$$V(Sc,t,Sf) = \gamma*(V(Sc,t+1,Sf)$$

```
#Fully booked scenario: you can't make money and no tickets can be sold
if Sc==0 and Sf==0:
    V[Sc,t,Sf]= delta * V[Sc,t+1,Sf]
```

We have the option to consolidate everything into a single Bellman equation by incorporating identity functions that account for the availability of tickets in either coach or first class. Nevertheless, in our analysis and structuring of the issue, we determined that segmenting the problem into distinct scenarios would provide the clearest path to a solution.

Terminal Conditions

Given the uncertainty surrounding the exact number of passengers who will show up for the flight, as it is governed by probability as outlined in the problem statement, it is necessary to establish terminal conditions related to the potential costs at departure. To address this, the expected costs on the departure day are calculated using a binomial probability distribution. We calculate the probabilities for every possible number of coach and first-class passengers showing up independently.

These probabilities are then factored into the cost variable, which is integrated into the value function for the corresponding number of sold coach (Sc) and first-class tickets (Sf). This process is applied to each potential number of actual coach (C) and first-class passengers (F) who arrive for the flight.

Taking into account the variables:

C represents the actual count of coach customers present for the flight,
F denotes the actual count of first-class customers present for the flight,
x corresponds to the likelihood of C coach customers showing up,

y corresponds to the likelihood of F first-class customers showing up.

As given in the problem statement previously, our policy entails the following:

- We upgrade a coach passenger to first class at a cost of $50 given that seats are available,
- We issue a $425 voucher to a coach passenger when the flight is fully booked and no seats are available,
- Our coach class has a capacity of 100 seats,
- Our first-class has a capacity of 20 seats,
- We track cost as a cumulative negative sum based on the number of coach and first-class tickets sold.

Certainly, here are the cost scenarios presented in an equation format:

- If the actual number of coach passengers (C) showing up is less than or equal to the number of available coach seats (100), there is no additional cost:

$$cost = cost - 0$$

- If the turnout of coach passengers exceeds the coach seats, but we can accommodate all overbooked passengers in first class, we upgrade them at a cost of $50 each: cost = cost - (number of upgrades * $50)

$$cost = cost - x{*}y{*}50(C-100)$$

- If the turnout of coach passengers exceeds both the coach seats and the available first-class seats, we upgrade as many as possible to first class and provide vouchers for any remaining overbooked passengers: cost = cost - (cost of upgrades + cost of vouchers)

$$cost = cost - x{*}y{*}(50(20-Y) + 425(C-100-(20-Y))$$

These costs, once computed for the actual number of coach and first-class tickets sold, will be incorporated into our value function as follows:

$$V(S_c,t,S_f) = cost$$

Code:

```
#****Cost Calculations-Loop:number of seats available******
for Sc in range(ScN):
    tc_booked = ScN - Sc-1 # number of tickets booked

    for Sf in range(SfN):
        tf_booked = SfN - Sf-1 # number of first-class tickets booked
        cost = 0

        #Iterating over each possible number of coach passengers actually showing up on the day of departure
        for tc_act_show in range(tc_booked+1):
            # probability of 'tc_act_show'/'tc_booked' number of people showing up
            prob_tc_act_show = scipy.stats.binom.pmf(tc_act_show,tc_booked,0.95)


            #**** Iterating over each possible number of First-Class passengers Actually****
            for tf_act_show in range(tf_booked+1):
            #showing up on the day of departure
            #probability of 'tf_act'/ 'tf number of people showing up
                prob_tf_act_show = scipy.stats.binom.pmf(tf_act_show,tf_booked,0.97)

                if tc_act_show<=Cc:
                    cost = cost+0 #no cost
                else:
                    # capacity of the coach < number of coach passengers who showed up< number of seats remainin
                    if tc_act_show-Cc <= Cf-tf_act_show:
                    # cost associated with bumping all of them to first-class
                        cost = cost - prob_tc_act_show*prob_tf_act_show*costs['ctof']*(tc_act_show-Cc)

                    else:
                    #number of coach passengers who showed up >  capacity of the coach
                    #and number of coach passengers who showed up > number of seats remaining in first-class
                    #cost linked to transferring some coach passengers to first-class and removing the remaining
                        cost = cost - prob_tc_act_show*prob_tf_act_show*(costs['ctof']*(Cf-tf_act_show)
                                                                  + costs['ctoout']*(tc_act_show-Cc-(Cf-tf_act_show)))
        V[Sc,tN-1,Sf] = cost
```

In our decision matrix, U, the departure day is designated as a point where no further ticket sales can occur, akin to a situation where all tickets for both first-class and coach have been sold out. Additional insights into the structure and function of our U matrix are elaborated further below:

Within our decision matrix, U, nine distinct outcomes are possible for each decision-making instance, corresponding to the combination of pricing strategies for coach and first-class tickets:

- Selling Low Price Coach, Low Price First Class: 11
- Selling Low Price Coach, High Price First Class: 12
- Selling Low Price Coach, First Class Sold Out: 13
- Selling High Price Coach, Low Price First Class: 21
- Selling High Price Coach, High Price First Class: 22
- Selling High Price Coach, First Class Sold Out: 23
- Coach Sold Out, Selling Low Price First Class: 31
- Coach Sold Out, Selling High Price First Class: 32
- Coach Sold Out, First Class Sold Out: 33

The optimal pricing strategy for coach and first-class tickets is identified by U, determined by the highest value across different price levels, taking into account the seats available in both classes and the time remaining until the flight.

- *If seats are available in both coach and first-class: Possible values include: 11, 12, 21, 22*

$$\mathrm{V}(\mathrm{S}c,t,\mathrm{S}f) = max(LL,LH,HL,HH)$$

$$U(\mathrm{S}c,t,\mathrm{S}f) = int((argmax(LL,LH,HL,HH))/2)+1)*10+$$

$$int((argmax(LL, LH, HL, HH))\%2) \,!= 0) + 1$$

<u>Code:</u>

```
V[Sc,t,Sf]=max(value_LL,value_LH,value_HL,value_HH) # value funciton maximizes expected profit

U[Sc,t,Sf]=(int(np.argmax([value_LL,value_LH,value_HL,value_HH])/2)+1)*10\
+int((np.argmax([value_LL,value_LH,value_HL,value_HH])%2)!=0)+1
```

- *If seats are available in coach but first-class is sold out: Possible values: 13, 23*

$$V(Sc,t,Sf) = max(Lc,Hc)$$

$$U(Sc,t,Sf) = (argmax(Lc,Hc) + 1)*10+3$$

<u>Code:</u>

```
V[Sc,t,Sf]=max(value_Lc,value_Hc) # value function maximizes expected profit
U[Sc,t,Sf]=(np.argmax([value_Lc,value_Hc])+1)*10+3
```

- *If seats are available in first-class but coach is sold out: Possible values: 31, 32*

$$\mathrm{V}(\mathrm{S}c,t,\mathrm{S}f) = max(Lf,Hf)$$

$$U(\mathrm{S}c,t,\mathrm{S}f) = 30+argmax(Lc,Hc) + 1$$

<u>Code:</u>

```
V[Sc,t,Sf]=max(value_Lf,value_Hf)
U[Sc,t,Sf]=30+np.argmax([value_Lf,value_Hf])+1
```

- *If both coach and first-class tickets are sold out: Possible values: 33*

$$V(S_c,t,S_f) = \gamma*(V(S_c,t+1,S_f)$$

$$U(S_c,t,S_f) = 33$$

Code:

```
V[Sc,t,Sf]= delta * V[Sc,t+1,Sf]
U[Sc,t,Sf]=33
```

Each decision is numerically coded to convey pricing strategy: the tens digit indicates the pricing for coach (if available), and the ones digit indicates the pricing for first class (if available).

Results - Strategy 1

The outcomes presented here reflect a single iteration and are not derived from simulation runs. The simulation-based results will be explored in the subsequent section.

Under strategy 1, we tested overbooking policies ranging from 5 to 15 tickets. By analyzing the profit outcomes for each policy, we can determine the most effective overbooking strategy.

The findings from these tests are detailed below:

When we set the overbook seats to be only 5, the expected profit is only $41,886.15, therefore, we recalled the function to check the profits throughout the seats up to 15.

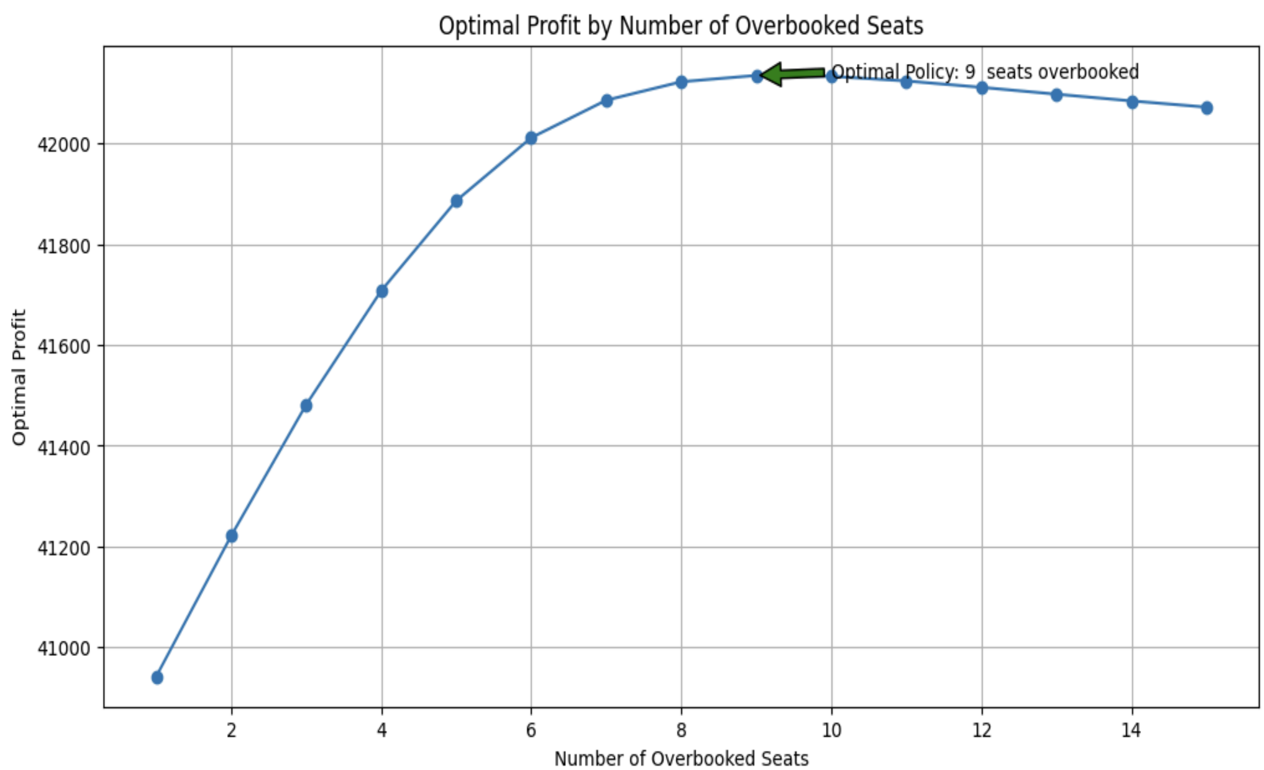
Then, we discovered that selling 9 overbooked tickets yields the highest expected profits. The precise figures are provided below:

Strategy 1’s Expected Profits

	Number of Overbook Coach Seats	Expected Profit
0	5	41886.158624
1	6	42011.221060
2	7	42085.536156
3	8	42122.167269
4	9	42134.624830
5	10	42132.900382
6	11	42123.665936
7	12	42111.029670
8	13	42097.419510
9	14	42084.111570
10	15	42071.742309

9 overbooked tickets, we get $42,134.62 in expected profits.

Based on the analysis, it is advisable to permit the overbooking of 9 coach seats, as this strategy yields the highest expected discounted profit. The profit increment begins to decline when more than 9 seats are overbooked. Therefore, a flexible overbooking approach is recommended, which can be adjusted according to real-time demand and other influencing factors throughout the year.



3. Strategy-2

Objective:
In strategy 2, we move away from a fixed limit on overbooked tickets and permit the dynamic programming algorithm to determine the optimal point to cease ticket sales. This aims to minimize costs and maximize profits by adding an option to halt sales of coach tickets at any stage, effectively treating them as sold out, similar to an aspect of strategy 1. Consequently, the differences between strategies 1 and 2 are minimal yet significant in approach.

State Variables:
These variables encompass the number of days until departure, the quantity of tickets sold in each class (coach and first-class), and the current overbooking level.

- Time (t): Representing the number of days until the flight departure.
- Coach seats Sold (Sc): Indicating the number of seats sold in coach class.
- First Class seats Sold (Sf): Reflecting the number of seats sold in first class.

Decision Variables/Choice Variables:

For each day and given state, the model decides on the ticket price for coach and first-class seats and whether to sell additional tickets beyond the plane's seating capacity.

- Pricing Strategy: High or Low pricing options for both first class (H or L) and coach class (H or L).

There would be an addition of 2 decision variables from strategy 1:

[(coach price low, first price low), (coach price high, first price low), (coach price low, first price low), (coach price high, first price high) , (don't sell coach, first low), (don't sell coach, first high]

Dynamics:

The dynamics could be viewed similarly, where an unsold coach ticket might be treated as if it was never offered for sale, factoring this as a decision in the process.

The potential subsequent states are characterized by:

- (Sc, t+1, Sf) - unsold coach and first-class ticket today
- (Sc-1, t+1, Sf) - sold coach ticket, but not first-class ticket today
- (Sc-1, t+1, Sf-1) - sold both coach and first-class ticket
- (Sc, t+1, Sf-1) - sold first-class ticket, but not coach

Value Function:

$$V(Sc, t, Sf) = \max\left(E\left(\sum_{i=0}^{T-t}(profits@t+1)\gamma i\right)\right.$$

Bellman Equation:

The Bellman equation would undergo minor modifications in two specific scenarios: when both coach and first-class tickets are up for sale, and when coach tickets are on sale but first-class tickets are not.

<u>Both coach and first-class tickets are available:</u>

LL, LH, HL, HH, as we have calculated before, we would introduce 0L and 0H

Simplifying the Price and Probability Equation

The expected revenue and price, from selling tickets can be broken down into four scenarios based on the probabilities of selling coach and first-class tickets. Let's denote A as the price of a coach ticket and B as the price of a first-class ticket.

- Price: We get A if the ticket sold in a coach and 0 if the ticket does not.
- For the above scenario i.e the coach Probability will be x and x-1 respectively.
- Price: We get B if the ticket sold in a first class and 0 if the ticket does not.
- For the above scenario i.e. the first-class Probability will be y and y-1 respectively.

4 Scenarios:

- Both coach and first-class tickets are sold: expected value- xy(A+B)
- Only the coach ticket is sold: expected value- x(1-y)A
- Only the first-class ticket is sold: expected value- (1-x)yB
- Neither ticket is sold: expected value- (1-x)(1-y)0

Bellman equation by also discounting future possible dates as well:

$$
\begin{aligned}
E[Price] &= xy(A+B) + x(1-y)A + (1-x)yB + (1-x)(1-y)0 \\
&= xA + yB
\end{aligned}
$$

$$V(Sc, t, Sf) = xA + xB + \gamma(..)$$

In this scenario, no coach tickets would be sold, leading to a situation where x equals 0. Thus, it follows that:

$$V(Sc, t, Sf) = 0A + yB + \gamma(..) = yB + \gamma(..)$$

$0L$ = $prob\_first\_sale\_low * price\_first\_low + \gamma * (prob\_coach\_no\_sale\_low * V(Sc, t+1, Sf) + prob\_first\_no\_sale\_low * V(Sc, t+1, Sf-1)$

$0H$ = $prob\_first\_sale\_high * price\_first\_high + \gamma * (prob\_coach\_no\_sale\_low * V(Sc, t+1, Sf) + prob\_coach\_no\_sale\_low * V(Sc, t+1, Sf-1))$

<u>Value when set both coach and first-class ticket prices low</u>

$LL = prob\_coach\_sale\_low * price\_coach\_low + prob\_first\_sale\_low * price\_first\_low + \gamma * (prob\_coach\_no\_sale\_low * prob\_first\_no\_sale\_low * V(Sc, t+1, Sf) + prob\_coach\_sale\_low * prob\_first\_no\_sale\_low * V(Sc-1, t+1, Sf) + prob\_coach\_sale\_low * prob\_first\_sale\_low * V(Sc-1, t+1, Sf-1) + prob\_coach\_no\_sale\_low * prob\_first\_sale\_low * V(Sc, t+1, Sf-1))$

<u>Value when set coach ticket price low and first-class ticket price high</u>

$LH = prob\_coach\_sale\_low * price\_coach\_low + prob\_first\_sale\_high * price\_first\_high + \gamma * (prob\_coach\_no\_sale\_low * prob\_first\_no\_sale\_high * V(Sc, t+1, Sf) + prob\_coach\_sale\_low * prob\_first\_no\_sale\_high * V(Sc-1, t+1, Sf) + prob\_coach\_sale\_low * prob\_first\_sale\_high * V(Sc-1, t+1, Sf-1) + prob\_coach\_no\_sale\_low * prob\_first\_sale\_high * V(Sc, t+1, Sf-1))$

<u>Value when set coach ticket price high and first-class ticket price low</u>

$HL = prob\_coach\_sale\_high * price\_coach\_high + prob\_first\_sale\_low * price\_first\_low + \gamma * (prob\_coach\_no\_sale\_high * prob\_first\_no\_sale\_low * V(Sc, t+1, Sf) + prob\_coach\_sale\_high * prob\_first\_no\_sale\_low * V(Sc-1, t+1, Sf) + prob\_coach\_sale\_high * prob\_first\_sale\_low * V(Sc-1, t+1, Sf-1) + prob\_coach\_no\_sale\_high * prob\_first\_sale\_low * V(Sc, t+1, Sf-1))$

<u>Value when set both coach and first-class ticket prices high</u>

$HH = prob\_coach\_sale\_high * price\_coach\_high + prob\_first\_sale\_high * price\_first\_high$
$+ \gamma * (prob\_coach\_no\_sale\_high * prob\_first\_no\_sale\_high * V(Sc, t+1, Sf)$
$+ prob\_coach\_sale\_high * prob\_first\_no\_sale\_high * V(Sc-1, t+1, Sf)$
$+ prob\_coach\_sale\_high * prob\_first\_sale\_high * V(Sc-1, t+1, Sf-1)$
$+ prob\_coach\_no\_sale\_high * prob\_first\_sale\_high * V(Sc, t+1, Sf-1))$

$$V(S_c,t,S_f) = max(LL,LH,HL,HH,0L,0H)$$

Code:

```
# value when decide not to sell coach and set first-class ticket price low
value_0L = prob_Lf[1]*priceLf + delta * (prob_Lf[0]*V[Sc,t+1,Sf]+prob_Lf[1]*V[Sc,t+1,Sf-1])

# value when decide not to sell coach and set first-class ticket price high
value_0H = prob_Hf[1]*priceHf + delta * (prob_Hf[0]*V[Sc,t+1,Sf]+prob_Hf[1]*V[Sc,t+1,Sf-1])

# value when both coach and first-class ticket prices low
value_LL = prob_Lc[1]*priceLc + prob_Lf[1]*priceLf + delta * (prob_Lf[0]*prob_Lc[0]*V[Sc,t+1,Sf]
                                                   +prob_Lf[0]*prob_Lc[1]*V[Sc-1,t+1,Sf]
                                                   +prob_Lf[1]*prob_Lc[1]*V[Sc-1,t+1,Sf-1]
                                                   +prob_Lf[1]*prob_Lc[0]*V[Sc,t+1,Sf-1])

# value when set coach ticket price low and first-class ticket price high
value_LH = prob_Lc[1]*priceLc + prob_Hf[1]*priceHf + delta * (prob_Hf[0]*prob_Lc[0]*V[Sc,t+1,Sf]
                                                   +prob_Hf[0]*prob_Lc[1]*V[Sc-1,t+1,Sf]
                                                   +prob_Hf[1]*prob_Lc[1]*V[Sc-1,t+1,Sf-1]
                                                   +prob_Hf[1]*prob_Lc[0]*V[Sc,t+1,Sf-1])

# value when set coach ticket price high and first-class ticket price low
value_HL = prob_Hc[1]*priceHc + prob_Lf[1]*priceLf + delta * (prob_Lf[0]*prob_Hc[0]*V[Sc,t+1,Sf]
                                                   +prob_Lf[0]*prob_Hc[1]*V[Sc-1,t+1,Sf]
                                                   +prob_Lf[1]*prob_Hc[1]*V[Sc-1,t+1,Sf-1]
                                                   +prob_Lf[1]*prob_Hc[0]*V[Sc,t+1,Sf-1])

# value when set both coach and first-class ticket prices high
value_HH = prob_Hc[1]*priceHc + prob_Hf[1]*priceHf + delta * (prob_Hf[0]*prob_Hc[0]*V[Sc,t+1,Sf]
                                                   +prob_Hf[0]*prob_Hc[1]*V[Sc-1,t+1,Sf]
                                                   +prob_Hf[1]*prob_Hc[1]*V[Sc-1,t+1,Sf-1]
                                                   +prob_Hf[1]*prob_Hc[0]*V[Sc,t+1,Sf-1])
```

If coach tickets are on sale but first-class tickets are not, we'll include an additional term, 0c, to account for the option of not selling any coach tickets:

$0_c = 0 + \gamma * (V(S_c,t+1,S_f))$

This is akin to treating it as if neither first-class nor coach tickets were available for sale. As a result, we arrive at the following formula:

$V(S_c,t,S_f) = max(L_c,H_c,0_c)$

Terminal Condition

Given the uncertainty surrounding the exact number of passengers who will show up for the flight, as it is governed by probability as outlined in the problem statement, it is necessary to establish terminal conditions related to the potential costs at departure. To address this, the expected costs on the departure day are calculated using a binomial probability distribution. We calculate the probabilities for every possible number of coach and first-class passengers showing up independently.

These probabilities are then factored into the cost variable, which is integrated into the value function for the corresponding number of sold coach (Sc) and first-class tickets (Sf). This process is applied to each potential number of actual coach (C) and first-class passengers (F) who arrive for the flight.

The terminal conditions remains same as strategy as the costs have not changed.

Additional Information:

Since adjustments were made to just two scenarios (when both coach and first-class tickets are accessible and when only coach tickets are available), modifications in the U matrix are required solely for these cases. As previously mentioned, the decision to halt the sale of coach tickets can be depicted as a situation where coach tickets are considered sold out, represented within the '30s' category in the matrix.

- *If seats are available in both coach and first-class: Possible values include: 11, 12, 21, 22,31,32*

$$V(S_c,t,S_f) = max(LL,LH,HL,HH,0L,0H)$$

$$U(S_c,t,S_f) = int((argmax(LL,LH,HL,HH,0L,0H))/2)+1)*10+$$

$$int((argmax(LL, LH, HL, HH,0L,0H))\%2) != 0) + 1$$

  <u>Code:</u>

```
V[Sc,t,Sf]=max(value_LL,value_LH,value_HL,value_HH,value_0L,value_0H) # value funciton maximizes
        
U[Sc,t,Sf]=(int(np.argmax([value_LL,value_LH,value_HL,value_HH,value_0L,value_0H])/2)+1)*10\
+int((np.argmax([value_LL,value_LH,value_HL,value_HH,value_0L,value_0H])%2)!=0)+1
```

- *If coach tickets are available but not first class: Possible values include: 13,23,33*

$$V(S_c,t,S_f) = max(Lc,Hc,0c)$$

$$U(S_c,t,S_f) = (argmax(Lc,Hc,0c) + 1)*10+3$$

Code:

```
V[Sc,t,Sf]=max(value_Lc,value_Hc,value_0c) # value function maximizes
U[Sc,t,Sf]=(np.argmax([value_Lc,value_Hc,value_0c])+1)*10+3
```

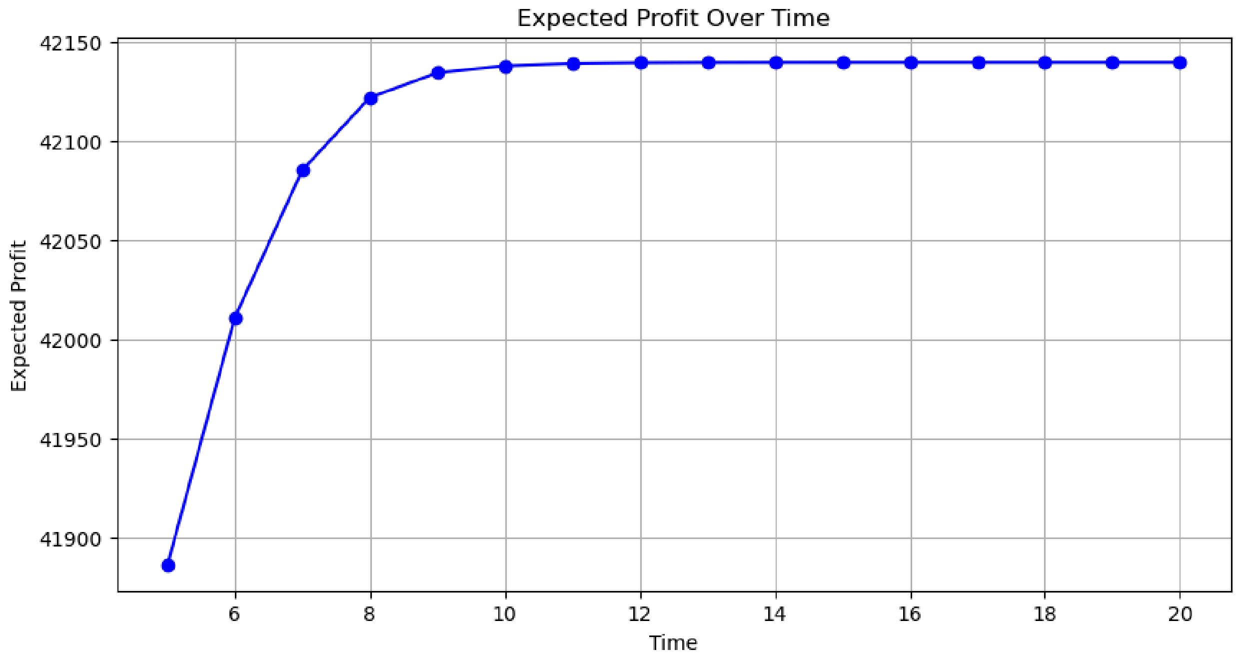
Results: Strategy-2

In strategy 2, the algorithm is given the discretion to determine the optimal point to cease overbooking coach tickets. We anticipate that, beyond a certain level of overbooking, the expected profits as illustrated by the algorithm would plateau.

Observations indicate that beyond approximately 10 tickets, ceasing further overbooking becomes advantageous for profit maximization. To pinpoint the precise threshold, a detailed examination of the expected profits associated with each level of overbooking is necessary. The relevant results is depicted below:

Strategy 2's Expected Profits

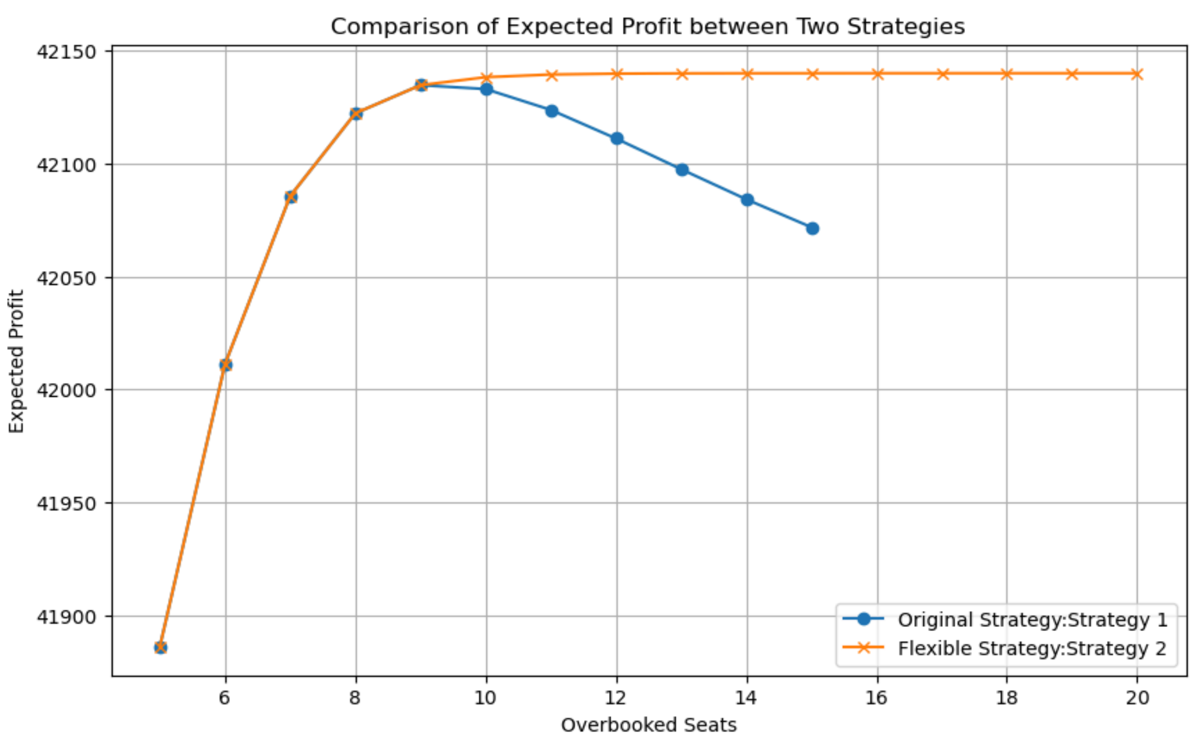
	Number of Overbook Coach Seats	Expected Profit
0	5	41886.158624
1	6	42011.221060
2	7	42085.536156
3	8	42122.167269
4	9	42134.624830
5	10	42138.140498
6	11	42139.327940
7	12	42139.708945
8	13	42139.831425
9	14	42139.872225
10	15	42139.886081
11	16	42139.890711
12	17	42139.892183
13	18	42139.892617
14	19	42139.892734
15	20	42139.892762



It becomes evident that beyond 15 tickets, there's no appreciable increase in profit down to the nearest cent. Under this strategy, the expected profit amounts to $42,139.89.

Comparison of Strategies:

	number of overbook	optimal profit_x	optimal profit_y
0	3	41481.575929	41481.575929
1	4	41707.623301	41707.623301
2	5	41886.158624	41886.158624
3	6	42011.221060	42011.221060
4	7	42085.536156	42085.536156
5	8	42122.167269	42122.167269
6	9	42134.624830	42134.624830
7	10	42132.900382	42138.140498
8	11	42123.665936	42139.327940
9	12	42111.029670	42139.708945
10	13	42097.419510	42139.831425
11	14	42084.111570	42139.872225
12	15	42071.742309	42139.886081
13	16	42060.591705	42139.890711
14	17	42050.750847	42139.892183
15	18	42042.211977	42139.892617
16	19	42034.906709	42139.892734
17	20	42028.737038	42139.892762



Under strategy 1, the anticipated profit stands at $42,134.62, while strategy 2 yields a slightly higher expected profit of $42,139.89, translating to a marginal gain of $5.27. Also, strategy 1 shows a diminishing return while allowing no demand would generate similar profit while increasing overbooking. This outcome suggests that strategy 2 marginally outperforms strategy 1 in terms of profit enhancement. However, since these figures result from a single iteration, it's essential to conduct multiple simulations to derive average profit values for each strategy, ensuring a more robust comparison.

Simulation and Analysis

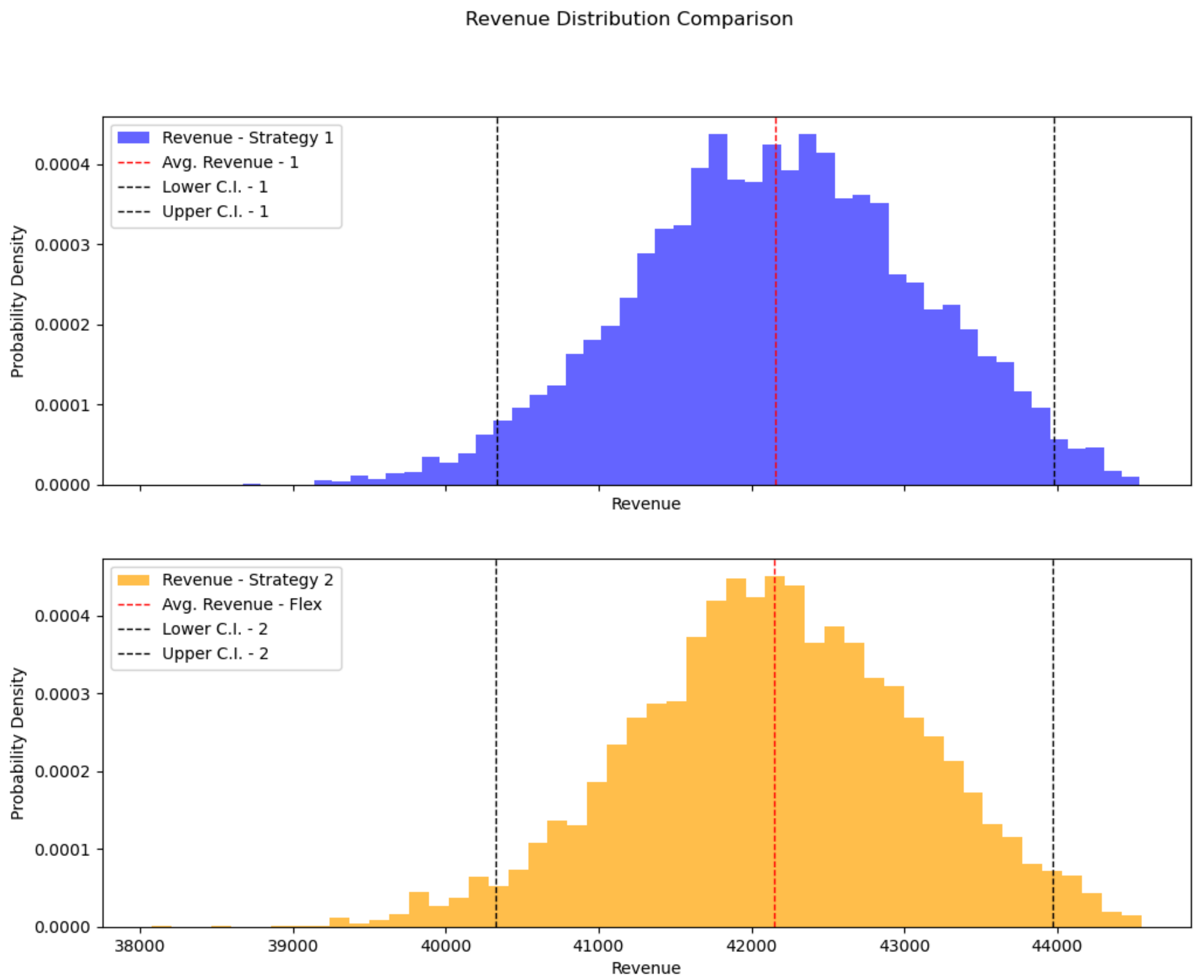
To evaluate the two strategies' performance, we conducted 5,000 simulation runs, comparing their profitability. Here's a summary of the optimal booking strategies identified for each:

- **Strategy 1**: Mandates selling coach tickets when available with an overbooking limit set to 9 seats.
- **Strategy 2**: Provides the flexibility to cease selling coach tickets on any given day, with a maximum overbooking limit of 20 seats.

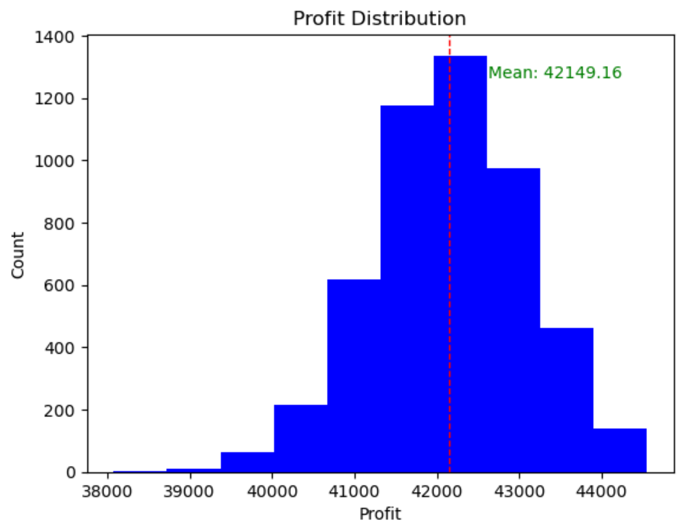
The simulations were executed by drawing samples from the probability distributions that govern ticket purchases across both classes, factoring in the ticket prices. Moreover, the simulations included sampling from a binomial distribution to estimate the actual turnout of passengers on the flight day. This was based on the number of tickets sold and the optimal pricing strategy determined via dynamic programming.

<u>Most Profitable Strategy:</u>

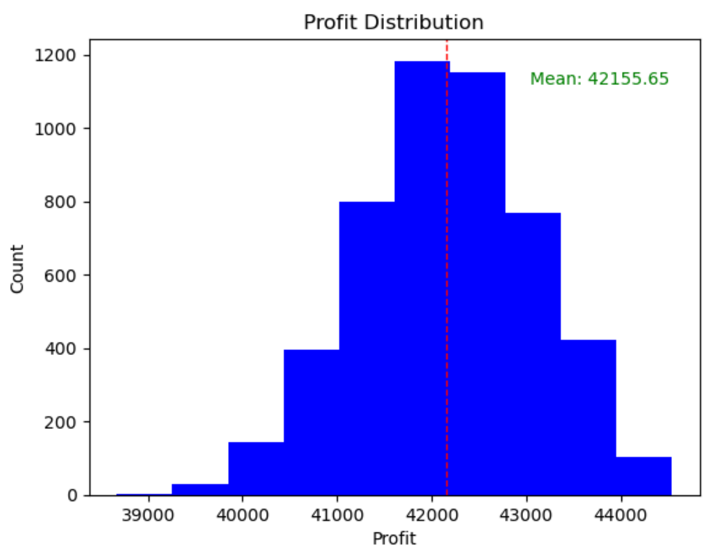
Comparing both Strategies' profits by Distribution graphs.



Strategy 1: Mean Profit of $42,155.65

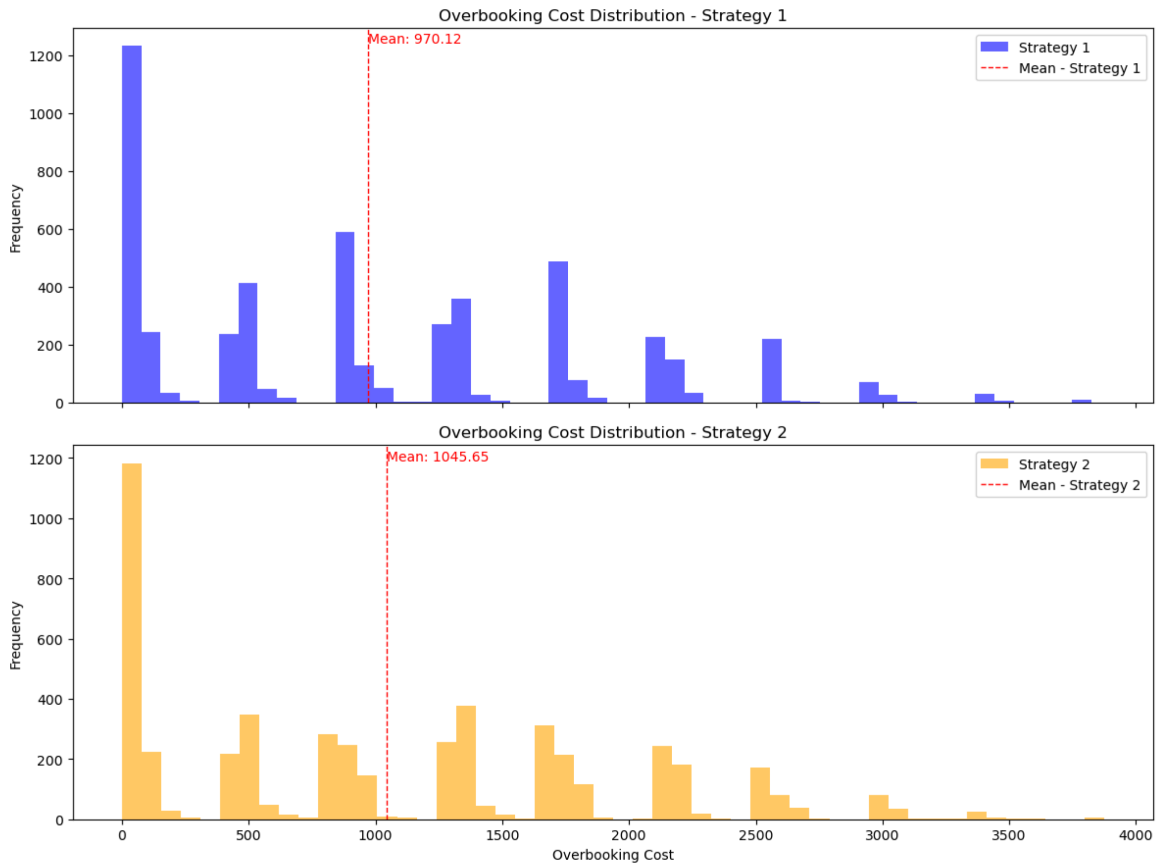


Strategy 2: Mean Profit of $42,149.16



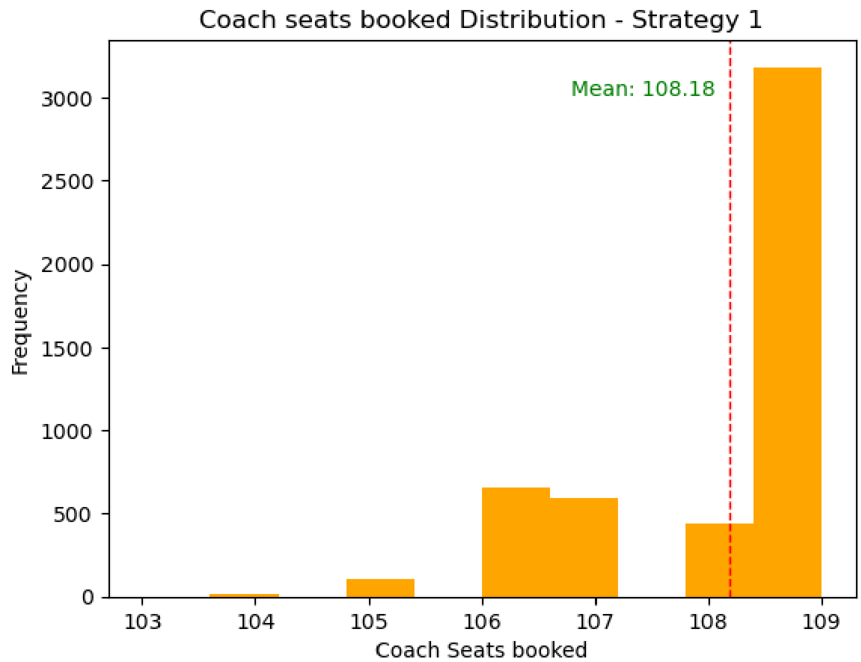
Although the 'first' strategy shows marginally higher average profits compared to the 'Optimal overbooking' strategy, the profit ranges for both strategies are quite similar. The profit distributions for the two approaches also appear nearly indistinguishable. This suggests that the performance difference between the two strategies is not statistically meaningful.

<u>Overbooked Cost Distribution</u>

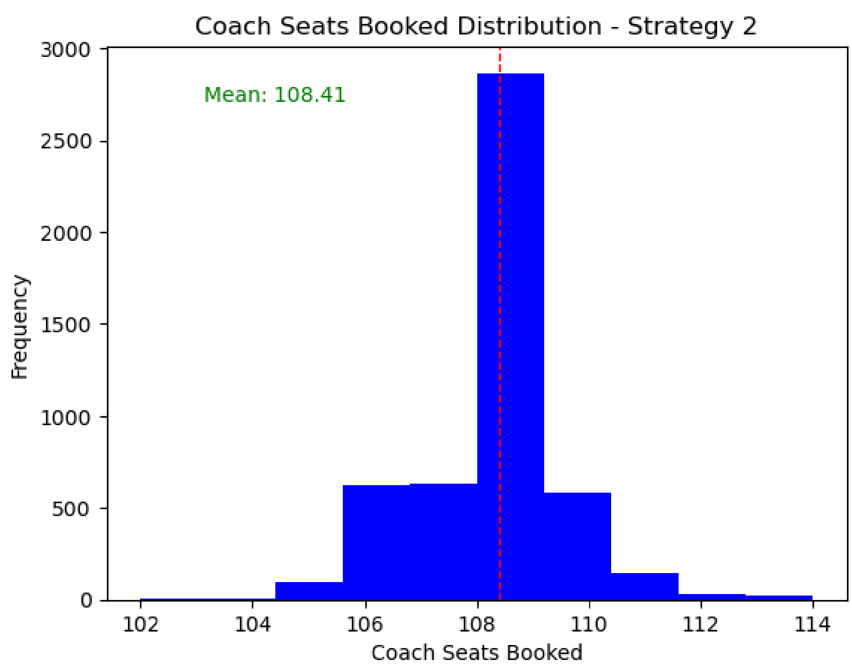


It's evident that the second strategy incurs significantly higher costs due to overbooking. The distribution plots further indicate that higher costs are more frequently encountered with this strategy. Hence, while the profits may be comparable between the two strategies, adopting the second strategy would lead to increased expenditures related to overbooking.

Number of Coach Seats Overbooked



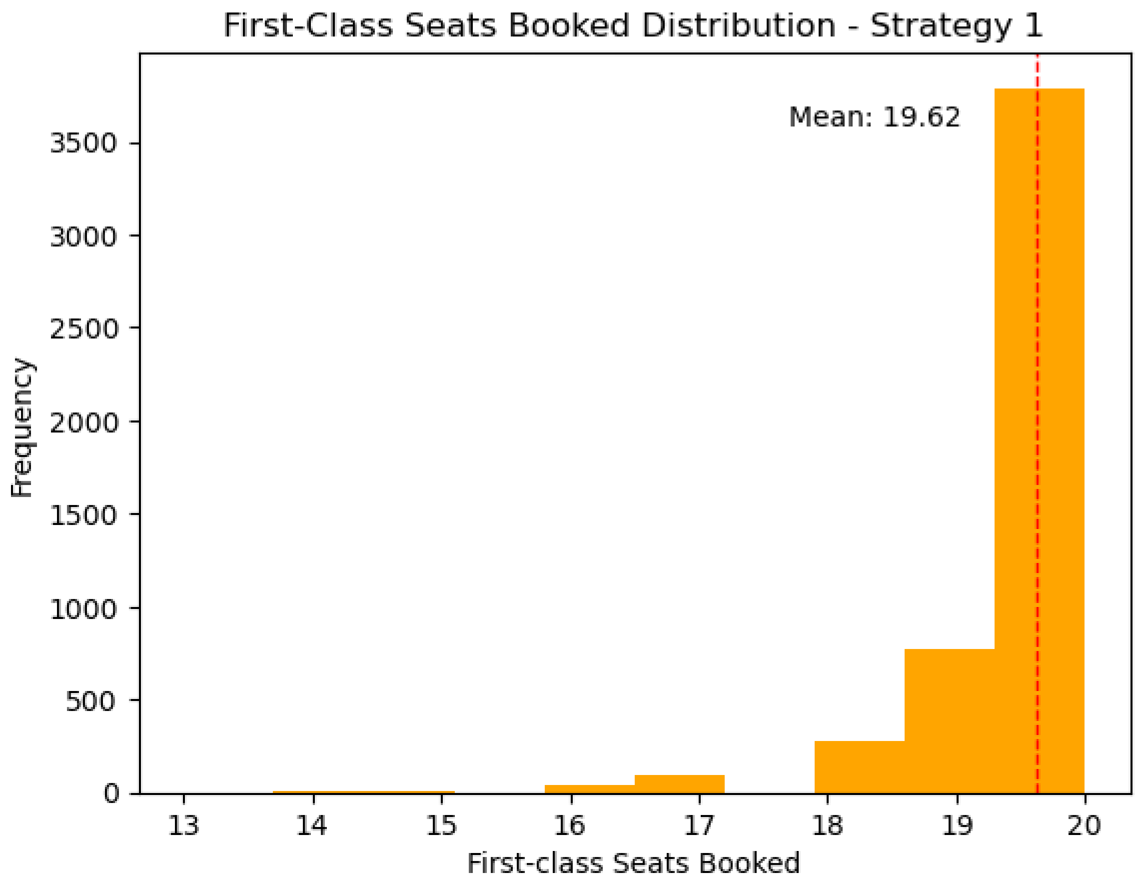
For Strategy 1, the coach is always overbooked if we have the option.



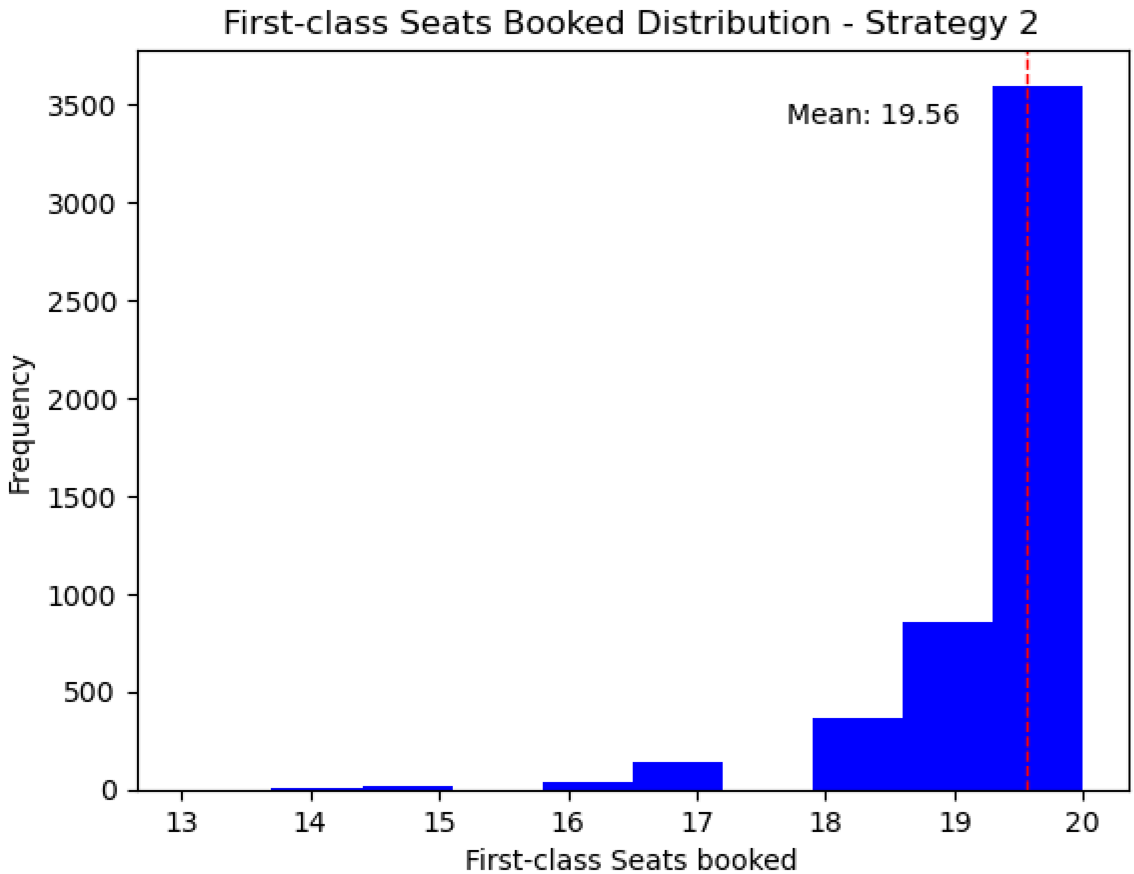
With Strategy 2, the number of coach seats booked is rarely higher than 109 as that would exceed the point where we get the highest expected profit.

It's apparent that the average number of coach seats booked under both strategies shows no substantial discrepancy. Nonetheless, the distribution patterns for each strategy diverge noticeably.

__Number of First-Class Seats Booked__



Even if it looks like the probability of selling a first-class ticket is low, it mostly gets fully booked due to the higher number of days to book them than the number of first-class tickets.

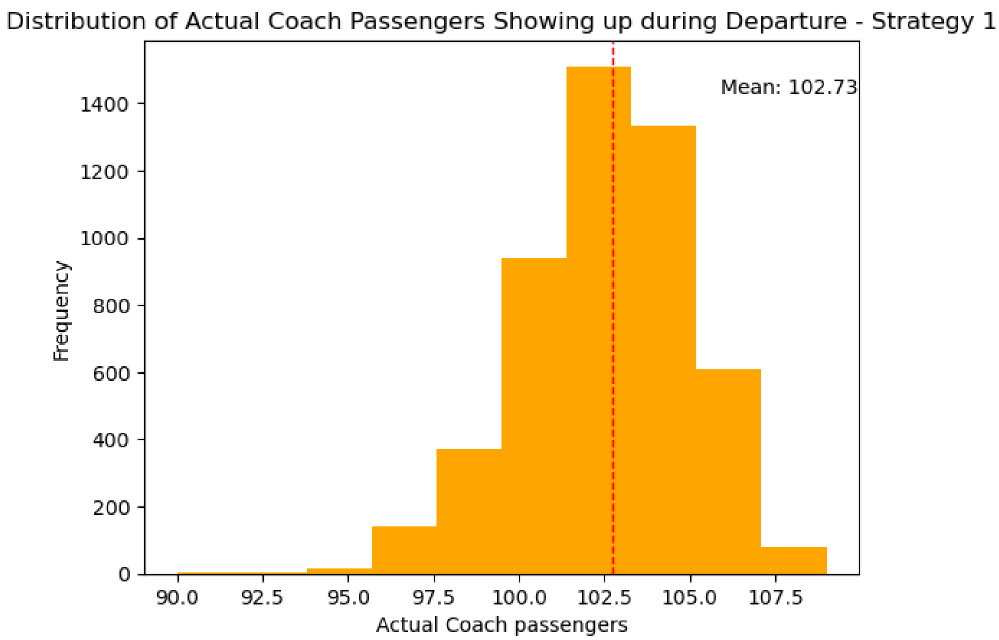


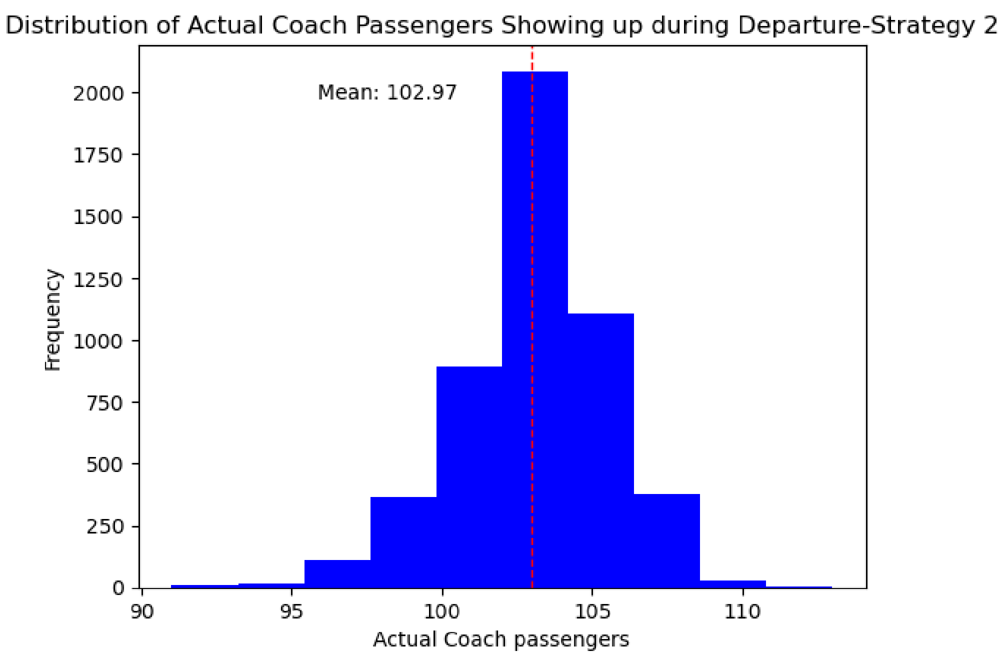
The frequent full booking of first-class seats is largely attributable to the greater window of time available for booking compared to the limited number of first-class tickets on offer.

In this case as well, there doesn't appear to be a notable difference in the average outcomes produced by the two strategies.

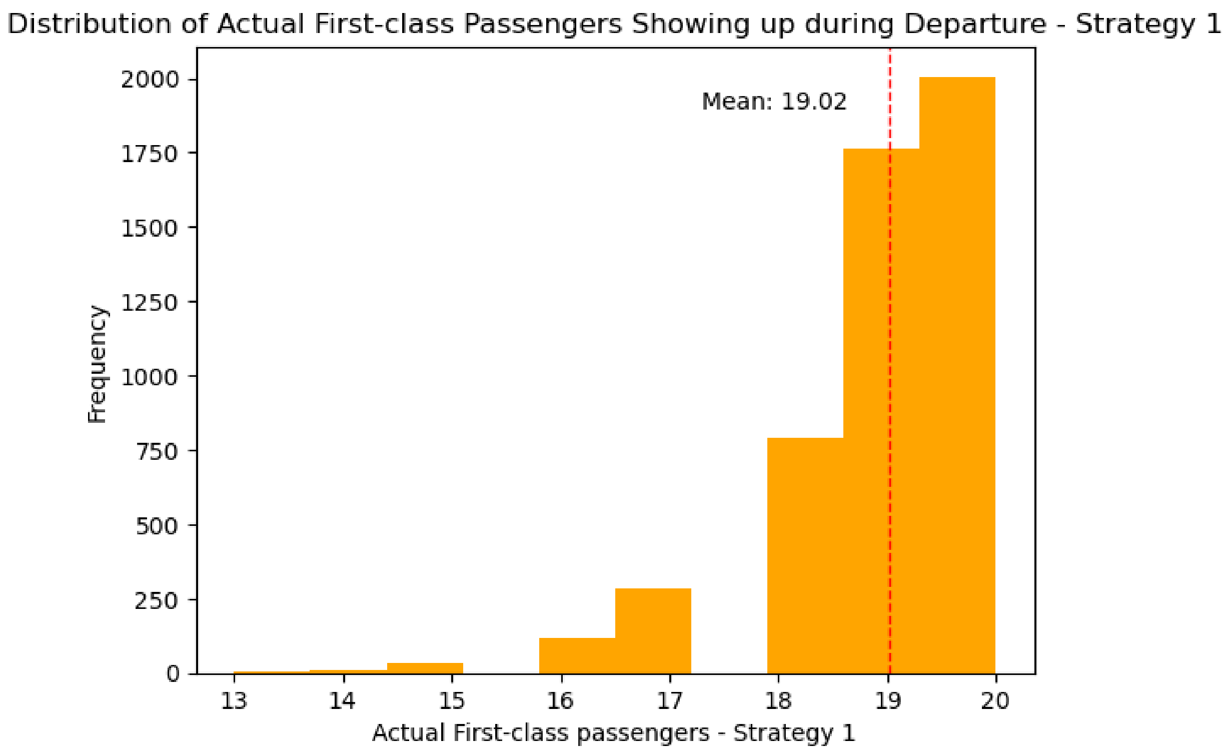
Number of Coach Passengers That Actually Show Up to the Flight

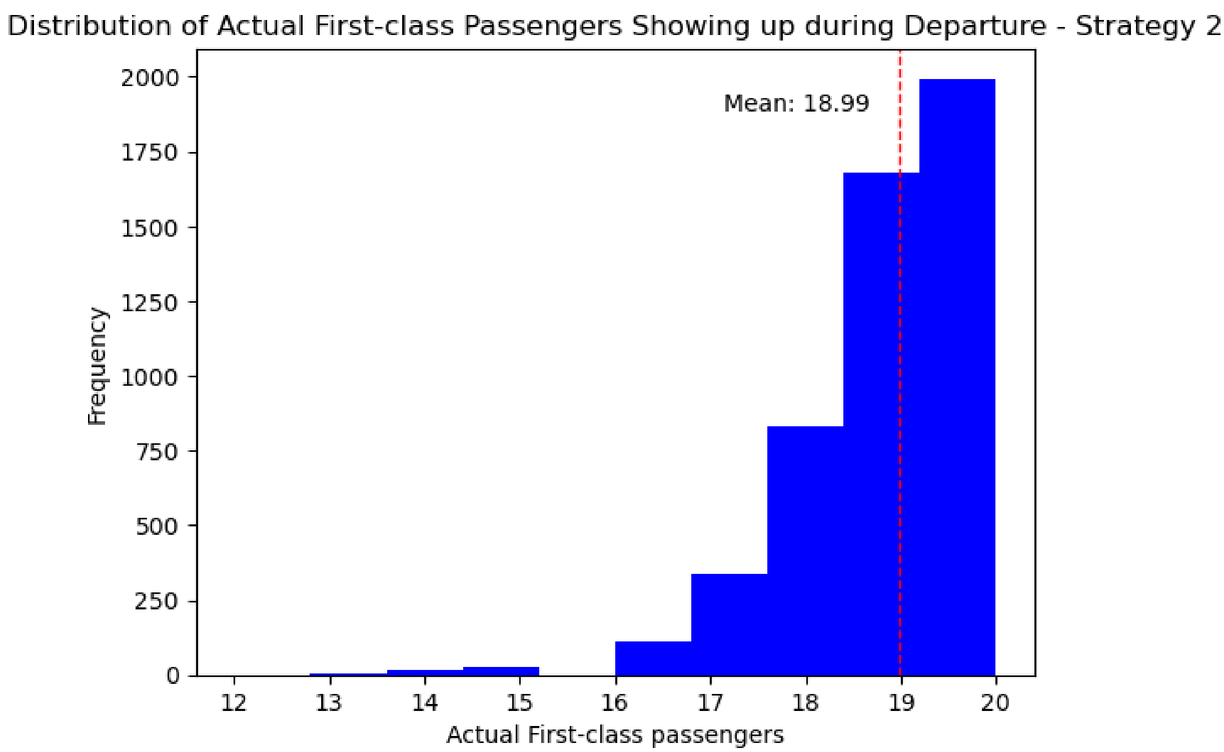
On average, a marginally higher number of passengers show up using the second strategy, though the difference is minimal. The distribution for the first strategy appears to be more concentrated around central values:





Number of First-Class Passengers That Actually Show Up to the Flight

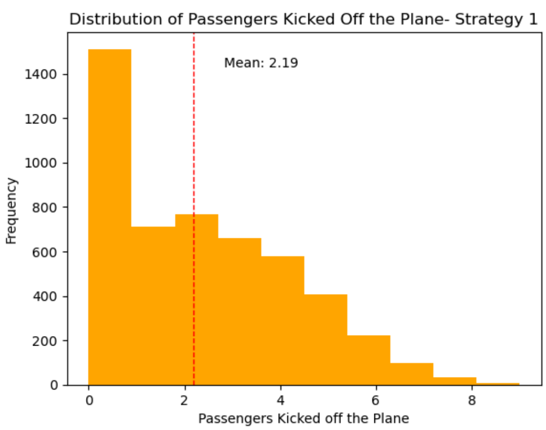


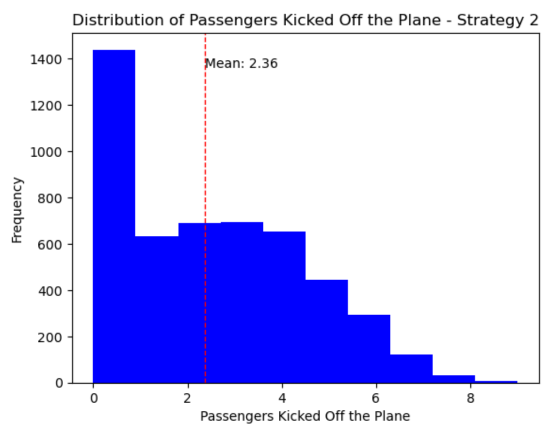


Number of Passengers that are Kicked-Off the Plane

We evaluate the incidence of passengers being denied boarding across the two strategies by examining the distribution of the number of passengers displaced during the 5,000 simulation runs.

The second strategy seems to result in a significantly higher frequency of passengers being denied boarding compared to the first strategy. As such, it's important to closely monitor the second strategy, since a higher volume of displaced passengers could lead to potential public relations challenges and adverse publicity.





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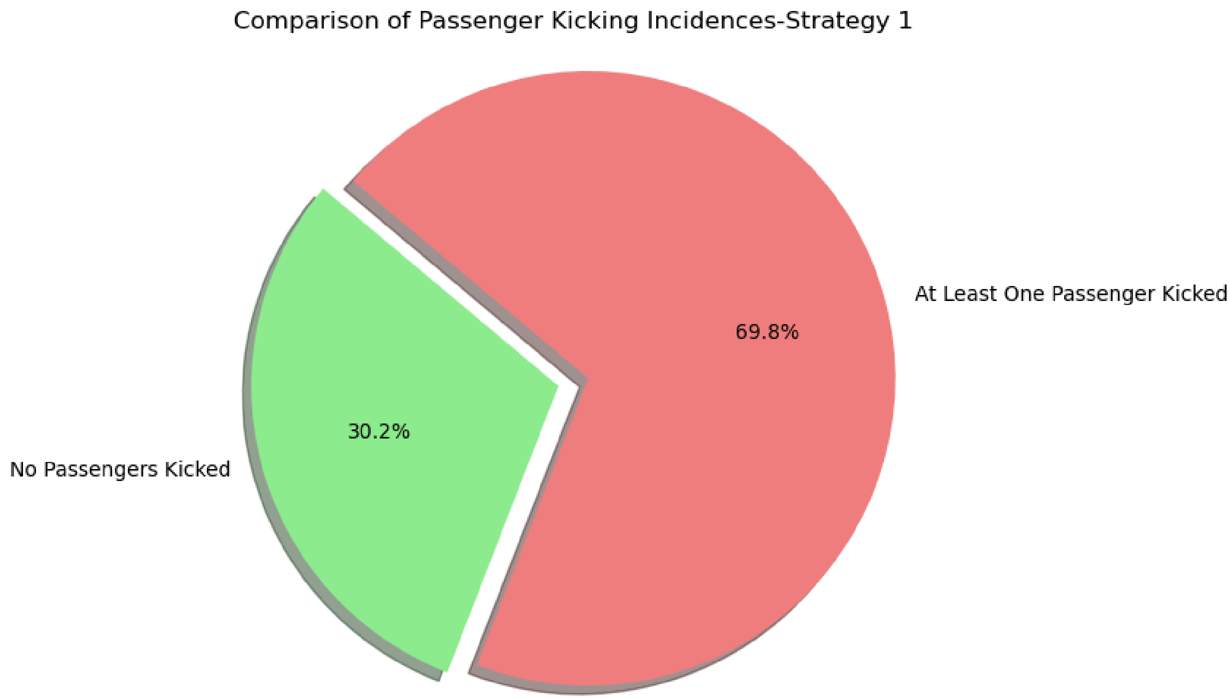
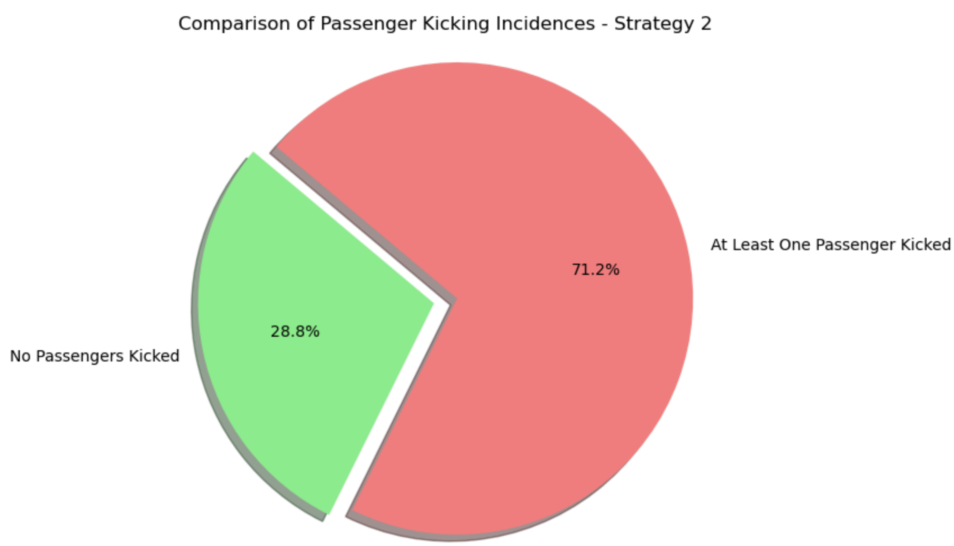
Strategy 1 - Among 5000 simulations, there were 1511 instances where no passenger was kicked off the plane. However, despite this relatively high occurrence, the instances where at least one passenger was kicked off the plane were more frequent, totaling 3489



Strategy 2- Out of 5000 flights, there are 1439 cases where no passengers are removed from the plane. Despite this substantial figure, the occurrences where at least one passenger is displaced are more common, numbering 3561.



Recommendation and Insights

Comparison of Overbooking Strategies

In this section, we provide an in-depth comparison between the traditional overbooking strategy, characterized by a fixed limit on the number of seats to overbook, and the flexible overbooking strategy, which allows for real-time adjustments in ticket sales based on demand and seat availability. Our analysis aims to delineate the advantages and drawbacks of each strategy, focusing on their effects on revenue, operational efficiency, and customer satisfaction.

Fixed Overbooking Strategy - Strategy 1

Advantages:

- Predictability: A fixed overbooking limit facilitates operational planning and resource allocation, as the expected number of overbooked passengers can be foreseen in advance.
- Simplicity: Implementation of a fixed overbooking strategy is relatively straightforward, necessitating less intricate systems and processes for managing ticket sales and passenger accommodations.

Disadvantages:

- Inflexibility: This approach overlooks fluctuations in demand or passenger show-up rates closer to departure, potentially resulting in higher instances of overbooking and passenger discontent.
- Revenue Opportunity Loss: By not adjusting the overbooking limit based on real-time data, the airline may forego opportunities to sell additional tickets on flights with a higher likelihood of no-shows.

Flexible Overbooking Strategy - Strategy 2

Advantages:

- Adaptability: Allows the airline to respond promptly to changes in demand and passenger behavior, optimizing revenue while mitigating the risk of overbooking.
- Customer Satisfaction: Potentially reduces the number of passengers denied boarding, thereby enhancing the overall travel experience and fostering customer loyalty.

Disadvantages:

- Complexity: Requires advanced data analytics and dynamic pricing systems to make daily adjustments to ticket sales, which could elevate operational costs.
- Resource Intensity: Constant monitoring and adjustment of ticket sales may necessitate more resources and training for sales and customer service teams.

Operational Impacts and Customer Experience:

Our simulation models suggest that the overbooking strategy I leads to fewer instances of overbooking and passenger displacement, while resulting a lower cost thereby slightly higher profit, which can factor in higher levels of customer satisfaction and potential reductions in compensation and rebooking costs. However, we should still stimulate to alternate between the two policies as the constraints changed, we also need to consider the increased complexity and resource requirements underscore the necessity for a methodical implementation strategy, balancing the benefits of flexibility with the costs of heightened operational demands with simulation.

Results

Analysis of Overbooking Policies: The primary aim of our analysis was to determine the ideal overbooking threshold that maximizes expected profits for airline companies, while minimizing potential negative impacts on passenger satisfaction. This section presents our findings, which were derived from dynamic programming and simulation models, and compares the outcomes of fixed versus flexible overbooking policies.

Optimal Overbooking Limit: Our dynamic programming model examined various overbooking thresholds, ranging from 5 to more seats beyond the coach capacity of 100 seats. It factored in daily ticket pricing decisions, passenger demand, and show-up probabilities to compute the expected discounted profit for each overbooking scenario.

Optimal Threshold: The optimal overbooking threshold was determined to be 9 seats according to the first policy, where the expected discounted profit peaked. This threshold strikes a balance between capturing additional revenue from overbooking and managing the costs and operational challenges associated with passenger bumping. However, the second strategy generates more profit after overbooking 9 seats when allowing no demand on coach ticket. However, we see that after 5000 simulation, strategy I demonstrated a slightly better cost and profit than strategy II, with better kickoff rate and show up rate as well.

Risk Assessment:

Our analysis also addressed the risks associated with overbooking, including potential negative customer experiences and impacts on the airline's reputation. While overbooking can boost revenue, it is crucial to balance this with maintaining high levels of customer service and satisfaction.

Managing Customer Expectations: Effective communication and transparent policies can help mitigate dissatisfaction among bumped passengers. Offering compensation, rebooking options, and additional services can also help uphold positive customer relations.

Strategic Recommendations: Based on the analysis, Airline should consider implementing a dynamic overbooking policy with forced demand on both classes and solve for optimal overbooking seats, complemented by robust customer service protocols, to manage risks and maximize the benefits of overbooking.

Conclusions:
The study conducted by our team offers crucial insights into optimizing overbooking strategies for airlines to enhance their expected discounted profits. Utilizing dynamic programming, we explored various fare strategies and ticket availability for specific flights. Our findings suggest that adopting the first strategy, which includes overbooking by 9 seats, can notably boost the expected discounted profit for this case.

It's essential to acknowledge that while overbooking can enhance airline profitability, it may also lead to customer dissatisfaction. Thus, airlines need to find a balance between profit maximization and ensuring a favorable customer experience. This can be achieved by maintaining transparency regarding overbooking practices, compensating volunteers who agree to give up their seats, and delivering outstanding service, therefore, it would be essential to also look deeper into the overbooking strategy for the passenger showup rate, the kickoff distribution, and the overall booking distribution than just analyze the profit and cost from dynamic pricing.

While both overbooking strategies offer avenues for revenue optimization, even the fixed overbooking strategy shows better business inclination, however, the strategy II approach affords Airline the opportunity to adapt more adeptly to market dynamics and passenger needs. Implementing this strategy will necessitate meticulous planning and investment in technology and training, but the potential benefits in terms of revenue and customer loyalty make it a compelling option for the future.

In conclusion, our analysis aids airlines in refining their overbooking practices to improve profitability. The strategy II approach affords Airline the opportunity to adapt more adeptly to market dynamics and passenger needs. Alternating both strategy up on its given constraints could necessitate meticulous planning and investment in technology and training, Nevertheless, it's critical for airlines to consider the implications of these practices on customer relations and to endeavor to uphold high levels of customer satisfaction and a strong brand reputation.