

Marketing Budget Optimization Project

Submitted to:

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Optimization I, RM 294

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Project Background: A recent CMO survey sponsored by the Fuqua School of Business at Duke University, Deloitte LLP, and the American Marketing Association found that marketing budgets now comprise 11% of total company budgets. However, the effectiveness of marketing varies greatly. On one hand, P&G cut more than \$100 million in digital marketing spending because their ads were largely ineffective, while on the other Netflix plans to increase advertising spending by 54% due to positive feedback in international markets.

In summary, while companies spend more and more on marketing, not all marketing strategies are created equal. Thus, it is important for companies to carefully track the results of their marketing campaigns and adjust as needed to ensure that they are getting the most out of their investment.

General Problem Statement: Our company is planning to spend \$10M as a marketing budget. As the data scientist in the marketing department, we are requested by the Chief Marketing Officer of the organization to prepare a report recommending how to spread this budget among several marketing mediums. The department has also employed outside consulting firms to estimate the return on investment (ROI) of each marketing medium under consideration.

Problem Statement 1: The consulting firm shared the below table as the estimated return on investment (ROI) of each of the marketing mediums under consideration:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	3.10%	4.90%	2.40%	3.90%	1.60%	2.40%	4.60%	2.60%	3.30%	4.40%

Additionally, we have add below constraints on the marketing budget:

1. The amount invested in print and TV should be no more than the amount spent on Facebook and Email.
2. The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords.
3. For each platform, the amount invested should be no more than \$3M.

We need to find the optimal marketing budget allocation to maximize return on investment (ROI)

Solution: The problem can be expressed in terms of Optimization as below

Objective:

$$ROI_{Print} * Y_{Print} + ROI_{TV} * Y_{TV} + ROI_{SEO} * Y_{SEO} + ROI_{AdWords} * Y_{AdWords} + ROI_{Facebook} * Y_{Facebook} + ROI_{LinkedIn} * Y_{LinkedIn} + ROI_{Instagram} * Y_{Instagram} + ROI_{Snapchat} * Y_{Snapchat} + ROI_{Twitter} * Y_{Twitter} + ROI_{Email} * Y_{Email} \leq 10$$

Where:

- ROI_x – Return of Investment from medium x
- Y_x – Marketing budget spent on medium x

Constraints:

1. The amount invested in print and TV should be no more than the amount spent on Facebook and Email

- $Y_{Print} + Y_{TV} \leq Y_{Facebook} + Y_{Email}$

2. The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords

- $Y_{Facebook} + Y_{LinkedIn} + Y_{Instagram} + Y_{Snapchat} + Y_{Twitter} \geq 2 * (Y_{SEO} + Y_{AdWords})$

3. For each platform, the amount invested should be no more than \$3M

- $0 \leq Y_{Print}, Y_{TV}, Y_{SEO}, Y_{AdWords}, Y_{Facebook}, Y_{LinkedIn}, Y_{Instagram}, Y_{Snapchat}, Y_{Twitter}, Y_{Email} \leq 3$

Code Snippet: The below code is run using Gurobi with the aim of maximizing the objective function keeping the constraints in mind.

```
import pandas as pd
import gurobipy as gp
from gurobipy import GRB

# Read ROI data from CSV file and set the 'Platform' column as the index
roi_data1 = pd.read_csv('ROI_data.csv', index_col="Platform")
# Extract platform names and ROI values
platforms1 = roi_data1.columns.tolist()
roi_values1 = roi_data1.iloc[0].tolist()

model1 = gp.Model("MarketingBudget1") # Give a unique name to the model
# Decision variables
x1 = {} # Amount to invest in each platform
for i, platform in enumerate(platforms1):
    x1[platform] = model1.addVar(lb=0, ub=3, vtype=GRB.CONTINUOUS, name=platform)

# Set objective function: Maximize ROI
model1.setObjective(gp.quicksum(roi_values1[i] * x1[platform] for i, platform in enumerate(platforms1)), GRB.MAXIMIZE)
# Constraints
model1.addConstr(x1['Print'] + x1['TV'] <= x1['Facebook'] + x1['Email'])
model1.addConstr(gp.quicksum(x1[p] for p in ['Facebook', 'LinkedIn', 'Instagram', 'Snapchat', 'Twitter']) >=
                 2 * gp.quicksum(x1[p] for p in ['SEO', 'AdWords']))
model1.addConstr(gp.quicksum(x1[platform] for platform in platforms1) <= 10)
# Budget constraint
budget = 10_000_000 # $10M
model1.addConstr(gp.quicksum(x1[platform] for platform in platforms1) <= budget)
model1.optimize()
if model1.status == GRB.OPTIMAL:
    print("Optimal budget allocation for all platforms (Code 1):")
    for platform in platforms1:
        print(f"{platform}: {x1[platform].x:.2f} million")
    objective_value_decimal = round(model1.objVal, 14) # Format with 14 decimal places
    print(f"Optimal objective: {objective_value_decimal:.14f}")
else:
    print("No optimal solution found (Code 1).")
```

Code Snippet 1

Results: Based on the results we should be spending \$3 million each on TV, Instagram and Email and \$1 million on AdWords in order to get a maximum return on investment of \$456,000

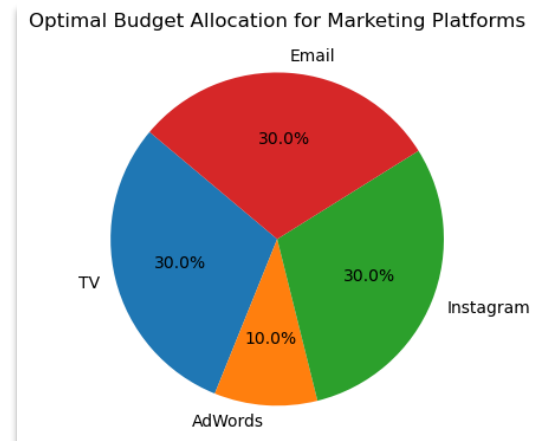
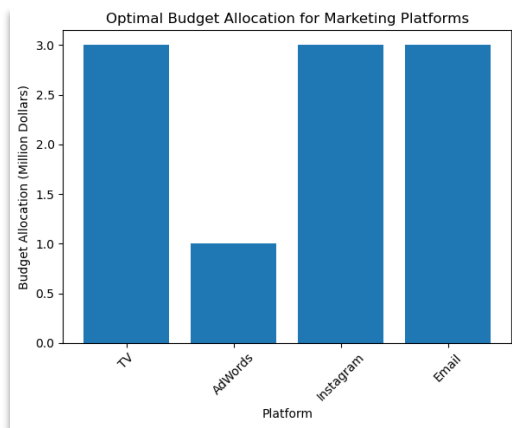


Fig 1. Optimal Budget allocation for Marketing Platforms

Problem Statement 2: Based on request we reached out to another consulting firm to get the ROI estimates and the firm shared the below table as the estimated return on investment (ROI) of each of the marketing mediums under consideration:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Second Firms ROI Estimate	4.90%	2.30%	2.40%	3.90%	4.40%	4.60%	2.60%	1.90%	3.70%	2.60%

Additionally, below constraints on the marketing budget still need to be kept in mind:

1. The amount invested in print and TV should be no more than the amount spent on Facebook and Email.
2. The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords.
3. For each platform, the amount invested should be no more than \$3M.

We need to find the optimal marketing budget allocation to maximize return on investment (ROI)

Solution: The problem can be expressed in terms of Optimization as below

Objective:

$$ROI_{Print} * Y_{Print} + ROI_{TV} * Y_{TV} + ROI_{SEO} * Y_{SEO} + ROI_{AdWords} * Y_{AdWords} + ROI_{Facebook} * Y_{Facebook} + ROI_{LinkedIn} * Y_{LinkedIn} + ROI_{Instagram} * Y_{Instagram} + ROI_{Snapchat} * Y_{Snapchat} + ROI_{Twitter} * Y_{Twitter} + ROI_{Email} * Y_{Email} \leq 10$$

Where:

- ROI_x – Return of Investment from medium x
- Y_x – Marketing budget spent on medium x

Constraints:

1. The amount invested in print and TV should be no more than the amount spent on Facebook and Email

- $Y_{Print} + Y_{TV} \leq Y_{Facebook} + Y_{Email}$

2. The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords

- $Y_{Facebook} + Y_{LinkedIn} + Y_{Instagram} + Y_{Snapchat} + Y_{Twitter} \geq 2 * (Y_{SEO} + Y_{AdWords})$

3. For each platform, the amount invested should be no more than \$3M

- $0 \leq Y_{Print}, Y_{TV}, Y_{SEO}, Y_{AdWords}, Y_{Facebook}, Y_{LinkedIn}, Y_{Instagram}, Y_{Snapchat}, Y_{Twitter}, Y_{Email} \leq 3$

Code Snippet: The below code is run using Gurobi with the aim of maximizing the objective function keeping the constraints in mind

```
import pandas as pd
import gurobipy as gp
from gurobipy import GRB

# Read ROI data from CSV file and set the 'Platform' column as the index
roi_data2 = pd.read_csv('ROI_data.csv', index_col='Platform')
# Extract platform names and ROI values for the second firm
platforms2 = roi_data2.columns.tolist()
roi_values2 = roi_data2.iloc[1].tolist()
model2 = gp.Model("MarketingBudget2") # Give a unique name to the model
# Decision variables
x2 = {} # Amount to invest in each platform
for i, platform in enumerate(platforms2):
    x2[platform] = model2.addVar(lb=0, ub=3, vtype=GRB.CONTINUOUS, name=platform)
# Maximize ROI for the second scenario
model2.setObjective(gp.quicksum(roi_values2[i] * x2[platform] for i, platform in enumerate(platforms2)), GRB.MAXIMIZE)
# Constraints
model2.addConstr(x2['Print'] + x2['TV'] <= x2['Facebook'] + x2['Email'])
model2.addConstr(gp.quicksum(x2[p] for p in ['Facebook', 'LinkedIn', 'Instagram', 'Snapchat', 'Twitter']) >=
                 2 * gp.quicksum(x2[p] for p in ['SEO', 'AdWords']))
model2.addConstr(gp.quicksum(x2[platform] for platform in platforms2) <= 10)
# Budget constraint
budget = 10_000_000 # $10M
model2.addConstr(gp.quicksum(x2[platform] for platform in platforms2) <= budget)
# Optimize the model
model2.optimize()
if model2.status == GRB.OPTIMAL:
    print("Optimal budget allocation for all platforms (Code 2 - Second Firm):")
    for platform in platforms2:
        print(f"{platform}: {x2[platform].x:.2f} million")
    objective_value_decimal = round(model2.objVal, 14) # Format with 14 decimal places
    print(f"Optimal objective: {objective_value_decimal:.14f}")
else:
    print("No optimal solution found (Code 2 - Second Firm).")
```

Code Snippet 2

Results: Based on the results we should be spending \$3 million each on Print, Facebook and LinkedIn and \$1 million on in order to get a maximum return on investment of \$456,000 i.e same as the one with earlier ROI data.

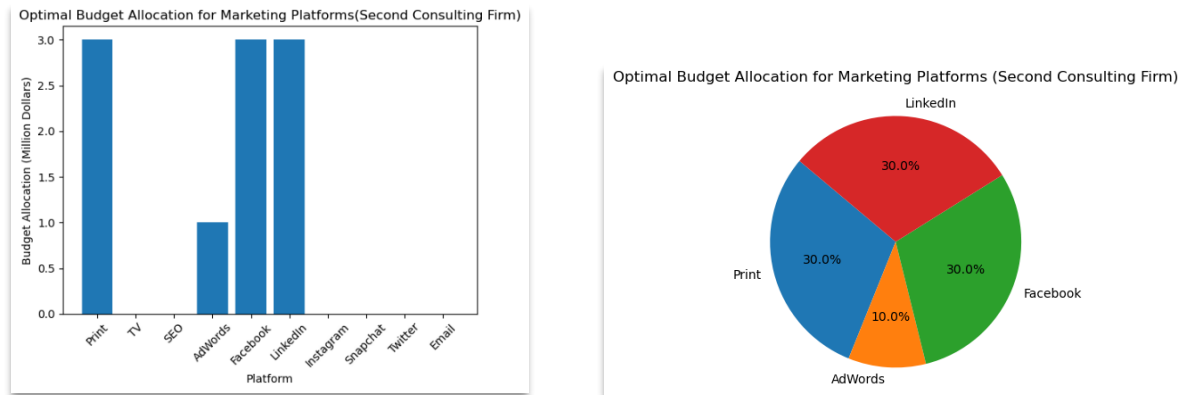


Fig 2. Optimal Budget Allocation for Marketing Platforms (Second Consulting Firm)

Problem Statement 3: We want to compare the budget allocations between the ROI estimates given by both consulting companies and see –

1. Are the allocations the same?
2. If we assume the first ROI data is correct and if we were to use the second allocation, how much lower would the objective be relative to the optimal objective, the one that uses the first ROI data and the first allocation?
3. Assuming the second ROI data is correct, if we used the first allocation how much lower would the objective be relative to the optimal objective?
4. Is the constraint “For each platform, the amount invested should be no more than \$3M” useful

Solution :

1. The optimization results for both the ROI scenario tells us that the allocation for both cases is not the same in general except for the AdWords. In the first case of ROI data TV, Instagram, and Emails should be getting a \$3M budget each while AdWords should be getting \$1M and in the second case of ROI data Print, Facebook and LinkedIn should be getting \$3M each as the budget and the remaining \$1M should go to AdWords for the maximum ROI in total. In both cases, the total return of investment is same i.e \$456,000 M hence the optimal value does not change.

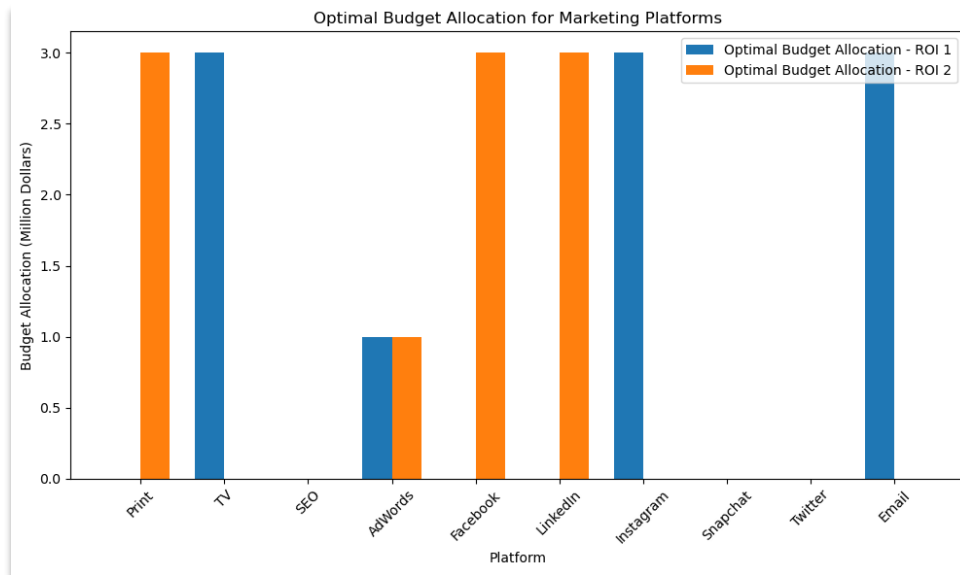


Fig 3. Optimal Budget Allocation for Marketing Platforms for ROI 1 and ROI 2

- If we assume the first ROI data is correct, the return obtained will be \$252,000 which is lower than the return obtained by 2nd consulting firm's ROI by \$204,000

```
roi1_true = int(np.dot(model1.Obj, model2.x) * 1000000)
print(f'Total return obtained will be: {roi1_true}; \n\
This return is lower than the return if 2nd firm\'s ROI by: {int(model2.objVal*1000000 - roi1_true)}')
```

Code Snippet 3

- If we assume the second ROI data is correct, the return obtained will be \$264,000 which is lower than the return obtained by 1st consulting firm's ROI by \$192,000

```
roi2_true = int(np.dot(model2.Obj, model1.x) * 1000000)
print(f'Total return obtained will be: {roi2_true}; \n\
This return is lower than the return if 1st firm\'s ROI by: {int(model1.objVal*1000000 - roi2_true)}')
```

Code Snippet 4

- Optimisation of the marketing channel investment allocation if the constraint on the Upper bound (\$3M) is removed:

First of all, the constraint decreases the objective hence we can say that the constraint is binding. Although it diversifies the risk on the other hand it reduces objective function. Thus, with regards to our aim of maximizing ROI this constraint is not useful.

	Platforms	Allocation1	Allocation2
0	Print	0.0	5.0
1	TV	5.0	0.0
2	SEO	0.0	0.0
3	AdWords	0.0	0.0
4	Facebook	0.0	5.0
5	LinkedIn	0.0	0.0
6	Instagram	0.0	0.0
7	Snapchat	0.0	0.0
8	Twitter	0.0	0.0
9	Email	5.0	0.0

Allocation Table after constrains removal

Above are the allocations for the problem when the upper bound on the marketing channel is removed. Allocations for the highest ROI marketing channel have been considered by the model.

```

1 print(f'Total ROI with first firms allocation and without the third constraint would be: ${model3.objVal}*1000000}\n\
2 Total ROI with first firms allocation and without the third constraint would be: ${model4.objVal}*1000000}')
3
4 print(f'\n\nThe ROIs are greater than the ROIs with the third constraint by:\n\
5 First: ${round((model3.objVal)*1000000-(model1.objVal)*1000000,2)}\n\
6 Second: ${round((model4.objVal)*1000000-(model2.objVal)*1000000,2)}')
```

Code Snippet 5

If the constraint is removed, there is an increase in the total ROI for the company. Therefore, the third constraint is **not useful** as without it there would be an increase in ROI by \$9000.

Problem Statement 4: Analysing the Impact of ROI data changes on Optimal Advertising Allocation based on a set of Return on Investment (ROI) data, we have refined our advertising allocation strategy. We must now understand how changes in ROI data may alter our ideal allocation approach. This research will assist us in determining the range of values within which the ROI of each advertising medium can increase or decrease while retaining the same optimal allocation.

Now, we will be determining how changes in ROI data affect our current ideal advertising allocation, which was calculated using step 3 ROI data.

Solution:

Code Snippet: This code in Gurobi aims in resulting a DataFrame q6 that contains four columns: 'LowerBound_ROI1', 'UpperBound_ROI1', 'LowerBound_ROI2', and 'UpperBound_ROI2', with values corresponding to the lower and upper bounds of ROI data for each platform from the respective x1 and x2 sets.

```

q6 = pd.DataFrame(index=platforms1)
q6['LowerBound_ROI1'] = [x1[platform].SAObjLow for platform in platforms1]
q6['UpperBound_ROI1'] = [x1[platform].SAObjUp for platform in platforms1]
q6['LowerBound_ROI2'] = [x2[platform].SAObjLow for platform in platforms2]
q6['UpperBound_ROI2'] = [x2[platform].SAObjUp for platform in platforms2]
q6
```

Code Snippet 6

Result: The allocation in the table below can change anywhere between the lower bound and upper bound without a change in the optimal ROI that was earlier calculated i.e. \$456,00.

	LowerBound_ROI1	UpperBound_ROI1	LowerBound_ROI2	UpperBound_ROI2
Print	-inf	0.049	0.039	0.052
TV	0.039	0.062	-inf	0.049
SEO	-inf	0.039	-inf	0.039
AdWords	0.033	0.046	0.0375	0.046
Facebook	-inf	0.029	0.029	inf
LinkedIn	-inf	0.039	0.039	inf
Instagram	0.039	inf	-inf	0.039
Snapchat	-inf	0.039	-inf	0.039
Twitter	-inf	0.039	-inf	0.039
Email	0.029	inf	-inf	0.02

Problem Statement 5: We have permission to reinvest half of the returns. What is the optimal allocation of budget each month given that the three constraints are still in place.

Solution:

The optimization model was run for each month, considering the monthly ROI and the set constraints. Using the provided data for each month, we estimate the optimal allocation across all marketing medium for each month. Below table represents the results which offer insights into how budgetary allocations shift based on part of the ROI reinvested into the budget.

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Returns	Budgets
Month												
January	3.000000	0.000000	0.000000	1.333333	0.000000	0.000000	2.666667	0.000000	0.000000	3.000000	0.373000	10.000000
February	3.000000	0.000000	0.000000	2.395500	3.000000	0.000000	0.000000	0.000000	1.791000	0.000000	0.406296	10.186500
March	0.000000	0.000000	0.000000	3.000000	0.000000	3.000000	1.203148	0.000000	3.000000	0.000000	0.407516	10.203148
April	0.000000	0.000000	0.000000	3.000000	0.000000	3.000000	3.000000	0.000000	1.203758	0.000000	0.400335	10.203758
May	1.200168	0.000000	0.000000	0.000000	0.000000	0.000000	3.000000	0.000000	3.000000	3.000000	0.411006	10.200168
June	3.000000	0.000000	0.000000	0.000000	0.000000	0.000000	3.000000	0.000000	1.205503	3.000000	0.423809	10.205503
July	0.000000	0.000000	0.000000	3.000000	1.211905	0.000000	3.000000	0.000000	3.000000	0.000000	0.428264	10.211905
August	2.714132	0.000000	0.000000	1.500000	0.000000	0.000000	0.000000	0.000000	3.000000	3.000000	0.437994	10.214132
September	0.609498	0.000000	0.000000	3.000000	0.000000	3.000000	0.000000	0.000000	3.000000	0.609498	0.402712	10.218997
October	0.000000	0.000000	0.000000	3.000000	0.000000	3.000000	3.000000	0.000000	0.000000	1.201356	0.371443	10.201356
November	3.000000	0.000000	0.000000	1.185722	0.000000	0.000000	3.000000	0.000000	0.000000	3.000000	0.441615	10.185722
December	3.000000	2.110404	0.000000	0.000000	3.000000	0.000000	0.000000	0.000000	0.000000	2.110404	0.432501	10.220807

The monthly allocation dynamics highlight the fluctuating significance of different marketing platforms. January saw maximum investments in Print and Email, each at \$3M, yielding a return of about \$0.373M. As the year progressed, platforms like Print, Facebook, and AdWords consistently received significant portions of the budget, as seen in months like February, March, and April, with returns ranging from \$0.400M to \$0.407M. The mid-year months, such as May through August, underscored the prominence of platforms like Instagram, Twitter, and Email, each frequently receiving the maximum allowable allocation, with returns peaking at approximately \$0.438M in August. Towards the year's end, in months like September through December, there was a diversified spread across platforms like AdWords, LinkedIn, Print, and Email, culminating in December's returns of around \$0.433M.

Despite the ubiquity of digital platforms, traditional media like Print maintained its stronghold, particularly in months like January, February, June, November, and December, suggesting its persistent relevance in the marketing mix. On the digital front, AdWords emerged as a stalwart, receiving substantial allocations throughout the year, indicative of its consistent ROI and importance in driving

online traffic. Social media, especially Instagram and Twitter, stood out, regularly securing full budget allocations, underscoring their pivotal role in contemporary marketing strategies.

Among the digital channels, Email's significance cannot be overstated. Its regular and substantial budget allocations, especially in months like January, May, June, August, November, and December, are a testament to its enduring effectiveness in reaching audiences directly. This reaffirms that despite the plethora of communication channels available today, Email continues to be an indispensable tool in the marketer's arsenal, offering both reach and personalization.

Below graph shows visual representation of monthly allocation across all marketing channels which provides a nice overview of changes across all channels.

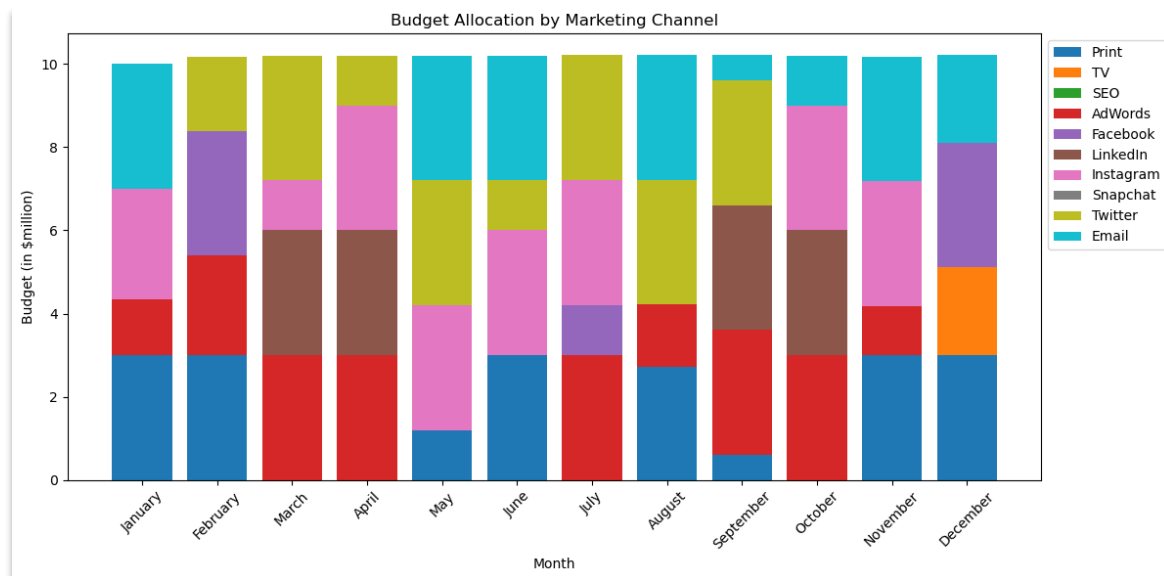


Fig 4. Budget Allocation by marketing Channel

Problem Statement 6: A stable budget is defined as a monthly allocation such that for each platform the monthly change in spend is no more than \$1M. Is the allocation you found stable?

Solution: A stable budget in marketing refers to a consistent and predictable allocation of funds across different marketing channels and activities over a specified period. Instead of drastic fluctuations in spending, a stable budget maintains a relatively uniform distribution, with minor adjustments based on performance, market conditions, or strategic changes.

Having a stable budget provides a foundation upon which strategies can be built, executed, and evaluated effectively. It strikes a balance between consistency and the agility to respond to changing market conditions.

Code Snippet: We set the stability threshold to \$1 million

```
# Create a copy of the df_results DataFrame to store stability information
stability_df = df_results.copy()

# Calculate monthly changes in spend for each platform
budget_changes = df_results[marketing_channels].diff()

# Define a stability threshold of $1 million
stability_threshold = 1

# Check if the budget changes exceed the stability threshold
is_stable = (budget_changes.abs() <= stability_threshold)

# Replace True with 1 and False with 0 in the is_stable DataFrame
is_stable = is_stable.astype(int)

# Create new column names for the stability information
stability_column_names = [f"{channel}_S" for channel in marketing_channels]

# Add the stability information to the stability_df DataFrame
stability_df[stability_column_names] = is_stable

# Display the resulting DataFrame
stability_df
```

Code Snippet 7

Print_S	TV_S	SEO_S	AdWords_S	Facebook_S	LinkedIn_S	Instagram_S	Snapchat_S	Twitter_S	Email_S
0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	1	0	1	0	0
0	1	1	1	0	0	0	1	0	1
1	1	1	1	1	1	0	1	0	1
0	1	1	0	1	0	1	1	0	0
0	1	1	1	1	1	1	1	0	1
0	1	1	0	0	1	1	1	0	0
0	1	1	0	0	1	0	1	1	0
0	1	1	0	1	0	1	1	1	0
1	1	1	1	1	1	0	1	0	1
0	1	1	0	1	0	1	1	1	0
1	0	1	0	0	1	0	1	1	1

In the above table, 1 represents that the allocation for the current month is stable while 0 represents that the allocation is unstable. The table shows a mixed bag of 0 and 1 which shows that not all our allocations are stable. Thus, we seem to have an **unstable budget**. This can have various implications in our marketing campaign. Sudden changes in budget can lead to halting or altering campaigns midway. This inconsistency can confuse customers and weaken brand recall and recognition. Fluctuating budgets prevent campaigns from gaining momentum or achieving their long-term objectives. Starting and stopping campaigns or constantly changing directions can lead to wasted resources, both in terms of money and time. Unpredictable budgets can strain relationships with vendors, agencies, and partners. They may become hesitant to offer favourable terms or even collaborate if they fear sudden budget cuts or payment delays.

Thus, while adaptability is essential in the ever-changing world of marketing, excessive budgetary instability can hinder effectiveness, strain relationships, and pose significant operational challenges. It's crucial for companies to strike a balance, ensuring that they maintain enough flexibility in their budgets without compromising stability.

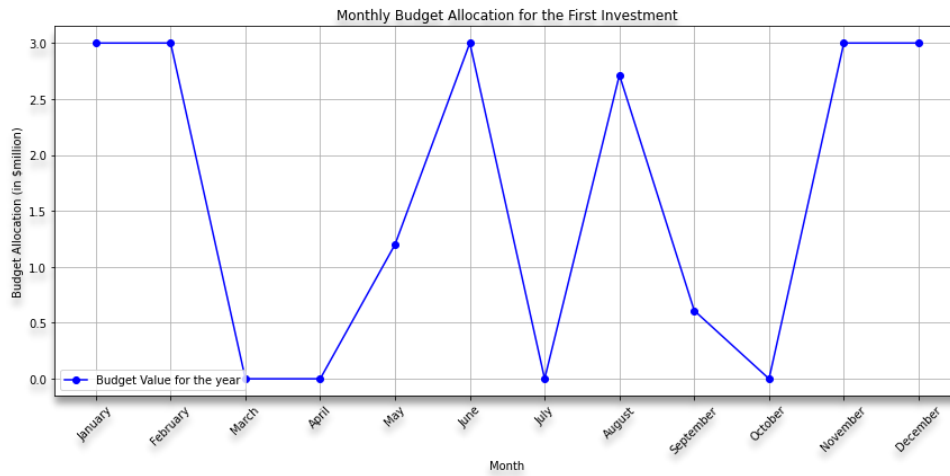


Fig 5. Monthly Budget Allocation for the First Investment

The fluctuation in stability can be clearly seen in the figure above.

Future Stability: To produce a stable budget, we would need to adjust the solution to add a constraint that directly states the change in budget allocations for each marketing medium must not exceed a change of \$1M from month to month. This would likely reduce the final ROI since we are adding more binding constraints.

This stable budget will essentially lay a foundation for more effective marketing operations, better financial management, and ultimately, more successful outcomes in reaching business objectives. It's an enabler that allows the us to function with foresight, efficiency, and creativity.

Is the allocation you found stable?

Given this table, the allocation found is not stable since there are multiple instances across various platforms where the month-to-month change exceeds \$1M

Describe how my might model this?

A more stable budget could be achieved by reducing the individual marketing channel investment, however due to this the overall ROI for the company may either decrease or remain the same. A more stable investments could be achieved by:

1. Introduce Change Variables:

- For each platform i and each month t (except the first month), we introduce variables $\Delta x_{i,t}$, which represent the change in allocation from the previous month to the current month.

$$\Delta x_{i,t} = x_{i,t} - x_{i,t-1}, \quad \text{for } i = 0, 1, 2, \dots, 9 \text{ and } t = 2, 3, 4, \dots, 12$$

Here, $x_{i,t}$ represents the monthly budget allocation for platform i in month t , and $x_{i,t-1}$ represents the allocation for the same platform in the previous month $t - 1$.

2. Constraints on Change Amount:

- We add constraints to bound the change variables $\Delta x_{i,t}$ to ensure that the absolute month-to-month change in allocation does not exceed \$1 million for any platform i .

$$\Delta x_{i,t} \leq 1, \quad \text{for } i = 0, 1, 2, \dots, 9 \text{ and } t = 2, 3, 4, \dots, 12$$

Here, $\Delta x_{i,t}$ represents the change in allocation from the previous month $t - 1$ to the current month t for platform i , and we ensure that it is limited to be less than or equal to \$1 million.

Conclusion:

In an effort to optimize marketing spend, a comprehensive analysis was conducted based on ROI estimates provided for various marketing mediums. The study aimed to determine the best allocation of a \$10M budget across these mediums while adhering to specific constraints. The initial optimization results highlighted the fluctuating significance of different channels throughout the year, with platforms like Print, AdWords, Instagram, and Email frequently securing substantial allocations. The importance of stability in budgeting was also explored, with a defined stable budget being one where the monthly allocation fluctuates by no more than \$1M. While the current allocations showed some instability, the implications of such variability were discussed, emphasizing the potential challenges and disruptions it can introduce. For future endeavors, the goal is to strike a balance between adaptability and stability, ensuring that marketing efforts are both responsive and consistent.