

**FINM 37400 Final Exam Solutions**

Fixed Income

February 5, 2025

**Name:** \_\_\_\_\_**UChicago ID:** \_\_\_\_\_

- This exam is **closed book** and **closed notes**.
- You are **not** allowed any electronics or calculator.

Section	Questions	Points per Question	Points Awarded	Points Possible
1	15	2		30
2	5	2		10
3	15	3		45
4	5	3		15
<b>Total</b>	40	-		<b>100</b>

## 1 True or False

1. The duration of a portfolio of bonds is the average of the durations of each bond in the portfolio.

☐ True  
☒ False

This unconditional statement is false, as the duration of a portfolio is the **weighted** average of the durations of each component of the portfolio.

2. For any outstanding U.S. treasury bond, the dirty price is greater than or equal to the clean price.

☒ True  
☐ False

Dirty price is the clean price plus the accrued interest. For a treasury bond with a non-negative coupon rate, the accrued interest must be non-negative. The Treasury has never issued a bond with a negative coupon rate. Even if one interpreted the question broadly about TIPS, those coupon rates are fixed and all non-negative.

3. We found the swap rate empirically has a small spread to account for the counterparty risk of lending (or borrowing) the notional amount.

☐ True  
☒ False

The swap never exchanges the notional. While there may be credit risk regarding the coupon or the market value of the swap, there is no credit risk on the notional.

4. We found the swap rate empirically has a small spread to account for the counterparty risk of lending (or borrowing) the notional amount.

☐ True  
☒ False

The swap never exchanges the notional. While there may be credit risk regarding the coupon or the market value of the swap, there is no credit risk on the notional.

5. We found that the first principal component, driving most variation in yields, is approximately a long level factor and short slope factor.

☐ True  
☒ False

It is not a combination of level and slope; those are the first two factors. The first principal component was over 90% correlated to a "level" factor, which simply averaged yields at different maturities.

6. Hedging a high-duration portfolio requires more frequent hedging adjustments.

✓ True

x False

Duration changes as rates change, and this duration sensitivity is higher for higher-duration bonds. Higher duration comes with higher convexity. (Even if convexity is hedged, that is only at a point.)

7. In Homework 3, we found that a regression did NOT give a good approximation of the average duration over a time series of two treasury bonds .

x True

✓ False

Regression was a good estimate of the average interest-rate sensitivity (duration) across time and rates. The simple regression we estimated would be a bad estimate of the duration at any given point in time.

8. If the spot discount rate curve is monotonically upward sloping (as a function of maturity) then we know the forward rates are higher than the spot rates at every maturity.

✓ True

x False

The forward curve relative to the spot curve indicates whether the spot curve will slope up or down.

9. A downward-sloping yield curve with zero term premium indicates that 1-year yields are expected to decrease over time.

✓ True

x False

The downward sloping yield curve indicates a downward sloping spot curve, which indicates forward rates are lower than today's 1-year yield. This may be due to rates being forecast to decrease or due to risk premium, in the form of term premium. Ruling out the latter means rates are indeed forecast to go down.

10. Duration is  $D$  in the following approximation,  $dP \approx -Ddr$  where  $dr$  is a parallel change in spot discount rates.

x True

✓ False

Duration approximation is for the percentage change in prices. Alternatively, this approximation would make sense if we used dollar duration rather than duration.

11. Before the 2008, the swap rate was typically higher than the maturity-matched treasury yield, but after 2008, the swap rate has been lower.

✓ True

x False

See the case study combined with the more recent data on the swap spread.

12. A forward rate quoted at  $t$  and in effect between  $T_1$  and  $T_2$  is determined by the time- $t$  yield curve and time  $T_1$  yield curve.

☐ True

☒ False

No, it is determined completely by the time- $t$  market data.

13. A Floating-Rate-Note has positive convexity which decreases toward zero as the reset date approaches.

☒ True

☐ False

Yes, the FRN acts as a very short fixed-coupon bond. Accordingly, it has small duration and convexity which approach zero as the reset date draws near.

14. The yield curve can be used to price any one-time cash-flow from the Treasury, whereas the spot discount rate can be used to price any one-time cash-flow from any source.

☐ True

☒ False

The yield can price any bundle of cashflows with the structure of the treasury (semiannual coupon.) The spot discount rate can be used to price any cashflow for a given maturity from the same source. (A treasury-generated spot discount rate cannot be used to price a cashflow with some other credit risk.)

15. If the yield curve is downward sloping, then the rate on a 2-year forward loan to the Treasury (locked in today) must be lower than the rate on an immediate two-year loan to the Treasury.

☒ True

☐ False

Another question on the relationship between the yield curve, spot curve, and forward curve. The decreasing yield curve implies a decreasing spot curve, which implies forward rates are lower than current spot rates.

## 2 Multiple Choice

Circle the bullet point of exactly ONE answer.

1. What causes Floating Rate Notes (FRNs) to be priced at par on reset dates?
  - ☐ It has no accrued interest.
  - ☒ The floating rate index represents the discount rate.
  - ☐ The current floating rate value equals the current discount rate.
  - ☐ The spot curve is completely flat.

This is worded somewhat ambiguously to where half credit for ticking the third option.

2. A parallel change in the coupon bond yield curve always implies:
  - ☐ A parallel change in the spot curve.
  - ☐ A parallel change in the forward curve.
  - ☐ A non-parallel change in the spot curve.
  - ☒ None of the above.

Typically a shift in YTM will cause a non-parallel shift in spot rates (and forward rates.) But for a flat yield curve, they'll all be parallel.

3. The standard convexity measure is defined to be...
  - ☐ change in duration for a change in rates.
  - ☐ second derivative of price to rates.
  - ☒ sensitivity of prices to a second-order change in rates, relative to the current price.
  - ☐ curvature of the price-yield relationship.

The second option are false because convexity involves a scaled second derivative (scaled by price.) It's not just the second derivative.

4. Using OLS to estimate spot rates often leads to forward rates which are inconsistent with...
  - ☐ the shape of the spot curve.
  - ☐ the shape of the yield curve.
  - ☒ other forward rates at small changes in maturity.
  - ☐ the pricing data of the reference bonds.

We saw that OLS-generated curves are extremely consistent with the in-sample data, and exactly so for an exactly-identified system. The problem is the variation in forward rates for small changes in maturity.

5. In the homework, we compared forward rates at a given time to the evolution of the one-year spot rates. We found that the forward rates...
  - ☒ overestimated the future spot rates.
  - ☐ underestimated the future spot rates.
  - ☐ were identical to the future spot rates.

See Homework 4.

### 3 Check All That Apply (if any)

Check the tick box for every true answer (and leave false answers unchecked.)

1. The return on a fixed coupon bond is guaranteed to equal the YTM (at purchase) on the bond if the following conditions all hold:

- ✓ The bond is held to maturity.
- ✓ The coupons are reinvested at the original YTM.
- ✗ The coupon rate equals the YTM.
- ✗ The YTM remains the same until expiry.

2. A flat **spot** curve implies:

- ✓ A flat yield curve.
- ✗ An increasing yield curve
- ✓ A flat forward curve.
- ✗ A flat discount factor curve.

Note that the discount factor varies with maturity due to the compounding of the discount rate, so it would only be flat if the spot rates were zero.

3. A flat **yield** curve implies...

- ✓ a flat spot curve.
- ✗ an increasing spot curve.
- ✓ a flat forward curve.
- ✗ a flat discount factor curve.

Note that the discount factor varies with maturity due to the compounding of the discount rate, so it would only be flat if the spot rates were zero.

4. Zero coupon bonds can be used to replicate:

- ✓ Fixed coupon bonds
- ✓ Forward Rate Agreements (FRAs)
- ✗ Swaps
- ✗ Floating Rate Notes (FRNs)

Static replication of fixed-coupon and FRAs. Would require dynamic, instantaneous, zero-transaction-cost) rebalancing to replicate FRNs and swaps.)

5. Two coupon bonds with the same maturity:

- ✓ can have the same YTM
- ✓ can have different YTMs

- ☐ must have the same convexity
- ☐ must have the same duration

6. The "swap rate"...

- ☒ is what the fixed leg of the swap pays (or receives.)
- ☐ equals the YTM of a treasury with the same maturity.
- ☐ sets the value of a new swap to 100.
- ☒ sets the value to par for a fixed coupon bond with coupon equal to the swap rate.
- ☐ equals the floating rate of the swap on swap (reset) dates.

Regarding the third statement, the value of a new swap is not 100, it is 0.

7. An **overestimation** of the entire spot discount curve (every point of the curve) will necessarily lead to **overestimation** of the...

- ☐ spot discount factors.
- ☒ forward rates.
- ☒ yields-to-maturity.
- ☐ price of a fixed-coupon bond.
- ☒ value of a paying-fixed swap.

8. A 30-year, 5% fixed-coupon, treasury bond's duration will necessarily **decrease** if, holding everything else constant,

- ☒ a (non-coupon-paying) day goes by.
- ☐ a coupon-paying day goes by.
- ☒ there is a parallel upward shift in the spot discount rate curve.
- ☐ only the 1-year spot discount rate increases.
- ☒ just before issuing the bond, the treasury decides to set the coupon higher, at a fixed 6%.

9. At any time,  $t$ , the fixed leg of a swap can be valued by discounting payments with the swap rate from...

- ☐ the date the swap was initialized.
- ☒ the current date.

10. Which of the following have positive duration:

- ☒ Receiving fixed in a swap.
- ☐ Selling a Floating Rate Note.
- ☐ Paying fixed in a Forward Rate Agreement.
- ☐ Selling a zero coupon bond.

11. A floating rate note necessarily has a value equal to par if the following statements are true.

- ✓ Floating rate equals the discount rate.
  - ✓ Valuation is immediately following a coupon payment
  - ✓ Valuation is immediately following a reset.
  - ✗ The floating coupon includes a fixed positive spread.
12. The following are advantages of using curve-fitting to model the spot discount rates relative to OLS. Curve-fitting...
- ✓ improves statistical power relative to OLS.
  - ✗ works with non-square cashflow matrices, unlike OLS.
  - ✗ works for cases where there are multiple gaps in maturity schedule, unlike OLS.
  - ✗ is more flexible in possible yield-curve shapes than OLS.
13. The long-short treasury convergence trade as seen in the case study had the following advantages.
- ✓ Convergence was guaranteed.
  - ✓ Duration was minimal.
  - ✓ Expected returns were high.
  - ✓ Historical context indicated the spread was typically small and converged quickly.
  - ✗ Short-term losses were limited by the trade construction.
14. Suppose we are long a treasury bond via repo. If the haircut goes from 2% to 5%,
- ✓ leverage goes down.
  - ✗ the repo rate increases.
  - ✓ expected return goes down.
- Decreasing leverage leads to decreasing expected return on the capital supporting the trade.
15. Suppose you download data for all outstanding U.S. treasury issues as of today. In selecting a subset of data to use for a pure bootstrapping of the yield curve, you must...
- ✓ keep only one treasury corresponding to any particular maturity date.
  - ✗ eliminate any treasury which is not precisely at an ex-coupon date.
  - ✓ eliminate any dates where multiple treasuries pay a coupon, yet none mature.
  - ✗ eliminate any dates where no treasury is maturing, and keep the securities with missing date columns.
  - ✓ eliminate any dates where no treasury is maturing, and eliminate the securities with missing date columns.



	coupon rate	price	YTM	swap rate	spread
Nov 2008	4.50%	105.21	4.19%	4.26%	0.06%
May 2009		102.31	4.36%	4.08%	-0.28%

Table 1: Scenario: Market according to the case.

## 4 Swap-Spread Trade

**Check the tick box for every true answer (and leave false answers unchecked.)**

In the homework, we analyzed a swap-spread trade with the following market data.

- Select the true statements regarding the Nov 2008 situation.
  - ✓ Coupon rates suggest positive cashflows for going long the treasury and paying-fixed in the swap at equal notionals.
  - ✗ The swap spread in Nov 2008 suggests going long the treasury and paying-fixed in the swap.
  - ✓ The hedging of dollar-duration meant that the notional was smaller in magnitude for the treasury position than the swap position.

The swap spread was initially slightly positive. If one thought the spread would go to zero, should be short the treasury and receiving-fixed. If one thought the spread would go up to historic values, should do the opposite. Thus, the second statement is ambiguous.

- Select the true statements regarding the evolution of the trade from Nov 2008 to May 2009.
  - ✗ Over these six months, both the treasury and receiving-fixed leg of the swap decreased in value.
  - ✗ The modified duration estimates were poor approximations for PnL, which is why the realized PnL was substantially different from zero, notwithstanding the hedge..
  - ✓ The revaluation of the positions was a larger impact on PnL than was the six-month cashflow.
  - ✓ The cashflow was a constant amount every six months, throughout the 30 years.
- Which of the following were substantial risks to the implementation of the trade we analyzed?
  - ✗ Monetary policy changing the market-wide level of interest rates.
  - ✗ Convexity.
  - ✗ Cashflow uncertainty.
  - ✓ Changes in the swap spread .
  - ✗ Changes in the repo rate.

- Suppose now that the swap-spread trade was implemented on a bond that was at par in Nov 2008, as shown in the following table.

Which of the following statements are necessarily true? In this version of the trade...

	coupon rate	price	YTM	swap rate	spread
Nov 2008	4.50%	100.00	4.50%	4.26%	-0.24%
May 2009		102.31	4.36%	4.08%	-0.28%

Table 2: Scenario: Market has the bond at par in Nov 2008.

- ✓ We go long the treasury and receiving-fixed on the swap.
- ✗ We go short the treasury and paying-fixed on the swap.
- ✗ The positioning is more unequal (in notional magnitude), if we once again size to be neutral in dollar duration.

The third statement is a bit ambiguous. The phrasing "more unequal" meant to compare to the version in Table 1, where the bond was not initially at par.

5. Which of the following statements are true about the expected return of the swap-spread trade?

- ✓ Speaking as of Nov 2008, the conditions in Table 2 indicate a higher expected return than the conditions in Table 1.
- ✓ Focusing just on Table 2, the expected return on a swap-spread trade is higher in May 2009 than in Nov 2008.

See the case study and related discussion.