FTCS Solver

number density

time step

$$\left(\frac{\partial n}{\partial t}\right)^{k} = \frac{n^{k+1} - n^{k}}{\Delta t} + O(1)$$

Diffusion coefficient

source term

$$\frac{n_{ij}^{k+1} - n_{ij}^{k}}{\Delta t} = D\left[\frac{n_{ij}^{k} - 2n_{ij}^{k} + n_{i+1,j}^{k}}{\Delta^2 x} + \frac{n_{i,j-1}^{k} - 2n_{i,j}^{k} + n_{i,j+1}^{k}}{\Delta^2 y}\right] + R_{i,j}^{k}$$

Forward time: forward Euler approximation for first order approx time

Central Space: central difference stencil (2nd order accurate in space)

Unsteady Diffusion Equation

$$\frac{\partial n}{\partial t} = \underbrace{-\nabla \cdot (n\vec{v})}_{\text{advection term}} + \underbrace{\nabla \cdot (D \nabla n)}_{\text{diffusion term}} + \underbrace{R}_{\text{source term}}$$

advection term: Movement caused by flow field (fluid carrying particles)

diffusion term: Spread due to random molecular motion

source term: local creation or destruction

Steady Diffusion Equation

1. $\vec{v} = 0$ no flow: removes advection term
2. $R = 0$ no sources: removes the source term
3. $\nabla D = 0$ constant diffusion coefficient

$$\nabla \cdot (D \nabla n) = D \nabla^2 n$$

4. $\frac{\partial n}{\partial t} = 0$ Steady State: no time dependence

$$0 = D \nabla^2 n$$

$D \neq 0 \quad \therefore \quad \nabla^2 n = 0$ ← identical to steady heat equation

Connection to FTCS

Unsteady diffusion equation

$$\frac{\partial n}{\partial t} = D \nabla^2 n + R$$

$$\frac{n_{i,j}^{k+1} - n_{i,j}^{k}}{\Delta t} = D \;\ldots$$

Heat-Sink Cooling for Pulsed Rocket Firing

$$\frac{dT}{dt} = \alpha \nabla^2 T$$

insulating far side wall

$$\frac{dT}{dt} = 0 \quad \text{(zero-derivative Neumann BC)}$$

hot side boundary condition $(x=0)$

$$h_g \left[T_{og} - T(x=0) \right] = K \frac{dT}{dx}$$

h_g: convective heat transfer coefficient

T_{og}: stagnation temperature of hot combustion gasses

K: thermal conductivity of the wall

$$\alpha = \frac{K}{\rho c_p}$$

ρ: density

c_p: specific heat capacity of wall material

When not firing

$$\frac{dT}{dx} = 0$$

When firing

h_g, T_{og} constant

K, α constant throughout the wall

Questions:

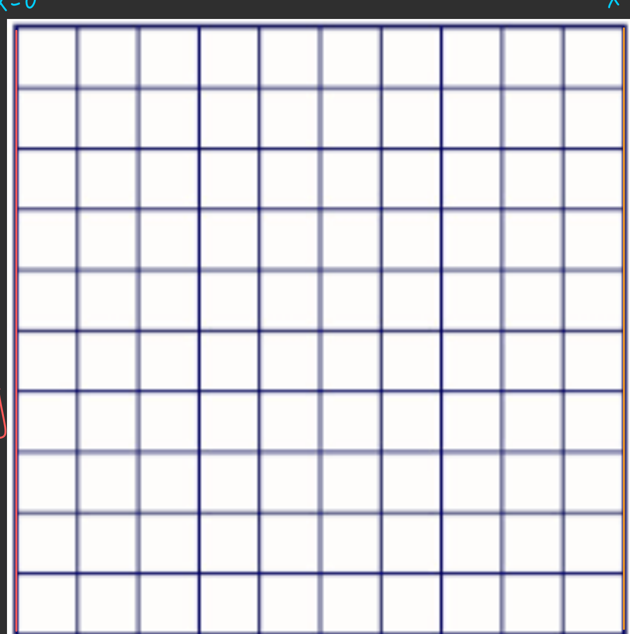
- Does this simulation have several frames like a video?
- What dimensions does the 2D plot have, 1 cm x 1cm?
- 1D to 2D?

Recipe:

1. Special part is finite difference
2. Time part is forward euler method

$x=0$

$x = L = 1\text{cm} = 0.01\text{m}$



$$\frac{\partial T(t,L)}{\partial x} = 0$$

y (m)

Thruster on:

$$\frac{\partial T(t,0)}{\partial x} = \frac{h_g}{K}\left[T_{og} - T(t,0)\right]$$

Thruster off:

$$\frac{\partial T(t,0)}{\partial x} = 0$$

x (m)

Walkthrough

For Laplace label

a

b f d

c

$\Delta x = \Delta y = \Delta h$

- Initial Condition: @ $t=0$ "step 0"
  $T = 300$ K, thruster on, whole grid same
- Step 1: ($dt = 1e{-4}$ s)

Left boundary (if thruster on)

$$T_1 - T_0 = \frac{hg}{K}[2500 - 300]$$

$$T_1 = \frac{hg}{K}[2200] + 300$$

Right boundary

$$T_1 = T_0 = 300$$

Inside (based on node i,j)

$$T_1 = \alpha \cdot dt \cdot \left[\frac{T_{0,b} + T_{0,c} + T_{0,d} + T_{0,a} - 4T_{0,f}}{(\Delta h)^2}\right] + T_0$$

- Step 2: ($dt = 1e{-4}$ s)

(i=0) Left boundary (if thruster on)

$$T_2 - T_1 = \frac{hg}{K}[2500 - T_{1,x=0}]$$

$$T_2 = \frac{hg}{K}[2500 - T_{1,x=0}] + T_{1,x=0}$$

Right boundary (i=10)

$$T_2 = T_1 = 300$$

Inside (based on node i,j)

$$T_2 = \alpha \cdot dt \cdot \left[\frac{T_{1,b} + T_{1,c} + T_{1,d} + T_{1,a} - 4T_{1,f}}{(\Delta h)^2}\right] + T_1$$