Population Distribution of Great White Sharks Over Time:
Math 42 Final Project Report by
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I. Introduction

Great white sharks are carnivorous fish growing up to 21 feet long and weighing up to 4,500 pounds (National Geographic, 2025). Infamous from their feature in the movie *Jaws*, these sharks are the world's largest predatory fish and can move through water at 35 miles per hour, making them fearsome hunters. Their habitat preferences change as they age and they are found across the planet, although they prefer warm and tropical waters near a coastline. Despite roaming vast oceanic ranges and earning a reputation as the ocean's apex predator, the survival of the great white shark depends on a fragile age structure shaped by survival and reproduction at each life stage. These animals face significant population pressure due to human activity, late and disjointed sexual maturity, and slow rates of reproduction. Small changes in survival rates have intense, long-term effects for the stability of their population. Understanding how these factors interact over time is ecologically critical.

In this project, we use mathematical modeling to answer the question: "*How are stage-specific survival and reproductive rates in a Lefkovitch matrix able to determine the long term population distribution of female great white sharks?*". Specifically, we modeled the distribution of sharks in their juvenile, mature, and adult stages using a life stage framework represented by a Lefkovitch matrix model. By analyzing the survival and birth rates across the stages, we were able to determine the long-term population distribution to truly understand age distribution patterns and causes of population decline.

II. Background

Great white sharks are classified as a vulnerable population by the IUCN Red List, with overall population rates declining due to overfishing, sport fishing, habitat degradation, and slow reproductive cycles. Since a female great white shark only reproduces every three years and juvenile sharks have very high mortality rates, population recovery is very slow. Pregnancies are very long, and each cycle only yields a few pups. However, migratory behavior and deep-ocean habitats make it extremely challenging to track shark populations and real estimates of birth and death rates by age group are vague approximations built on the shark behavior that is observable by humans. For example, we know that adult female great white sharks produce roughly 2-10 pups every three years; however, such a wide range without any real way of tracking shark reproduction in the wild means that our estimates of birth rates vary extremely. There is also no way of credibly gauging shark age, with length being used as an estimate.

Given that great white sharks cannot be bred in captivity and thus cannot experience population recovery through a hatchery or catch-and-release system, we were interested in seeing which factors would be most effective in maintaining stable populations. Because juveniles have a low survival rate, simply increasing the number of pups being born or released into the wild will not necessarily promote population recovery. By analyzing survival and birth rates across age groups, we hope to give a clearer picture of shark lifespans and survival to understand why populations are so at risk across the Pacific Northwest.

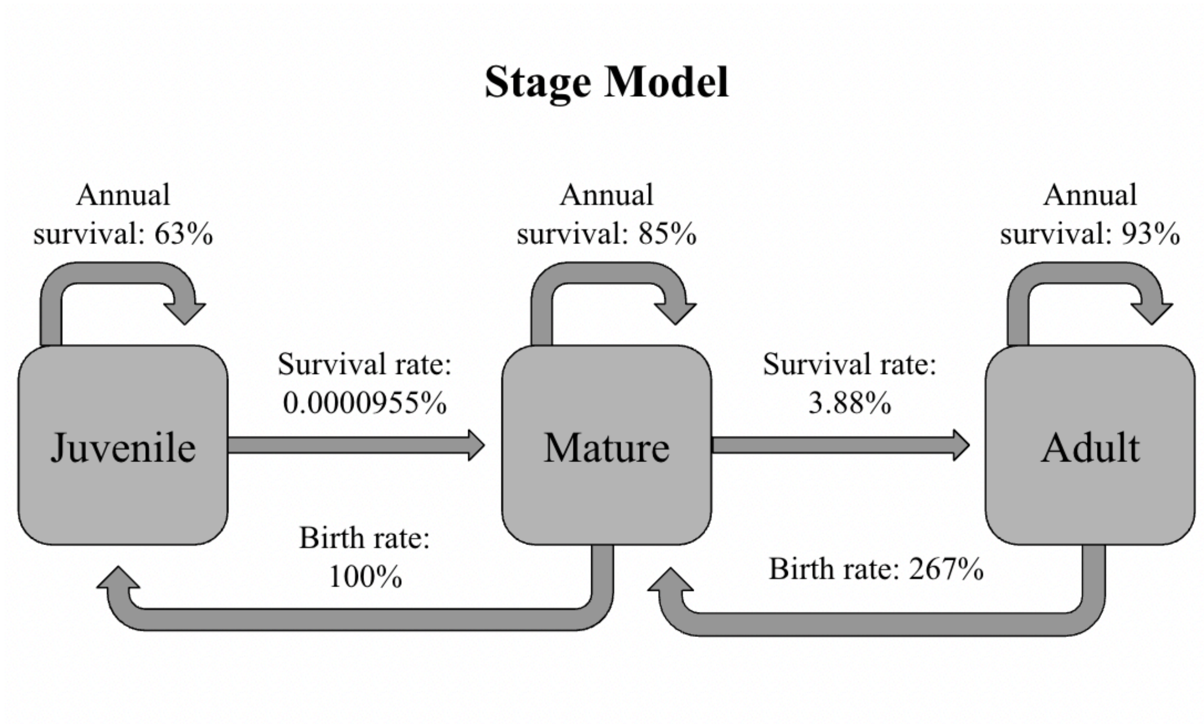
Male and female great white sharks have different ages of sexual maturity, with males beginning to reproduce at around 26 years old and females at around 33 years old. There is not enough data to determine whether or not male and female great white sharks have different lifespans. For the sake of this project, we will be analyzing population levels for female great white sharks since they dictate pup births. Juvenile sharks are considered sharks who have not yet sexually matured; mature sharks span an age range of roughly 33-55 years old while adult sharks make up the 55-70 age bracket. Great white sharks interestingly experience lifespans similar to that of a human, provided that juveniles survive to sexual maturity.

III. Data Sources + Lefkovitch Matrix

Accurate data on great white sharks is unfortunately scarce, making it challenging to find reliable sources. Despite this, several credible references are able to provide valuable insights, including legislation from the Convention on International Trade in Endangered Species of Wild Fauna and Flora (CITES), ScienceDaily, CSIRO (Commonwealth Scientific and Industrial Research Organisation), the British Ecological Society, and NOAA Fisheries. We are additionally estimating cumulative survival rates between age states by raising the annual survival rate for juveniles reaching maturation to the power of 30 and for mature sharks reaching adulthood to the power of 20. From these sources, the following life history parameters were obtained:

- Juvenile annual survival: 63%
- Mature annual survival: 85%
- Adult annual survival: 93%
- Proportion of juveniles who survive to maturation: 0.0000955%
- Proportion of mature sharks surviving to adulthood: 3.88%
- Juvenile annual birth rate: 0%
- Mature estimated annual birth rate: 100%
- Adult estimated annual birth rate: 267%

Our assumptions in creating this model are that birth rates remain constant for each age group. This is not necessarily the case as the number of pups birthed will not consistently be two or eight for each age group, but we are using these numbers as rough estimates of the number of pups produced over time. We are also assuming that there aren't any specific ecological or habitat catastrophes that drastically impact survival rate. In essence, we are assuming that our birth and survival rates remain roughly constant even with population evolution. Additionally, we are assuming fixed time steps for each age class even though the age of sexual maturity for great white sharks can vary by a few years.



The survival rates are annual, whereas the birth rates are annual averages of sharks birthing 2-10 pups every three years (varying by age and length). We are estimating that mature great white sharks at the beginning of their sexual maturity are producing roughly three viable pups every three years, whereas adult sharks are producing eight babies in three years.

Variables:

- $P_{i,j}$ = the probability that an individual of age class i will move to age class j
- $P_{i,i}$ = the probability that an individual of age class i will survive to the next year while remaining in the same age class
- $F_{i,j}$ = birth rate

We chose to use a Lefkovitch Matrix model, which allows us to take into account annual survival rate and survival rate between age classes. Since birth rates are triannual but our time steps between age classes last 30 years, this allows us to rectify our time step issue by using two different survival rates. This problem could also be modeled using a 70x70 Leslie matrix covering annual time steps with consistent survival rates for each age group. This model follows the structure outlined as such:

$$\begin{vmatrix} P_{j,j} & F_{m,m} & F_{a,a} \\ P_{j,m} & P_{m,m} & 0 \\ 0 & P_{m,a} & P_{a,a} \end{vmatrix}$$

With this, the following Lefkovitch Matrix is constructed with birth rates given in the top row, survival probabilities from one stage to the next given on the subdiagonal and annual survival rates given on the diagonal:

$$\begin{vmatrix} 0.63 & 1 & 2.67 \\ 0.000000955 & 0.85 & 0 \\ 0 & 0.0388 & 0.93 \end{vmatrix}$$

This Lefkovitch matrix allows us to analyze the population dynamics and visualize the long-term distribution of the great white sharks.

IV. Numerical Analysis

A. Population distribution at time t + 1

If we started with 100 juvenile sharks, no mature sharks and no adults, we would naturally end up with roughly 63 juveniles, no mature sharks and no adults after one year given that it takes 30ish years for a female shark to mature.

$$\begin{bmatrix} 0.63 & 1 & 2.67 \\ 0.000000955 & 0.85 & 0 \\ 0 & 0.0388 & 0.93 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 63 \\ 0 \\ 0 \end{bmatrix}$$

If we started with an equal distribution of juvenile, mature and adult sharks, we can see that we end up with many more juvenile sharks than we do mature sharks and adults.

$$\begin{bmatrix} 0.63 & 1 & 2.67 \\ 0.000000955 & 0.85 & 0 \\ 0 & 0.0388 & 0.93 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix} \approx \begin{bmatrix} 430 \\ 85 \\ 96 \end{bmatrix}$$

As the time steps continue to increase, we can predict that the number of pups will continue to largely outnumber mature and adult sharks.

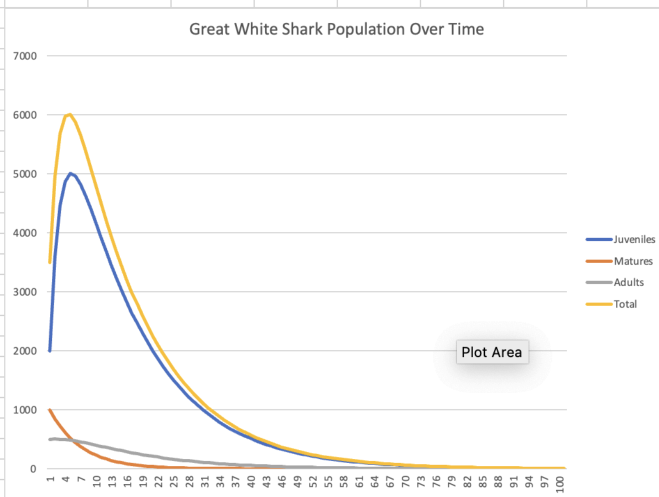
Time specific growth rate is given by $\lambda_t = N_{t+1} / N_t$. The time specific growth rate of the population modeled above is (430 + 85 + 96) / (100 + 100 + 100) = 2.036.

B. Steady State Distribution

By plugging our matrix into Excel, we can calculate growth rates, total population and population distribution over time. We can see that even though there is a brief spike in population initially, population eventually declines rapidly as the number of reproducing mature and adult sharks declines.

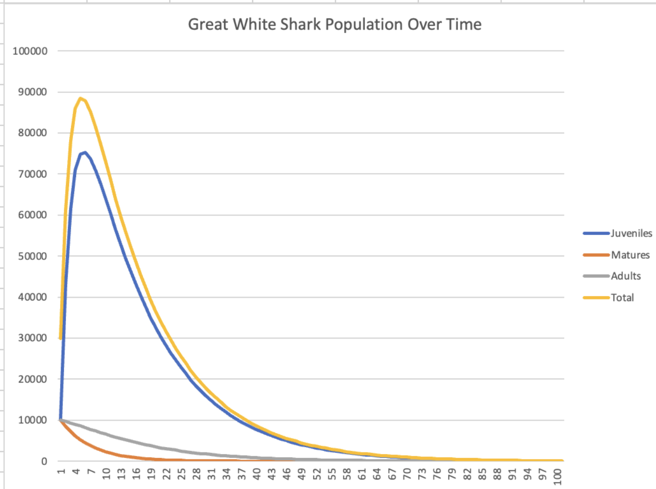
	F(j)	F(m)	F(a)		vector
	0.63	1	2.67		2000
	0.00000095	0.85	0		1000
	0	0.0388	0.93		500

Year	Juveniles	Matures	Adults	Total	λt
0	2000	1000	500	3500	1.4139434
1	3595	850.0019	503.8	4948.8019	1.14856426
2	4459.9979	722.50503	501.514074	5684.017	1.05205897
3	4871.34628	614.133513	494.441284	5979.92108	1.00484884
4	5003.2399	522.018114	483.658774	6008.91679	0.9784135
5	4965.42818	443.72015	470.056963	5879.20529	0.9624648
6	4826.99199	377.166844	454.369317	5658.52815	0.95239109
7	4631.33788	320.596403	437.197539	5389.13182	0.94582969
8	4405.65669	272.511343	419.032851	5097.20089	0.94145899
9	4166.89277	231.638827	400.273992	4798.80559	0.9384944
10	3925.51283	196.896961	381.242399	4503.65219	0.93645074
11	3687.88725	167.366146	362.195033	4217.44843	0.93501932
12	3457.79585	142.264728	343.335187	3943.39577	0.93399983
13	3237.38106	120.928304	324.821596	3683.13096	0.9332604
14	3027.75203	102.792133	306.776102	3437.32027	0.93271332
15	2829.36811	87.3761898	289.29011	3206.03441	0.93229977
16	2642.28269	74.2724493	272.429998	2988.98514	0.93198001
17	2466.29864	63.134092	256.241669	2785.6744	0.93172704
18	2301.06749	53.6663212	240.754355	2595.48817	0.93152239
19	2146.15297	45.618559	225.983804	2417.75533	0.93135331
20	2001.07169	38.777814	211.934937	2251.78444	0.93121097
21	1865.31926	32.9630429	198.604071	2096.88637	0.93108915
22	1738.38705	28.0203586	185.980752	1952.38816	0.93098344
23	1619.77281	23.8189562	174.049289	1817.64105	0.93089067
24	1508.98743	20.2476516	162.790015	1692.02509	0.93080849
25	1405.55907	17.2119374	152.180322	1574.95133	0.93073519
26	1309.03561	14.6314821	142.195523	1465.86262	0.93066942



This remains true regardless of the initial distribution: we see a similar curve when starting with an equal number of juvenile, mature and adult sharks as we do with a higher proportion of initial juveniles.

	F(j)	F(m)	F(a)		vector
	0.63	1	2.67		10000
	0.00000095	0.85	0		10000
	0	0.0388	0.93		10000
Year	Juveniles	Matures	Adults	Total	λt
0	10000	10000	10000	30000	2.03960032
1	43000	8500.0095	9688	61188.0095	1.27511353
2	61456.9695	7225.04893	9339.64037	78021.6588	1.1020956
3	70879.7795	6141.34997	8966.19744	85987.3269	1.02959848
4	74735.3582	5220.21481	8576.848	88532.421	0.99195233
5	75203.6746	4437.25359	8179.01297	87819.9412	0.97021151
6	73653.5333	3771.73699	7778.6475	85203.9178	0.95686896
7	70942.4518	3206.04641	7380.48557	81528.9838	0.94836385
8	67605.6875	2725.20685	6988.24619	77319.1406	0.94280283
9	63975.4073	2316.49005	6604.80698	72896.7043	0.9390995
10	60255.8313	1969.07732	6232.3503	68457.2589	0.93659732
11	56570.6263	1673.77296	5872.48598	64116.8853	0.93488509
12	52992.8051	1422.76076	5526.35435	59941.9202	0.93369888
13	49563.5941	1209.39699	5194.71267	55967.7038	0.93286632
14	46304.3441	1028.03453	4878.00738	52210.386	0.93227349
15	43224.051	873.873336	4576.43461	48674.359	0.93184442
16	40324.1059	742.833398	4289.99047	45356.9297	0.9315281
17	37601.2947	631.446697	4018.51307	42251.2544	0.9312901
18	35049.6922	536.765413	3761.71729	39348.1749	0.93110708
19	32661.8567	456.283899	3519.22358	36637.3642	0.93096311
20	30429.5806	387.872343	3290.58174	34108.0346	0.93084731
21	28344.3613	329.720399	3075.29047	31749.3722	0.93075214
22	26397.6936	280.289267	2872.81329	29550.7961	0.9306724
23	24581.2477	238.270954	2682.59158	27502.1102	0.93060442
24	22886.9765	202.553663	2504.05508	25593.5853	0.93054562
25	21307.1759	172.192356	2336.63031	23815.9986	0.93049414
26	19834.5161	146.383745	2179.74725	22160.6471	0.93044863



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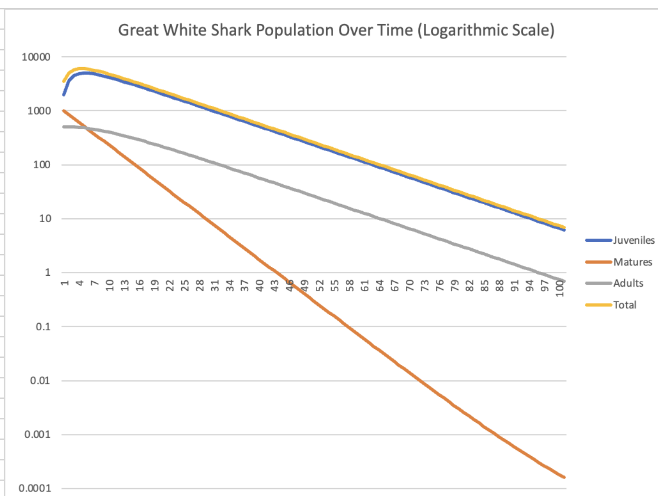
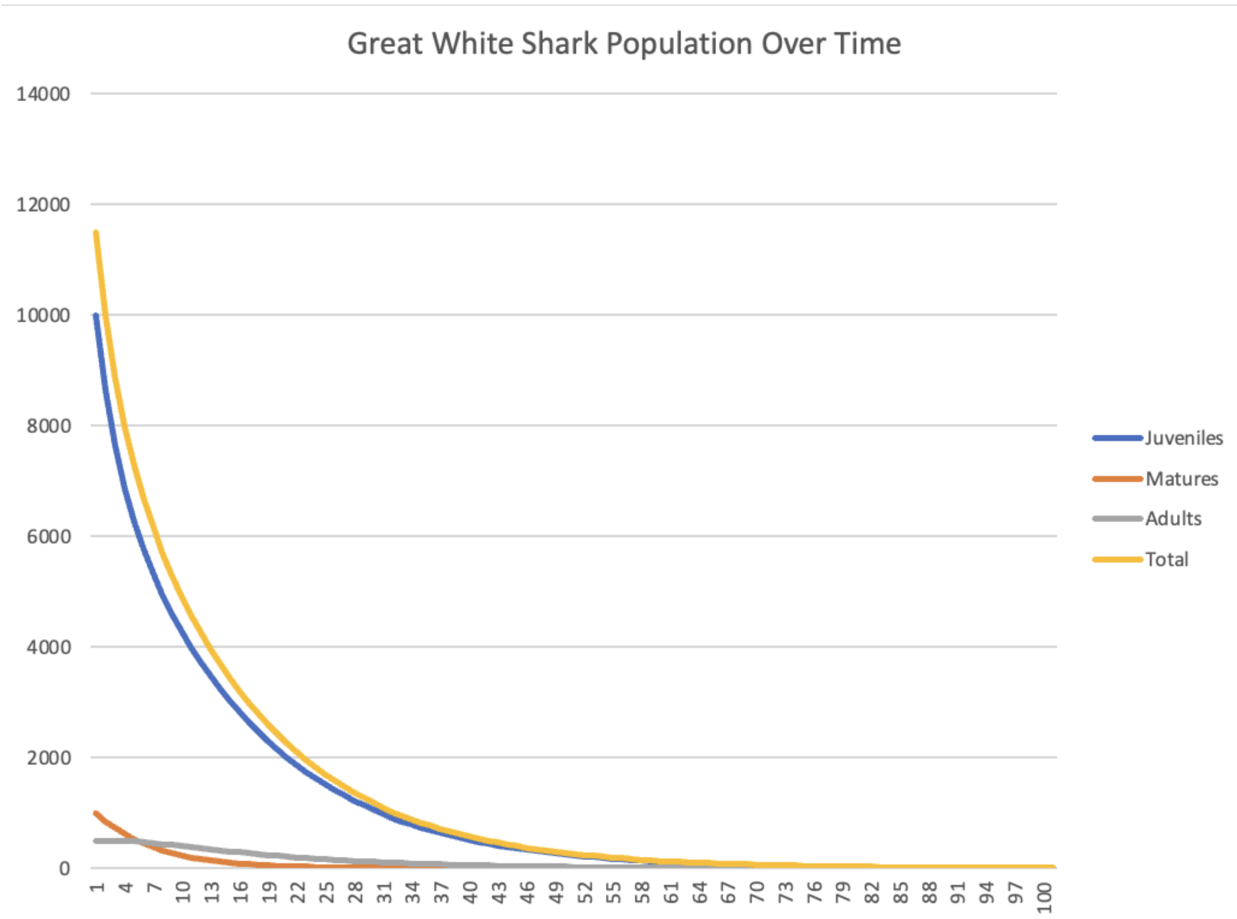


V. Discussion

Given our birth and death rates, it makes quite a lot of sense that we end up with a primarily juvenile population. With a relatively solid birth rate but very low survival from juvenile age to maturity, very few great white sharks reach an age where they can reproduce and grow old. Increasing the number of juveniles does not, however, save the population from reaching extinction over time. This is seen in the visualization below, where we begin with 10000 juveniles instead of 2000.



What is much more effective is to increase the survival rate of juveniles over time. If we begin with the same 2000 juveniles, 1000 mature sharks and 500 adults but adjust the juvenile survival rate from 0.63 to 1, we can see that the population actually increases then stabilizes at around 375000 sharks. In the same vein, if we maintain the yearly survival rate of 0.63 then note that 0.1 percent of juveniles make it to maturity, the population increases exponentially. This indicates that conservation methods such as reducing overfishing and bycatch issues, which would increase juvenile survival rate, are much more effective that a farm or hatchery system in which more juveniles are released into the wild each year.

VI. Limitations and Future Work

While the Lefkovitch model provides significant insight into the long-term population dynamics of great white sharks, it is limited in various aspects. Firstly, the model relies on uncertain demographic parameters. Due to the lack of data about great white sharks, many of the parameters are derived from estimates of indirect evidence. Since the evidence obtained are approximations, rather than precise measurements, we can not be certain of our model's credibility in the population's real dynamics.

The model also assumes fixed stage lengths of around thirty years for juveniles and twenty for mature sharks. It is important to note that sexual maturity varies considerably upon each shark and its environmental conditions, food availability, and level of human interactions. The simplification done to create this model may affect the accuracy of transition probabilities between stages, and therefore the steady-state distribution in its entirety. Furthermore, the Lefkovitch matrix assumes a closed population, despite likely migration to and from the system.

Additionally, the model exclusively focuses on female sharks. Though appropriate for analyzing birth dynamics, it is important to consider the effects of the male great white sharks reaching sexual maturity at a younger age than female sharks. Along with this, potential effects of imbalance in the ratio between the sexes and other mating limitations could influence population growth.

Future work that could improve this model includes incorporating sex-specific stage structures dependent on the different rates of sexual maturity. A two-sex model would allow separate survival and transition probabilities for males and females, and would determine whether population growth is limited by female survival or by imbalance in sex ratios. It would provide a more realistic representation of how later female maturity has a role in long-term population recovery, and how that can be combatted through coupling on the basis of reproduction in mature and adult sharks.

VII. Conclusions

In this project, we used a stage-structured Lefkovitch matrix model to examine the survival and reproduction rates of female great white sharks across life stages to determine how the long-term population distribution of these animals is shaped. As a result of our analysis, it is clear that increasing the number of baby sharks can not alone prevent population decline. Although juvenile sharks dominate the population as per high birth rates from mature and adult sharks, the extremely low survival rate from juvenile to mature will limit the long-term population stability.

The steady-state behavior of the model highlights the importance of juvenile survival: with even a small increase in the proportion of juvenile sharks reaching maturity will lead to dramatic improvements in population growth. This emphasizes the need for conservation strategies focusing on reducing juvenile mortality, by means of regulating fishing practices and protecting nursery habitats, will be far more effective than artificial increases in juvenile births; which have comparatively little success.

The mathematical framework we have provided illustrates how delayed maturity and low survival rates make great white sharks incredibly vulnerable to population collapse. In our model, we were able to gain a clear understanding of why their recovery is difficult to achieve, and which interventions are most likely to garner success.

VIII. References

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