Lecture 3: Autoregressive Distributed Lag Model (ADL)

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EESP-FGV

Econometrics 2

Administrative

- Recommended Reading: Hanck, Arnold, Gerber and Schmelzer: Chapter 14.5
- Problem Set 3 - Deadline: June 4th at 9:00 am

Outline

1. Motivation
2. Definitions and Models

Motivation

Motivation

- ARMA(p,q) models uses a variable's own lags to forecast its future values.
- ARMA(p,q) is too simple and ignores data that may be readily available.
- An ADL model uses lags of other variables for forecasting.

Motivation

- Example 1: We may use inflation and exchange rates to forecast GDP growth in Brazil. (Problem Set)

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- Example 2: What is the relationship between the population sizes of jaguatiricas and cutias?

Definitions and Models

Outline

Model 1: ADL with a single extra predictor

An $ADL(p, q)$ model assumes that a time series $\{Y_t\}$ can be represented by a linear function of p of its lagged values and q lags of another time series $\{X_t\}$:

$$Y_t = \alpha + \left(\sum_{l=1}^{p} \phi_l \cdot Y_{t-l} \right) + \left(\sum_{m=1}^{q} \beta_m \cdot X_{t-m} \right) + \epsilon_t,$$

where

$$\mathbb{E}\left[\epsilon_t \middle| Y_{t-1}, Y_{t-2}, \ldots, X_{t-1}, X_{t-2}, \ldots \right] = 0.$$

Definition: Joint Stationarity

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Two stochastic processes $\{Y_t\}$ and $\{X_t\}$ are jointly stationary if, for any values of $j_1, j_2, \ldots, j_n$, the joint distribution of $Y_t, X_t, Y_{t+j_1}, X_{t+j_1}, \ldots, Y_{t+j_n}, X_{t+j_n}$ depends only on the intervals separating the dates $(j_1, j_2, \ldots, j_n)$ and not on the date itself (t).

Model 2: General Time Series Regression

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The general time series regression model extends the ADL model such that multiple regressors and their lags are included. It uses P lags of the dependent variable and Q_l lags of L additional predictors, where $l \in \{1, \ldots, L\}$:

$$Y_t = \alpha + \left(\sum_{p=1}^{P} \phi_p \cdot Y_{t-p} \right) + \left[\sum_{l=1}^{L} \left(\sum_{q=1}^{Q_l} \beta_{l,q} \cdot X_{l,t-q} \right) \right] + \epsilon_t$$

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For estimation, we will make five assumptions.

Model 2: General Time Series Regression

Assumption 1 (Exogeneity)

$\mathbb{E}\left[\epsilon_t \middle| Y_{t-1}, Y_{t-2}, \ldots, X_{1,t-1}, X_{1,t-2}, \ldots, X_{L,t-1}, X_{L,t-2}, \ldots\right] = 0$

Assumption 2 (Stationarity)

$\{(Y_t, X_{1,t}, \ldots, X_{L,t})\}$ are jointly stationary.

Assumption 3 (Weak Dependence Formal Definition)

$(Y_t, X_{1,t}, \ldots, X_{L,t})$ and $(Y_{t-j}, X_{1,t-j}, \ldots, X_{L,t-j})$ become independent as j gets large.

Model 2: General Time Series Regression

Assumption 4 (Large Outliers are Unlikely)

$(Y_t, X_{1,t}, \ldots, X_{L,t})$ have nonzero, finite fourth moments.

Assumption 5

No perfect multicollinearity.

Thank you!

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References

Formal Definition: Weak Dependence

Fix a set S, a sequence of sets of measurable functions $\{\mathcal{F}_d\}_{d=1}^{\infty} \in \prod_{d=1}^{\infty} \left(S^d \to \mathbb{R}\right)$, a positive sequence $\{\theta_\delta\}_{\delta=1}^{\infty} \to 0$, and a function $\psi \in \mathcal{F}^2 \times (\mathbb{Z}^+)^2 \to \mathbb{R}^+$. A sequence $\{X_n\}_{n=1}^{\infty}$ of random variables is $(\{\mathcal{F}_d\}_{d=1}^{\infty}, \{\theta_\delta\}_\delta, \psi)$-weakly dependent if, for all $j_1 \leq j_2 \leq \cdots \leq j_d < j_d + \delta \leq k_1 \leq k_2 \leq \cdots \leq k_e$, for all $\phi \in \mathcal{F}_d$, and $\tau \in \mathcal{F}_e$, we have

$$|\operatorname{Cov}(\phi(X_{j_1}, \ldots, X_{j_d}), \tau(X_{k_1}, \ldots, X_{k_e}))| \leq \psi(\phi, \tau, d, e) \cdot \theta_\delta.$$

Intuition: The covariance between any function of the variables goes to zero when the distance between the time periods grows.

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