Lecture 6B: General Method of Moments (GMM) in Practice

Vitor Possebom

EESP-FGV

Econometrics 2

Administrative

- Recommended Reading: Hamilton's Chapter 14
- Soft Reading: Chaussé (2021)
- Optional Reading: Hayashi's Chapters 3 and 4
- Problem Set 5 - Deadline: June 20th at 9:00 am (Be careful with deadlines here.)

Outline

1. Recap
2. Example 0: OLS, IV and MLE
3. Example 1: Heteroscedasticity as a Source of Identification
4. Example 2: Infinite Horizon Consumption Problem under Uncertainty
5. Example 3: CAPM Model
6. Example 4: Nonlinear System of Simultaneous Equations
7. Optimal Weighting Matrix May Be Problematic

Recap

Recap

- W_t: $h \times 1$ vector of variables observed at date t.
- θ: unknown $a \times 1$ vector of coefficients
- $h : \mathbb{R}^a \times \mathbb{R}^h \to \mathbb{R}^r$: $h(\theta, W_t)$ is a vector-valued function.

Recap

- W_t: $h \times 1$ vector of variables observed at date t.
- θ: unknown $a \times 1$ vector of coefficients
- $h : \mathbb{R}^a \times \mathbb{R}^h \to \mathbb{R}^r$: $h(\theta, W_t)$ is a vector-valued function.

True value θ_0 satisfies the ortogonality conditions

$$\mathbb{E}\left[h\left(\theta_0, W_t\right)\right] = 0.$$

- $Y_T := \left(W_T', W_{T-1}', \ldots, W_1'\right)'$ is a $Th \times 1$ vector with all the observation in a sample of size T.

Recap

- $Y_T := \left(W_T', W_{T-1}', \ldots, W_1'\right)'$ is a $Th \times 1$ vector with all the observation in a sample of size T.
- $g : \mathbb{R}^a \to \mathbb{R}^r$ such that $g\left(\theta, Y_T\right) = \frac{\sum_{t=1}^{T} h\left(\theta, W_t\right)}{T}$

Recap

- $Y_T := (W'_T, W'_{T-1}, \ldots, W'_1)'$ is a $Th \times 1$ vector with all the observation in a sample of size T.
- $g : \mathbb{R}^a \to \mathbb{R}^r$ such that $g(\theta, Y_T) = \frac{\sum_{t=1}^{T} h(\theta, W_t)}{T}$

GMM estimator $\hat{\theta}_T$ satisfies

$$\hat{\theta}_T := \underset{\theta \in \mathbb{R}^a}{\operatorname{argmin}} \left[g(\theta, Y_T) \right]' W_T \left[g(\theta, Y_T) \right],$$

where $\{W_T\}_{T=1}^{+\infty}$ is a sequence of $(r \times r)$ positive definite weighting matrices.

Recap

Key Identifying Condition,

$$\mathbb{E}\left[h\left(\theta_0, W_t\right)\right] = 0,$$

Recap

Key Identifying Condition,

$$\mathbb{E}\left[h\left(\theta_0, W_t\right)\right] = 0,$$

explains why the GMM estimator,

$$\hat{\theta}_T := \underset{\theta \in \mathbb{R}^a}{\text{argmin}} \left[\frac{\sum_{t=1}^{T} h\left(\theta, W_t\right)}{T}\right]' W_T \left[\frac{\sum_{t=1}^{T} h\left(\theta, W_t\right)}{T}\right],$$

makes intuitive sense.

Recap

Key Identifying Condition,

$$\mathbb{E}\left[h\left(\theta_0, W_t\right)\right] = 0,$$

explains why the GMM estimator,

$$\hat{\theta}_T := \underset{\theta \in \mathbb{R}^a}{\text{argmin}} \left[\frac{\sum_{t=1}^{T} h\left(\theta, W_t\right)}{T}\right]' W_T \left[\frac{\sum_{t=1}^{T} h\left(\theta, W_t\right)}{T}\right],$$

makes intuitive sense.

We need to understand how can we find those identifying conditions!

Moment Restrictions in the Wild

Outline

1. Recap
2. Example 0: OLS, IV and MLE
3. Example 1: Heteroscedasticity as a Source of Identification
4. Example 2: Infinite Horizon Consumption Problem under Uncertainty
5. Example 3: CAPM Model
6. Example 4: Nonlinear System of Simultaneous Equations
7. Optimal Weighting Matrix May Be Problematic

Example 0: OLS, IV and MLE

Example 0: OLS, IV and MLE

OLS: Your covariates are not correlated with the unobservables variables in your model.

Example 0: OLS, IV and MLE

OLS: Your covariates are not correlated with the unobservables variables in your model.

$$\mathbb{E}\left[X_t' \cdot (Y_t - X_t \cdot \beta)\right] = 0$$

Example 0: OLS, IV and MLE

OLS: Your covariates are not correlated with the unobservables variables in your model.

$$\mathbb{E}\left[X_t' \cdot (Y_t - X_t \cdot \beta)\right] = 0$$

IV: Your instruments are not correlated with the unobservables in the second stage.

Example 0: OLS, IV and MLE

OLS: Your covariates are not correlated with the unobservables variables in your model.

$$\mathbb{E}\left[X_t' \cdot (Y_t - X_t \cdot \beta)\right] = 0$$

IV: Your instruments are not correlated with the unobservables in the second stage.

$$\mathbb{E}\left[Z_t' \cdot (Y_t - X_t \cdot \beta)\right] = 0$$

Example 0: OLS, IV and MLE

OLS: Your covariates are not correlated with the unobservables variables in your model.

$$\mathbb{E}\left[X_t' \cdot (Y_t - X_t \cdot \beta)\right] = 0$$

IV: Your instruments are not correlated with the unobservables in the second stage.

$$\mathbb{E}\left[Z_t' \cdot (Y_t - X_t \cdot \beta)\right] = 0$$

MLE: Your functional form assumptions are correct. You may use the score function (MLE's first-order conditions) as your moment condition.

Example 0: MLE for Binary Variables

Example 0: MLE for Binary Variables

Let Y_t be a binary variable.

Example 0: MLE for Binary Variables

Let Y_t be a binary variable. We impose that

$$\mathbb{P}\left[\, Y_t = 1 | \, X_t \right] = G\left(X_t \cdot \beta\right),$$

for a known $G\left(.\right)$.

- Link function $G\left(.\right)$ is usually a CDF.

Example 0: MLE for Binary Variables

Let Y_t be a binary variable. We impose that

$$\mathbb{P}\left[\, Y_t = 1 \middle|\, X_t\right] = G\left(X_t \cdot \beta\right),$$

for a known $G\left(.\right)$.

- Link function $G\left(.\right)$ is usually a CDF.
    - Probit model: $G\left(z\right) = \Phi(z) = \int_{-\infty}^{z} \phi(v)dv$

Example 0: MLE for Binary Variables

Let Y_t be a binary variable. We impose that

$$\mathbb{P}\left[\, Y_t = 1 \middle| \, X_t\right] = G\left(X_t \cdot \beta\right),$$

for a known $G\left(.\right)$.

- Link function $G\left(.\right)$ is usually a CDF.
    - Probit model: $G\left(z\right) = \Phi(z) = \int_{-\infty}^{z} \phi(v)dv$
    - Logit model: $G\left(z\right) = \dfrac{\exp(z)}{1 + \exp(z)}$

Example 0: MLE for Binary Variables

Let Y_t be a binary variable. We impose that

$$\mathbb{P}\left[Y_t = 1 | X_t \right] = G\left(X_t \cdot \beta \right),$$

for a known $G\left(.\right)$.

- Link function $G\left(.\right)$ is usually a CDF.
  - Probit model: $G\left(z\right) = \Phi(z) = \int_{-\infty}^{z} \phi(v)dv$
  - Logit model: $G\left(z\right) = \dfrac{\exp(z)}{1+\exp(z)}$

We can estimate partial effects (continuous vs discrete covariates).

Example 0: MLE for Binary Variables

We can estimate partial effects (continuous vs discrete covariates).

Example 0: MLE for Binary Variables

We can estimate partial effects (continuous vs discrete covariates).

- Continuous:

Example 0: MLE for Binary Variables

We can estimate partial effects (continuous vs discrete covariates).

- Continuous:
    - At a specific value of the covariates: $\dfrac{\partial \mathbb{P}\left[\, Y_t = 1 \middle|\, X_t = x\right]}{\partial x_k} = g\left(x \cdot \beta\right) \cdot \beta_k$

Example 0: MLE for Binary Variables

We can estimate partial effects (continuous vs discrete covariates).

- Continuous:
  - At a specific value of the covariates: $\frac{\partial \mathbb{P}\left[\, Y_t = 1 \middle|\, X_t = x\right]}{\partial x_k} = g\left(x \cdot \beta\right) \cdot \beta_k$
  - Average Partial Effect: $\mathbb{E}\left[g\left(X_t \cdot \beta\right) \cdot \beta_k\right]$

Example 0: MLE for Binary Variables

We can estimate partial effects (continuous vs discrete covariates).

- Continuous:
  - At a specific value of the covariates: $\dfrac{\partial \mathbb{P}[\,Y_t = 1 |\, X_t = x]}{\partial x_k} = g(x \cdot \beta) \cdot \beta_k$
  - Average Partial Effect: $\mathbb{E}[g(X_t \cdot \beta) \cdot \beta_k]$
- Discrete:

Example 0: MLE for Binary Variables

We can estimate partial effects (continuous vs discrete covariates).

- Continuous:
  - At a specific value of the covariates: $\dfrac{\partial \mathbb{P}\left[\, Y_t = 1 \middle| \, X_t = x\right]}{\partial x_k} = g\left(x \cdot \beta\right) \cdot \beta_k$
  - Average Partial Effect: $\mathbb{E}\left[g\left(X_t \cdot \beta\right) \cdot \beta_k\right]$

- Discrete:
  - At a specific value of the covariates:
    $\mathbb{P}\left[\, Y_t = 1 \middle| \, X_{t,-k} = x_{-k}, X_{t,k} = 1\right] - \mathbb{P}\left[\, Y_t = 1 \middle| \, X_{t,-k} = x_{-k}, X_{t,k} = 0\right] =$
    $G\left(x_{-k} \cdot \beta_{-k} + \beta_k\right) - G\left(x_{-k} \cdot \beta_{-k}\right)$

Example 0: MLE for Binary Variables

We can estimate partial effects (continuous vs discrete covariates).

- Continuous:
    - At a specific value of the covariates: $\frac{\partial \mathbb{P}\left[Y_t = 1 \middle| X_t = x \right]}{\partial x_k} = g\left(x \cdot \beta\right) \cdot \beta_k$
    - Average Partial Effect: $\mathbb{E}\left[g\left(X_t \cdot \beta\right) \cdot \beta_k\right]$
- Discrete:
    - At a specific value of the covariates: $\mathbb{P}\left[Y_t = 1 \middle| X_{t,-k} = x_{-k}, X_{t,k} = 1\right] - \mathbb{P}\left[Y_t = 1 \middle| X_{t,-k} = x_{-k}, X_{t,k} = 0\right] = G\left(x_{-k} \cdot \beta_{-k} + \beta_k\right) - G\left(x_{-k} \cdot \beta_{-k}\right)$
    - Average Partial Effect: $\mathbb{E}\left[G\left(X_{-k} \cdot \beta_{-k} + \beta_k\right) - G\left(X_{-k} \cdot \beta_{-k}\right)\right]$

Example 0: MLE for Binary Variables

We can estimate partial effects (continuous vs discrete covariates).

- Continuous:
  - At a specific value of the covariates: $\dfrac{\partial \mathbb{P}\left[\left. Y_t = 1 \right| X_t = x\right]}{\partial x_k} = g\left(x \cdot \beta\right) \cdot \beta_k$
  - Average Partial Effect: $\mathbb{E}\left[g\left(X_t \cdot \beta\right) \cdot \beta_k\right]$
- Discrete:
  - At a specific value of the covariates:
    $\mathbb{P}\left[\left. Y_t = 1 \right| X_{t,-k} = x_{-k}, X_{t,k} = 1\right] - \mathbb{P}\left[\left. Y_t = 1 \right| X_{t,-k} = x_{-k}, X_{t,k} = 0\right] =$
    $G\left(x_{-k} \cdot \beta_{-k} + \beta_k\right) - G\left(x_{-k} \cdot \beta_{-k}\right)$
  - Average Partial Effect: $\mathbb{E}\left[G\left(X_{-k} \cdot \beta_{-k} + \beta_k\right) - G\left(X_{-k} \cdot \beta_{-k}\right)\right]$

Outline

1. Recap
2. Example 0: OLS, IV and MLE
3. Example 1: Heteroscedasticity as a Source of Identification
4. Example 2: Infinite Horizon Consumption Problem under Uncertainty
5. Example 3: CAPM Model
6. Example 4: Nonlinear System of Simultaneous Equations
7. Optimal Weighting Matrix May Be Problematic

Example 1: Heteroscedasticity as a Source of Identification

Example 1: Heteroscedasticity as a Source of Identification [Lewbel, 2012]

Consider the linear triangular model:

$$
\begin{aligned}
Y_1 &= X'\beta_{10} + Y_2\gamma_{10} + \epsilon_1 \\
Y_2 &= X'\beta_{20} + \epsilon_2
\end{aligned}
$$

Assume that:

1. $Y = (Y_1, Y_2)'$ and X are random vectors. $\mathbb{E}[XY']$, $\mathbb{E}[XY_1Y']$, $\mathbb{E}[XY_2Y']$ and $\mathbb{E}[XX']$ are finite and identified from the data. $\mathbb{E}[XX']$ is nonsingular.
2. $\mathbb{E}[X\epsilon_1] = 0$, $\mathbb{E}[X\epsilon_2] = 0$ and, for some random vector Z, $\operatorname{cov}(Z, \epsilon_1\epsilon_2) = 0$.
    - Some or all of the elements of Z can also be elements of X.
3. $\operatorname{cov}\left(Z, \epsilon_2^2\right) \neq 0$.

Example 1: Heteroscedasticity as a Source of Identification [Lewbel, 2012]

Then, the moment conditions

$$\mathbb{E}\left[\begin{array}{c} X\left(Y_2 - X'\beta_{20}\right) \\ X\left(Y_1 - X'\beta_{10} + Y_2\gamma_{10}\right) \\ \left(Z - \mathbb{E}\left[Z\right]\right)\left(Y_2 - X'\beta_{20}\right)\left(Y_1 - X'\beta_{10} + Y_2\gamma_{10}\right) \end{array}\right] = 0$$

identify β_{10}, β_{20} and γ_{10}.

Intuition: The assumption that Z is uncorrelated with $\epsilon_1\epsilon_2$ means that $\left(Z - \mathbb{E}\left[Z\right]\right)\epsilon_2$ is a valid instrument for Y_2. This instrument's strength (its correlation with Y_2 after controlling for the other instruments X) is proportional to the covariance of $\left(Z - \mathbb{E}\left[Z\right]\right)\epsilon_2$ with ϵ_2, which corresponds to the degree of heteroscedasticity of ϵ_2 with respect to Z.

Example 1: Heteroscedasticity as a Source of Identification [Lewbel, 2012]

Possible applications:

Example 1: Heteroscedasticity as a Source of Identification [Lewbel, 2012]

Possible applications:

- Classical Measurement Error
- Supply and Demand
- Returns to Schooling

Example 1: Heteroscedasticity as a Source of Identification [Lewbel, 2012]

Possible applications:

- Classical Measurement Error
- Supply and Demand
- Returns to Schooling
- Engel Curves: Food Consumption as a function of Total Consumption

Example 1: Heteroscedasticity as a Source of Identification [Lewbel, 2012]

Possible applications:

- Classical Measurement Error
- Supply and Demand
- Returns to Schooling
- Engel Curves: Food Consumption as a function of Total Consumption
  - $Y_1 =$ Food budget share
  - $Y_2 =$ Log real total expenditures (classical measurement error)
  - $X = Z =$ age, spouse's age, squared ages, seasonal dummies, spouse working, gas central heating, washing machine, cars

Example 2: Infinite Horizon Consumption Problem under Uncertainty

Outline

Example 2: Infinite Horizon Consumption Problem under Uncertainty

In Macroeconomics, you analyzed the Infinite Horizon Consumption Problem under Uncertainty:

$$\max_{\{c_t\}_{t=0}^{+\infty}} \mathbb{E}\left[\sum_{t=0}^{T} \beta_0^t u\left(c_t\right)\right]$$

$$\text{s.t. } b_{t+1} = \left(w_t + b_t - c_t\right) \cdot R_t$$

where labor income w_t is uncertain, c_t is consumption at time t, β_0 is the discounting factor, $u\left(\cdot\right)$ is the utility function, b_t is the amount of savings and R_t is the gross ex-post rate of return.

Example 2: Infinite Horizon Consumption Problem under Uncertainty

Solving this model, we find the Euler equation:

$$\mathbb{E}\left[R_{t+1} \frac{\beta_0 u'(c_{t+1})}{u'(c_t)} \middle| I_t \right] = 1,$$

where I_t is the information available at date t.

Example 2: Infinite Horizon Consumption Problem under Uncertainty

Solving this model, we find the Euler equation:

$$\mathbb{E}\left[R_{t+1}\frac{\beta_0 u'(c_{t+1})}{u'(c_t)}\middle| I_t\right]=1,$$

where I_t is the information available at date t.

Assuming that $u(c)=\dfrac{c^{1-\alpha_0}}{1-\alpha_0}$, the Euler equation simplifies to:

$$\mathbb{E}\left[R_{t+1}\cdot\beta_0\cdot\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha_0}\middle| I_t\right]=1.$$

Example 2: Infinite Horizon Consumption Problem under Uncertainty

Assuming that X_t is a vector of variables whose values are known at date t, we have that

$$\mathbb{E}\left[\mathsf{X}_t \cdot \left(R_{t+1} \cdot \beta_0 \cdot \left(\frac{c_{t+1}}{c_t}\right)^{-\alpha_0} - 1\right)\right] = 0.$$

For example, X_t may be a vector of lagged values of consumption and rates of return.

Example 3: CAPM Model

Outline

Example 3: CAPM Model

The CAPM model implies that

$$\mu_i - R_f = \beta_i \left(\mu_m - R_f \right),$$

where i indexes different stocks, μ_i is the expected value of stock i's return, R_f is the risk-free rate and μ_m is the expected value of the market portfolio's return.

Example 3: CAPM Model

We can rewrite the CAPM model in terms of moment conditions:

$$\mathbb{E}\left[\{(R_{i,t} - R_{f,t}) - \alpha_i - \beta_i \cdot (R_{m,t} - R_{f,t})\} \cdot (R_{m,t} - R_{f,t})\right] = 0,$$

where $\alpha_i = 0$ for every stock i if the CAPM model is valid.

- Legend: $R_{i,t}$ is the return of stock i in day t; $R_{f,t}$ is the risk-free rate in day t; $R_{m,t}$ is the market portfolio's return in day t.

Example 3: CAPM Model — Heated Debate

Example 3: CAPM Model — Heated Debate

- The original CAPM model generated a heated debate in Finance.

Example 3: CAPM Model — Heated Debate

- The original CAPM model generated a heated debate in Finance.
- Since it did not fit the data very well, it was criticized and modified.

Example 3: CAPM Model — Heated Debate

- The original CAPM model generated a heated debate in Finance.
- Since it did not fit the data very well, it was criticized and modified.
- The CAPM was extended to the Consumption-based capital asset pricing model (C-CAPM).

Example 3: CAPM Model — Heated Debate

- The original CAPM model generated a heated debate in Finance.
- Since it did not fit the data very well, it was criticized and modified.
- The CAPM was extended to the Consumption-based capital asset pricing model (C-CAPM).
- The C-CAPM still did not explain the equity premium puzzle and a few extensions were proposed:
    - Long-run risk, recursive preferences, habit formation, and limiting participation

Example 3: CAPM Model — Heated Debate

- The original CAPM model generated a heated debate in Finance.
- Since it did not fit the data very well, it was criticized and modified.
- The CAPM was extended to the Consumption-based capital asset pricing model (C-CAPM).
- The C-CAPM still did not explain the equity premium puzzle and a few extensions were proposed:
  - Long-run risk, recursive preferences, habit formation, and limiting participation
- These extensions did a better job at explaining the equity premium puzzle, but they did not explain Euler Equation Errors.

Example 3: CAPM Model — Euler Equation Errors

Example 3: CAPM Model — Euler Equation Errors

- Euler Equation Errors are large and persistent pricing errors found empirically.

Example 3: CAPM Model — Euler Equation Errors

- Euler Equation Errors are large and persistent pricing errors found empirically.
- Recently, Parra-Alvarez et al. [2022] proposed an extension to the C-CAPM that can explain Euler Equation Errors.
    - They allowed for low-probability events that cause infrequent but sharp contractions in aggregate consumption.

Example 3: CAPM Model — Euler Equation Errors

- Euler Equation Errors are large and persistent pricing errors found empirically.
- Recently, Parra-Alvarez et al. [2022] proposed an extension to the C-CAPM that can explain Euler Equation Errors.
  - They allowed for low-probability events that cause infrequent but sharp contractions in aggregate consumption.
  - Allowing for rare disasters rationalizes the large pricing errors found empirically.

Example 3: CAPM Model — Euler Equation Errors

- Euler Equation Errors are large and persistent pricing errors found empirically.
- Recently, Parra-Alvarez et al. [2022] proposed an extension to the C-CAPM that can explain Euler Equation Errors.
    - They allowed for low-probability events that cause infrequent but sharp contractions in aggregate consumption.
    - Allowing for rare disasters rationalizes the large pricing errors found empirically.
- This new model is also estimated via GMM.

Example 4: Nonlinear System of Simultaneous Equations

Outline

Example 4: Nonlinear System of Simultaneous Equations

The CAPM Moment Conditions are a particular case of a (linear) system of simultaneous equations.

We can generalize this type of model to a nonlinear system of simultaneous equations.

We want to estimate a system of n nonlinear equations of the form

$$Y_t = f(\theta, Z_t) + U_t.$$

Example 4: Nonlinear System of Simultaneous Equations

Suppose that X_{it} is a vector of instruments that are uncorrelated with the i-th element of U_t.

The following moment conditions hold

$$\mathbb{E}\begin{bmatrix} \{Y_{1t} - f_1(\theta, Z_t)\} X_{1t} \\ \{Y_{2t} - f_2(\theta, Z_t)\} X_{2t} \\ \vdots \\ \{Y_{nt} - f_n(\theta, Z_t)\} X_{nt} \end{bmatrix} = 0.$$

Optimal Weighting Matrix May Be Problematic

Outline

Optimal Weighting Matrix May Be Problematic

Altonji and Segal [1996] finds that optimally weighted GMM estimators are biased in small samples. Consequently, their estimated confidence intervals cover the true parameter with a probability that is smaller than the nominal confidence level.

Optimal Weighting Matrix May Be Problematic

Altonji and Segal [1996] finds that optimally weighted GMM estimators are biased in small samples. Consequently, their estimated confidence intervals cover the true parameter with a probability that is smaller than the nominal confidence level.

They argue that this problem is caused by the correlation between the sampling errors in the second moments and in the weighting matrix.

Optimal Weighting Matrix May Be Problematic

Altonji and Segal [1996] finds that optimally weighted GMM estimators are biased in small samples. Consequently, their estimated confidence intervals cover the true parameter with a probability that is smaller than the nominal confidence level.

They argue that this problem is caused by the correlation between the sampling errors in the second moments and in the weighting matrix.

Curiously, they also find that the equally weighted GMM estimator seems to work fine.

Optimal Weighting Matrix May Be Problematic

Altonji and Segal [1996] finds that optimally weighted GMM estimators are biased in small samples. Consequently, their estimated confidence intervals cover the true parameter with a probability that is smaller than the nominal confidence level.

They argue that this problem is caused by the correlation between the sampling errors in the second moments and in the weighting matrix.

Curiously, they also find that the equally weighted GMM estimator seems to work fine.

Take-home Lesson: When working with more complicated estimators, running a few Monte Carlo simulation can help you sleep better. [Ferman, 2021]

Thank you!

Contact Information:

Vitor Possebom

E-mail: vitor.possebom@fgv.br

Website: `sites.google.com/site/vitorapossebom/`

References

J. G. Altonji and L. M. Segal. Small-Sample Bias in GMM Estimation of Covariance Structures. *Journal of Business and Economic Statistics*, 14(3):pp. 353–366, 1996.

B. Ferman. Assessing Inference Methods. Available at `https://www.dropbox.com/s/iofibyr1x96iowf/ferman_assessment.pdf?dl=0`, Nov. 2021.

A. Lewbel. Using Heteroscedasticity to Identify and Estimate Mismeasured and Endogenous Regressor Models. *Journal of Business and Economic Statistics*, 30(1):p. 67–80, 2012.

J. C. Parra-Alvarez, O. Posch, and A. Schrimpf. Peso problems in the estimation of the C-CAPM. *Quantitative Economics*, 13:pp. 259–313, 2022.