Lecture 7: Dynamic Panel Data Models

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EESP-FGV

Econometrics 2

Administrative

- Recommended Reading: Bond (2002)
- Problem Set 6 - Deadline: June 20th at 5:00 pm

Outline

1. Motivation

2. Autoregressive Models

3. Multivariate Dynamic Models

4. Frontier in Panel Data Research and Dynamic Models

Motivation

Motivation

We want to estimate dynamic models.

- Relationship between a treatment in the past and an outcome today.
- Relationship between a endogenous variable in the past and its value today.

To estimate this type of model, we need a time dimension. However, time series data may be insufficient.

- Aggregate time series data may obscure microeconomic relationships.
    - Robinson [1950]: In the U.S. in 1930, immigrants were less likely to be literate than native citizens, but states with more immigrants had higher literacy rates.

Motivation

Panel data $\rightarrow$ microeconomic heterogeneity: firms, households or individuals.

Examples:

- Lecture 1 - Income Dynamics
- Household consumption - Euler Equations
- Adjustment cost models for firm's factor demands
- Economic growth
- Production functions with serially correlated productivity shocks
- Company investment rates

Motivation

Goal:

- Estimation of single equation, autoregressive-distributed lag models from panels with a large number of cross-section units, each observed for a small number of time periods.
- Estimation methods that do not require the time dimension to become large in order to obtain consistent parameter estimates.
- Focus on micro panel data on individuals or firms.

Identification depends on limited serial correlation in the error term.

- It relies on somewhat hard-to-justify assumptions.

Autoregressive Models

Outline

1. Motivation
2. Autoregressive Models
3. Multivariate Dynamic Models
4. Frontier in Panel Data Research and Dynamic Models

Autoregressive Models

We start with a simple $AR(1)$ model:

$$Y_{i,t} = \alpha \cdot Y_{i,t-1} + \eta_i + \nu_{i,t},$$

where $|\alpha| < 1$, $i \in \{1, 2, \ldots, N\}$ and $t \in \{2, 3, \ldots, T\}$.

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    - heterogeneity in the means of the $Y_{i,t}$ series across individuals

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Cross-sectional asymptotics: N goes to infinite, while T is fixed.

Autoregressive Models

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   $\Rightarrow \; Cov\left(\eta_i, Y_{i,t-1}\right) \neq 0$

OLS estimator is inconsistent.

Autoregressive Models

"Are we in the woods without a dog?"

Autoregressive Models

“Are we in the woods without a dog?”

“If we don’t have a dog, we hunt with a cat.”

Within-groups estimator:

- Run OLS in the demeaned model

$$\tilde{Y}_{i,t} = \alpha \cdot \tilde{Y}_{i,t-1} + \tilde{\nu}_{i,t},$$

where $\tilde{Y}_{i,t} := Y_{i,t} - \frac{\sum_{\tau=2}^{T} Y_{i,\tau}}{T-1}$, $\tilde{Y}_{i,t-1} := Y_{i,t-1} - \frac{\sum_{\tau=1}^{T-1} Y_{i,\tau}}{T-1}$ and $\tilde{\nu}_{i,t} := \nu_{i,t} - \frac{\sum_{\tau=2}^{T} \nu_{i,\tau}}{T-1}$.

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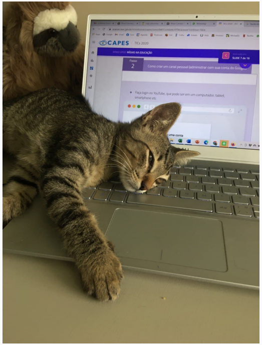
$$\Rightarrow \; Cov\left(\tilde{Y}_{i,t-1}, \tilde{\nu}_{i,t}\right) \neq 0$$

Within-groups estimator is inconsistent.

Autoregressive Models

“It was a house cat!”



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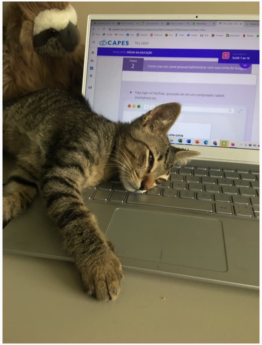

Let's try something different.

First-differences estimator:

- Run OLS in the first-differences model

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Autoregressive Models

"It was another house cat! A sleepy one!"

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We will need a dog!

Anderson-Hsiao estimator!

Autoregressive Models

Anderson-Hsiao estimator:

- Run 2SLS in the first-differences model

$$\Delta Y_{i,t} = \alpha \cdot \Delta Y_{i,t-1} + \Delta \nu_{i,t}.$$

- Instrument is $Y_{i,t-2}$.

Anderson-Hsiao estimator:

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- Extra Assumption - Predetermined Initial Conditions:

$$Cov\left(Y_{i,1}, \nu_{i,t}\right) = 0 \text{ for } t \in \{2, 3, \ldots, T\}$$

Anderson-Hsiao estimator: For $t \in \{3, 4, \ldots, T\}$, we have that

$$\begin{aligned}
Cov\left(\Delta\nu_{i,t}, Y_{i,t-2}\right) &= Cov\left(\nu_{i,t} - \nu_{i,t-1}, Y_{i,t-2}\right) \\
&= Cov\left(\nu_{i,t}, Y_{i,t-2}\right) - Cov\left(\nu_{i,t-1}, Y_{i,t-2}\right) \\
&= Cov\left(\nu_{i,t}, \alpha^{t-3} \cdot Y_{i,1} + \left\{\sum_{\tau=0}^{t-4} \alpha^{\tau} \cdot \nu_{i,t-2-\tau}\right\}\right) \\
&\qquad - Cov\left(\nu_{i,t-1}, \alpha^{t-3} \cdot Y_{i,1} + \left\{\sum_{\tau=0}^{t-4} \alpha^{\tau} \cdot \nu_{i,t-2-\tau}\right\}\right) \\
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&= 0
\end{aligned}$$

Anderson-Hsiao estimator is large-N consistent if $T \geq 3$.

Autoregressive Models

We can do better if $T > 3$.

Arellano-Bond Estimator

- Period $t = 3$: $Y_{i,1}$ is the only valid instrument.
- Period $t = 4$: $Y_{i,1}$ and $Y_{i,2}$ are valid instruments.

$$\vdots$$

- Period $t = T$: $Y_{i,1}, Y_{i,2}, \ldots, Y_{i,T-2}$ are valid instruments.

Arellano-Bond Estimator ($T > 3$):

- Efficient GMM estimator based on $\mathbb{E}\left[Z_i' \Delta \nu_i\right] = 0$,

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$$Z_i := \begin{bmatrix} Y_{i,1} & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & Y_{i,1} & Y_{i,2} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & Y_{i,1} & \cdots & Y_{i,T-2} \end{bmatrix}$$

$$\Delta\nu_i := (\Delta\nu_{i,3}, \Delta\nu_{i,4}, \ldots, \Delta\nu_{i,T})'$$

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  - Older values of $Y_{i,t-j}$ are valid instruments.

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  - Large number of instruments $\Rightarrow$ Poor Small Sample Performance and Weak Instruments.

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  - Older values of $Y_{i,t-j}$ are valid instruments.
- Drawbacks:
  - Large number of instruments $\Rightarrow$ Poor Small Sample Performance and Weak Instruments.
  - Too few AR terms $\Rightarrow$ Arellano-Bond estimator is inconsistent.
  - Too many AR terms $\Rightarrow$ Weak Instruments

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- η_i: firm-specific depreciation rates

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Table 1. Alternative estimates of the $AR(1)$ specification for company investment rate

Dependent variable: $(I/K)_t$	OLS levels	Within groups	2SLS DIF	GMM DIF	GMM DIF
$(I/K)_{t-1}$	0.2669	-0.0094	0.1626	0.1593	0.1560
	(.0185)	(.0181)	(.0362)	(.0327)	(.0318)
Sargan			(p-value)	.36	.43
Instruments			$(I/K)_{t-2}$	$(I/K)_{t-2}$	$(I/K)_{t-2}$
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- Within-Groups is likely to be biased downwards. (See PSet 6)

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- 2SLS and GMM: between OLS and Within-Groups

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- 2SLS and GMM: close to each other
- Sargan Test: do not reject
- Many IVs: Column (4) = 23, Column (5) = 78

Multivariate Dynamic Models

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4. Frontier in Panel Data Research and Dynamic Models

Now, we analyze a slightly more complicated model:

$$Y_{i,t} = \alpha \cdot Y_{i,t-1} + \beta \cdot X_{i,t} + \eta_i + \nu_{i,t},$$

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where $|\alpha| < 1$, $i \in \{1, 2, \ldots, N\}$ and $t \in \{2, 3, \ldots, T\}$.

- $X_{i,t}$: current and lagged values of additional explanatory variables.
- No need to model the time series $X_{i,t}$.
- $X_{i,t}$ can be correlate with η_i.

Multivariate Dynamic Models

First-differences model:

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- $X_{i,t}$ is endogenous: $Cov\left(X_{i,t}, \nu_{i,\tau}\right) \neq 0$ if $\tau \leq t$ and $Cov\left(X_{i,t}, \nu_{i,\tau}\right) = 0$ if $\tau > t$.
  $\Rightarrow$ $X_{i,t-2}$, $X_{i,t-3}$ and longer lags are valid instruments.

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  $\Rightarrow$ $X_{i,t-2}$, $X_{i,t-3}$ and longer lags are valid instruments.
- $X_{i,t}$ is predetermined: $Cov\left(X_{i,t}, \nu_{i,\tau}\right) \neq 0$ if $\tau < t$ and $Cov\left(X_{i,t}, \nu_{i,\tau}\right) = 0$ if $\tau \geq t$.
  $\Rightarrow$ $X_{i,t-1}$ is a valid instrument too.

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  $\Rightarrow$ $X_{i,t-2}$, $X_{i,t-3}$ and longer lags are valid instruments.
- $X_{i,t}$ is predetermined: $Cov\left(X_{i,t}, \nu_{i,\tau}\right) \neq 0$ if $\tau < t$ and $Cov\left(X_{i,t}, \nu_{i,\tau}\right) = 0$ if $\tau \geq t$.
  $\Rightarrow$ $X_{i,t-1}$ is a valid instrument too.
- $X_{i,t}$ is strictly exogenous: $Cov\left(X_{i,t}, \nu_{i,\tau}\right) = 0$ for any $\tau \in \{1, 2, \ldots, T\}$.
  $\Rightarrow$ $X_i' := (X_{i,1}, \ldots, X_{i,T})'$ is a valid instrument.

Frontier in Panel Data Research and Dynamic Models

Outline

1. Motivation
2. Autoregressive Models
3. Multivariate Dynamic Models
4. Frontier in Panel Data Research and Dynamic Models

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- Arellano and Bonhomme [2017]: Survey paper on Non-linear Panel Data Models

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- Both authors rely on control function arguments
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- In large-N scenarios, the estimator of the fixed-effect parameters is very imprecise.
- Shrinkage methods increases precision by allowing for some bias.
- Navigating this trade-off: Minimize $MSE \rightarrow$ Kwon [2021].

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Choice-based Treatment Effect Models:

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- Heckman et al. [2016]:
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- Han [2021]:
    - Sequences of treatment allocations.
    - Optimal treatment regimes.

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Frontier in Panel Data Research and Dynamic Models

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Synthetic Control Method:

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- Abadie and Gardeazabal [2003], Abadie et al. [2010] and Abadie et al. [2015]
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- Ben-Michael et al. [2021], Cattaneo et al. [2023]:
    - Synthetic Controls with Staggered Treatment Adoption.

Thank you!

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References

A. Abadie and J. Gardeazabal. The Economic Costs of Conflict: A Case Study of the Basque Country. *American Economic Review*, 93(1):113–132, 2003.

A. Abadie, A. Diamond, and J. Hainmueller. Synthetic Control Methods for Comparative Case Studies: Estimating the Effect of California's Tobacco Control Program. *Journal of the American Statiscal Association*, 105(490):493–505, 2010.

A. Abadie, A. Diamond, and J. Hainmueller. Comparative Politics and the Synthetic Control Method. *American Journal of Political Science*, 59(2):495–510, 2015.

M. Arellano and S. Bonhomme. Nonlinear Panel Data Methods for Dynamic Heterogeneous Agent Models. *Annual Review of Economics*, 9:pp. 471–496, 2017.

E. Ben-Michael, A. Feller, and J. Rothstein. Synthetic Controls with Staggered Adoption. *Journal of the Royal Statistical Society - Series B*, Jan. 2021. Available at https://arxiv.org/abs/1912.03290.

B. Callaway and P. H. Sant'anna. Difference-in-Differences with Multiple Time Periods. *Journal of Econometrics*, 225(2):pp. 200–230, Mar. 2021. Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3148250.

B. Callaway, A. Goodman-Bacon, and P. H. Sant'Anna. Difference-in-Differences with a Continuous Treatment. Available at https://arxiv.org/abs/2107.02637., July 2021.

M. D. Cattaneo, Y. Feng, F. Palomba, and R. Titiunik. Uncertainty Quantification in Synthetic Controlswith Staggered Treatment Adoption. Available at https://mdcattaneo.github.io/papers/Cattaneo-Feng-Palomba-Titiunik_2024_wp.pdf., 2023.

L. Davezies, X. DHaultfoeuille, and L. Laage. Identification and Estimation of Average Marginal Effects in Fixed Effects Logit Models. Available at `https://arxiv.org/abs/2105.00879.`, May 2021.

B. Ferman, C. Pinto, and V. Possebom. Cherry Picking with Synthetic Controls. *Journal of Policy Analysis and Management*, 39(2):pp. 510–532, 2020.

A. F. Galvao. Quantile Regression for Dynamic Panel Data with Fixed Effects. *Journal of Econometrics*, 164(1):pp. 142–157, 2011. Available at `https://doi.org/10.1016/j.jeconom.2011.02.016.`

W. Y. Gao and M. Li. Robust Semiparametric Estimation in Panel Multinomial Choice Models. Available at `https://www.waynegao.com/uploads/8/1/4/6/81465138/gl_210327.pdf.`, Mar. 2021.

A. Goodman-Bacon. Difference-in-Differences with Variation in Treatment Timing. *Journal of Econometrics*, 225(2):pp. 254–277, 2021.

S. Han. Identification in Nonparametric Models for Dynamic Treatment Effects. *Journal of Econometrics*, 225, 2021.

J. J. Heckman, J. E. Humphries, and G. Veramendi. Dynamic Treatment Effects. *Journal of Econometrics*, 191(2):pp. 276–292, 2016.

S. Kwon. Optimal Shrinkage Estimation of Fixed Effects in Linear Panel Data Models. Available at `https://soonwookwon.github.io/files/Soonwoo_Kwon_JMP.pdf`., Jan. 2021.

L. Laage. A Correlated Random Coefficient Panel Model with Time-Varying Endogeneity. Available at `https://arxiv.org/abs/2003.09367`., Mar. 2020.

M. Li. A Time-Varying Endogenous Random Coefficient Model with an Application to Production Functions. Available at https://uc6c705c8f483c94e338df344781.dl.dropboxusercontent.com/cd/0/inline2/BfB6ZQh8j0BkLg17Cy2aZgLPf4o3x6Wj-EX5ddq_82dp8wn4Iki13tJYlcb8VRelF8u9Y2H6wlz4JUMNlIV17_qRGgrdAApEfXjfAiEO5bRgeIgT03T1ZKjIEGn209tXrg1yOhxdFMrhAtpVKv3MaXxclaqRsDBa
mYbsAJp5v-8G7kS7d8f2AM0fJULA8NkzHTT3PVKSYNhXjvXPHLrdX_LRYPRYIWLjfdwCSKsI5gy7lGCrYRszfySIT0Xys5L86H1qINt1flu0Ob8_yNy9ZNnxvH5hQKxnl4VQul24v3pPGc480Id_4JhPIL2PCF0kQAEDxxbN5i8vU4QTyP2-t8RuB0dNmisSD4/file#., Oct. 2021.

W. S. Robinson. Ecological Correlations and the Behavior of Individuals. *American Sociological Review*, 15(3):pp. 351–357, 1950.