Probability Practice Assignment

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2024-08-07

R Probability Practice

Part A. Visitors to your website are asked to answer a single survey question before they get access to the content on the page. Among all of the users, there are two categories: Random Clicker (RC), and Truthful Clicker (TC). There are two possible answers to the survey: yes and no. Random clickers would click either one with equal probability. You are also giving the information that the expected fraction of random clickers is 0.3. After a trial period, you get the following survey results: 65% said Yes and 35% said No. What fraction of people who are truthful clickers answered yes? Hint: use the rule of total probability.

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1. Define the probabilities and given data:

- P(RC) = 0.3 #Random Clickers
- P(TC) = 1 - P(RC) = 0.7 #Truthful Clickers
- P(Yes|RC) = 0.5
- (Yes) = 0.65

2. Rule of total probability (What fraction of people who are truthful clickers answered yes)

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

$$P(Yes) = P(Yes|RC)P(RC) + P(Yes|TC)P(TC)$$

- 0.65 = (0.5 * 0.3) + P(Yes|TC) * 0.7
- 0.65 = 0.15 + P(Yes|TC) * 0.7
- 0.50 = P(Yes|TC) * 0.7
- P(Yes|TC) = 0.7143

Fraction of people who are truthful clickers that answered yes is 71.43%

Part B. Imagine a medical test for a disease with the following two attributes:

- The sensitivity is about 0.993. That is, if someone has the disease, there is a probability of 0.993 that they will test positive.

- The specificity is about 0.9999. This means that if someone doesn't have the disease, there is probability of 0.9999 that they will test negative.

- In the general population, incidence of the disease is reasonably rare: about 0.0025% of all people have it (or 0.000025 as a decimal probability).

Suppose someone tests positive. What is the probability that they have the disease?

1. Define the probabilities and given data:

- P(Positive Test|Has Disease) = 0.993

- P(Negative Test|Doesn't Have Disease).= 0.9999
- P(Has Disease) = 0.000025
- P(Doesn't Have Disease) = 1 - P(Has Disease) = 0.999975

2. What is the probability that hey have the disease if the person tests positive?

$$P(HasDisease|PositiveTest)$$

$$P(\text{Has Disease}|\text{Positive Test}) = \frac{P(\text{Positive Test}|\text{Has Disease}) \cdot P(\text{Has Disease})}{P(\text{Positive Test})}$$

- First find P(Positive Test) using rule of total probabilities

$$P(\text{Positive Test}) = P(\text{Positive Test}|\text{Has Disease}) \cdot P(\text{Has Disease}) + P(\text{Positive Test}|\text{Doesnt Have Disease}) \cdot P(\text{Doesnt Ha}$$

- P(Positive Test) = 0.0001248
- Use Bayes Theorem
- P(Has Disease|Positive Test) = 0.1988

The probability that a person who tested positive actually has the disease is around 19.88%