The impact of syntrophic interaction on microbial communities

Léo Buchenel & Alberto Pascual-García

ETH Zürich

May 23, 2020

Presentation plan

Introduction to the model

Syntrophic interaction

INSERT DRAWING OF SYNTROPHIC MCs

Introduction to the model

Processes at play

INSERT EXPLANATION OF ALL DIFFERENT PROCESSES AT PLAY (DRAWING)

Introduction to the model

Dynamical Equations

Temporal evolution of the resources R_μ ($\mu = 1, \ldots, N_R$) and the consumers S_i ($i = 1, \ldots, N_S$):

$$\begin{cases} \dot{R}_\mu = \left(\sum_{j=1}^{N_S} \gamma_{j\mu} S_j - m_\mu \right) R_\mu + \sum_{j=1}^{N_S} \alpha_{\mu j} S_j + l_\mu & \text{(1a)} \\ \dot{S}_i = \left(\sum_{\nu=1}^{N_R} \sigma_{i\nu} \gamma_{i\nu} R_\nu - d_i - \sum_{\nu=1}^{N_R} \alpha_{\nu i} \right) S_i & \text{(1b)} \end{cases}$$

Introduction to the model

Metaparameters framework

INSERT DIAGRAM WHERE EXPLAIN METAPARAMETERS

Feasibility

Physical Requirements

Impose two conditions:

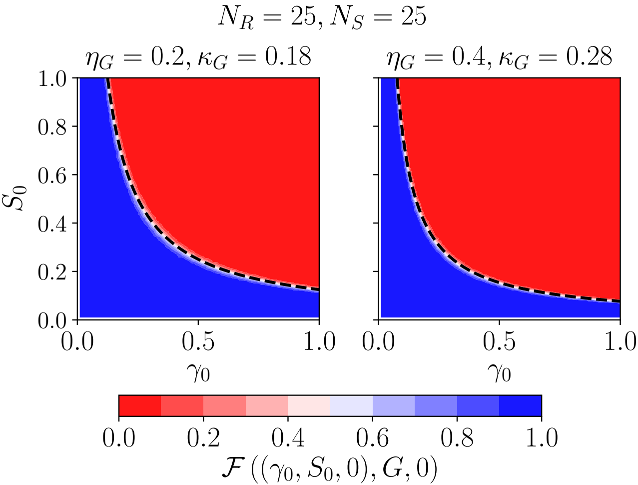
- Conservation of biomass
- Positivity of the parameters

$\rightarrow$ restrictions on parameters $\rightarrow$ restrictions on metaparameters

Feasibility

Feasibility profile

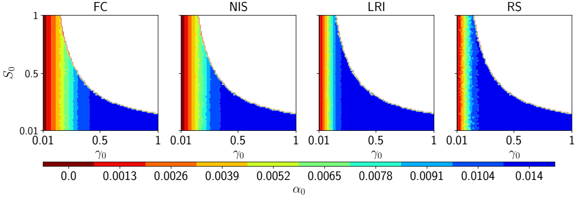
Without syntrophy: full feasibility for γ_0 below curve $\sim S_0$



Feasibility

Fully feasible region

Addition of syntrophy → only high γ_0, low S_0 remain feasible



Dynamical stability

Structural stability

Backup slides – Feasibility

Physical Requirements

- We impose that – at equilibrium – no biomass can be created out of nothing, which is translated mathematically by the constraint:

$$\sum_{\nu=1}^{N_R} (1-\sigma_{i\nu})\,\gamma_{i\nu} R^*_\nu \geq \sum_{\nu=1}^{N_R} \alpha_{\nu i} \quad \forall i = 1, \ldots, N_S. \tag{2}$$

- We need every parameter in the model to be positive. In total there are $3N_R + 2N_S + 3N_R N_S$ parameters, constrained by the $N_R + N_S$ fixed points equations. So we pick $2N_R + N_S + 3N_R N_S$ positive parameters and we make sure they verify:

$$\begin{cases} d_i = \sum_{\nu=1}^{N_R} (\sigma_{i\nu}\gamma_{i\nu} R^*_\nu - \alpha_{\nu i}) > 0 \quad \forall i = 1, \ldots, N_S, & \text{(3a)} \\ m_\mu = \dfrac{l_\mu - \sum_{j=1}^{N_S} \left(\gamma_{j\mu} R^*_\mu - \alpha_{\mu j}\right) S^*_j}{R^*_\mu} > 0 \quad \forall \mu = 1, \ldots, N_R. & \text{(3b)} \end{cases}$$

Backup slides – Feasibility

Theoretical predictions

- **Fully feasible region:** Transforming Eqs.(2) and (3) in their metaparameters form, we get an equation that roughly describes the fully feasible zone:

$$\min(1-\sigma_0, \sigma_0)\gamma_0 R_0 \gtrapprox \max_i \left\{ \frac{\deg(A,i)}{\deg(G,i)} \right\} \alpha_0 \tag{4a}$$

$$\gamma_0 R_0 \lessapprox \min_\nu \left\{ \frac{l_0}{\deg(G,\nu) S_0} + \frac{\deg(A,\nu)}{\deg(G,\nu)} \alpha_0 \right\} \tag{4b}$$