As explained in Methods **??**, before addressing the question of the stability of a system, be it dynamical or structural, it is important to study whether that system is *feasible*. In short we must answer the question : “does it make sense to talk about this system? Does it even exist?”.

We will say that it makes sense to talk about a system if it is *feasible* (see Methods **??**). In order to be feasible, a system should respect two conditions : it must conserve biomass and its parameters must have a direct biological interpretation.

0.0.1 The feasibility volume $\mathcal{V}_x^{G,A}$

Formally we can define $\forall x \in [0,1]$ the x-feasibility volume $\mathcal{V}_x^{G,A} \subset \mathcal{M}$ of the consumption network coupled with the syntrophy network $(G,A) \in \mathcal{B}_{N_S \times N_R} \times \mathcal{B}_{N_R \times N_S}$ (see Methods **??**). Every metaparameter set $m \in \mathcal{M}$ contained in the x-feasibility volume $\mathcal{V}_x^{G,A}$ will give rise to a percentage x of feasible systems. A first order approximation of the fully feasible volume $\mathcal{V}_1^{G,A}$ is given by Eq.(**??**). In the absence of syntrophy $\alpha_0 = 0$, it becomes :

$$\gamma_0 R_0 \lessapprox \frac{l_0}{\max_\nu\{\deg(G,\nu)\} S_0}. \tag{1}$$

This relation is interesting in many ways. First of all it tells us that at fixed consumption rate γ_0 and resource equilibrium abundance R_0, feasibility increases when :

- the external resource input rate increases. This result was somewhat expected : if you give more food to a goldfish you expect it to thrive more.
- the consumer equilibrium abundance S_0 increases. What this means is that if you want to maintain the same consumption interaction but get a higher abundance of resources at equilibrium, you must at the same time decrease the consumers equilibrium abundance.
- the largest column-degree of the consumption matrix decreases. The degree of a given column ν of the consumption matrix tells you how many species eat from resource ν. This encourages communities of specialists, where each consumer eats from its own and no other resource.

Overall we see that feasibility increases when the *consumption flow* $\simeq \gamma_0 S_0 \deg(G,\nu)$ is low $\forall \nu$. Eq.(**??**) can be confronted to simulations. Figure **??** shows the proportion of feasible systems without syntrophy $\mathcal{F}(\gamma_0, S_0, \alpha_0 = 0, G)$ and $R_0 = l_0 = 1$, $\sigma_0 = 0.25$ (see Methods **??**), for two matrices G_1 and G_2 of our set. G_1 has connectance $\kappa_1 = 0.17$ and nestedness $\eta_1 = 0.2$, G_2 is more connected and more nested : $\kappa_2 = 0.37$ and $\eta_2 = 0.4$.

We observe a very sharp transition from a fully feasible to a fully unfeasible regime. Theoretically, this sharp transition happens when both sides of the inequality (**??**) are equal, *i.e.* at $\gamma_0 R_0 = l_0 / \max_\nu\{\deg(G,\nu)\} S_0$. Numerically we fit the points which are at “the boundary” of the common feasible volume, *i.e.* points where $0.4 \leqslant \mathcal{F}(\gamma_0, S_0, G) \leqslant 0.6$.

For G_1, the theoretical expectation is $S_0 = 0.125/\gamma_0$ and a fit on the numerical results gives $S_0 = (0.124 \pm 7 \times 10^{-8})/\gamma_0 - (6.8 \times 10^{-4} \pm 3 \times 10^{-7})$ so the theoretical relation is

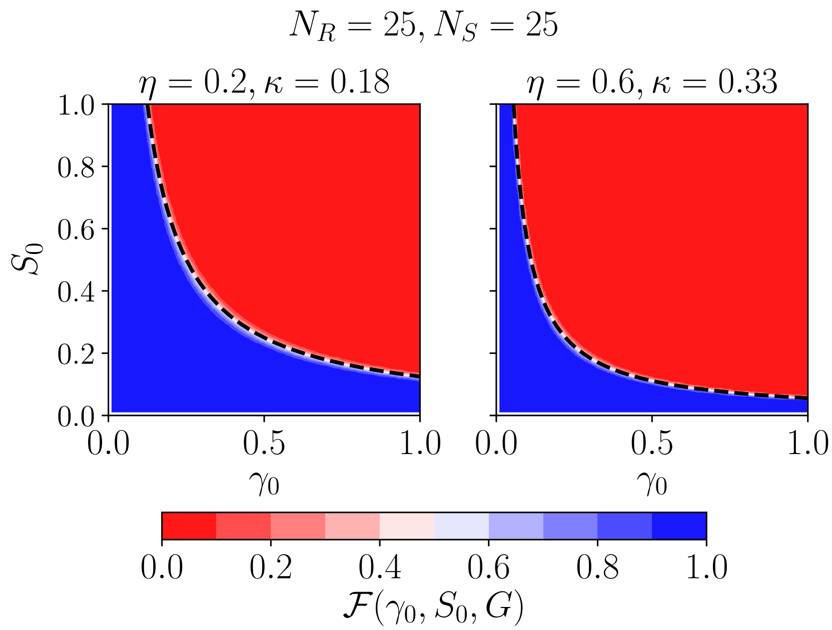

Figure 1: Plot of the feasability region. The color curve indicicates the feasibility function $\mathcal{F}(\gamma_0, S_0, \alpha_0 = 0, G)$ for G_1 (left) and G_2 (right). We observe a steep descent which marks a very clear transition from a totally feasible regime to a totally unfeasible regime, which allows us to precisely get the boundary of $\mathcal{V}_1^G$. The dashed lines indicate the theoretical predictions, which for both G_1 and G_2 are accurate to the order of 0.1%.

already very good. For G_2, we expect $S_0 = 0.077/\gamma_0$. A fit gives $S_0 = (0.075 \pm 2 \times 10^{-8})/\gamma_0 + (3.6 \times 10^{-4} \pm 1 \times 10^{-7})$. Again, the theoretical value is very close to the measured value.

The numerical estimate does not always match that well with the theoretical value. Fig.**??** shows the relative error $\Delta_G = 1-$ (theoretical value)/(numerical estimate). We see that in general the theoretical expectation tends to overestimate the fully feasible region. This is probably due to the noise (*i.e.* the deviations away from the metaparameters) in the actual systems and the structure of the G matrix. Indeed Fig.**??** shows that the lower the nestedness and connectance of G, the worse the theoretical estimate. In the future a better approximation can surely be found taking into account the variance of the metaparameters and the nestedness of G.

We can similarly measure the common fully feasible volume $\mathcal{V}^*$, which according to Eq.(**??**) is inversely proportional to the largest maximal row degree of the matrix set. For the set we considered, this yields in theory : $S_0 = 0.053\gamma_0$. A fit on the points at the edge yields the critical boundary $S_0 = (0.043 \pm 10^{-8})/\gamma_0 - (4.6 \times 10^{-3} \pm 3 \times 10^{-8})$. The theoretical prediction is not as good as before with an error of ~ 20 %. The discrepancy is probably due to the fact that numerically we determine the common feasibility volume by counting the points for which $\mathcal{F}(\gamma_0, S_0, G) = 1\ \forall G \in S_G$. while the theoretical value matches better a fit of the points $0.4 \leqslant \mathcal{F} \leqslant 0.6$ **make this more understandable**.

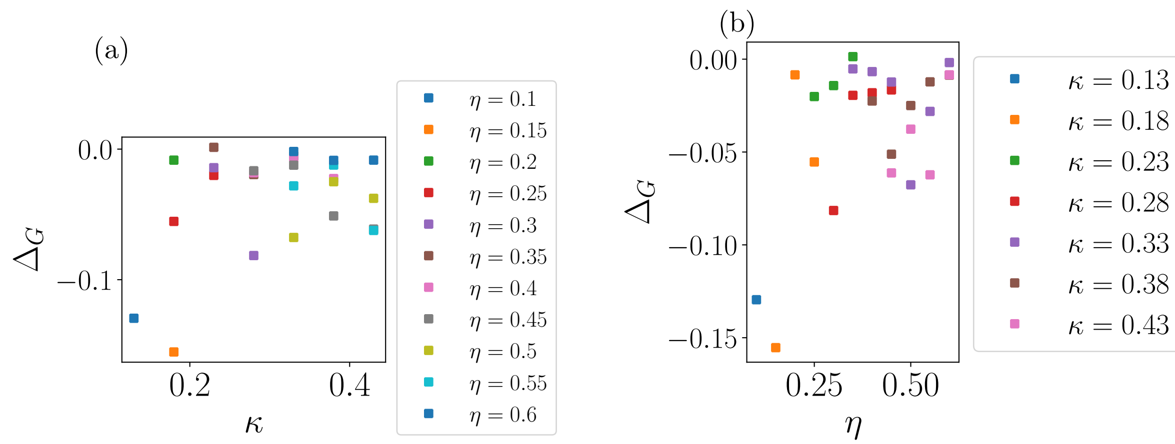

Figure 2: Relative error in the determination of the boundary of $\mathcal{V}_1^{G,0}$ (a) varying connectance at fixed nestedness and (b) varying nestedness at fixed connectance. The theoretical prediction tends to overestimate the measured value. The larger the nestedness and connectance, the better the estimate.

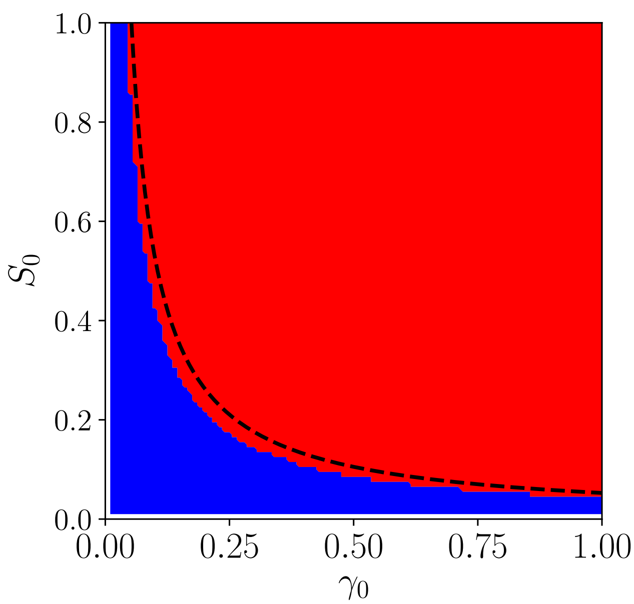

Figure 3: Plot of the common feasability region. The blue area indicates the common feasibility volume, computed numerically, while the dashed line shows the analytical prediction. Although the match is not as good as before, the relative error is only of the order of 20%. The red part is the area where not all matrices are fully feasible. From now on, it will not be considered anymore.