Erosion

Perfect Match $\rightarrow$ set pixel value to 1 otherwise `0`.

The Blacks increase and the white decreases.

Shrinks or thins the object

Image details smaller than the structuring element are filtered out.

Dilation

White increase and black decreases.

Perfect Match and partial match $\rightarrow$ set pixel value to 1 of the structure element otherwise `0`.

Grows or thickens objects.

Erosion and dilation are complements of each other: DUALITY

Opening

Erosion followed by Dilation.

Closing

Dilation followed by erosion.

Opening and closing are duals of each other.

Properties Of Opening

- Performing opening of A by B results in an image which is a subset of original image.
- Performing opening of A by B and the result by B and the result will the same as opening obtained first (will have no effect if we apply opening more than once).

Hit or Miss Transform

- Basic tool for shape detection.
- utilizes two structuring elements, one for detecting shapes in the foreground and one for detecting shapes in the background.
- Perform erosion of foreground by first SE and then perform erosion of background with the second SE and take the union of them.

In a single SE

Define a SE identical to the object to detect but having in addition border of the background elements with a width of 1 pixel. Now we see if the SE is subset of the image.

Boundary Extraction

First Erode A by B (SE), then perform the difference between A and the erosion obtained.

Hole Filling

- Form an array X_0 of the same size as I and set the values to 1 where there is a hole
- Perform dilation of X_{k-1} with B and taking the intersection with I^c to obtain X_k.
- Perform the above until $X_k = X_{k-1}$.
- X_k contains all the filled holes and $X_k \cup I$ contains all the filled holes and their boundaries.

$$X_k = \left(X_{k-1} \oplus B\right) \cap I^c \qquad k = 1, 2, 3, \ldots$$

Extraction of Connected Components

$$X_k = \left(X_{k-1} \oplus B\right) \cap I \qquad k = 1, 2, 3, \ldots$$

Convex Hull

A set A is set to be **convex**, if any two points in A are forming a straight line which is entirely in A.

The **convex hull** H of an arbitary set S is the smallest convex set containing S.

- Four SEs are used.
- We perform hit or miss transform of X_{k-1} with every SE and perform union with A to obtain X_k^i.
- Stop when $X_k^i = X_{k-1}^i$
- Take $X_0^i = A$.
- Perform union of every $X_k^i, for\ i = 1, 2, 3, 4.$

$$X_k^i = (X_{k-1} \circledast B^i) \cup A$$
$$i = 1,2,3,4 \quad \text{and} \quad k = 1,2,3,\ldots$$

with $X_0^i = A$.

When the procedure converges, or $X_k^i = X_{k-1}^i$, let $D^i = X_k^i$, the convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i$$

Thinning

- First perform hit or miss tranform of A with B
- Then subtract the transform from A.

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

where B^i is a rotated version of $B^{i\text{-}1}$

The thinning of A by a sequence of structuring element $\{B\}$

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

Thickening

- Perform hit or miss transform of A with B
- Take union of A and the transform.

Thinning and thickening are duals of each other.

Pruning

Thickening and skeletonizing tends to leave parasitic components, pruning eliminates them.

Morphological Reconstruction

Involves two images and a SE.

- One image consists the starting points for the transformation (Marker)
- Another image (mask) constrains the transformation
- The structuring element is used to define connectivity.

Geodesic Dilation

- Perform dilation of Marker with SE
- Take intersection with the mask.

$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

The geodesic dilation of size n of the marker image F with respect to G, denoted by $D_G^{(n)}(F)$, is defined as

$$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$$

with $D_G^{(0)}(F) = F$.

Digital Image Processing

Perform geodesic dilation one time with the same $n-1$ times.

Geodesic Erosion

- Perform Erosion of marker with SE
- Take union with the mask.

$$E_G^{(1)}(F) = \left(F \ominus B\right) \cup \mathrm{G}$$

Perform these operations until stability is achieved (we can go no further reconstruction wise or no changes afterwards)

Opening by Reconstruction

- Perform n erosions of the image
- Reconstruct the result by dilation with mask as the image itself.
- This will give the opening

$$O_R^{(n)}(F) = R_F^D\left[\left(F \ominus nB\right)\right]$$

where $\left(F \ominus nB\right)$ indicates n erosions of F by B.

n is in the subscript of the operator.

Filling Holes with reconstruction

- Form F which is having `0` everwhere except the image border where it is set ti $1 - I$ where I is the original binary image.
- Then perform reconstruction by dilation of F with mask as complement of I
- Take the complement of the above.

$$H = \left[R_{I^c}^D(F)\right]^c$$

Border Clearing

$$X = I - R_I^D(F)$$

$$F(x,y) = \begin{cases} I(x,y) & \text{if } (x,y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

- Take F as I but having only borders.
- Perform reconstruction by dilation with mask as original image.
- Subtract the result from the original image.

Morphological Smoothing

- Opening supresses bright details smaller than the spcified SE.
- Closing supresses dark details.
- Opening and closing are often used in combination for image smoothing and noise removal.

Morphological Gradient

- Perform dilation with SE
- Perform erosion with SE
- subtract the two.

$$g = (f \oplus b) - (f \ominus b)$$

The **edges are enchanced** and the **contribution of the homogeneous areas are suppressed**, producing a derivative like effect.

Top Hat

An image minus its opening.

Bottom Hat

Closing of an image minus the image itself.

USE: Removing objects by using SE in opening or closing operation.

Granulometry

- Determining the size of distrbution of particles in an image
- Opening will have the most effect on areas of particals of similar size.
- For each opening, the sum of intensity values in the opening is calculated.

Morphological

- Perform opening by reconstruction
- Subtract the opening from the original image
- Again perform opening by reconstruction of the above result using an alternate SE.