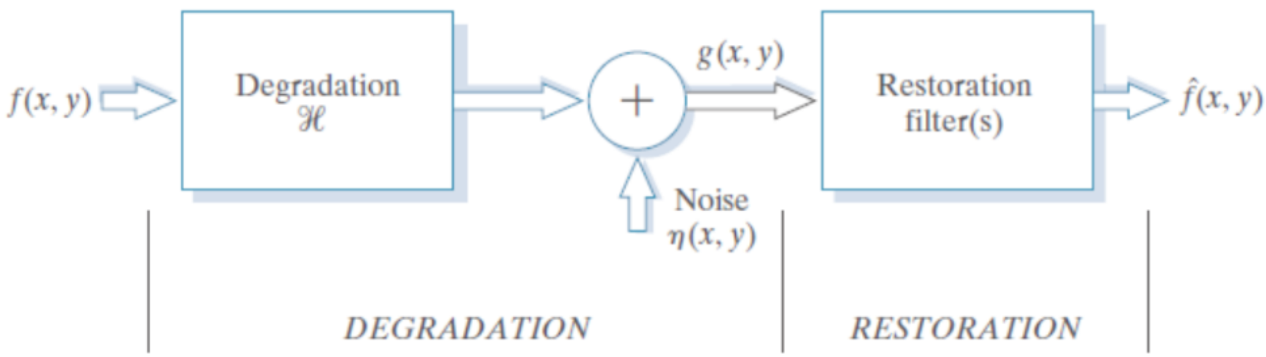
Image Restoration and Reconstruction

A Model of Image Restoration



Degradation

- Degradation Function ($\mathcal{H}$)
- Additive Noise $\eta(x, y)$

If $\mathcal{H}$ is a linear, position-invariant process, then the degraded image in the spatial domain is given by

$$g(x, y) = \mathcal{H}(x, y) * f(x, y) + \eta(x, y)$$

The Model of the degraded image in the frequency domain is given by:

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Noise Models

Principle Sources Noise

- During image acquisition
- During transmission

When the fourier spectrum of noise is constant, the noise is called **white noise**.

Gaussian Noise

The PDF of a Gaussian random variable, z, is defined by the following

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}}, \quad \infty < z < \infty$$

where,

$z = intensity$
$\bar{z} = mean$
$\sigma = standard\ deviation$

The probability that values of z are in the range $\bar{z} \pm \sigma$ is approximately 0.68; the probability is about 0.95 in the range $\bar{z} \pm 2\sigma$.

Rayleigh Noise

The PDF of Rayleigh Noise is given by:

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$$

The mean and variance of z when this random variable is characterised by a Rayleigh Noise PDF are:

$$\bar{z} = a + \sqrt{\pi b/4}$$

and

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

The Rayleigh density can be quite useful for modeling the shape of skewed histograms.

Erlang (Gamma) Noise

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!}e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

where the parameters are such that $a > b$, b is a positive integer.

$$\bar{z} = \frac{b}{a}$$

and

$$\sigma^2 = \frac{b}{a^2}$$

Exponential Noise

$$p(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

where $a > 0$

$$\bar{z} = \frac{1}{a}$$

$\sigma^2 = \frac{1}{a^2}$

This is a special case of erland PDF with $b = 1$.

Uniform Noise

$$p(z) = \begin{cases} \frac{1}{b-a} & a \leq z \leq b \\ 0 & otherwise \end{cases}$$

$\bar{z} = \frac{a+b}{2}$

$\sigma^2 = \frac{(b-a)^2}{12}$

Salt-and-Pepper Noise

$k = Number\ of\ bits$

Range of possible intensity values $= [0, 2^k - 1]$

$$p(z) = \begin{cases} P_s & z = 2^k - 1 \\ P_p & z = 0 \\ 1 - (P_s + P_p) & z = V \end{cases}$$

where $V \in (0, 2^k - 1)$

Let $\eta(x, y)$ denote the salt-and-pepper noise.

Given an image $f(x, y)$ and $\eta(x, y)$ of same sizes.
We corrupt the image by assigning 0 in f where there is a 0 in η and do the same for $2^k - 1$.
Rest values we leave unchanged.

If neither P_s nor P_p is zero, and especially if they are equal, noise will be white $(2^k - 1)$ or black(0) and will resemble salt and pepper granules distributed randomly over the image.

The probability, $\mathcal{P}$, that a pixel is corrupted by salt or pepper noise is $\mathcal{P} = P_s + P_p$.

$$\bar{z} = (0)P_p + K(1 - P_s - P_p) + (2^k - 1)P_s$$

and

$$\sigma^2 = (0 - \bar{z})^2 P_p + (K - \bar{z})^2 (1 - P_p - P_s) + (2^k - 1)^2 P_s$$

Periodic Noise

- Arises typically from electrical or electromagnetic interference during image acquisition.
- Type of spatial dependent noise
- Can be reduced by **frequency domain filtering**.

Estimation of Noise Parameters

- By using Graph and comparing with different noise models
- By mean and variance

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_s(z_i)$$

and

$$\sigma^2 = \sum_{i=0}^{L-1} (z - \bar{z})^2 p_s(z_i)$$

Presence of Noise Only : Spatial Filtering

Noise Model without degradation

$$g(x, y) = f(x, y) + \eta(x, y)$$

and

$$G(u, v) = F(u, v) + N(u, v)$$

Spatial Filtering : Mean Filtering

Let S_{xy} represent the set of coordinates in a rectangle subimage **window** of size $m \times n$, centered at (x, y)

Arithmetic mean filter:

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(r,c) \in S_{xy}} g(r, c)$$

Geometric mean filter:

$$\hat{f}(x, y) = [\prod_{(r,c) \in S_{xy}} g(r, c)]^{\frac{1}{mn}}$$

Generally, a geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process.

Harmonic mean filter:

$$\hat{f}(x, y) = \frac{mn}{\sum_{(r,c) \in S_{xy}} \frac{1}{g(r,c)}}$$

Works well for salt noise and gaussian noise, but fails for pepper noise.

Contraharmonic mean filter:

$$\hat{f}(x, y) = \frac{\sum_{(r,c) \in S_{xy}} g(r,c)^{Q+1}}{\sum_{(r,c) \in S_{xy}} g(r,c)^{Q}}$$

$Q = order\ of\ the\ filter$

Well suited for reducing or virtually eliminating the effects of salt-and-pepper noise.

$Q = +ve \rightarrow$ eliminates pepper noise
$Q = -ve \rightarrow$ eliminates salt noise
$Q = 0 \rightarrow$ arithmetic mean
$Q = -1 \rightarrow$ harmonic mean

Median Filter

- Best known order-statistic filter.
- Considerbaly less blurring with excellent results.
- Particularly effective in the presence of both bipolar and unipolar impulse noise

$$\hat{f}(x, y) = median_{(r,c)\in S_{xy}}\{g(r, c)\}$$

Replaces the pixel with median of the intensity values in the predefined neighborhood.

Min and Max filters

$$\hat{f}(x, y) = \max_{(r,c)\in S_{xy}}\{g(r, c)\}$$

- Useful for finding brightest spots or for eroding dark regions next to those bright spots.
- Since Pepper noise has very low values, it is reduced by this filter.

$$\hat{f}(x, y) = \min_{(r,c)\in S_{xy}}\{g(r, c)\}$$

- Useful for finding darkest spots or for eroding light spots next to those regions.
- Reduces salt noise.

Midpoint filter

$$\hat{f}(x, y) = \frac{1}{2}(\max_{(r,c)\in S_{xy}}\{g(r, c)\} + \min_{(r,c)\in S_{xy}}\{g(r, c)\})$$

- Works best for randomly distributed noise.

Alpha-trimmed Mean filter

Suppose we delete $d/2$ lowest and $d/2$ highest intensity values of $g(r, c)$. The remaining pixels $= mn - d$.

$$\hat{f}(x, y) = \frac{1}{mn-d}\sum_{(r,c)\in S_{xy}} g_R(r, c)$$

where d can range from 0 to $mn - 1$.

If $d = 0 \rightarrow$ mean filter
If $d = mn - 1 \rightarrow$ median filter

- Useful where there is multiple type of noises, such as combination of salt-and-pepper and gaussian noise.