Bio 417 Homework 1

Part 2 (for the week of Feb 4th, due Feb 11th)

1. (Condition for growing population) As mentioned in the lecture, the expected number of offspring over the lifetime of an individual is given by $R_0 = \int_0^k l(x)m(x)dx$, but this is not equivalent to the population growth rate r, found from the Euler-Lotka equation. Show that, nonetheless, $R_0 > 1$ gives the condition for the population to grow. In other words, establish that $r > 0$ whenever $R_0 > 1$ and not otherwise. (Hint: think about the derivative of the right-hand side of the Euler-Lotka equation with respect to r).

Following the hint, the derivative of the right-hand side of the of the Euler-Lotka equation is:

$$\frac{d}{dr}\int_0^k e^{-rt}l(t)m(t)dt = \int_0^k -te^{-rt}l(t)m(t)dt < 0 \, ,$$

since every term in the integral is positive. Now, observe that, from the definition of R_0,

$$R_0 = \int_0^k e^{-0*t}l(t)m(t)dt = \int_0^k l(t)m(t)dt \, .$$

In other words, R_0 is the value of the right-hand side when $r = 0$. Therefore if $r = 0$ solves the Euler-Lotka equation, $R_0 = 1$. Now, if $r > 0$ solves the Euler-Lotka equation, that means the right-hand side of the E-L is greater than 1 for $r = 0$, meaning $R_0 > 1$. Conversely, if the E-L equation holds for $r < 0$, that means for $r = 0$, the RHS is less than 1 when $r = 0$, meaning $R_0 < 1$.

2. (US population) Consider the population growth rate calculated in the iPython notebook “Demography and age structured population growth.” Convert it to a annual change (as percentage of current population size). Google “us population growth rate,” and compare the number that Google displays on top to the number you found. Which is greater? What is your explanation?

The r found in the iPython notebook is roughly 0.05, which translates into an annual growth rate of 5% ($e^{(0.05)} \approx 1.05$). Google should tell you that the actual US growth rate is 0.7%, an order of magnitude lower than the one we calculated. So, what gives? Here are two big explanations, though there are many other reasonable ones:

1. The growth rate r we get from the Euler-Lotka equation is the growth rate of a population at its stable age distribution. The actual US age distribution has a lot more older, post-reproductive individuals than this stable age distribution predicts (see Fig 1.). In other words, the US is in the transient dynamics of its demography (e.g., WW2 and the baby boomers represent a major one-time perturbation to the demography which you can see in the age pyramid of the US in both the baby boomer generation, and the aftershock that represents their offspring.) Note that to reach the stationary distribution, you need to have a long period where the vital rates stay approximately constant. This has not been the case in the past (e.g., the 20th century saw large reductions in mortality at young and old ages, as welll as overall decrease in fertility).

2. The growth rate is calculated assuming everyone in the population is a female, and is capable of giving birth. But in reality, roughly half the population is male, and does not contribute to the population growth by giving birth (and roughly half the offspring is also male, and therefore will not contribute to births as the Euler-Lotka equation assumes). Another way of saying that is that we should've really divided the number of offspring per capita by 2, to count only female offspring.

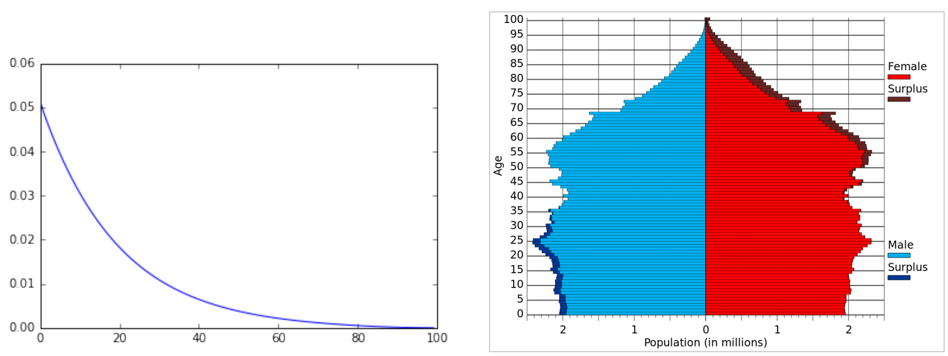

Figure 1: The stable age distribution calculated from the demographic data on the left, and the actual age distribution of the US in 2015 on the right. Notice that the actual distribution has two features: there are both males and females, one of which don't give birth, and there are a lot more older, non-reproductive age individuals of both sexes in the actual distribution.