# Bio 417 Homework 4

## (for the weeks of Mar 11th and 18th, due April 1st)

1. (Selection vs. drift) Consider the two-allele haploid model in a finite population, as simulated in the Jupyter notebook.
a) Set $w_A = 1.01$, and $w_B = 1$. Run 100 replicate populations for 1000 generations, starting with an initial allele frequency for A $p = 0.01$. Run simulations for population sizes $N = 10, 100, 500, 1000, 5000, 10000$ and $100000$. For each population size, record the average final frequency of A in the 100 replicate populations. Plot this final average frequency against population size (use a log-scale for population size). What do you observe? What does that mean for natural selection?
b) Now set $w_A = 1.1$, leaving $w_B = 1$, and do the same as above. What do you observe?

2. (Marginal fitness) Show that you can write the expression for new allele frequency $p'$ in the diploid case (eq 4 in the notes) in an analogous way to the haploid case (eq. 1 in the notes), by defining a composite fitness measure $w_{A*}$. What is this fitness measure, and what is its interpretation?

3. (Long-term evolution in the iterated PD) Assume that inequality 22 in the notes holds (and $s < p < r < t$ as in the notes).
a) Consider a population composed of ALLC at frequency $q$ and TFT at frequency $(1-q)$. Suppose a mutant arises that plays ALLD. Under which conditions, if any, will it be able to increase?
b) Is the TFT strategy evolutionarily stable? Prove your answer.
c) If you have a finite but large population composed of all TFT initially, but ALLC and ALLD can spontaneously arise through mutation, what will happen in the long run?

4. (Fisher's fundamental theorem) Calculate the "transmission bias" ($E[w_i \delta_i]$) term in the Price equation for fitness in the (one shot, i.e., not iterated) Prisoner's Dilemma game model (set $r = b - c$, $t = b$, $p = 0$, and $s = -c$, as in the donation game, with $b > c$), and show that it is always negative and bigger than the covariance term, so that fitness always goes down.