Bio 417 Homework 3

(for the week of Feb 25th, due Mar 25th)

1. (Limiting similarity) Consider the Lotka-Volterra competition equation:

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1 - \alpha_{12} N_2}{K_1} \right) \tag{1}$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{K_2 - N_2 - \alpha_{21} N_1}{K_2} \right) , \tag{2}$$

where α_{12} is the competitive effect of species 2 on 1, and α_{21} is the competitive effect of 1 on 2.

a) Write down the Jacobian matrix, calculate its trace and determinant, and derive the conditions for the internal equilibrium to be stable.

b) Suppose that $K_2 > K_1$ and that $\alpha_{12} = \alpha_{21} = \chi(d)$, where $\chi(d)$ is a strictly decreasing function of a variable d, which measures the difference in the ecological requirements of the two species (e.g., d might be the difference between the optimal soil temperature for growth of two plant species). In other words, the competitive effect of the two species on each other is symmetric, and the more different they are, the less competition they experience from each other. What is the condition for the two species to coexist stably (i.e., for the internal equilibrium to be stable)?

2. (Community stability) Consider again the symmetric Lotka-Volterra competition as above, with $\alpha_{12} = \alpha_{21} = \alpha$. To make things simpler, assume $r_1 = r_2 = K_1 = K_2 = 1$.

a) Show that in this special case, the condition for stable co-existence is $\alpha < 1$.

b) Now imagine a third species, that competes with both of the existing species. In particular, assume that the competition coefficient for the third species is $\alpha_{13} = \alpha_{31} = \alpha_{23} = \alpha_{32} = \beta$, and that $K_3 = r_3 = 1$. Write down the new system of equations.

c) Now, write the Jacobian for this three-dimensional system, and evaluate the Jacobian at the equilibrium where $N_3 = 0$ but $N_1, N_2 > 0$ (which corresponds to the internal equilibrium for the two-dimensional case in part (a).)

d) Show that the condition for the third species to invade (increase from zero) is:

$$\frac{1+\alpha}{2} > \beta\,.$$

(Hint, you don't have to [but can] use the Jacobian to answer this question.)

3. (Forward simulation) Consider the two-locus haploid model