Bio 417 Homework 1

Part 1 (for the week of Jan 28th, due Feb 11th, 2019)

1. (Gompertz equation) Another model of negative density dependence is the Gompertz equation, which is given by:

$$\frac{dN}{dt} = -aN \ln(bN) \, ,$$

where a and b are positive constants.

a) Plot the flow diagram of the system, identify the equilibria and their stability, both using the graphical method, and using linear stability analysis.

b) What is the biological interpretation of the coefficients a and b?

c) This equation is found to be a good approximation for tumor growth in cancer, except for very small tumor sizes (small N). Explain, based on the equation, why the failure at small tumor sizes is not surprising.

2. (Intermittency) Consider the discrete logistic map:

$$x_{t+1} = \lambda x_t (1 - x_t)$$

a) Using the code in the iPython notebook, plot some trajectories for $\lambda = 3.828$. What do you observe?

b) Plot the map resulting from applying the logistic map 3 times in succession. How does this map explain the behavior you observe in the trajectories? What happens when you increase λ slightly (say, by 0.001)?

c) Discuss the potential biological consequences of the dynamical behavior you observed in part (a).

3. (Limits of the logistic map) The logistic map is customarily considered with λ between 0 and 4, and x between 0 and 1. Explain why these limits make sense. (Hint: show that for $x_0 > 1$, x will eventually go to $-\infty$.)