Bio 417 Homework 1

Part 2 (for the week of Feb 4th, due Feb 11th)

1. (Condition for growing population) As mentioned in the lecture, the expected number of offspring over the lifetime of an individual is given by $R_0 = \int_0^k l(x)m(x)dx$, but this is not equivalent to the population growth rate r, found from the Euler-Lotka equation. Show that, nonetheless, $R_0 > 1$ gives the condition for the population to grow. In other words, establish that $r > 0$ whenever $R_0 > 1$ and not otherwise. (Hint: think about the derivative of the right-hand side of the Euler-Lotka equation with respect to r).

2. (US population) Consider the population growth rate calculated in the iPython notebook "Demography and age structured population growth." Convert it to a annual change (as percentage of current population size). Google "us population growth rate," and compare the number that Google displays on top to the number you found. Which is greater? What is your explanation?

3. (Life history evolution) Suppose you have the a stage structured population, whose dynamics are described by the projection matrix:

$$A = \begin{pmatrix} 0 & 1.5 & 2.5 \\ 0.5 & 0.2 & 0.5 \\ 0 & 0.4 & 0.1 \end{pmatrix}$$

a) Calculate the asymptotic growth rate, stable stage distribution, and reproductive values of the classes.
b) Calculate the matrix of sensitivities of the leading eigenvalue to each element.
c) Suppose that individuals in age class 3 can trade-off investment into survival and reproduction, such that the elements a_{13}, a_{23}, and a_{33} tradeoff as follows:

$$a_{23} = 5a_{33}$$
$$a_{13} = 2.55 - a_{33}/2$$

Given your answer in b), do you expect this species evolve to increase or decrease its reproductive output in age-class 3?