Climate in the Light of Mathematical Equations

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29 June - 5 July 2025

Goal and Approaches

Predicting weather features for Kaunas on a certain day (31st of December 2024)

Approaches:

- **PDEs-Based Simulation:** Modeling weather evolution using the 1D Euler equations with solar forcing.
- **Hybrid SARIMAX–LSTM Model:** Statistical and machine learning methods for time series forecasting.

Governing PDEs: Conservation Laws for Fluid Dynamics

Fluid behavior is modeled using the 1D Euler equations, initialized and constrained by real meteorological data.

$$
\begin{cases}
\partial_t \rho + \partial_x m = 0 & \text{(Mass conservation)} \\
\partial_t m + \partial_x (mu + P) = 0 & \text{(Momentum conservation)} \\
\partial_t E + \partial_x ((E + P)\, u) = Q(t) & \text{(Energy conservation)}
\end{cases}
$$

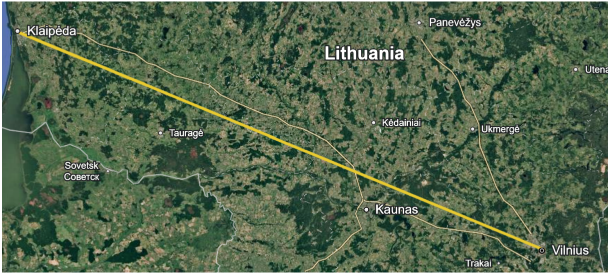
where

- $E = \frac{P}{\gamma - 1} + \frac{1}{2}\rho v^2$: energy density
- ρ: air density
- v: velocity
- P: pressure
- $\gamma \approx 1.4$: air heat capacity ratio
- $Q(t)$: time-dependent solar heating source.

For a numerical solution, apply the Lax–Friedrichs scheme.

Weather Data: Spatial Distribution

To validate the simulation, and to set initial and boundary conditions on the simulation, we use real atmospheric data collected on the specified day (31 December 2024).



- 23 weather stations
- almost in a straight line
- ≈ 13 *km* apart

Results from PDEs-based approach (1)

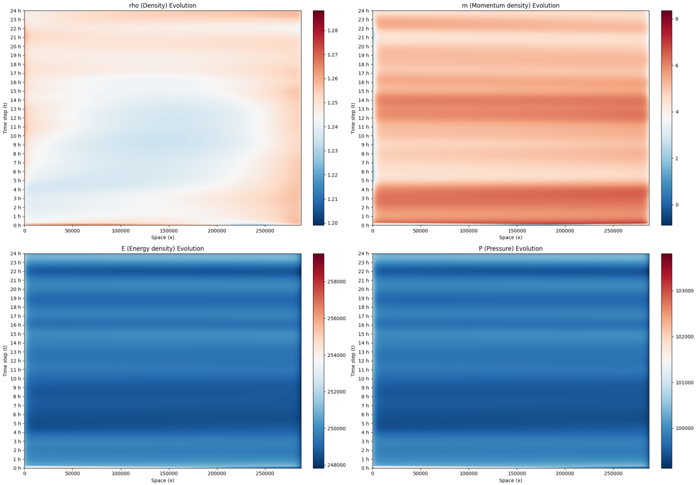


Figure: Heatmap of simulated variables on 31 December 2024 for whole domain

Results from PDEs-based approach (2)

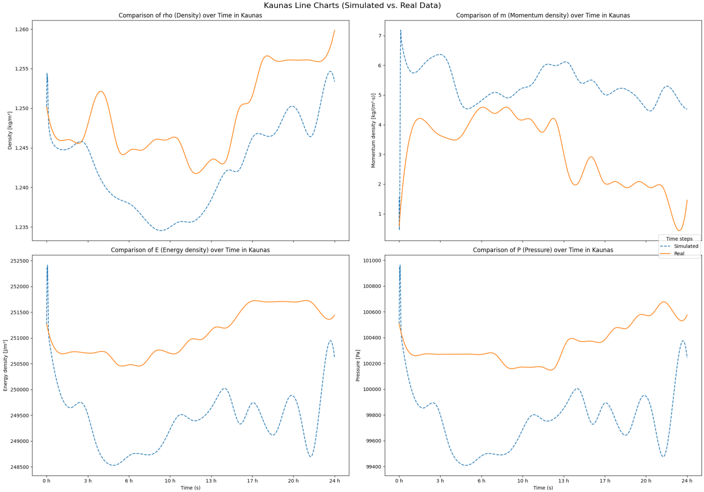
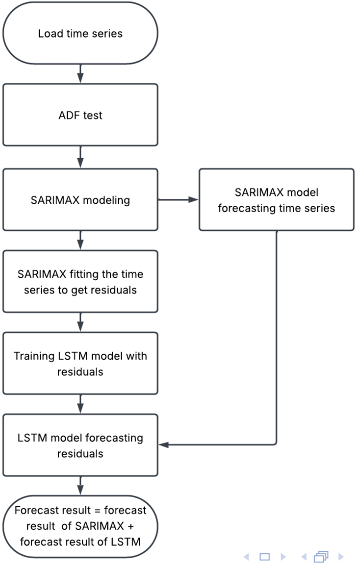


Figure: Comparison between simulated and real data for Kaunas on 31 December 2024

Machine Learning: Hybrid SARIMAX–LSTM Model

- **SARIMAX** (Seasonal Autoregressive Integrated Moving Average with Exogenous Regressors) – extends ARIMA framework - which combines autoregressive (AR), differencing (I), and moving average (MA) components — by modeling seasonality and incorporating external variables.
- **LSTMM** (Long-Short-Term Memory Model) – is a recurrent neural network (RNN) architecture widely used in Deep Learning. It excels at capturing long-term dependencies, making it ideal for sequence prediction tasks.

Hybrid SARIMAX–LSTM Model Algorithm



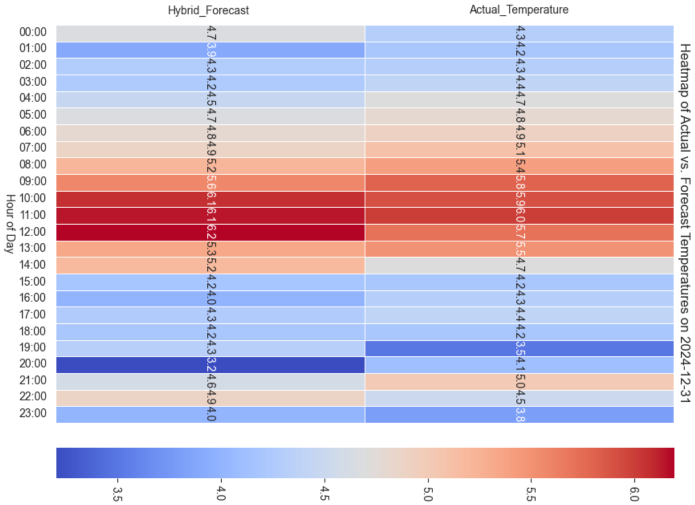


Figure: Heat map for comparison of real and predicted data (31/12/2024)

Results from Hybrid Machine Learning Approach (2)

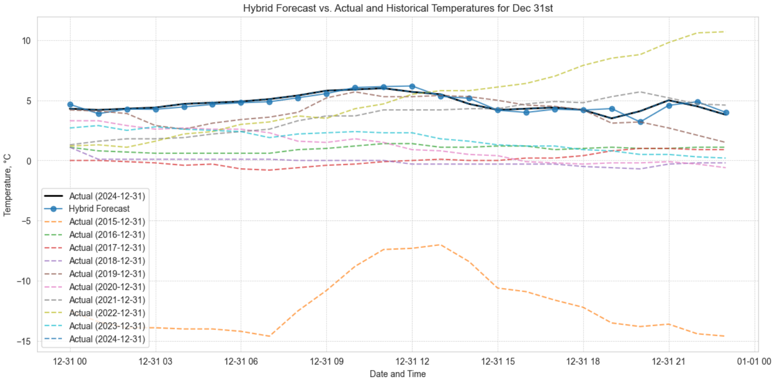


Figure: Comparison by years for 31 December, Kaunas

PDEs-Based Simulation vs Hybrid ML Method Results

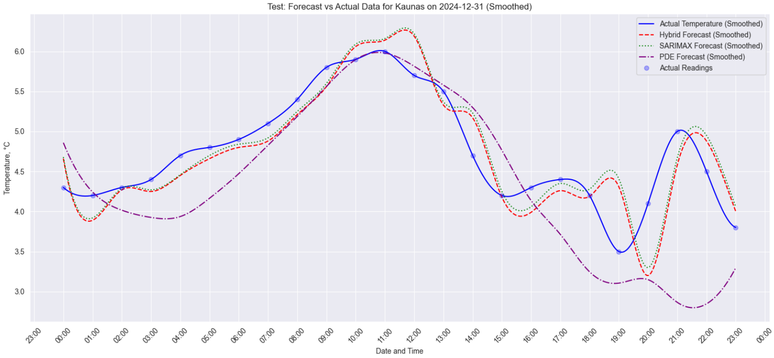


Figure: Comparison of both model testing results with real data and predicted data. Date that exists in the real data records (31 December 2024)

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