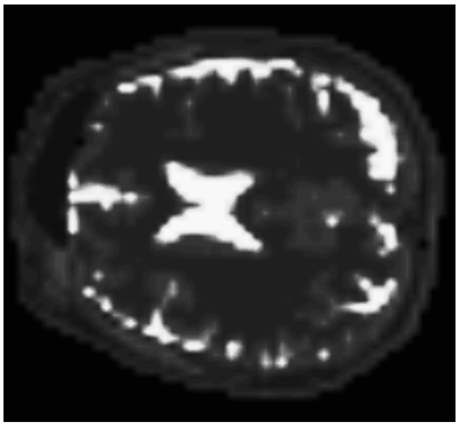
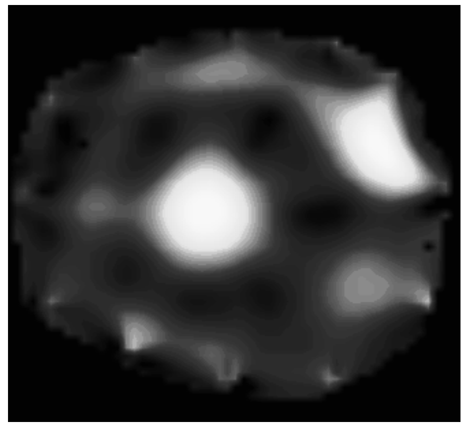
Medical Imaging

Image Reconstruction from Projections





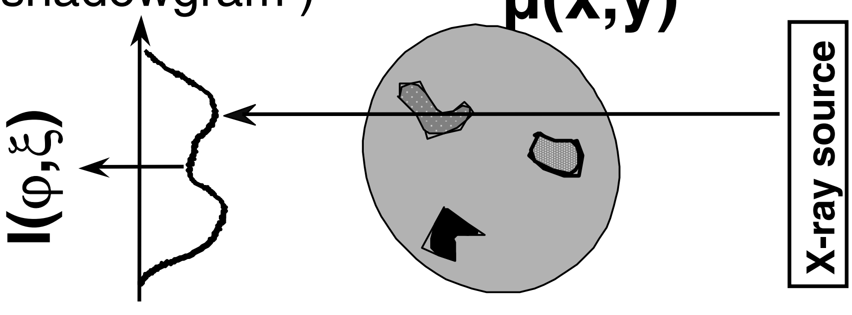
Prof Ed X. Wu

Transmission Measurement

(measurable attenuation, shadowgram)

unknown **absorption** cross-section

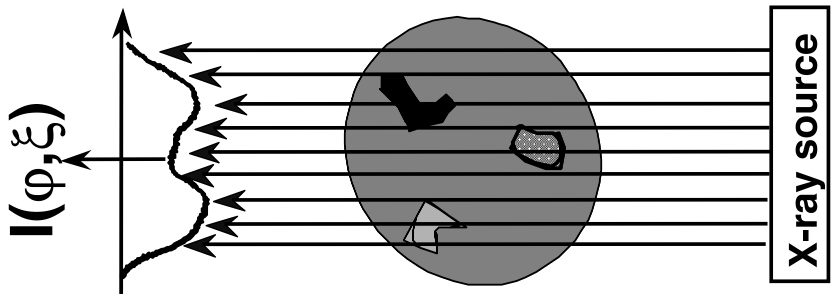
$\mu(x,y)$



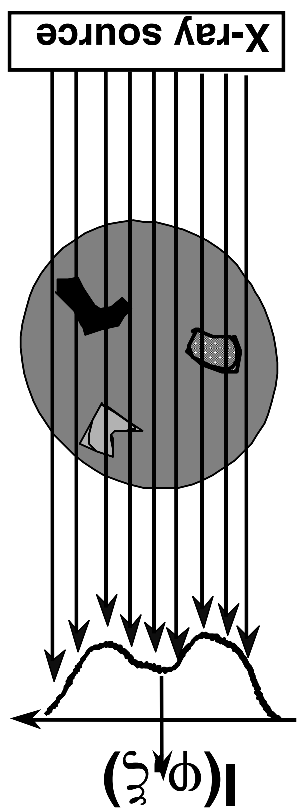
$$I = I_o \exp\left(-\int_L A(x,y)\,dl\right)$$

X-Ray Tomography

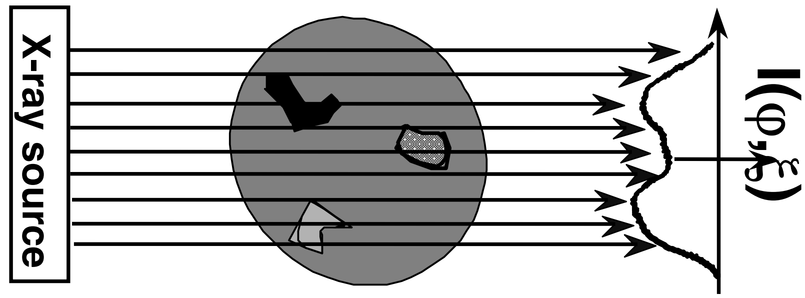
Uses X-rays to generate several shadowgrams I(φ,ξ).



X-Ray Tomography

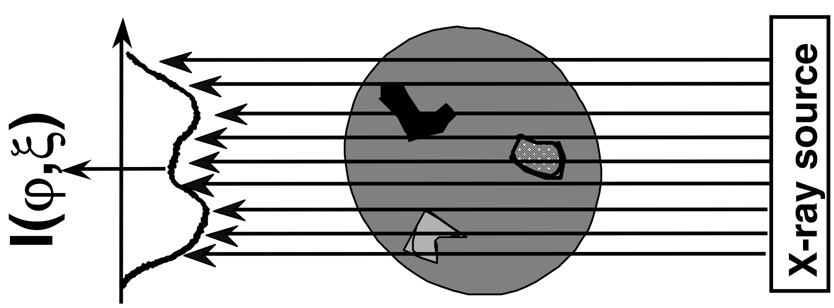


X-Ray Tomography



X-Ray Tomography

unknown
absorption cross-section
$\mu(x,y)$



To obtain image from projection data use inverse radon transform.

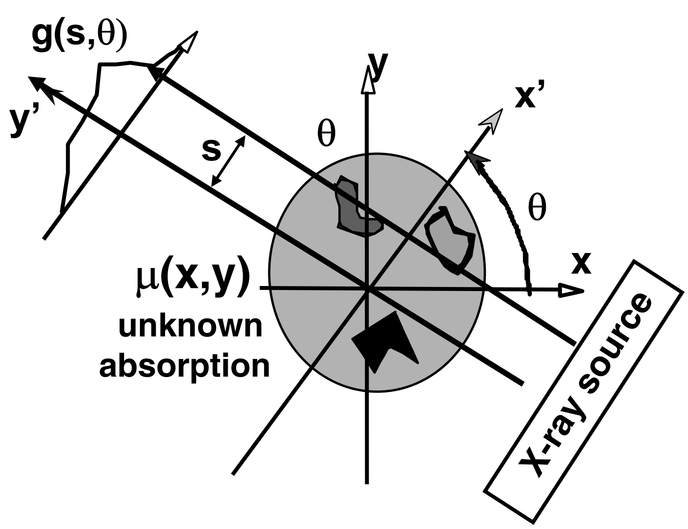
Overview

Image Reconstruction from Projections:

- **Radon Transform**
- **Projection Slice Theorem**
- **<u>Image Reconstruction from Projection Data</u>**
  - Method I: Fourier Reconstruction
  - Method II: Backprojection Filtering
  - Method III: Fourier Filtered Backprojection
  - Method IV: Convolution Filtered Backprojection

Radon Transform

$$\mu(x,y) \rightarrow g(s,\theta)$$



$$I = I_o \exp\left(-\int_L \mu(x,y)dl\right)$$

$$g \equiv \ln\left(\frac{I_o}{I}\right) \qquad (Signal)$$

$$\boxed{g(s,\theta) = \int_L \mu(x,y)dl}$$

The Radon transform $g(s,\theta)$ of a function $\mu(x,y)$ is defined as its line integral along a line inclined at an angle θ from the y-axis and at a distance s from the origin.

Radon Transform

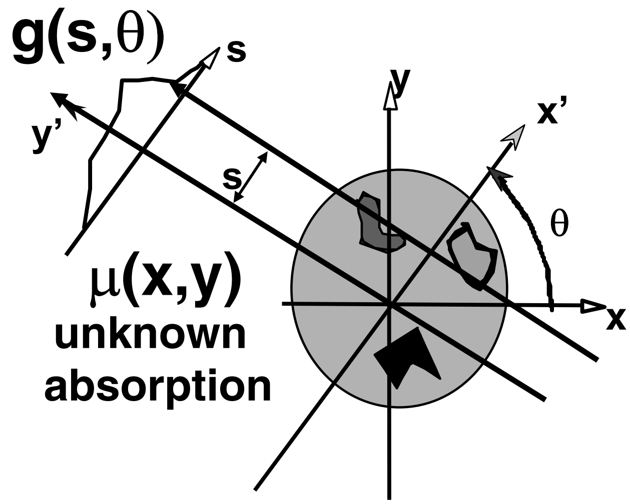
The Radon transform g(s,θ) of a function μ(x,y) is the one-dimensional projection of μ(x,y) at an angle θ.

$$g(s,\theta) = \int_L \mu(x,y)dl$$

The Radon transform maps the spatial domain (x,y) to the domain (s,θ).

μ(x,y) ➡ **g(s,θ)**

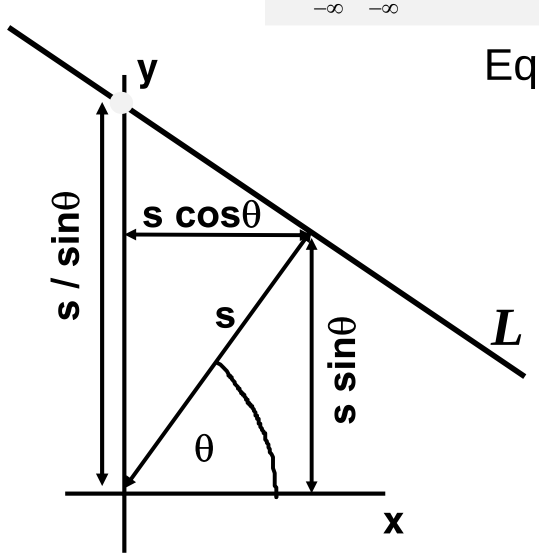
Each point in the (s,θ) space corresponds to a line in the spatial domain (x,y).



Radon Transform

$$g(s,\theta) = \int_L \mu(x,y)dl = \int_{x\cos\theta + y\sin\theta - s = 0} \mu(x,y)dl$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(x,y)\delta(x\cos\theta + y\sin\theta - s)dxdy$$



Equation of line (red) given s and θ;

y(x) = m x + c

m = - cosθ / sinθ

c = y(0) = s/cos(90- θ)

= s/sinθ

=> y = - cosθ /sinθ x + s/sinθ

0 = - sin θ y - cosθ x + s

0 = x cosθ + y sinθ - s

More Radon Transform

$$g(s,\theta) = \iint \mu(x,y)\delta(x\cos\theta + y\sin\theta - s)dxdy$$

$$= \iint \mu(x = x'\cos\theta - y'\sin\theta, y = x'\sin\theta + y'\cos\theta)\delta(x'-s)dx'dy'$$

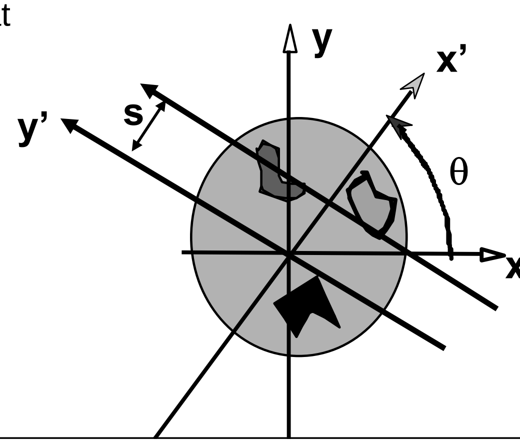
$$= \int \mu(x = x'\cos\theta - y'\sin\theta, y = x'\sin\theta + y'\cos\theta)\ \Big|_{x'=s}\ dy'$$

$$= \int \mu(s\cos\theta - y'\sin\theta, s\sin\theta + y'\cos\theta)dy'$$

Achieved by rotating coordinate system so that the integration line is along y'-axes:

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

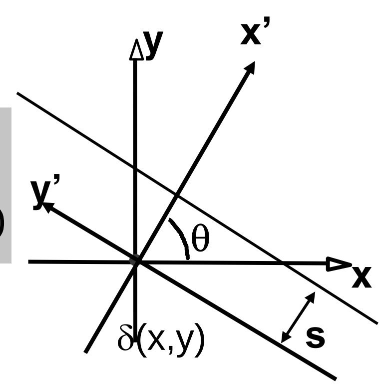
$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$



Radon Transform: Example 1

A(x,y) = δ(x, y) (point at center)

$$g(s,\theta) = \iint \delta(x,y)\delta(x\cos\theta + y\sin\theta - s)dxdy$$
$$g(s,\theta) = \left[\delta(x\cos\theta + y\sin\theta - s)\right]_{x=0,y=0} = \delta(-s) = \delta(s)$$



Sinogram

Line in Radon space (s,θ)

S=0

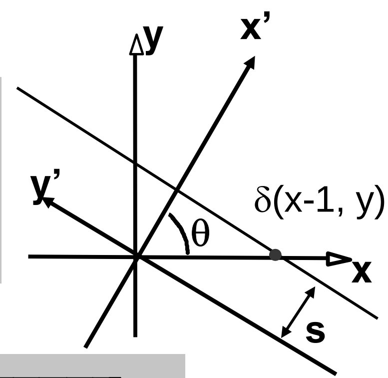
Radon Transform: Example 2

A(x,y) = δ(x-1,y) (point on x-axis)

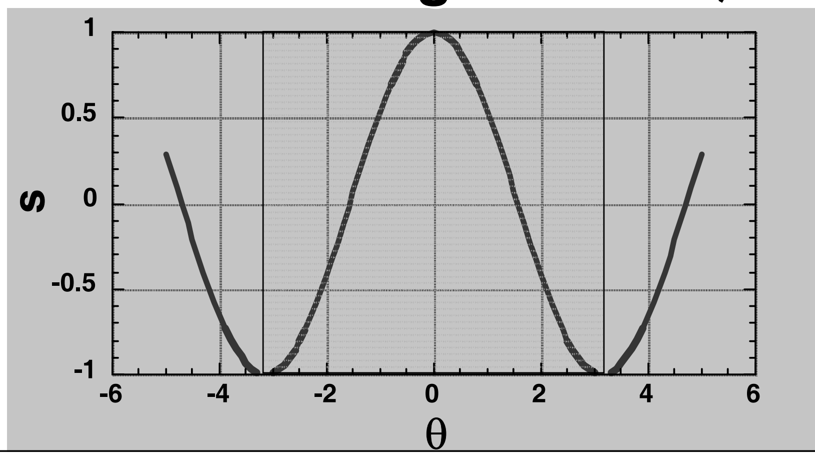
$$g(s,\theta) = \iint \delta(x-1, y)\delta(x\cos\theta + y\sin\theta - s)dxdy$$

$$g(s,\theta) = \left[\delta(x\cos\theta + y\sin\theta - s)\right]_{x=1,y=0} = \delta(\cos\theta - s)$$

$$Non-zero \Rightarrow \cos\theta = s$$



Sinogram



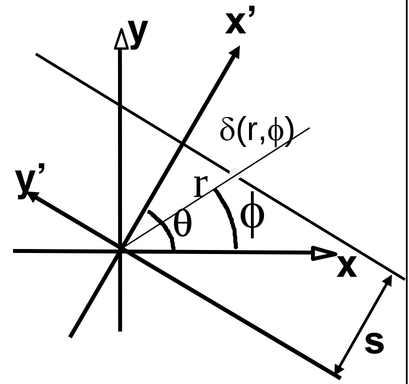
Radon Transform: Example 3

Example II: A(x,y) = δ(r,ϕ) (arbitrary point)

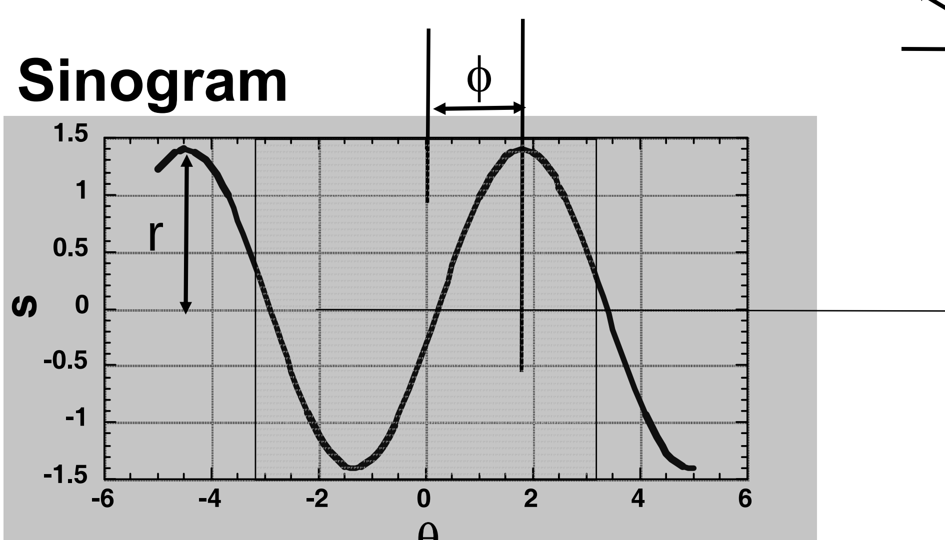
$$g(s,\theta) = \iint A(x,y)\delta(x\cos\theta + y\sin\theta - s)dxdy$$

$$g(s,\theta) = \left[\delta(x\cos\theta + y\sin\theta - s)\right]_{x=r\cos\phi,y=r\sin\phi} = \delta\left[r\cos(\theta-\phi) - s\right]$$

$$Non-zero \Rightarrow r\cos(\theta-\phi) = s$$

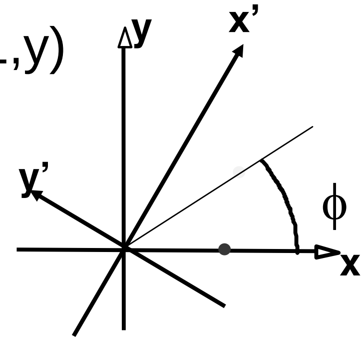


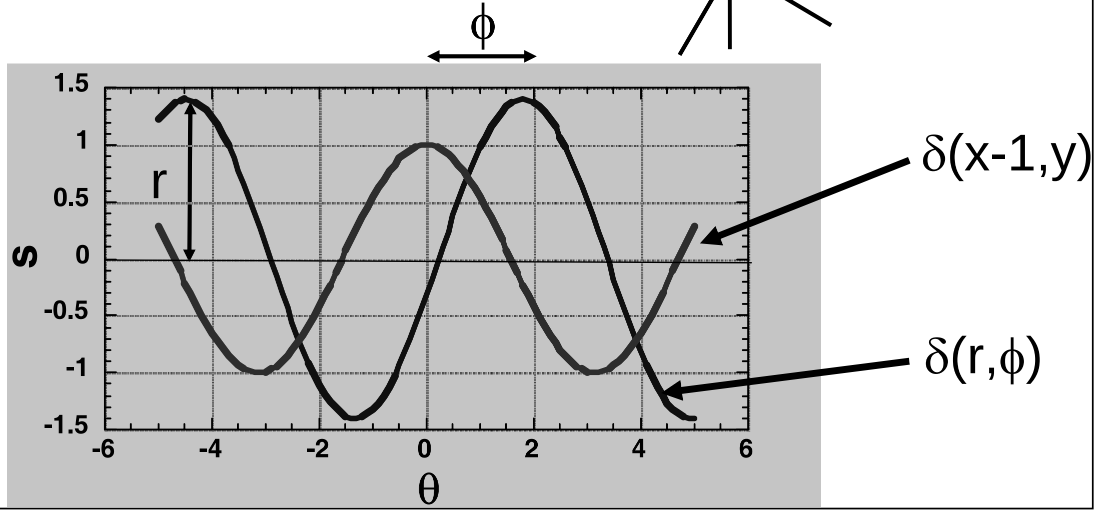
Sinogram



Radon Transform: Example 4

Example III: $A(x,y) = \delta(r,\phi) + \delta(x-1,y)$

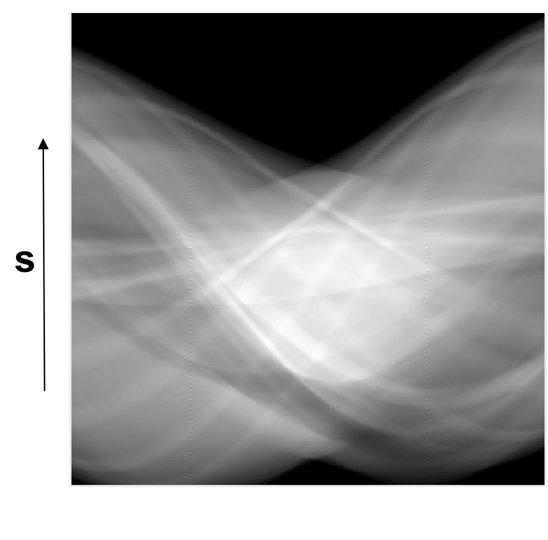




Radon Transform

2D Real Space

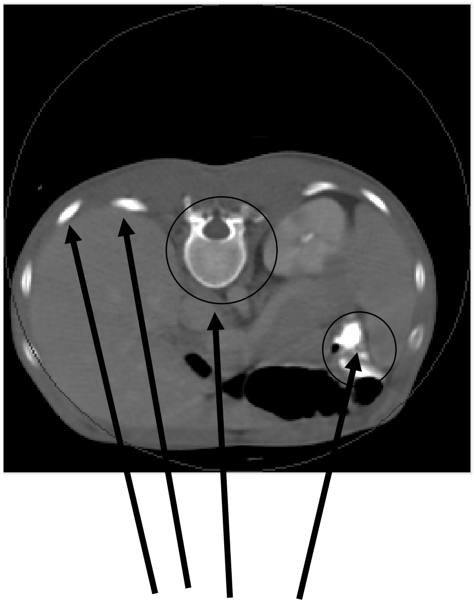
2D Radon Space



θ
(0 to 180 degree)

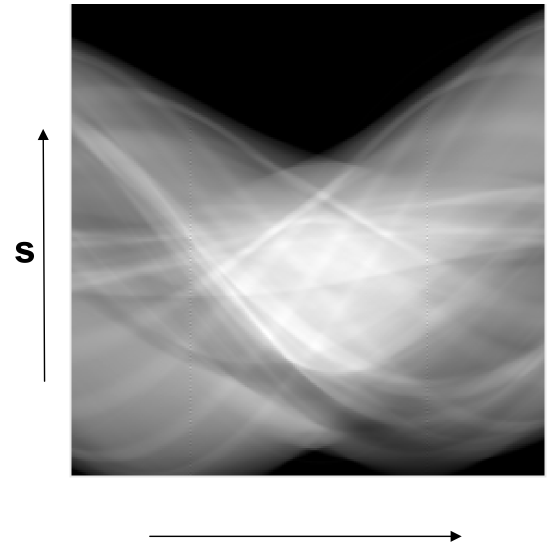
Radon Transform

2D Real Space



Where these structures are in Radon space?

2D Radon Space



θ
(0 to 180 degree)

Properties of Radon Transform

	Function	Radon Transform
	$f(x, y) = f_p(r, \phi)$	$g(s, \theta)$
1	Linearity: $a_1 f_1(x, y) + a_2 f_2(x, y)$	$a_1 g_1(s, \theta) + a_2 g_2(s, \theta)$
2	Space limitedness: $f(x, y) = 0, \lvert x\rvert > \frac{D}{2}, \lvert y\rvert > \frac{D}{2}$	$g(s, \theta) = 0, \quad \lvert s\rvert > \frac{D\sqrt{2}}{2}$
3	Symmetry: $f(x, y)$	$g(s, \theta) = g(-s, \theta \pm \pi)$
4	Periodicity: $f(x, y)$	$g(s, \theta) = g(s, \theta + 2k\pi)$, $k =$ integer
5	Shift: $f(x - x_0, y - y_0)$	$g(s - x_0 \cos\theta - y_0 \sin\theta, \theta)$
6	Rotation by θ_0: $f_p(r, \phi + \theta_0)$	$g(s, \theta + \theta_0)$
7	Scaling: $f(ax, ay)$	$\frac{1}{\lvert a\rvert} g(as, \theta), \quad a \neq 0$
8	Mass conservation: $M = \iint_{-\infty}^{\infty} f(x, y)\, dx\, dy$	$M = \int_{-\infty}^{\infty} g(s, \theta)\, ds, \quad \forall\theta$

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Projection Theorem

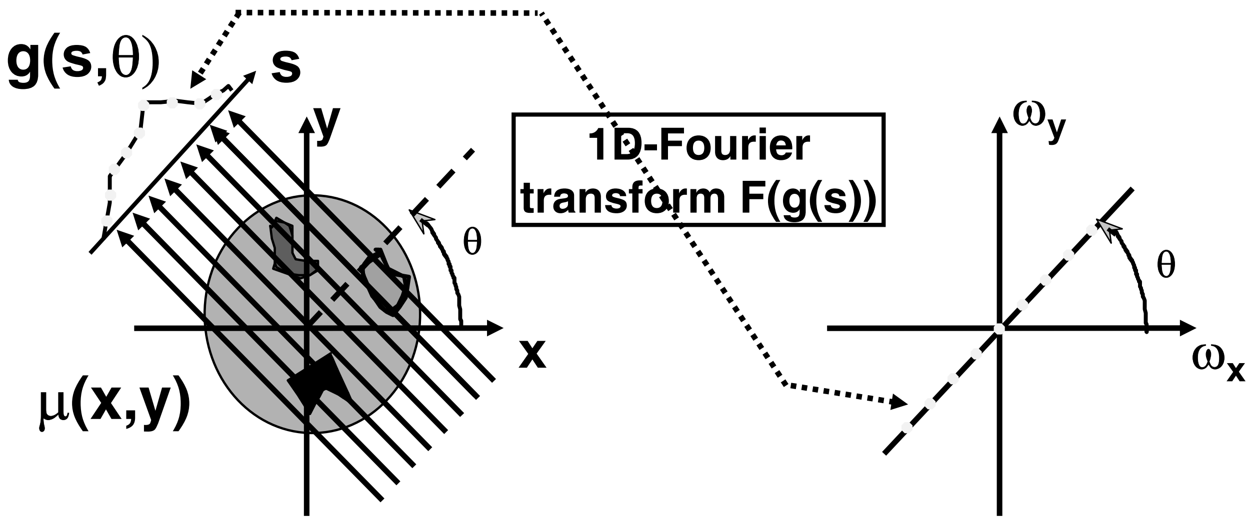
(also “Central Slice Theorem” or Projection Slice Theorem)

If $g(s,\theta)$ is the Radon transform of a function $f(x,y)$, then the one-dimensional Fourier transform $G(\omega_s,\theta)$ with respect to s of the projection $g(s,\theta)$ is equal to the central slice, at angle θ, of the two dimensional Fourier transform $F(\omega_x, \omega_y)$ of the function $f(x,y)$.

Projection Theorem

(also “Central Slice Theorem” or Projection Slice Theorem)

2D-space domain of $\mu(x,y)$ **2D-frequency domain of $\mu(x,y)$**

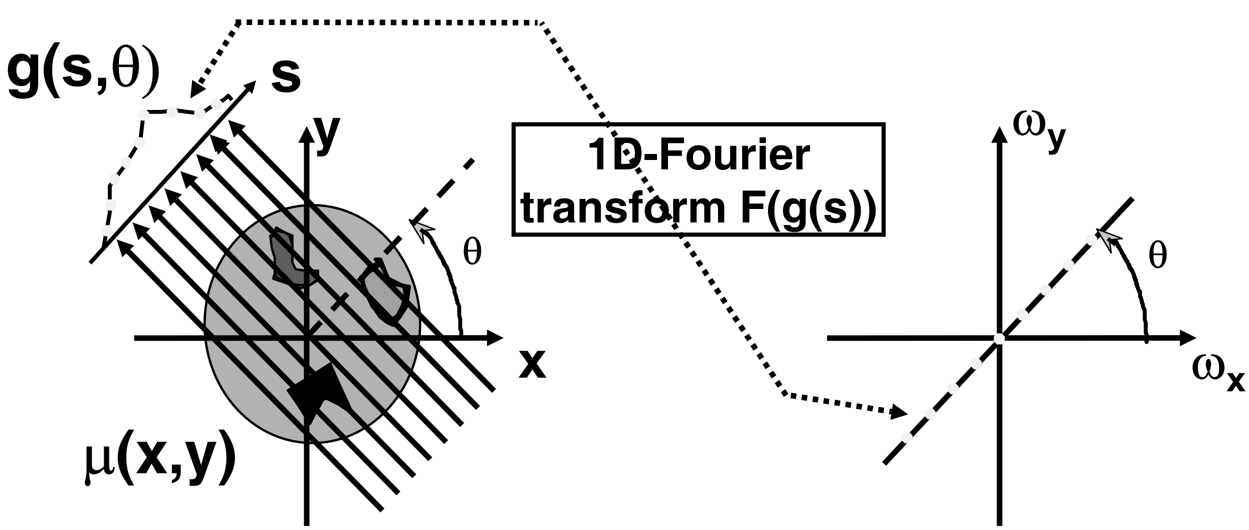


If $g(s,\theta)$ is the Radon transform of a function $f(x,y)$, then the one-dimensional Fourier transform $G(\omega_s,\theta)$ with respect to s of the projection $g(s,\theta)$ is equal to the central slice, at angle θ, of the two dimensional Fourier transform $F(\omega_x, \omega_y)$ of the function $f(x,y)$.

Projection Theorem

(also “Central Slice Theorem” or Projection Slice Theorem)

Proof: Assignment

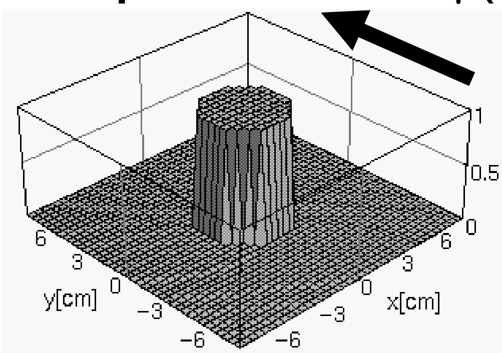


Clues ?

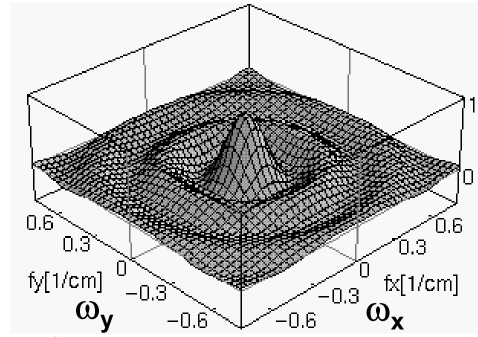
Projection Theorem

(also “Central Slice Theorem” or Projection Slice Theorem)

2D-space domain of $\mu(x,y)$ **2D-frequency domain of $\mu(x,y)$**

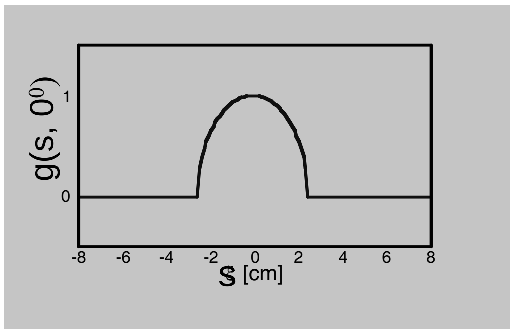


2D-FT
$F(\mu(x,y))$

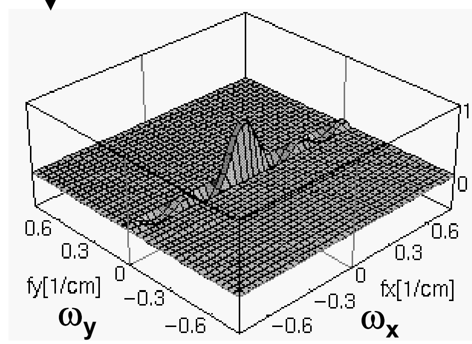


under viewing angle $\theta = 0^0$

Projection Theorem



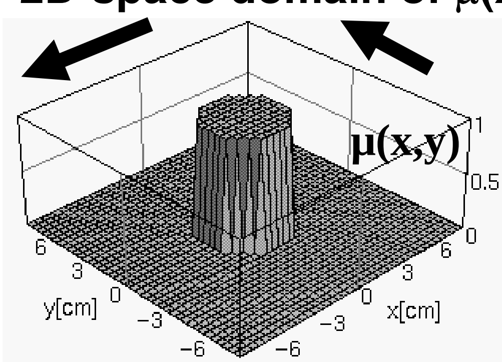
1D-FT
$F(g(s))$



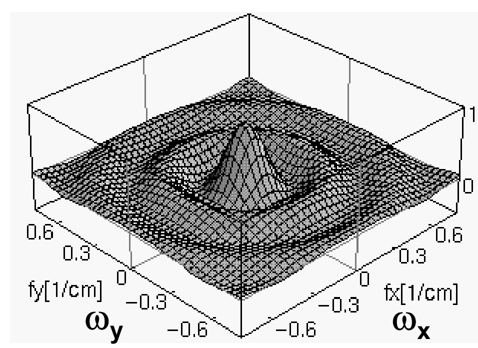
Projection Theorem

(also “Central Slice Theorem” or Projection Slice Theorem)

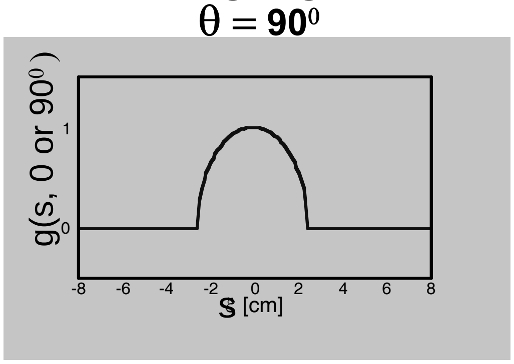
2D-space domain of $\mu(x,y)$ **2D-frequency domain of $\mu(x,y)$**



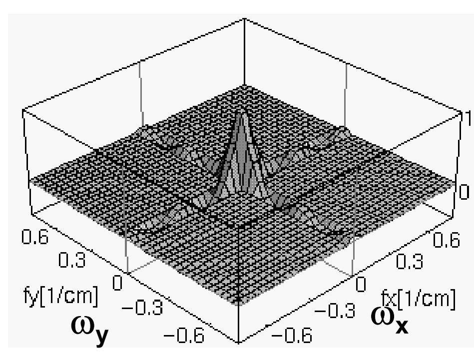
2D-FT
$F(\mu(x,y))$



under viewing angles $\theta = 0^0$ & $\theta = 90^0$



1D-FT
$F(g(s))$



Preparing for Class Project

- **Computer skills (MatLab or C *from ELEC2201*) ?**
- **Skills in numerical computation ?**

Overview

Image Reconstruction from Projections:

- Radon Transform
- Projection Slice Theorem
- **<u>Image Reconstruction from Projection Data</u>**
  - **Method I: Fourier Reconstruction**
  - **Method II: Backprojection Filtering**
  - **Method III: Fourier Filtered Backprojection**
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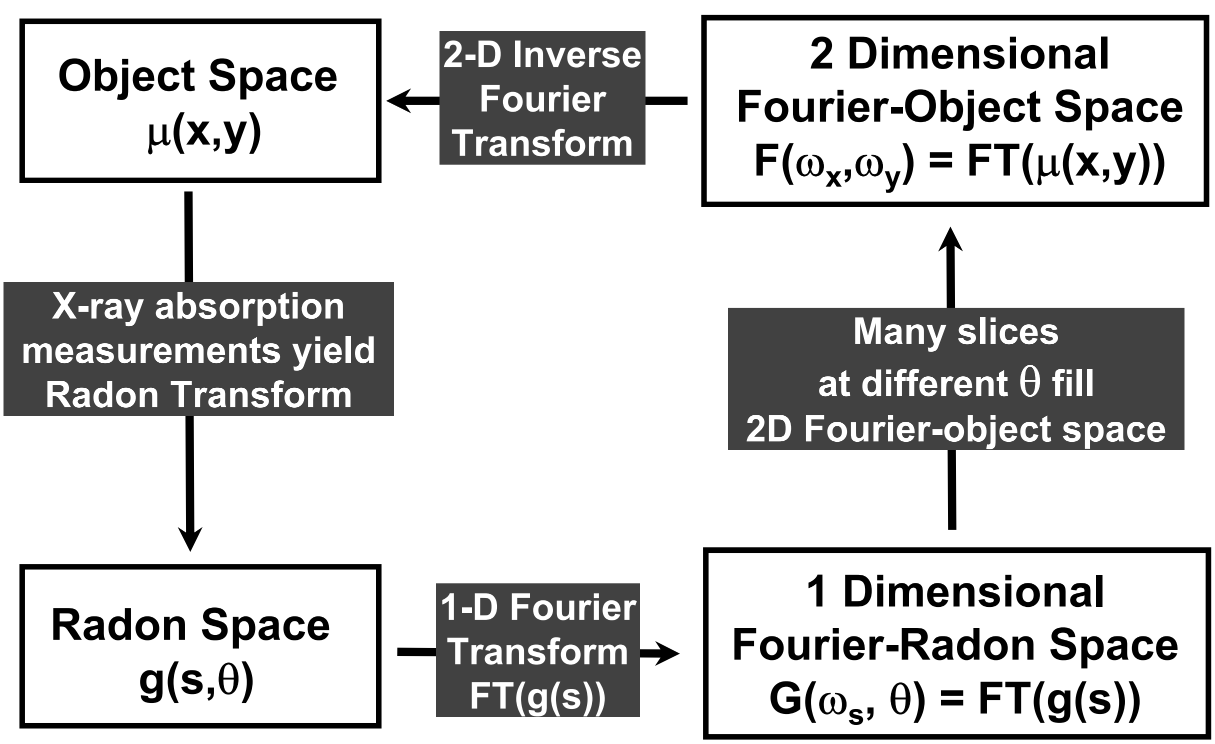
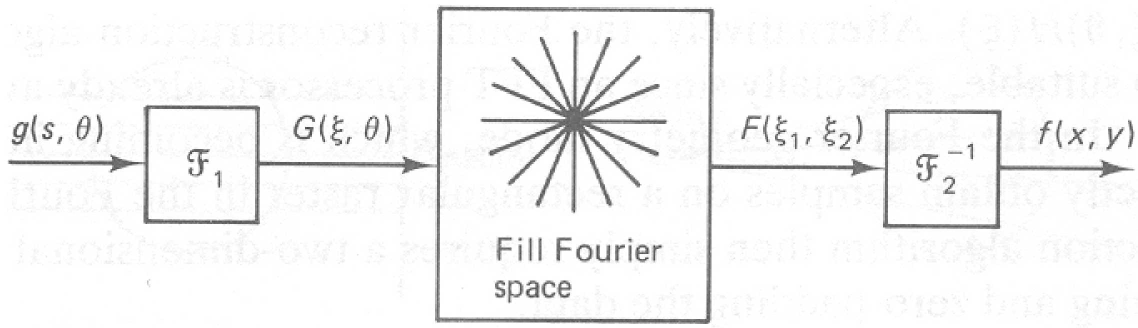
Projection Theorem

How can we use Projection slice theorem to reconstruction spatial distribution of absorption profile $\mu(x,y)$?

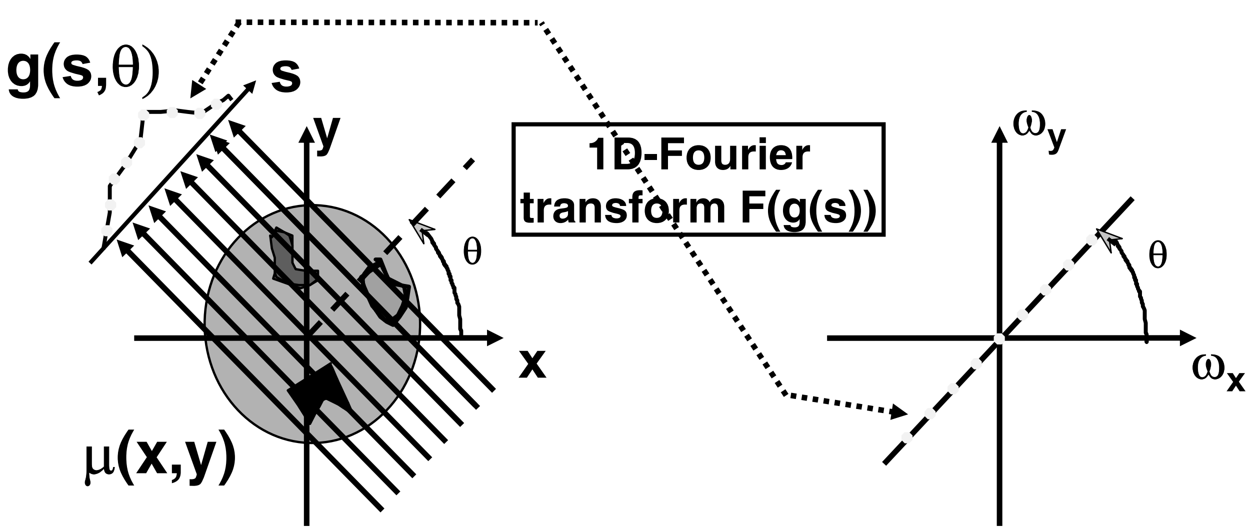
I. Fourier Reconstruction



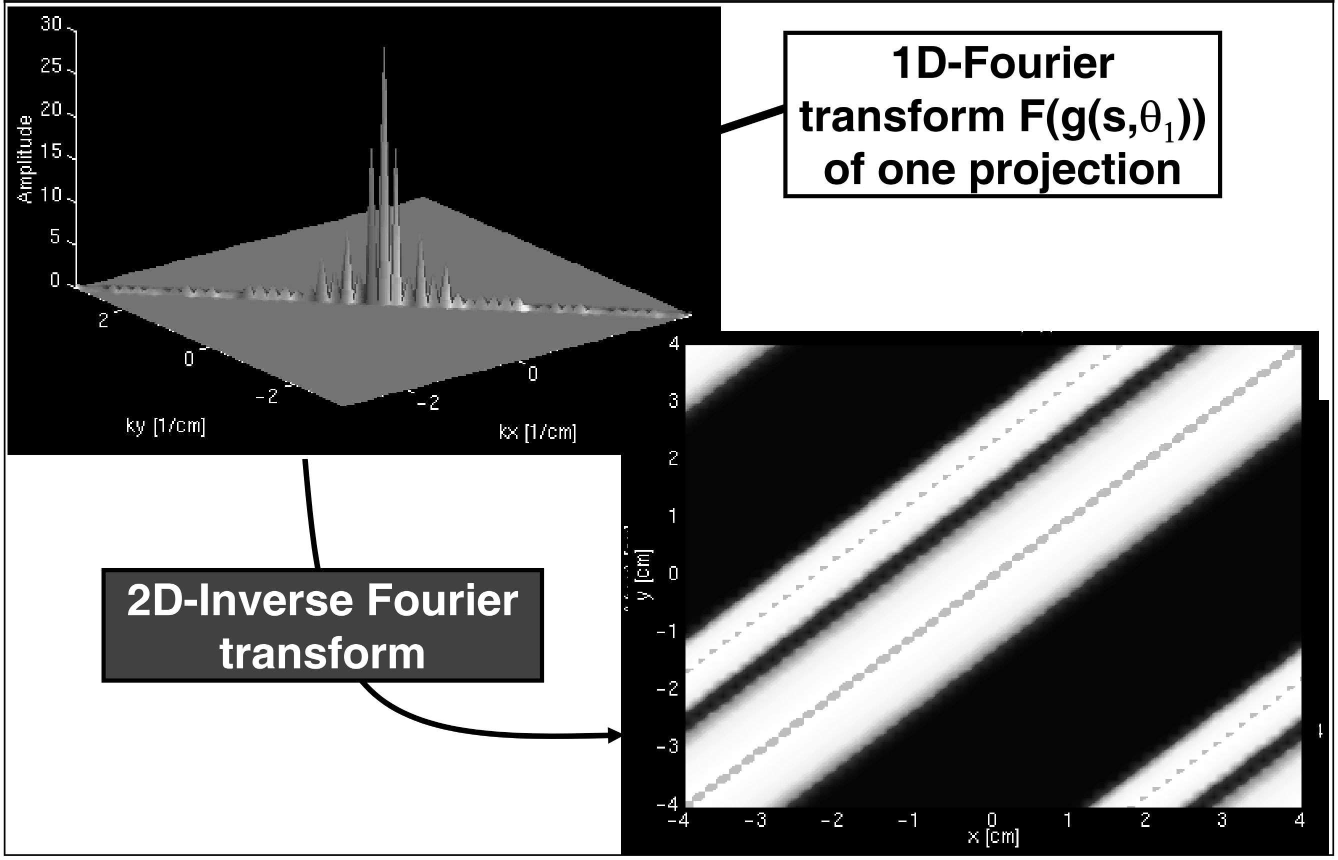
Fourier Image Reconstruction with Projection Theorem



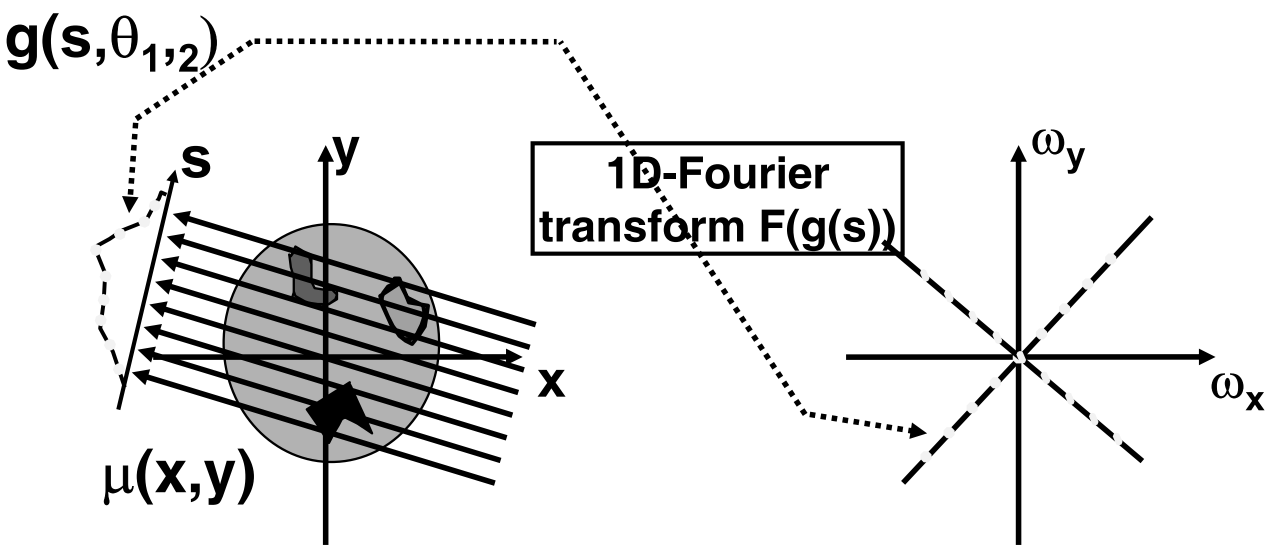
Fourier Reconstruction



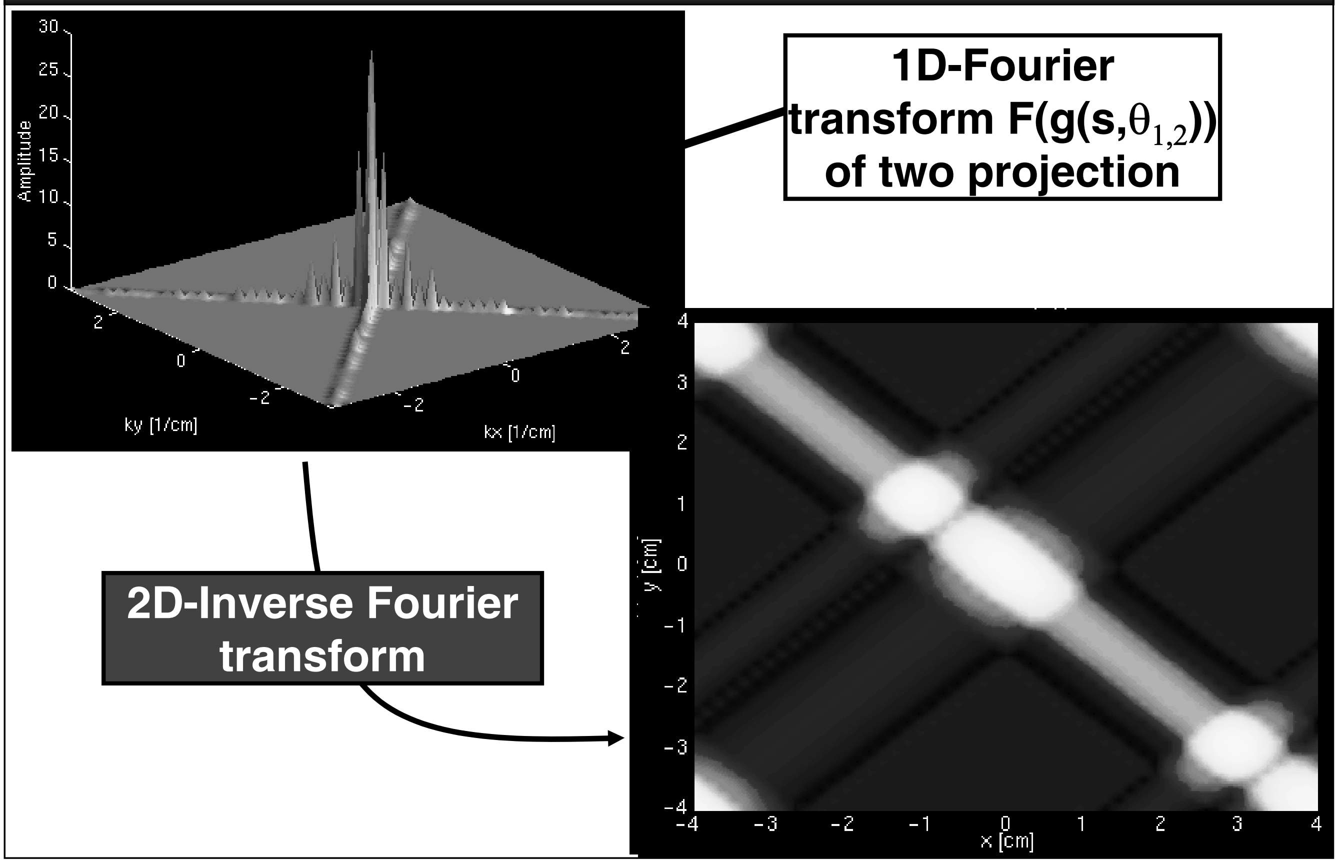
Fourier Reconstruction



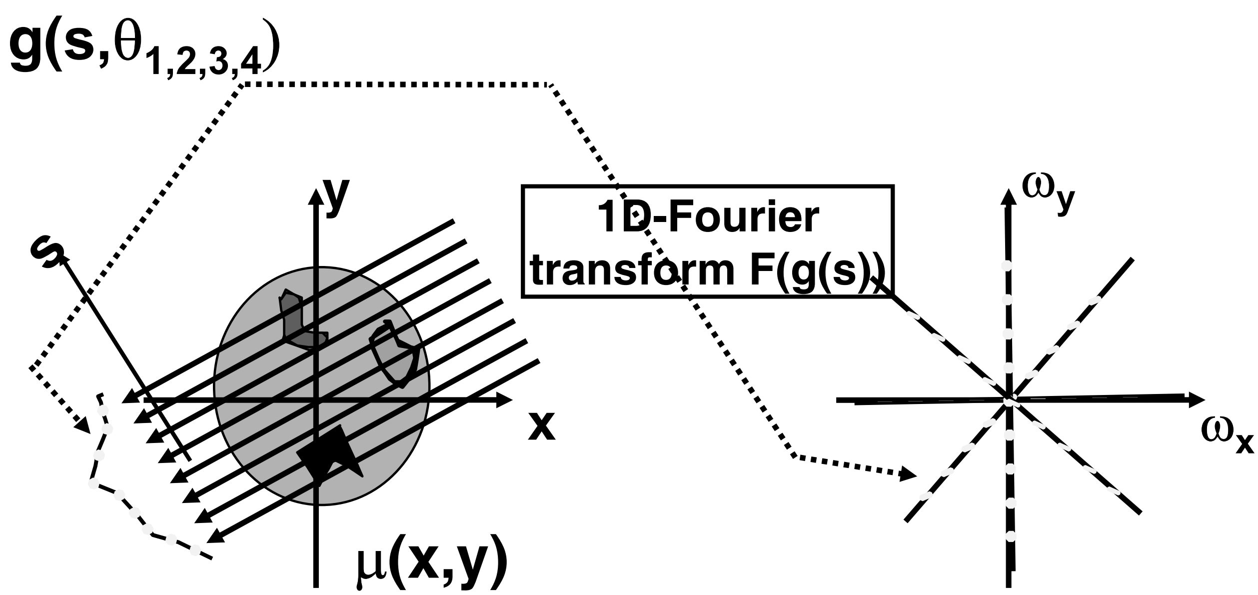
Fourier Reconstruction



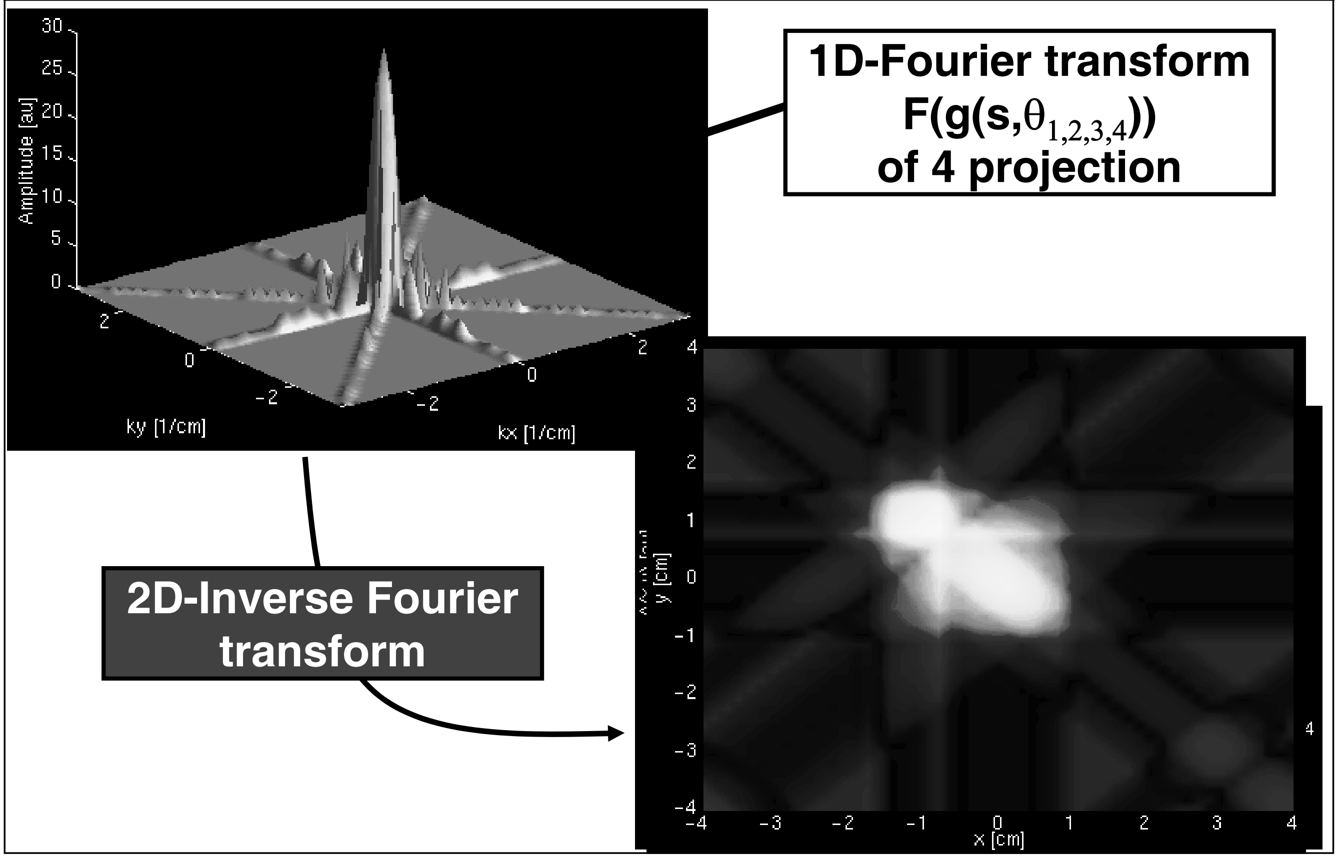
Fourier Reconstruction



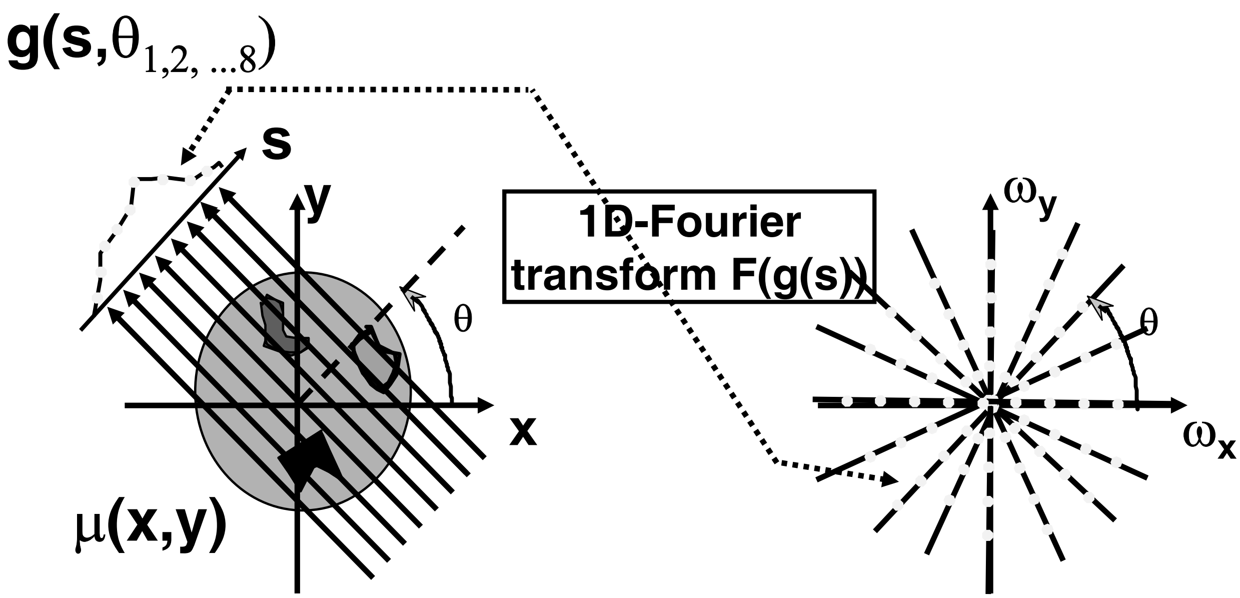
Fourier Reconstruction



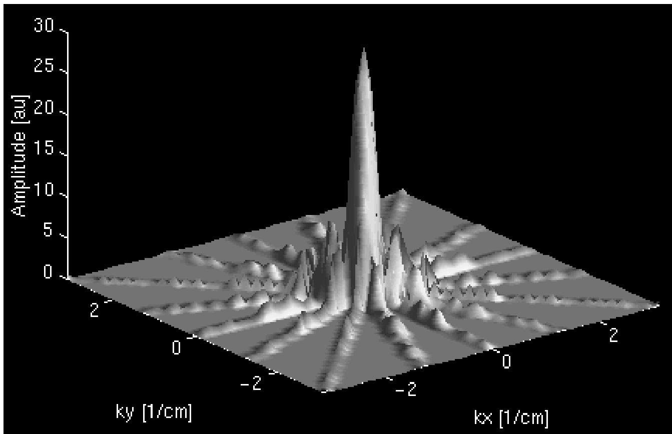
Fourier Reconstruction



Fourier Reconstruction

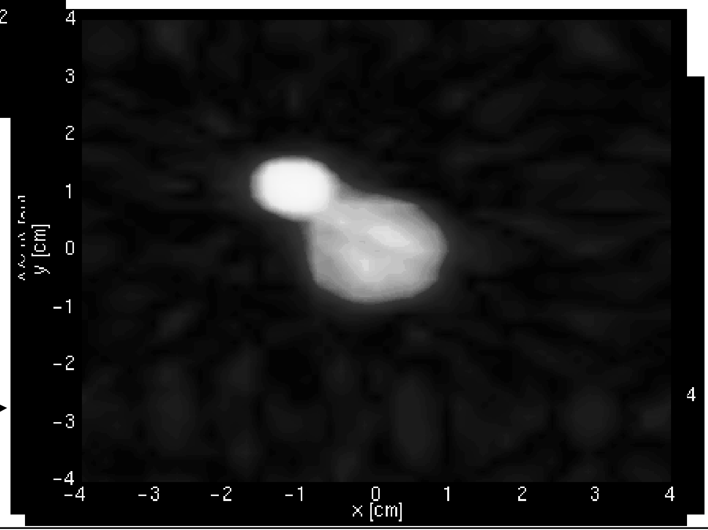


Fourier Reconstruction

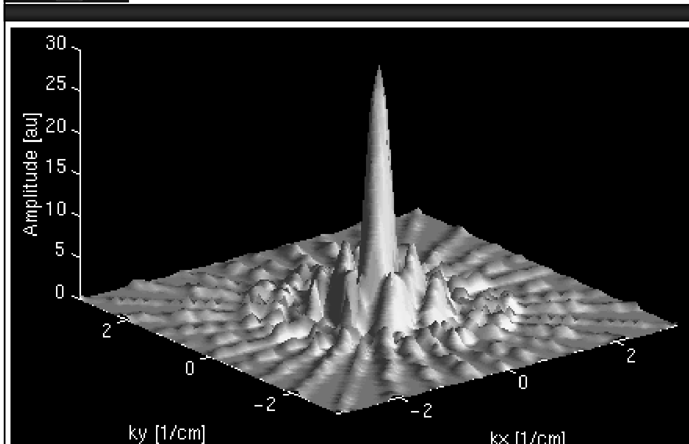


1D-Fourier transform
F(g(s,$\theta_{1,2,\ldots,8}$))
of 8 projection

2D-Inverse Fourier transform



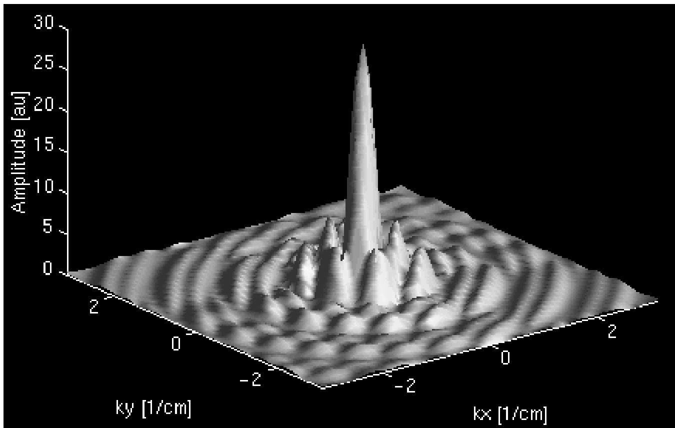
Fourier Reconstruction



1D-Fourier transform
F(g(s,$\theta_{1,2,\ldots,16}$))
of 16 projection

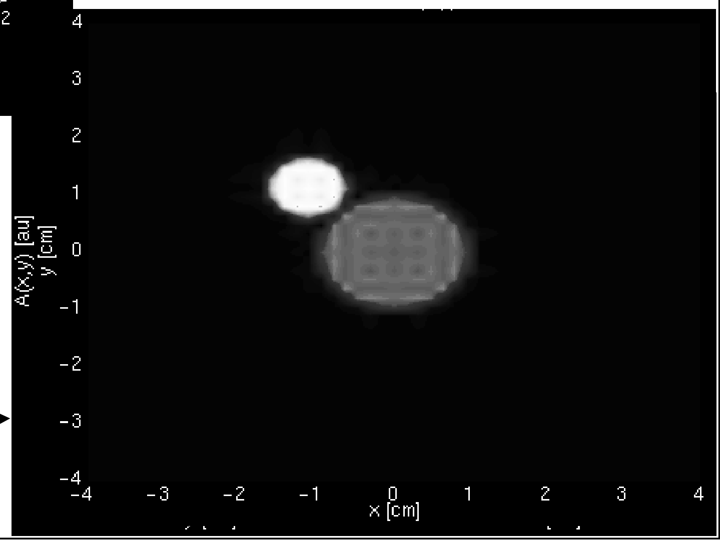
2D-Inverse Fourier transform

Fourier Reconstruction

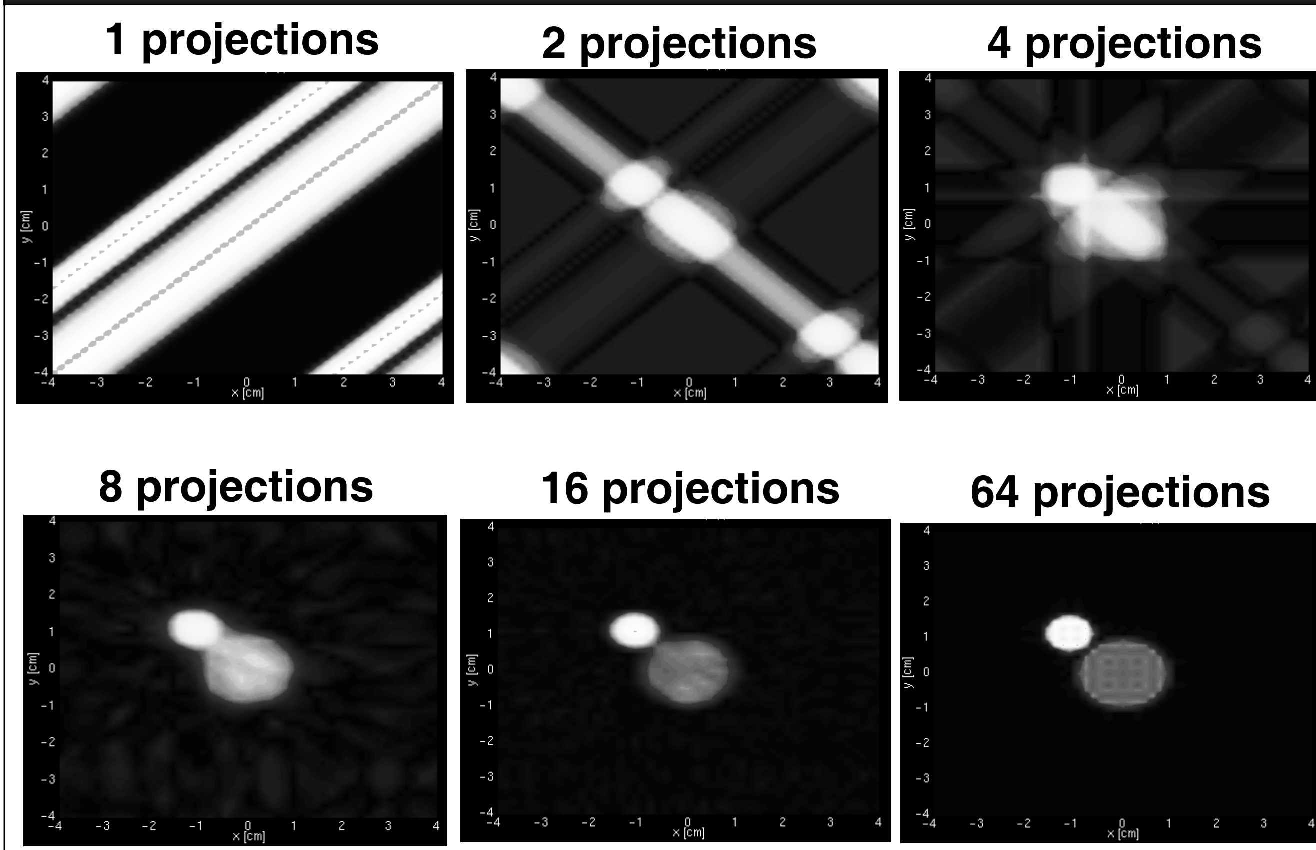


1D-Fourier transform
$F(g(s,\theta_{1,2,\ldots,64}))$
of 64 projection

2D-Inverse Fourier transform

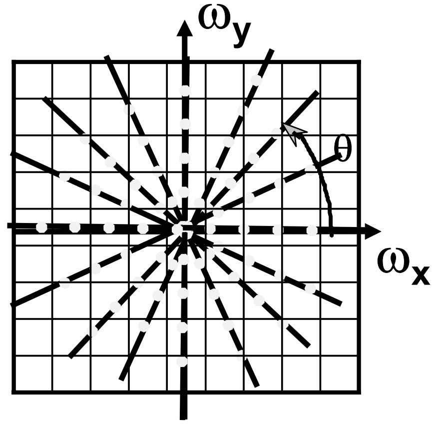


Influence of # of Projections



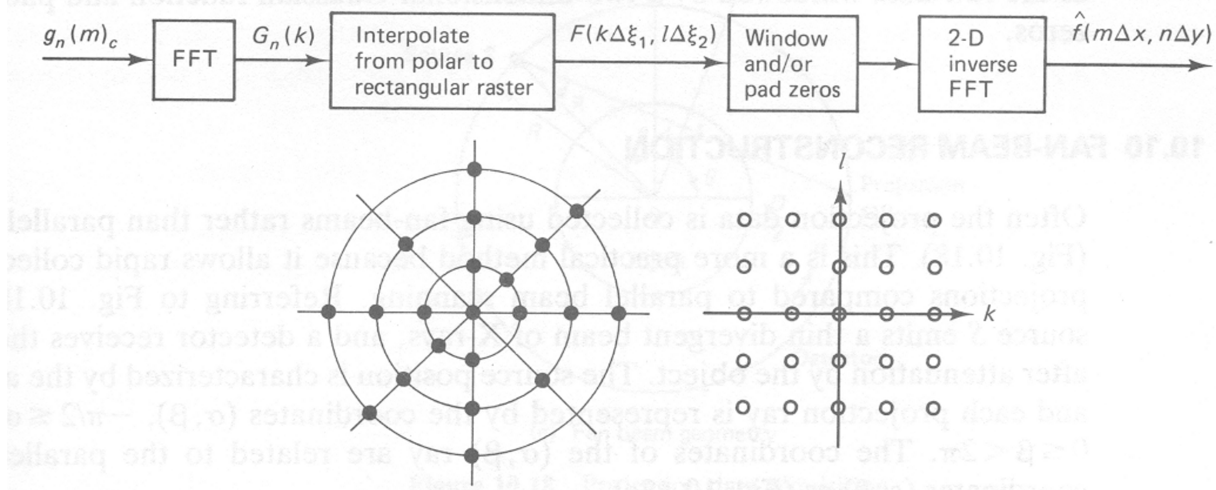
Fourier Image Reconstruction with Projection Theorem

Problem:
Points in 2D Fourier Space are not on rectangular grid.
=> Inverse Fourier transform not trivial.



Fourier Image Reconstruction with Projection Theorem

A practical algorithm:

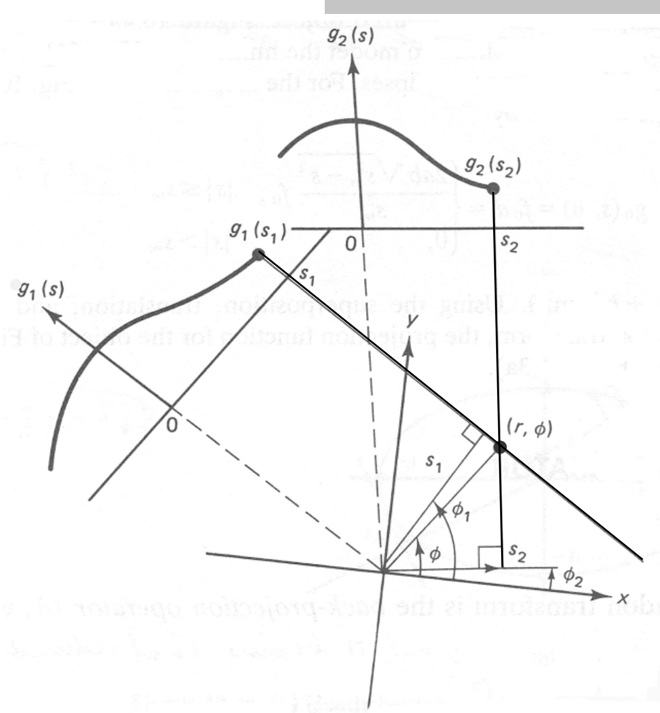


Fourier Image Reconstruction with Projection Theorem

How is the Fourier reconstruction method connected with the backprojection method?

II. Backprojection Filtering

Backprojection operator $$b(x,y) \equiv \hat{f}(x,y) \equiv Bg = \int_0^{\pi} g(s = x\cos\theta + y\sin\theta, \theta)\, d\theta$$



The value of the backprojection *Bg* is evaluated by integrating g(s,θ) over θ for all lines that pass through that point.

Example:

Backproject 2 projections (g_1 and g_2) only

2 projections represented in Randon space

$$g(s,\theta) = g_1(s)\delta(\theta - \phi_1) + g_2(s)\delta(\theta - \phi_2)$$

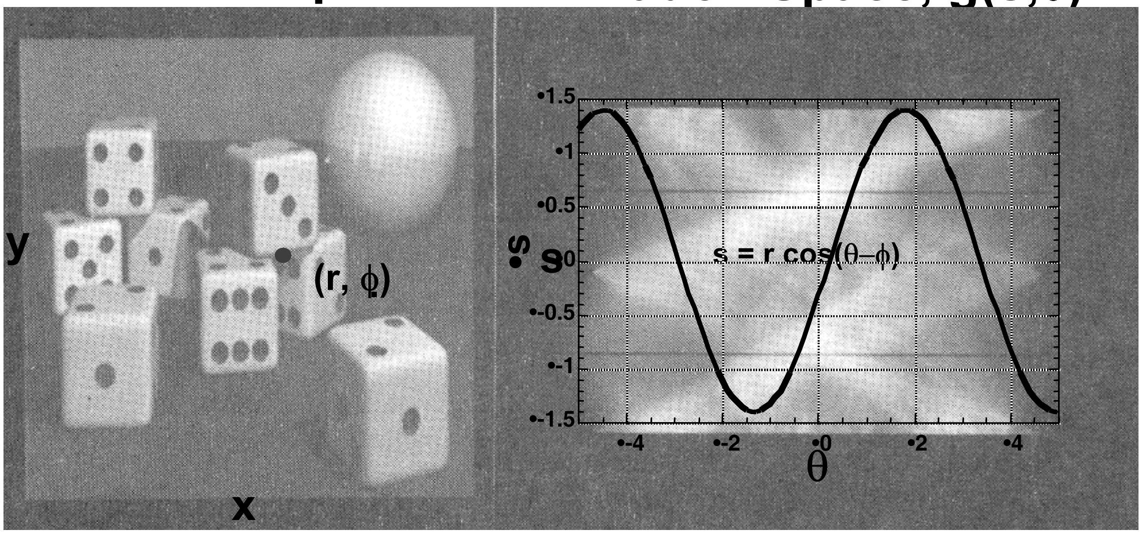
Backproject 2 Projections

$$b(x,y) = g_1(s_1) + g_2(s_2) = b(r,\phi)$$

$$s_1 = r\cos(\phi_1 - \phi),\ s_2 = r\cos(\phi_2 - \phi)$$

Backprojection & Radon Transform

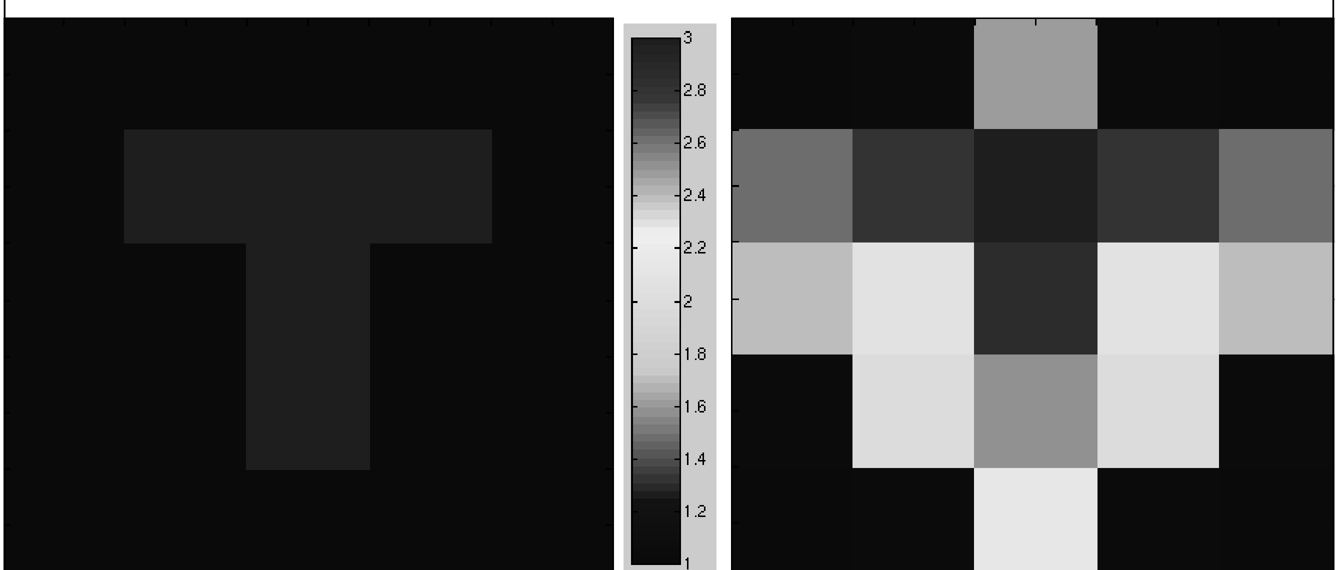
Real Space | Radon Space, $g(s,\theta)$



The backprojection at (r, ϕ) is the integration of $g(s,\theta)$ along the sinusoid $s = r \cos(\theta - \phi)$

WHY? (Optional)

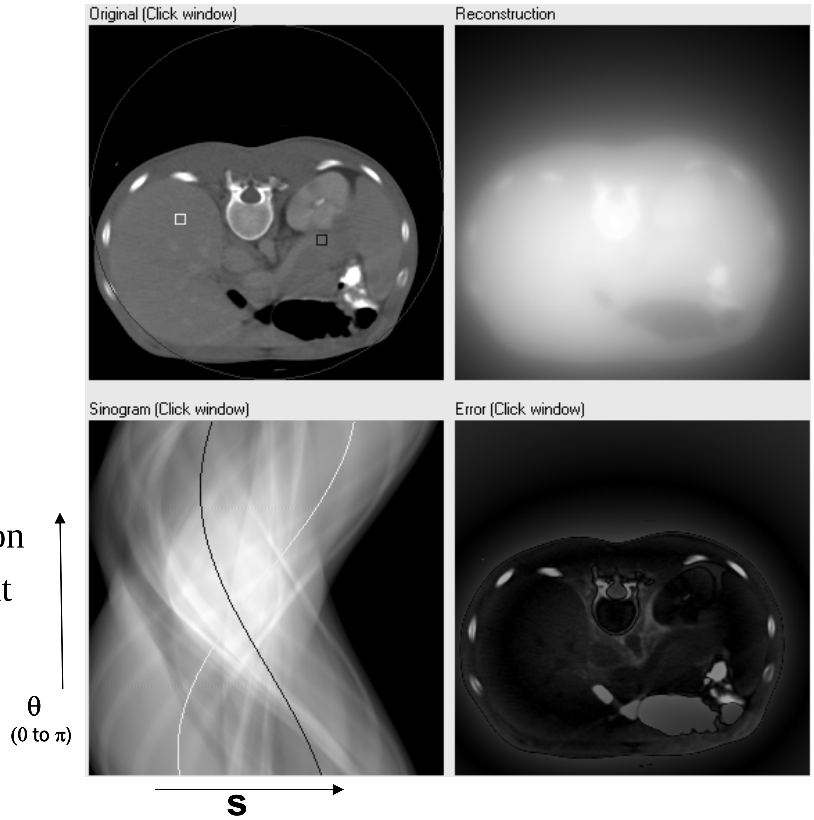
This is why we saw…



Backprojection & Radon Transform

Object

Simple Backprojection (BP)



Sinogram Backprojection Measurement

Difference between Object & BP image

Backprojection Operator: Mathematics

It can be shown from

$$\hat{f}(x,y) \equiv Bg = \int_0^{\pi} g(s = x\cos\theta + y\sin\theta, \theta)\, d\theta$$

$$g(s,\theta) = \int_L f(x,y)\, dl$$

$$= \iint f(x,y)\,\delta(x\cos\theta + y\sin\theta - s)\, dxdy$$

that the backprojected Radon transform data *g* or R*f* (i.e., the simple backprojection image)

$$\hat{f}(x,y) \equiv Bg = BRf$$

$$= f(x,y) \otimes \left(\frac{1}{\sqrt{x^2+y^2}} \right)$$

Proof?

Assignment

Therefore backprojection of radon transform gives the original image convolved with 1/sqrt(x^2+y^2). This results in blurred image.

What could you do to get back *f*(*x*,*y*)?

Use a filter! - But what filter?

$$\hat{f}(x,y) \equiv Bg = BRf$$
$$= f(x,y) \otimes \left(x^2+y^2\right)^{-1/2}$$

Use convolution theorem:

$$F\left(\hat{f}(x,y)\right) = F\left(f(x,y) \otimes \left(x^2+y^2\right)^{-1/2}\right) = F\left(f(x,y)\right) F\left(\left(x^2+y^2\right)^{-1/2}\right)$$
$$= F\left(f(x,y)\right)\left(\omega_x^2+\omega_y^2\right)^{-1/2}$$

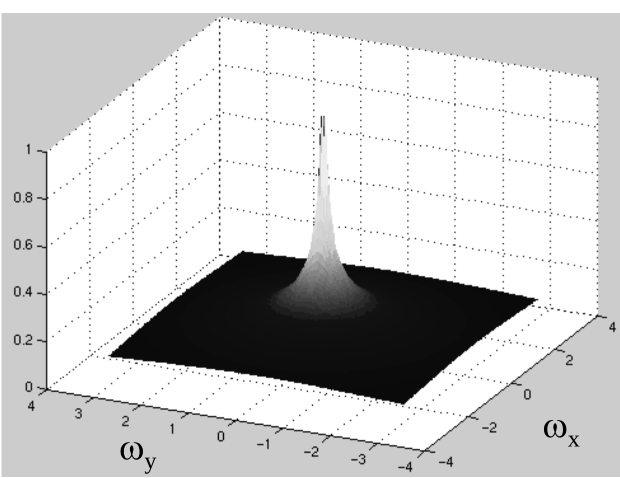
$$F\left(\hat{f}(x,y)\right)\left(\omega_x^2+\omega_y^2\right)^{1/2} = F\left(f(x,y)\right)\left(\omega_x^2+\omega_y^2\right)^{-1/2}\left(\omega_x^2+\omega_y^2\right)^{1/2}$$
$$= F\left(f(x,y)\right)$$

$$IF\left(F\left(\hat{f}(x,y)\right)\left(\omega_x^2+\omega_y^2\right)^{1/2}\right) = IF\left(F\left(f(x,y)\right)\right) = f(x,y)$$

SQRT - Filters

$$|\omega|^{-1} = \left(\omega_x^2+\omega_y^2\right)^{-1/2}$$ **Filter (LP)**

$$|\omega| = \left(\omega_x^2+\omega_y^2\right)^{1/2}$$ **Filter (HP)**



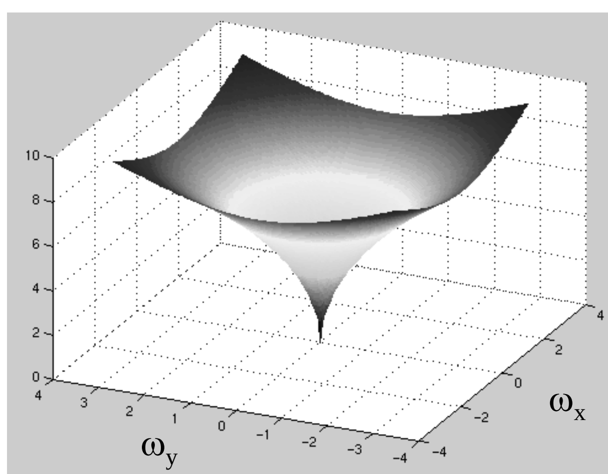


Image Reconstruction by Backprojection Filtering

Backprojection Filtering Algorithm:

(1) Get Radon transform $g(s,\theta)$ of $f(x,y)$ by performing tomographic X-tray imaging.

(2) Backproject the Radon transform data.

(3) Take Fourier transform of backprojected data.

(4) Multiply with filter $\mathrm{sqrt}(\omega_x^2 + \omega_y^2)$

(5) Perform inverse Fourier Transform to obtain $f(x,y)$

$$f(x,y) = IF_2\left(|\omega| \bullet F_2\left(B(Rf)\right)\right)$$

Backprojection Filtering Reconstruction & Fourier Reconstruction

Backprojection Filtering Method:

$$f(x,y) = IF_2\left(|\omega| \bullet F_2\left(B(Rf)\right)\right)$$

Fourier Reconstruction Method:

$$f(x,y) = IF_2\left(\left(PST(Rf)\right)\right) = IF_2\left(\sum_{n=1}^{N} F_{1,s}\left(g(s,\theta_n)\right)\right)$$

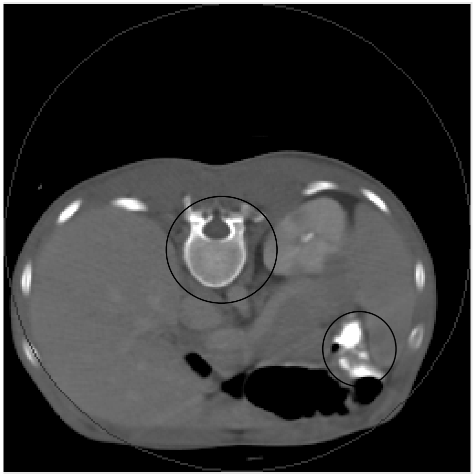
Note that re-griding is required

There are also other techniques !!!!

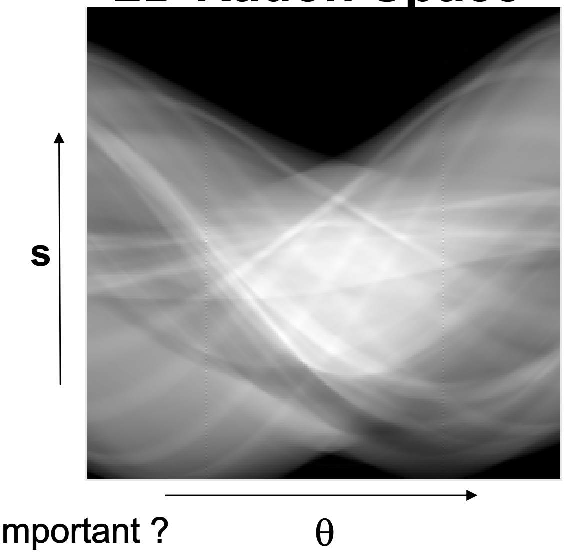
PST := Projection Slice Theorem or Central Slice Theorem

Radon Transform

2D Real Space



2D Radon Space



Why such 2D Radon Transform is important ?

- Formulation of x-ray attenuation measurement in CT
- Mathematics for later use in image reconstruction
- Inverse Radon Transform possible?

$$f(x,y)=\int_0^{\pi}\hat{g}(s=x\cos\theta+y\sin\theta,\theta)\,d\theta$$

$$\text{with } \hat{g}(s,\theta)=\int_{-\infty}^{\infty}|\omega_s|\,G(\omega_s,\theta)\exp(i\omega_s s)\,d\omega_s$$

$$\text{with } G(\omega_s,\theta)=F_{1,s}\big(g(s,\theta)\big)$$

III. Fourier Filtered Backprojection

Inverse Radon Transform Theorem:

$$f(x,y)=\int_0^{\pi}\hat{g}(s=x\cos\theta+y\sin\theta,\theta)\,d\theta$$

$$\text{with } \hat{g}(s,\theta)=\int_{-\infty}^{\infty}|\omega_s|\,F_1(\omega_s,\theta)\exp(i\omega_s s)\,d\omega_s$$

$$\text{with } F_1(\omega_s,\theta)=F_{1,s}\big(g(s,\theta)\big)$$

The inverse Radon transform is obtained in two steps:

(1) Each projection is filtered by a one dimensional filter whose frequency response is $|\omega_s|$.

(2) The result of step (1) is backprojected to yield *f*(*x*,*y*).

Proof ?

Proof

The inverse Fourier transform is given by:

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(\omega_x,\omega_y)\exp[i(\omega_x x+\omega_y y)]d\omega_x d\omega_y$$

Rewriting in polar coordinates results in:

$$f(x,y)=\int_{0}^{2\pi}\int_{0}^{\infty}F_p(\omega_s,\theta)\exp[i\omega_s(x\cos\theta+y\sin\theta)]\omega_s d\omega_s d\theta$$

Changing the limits of integration we get:

$$f(x,y)=\int_{0}^{\pi}\int_{-\infty}^{\infty}|\omega_s|F_p(\omega_s,\theta)\exp[i\omega_s(x\cos\theta+y\sin\theta)]d\omega_s d\theta$$

Since ***the*** Projection Slice Theorem $F_p(\omega_s,\theta)=G(\omega_s,\theta)$

(1D Fourier transform with respect to s of Radon transform equals slice through 2D Fourier transform at angle θ of the object function f)

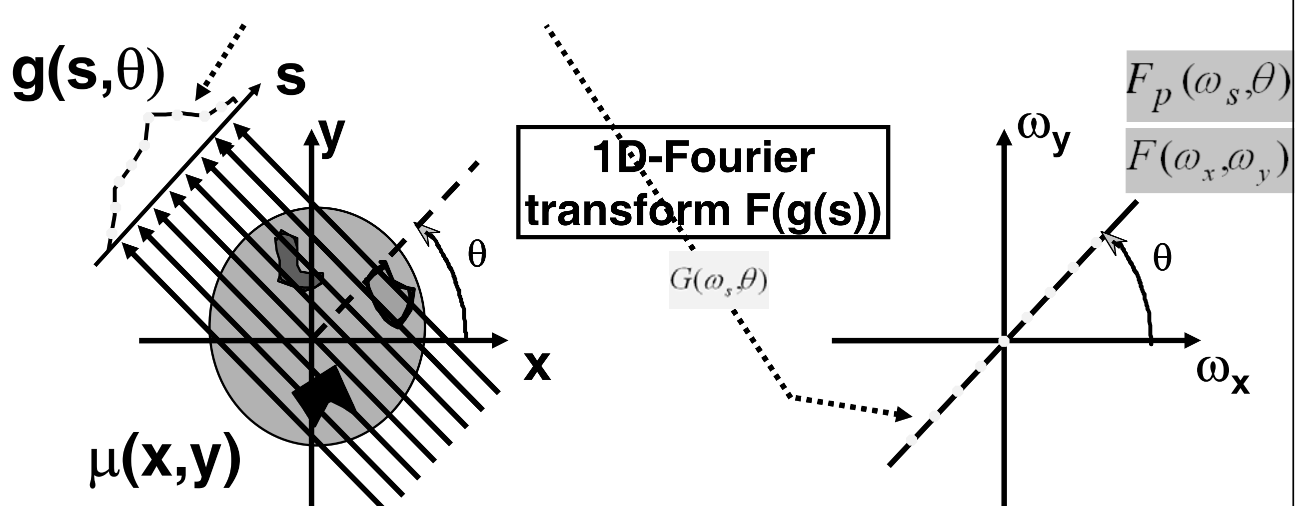
$$f(x,y)=\int_{0}^{\pi}\left\{\int_{-\infty}^{\infty}|\omega_s|G(\omega_s,\theta)\exp[i\omega_s s]d\omega_s\right\}d\theta$$

$$f(x,y)=\int_{0}^{\pi}\hat{g}(x\cos\theta+y\sin\theta,\theta)d\theta=\int_{0}^{\pi}\hat{g}(s,\theta)d\theta$$

Projection Theorem

(also “Central Slice Theorem” or Projection Slice Theorem)

$$G(\omega_s,\theta) \qquad = \qquad F_p(\omega_s,\theta)$$



Fourier Filtered Backprojection Reconstruction & Backprojection Filtering Reconstruction

$$f(x,y)=\int_0^{\pi}\hat{g}(s=x\cos\theta+y\sin\theta,\theta)d\theta$$

$$with\ \hat{g}(s,\theta)=\int_{-\infty}^{\infty}|\omega_s|F_1(\omega_s,\theta)\exp(i\omega_s s)d\omega_s$$

$$with\ \ F_1(\omega_s,\theta)=F_{1,s}\left(g(s,\theta)\right)$$

Fourier Filtered Backprojection Method

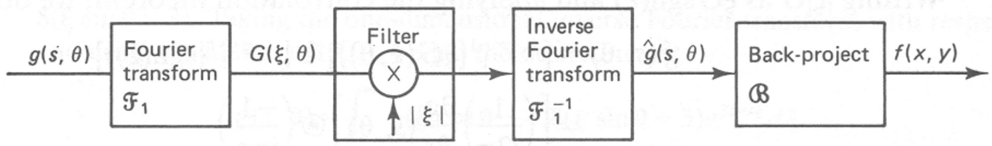
$$f(x,y)=B\Big(IF_{1,s}\big(|\omega_s|\bullet F_{1,s}(Rf)\big)\Big)$$

Backprojection Filtering Method

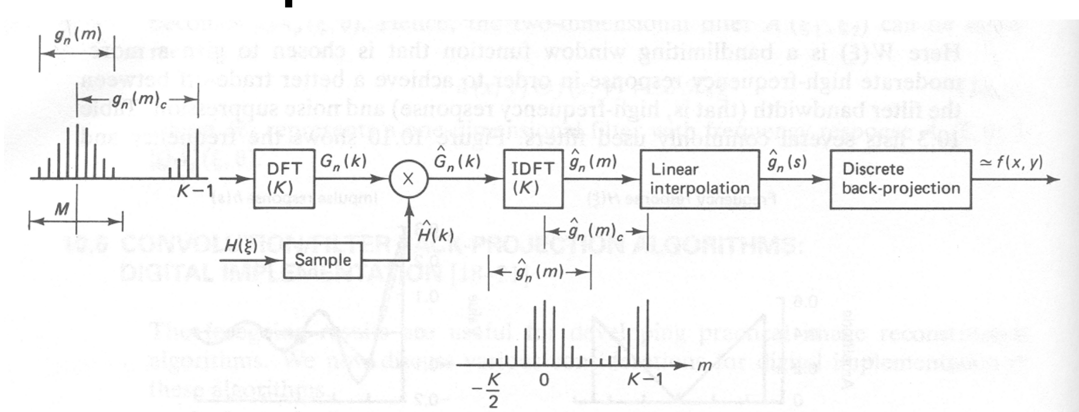
$$f(x,y)=IF_2\Big(|\omega|\bullet F_2\big(B(Rf)\big)\Big)$$

Fourier Filtered Backprojection Reconstruction

basic concept:

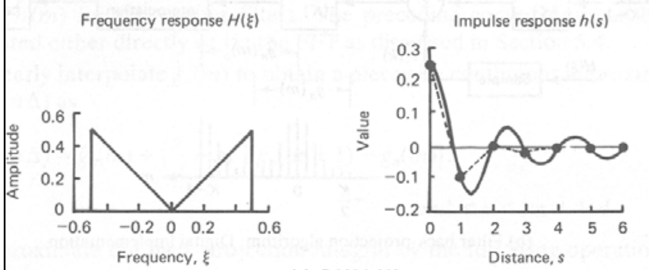


discrete implementation:



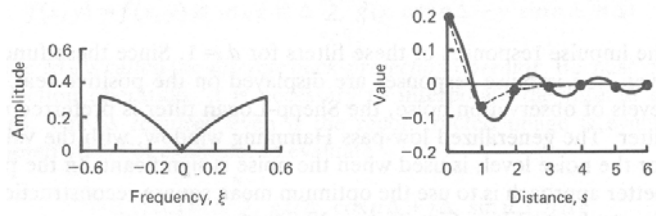
|ω| - Filters

RAM - LAK



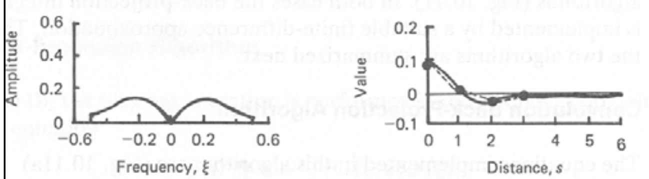
(a) RAM-LAK

Shepp-Logan



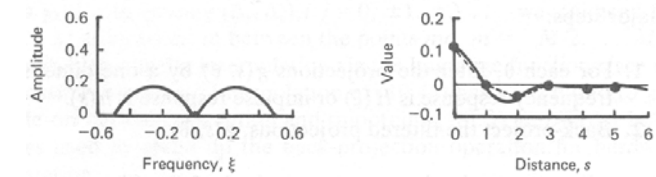
(b) Shepp-Logan

Hamming



(d) Generalized hamming

Lowpass Cosine



(c) Lowpass cosine

IV. Convolution Filtered Backprojection

$$\underline{f(x,y)} = \int_0^{\pi} \hat{g}(s = x\cos\theta + y\sin\theta, \theta)\, d\theta$$

$$\text{with } \hat{g}(s,\theta) = \int_{-\infty}^{\infty} |\omega_s| G(\omega_s,\theta) \exp(i\omega_s s)\, d\omega_s \qquad \text{with } G(\omega_s,\theta) = F_{1,s}(g(s,\theta))$$

$$= \int_{-\infty}^{\infty} \underline{\omega_s G(\omega_s,\theta)}\, \underline{\operatorname{sgn}(\omega_s)} \exp(i\omega_s s)\, d\omega_s$$

convolution theorem

$$= \left[IF_1\{\omega_s G(\omega_s,\theta)\}\right] \otimes \left[IF_1\{\operatorname{sgn}(\omega_s)\}\right]$$

$$= \underline{\left[\left(\frac{1}{i2\pi}\right)\frac{\partial g(s,\theta)}{\partial s}\right] \otimes \left[\frac{-1}{i\pi s}\right]}$$

$$= \left(\frac{1}{2\pi^2}\right) \int_{-\infty}^{-\infty} \left[\frac{\partial g(t,\theta)}{\partial t}\right] \frac{1}{s-t}\, dt$$

Hilbert Transform

	Function	Fourier transform unitary, ordinary frequency	Fourier transform unitary, angular frequency	Fourier transform non-unitary, angular frequency
	$f(x)$	$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi}\, dx$	$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$	$\hat{f}(\nu) = \int_{-\infty}^{\infty} f(x) e^{-i\nu x}\, dx$
101	$a \cdot f(x) + b \cdot g(x)$	$a \cdot \hat{f}(\xi) + b \cdot \hat{g}(\xi)$	$a \cdot \hat{f}(\omega) + b \cdot \hat{g}(\omega)$	$a \cdot \hat{f}(\nu) + b \cdot \hat{g}(\nu)$
102	$f(x-a)$	$e^{-2\pi i a \xi} \hat{f}(\xi)$	$e^{-ia\omega} \hat{f}(\omega)$	$e^{-ia\nu} \hat{f}(\nu)$
103	$e^{2\pi i a x} f(x)$	$\hat{f}(\xi - a)$	$\hat{f}(\omega - 2\pi a)$	$\hat{f}(\nu - 2\pi a)$
104	$f(ax)$	$\frac{1}{\lvert a\rvert} \hat{f}\left(\frac{\xi}{a}\right)$	$\frac{1}{\lvert a\rvert} \hat{f}\left(\frac{\omega}{a}\right)$	$\frac{1}{\lvert a\rvert} \hat{f}\left(\frac{\nu}{a}\right)$
105	$\hat{f}(x)$	$f(-\xi)$	$f(-\omega)$	$2\pi f(-\nu)$
106	$\frac{d^n f(x)}{dx^n}$	$(2\pi i \xi)^n \hat{f}(\xi)$	$(i\omega)^n \hat{f}(\omega)$	$(i\nu)^n \hat{f}(\nu)$
107	$x^n f(x)$	$\left(\frac{i}{2\pi}\right)^n \frac{d^n \hat{f}(\xi)}{d\xi^n}$	$i^n \frac{d^n \hat{f}(\omega)}{d\omega^n}$	$i^n \frac{d^n \hat{f}(\nu)}{d\nu^n}$
108	$(f * g)(x)$	$\hat{f}(\xi)\hat{g}(\xi)$	$\sqrt{2\pi} \hat{f}(\omega)\hat{g}(\omega)$	$\hat{f}(\nu)\hat{g}(\nu)$
109	$f(x)g(x)$	$(\hat{f} * \hat{g})(\xi)$	$\frac{(\hat{f} * \hat{g})(\omega)}{\sqrt{2\pi}}$	$\frac{1}{2\pi}(\hat{f} * \hat{g})(\nu)$
110	For $f(x)$ a purely real even function	$\hat{f}(\omega)$, $\hat{f}(\xi)$ and $\hat{f}(\nu)$ are purely real even functions.		
111	For $f(x)$ a purely real odd function	$\hat{f}(\omega)$, $\hat{f}(\xi)$ and $\hat{f}(\nu)$ are purely imaginary odd functions.		

Convolution Filtered Backprojection

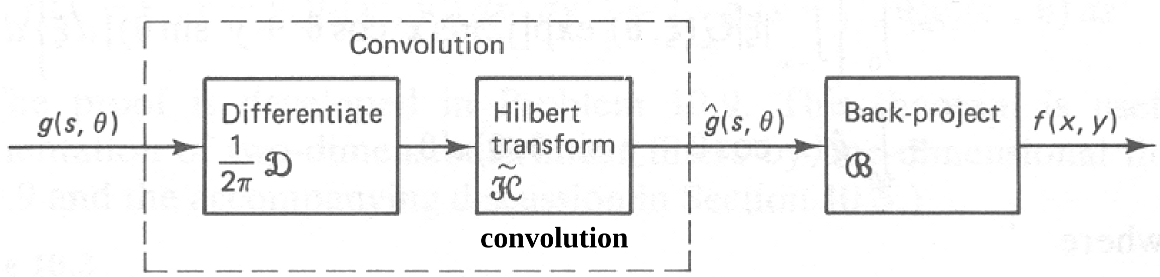
The inverse Radon transform is obtained in three steps:

(1) Each projection is differentiated with respect to s.

(2) A Hilbert transformation is performed with respect to s.

(3) The result of step (2) is backprojected to yield *f*(*x*,*y*).

$$f(x,y) = (1/2\pi) B\left(H_S\left(D_S\left(Rf\right)\right)\right)$$



Summary

Fourier Reconstruction - Method I:

$$f(x,y) = IF_2\left(\sum_{n=1}^{N} F_{1,s}\left(g(s,\theta_n)\right)\right)$$

Backprojection Filtering - Method II:

$$f(x,y) = IF_2\left(|\omega| \bullet F_2\left(B(Rf)\right)\right)$$

Fourier Filtered Backprojection - Method III

$$f(x,y) = B\left(IF_{1,s}\left(|\omega_s| \bullet F_{1,s}(Rf)\right)\right)$$

Convolution Filtered Backprojection – Method IV:

$$f(x,y) = (1/2\pi)B\left(H_s\left(D_s(Rf)\right)\right)$$

Projects

Groups of 3-4 work on the same problem but with different approaches. Consult each other and divide work whenever possible.

Presentation in class

Will be graded as ~6 homeworks (or ~10% Grade).

Group Projects

25 students => **5** groups of **5**

Each group will write reconstruction program in Matlab:

A. Fourier Reconstruction - Method I:

$$f(x,y) = IF_2\left(\sum_{n=1}^{N} F_{1,s}\left(g(s,\theta_n)\right)\right)$$

B. Backprojection Filtering - Method II

$$f(x,y) = IF_2\left(|\omega| \bullet F_2\left(B\left(Rf\right)\right)\right)$$

C. Fourier Filtered Backprojection - Method III:

$$f(x,y) = B\left(IF_{1,s}\left(|\omega_s| \bullet F_{1,s}(Rf)\right)\right)$$

D. Convolution Filtered Backprojection - Method IV:

$$f(x,y) = (1/2\pi)B\left(H_s\left(D_s\left(Rf\right)\right)\right)$$

E. Iterative Reconstruction