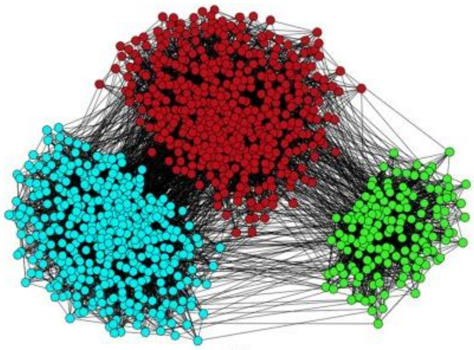
INSTITUTO SUPERIOR TÉCNICO | UNIVERSIDADE DE LISBOA
NETWORK SCIENCE 2023/24
LIST OF SUGGESTIONS FOR YOUR PROJECT
(VERSION 1.0, SEPT. 2023)

Below please find some suggestions for your assignment/project. Importantly, please feel free to propose your own idea and let us know about it. For instance, if you are already involved in a particular project (say, MSc or PhD thesis) related with complex systems or network science, please let us know. Please be aware that choosing a project/challenge may involve some reading and a bit of time… Start as soon as possible. Thank you in advance!

A. Analysis of existing datasets



Here, the idea is to analyze a network dataset using the concepts you learn in the course. The topic is rather open such that you get the opportunity to explore what interests you most. You may choose a relatively small network or a large-scale graph. Please explore our "problem sets and laboratories" section for a list of datasets and empirical networks (e.g., SNAP[1]). Please note that these networks may come in different formats. Depending on your choice of dataset and domain, you may address this project resorting to different tools, while addressing distinct technical obstacles. Try to find and follow the articles associated with the dataset repeating or extending their analysis and discussion. Often these articles include the characterization of the network, to evaluate the importance (centrality) of each node using relevant centrality measure for the chosen dataset and problem, or to suggest a set of principles responsible for the self-organization and creation of such network topology/topologies. Here are a few examples:

1. Online social media sites and users create fabulous networks of knowledge and cooperation. Wikipedia is a fantastic example of it. Here you create your own dataset through existing APIs, or use an existing one to assess what is the most important article on a sub area, what communities are there, what is the difference on the network topology between different languages, etc. Moreover, relations between users often reflect a mixture of positive (friendly) and negative (antagonistic) interactions, translated into networks where edges have an associated sign. Thus, you can use this type of platform to dive into theories of signed networks from social psychology, such as social balance, and others [1] or to study link prediction in these contexts [2]. For datasets, please check Stanford's' Large Network Data Collection (SNAP).

2. Study the topology or intercommunity interactions on Reddit, examining cases where users of one community are mobilized by negative sentiment to comment in another community. For an interesting reference on this topic, please see [3] and associated dataset available in Stanford's' Large Network Data Collection (SNAP).

3. Urban mobility increasingly relies on multimodality, combining the use of bicycle paths, streets, and rail networks. These different modes of transportation are well described by multiplex networks. Here we propose that you follow the method proposed in [4] to describe the multimodal profile from a city's multiplex transportation network. To perform this kind of extraction an analysis, you may use OSMnx (install:

[1] https://snap.stanford.edu/data/

https://osmnx.readthedocs.io/en/stable/, examples: https://github.com/gboeing/osmnx-examples/tree/main/notebooks).

4. Temporal networks are networks whose topology evolves in time, e.g., where links and nodes are active only at certain points in time. As a result, all metrics need to be revised — for a review see here [5]. You may analyze networks available in the abovementioned repositories from this perspective, assessing time-invariant topologies, link prediction or Motif finding in these contexts. Often this time-dependence is somewhere hidden in the raw data (see Reddit [3], Stack/Math Overflow, Wiki-talk [1], etc., in the SNAP website). For instance, in these examples, link carries information on when it is active, along with other characteristics such as a weight.

5. Natural language is an evolving system whose present structure can doubtlessly be considered a product of the long history of self-organization. Many attempts to study language from the point of view of network science express words as nodes and their relations by edges. If you are interested in knowing more about natural languages, you are invited to explore the differences between network characteristics of the texts belonging to different novels/languages/fields. See, e.g., [6, 7].

6. Scientometrics or Science of Science is the field of study which concerns itself with measuring and analyzing scholarly literature. Here you are invited to analyze the role of inequality and hierarchy in faculty hiring networks, following the analysis performed in Ref. [8]. The datasets can be found here: https://aaronclauset.github.io/facultyhiring/ , including data on 18,924 tenure or tenure-track faculty collected between May 2011 and August 2013 for the disciplines of Computer Science, Business, and History.

7. Network science has a strong impact in biological and life sciences. Here we propose that you find a biology network (ex: protein interaction, gene regulation, brain networks, ...) and explore the papers associated. This is a nice opportunity to dive into an entire new domain…

8. Complex networks theory has solid applications within the area of airline transportation networks. You may take flights and explore airport importance, country importance, communities, relate it to the flux of people, diseases, etc.

9. Follow Evelina Gabasova’s footsteps [9] and analyze the Star Wars social network. You may also pick an user-product review dataset (ex: imdb) and analyze it. Example of tasks: analyze degree distributions, communities, time-evolution of network properties, predict links/scores, etc.

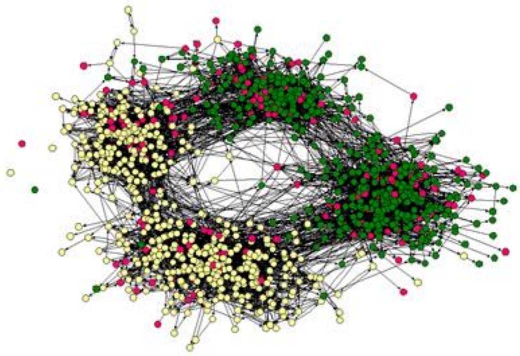
10. Recommendation networks in Amazon. The network available here is based on *Customers Who Bought This Item Also Bought* feature of the Amazon website. If a product i is frequently co-purchased with product j, the graph contains a directed edge from i to j. You may wish to study this dataset following Ref. [10].

11. Take the flights in https://openflights.org/data.html and explore airport importance, country importance, communities, etc.

12. Instead of analyzing an existing dataset, you may dedicate your project to contribute to the community by creating a novel dataset. Please let us know if you have an idea.

B. Community finding, graph clustering and ranking

The problem of finding clusters in graphs is a classic problem, not only for network science researchers, but also for data scientists as a suitable method for large-scale (unsupervised) clustering method. As a result, community finding algorithms have been studied by mathematicians, computer scientists, physicists, sociologists, among others. You may pick one of the datasets above (see, e.g., https://snap.stanford.edu/data/#communities) and identify the communities associated. Before that, read Chapter 9 of A. L. Barabási's book[2]. Below please find some examples of projects related to this topic.



13. Implement and compare two or three algorithms for finding communities in different graphs.

14. Consider the use of network clustering/partitioning methods for vertex reordering and its use for compressing graphs representations, as it is the case of Webgraph [11]. Try different partitioning methods and compare compression results with LLP (Layered Label Propagation), which is presently used in Webgraph.

15. Implement the local optimization method based on Heat-Kernel and sweeping on top of Webgraph [11]. Alternatively, you may conduct an experiment in order to compare Page-Rank and Heat-Kernel, comparing obtained rankings and convergence ratios (for the PageRank you can see the book "Mining of Massive Datasets" chapter 5 and for the HeatKernel you can check these two papers [12, 13]). We suggest that you consider a Web graph in this study and the application on personalized ranking.

16. Explore the resolution limit of modularity in community detection [14].

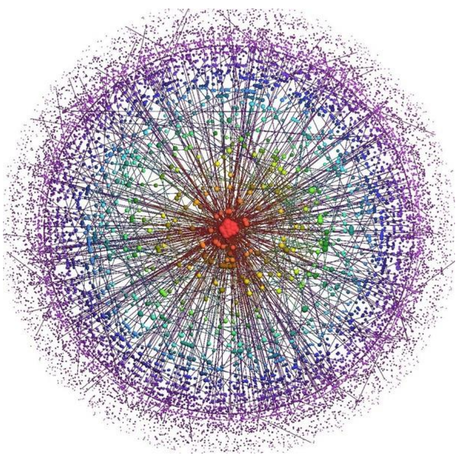
17. Consider the alternative method InfoMap for finding communities and compare it with studied methods. We recommend that you rely on known benchmarks and on Normalized mutual information (NMI) for comparing partitions.

18. Explore the clique percolation method for uncovering the overlapping community structure of complex networks.

19. Explore how to use ranking for the local partitioning of graphs (directed and undirected) [15, 16].

C. Mining large graphs and sampling effects

Here, the focus should be given to the efficiency of the implemented algorithms (e.g., showing execution times on different datasets) and exploration of different algorithmic approaches. These projects should be used as an opportunity (or a good excuse!) to learn more about advanced algorithms on graphs. Alternatively, you may also opt to discuss the relationship between the network properties of a sampled graph and the real underlying network.



20. Since exact computation in large networks is prohibitively expensive, we present two efficient randomized algorithms for betweenness estimation. Explore how to compute vertex betweenness centrality for large networks efficiently through sampling. You may follow this paper by Riondato and Kornaropoulos [17].

21. Explore the gtrie approach to enumerate motifs on graphs [18].

[2] http://networksciencebook.com/

22. Implement and compare two or three algorithms for finding k-cores of a graph (you can see, for instance, this paper [19]). Alternatively, you may wish to propose an algorithm for finding k-cores in linear time on top of Webgraph [11].

23. Explore how the Average Path Length (APL) can be computed approximately through the use of approximated counters [20, 21]. You may take a look on Webgraph HyperBall implementation, namely the use of HyperLogLog algorithm.

24. Many complex networks' studies are grounded on subsets of the complete network. Here, we invite you to discuss the relationship between a sampled graph and the real underlying network under simple sampling schema. For instance, is a sample of a scale-free network, also a scale-free network? In this project you are expected to simulate computationally 2 sampling methods (random & percolated) on a given network (regular, random and scale-free) of large dimensions. To get into the general problem, you may start by reading Ref. [22].

25. Accuracy and scaling phenomena in Internet mapping. Analyze through computer simulations the advantages and problems of *traceroute* [23] as a sampling method through computer simulations. *Traceroute* has been used to extract the Internet graph Simulate *traceroute* on a given network (regular, random and scale-free) of large dimensions. Test the quality of the sampling resorting from a single source and multiple TTLs and multiple source with a given TTL. As an alternative, follow the analysis taken by Clauset et al [24-26] on this very same problem.

D. Robustness and cascading effects in complex networks

For many physical networks, the removal of nodes can have a much more devastating consequence then the loss of vertices and links whenever the intrinsic flows and maximum loads are considered (e.g., think about power transmission grid, airport networks, etc.). Here, you're invited to model this type of process and engineer some control measures capable of halting a cascade.

26. Robustness of a Network of Networks. Network research has been focused on studying the properties of a single isolated network. Here you test through numerical simulations the analytical framework proposed in Refs. [27, 28] to understand the resilience *n* interdependent networks under random and targeted attacks.

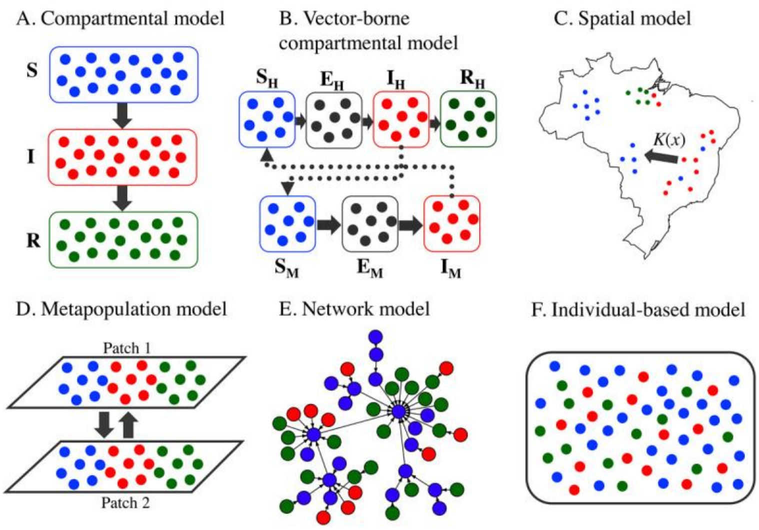
27. Global cascades in social and economic systems, as well as cascading failures in engineered networks, display two striking qualitative features: they occur rarely, but by definition are large when they do. Here you shall discuss a computational model (see here [29]) designed to explain why this type of behavior is observed, offering testable predictions about cascades in real systems.

28. In many realistic situations the flow of physical quantities in the network, as characterized by the loads on nodes, is important. Here you're invited to implement the Motter-Lai model (2002) [30] which shows that intentional attacks can lead to a cascade of overload failures. The project involves the creation of a computer model that shows that the heterogeneity of real-world networks makes them particularly vulnerable to attacks in that a large-scale cascade may be triggered by disabling a single key node.

29. How can we halt cascading failures? In line with the previous project, here you shall investigate an efficient strategy of defense based on a selective removal of nodes and edges, right after the initial attack or failure. The project involves the creation of a computer model of load balance (see [31]).

E. Disease spreading

The properties of real-world networks have a profound impact on dynamical processes occurring in various systems. The study of epidemic spreading is perhaps one the most evident examples. Here you are invited to explore several simulation and mathematical approaches related with network epidemics. For a general overview of this topic, please see Ref. [32].



30. Implement a computer simulation of the SIS model in a given network. Compute the epidemic threshold for lattices, random graphs, BA model networks and minimal model. Discuss the results. Suggested reading: [32, 33] (see also [34, 35]).

31. Simulate and analyze (numerically or analytically) the impact of degree-degree correlations on the epidemic threshold of random and scale-free networks (see [36] to create artificial networks with a given assortativity).

32. Epidemics. Discuss analytically the expected epidemic threshold in Random graphs and SF networks. In the later, analyze the epidemic threshold as a function of the exponent of the degree distribution. Assume that networks are infinitely large. Suggested reading: [32, 33] (see also [34, 35]).

33. Epidemics. Discuss and test (numerically or analytically) other forms of targeted immunization which do not assume global knowledge (e.g., please follow Refs. [37-39]).

34. Analytical methods for stochastic epidemic dynamics. Describe the stochastic SIS model through a Fokker-Plank Equation or, alternatively, as a Master equation (also known as Kolmogorov-forward equations). These equations provide an exact solution equivalent to multiple simulations of a model with stochasticity. See, for instance, Ref. [40], Section 6.6.

35. Epidemic spreading and temporal networks. Measurements indicate pairs of individuals will have periods of frequent interactions, when multiple contacts follow each other within a relatively short time frame, and long periods without any further contact. In this project the challenge will be to analyze the impact of bursts of interactions in disease spreading when compared to interaction patterns that are uniformly distributed in time through computer simulations. For more information please check [41-43]. As an alternative you may also discuss the origins of such bursts of interaction in networks [41, 44]. See also Ref. [45] and the dataset available. You may also explore the contradictory conclusions obtained in Refs. [45, 46] and [47].

36. Try to implement a simple computer model of co-evolution of disease states (e.g., SIS) and network structure. Assume that, at each time-step, individuals connected to infected nodes will try to rewire their links. In other words, network evolution is a natural outcome of information disease states at each neighborhood. Try out an agent-based version of the minimal model discussed in Ref. [48].

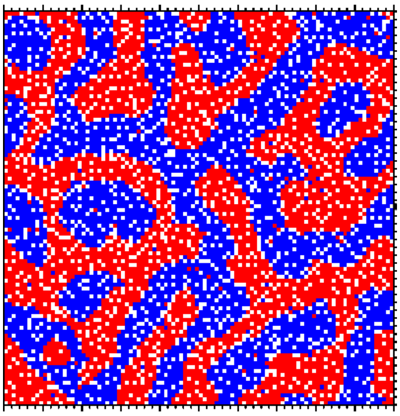
37. Message passing and Covid-19 related apps [49]. Here you're invited to present a simulation on the impact of contact tracing in mitigating an epidemic wave. You will try to answer how the increase of the app adoption level modified the value of the epidemic threshold..

38. The recent Zika epidemic poses a major global public health emergency. While Zika is transmitted from human to human by bites of mosquitoes, recent evidence indicates that it can also be transmitted via sexual contact. In this project you're invited to explore the analytical model proposed in ref. [50] to investigate the impact of mosquito-borne and sexual transmission on the spread and control of Zika.

39. Illustrate the use of the GLEAMviz (www.gleamviz.org) large-scale simulation tool [51], creating and analyzing the results of a new simulation and providing a step-by-step tutorial on the use of this new platform.

F. Racial segregation models

Racial segregation has always been a pernicious social problem. Why is segregation such a difficult problem to eradicate? In 1971, the American economist Thomas Schelling created an agent-based model that might help explain why segregation is so difficult to combat. His model of segregation showed that even when individuals (or "agents") didn't mind being surrounded or living by agents of a different race, they would still choose to segregate themselves from other agents over time! Although the model is quite simple, it gives a fascinating look at how individuals might self-segregate, even when they have no explicit desire to do so.



40. In this project it is expected that you create a computer simulation and analyze Schelling's model. For details please see here [52-55].

41. The Schelling model of segregation nicely illustrates how individual incentives and individual perceptions of difference can lead collectively to segregation. What if, on top of Schelling original assumptions, we allow agents to adapt their tolerance to others in response to their local environment? Let's say that when agents are exposed to the out-group their tolerance increases if they are currently satisfied with their environment, but otherwise it decreases. Does adaptive tolerance increases segregation? Try to answer to this question following Ref. [56].

G. Cooperation, reciprocity and Reputation dynamics

Being an essential ingredient of evolution, cooperation has played a key role in the shaping of species, from the simplest organisms to vertebrates. In this context, one of the most fascinating challenges has been to understand how cooperation may survive in communities of self-regarding agents, a problem which has been typically formalized in the framework of Evolutionary Dynamics and Game Theory. In the last decades, several mechanisms have been identified as cooperation promotors. Here you're invited to test (by means of simulations or analytical approaches) how if and how some these mechanisms work.

42. Reproduce the classical computer simulations by Nowak & Sigmund [57] which showed, for the first time, the evolution of cooperation through reputation dynamics. This is the first model of indirect reciprocity and reputation dynamics. It is funny to implement and brings interesting (unexplored) questions in complex networks. For a recent review on the marvelous topic of indirect reciprocity, see [58].

43. In ref. [57] (see previous project), the authors propose a social norm which, in general, fails to promote cooperation [59]. Here you should implement a simple computer simulation which shows that result and helps to offer a suitable alternative (please follow Ref.[59]).

44. Indirect reciprocity and dynamics of cooperation from an analytical / dynamical systems perspective. In this project you are invited to discuss two analytical models of indirect reciprocity extracting the phase diagrams for 2 famous social norms (image-scoring and standing). See [58] and references within.

45. Evolution of cooperation by multi-level selection. Competition between groups can lead to selection of cooperative behavior. This idea can be traced back to Charles Darwin. In this project, you should repeat the analytical approach performed in ref. [60].

H. Cooperation in networked populations

Following the previous challenge of how cooperation emerges in large populations, here you are invited to assess the role of population structure in the final outcome of cooperation and fairness. First, you can find projects where the goal is to explore the consequences of placing these players in a two-dimensional spatial array, mimicking the spatially embedded constraints commonly find in real-world settings. Secondly, you are invited to evaluate the impact of heterogeneity, in which some individuals have many more contacts than others. This fact contrasts with the traditional well-mixed setting used in analytical studies of evolutionary game dynamics where all individuals are equally likely to interact.

46. Spatial dynamics of cooperation. Spatial constraints have been shown to deeply influence self-organized behavior. In this project, it is expected that you reproduce the classical results obtained by Nowak & May in Nature paper from 1992 (see Ref. [61]). If you like this type of challenge, this project begs for a nice 2D visualization tool of a computer simulation of evolution of traits on lattices.

47. Spatial dynamics in co-existence games. Project similar to the previous one, but with a different social dilemma (the Chicken or Snowdrift game [62]). In this case, you will be dealing with a co-existence dynamics and you're expected to replicate the results obtained in Ref. [63]. Again, this project begs for a nice 2D visualization of a computer simulation of the evolution of traits on lattices.

48. Even if social structure may sometimes promote cooperation, not all agents have access to the same level of resources. In this project, following Ref. [64], you shall investigate (through computer simulations) how inequality of resources among agents influence the emergence of cooperation.

49. Literature suggests that humans may cooperate due to repeated interaction with the same individuals, suggesting the concept of Direct Reciprocity [65]. Here, you are invited to explore the impacts of different network topologies (BA models, Small-World, lattice, random networks, etc.) on Direct Reciprocity through agent-based computer simulations. To this end, please follow [66] or [67] and try to understand how the inclusion of additional cognitive skills may increase or decrease the overall levels of cooperation. You're also invited to check https://ncase.me/trust/ out as inspiration.

50. Population genetics and evolution on networks. Compute numerically (i.e., by means of agent-based computer simulations) the average fixation probability [68] of a mutant as a function of the degree of a network. Assume that the mutant has a fitness r>1, where resident traits share a fitness=1. Study this dependence as a function of *r*. Start by considering regular, random and scale-free networks (BA model). Randomize a BA model and repeat the simulations, checking if the degree is the only important factor.

51. Evolution of cooperation in networked populations through computer simulations. Real populations have been shown to be heterogeneous, in which some individuals have many more contacts than others. Here you shall incorporate heterogeneity in the population by studying games on graphs with various topologies, in which the variability in connectivity ranges from single-scale graphs, for which heterogeneity is small and associated degree distributions exhibit a Gaussian tale, to scale-free graphs, for which heterogeneity is large with degree distributions exhibiting a power-law behavior. Consider a simple Prisoner's dilemma game [69](T, R=1, P=0, S), studied in the framework of evolutionary game theory [70]. Compute the stationary fraction of cooperators for lattices, random graphs and scale-free networks. Suggested reading list: Ref. [34] (section 10.5), and Ref. [71].

52. Evolution of cooperation in temporal networks. The structure of social networks is a key determinant in fostering cooperation and other altruistic behavior among naturally selfish individuals. However, most real social interactions are temporal, being both finite in duration and spread out over time. In this project you shall investigate the impact of temporal patterns on self-organized cooperation. Suggested reading: [72].

53. Collective action in heterogeneous political networks. Analyze the impact of structural diversity in the evolution of cooperative behavior in political networks. Check Fig. 3 in Ref. [73] and extend the methodology to other classes of N-person games (N-person Snowdrift-Game and N-person Stag-hunt game). [Involves computer simulations]

54. Develop a computer model to analyze the role of punishment and reputation in spatial public goods games (graph=lattice) (please follow [74]). Extend it to other networks if you have the time.

55. Information sharing, interdependent networks and prosocial behaviors. Implement a computer simulation with two interdependent lattices (you may extend it to a general network). The goal is to assess the impact of information sharing about strategy choice between players residing on two different networks on the evolution of cooperation. See [75] for further details.

56. Leadership and conformism in social dilemmas. Develop a computer simulation in order to assess the role of conformism in social dilemmas played in heterogeneous networks. You may follow the recent discussion presented in Ref. [76].

57. Try to implement a simple computer model of co-evolution of strategies and network structure. For an agent-based example see Ref. [77].

58. Try to implement a simple analytical model of co-evolution of strategies and network structure. Discuss the “Active Linking” model introduced in Ref. [78] and address the game transformations also discussed in the same paper.

I. The evolution of fairness

The decision-making process associated with fairness is often framed within the Ultimatum game. In the ultimatum game, two players are asked to split a certain sum of money. The proposer has to make an offer. If the responder accepts the offer, the money will be shared accordingly. If the responder rejects the offer, both players receive nothing. The rational solution is for the proposer to offer the smallest possible share, and for the responder to accept it. Human players, in contrast, usually prefer fair splits. Here we shall try to understand why.

59. The evolution of fairness and the ultimatum game [79-81]: In this project you shall investigate (through computer simulations) how a spatial setting drives (or not) evolution towards widespread fairness (for more information see [82]). If possible explore the possibility of having punishment of low offers and its impact in the evolution of fairness.

60. Following the ultimatum game (1) proposal suggested above [79-81], here you are expected to implement the ultimatum game on scale-free networks and analyze the levels of fairness obtained (you may follow the approach proposed in Ref. [83]) [Involves computer simulations].

61. Here you are expected to analyze the emergence of fairness in populations of agents interacting following a N-person version of the ultimatum game. Instead of having 2 individuals, proposal are made to a collective of individuals who may collectively accept and reject an offer [84]. This type of situations are similar to the ones observed in international negotiations [Involves computer simulations]. You may also evaluate the role of networks in this context [85].

J. Public goods, climate action and N-player interactions

Most human interactions involve dilemmas that occur in large groups with a complexity that largely surpasses 2-player interactions and game, and that only recently started to be reasonably understood. Here we suggest a few options that would allow you to learn more about it.

62. In the natural world, performing a given task which is beneficial to an entire group often requires the cooperation of several individuals of that group who often share the workload required to perform the task. Here you are invited to study this problem of collective action in well-mixed populations (analytical project). You may compute analytically (through, for instance, a *Mathematica* notebook) the gradient of selection for N-person games with thresholds. Check the following two refs: [86, 87].

63. Game theory has been used to investigate possible climate negotiation solutions and strategies for accomplishing them. Here you shall analyze the evolutionary dynamics of cooperation in climate dilemmas through a stochastic process (see ref. [73]). You shall identify the role or risk, group size and diversity in the chances of reaching to cooperation. This project can be dealt both through computer simulations and analytically, computing a stationary distribution of a Markov chain.

64. Tourism and cooperation. Tourists and traditional divers in a common fishing ground (try to reproduce analytically or by means of computer simulations some of the results in Ref. [88]). Discuss the general approach to the problem.

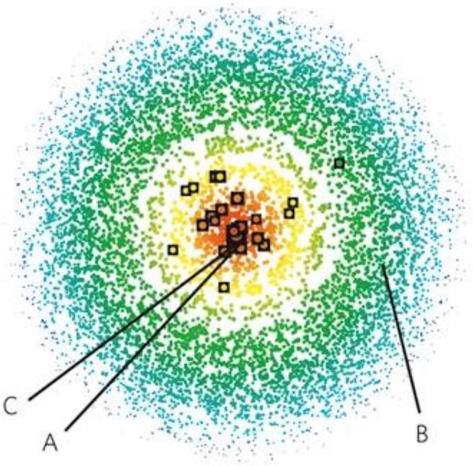
65. Cooperation and Ostracism. Analyze a game-theoretical model of social exclusion in which a punishing cooperator can exclude free-riders from benefit sharing. You may opt to repeat the calculations described in Ref. [89] or propose a computer model replicating the same idea.

66. Social diversity and Public goods games in complex networks. Repeat the computer simulations proposed in Ref. [90] and discuss the results obtained.

67. Here we you shall implement an N-player bargaining game in an agent-based model (see here [91]) to examine the past failures of and future prospects for a robust international climate agreement. This project involves computer simulations.

K. Opinion dynamics and information spreading in social networks

Agreement among peers is one of the most important aspects of social dynamics. We find many situations in which it is necessary for a group to reach shared decisions. Moreover, the knowledge of the spreading pathways through the network of social interactions is crucial for developing efficient methods to either hinder spreading in the case of diseases (see also topics above), or accelerate spreading in the case of information dissemination. Below please find a few examples of related projects, but you may also check section III of [92] for more ideas.



68. Identifying the most efficient 'spreaders' in a network is an important step towards optimizing the use of available resources and ensuring the more efficient spread of information. Here you will show that often the best spreaders do not correspond to the most connected or central nodes [93]. The simulation of this problem may provide a route for an optimal design of efficient dissemination strategies, which you may also explore.

69. Rumor and information spreading in complex networks (analytical or numerical simulations). Implement one of the models discussed in [34](see section 10.2) on a network. Discuss the efficiency of rumor spreading in 3 network classes (e.g., WS strogatz, BA model and Minimal model). As an alternative, discuss the impact of community structures in the overall efficiency of information spreading.

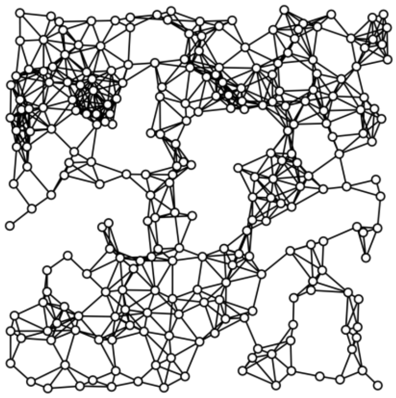
70. The origin of 3-degrees of influence. In 2007 it was found that social influence does not end with the people to whom a person is directly tied [94, 95]. We influence our friends, who in their turn influence their friends, and so our actions can influence people we have never met, to whom we are only indirectly tied. As Fowler and Christakis, the authors of this idea posit, "ripple through our network, having an impact on our friends (one degree), our friends' friends (two degrees), and even our friends' friends' friends (three degrees). Our influence gradually dissipates and ceases to have a noticeable effect on people beyond the social frontier that lies at three degrees of separation". Implement the Voter Model in a random graph. Analyze the emergent correlations among nodes at different distances and compare it with the correlations obtained in the random case. Try to understand the emergence of the phenomena through this simple computational model. For details please see [96].

71. Complex contagion is the phenomenon in social networks in which multiple sources of exposure to an innovation are required before an individual adopts the change of behavior. In this project you shall propose a new computational model for complex contagion and discuss one of the following options: i) adapt the SIS model to consider a complex contagion and discuss the emergence of an endemic state in this case; ii) adapt the SIR model for complex contagion and analyze the if in this case one may also obtain the same pattern of 3-degrees of influence [94, 95] modeled here [96].

72. In this project, you shall study the co-evolution of opinions and networks, and analyze (through a computational model) how the time-evolution of social ties can influence diversity and uniformity in individuals' preferences. You may also focus your attention on the emergence of the identification of the conditions under which partisan echo chambers emerge. Please check [97, 98]. This project involves computer simulations.

L. Spatially embedded networks

Discuss the possibility of having scale-free spatial networks (see [99, 100]) and implement one these models through computer simulations or discuss the models proposed in refs. [99, 101]. For a review on spatial networks please check ref. [102].



73. Networks and population of robots. Consider a spatial network of moving agents. Implement a computer simulation where a scale-free ad-hoc network emerges from such interacting system.

References

Some of the references below have an associated URL. In principle, all papers can be downloaded via Google Scholar (for each entry, see also "All versions"). If you still cannot find a way to download a paper please let us know (or try it via https://sci-hub.se/).

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