Collective action in political networks: How risk affects cooperation

Group 16

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Abstract

The tragedy of the commons and public goods games are problems of large interest in evolutionary game theory and economics. Given the option to contribute a share of resources to a common cause, most agents will prefer to let others contribute instead, even if that might be detrimental to the group as a whole. We replicate the results of previous studies on how changes in the ratio between collective risk and contribution requirements change the evolutionary dynamics of cooperation for populations playing an N-person Stag hunt Game and extend the methodology for an N-person Snowdrift Game. We show that previous findings on the benefits of a high perception of risk for cooperation levels also apply to this type of game. When addressing real-world public goods, it might be advantageous to increase awareness of risk within the population.

I. Introduction

Public goods games can be used to describe several problems of major interest to human society, from facing climate change[1], to cooperation amongst farmers and maintenance of shared infrastructures[2] or vaccination of a population[3][4]. Since these problems require that individuals contribute a fraction of their endowment for a common objective, most agents will choose to defect, unless that creates a significant risk for themselves, leaving others to contribute for them, leading to the tragedy of the commons[12].

As a result, if we expect to evade the tragedy of the commons, it is imperative that the agents have a reason to want to cooperate, be it some sort of punishment, [10] reward or risk that they inherently accept by adopting a given strategy. In many games[1], risk might really be the most natural reason why someone would want to cooperate. In the climate change example, if most agents choose to defect, there may not be enough contributions to prevent the increase in climate related catastrophes[18, 19], hurting all agents. Likewise, when maintaining common infrastructures, [2] if certain goals aren't met, those infrastructures won't have the necessary conditions to be of any use to the population. In these scenarios, there is a collective risk, since all individuals suffer together, instead of just the ones who pick a certain strategy.

Although we can analyze cooperation between two players, by playing classic two player games in game theory[5], most real life games of interest are played in much larger groups[1, 2, 3, 4]. Some work has already been done on cooperation in N-person Prisoner's Dilemma (PD) and Stag Hunt (SH) [6, 8] games. Here, we extend previous work on the role of risk of collective failure on the evolutionary dynamics of cooperation[8] to the case of N-person Snowdrift games (SG), adapting from [7].

II. Methods

I Games and Payoffs

In order to define the outcome of a game, we build $\Pi_D(k)$ and $\Pi_D(k)$, expressing the payoffs for defectors and cooperators, respectively, considering N players and k cooperators.

The payoffs are determined by the type of game and generalized for a public goods game involving N players. Given a cooperation threshold M that must be achieved, the Heaviside step function θ with input (k-M) (1) helps us define the payoffs of coordenators (Cs) and defectors (Ds). [6]

$$\theta(k - M) = \begin{cases} 0 & \text{if } k < M \\ 1 & \text{if } k \ge M \end{cases} \tag{1}$$

A PD game with risk involved becomes a SH one, meaning there's hope for a cooperative equilibrium. [6] A SG game is said to be dominated by greed, but not fear like in the PD. [7] Risk (r) is the probability of losing all endowments in case the condition $k \geq M$ isn't met. Constant b=1 defines the initial endowment and c=0.1 the fraction of b that Cs contribute with. Payoffs for different types of N-person dilemmas [6, 7] are described in equations 2 and 3.

$$SH \begin{cases} \Pi_D(k) &= b(\theta(k-M) + (1-r)[1-\theta(k-M)]) \\ \Pi_C(k) &= \Pi_D(k) - cb \end{cases}$$
(2)

$$SG \begin{cases} \Pi_D(k) &= b(\theta(k-M) + (1-r)[1-\theta(k-M)]) \\ \Pi_C(k) &= \Pi_D(k) - \frac{cb}{k}\theta(k-M) - \frac{cb}{M}(1-\theta(k-M)) \end{cases}$$
(3)

Note that the SG payoffs here are not the typical ones, [7] the difference being the introduction of M and r. When the threshold is not met, each C gives their best contribution to achieve the cooperation goal, each giving $\frac{1}{M}$ of the total cb.

II Infinite Well-Mixed Populations

We considered an infinite well-mixed population, meaning every individual can potentially interact with any other individual in the population and that the fitness of a C (D) is the same for all Cs (Ds). x denotes the fraction of cooperators C in the population and (1-x) of D, such that x=k/Z. Using a binomial distribution, we can compute the average fitness [6, 8] of cooperators $f_C = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \Pi_C(k+1)$ and that of defectors $f_D = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \Pi_D(k)$.

Cost to risk ratio is $\gamma = \frac{c}{r}$. For sufficiently high risk, internal roots appear in the replicator equation $\dot{x} = x(1-x)(f_C - f_D)$, [8] associated with the roots of $f_C - f_D$ in $x \in (0,1)$. To find the (relevant) zeros of \dot{x} is, then, the same as solving $f_C = f_D$ for $x \in (0,1)$. The zeros \bar{x} of \ddot{x} are critical points in the game dynamics where cooperation is not decelerating nor accelerating. If $\ddot{x} > 0$ ($\ddot{x} < 0$), the rate of cooperation is increasing (decreasing), therefore, cooperation becoming more (less) stable.

For the SH game, we have $f_C - f_D = b(\Gamma(x) - \gamma)$, where r is the collective risk of failure and c is how much of their endowment Cs contribute with. Γ is defined in (4). For the SG case, instead of finding a closed form for $f_C - f_D$, we simply computed the difference numerically.

$$\Gamma(x) = \binom{N-1}{M-1} x^{N-1} (1-x)^{N-M} \tag{4}$$

In this game, the zeros \bar{x} of \ddot{x} are the internal roots of $\dot{\Gamma}$. We have $\dot{\Gamma} = \binom{N-1}{M-1} x^{M-2} (1-x)^{N-M-1} s(x)$, with s(x) = 1 + (N-1)x - M. For N=6>2 and 1 < M=3 < N, $\dot{\Gamma}$ has a single internal root for $\bar{x} = \frac{M-1}{N-1}$. [8] Since s(x) is positive (negative) for $x < \bar{x}$ $(x > \bar{x})$, then $x = \bar{x}$ is a global maximum of $\Gamma(x)$. Finally, we have $\bar{\gamma} = \Gamma(\bar{x})$ [8].

III Finite Well-Mixed Populations

For finite well-mixed populations of size Z, the groups were sampled according to a hypergeometric distribution. Average fitness of contributors (5) and defectors (6) are rewritten [6] as

$$f_C = {\binom{Z-1}{N-1}}^{-1} \sum_{j=0}^{N-1} {\binom{k-1}{j}} {\binom{Z-k}{N-j-1}} \Pi_C(k+1)$$
 (5)

$$f_D = {\binom{Z-1}{N-1}}^{-1} \sum_{j=0}^{N-1} {\binom{k}{j}} {\binom{Z-k-1}{N-j-1}} \Pi_D(k)$$
 (6)

I Stochastic Effects

To simulate the dynamics of cooperators and defectors, we investigate the stochastic effects of copying based on the imitation rule and behavioral mutations. To better understand the implications of these probabilistic events we employ a Markov chain model. Specifically, we deal with a birth-death process, where the "birth" corresponds to an increase in C

by 1 and "death" a decrease by 1. [6] The transition probabilities $p_{k,j}$ denote the probability of the process moving from k Cs to j Cs.

The copying mechanism is based on a pair-wise comparison rule in which each individual i adopts the strategy of a randomly selected member l of the population, with probability p given by the Fermi function $p = Fermi(\beta, j, i) = [1 + e^{-\beta(f_j - f_i)}]^{-1}$, where f_j and f_i are the fitness values of individuals j and i. In this context, β controls the intensity of selection[8] and a value of β =5 was used. Based merely on this mechanism, we define the probabilities to increase $T^+(k)$ (8) and decrease $T^-(k)$ (9) the number of Cs in the population by one individual. (7)

$$T^{+}(k) = \frac{k}{Z} \frac{Z - k}{Z} [1 + e^{-\beta(f_{j} - f_{i})}]^{-1} = x(1 - x) Fermi(\beta, j, i)$$

$$T^{-}(k) = \frac{k}{Z} \frac{Z - k}{Z} [1 + e^{+\beta(f_{j} - f_{i})}]^{-1} = x(1 - x) Fermi(-\beta, j, i)$$
(7)

Since we are dealing with finite populations, the gradient of selection G(k) can be written as the difference between the probabilities to increase $T^+(k)$ and decrease $T^-(k)$ the number of Cs in the population by one individual, given the current k. (8)

$$G(k) = T^{+}(k) - T^{-}(k) = x(1-x)\tanh\left(\frac{\beta}{2}[f_{C}(xZ) - f_{D}(xZ)]\right)$$
(8)

Secondly, we simulate behavioral mutations given a mutation rate $\mu=0.01$. In the presence of such events, and since the population is finite, it will never converge to a state of only cooperators or only defectors[13]. Combining the probabilities related to both the copying and mutation mechanisms, we get the transition probabilities for a mutation rate μ . We define $T_{\mu}^{+}(\mathbf{k})$ as the probability to increase from k to k+1 Cs and $T_{\mu}^{-}(\mathbf{k})$ as the probability to decrease from k to k-1 Cs. (9)

$$T_{\mu}^{+}(k) = (1 - \mu)T^{+}(k) + \mu \frac{Z - k}{Z}$$

$$T_{\mu}^{-}(k) = (1 - \mu)T^{-}(k) + \mu \frac{k}{Z}$$
(9)

II Stationary Distribution

To aggregate the previously defined probabilities $T_{\mu}^{-}(k)$ and $T_{\mu}^{+}(k)$, we construct the tridiagonal transition matrix S, defined as $S = [p_{i,j}]^T$. The entries are computed using the transition probabilities for each value of k. (10) Note that $p_{k,k}$ defines the probability to have the same number of Cs after one time step, therefore, defined as 1 minus the probability of changing k. S accounts for transitions due to the imitation mecanism $[(1-\mu)T^{-/+}(k)]$ and due to the behavioral mutations $[\mu \frac{k}{Z}]$.

$$p_{k,j} = \begin{cases} T_{\mu}^{+}(k) & \text{if } j = k+1 \\ T_{\mu}^{-}(k) & \text{if } j = k-1 \\ 1 - p_{k,k-1} - p_{k,k+1} & \text{if } j = k \\ 0 & \text{otherwise} \end{cases}$$
(10)

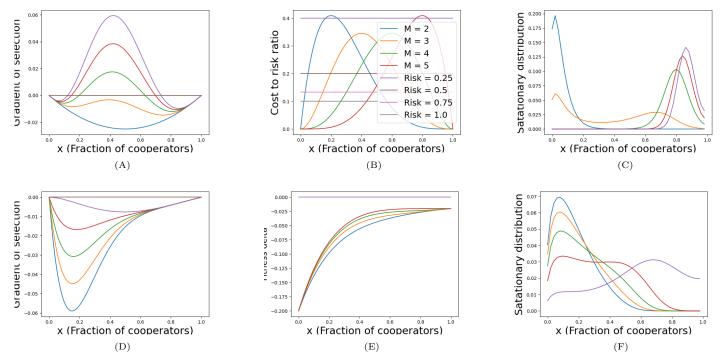


Figure 1: Evolutionary dynamics for different values of risk for Stag Hunt (A-C) and Snowdrift (D-F) games. (A and D) If the gradient of selection is positive (negative), the fraction of Cs will increase (decrease). N = 6, M = 3. (B and D) Internal equilibrium of the replicator dynamics. In B, the internal roots of the gradient of selection are the intersection points of the cost to risk ratio γ line with each curve $\Gamma(x)$ with N = 6. 4 In E, the internal equilibrium points are the roots of the curve fitness delta $(f_C - f_D)(x)$ with $\gamma = 0.125$ and N = 6. (C and F) Stationary distribution describing the prevalence of the fraction of cooperators x in finite populations, in the presence of stochastic effects. N = 6, M = 3, Z = 50.

In order to analyze the evolution of the population, while overcoming the stochastic nature of the mutations and social dynamics, we computed it's stationary distribution $P(\frac{k}{Z})$, a probability distribution that remains unchanged in time (if the transition to a certain state i has probability p at time t, then it will also have probability p at time $t+\epsilon$) [14].

 $P(\frac{k}{Z})$ characterizes the pervasiveness in time of a given composition of the population. This distribution is defined as the eigenvector of S with eigenvalue 1 and sum of elements 1. Therefore, it was obtained by computing the eigenvectors of S, picking the eigenvector v associated with the eigenvalue closest to 1 and then normalizing v, such that:

$$P(\frac{k}{Z}) = \frac{v}{\sum_{j=0}^{Z} v_j} \tag{11}$$

III. Results and Discussion

We can model such games by considering a population of size Z, with groups of N players, where each agent starts with an endowment b, of which a fraction c must be spent, if they decide to cooperate. Some groups may have overlapping players. The objective of the population, as a whole, is to reach a threshold of M cooperators, otherwise all players lose their endowment with a probability r, which is the risk.

Although a player may start with a given strategy, populations are dynamic objects and most behaviors involve interaction between agents, so players will tend to adapt their strategies to improve their fitness: [5] if we notice that we have some very successful friends, we are more likely to imitate them. Agents might also mutate their behaviour for, seemingly, arbitrary reasons. We can simulate such dynam-

ics by considering an evolutionary game model. In such a framework, the evolution of the fraction x of Cs and 1 - x of Ds is governed by the gradient of selection associated with the replicator dynamics equation [11] $\dot{x} = x(1-x)(f_C - f_D)$, where f_C and f_D are the fitness of cooperators and defectors, respectively. The number of cooperators decreases for $\dot{x} < 0$ and increases for $\dot{x} > 0$. [8]

I Stag Hunt

Infinite Well-Mixed Populations

As shown in previous studies, [8] we find that in the absence of risk (r=0), \dot{x} will always be negative, leading to the extinction of cooperators, as seen in Fig. 1. In the presence of risk, we notice the emergence of two mixed internal equilibria, rendering cooperation viable, and that collective cooperation becomes easier to achieve in games with high risk. [8]

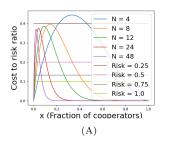
In Fig. 2, we confirm the location of these equilibria as a function of the cost to risk ratio γ , defined as $\gamma = \frac{c}{r}$, and coordination threshold M. Depending on whether γ is smaller than, larger than, or equal to a critical value $\bar{\gamma}$, there will be 0, 1 or 2 interior fixed points[8], respectively. This implies that higher risks and lower costs for contributions favor higher cooperation. We also note that, for fixed γ (drawing an horizontal line for a value of γ), increasing M will maximize cooperation (the largest value of equilibria(x^*) for which the function intercepts the previously mentioned horizontal line), at the expense of making it more difficult to emerge (getting past the first value of equilibria(x^*) for which there is an interception, shifting the curve to the right).

Finite Well-Mixed Populations

In real world scenarios, populations are finite. In this context, stochastic events such as genetic drift and social dynamics become much more prominent, [9] specially for smaller populations. In such populations, behavioral dynamics are best described by a finite population gradient of selection $G(\frac{k}{Z})$ and the stationary distribution of the population. [8]

In Fig. 1, we show the stationary distributions for different values of risk and for a population of size Z=50, where N=2M=6. Here, we note that the population spends most of the time in configurations where Cs prevail, for risk > 0.25, regardless of the initial condition.

Something else that ought to be taken into account is the size of the groups playing the game. Imagine we were to stop a pandemic by vaccinating the global population. One solution could be to come to a consensus on a single world-wide vaccination policy. This might prove to be a hard or prolonged process, as all parties would have to agree. Another way might be to define policies at smaller levels (for example, national policies), that might still be a good approximation to the optimal solution. In fact, this seems to hold true. We confirm in Fig. 4, as has been previously shown, [8] that cooperation seems to improve with smaller group sizes. At the same time, the positive effect of a higher perception of risk and increased minimum cooperation is counterbalanced by the increase in the coordination barrier. This is clear for cases in which N/M=2 (Fig. 5).



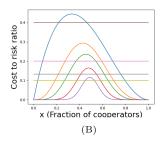


Figure 2: Stag Hunt (SH) game: Replicator equation internal roots for different values of N, M and risk. Z=50. (A) M=2 (B) N/M=2

II Snowdrift

As shown in Fig. 3 and 6, a SG game, even with risk = 0.4, generally does not favor overall cooperation. For M = 3, independently of the initial fraction of cooperators, there's $f_C < f_D$, a player aware of their conditions will choose to defect and the tendency is for the fraction of defectors to increase. However, when the risk or the threshold M increases (N/M = 2), there's chance for a stable fraction of cooperators x at equilibrium. This means a dynamic where $f_C = f_D$ is achieved and there's negative feedback involved.

Even when \dot{x} is positive, it isn't very high, meaning the rate at which new cooperators emerge is rather small, which relates to the spreadness of the stationary distribution in Fig. 7. Generally, achieving stable cooperation is not possible, except under highly specific combinations of N, M and risk. This specificity serves to express the profound nature

of the SG and the challenging potential for coordination failure.

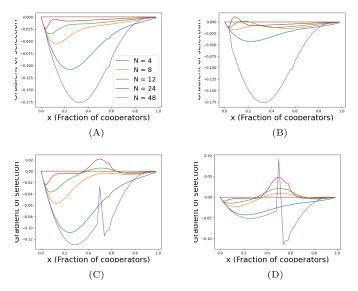


Figure 3: Snowdrift (SG) game: Gradient of selection for different values of N, M and risk. Z = 50. (A) M = 2, risk = 0.4 (B) M = 2, risk = 0.8 (C) N/M = 2, risk = 0.4 (D) N/M = 2, risk = 0.8

IV. Conclusion

For populations playing a N-person Stag Hunt game, the perception of risk really seems to help us avoid the tragedy of the commons, turning cooperation, an otherwise poorly fit strategy, into a more appealing one. That notion of perception of risk is key here. We assumed that all players shared the same risk and were equally aware of it. If some agents had a much lower (possibly null) perception of the collective risk, they would likely fall back to their tendency to become free-riders. Besides this, small group sizes also seem to contribute significantly to reaching higher levels of cooperation. A cooperation threshold increasing with the group size raises the coordination barrier, and once achieved, it promotes higher levels of cooperation.

In case players are dealing with a N-person Snowdrift game, only a very high perception of risk will lead to more cooperation, specially if it includes a demanding threshold. In such cases, the correlation between group size and the prevalence of cooperators is not clear.

${ m V.}~~{ m Future~work}$

We examined the risk of collective failure, in *public goods* games, for several types of N-person games and how it effected the evolutionary dynamics of populations, however, either the risk r or the group size N were always fixed. In real life, these parameters aren't always necessarily fixed.

If we go back to the climate change example, one might note that agents (countries, organizations, individuals, etc) might have different endowments or simply be willing to contribute more or less than some other agent, even if all of them choose to cooperate. How might the results presented in this paper change if c becomes a free variable? Likewise, even though failure to meet the cooperation threshold will prove

detrimental for all players, some might suffer more than others. If we consider two countries and how they are effected by climate change, we'd have to take into account factors like their economic and technological power, their proximity to the ocean, likelihood of natural disasters, among other things. Will these results hold if we allow each player to have a different perception of the the risk? Countries might also see their ability to contribute or risk perception change, following major changes in their economy or political events. We suspect that if both c and r are allowed to vary among different agents, the presented framework of evolutionary dynamics could change to one where the Neutral Theory[15] might be witnessed, to some small degree.

We could also have considered a parameter of punishment, [10, 16, 17] for example, a nation that refuses to enforce mandatory vaccination and in return is threatened with economical sanctions by their trade partners, though this could be achieved by extending the presented framework to allow the risk to increase for defectors, while staying the same for cooperators.

Another model could include collaboratively enhanced benefits, meaning each additional cooperator would provide an incremental benefit to all. In order to do that, our payoff would have to include a factor potentiated by the number or fraction of cooperators. This could be applied, for example, to community gardens. The more neighbors who contribute, the larger the harvest becomes, benefiting everyone involved in the game. [20]

VI. Acknowledgements

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A Appendix

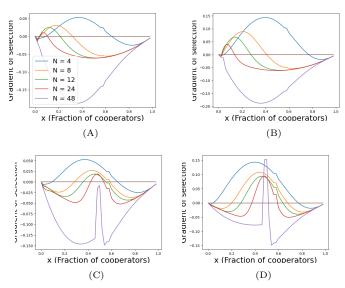


Figure 4: Stag Hunt (SH) game: Gradient of selection for different values of N, M and risk. Z=50. (A) M=2, risk = 0.4 (B) M=2, risk = 0.8 (C) N/M=2, risk = 0.4 (D) N/M=2, risk = 0.8

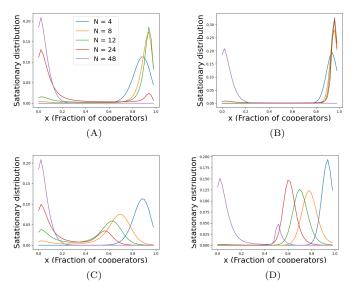


Figure 5: Stag Hunt (SH) game: Stationary distributions describing the prevalence of each fraction of cooperators x for different values of N, M and risk. Z = 50. (A) M = 2, risk = 0.4 (B) M = 2, risk = 0.8 (C) N/M = 2, risk = 0.4 (D) N/M = 2, risk = 0.8

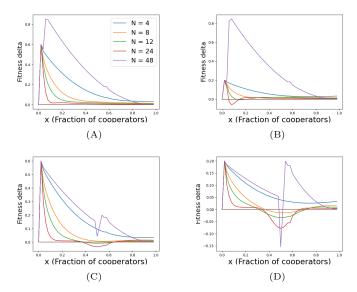


Figure 6: Snowdrift (SG) game: Internal roots of the replicator equation for different values of N, M and risk (r). Z = 50. (A) M = 2, r = 0.4 (B) M = 2, r = 0.8 (C) N/M = 2, r = 0.4 (D) N/M = 2, r = 0.8

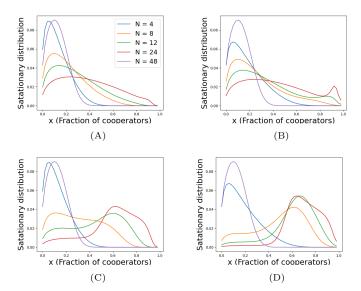


Figure 7: Snowdrift (SG) game: Stationary distributions describing the prevalence of each fraction of cooperators x for various values of N, M and risk (r). Z = 50. (A) M = 2, r = 0.4 (B) M = 2, r = 0.8 (C) N/M = 2, r = 0.4 (D) N/M = 2, r = 0.8