

# Exploring uncertainty in moment estimation for small earthquakes using the coda envelope method

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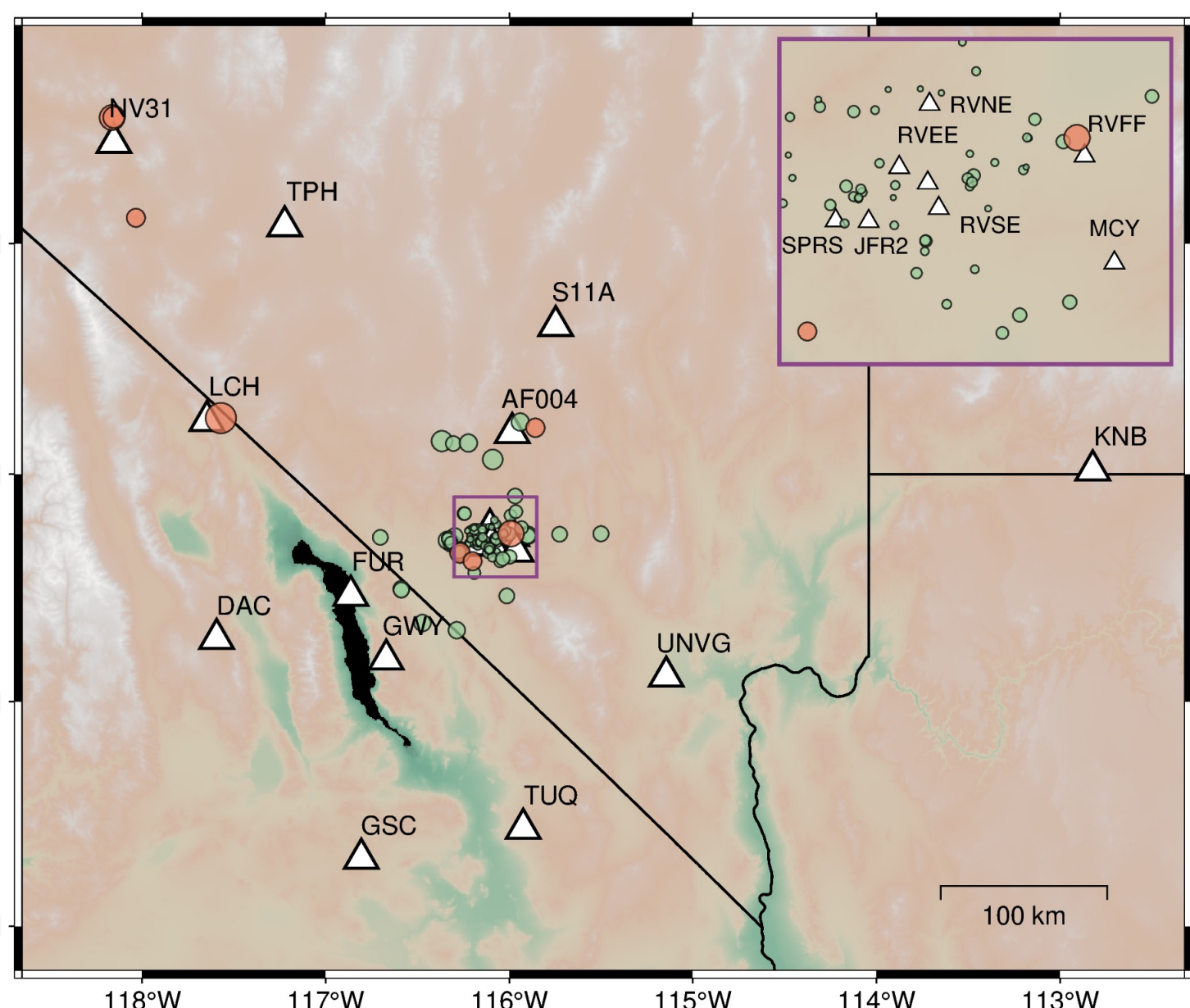
## Introduction

Accurate moment magnitude estimation using waveform modeling begins to fail below  $M3.5$  due to coarse velocity models and extreme computational requirements. But the abundance of data from small events presents countless opportunities to thoroughly characterize regions with high seismicity rates. So, how can we study the source of these small events? Here we use the seismic coda to estimate  $M_W$  using the Coda Calibration Tool (CCT; Barno, 2017), which streamlines the method of Mayeda and Walter (1996) and Mayeda et al. (2003). The seismic coda averages the heterogeneities in the crust, thereby eliminating effects from the radiation pattern and local geology (Figure 1).

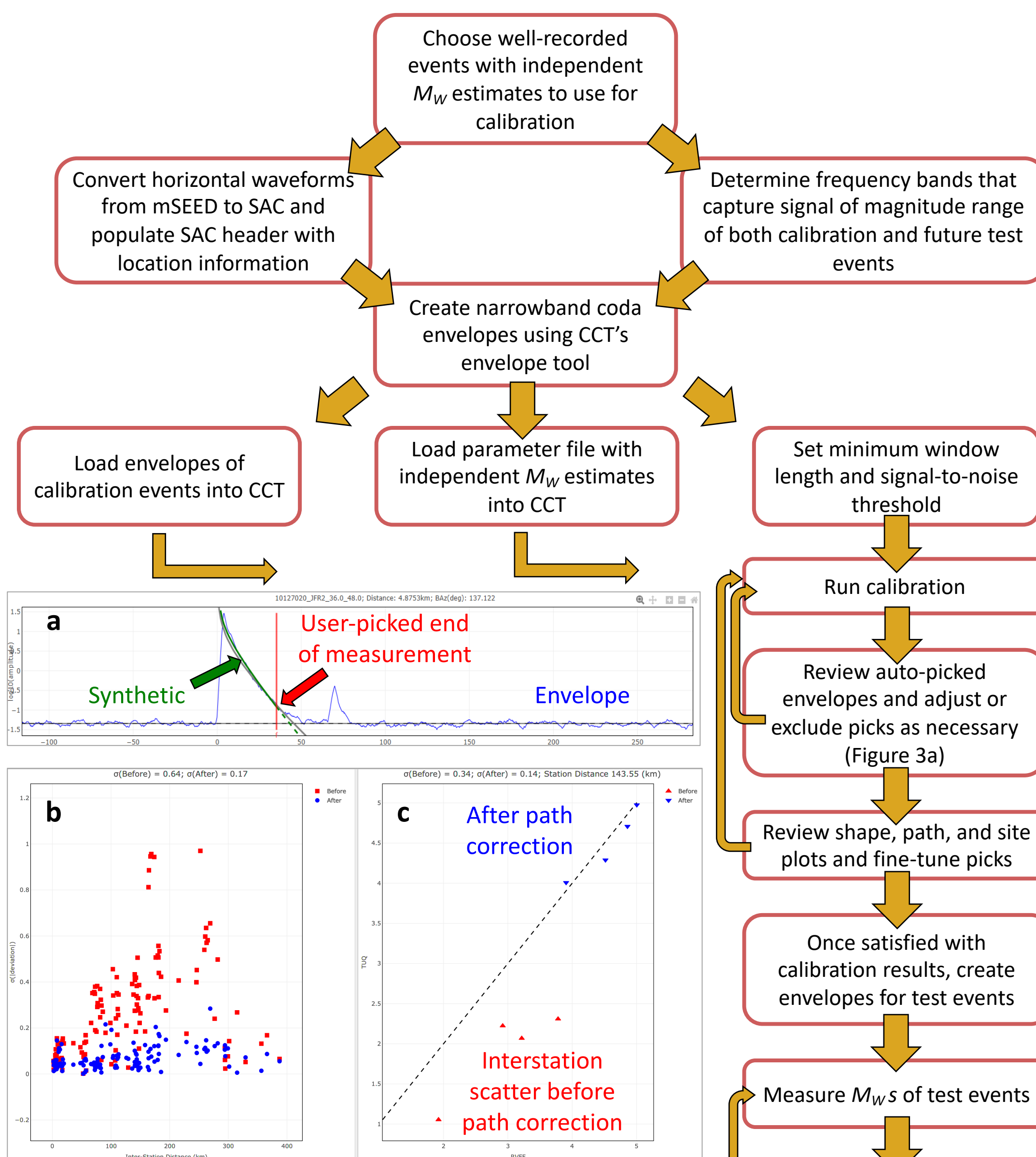
**Figure 1 (left).** This schematic shows a direct wave (blue) arriving at a station after passing through a heterogeneous area (red) that causes an inconsistency in amplitude. The scattered coda wave (green) passes through all anomalies in the crust, canceling out effects from local geology.

## Study Area: Rock Valley, Nevada

**Figure 2 (right).** Map of the study area primarily in southern Nevada and Rock Valley (inset), featuring the eight calibration events (orange), ~85 test events (green), and the stations used in the calibration (white triangles).



## CCT Workflow

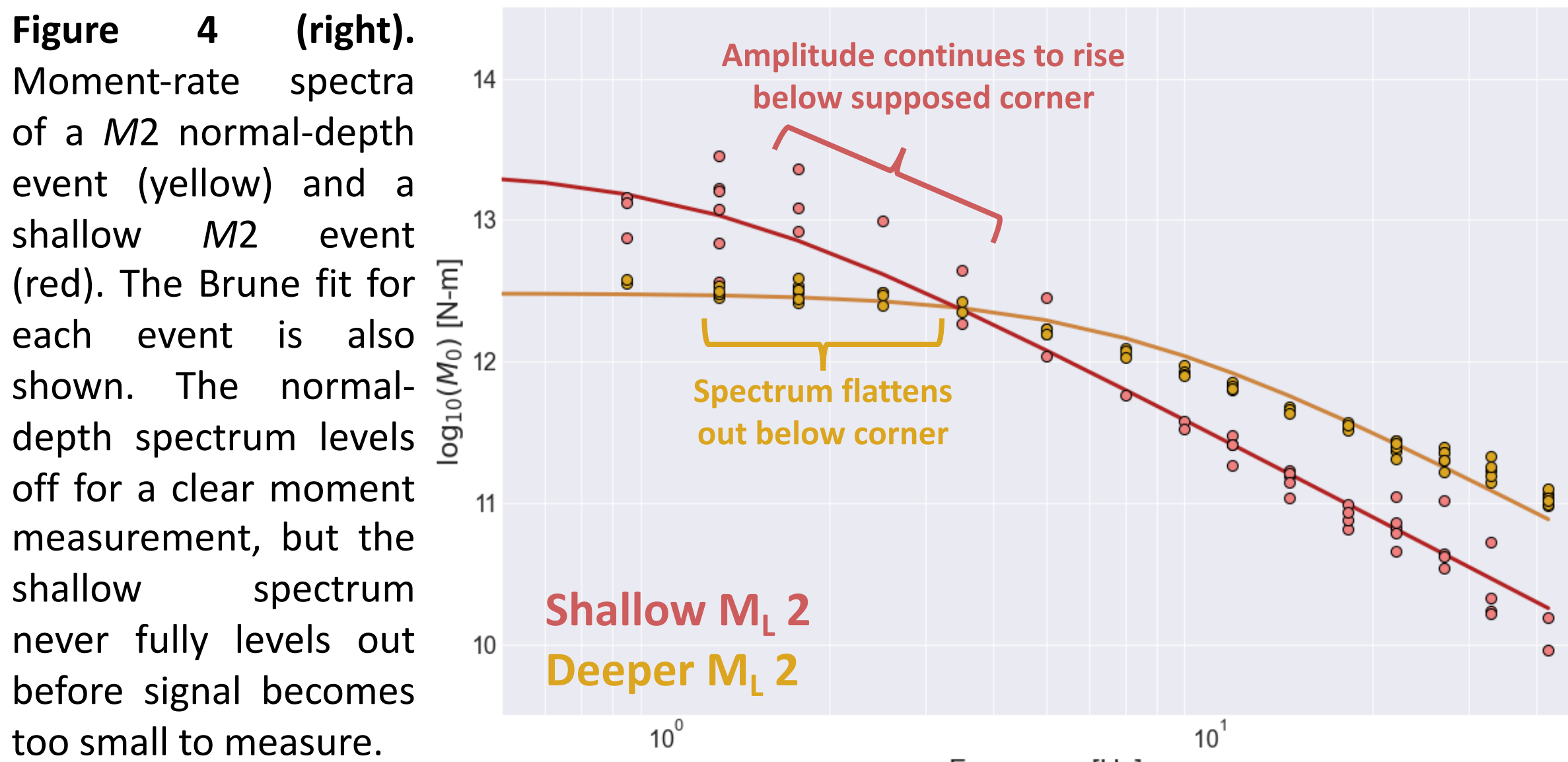


**Figure 3 (above).** (a) Coda envelope (blue) and synthetic (green) for a top frequency band of 36-48 Hz. (b) Relative path amplitudes for each pair of stations before (red) and after (blue) path correction for the 6-8 Hz band. (c) Amplitudes at stations TUQ and RVFF before (red) and after (blue) path corrections.

## Results

After reviewing the test event spectra, we fit a line to convert  $M_L$  to  $M_W$ . Originally, the conversion had a slope of  $\sim 0.9$ . However, after plotting the depth of these events, we noticed that the scatter in Figure 5 was largely due to shallow earthquakes. Upon comparing shallow- and normal-depth spectra (Figure 4), it became clear that many of the shallow events were exhibiting peaking in their spectra near the corner frequency, causing the Brune spectral model to overestimate the moment. Upon removing the shallow events (chosen to be less than 3 km depth), the scatter in the conversion was greatly reduced. Additionally, the slope of the line using only normal-depth events was reduced to  $\sim 0.8$  between  $M_W$  0 and 3, which agrees better with previous studies seeking to convert  $M_L$  to  $M_W$ .

**Figure 4 (right).**

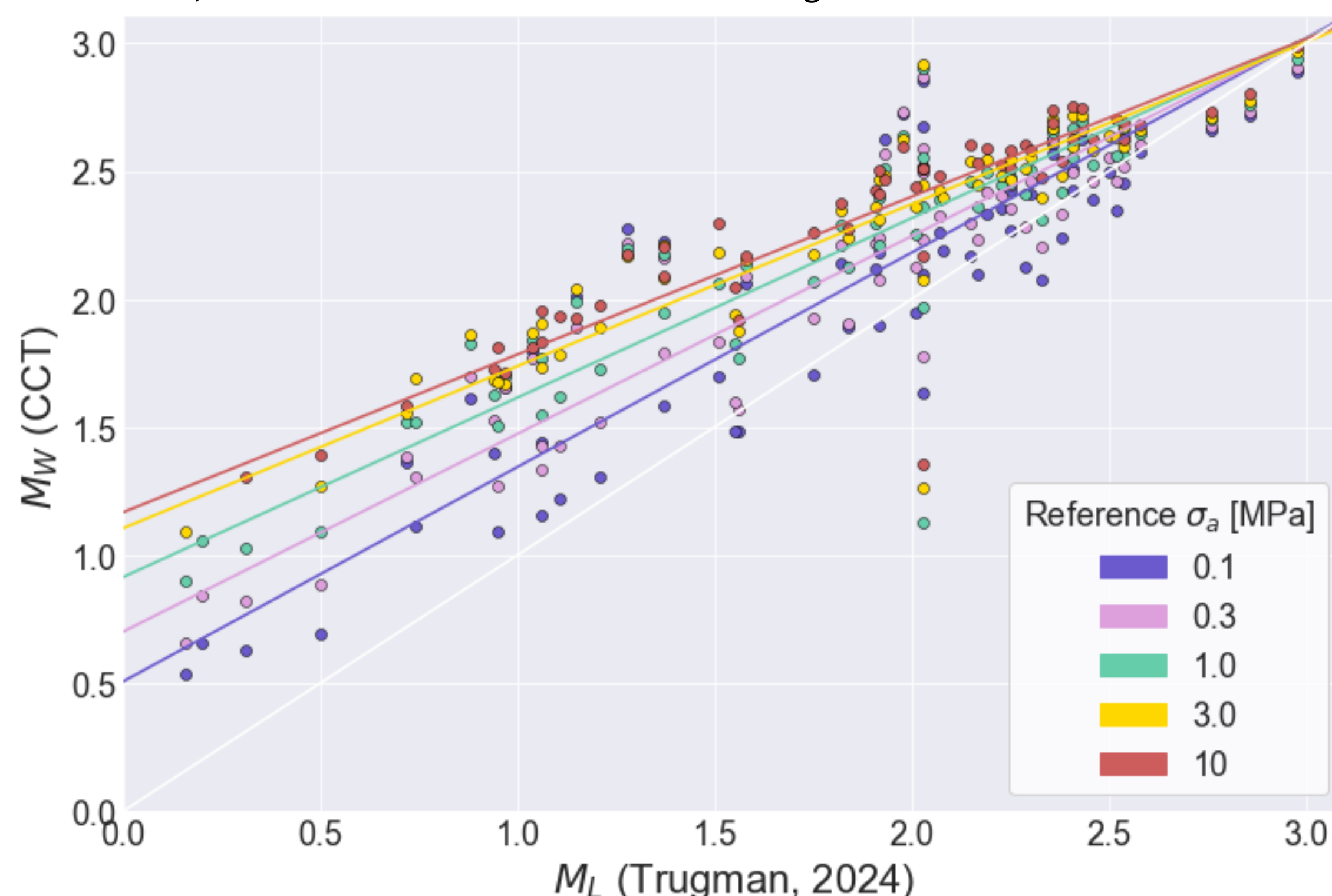


Moment-rate spectra of a M2 normal-depth event (yellow) and a shallow M2 event (red). The Brune fit for each event is also shown. The normal-depth spectrum levels off for a clear moment measurement, but the shallow spectrum never fully levels out before signal becomes too small to measure.

## How does the assumed apparent stress affect $M_W$ ?

In CCT, the user has the option to include an independent estimate of apparent stress ( $\sigma_a$ ) to better constrain the corner frequency and apparent stress estimates of the dataset. But because these calibration events were quite small, they do not have independent  $\sigma_a$  estimates. To explore the effect of  $\sigma_a$  assumptions on  $M_W$ , we re-calculated  $M_W$ s for the test dataset after assigning a series of values of apparent stress to the calibration events. These trials resulted in appreciable changes in  $M_W$  (Figures 6 & 8), with the effects increasing with decreasing magnitude.

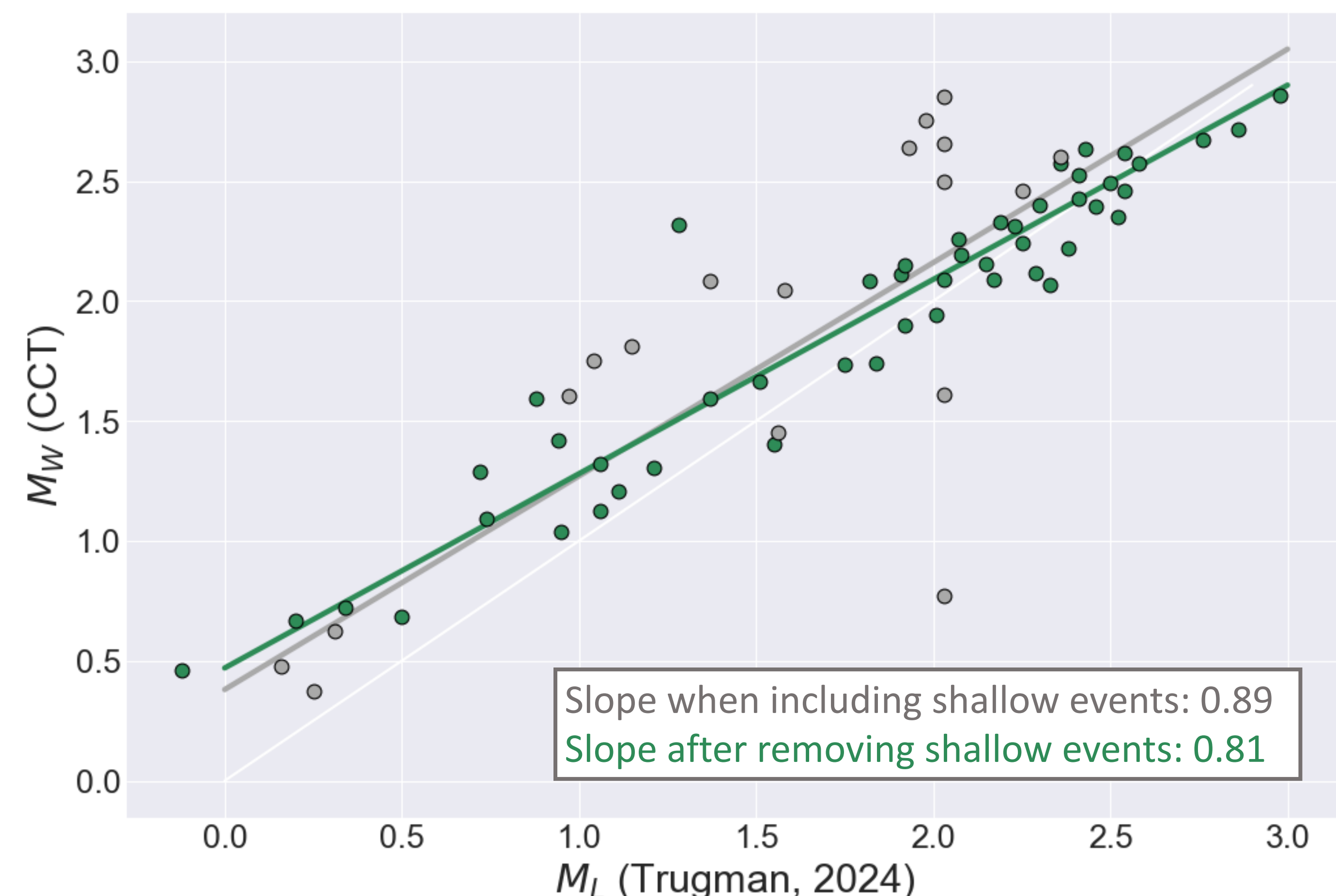
**Figure 7 (below).** New conversion relationships for each trial of reference apparent stress. Below M2,  $M_W$ s calculated by CCT consistently increased as the reference apparent stress was increased, with more variation at the smallest magnitudes.



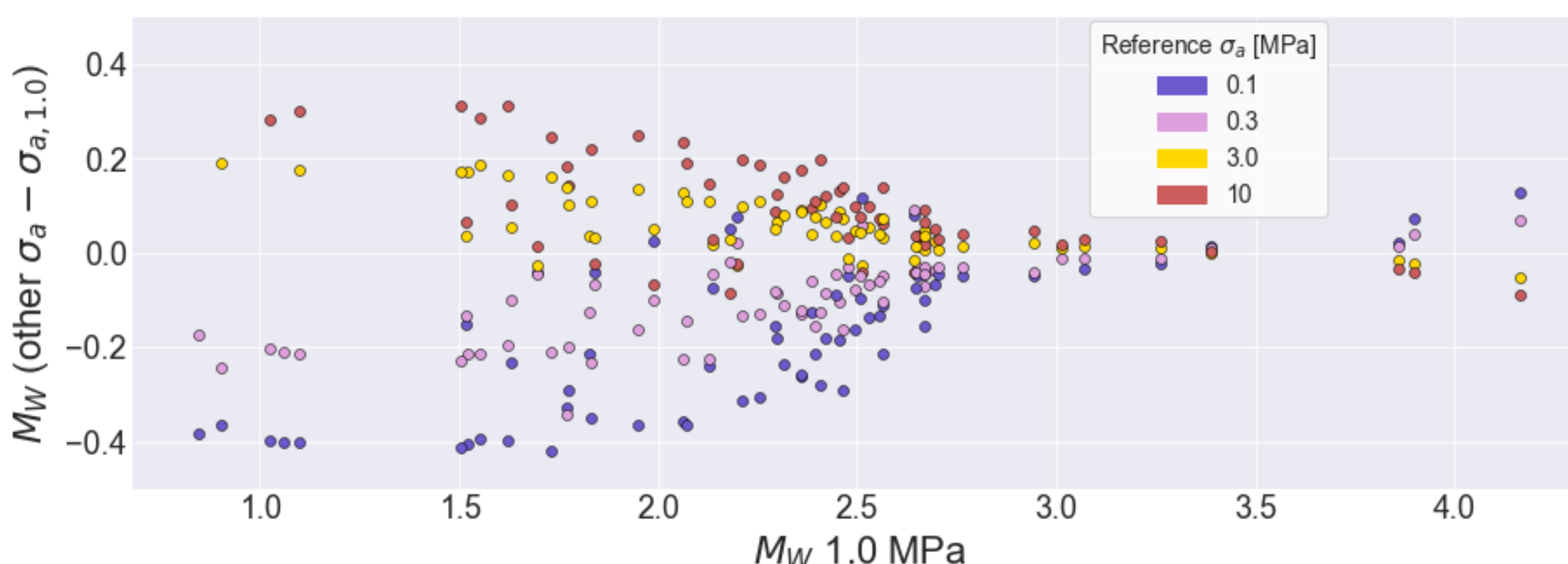
## Conclusions & Future Directions

We conclude that (1) CCT produces reasonable  $M_W$  estimates for very small earthquakes. (2) A linear relationship exists between CCT's  $M_W$  and the Nevada Seismological Laboratory's  $M_L$  for  $M < 3$  near Rock Valley. (3) Very shallow events may require a new spectral model if we want to measure their source parameters accurately. (4) Changing reference apparent stress values impacts  $M_W$  estimates and the  $M_W$ - $M_L$  conversion. Future study will:

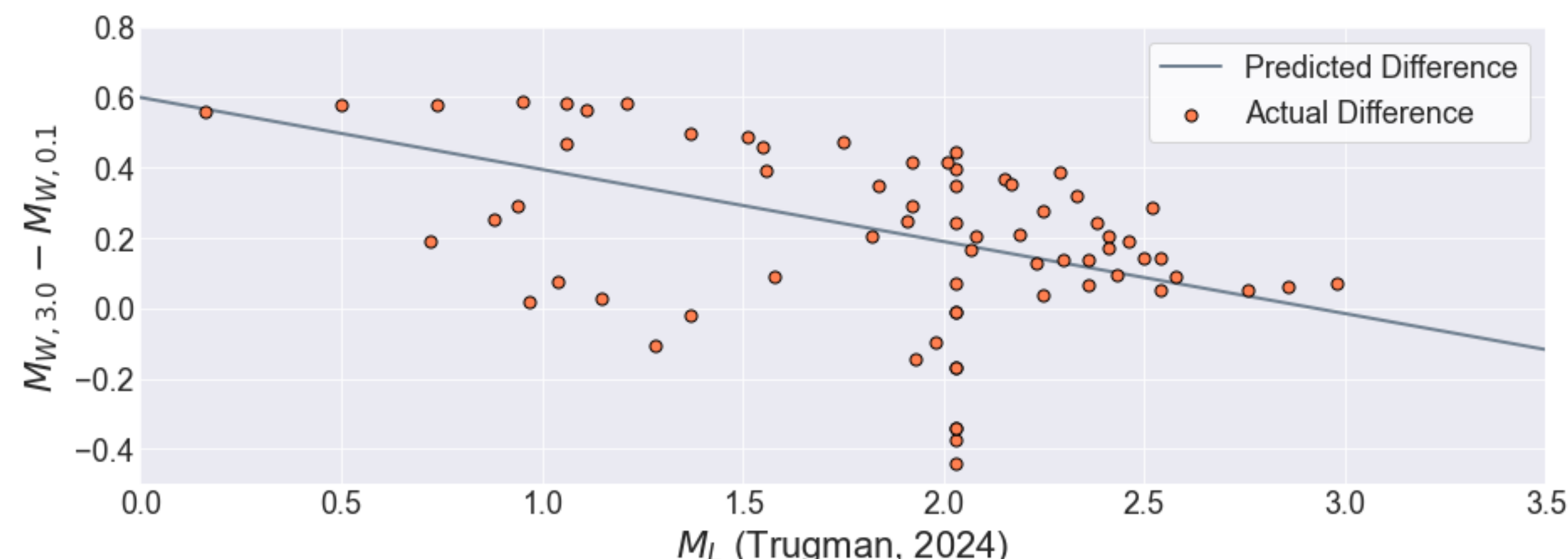
- Quantify tradeoffs between earthquake size, coda length, and distance to nearest station
- Expand test dataset to include more  $M$  0-2 events



**Figure 5 (above).**  $M_W$  versus  $M_L$  for the test dataset using  $M_L$  0 - 3 from Trugman (2024). The shallow events ( $< 3$  km) are in gray, and the normal-depth events ( $> 3$  km) are in green. Before removing the shallow events, the conversion was  $M_W = 0.89 M_L + 0.38$  (gray line). After removing the shallow events, the scatter was reduced, and the conversion equation became  $M_W = 0.81 M_L + 0.47$  (green line). The 1:1 line is shown in white.



**Figure 6 (above).**  $M_W$ s for different trials of reference apparent stress relative to  $M_W$  at 1 MPa (y-axis) compared with  $M_W$ s at 1 MPa (x-axis). The deviation from 1 MPa increases as the events get smaller.



**Figure 8 (above).** The theoretical variation in  $M_W$  (line, gray) and the actual difference in  $M_W$  (dots, red) between the 3 MPa trial and the 0.1 MPa trial as a function of  $M_L$  from Trugman (2024). The 3 MPa and 0.1 MPa values were used to represent the high and low ends of reasonable values for apparent stress in this area. The variation in  $M_W$  is inversely correlated with event size.

## References

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