СП на (Ω, F, P)

$$X(t) = X(\omega, t) \qquad \omega \in \Omega,\ t \in T$$

при $\omega = \omega_0$ $\quad X(\omega_0, t)$ — траектория СП

при $t = t_0$ $\quad X(\omega, t_0)$ — СВ

ФР СП $\quad F_n(x_1, \dots, x_n; t_1, \dots, t_n) = P(x(t_1) < x_1, \dots, x(t_n) < x_n)$

для одномерн.

МО СП $\quad m(t) = E\,x(t) = \int_{\mathbb{R}} x\, dF(x; t)$

Дисперсия СП $\quad V(t) = V\,x(t) = E((x(t) - m(t))^2) =$

$$= \int_{\mathbb{R}} (x - m(t))^2\, dF(x; t)$$

Корр. ф СП $\quad r(t_1, t_2) = E(x(t_1) \cdot x(t_2)) = \iint_{\mathbb{R}^2} x_1 \cdot x_2\, dF_2(x_1, x_2; t_1, t_2)$

Ковар. ф СП $\quad R(t_1, t_2) = E((x(t_1) - m(t_1)) \cdot (x(t_2) - m(t_2))) =$

$$= \iint_{\mathbb{R}^2} (x_1 - m(t_1)) \cdot (x_2 - m(t_2))\, dF_2(x_1, x_2; t_1, t_2)$$

Стационарный СП — МО не зав. от времени t

$$m(t) = m \quad \forall t$$

Внутренне стационарный СП — если

1) $E(x(t_1) - x(t_2)) = 0$

2) $V(x(t_1) - x(t_2)) = 2\gamma(t_1 - t_2)$

для любой пары одинаково отстоящих моментов времени приращения значений проц. одинаковы

Вариограмма $\quad 2\gamma(h) = V(x(s+h) - x(s)) =$ $\quad h = t_1 - t_2$

(дисперсия приращения) $= E((x(s+h) - x(s) - E(x(s+h) - x(s)))^2)$

$\gamma(h)$ — семивариограмма

Все внутренние стационарные СП всегда стационарные

$$2\gamma(h) = D\{X(s+h) - X(s)\}, \quad s, h \in \mathbb{Z}$$

Рассмотрим статистику вида:

$$2\tilde{\gamma}(h) = \frac{1}{n-h}\sum_{s=1}^{n-h}(X(s+h) - X(s))^2$$

$$h = \overline{0, n-1}; \quad \tilde{\gamma}(-h) = \tilde{\gamma}(h), \ h = \overline{0, n-1}$$

$$\tilde{\gamma}(h) = 0, \quad |h| \geq n$$

$$M(2\tilde{\gamma}(h)) = \frac{1}{n-h}\sum_{s=1}^{n-h}\underbrace{M(X(s+h) - X(s))^2}_{2\gamma(h)\ \text{вн. стац.}} =$$

$$\cancel{= \left| \begin{array}{l} \{X(s+h) - X(s)\}^2 = 2\gamma(h)\cdot\chi_1^2 \\ M(\ \ ) = 2\gamma(h) \\ D(\ \ ) = 2\cdot(2\gamma(h))^2 \end{array} \right| =}$$

$$= \frac{1}{n-h}\sum_{s=1}^{n-h}(2\gamma(h)) = 2\gamma(h)$$

$$\cancel{\operatorname{cov}(2\tilde{\gamma}(h_1), 2\tilde{\gamma}(h_2)) = M\{(2\tilde{\gamma}(h_1) - M(2\tilde{\gamma}(h_1)))(2\tilde{\gamma}(h_2) - M\{2\tilde{\gamma}(h_2)\})\} = M\{(2\tilde{\gamma}(h_1) - 2\gamma(h_1))(2\tilde{\gamma}(h_2) - 2\gamma(h_2))\}}$$

$$\operatorname{cov}(2\tilde{\gamma}(h_1), 2\tilde{\gamma}(h_2)) = E((2\tilde{\gamma}(h_1) - E(2\tilde{\gamma}(h_1)))\cdot(2\tilde{\gamma}(h_2) - E(2\tilde{\gamma}(h_2)))) = E((2\tilde{\gamma}(h_1) - 2\gamma(h_1))\cdot(2\tilde{\gamma}(h_2) - 2\gamma(h_2))) =$$

$$= E\left(\left(\frac{1}{n-h_1}\sum_{s=1}^{n-h_1}(x(s+h_1) - x(s))^2 - 2\gamma(h_1)\right)\cdot\left(\frac{1}{n-h_2}\sum_{s=1}^{n-h_2}(x(s+h_2) - x(s))^2 - 2\gamma(h_2)\right)\right) =$$

= [раскрываем скобки под МО, затем подносим МО] =

$$= \frac{1}{(n-h_1)(n-h_2)}\cdot\sum_{t=1}^{n-h_1}\sum_{s=1}^{n-h_2}\underbrace{E((x(s+h_1) - x(s))^2\cdot(x(s+h_2) - x(s))^2)} - \frac{2\gamma(h_2)}{n-h_1}\sum_{s=1}^{n-h_1} 2\gamma(h$$

$$- \underbrace{\frac{2\gamma(h_1)}{n-h_2} \cdot \sum_{s=1}^{n-h_2} 2\gamma(h_2)}_{2\gamma(h_1)\cdot 2\gamma(h_2)} + 2\gamma(h_1) \cdot 2\gamma(h_2)$$

$$\operatorname{cov}(2\tilde{\gamma}(h_1), 2\tilde{\gamma}(h_2)) = E\big((2\tilde{\gamma}(h_1) - E(2\tilde{\gamma}(h_1)))\cdot$$

$$\cdot (2\tilde{\gamma}(h_2) - E(2\tilde{\gamma}(h_2)))\big) =$$

$$= E\Big(\frac{1}{n-h_1}\sum_{s=1}^{n-h_1}\big((x(s+h_1) - x(s))^2 - E((x(s+h_1) - x(s))^2)\big)\cdot$$

$$\cdot \frac{1}{n-h_2}\sum_{t=1}^{n-h_2}\big((x(t+h_2) - x(t))^2 - E((x(t+h_1) - x(t))^2)\big)\Big) =$$

$$= \frac{1}{(n-h_1)(n-h_2)}\cdot\sum_{s=1}^{n-h_1}\sum_{t=1}^{n-h_2}\operatorname{cov}\big((x(s+h_1) - x(s))^2, (x(t+h_2) - x(t))^2\big) =$$

= [по опр. $\operatorname{cov}(a,b) = \operatorname{corr}(a,b)\cdot\sqrt{Da\cdot Db}$] =

$$= \frac{1}{(n-h_1)(n-h_2)}\cdot\sum_{s=1}^{n-h_1}\cdot\sum_{t=1}^{n-h_2}\operatorname{corr}\big((x(s+h_1) - x(s))^2, (x(t+h_2) - x(t))^2\big)\cdot$$

$$\cdot\sqrt{D(x(s+h_1) - x(s))^2 D(x(t+h_2) - x(t))^2} =$$

= [исп. равенство (a), упр. 2] =

$$= \frac{1}{(n-h_1)\cdot(n-h_2)}\sum_{s=1}^{n-h_1}\cdot\sum_{t=1}^{n-h_2}\operatorname{corr}\big((x(s+h_1) - x(s))^2, (x(t+h_2) - x(t))^2\big)\cdot 2\cdot 2\gamma(h_1)\cdot 2\gamma(h_2)$$

$$\left(\frac{2\gamma(\cancel{s}+h_1-\cancel{t}) + 2\gamma(\cancel{s}-\cancel{t}-h_2) - 2\gamma(\cancel{s}+h_1-t-h_2) - 2\gamma(s-t)}{2\sqrt{2\gamma(h_1)}\sqrt{2\gamma(h_2)}}\right)^2$$

$$= \left(\frac{\mathcal{D}(x(s+h_1)-x(t)) + \mathcal{D}(x(s)-x(t+h_2)) - \mathcal{D}(x(s+h_1)-x(t+h_2)) - \mathcal{D}(x(s)-x(t))}{2\sqrt{2\gamma(h_1)}\cdot\sqrt{2\gamma(h_2)}}\right)^2 =$$

$$= \left(E((x(s+h_1)-x(t))^2) + E((x(s)-x(t+h_2))^2) - E((x(s+h_1)-x(t+h_2))^2) - E((x(s)-x(t))^2)\right)^2 =$$

$$[\cancel{x^2(s+h_1)} - 2x(s+h_1)x(t) + \cancel{x^2(t)} + \cancel{x^2(s)} -$$
$$- 2x(s)x(t+h_2) + \cancel{x^2(t+h_2)} - \cancel{x^2(s+h_1)} + 2x(s+h_1)\times x(t+h_2) -$$
$$- \cancel{x^2(t+h_2)} - \cancel{x^2(s)} + 2x(s)x(t) - \cancel{x^2(t)}] =$$

$$= \left(E\left(\frac{x(s)x(t) - x(s+h_1)x(t) - x(s)x(t+h_2) + x(s+h_1)x(t+h_2)}{\sqrt{2\gamma(h_1)}\sqrt{2\gamma(h_2)}}\right)\right)^2$$

$$\left(\frac{E(x(s)x(t) - x(s+h_1)x(t) - x(s)x(t+h_2) + x(s+h_1)x(t+h_2))}{\sqrt{2\gamma(h_1)}\sqrt{2\gamma(h_2)}}\right)^2$$

$$\left(\frac{E(\cancel{x(s)}\ \mathrm{corr}(x(s+h_1)-x(s))\cdot(x(t+h_2)-x(t))}{}\right)^2$$

$$\mathrm{corr}\left((x(s+h_1)-\cancel{x}(s))^2, (x(t+h_2)-x(t))^2\right) =$$

$$= E\left((x(s+h_1)-x(s))^2 \cdot (x(t+h_2)-x(t))^2\right) =$$

$$= E\left((x(s+h_1)-x(s))(x(t+h_2)-x(t)) \right)^2 =$$

$$= E\left(x(s+h_1)\,\xi\, x(t+h_2) - x(s+h_1)\,\xi\, X(t) - \right.$$

$$\left. - \xi\, x(t+h)\,x(s) + x(s)\,\xi\, x(t) \right)^2$$