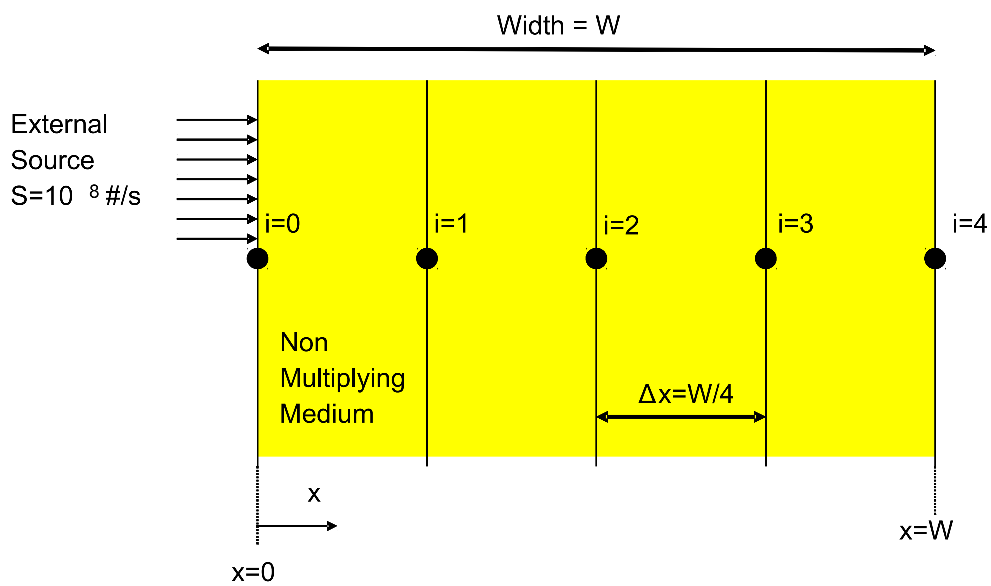
NE 470 **Nuclear Reactor Theory I** **Fall 2016**

Due on September 27th, 2016

PROJECT #1

Consider a one dimensional slab of width W of a non-multiplying medium with an external source located at x=0, as shown below.



Assume the steady-state one-speed neutron diffusion equation applies to this arbitrary region or composition “m”:

$$-D_m \frac{d^2\varphi}{dx^2} + \Sigma_a^m \varphi = \begin{cases} S(x=0) \\ 0(x>0) \end{cases}$$

1. Solve this problem analytically. Provide an expression for the flux as a function of x. Plot the flux between x=0 and x=W. Assume a source strength of $S=1\text{x}10^8$ #/s.
2. Assuming a 5-node numerical discretization as shown above, derive the finite-difference equation for an arbitrary *internal node i*.
3. Derive the finite-difference equation for the *left-most node* (x=0) assuming the following boundary condition applies:

$$-D_m \frac{d\varphi}{dx}\Big|_{x=0} = J = \frac{S}{2}$$

4. Assume that the following boundary condition applies to the *right-most node* (x=W):

$$\varphi(W) = 0$$

5. Assuming the 5-node system shown above, define the matrix system of equations that would have to be solved. Carefully define all matrix entries and vectors involved. Describe the primary *unknowns*? (i.e., what are you trying to solve for?)
6. Modify the program to solve a cylinder of radius R=W, instead of the slab, and a sphere of radius R=W, instead of the slab.

TABLE 5-3 Macroscopic Cross Sections

Material	$\Sigma_{tr}(cm^{-1})$	$\Sigma_a(cm^{-1})$	$\nu\Sigma_f(cm^{-1})$	*Relative Absorption*
H	1.79×10^{-2}	8.08×10^{-3}	0	0.053
O	7.16×10^{-3}	4.90×10^{-6}	0	0
Zr	2.91×10^{-3}	7.01×10^{-4}	0	0.005
Fe	9.46×10^{-4}	3.99×10^{-3}	0	0.026
^{235}U	3.08×10^{-4}	9.24×10^{-2}	0.145	0.602
^{238}U	6.95×10^{-3}	1.39×10^{-2}	1.20×10^{-2}	0.091
^{10}B	8.77×10^{-6}	3.41×10^{-2}	0	0.223
	3.62×10^{-2}	0.1532	0.1570	1.000

Deliverables:

1. Using the above one-group constants representative of a light water reactor, write a short program to solve the analytical and numerical problem. Compare to the analytical solution that is available from your textbook (plot numerical versus analytical solution side by side).

2. Initially, use the 5-node representation provided, however, subsequently adjust your computer program to handle an arbitrary number of nodes (N) and compare your numerical results versus your analytical results as you increase the number of nodes from N=5, 10, 20, etc. Provide comparisons of the numerical solution versus the analytical solution as you increase the number of nodes.

3. Substitute the boundary condition on the right hand side at x=W with another external source of strength f*S, where f is a factor that can be varied (0.5, 1.0, 1.5, etc.)

4. The program should ask the user which geometry (slab, cylinder, or sphere) to consider, and setup the matrix accordingly.
   See the discussion on pages 176-182 of D&H, in particular equations 5-109, 5-124, 5-125, 5-126, 5-127, for how to setup the matrix relevant to these three cases.
   Compare the fluxes for slab, cylinder, and spherical geometry for N=10 and f=1.0.

Your project should be presented in a <u>short report</u> format. This usually implies a write-up that includes the following sections: Introduction & Background (project description), Methodology (derivations and approach to solution), Results, and Conclusions. Note, this need not be long, just formatted properly, using a word processing program instead of hand-written. Submit as PDF.