Econ. Theory 2, 389–426 (1992)

Economic Theory

Competitive equilibria in overlapping generations experiments*

Charalambos D. Aliprantis[1] and Charles R. Plott[2]

[1] Department of Mathematics, IUPUI, Indianapolis, IN 46205–2820, USA
[2] Division of Humanities and Social Sciences, Caltech, Pasadena, CA 91125, USA

Received: April 5, 1991; revised version August 21, 1991

Summary. The paper presents the results of four overlapping generations experiments performed at the California Institute of Technology. Overlapping generations markets were created in which each agent had a two period life span. With the exception of the first period, there were eight agents trading in each period; four buyers (two young and two old) and four sellers (two young and two old). Parameters were selected so that a "small" set of equilibria existed. The markets were open for twenty-nine periods with a demand shift occurring at the fifteenth and sixteenth periods.

This work provides a method of computing all competitive equilibria for a class of environments—called the opposing shift environments. The main conclusion of the experiments is that the experimental price data converge to near the stationary portions of the competitive equilibria. "Demographic" dynamics are also explored as part of the price adjustment process. Dynamic features found in non-overlapping environments are also observed in the overlapping generations case.

1. Introduction

This paper initiates an investigation into the behavior of competitive markets with overlapping generations without any special role played by fiat money[1]. The focus is on finite lived experimental economies that can be represented in a natural way

* We thank E. J. Nanale, a Caltech undergraduate student, who helped design and conduct the experiments. Special thanks are due to Hsing Yang Lee for his help in adapting the Multiple Unit Double Auction Program to accomodate the overlapping generations environment and for his help with the data processing. We also thank David Grether who found a serious problem with the statistical analysis in an early draft. The research assistance of Kay-Yut Chen was also very valuable to the completion of this work. The comments and suggestions of P. Rangazas were very helpful. The research of the second author was supported in part by a National Science Foundation Grant and the California Institute of Technology Laboratory for Experimental Economics and Political Science.

[1] Equilibria in economies with fiat money have been studied experimentally by Lim et al. [7] and Marimon and Sunder [8, 9]. The features of these economies necessitated by the existence of a fiat money make direct comparisons with our study very difficult to implement.

by models in which nondegenerate competitive equilibria exist. A rich body of theoretical literature explores the properties of the competitive equilibria in overlapping generations (OLG) models, identifies welfare economics paradoxes and also suggests the possible existence of interesting economic phenomena, e.g., sunspot equilibria. While this study does not pursue the questions typically posed by the models in the OLG literature, it does pose questions about a fundamental assumption of those models. The models in the literature typically assume, directly or indirectly, that the competitive equilibria of the mathematical model will be closely related to the actual prices observed in markets. This paper provides the first evidence on the empirical foundation for that assumption in OLG environments.

The narrow focus on the empirical relationships between the overlapping generations structure of a real economy and a competitive equilibrium model of that economy necessitated some trade offs. The economy we created is one in which the natural competitive model has a unique equilibrium price path. The existence of multiple equilibria can substantially complicate any assessment of the errors of the model so uniqueness was considered to be very important. As a consequence of this choice many features of economies that figure prominently in the OLG literature are absent. For example, money plays no particular role. However, assets exist and Walras' law need not be satisfied. In any case the choice of environment is not what one might expect from a reading of the literature.

The research strategy reflects four important considerations. First, an attempt was made to use experimental conditions in which the competitive equilibrium model is known to work. For example, an incomplete information condition (about the preferences of others and about the termination date of the economy), which is standard in the experimental literature, was imposed. Complete information and perfect foresight are by no means necessary for competitive equilibrium behavior. A natural expectation would be that the mysterious phenomena of equilibration observed in the Arrow–Debreu world would carry over to the OLG world.

Second, the dynamics of non OLG markets are not understood. Furthermore, markets with asset structure, aspects of which are necessarily present in the overlapping generations environment, can yield very unexpected behavior such as bubbles; see King et al. [6], Smith et al. [15]. The simplest overlapping generations structure is very similar to environments in which bubbles are known to occur. Thus, existing experiments hold contradictory implications for actual behavior in the OLG case.

Third, the overlapping generations models themselves tell us that surprising things can happen. The parametric environments studied here will not be any of those in which theory suggests that surprises might exist. The environments studied in the literature contain such rich sets of equilibria that disequilibrium behavior with equilibration cannot be distinguished easily from equilibrium behavior along some path. Nevertheless, the fact that overlapping generations environments are involved demands an extraordinary degree of conservatism.

Finally, part of this paper is devoted to computational techniques. Almost all of the theoretical literature on OLG models consists of existence results. Little is known about the uniqueness or about computational methods. As was mentioned above, an essential feature of an experiment is that the equilibrium paths do not fill too much of the observation space. Otherwise, anything observed could be interpreted as part of some equilibrium path. The focus on computational methods for a class of environments reflects the requirement posed by the experimental approach that relevant models produce observable and falsefiable implications. These computational techniques are important for additional experimental studies within the OLG environment.

The research strategy was as follows. A finite time economy was designed and parameters were chosen such that a well defined, nondegenerate set of equilibria exist in the natural OLG model of the environment. The environment involves a one time change of parameters which causes demand and supply to shift in opposite dirrections (the "opposing shift" case). The parameters are in Sect. 3 of the paper. Section 4 contains a statement of the model and Sect. 5 contains the equilibrium of the model. All theorems and proofs to support this result are in the Appendix. Section 6 contains a special section on disequilibrium and convergence behavior as opposed to equilibrium time paths. The experimental results are in Sect. 7.

2. Theoretical literature

While the environment we study is not one of those studied in the literature, a brief outline of the literature might help to orientate the work. The reader should keep in mind that the experimental environment is connected to this literature in two important ways. First, the definition of the competitive equilibrium is the same in all cases. Secondly, the unique equilibrium price time path characterized in the sections below is the same whether the environment involves sufficiently large finite time or (countably) infinite time. Consequently, the general theories found in the literature provide an unambiguous prediction when applied to the environment of the experiments.

In his 1958 classic work on the consumption loan model [14] Samuelson astounded the economic world by exhibiting a Walrasian equilibrium that failed to be Pareto optimal. This example was in direct contradiction with the Arrow–Debreu model of general equilibrium theory according to which Walrasian equilibria are always Pareto optima (the first welfare theorem). Since then, Samuelson's example is known as *Samuelson's Paradox* and several attempts have been made to explain its failure. The research efforts associated with these attempts created the second class of economic models (the first class consists, of course, of the Arrow–Debreu models) of general equilibrium theory referred to collectively as **Overlapping Generations Models**, or briefly as OLG models.

An OLG model differs substantially from the Arrow–Debreu model. To start with, an OLG model has countably many overlapping generations and each generation consists of a finite number of agents. Each agent is endowed with an initial endowment, lives a finite number of periods, and his/her utility function depends only upon the consumption bundles available to him during his/her lifetime. In this setting, Aliprantis et al. [1] build up a model along the Arrow–Debreu analytic framework employing functional analytic techniques and especially the mathematical theory of inductive limits. In this OLG model, the commodity space is an inductive limit of AM-spaces and the price space is a projective limit of AL-spaces. It was shown that in this OLG model Walrasian equilibria always exist. This work also highlights the following main difference between the OLG model and the Arrow–Debreu model. *The commodity space for the OLG model does not contain the social endowment*. Consequently, Walras' law (which plays a fundamental role in the Arrow–Debreu model) is not applicable. This failure provides a natural explanation to Samuelson's Paradox!

It is important to mention that the Aliprantis–Brown–Burkinshaw model provides two possibilities for price normalization. One can normalize prices such that the total endowment has unit value, in which case the equilibrium prices may be nonzero singular linear functionals that assign zero value to each agent's initial endowment. Or, one can normalize prices according to Wilson [16] so that the initial endowment of a fixed agent has unit value, in which case the equilibrium price may assign infinite value to the social endowment. The reader can find a complete description of this OLG model in Chap. 5 of [2]. For an excellent survey of OLG models available in the literature see the article of Geanakoplos [4].

In this work, we shall distinguish between two types of OLG models—the *long-run* and the *short-run*. A *long-run OLG model* is the ideal mathematical OLG model with a countable number of periods; these OLG models have been studied in a rigorous mathematical manner in the works cited above. In contrast to this, a *short-run OLG model* is an OLG model with a finite number of periods and several experimental constraints. Complete descriptions of these models will be given in the sequel.

Most of the works cited above demonstrate the existence of equilibria in long-run OLG models but they do not provide an explicit algorithm of how to obtain (or even how to approximate) these equilibria. In other words, there are very few computational methods or techniques that produce equilibria in OLG models.

In this work, we shall compute OLG equilibria by (essentially) taking advantage of the following theoretical consideration. The existence proofs of equilibria in a long-run OLG model consider a sequence of economies $\{\mathcal{E}_n\}$, where $\mathcal{E}_n$ denotes the finite economy up to period n. For each such economy $\mathcal{E}_n$ a competitive equilibrium $(\mathbf{x}_n, \mathbf{p}_n)$ is obtained, and then an "appropriate limit" of the sequence $\{(\mathbf{x}_n, \mathbf{p}_n)\}$ is taken to yield a Walrasian equilibrium for the OLG model. The finite nature of these subeconomies allows us to get a "computational-handle" on the OLG model and, in fact, to compute the sequence of Walrasian equilibria $\{(\mathbf{x}_n, \mathbf{p}_n)\}$ in the experimental environment. This is a remarkable fact because although Walras' Law is not applicable in the long-run OLG model it is valid in any finite horizon economy and guarantees the existence of competitive equilibria. Our experimental setting takes advantage of this important behavior.

3. The experimental opposing shift environment

The experimental economy will consist of a single commodity. Unlike the literature, cash will play no role in savings but the commodity can be viewed as an asset with a life over time once supplied to the system. During the life of the economy a single (unanticipated) shift in demand and supply will occur. In the exercises that follow computations of the competitive equilibrium will proceed *as if* the time path of prices was publicly known. This structure will allow ordinary demand and supply functions to be used.

We shall say that a pair of demand and supply functions $\big(D(p), S(p)\big)$ and $\big(D^*(p), S^*(p)\big)$ satisfies the **opposing shift property** whenever

$$D^*(p) \geq D(p) \quad \text{and} \quad S^*(p) \leq S(p)$$

hold for each price p. Graphically, a pair of supply and demand curves with the opposing shift property should look like the ones in Fig. 1 below. As will be made explicit in Appendix B, if the parameters remain at $\big(D(p), S(p)\big)$ for $k-1$ periods and then at the shift to $\big(D^*(p), S^*(p)\big)$ where they remain, then a unique competitive equilibrium price exists. It is approximately (depending on k and N) p_0 for the first

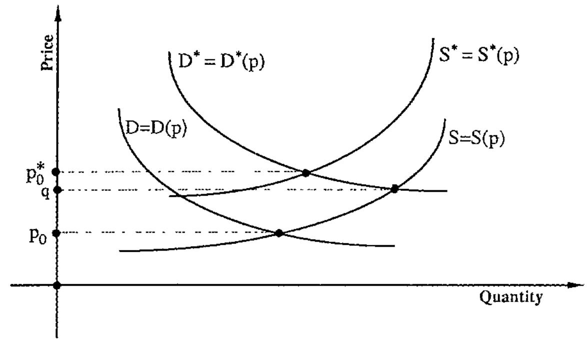

Fig. 1. The opposing shift property

$k-1$ periods. At period k it becomes q and at $k+1$ it jumps to approximately p_0^* where it remains until N. The allocations supporting this unique price path are not unique.

Five experiments were conducted at the California Institute of Technology all of which had the opposing shift property of Fig. 1. The first was a pilot experiment and the results of the other four are reported here. Each experiment consisted of eight individuals who participated in a single market that lasted for up to twenty-nine periods. The markets were organized by a computerized, multiple-unit double-auction market system; Plott and Gray [13] and Johnson et al. [5]. Most subjects had previous experience in electronic markets but not in the overlapping generations environment.

The incentives had an overlapping generations structure of the following form. Each generation had a lifetime of two periods and were designed as the "odd generation" or the "even generation." In any given period there were two generations trading in the market (except in period one). Each generation had two buyers and two sellers. Thus, four people were the odd generations (generations $1, 3, 5, \ldots$) and the other four were the even generations (generations $2, 4, 6, \ldots$). Of course the fact that agents were "reincarnated" immediately after "death" as a new generation and the fact that their lives were only two periods would both seem to be important and the reasons for these procedures will be discussed in the final section.

Each generation's buyers and sellers were given a redemption/cost value function for that generation. These were given in terms of "franc" units, which were worth 0.001 dollars for experiments 1 and 2, 0.0007 dollars for experiments 3 and 4, and 0.008 dollars for experiment 5. The use of "francs" gives the experimenter a way to attempt to mask the fact that two experiments have the same dollar parameters when the same subjects are involved. The numbers reported here will always be in the "franc" units since the data are in that form. However some of the results will be analyzed relative to the "dollar" scale.

Redemption values and costs were such that individuals were indifferent between the two periods of their life that they had the opportunity to buy or sell. Suppose, for example, the redemption value for an individual in generation t was $900f$ for the first unit redeemed and $800f$ for the second. It would make no difference if the unit was redeemed in period t in which the individual was "born" or in period $t+1$ in which the individual "died." Thus, according to the competitive model, the buyer would never purchase in one period of his/her life if prices were for certain lower in the next period. Similarly, a supplier faced with two different prices in the two

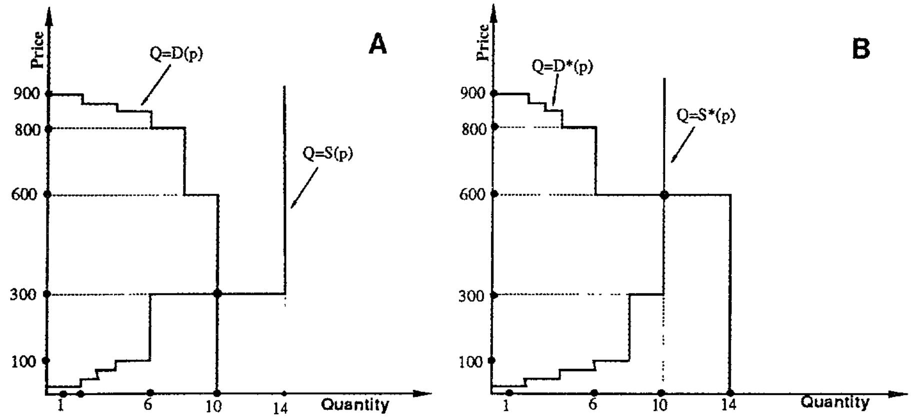

Fig. 2A,B. Generation aggregate demand and supply; **A** before switch (periods 1–14); **B** after switch (periods 15 on)

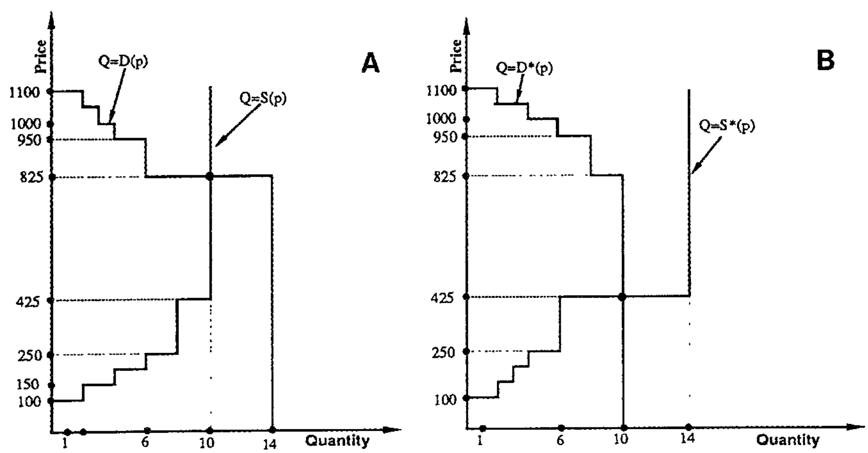

Fig. 3A,B. Generation aggregate demand and supply; **A** before switch (periods 1–14); **B** after switch (periods 15 on)

periods of his/her life would always sell their entire lifetime supply in the period with the highest price and sell nothing in the other period.

Figures 2 and 3 provide further insights about the parameters. Shown in Fig. 2A is, loosely speaking, the aggregate demand and aggregate supply for a generation used in experiments 1 and 2. That is, if the lifetime redemption values/costs for both buyers/sellers of the generation are aggregated into a single demand/supply function the curves would look as shown in Fig. 2. These are reproduced in Table 1. The aggregate generation curves (but not the individual curves) were the same for each of the first 14 lifetimes. With the fifteenth lifetime the aggregate curve shifted to the values represented in Fig. 2B. Notice that the generation demand function in Fig. 2A is essentially the "mirror image" of the generation supply function in Fig. 2B. Furthermore, the supply function of Fig. 2A becomes the mirror image of the demand function in Fig. 2B. Thus, at the generation level the system was constant for the first

Table 1. Parameters for a generation[a]

Unit	Experiments 1 and 2				Experiments 3 and 4			
	Before Shift		After Shift		Before Shift		After Shift	
	Redemption Values	Costs	Redemption Values	Costs	Redemption Values	Costs	Redemption Values	Costs
1	900	25	900	25	1100	100	1100	100
2	900	25	900	25	1100	100	1100	100
3	875	50	875	50	1050	150	1050	150
4	875	75	850	50	1000	150	1050	200
5	850	100	800	75	950	200	1000	250
6	850	100	800	75	950	200	1000	250
7	800	300	600	100	825	250	950	425
8	800	300	600	100	825	250	950	425
9	600	300	600	300	825	425	825	425
10	600	300	600	300	825	425	825	425
11	0	300	600	∞	825	∞	0	425
12	0	300	600	∞	825	∞	0	425
13	0	300	600	∞	825	∞	0	425
14	0	300	600	∞	825	∞	0	425
15	0	∞	0	∞	0	∞	0	∞
16	0	∞	0	∞	0	∞	0	∞

[a] Each generation had two agents and was labeled as odd or even. The units were divided between the agents and rotated each period with the biggest variation in the rotation concentrated in the marginal units.

14 lifetimes after which an opposing shift occurred, i.e., an increase in "demand" and a decrease in "supply." In experiments 3 and 4 the opposite shift occurred as is shown in Fig. 3.

While the curves were constant at the generation level, they were not constant at the individual level. At the beginning of a lifetime the redemption/cost values were known to the individual for that lifetime and not those of future lifetimes. Furthermore, the aggregate values were split between the two individuals and "rotated" each period so no individual saw two identical lifetime redemption/cost values side by side. As a result, the new parameters that occurred in the fifteenth lifetime were not clearly detectable as being new[2].

Thus each individual was aware of his/her own parameters only. These parameters were changing constantly at the individual level. The one-time changes in the

[2] The values were given in a stapled stack of pages. A page was removed to reveal the new values only at "birth." The rotating of individual values, as opposed to constant individual values, is thought to cause a higher variance in price behavior and perhaps a somewhat slower convergence to the competitive equilibrium. Convergence is known to occur; see Daniels and Plott [3].

aggregate parameters came without warning. All individuals knew that the experiment's final period would be announced three periods before it actually occurred. Of course, all of these decisions about information could have been made differently. The reasons will be discussed in the final sections.

The detailed instructions which were read to agents, were identical to those found in the literature with the exception of one paragraph added at the end to explain the generation structure[3].

The format used in the experiment was reproduced on the chalkboard and reviewed as part of the instructions. Accounting procedures were checked between periods. A change in instructions was made in experiments 3 and 4 to clarify a point that was misunderstood by a subject in experiment 2. The subject failed to realize that the single redemption schedule was to be used during both periods of a lifetime. He thought he could use the same redemption schedule a second time during the second period of life, thus effectively doubling his own demand. The effect of this is discussed in the results section.

The first trading period (which had only generation 1 trading) lasted five minutes. After the first few periods the time per period was reduced to three minutes per period. The accounting of all subjects was checked frequently during the first several periods after the problem encountered in experiment 2.

4. The OLG models

The appropriate intuition for the model can be obtained from the above section. This section simply provides a formal development and appropriate notations.

The natural OLG model needed to apply to the experimental environment has two types of agents; the usual buyers and sellers. Their life-span consists of two periods. That is, a buyer (or a seller) born in period t will live all his/her life and trade in periods t and $t+1$. An agent born in period t will be referred to as a *young* agent and the same agent in period $t+1$ will be called an *old* agent. Agents born at a period t will be referred to collectively as the *agents (buyers or sellers) of generation* t. Thus, in this terminology, at period $t > 1$ only the agents of generations t and $t-1$ are participating in the trading. We assume the following.

1. There are two new buyers and two new sellers entering the market at each period. Thus, with the exception of period one, there are eight agents trading in each period; four buyers (two young and two old) and four sellers (two young and two old).

   The period one is exceptional. In this period, we have only four young agents trading; two young buyers and two young sellers.

[3] Instructions are those reproduced in Plott [11]. The added material was the following.
Note: If you look on your redemption/cost sheets, you will notice that there are two periods indicated at the top of the page. You are to continue using the same sheet for the two periods. For example, let's say your sheet was for periods 2 and 3. If you bought two units in period 2, you would record them (as usual) in the first two positions on the page. If you bought another three units in period 3, you would record them in the third, fourth, and fifth positions *on the same page*. Only at the end of period 3 you total your earning and turn to the next page. Do not look at the sheets for a period until you actually need them.

2. The OLG model has countably many periods $(1, 2, \dots)$. However, the short-run OLG model has, of course, a finite number of periods N; in our case here $N = 29$.
3. The two new buyers entering the market at a period t are given a demand function $D_t(p)$. Similarly, the two new sellers entering the market at period t are given a supply function $S_t(p)$. Typical examples of the demand and supply functions considered in this paper are shown in Fig. 4.
4. All agents are profit maximizers. The profit maximizing behavior is the driving force behind their trading actions. We shall employ the following notation.
   a) The buyers born at period t buy b_t^t units at period t and b_{t+1}^t units at period $t+1$. In other words, the trading actions of the new buyers entering the market at period t are described by a vector (b_t^t, b_{t+1}^t).
   b) The trading actions of the new sellers entering the market at period t are described by a vector (s_t^t, s_{t+1}^t). That is, the sellers born at period t sell s_t^t units at period t and s_{t+1}^t units at period $t+1$.

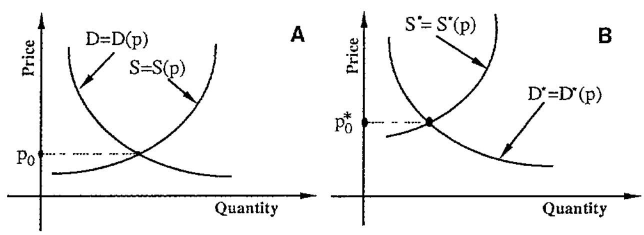

Fig. 4A,B. Examples of demand and supply functions

It is important to keep in mind that the profit maximization behavior of the agents has the following consequence: If p_t and p_{t+1} denote the prevailing prices at period t and $t+1$, respectively, and $p_t < p_{t+1}$, then $b_{t+1}^t = 0$ and $s_t^t = 0$. Similarly, $p_{t+1} < p_t$ implies $b_t^t = 0$ and $s_{t+1}^t = 0$.

An OLG model that satisfies the above properties will be referred to as a **long-run OLG model**—as opposed to our "short-run" OLG model. The notion of a competitive equilibrium in a long-run OLG model is as follows.

Definition 4.1. *A* **competitive equilibrium** *(or simply an* **equilibrium***) for the long-run OLG model consists of two sequences*

$$(p_1, p_2, \dots) \quad \textit{and} \quad \left(\begin{bmatrix} b_1^1 \\ s_1^1 \end{bmatrix}, \begin{bmatrix} b_2^1 & b_2^2 \\ s_2^1 & s_2^2 \end{bmatrix}, \begin{bmatrix} b_3^2 & b_3^3 \\ s_3^2 & s_3^3 \end{bmatrix}, \dots \right).$$

such that:

1. *The sequence $(p_1, p_2, \dots)$ is called the* **price path** *and each price p_t represents the prevailing price at period t.*
2. *The sequence* $\left(\begin{bmatrix} b_1^1 \\ s_1^1 \end{bmatrix}, \begin{bmatrix} b_2^1 & b_2^2 \\ s_2^1 & s_2^2 \end{bmatrix}, \begin{bmatrix} b_3^2 & b_3^3 \\ s_3^2 & s_3^3 \end{bmatrix}, \dots \right)$ *is the (trade) allocation sequence. The component $T_t = \begin{bmatrix} b_t^{t-1} & b_t^t \\ s_t^{t-1} & s_t^t \end{bmatrix}$ represents the trading actions of the four types of agents in period t.*

- *The vector* (b_t^t, b_{t+1}^t) *represents the trading actions of the buyers entering the market at period* t. *That is, the new buyers entering at period* t *buy* b_t^t *units at period* t *and* b_{t+1}^t *units at period* $t+1$.
- *The vector* (s_t^t, s_{t+1}^t) *represents the trading actions of the sellers entering the market at period* t. *That is, the new sellers entering at period* t *sell* s_t^t *units at period* t *and* s_{t+1}^t *units at period* $t+1$.

3. *The bought and sold units satisfy*
   a) *the "clearance conditions"*

$$b_1^1 = s_1^1 \ \textit{and} \ b_t^t + b_t^{t-1} = s_t^t + s_t^{t-1} \ \textit{for} \ t \geq 2\,, \ \textit{and}$$

   b) *the profit maximizing "budget constraints"*

$$b_t^t + b_{t+1}^t = D_t\big(\min\{p_t, p_{t+1}\}\big) \ \textit{and} \ s_t^t + s_{t+1}^t = S_t\big(\max\{p_t, p_{t+1}\}\big)$$

   for each $t = 1, 2, \ldots$.

A **sequence of equilibrium prices** is any sequence $(p_1, p_2, \ldots)$ of prices that is a part of an equilibrium as defined in Definition 4.1. Following standard economic terminology, we shall also say that a sequence of prices $(p_1, p_2, \ldots)$ can be **supported by trades** whenever it is the sequence of prices for an equilibrium. Our short-run OLG model has a finite number of periods. It is defined as follows.

Definition 4.2. *A* **short-run opposing shift OLG model** *is an OLG model with* $k+m$ *periods such that:*

a) *The aggregate demand and supply functions for the new buyers and sellers entering the market at periods* $1, 2, \ldots, k-1$ *are all identical.*
b) *The aggregate demand and supply functions for the new buyers and sellers entering the market at periods* $k, k+1, \ldots, k+m$ *are also all identical.*
c) *The pair of supply and demand functions* $\big(D(p), S(p)\big)$ *and* $\big(D^*(p), S^*(p)\big)$ *satisfies the opposing shift property.*

In other words, an OLG model is short-run whenever it has $k+m$ periods and the new buyers and sellers in the first $k-1$ periods have identical demand and supply functions and from period k on (the **switch period**) they also have identical demand and supply functions which are different than those of the first $k-1$ periods but they satisfy the opposing shift property.

In general, we can allow demands and supplies to be compact-valued correspondences. In this case, property 3(b) of Definition 4.1 should be written in a "set form" rather than in an "equation form." That is, property 3(b) of Definition 4.1 should read:

$$b_t^t + b_{t+1}^t \in D_t\big(\min\{p_t, p_{t+1}\}\big) \ \text{ and } \ s_t^t + s_{t+1}^t \in S_t\big(\max\{p_t, p_{t+1}\}\big)\,.$$

Also, the opposing shift property for correspondences should read: For each p we have

$$\max D(p) \geq \max D^*(p) \quad \text{and} \quad \min D(p) \geq \min D^*(p)$$

and

$$\max S^*(p) \geq \max S(p) \quad \text{and} \quad \min S^*(p) \geq \min S(p)\,.$$

In our experiments the supplies and demands are correspondences. They are the demand and supply correspondences $D(p)$ and $S(p)$ shown in Figs. 2A and 3A. However, they can be "approximated" easily by demand and supply functions.

5. Equilibria in the experimental environment

In this section the competitive equilibrium is treated as a technical property of a model as required by the formal definition. It is computed *as if* agents were aware of the time path prices. The fact that agents in the actual economy have no knowledge of the demand and supply shifts does not effect this assumption. At issue of course is whether or not a model based on such computations should be applied but that is a matter to be decided by the data.

Figure 5 summarizes the results of the analysis that follow. Given the parameters and procedures of these markets there exists a well defined set of competitive equilibria. In Fig. 5A the theoretical equilibria for experiments 1 and 2 are shown. The price should start at 300 and remain constant for at least 14 periods. At period 15 the price can take any value in the closed interval [300, 600]. Beginning with period 16 the price stays constant at 600.

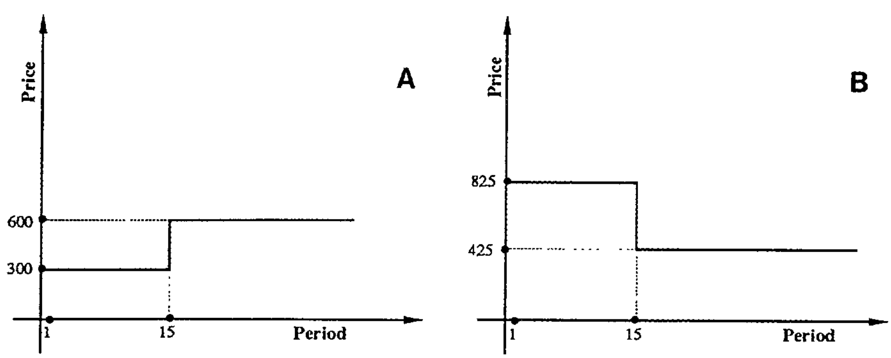

Fig. 5A,B. Competitive equilibrium experiments; **A** 1 and 2; **B** 3 and 4

Figure 5B shows the set of theoretical competitive equilibrium prices for the parameters of experiments 3 and 4. Prices should remain constant at 825 for 14 periods. In period 15 the price can take any value on the closed interval [425, 825]. Beginning with period 16 prices should remain constant at 425.

Allocations have a large range of possibilities. The most prominent feature is that those generations that span a price change have specific requirements. If prices increase between periods of a lifetime, buyers all buy when they are young and sellers all sell when they are old. If price decreases during a lifetime, buyers all buy when they are old and sellers all sell when they are young. Thus the competitive equilibria is accompanied by specific predictions about the age distribution of those who trade.

This section is used to demonstrate that price time paths can be supported by a time path of allocations such that the pair satisfies the definition of a competitive equilibrium. The next section is developed to show that those time paths are the only paths that can be so supported. Only expected parameters of experiments 1 and

2 will be considered formally. The results for experiments 3 and 4 follow from symmetry.

Theorem 5.1. *Let* $p_0 = 300$, $p_0^* = 600$, *and let* $q \in [p_0, p_0^*]$ *be arbitrary. Then in experiments* 1 *and* 2 *all price paths* $(p_1, p_2, \ldots)$ *of the form*

$$p_t = \begin{cases} p_0, & \text{if } t < 15; \\ q, & \text{if } t = 15; \\ p_0^*, & \text{if } t > 15\,, \end{cases}$$

can be supported as competitive equilibria.

Proof. From the definition of equilibrium, we must show that there exist allocations $T_1 = \begin{bmatrix} b_1^1 \\ s_1^1 \end{bmatrix}$ and $T_t = \begin{bmatrix} b_t^{t-1} & b_t^t \\ s_t^{t-1} & s_t^t \end{bmatrix}$ $(t = 2, 3, \ldots)$ such that:

$$b_1^1 = s_1^1 \quad \text{and} \quad b_t^{t-1} + b_t^t = s_t^{t-1} + s_t^t \ \text{ for } t \geq 2\,, \tag{$\star$}$$

$$b_{t-1}^{t-1} + b_t^{t-1} \in D\big(\min\{p_{t-1}, p_t\}\big)\,, \tag{$\star\star$}$$

and

$$s_{t-1}^{t-1} + s_t^{t-1} \in S\big(\max\{p_{t-1}, p_t\}\big)\,. \tag{$\star\star\star$}$$

Next, consider the sequence of allocations

$$\begin{aligned}
&T_1 = \begin{bmatrix} 10 \\ 10 \end{bmatrix},\ T_2 = \begin{bmatrix} 0 & 10 \\ 0 & 10 \end{bmatrix} = T_3 = \cdots = T_{11} = \begin{bmatrix} 0 & 10 \\ 0 & 10 \end{bmatrix}, \\
&T_{12} = \begin{bmatrix} 0 & 10 \\ 2 & 8 \end{bmatrix},\ T_{13} = \begin{bmatrix} 0 & 10 \\ 6 & 4 \end{bmatrix},\ T_{14} = \begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix},\ T_{15} = \begin{bmatrix} 0 & 14 \\ 14 & 0 \end{bmatrix}, \\
&T_{16} = \begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix},\ T_{17} = \begin{bmatrix} 4 & 14 \\ 10 & 8 \end{bmatrix},\ T_{18} = \begin{bmatrix} 0 & 12 \\ 2 & 10 \end{bmatrix},\ T_{19} = \begin{bmatrix} 0 & 10 \\ 0 & 10 \end{bmatrix}, \\
&T_{20} = \begin{bmatrix} 0 & 10 \\ 0 & 10 \end{bmatrix},\ T_{21} = \begin{bmatrix} 0 & 10 \\ 0 & 10 \end{bmatrix}, \ldots\,.
\end{aligned}$$

A direct verification shows that these allocations satisfy ($\star$). Now notice that from the parameters in Fig. 2, we have

$$\begin{aligned}
D(p_0 = 300) &= \{10\},\ \ D(p_0^* = 600) = \{9, 10\}; \\
D^*(p_0 = 300) &= \{14\},\ \ D^*(p_0^* = 600) = \{7, 8, 9, 10, 11, 12, 13, 14\}; \\
S(p_0 = 300) &= \{7, 8, 9, 10, 11, 12, 13, 14\},\ \ S(p_0^* = 600) = \{14\}; \\
S^*(p_0 = 300) &= \{9, 10\},\ \ S^*(p_0^* = 600) = \{10\}\,.
\end{aligned}$$

(Keep in mind that we treat our experimental model as a discrete model!) Again, a direct verification shows that for all generations facing constant prices of our hypotheses at periods $1 \leq t \leq 13$ and $16 \leq t \leq 28$ both ($\star\star$) and ($\star\star\star$).

Consider now the two remaining generations at periods 14 and 15 which might experience price increases. Notice that for both ($\star\star$) and ($\star\star\star$) are satisfied as long as price does not go down during a lifetime. Buyers in these two generations purchase only when they are young and sellers sell only when they are old. Since for all generations, T_t satisfies ($\star$), ($\star\star$), and ($\star\star\star$) under the above price path $(p_1, p_2, \ldots)$, it follows that the sequence $\{(p_t, T_t)\colon t = 1, 2, \ldots\}$ is a competitive equilibrium. ■

The preceding theorem establishes that the family of time paths in Figs. 4A and 4B can be supported by competitive equilibrium trade allocations. In the Appendix, we shall establish that when supply and demand are given by functions, then the competitive equilibrium path is uniquely determined. Since our correspondences can be approximated by functions, it will follow that the competitive equilibrium path of Theorem 5.1 is essentially the only one that can be observed experimentally. It will also be shown that the price time path remains unique and unchanged over the short-run economy, when the time periods are increased to countably many.

6. An observation about dynamics

The discussion in the preceding section and in the Appendix suggests a question about the nature of price changes. The dynamics of price adjustments along a competitive equilibrium time path is different from classical ideas about the behavior of price adjustments which might be off of an equilibrium path. This brief section identifies how these two models of the time path of price differ in terms of observables in the experimental environment. For our discussion here and in the next section, we shall employ the following notation:

P = Generic notation for observed prices,

P_k = The k[th] observed price in some well defined (when needed) sequence,

P_t^c = The observed price of contract number c in period t,

$\overline{P}_t$ = The average observed price of contracts in period t, and

p_t = The competitive equilibrium price in period t.

The classical hypothesis, when put in a form that can be estimated[4], is

$$P_{t+1} - P_t = \alpha + \beta\big[D(P_t) - S(P_t)\big] + \varepsilon_t\,,$$

where $\alpha = 0$, $\beta > 0$, and $\varepsilon_t \sim N(0, \sigma^2)$. The model has empirical content in an experiment because the relevant aspects of $D(P)$ and $S(P)$ can be observed and thus used as independent variables. A difficulty exists in extending this model to the overlapping generations case because a notion of excess demand is not so clearly distinguishable as it is when agents only have a one-period life span.

A natural generalization of the concept of excess demand to the overlapping generations case is as follows—which, in order to emphasize its difference with the static measure, we shall refer to as the *instantaneous excess demand* and denote it by Z. Formally, the **instantaneous excess demand** at period t is given by the formula

$$Z_t(P) = \big[(D_{t-1}(P) - b_{t-1}^{t-1}) + D_t(P)\big] - \big[(S_{t-1}(P) - s_{t-1}^{t-1}) + S_t(P)\big]\,,$$

where the first bracketed term represents a concept of "potential demand" and the second is a concept of "potential supply" at an arbitrary observed price P. The instantaneous excess demand is defined to be the difference between the two. The potential demand is the demand from the old generation, the total amount the old generation wants at the current price minus the amount that it has purchased so far plus

[4] See Plott and George [12] for an analysis and demonstration of the power of this model and the closely related Marshallian model of price adjustment.

the demand of the current generation, measured as the amount the current generation would want at the price given that the next period price is not lower. Potential supply involves similar concepts for the sellers. In both cases the interpretation of demand and supply from the current generation has obviously troublesome features involving possibly incorrect expectations.

By using the above definition of excess demand, we define the **Extended Classical Adjustment Model** to be

$$\overline{P}_{t+1} - \overline{P}_t = \alpha + \beta Z_{t+1}(\overline{P}_t) + \varepsilon_t ,$$

where $\alpha = 0$, $\beta > 0$, and $\varepsilon_t \sim N(0, \sigma^2)$. The model rests on the idea that the movement of price during period $t+1$ depends upon the "excess demand" that exists at the beginning of period $t+1$ and that excess demand is measured at the average price that occurred in period t.

In the experimental environment under examination here the Extended Classical Adjustment Model suggests observations that are the opposite of the competitive equilibrium dynamics. In the Extended Classical Adjustment Model upward price movements are associated with positive excess demand and downward price movements are associated with negative excess demand. Price movements and excess demand are positively related but along the competitive equilibrium time path the positive relationship does not hold. To the extent that price movements exist in the experimental environment upward price movement along a competitive equilibrium time path is associated with negative instantaneous excess demand and downward price movement along a competitive equilibrium time path is associated with positive instantaneous excess demand.

In order to see the relationships, consider periods 14, 15, and 16 which are the only periods in which price movements can exist in the competitive equilibrium path. Assume $P_{13} = P_{14} < P_{15} < P_{16}$. So, from the equilibrium market clearing conditions, we have

$$D_{13}(P_{14}) - b_{14}^{13} + D_{14}(P_{14}) - \left[S_{13}(P_{14}) - s_{13}^{13}\right] = 0 , \tag{6}$$

$$D_{15}(P_{15}) - S_{14}(P_{15}) = 0 , \text{ and} \tag{7}$$

$$D_{16}(P_{16}) - S_{16}(P_{16}) = 0 . \tag{8}$$

Using (6), (7), and (8), the fact that in the range of the competitive equilibrium prices $S_{14}(P) > S_{15}(P)$, and the definition of instantaneous excess demand, we see that

$$\begin{aligned}
Z_{14}(P_{13}) &= \left[D_{13}(P_{13}) - b_{13}^{13} + D_{14}(P_{13})\right] - \left[S_{13}(P_{13}) - s_{13}^{13} + S_{14}(P_{13})\right] < 0 , \\
Z_{15}(P_{14}) &= \left[D_{14}(P_{14}) - b_{14}^{14} + D_{15}(P_{14})\right] - \left[S_{14}(P_{14}) - s_{14}^{14} + S_{15}(P_{14})\right] < 0 , \text{ and} \\
Z_{16}(P_{15}) &= \left[D_{15}(P_{15}) - b_{15}^{15} + D_{16}(P_{15})\right] - \left[S_{15}(P_{15}) - s_{15}^{15} + S_{16}(P_{15})\right] < 0 .
\end{aligned}$$

Thus, at the beginning of each of periods 14, 15, and 16 excess demand is negative. However, along the competitive equilibrium time path (for experiments 1 and 2), we have

$$P_{14} - P_{13} = 0, \quad P_{15} - P_{14} > 0, \quad \text{and} \quad P_{16} - P_{15} > 0 .$$

The periods of increasing prices are associated with negative instantaneous excess demand. The Extended Classical Adjustment Model asserts the existence of a positive relationship. In this sense the models are in direct opposition. This observation will form the basis for one of the questions posed in the next section.

7. Experimental results

Figures 6 through 9 provide a time series of all contracts in all periods in each of the four experiments. Each dot represents the price of a contract in the sequence in which it occurred. The figures also contain the average price for each period and the price of the last contract in a period. The time paths of the competitive equilibrium prices taken from Figs. 4A and 4B are also displayed.

In all experiments the parameters are as discussed in Sect. 3 except for experiment 2 in which a buyer mistakenly used the same redemption value sheet two times during each of his first seven lifetimes as opposed to once. The effect of this was an increase in demand for the first eight periods. However, when the parameters are adjusted for this behavior using the reparametrization techniques outlined in Plott [10] the price paths which support the competitive equilibria in this experiment remain unchanged from the paths that would have existed without the buyer error. Since the major features of this experiment are similar to the other experiments we chose to report the data rather than conduct a new experiment to replace this one.

The first result is not particularly surprising. All experimental markets involve a convergence pattern that is not captured by the competitive equilibria. Almost all complicated models have no random variables that can be used to capture the natural variation in the data. The overlapping generations markets are similar in both regards. So, it is not surprising that the competitive equilibrium model can be rejected as a faithful representation of the data. Nevertheless the result is reported for completeness in order to avoid any possible misinterpretations of the context of the results reported in the body of this section.

Result 1. *The observed time path of contract prices is not confined to the competitive equilibria.*

Support. Compare the actual time series with the competitive equilibria in Figs. 6 through 9. In none of the time series do the data lie on a competitive equilibrium path. The equilibrium time path is not immediately attained. When the shift occurs, price changes do not begin to adjust immediately in period 15, and when the price changes do begin they last for more than one period. In this sense the competitive equilibria can be rejected as a representation of the data. ■

The next two results are more in keeping with the behavior characteristic of non-overlapping generations environments. The data always converge to near the competitive equilibrium steady states.

Result 2. *Prices are converging to the competitive equilibrium steady states.*

Support. Two models are applied to capture the nature of price movements. The first involves only the sign of price changes and not the magnitudes. The second is a partial adjustment model that factors in the magnitude of movements.

Let P_t^c denote the price of the c^{th} contract in period t. If

$$(P_t^c - p_t)(P_t^{c+1} - P_t^c) < 0\,,$$

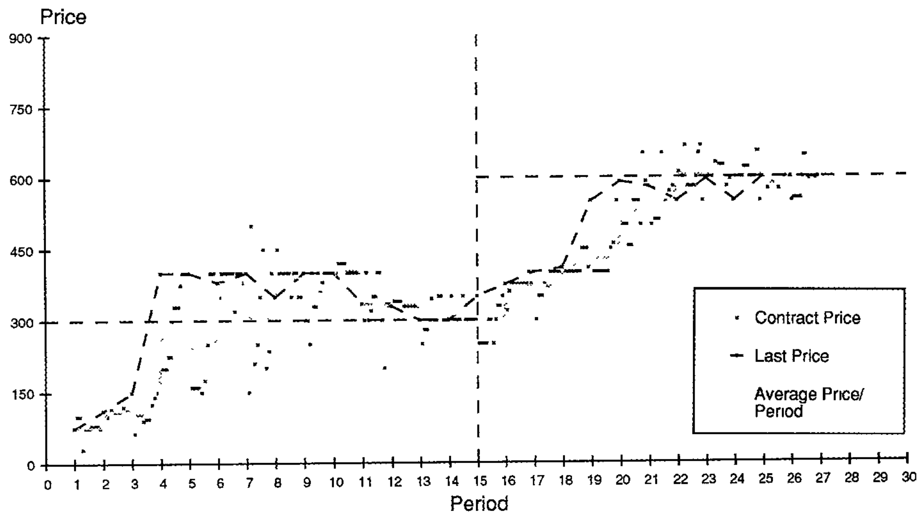

Fig. 6. All contract prices and average prices for all periods for experiment #1 (12/01/89)

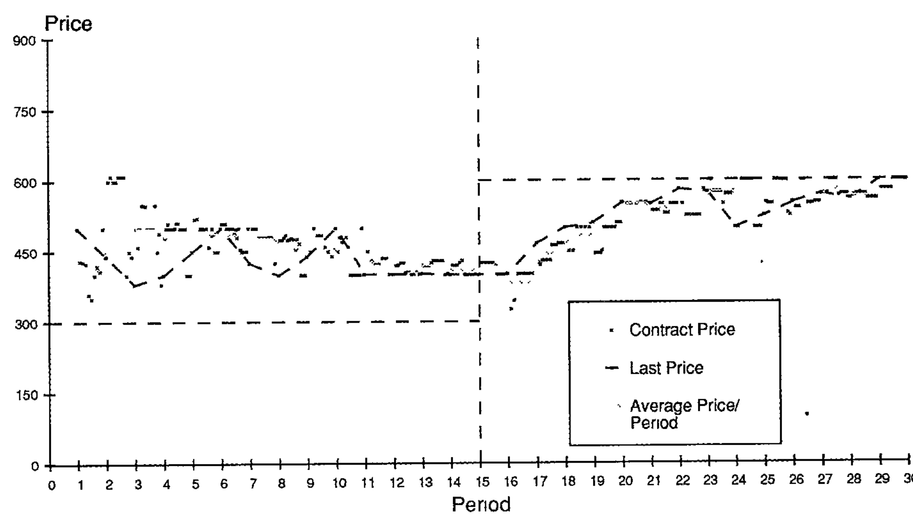

Fig. 7. All contract prices and average prices for all periods for experiment #2 (01/11/90)

where p_t is the equilibrium price at period t, then the movement from P_t^c to P_t^{c+1} is in the direction of the equilibrium of the competitive model. Table 2 contains for each experiment the percentage of price changes which were toward p_t. The percentage of price changes that were zero and the percentage of changes that were away from p_t are also listed in Table 2 for each of the four experiments. As can be seen from the table, the most frequent event is no change in price. The likelihood ratio test of equally likely price changes toward and away from the competitive equilibrium, *given* that a non-zero price change occurs, can be rejected at the 0.05 level of significance in all experiments except in experiment 2. Since the proportions favor movement

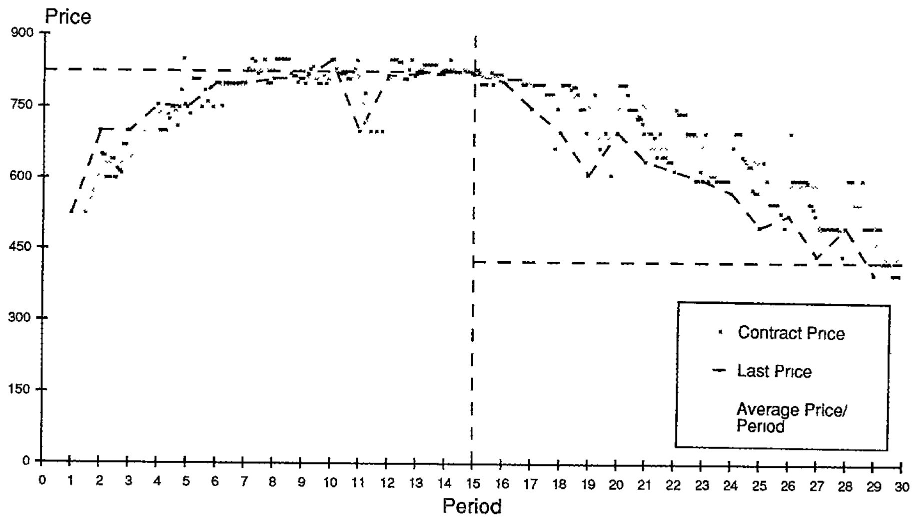

Fig. 8. All contract prices and average prices for all periods of experiment #3 (01/30/90)

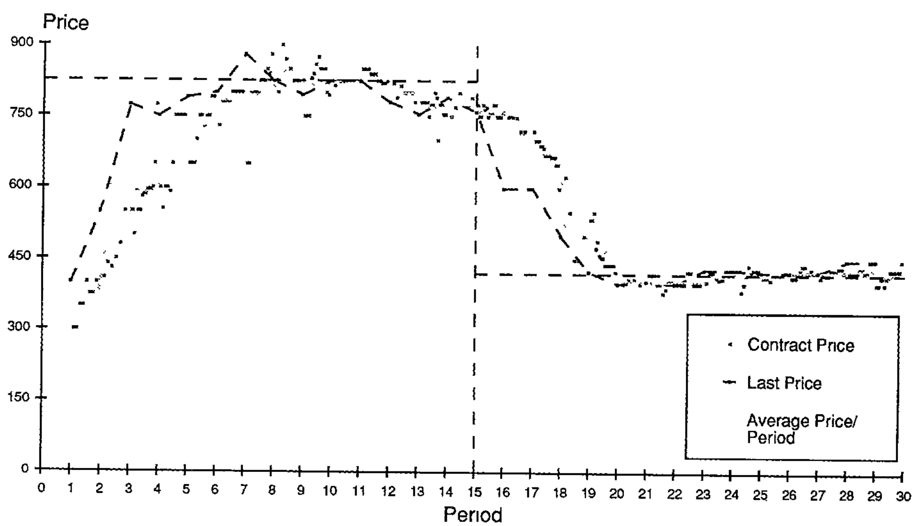

Fig. 9. All contract prices and average prices for all periods of experiment #4 (03/01/90)

toward equilibrium, the non-quantitative measurements indicate that movement is in the direction of the equilibrium.

The model[5]

[5] A model with a more natural interpretation would be the Partial Adjustment Model

$$P_t^{c+1} - P_t^c = \alpha + \beta(P_t^c - p_t) + \varepsilon_t^c$$

However, the relevant parameter β and the parameter capturing seriel correlation enter the model symmetrically and therefore can not be identified. This problem forced the use of Eq. (9) and the related statistical tests.

Table 2. Price changes between contracts

Experiment	Percent of Contract Movements			n
	Toward Equilibrium	No Change	Away from Equilibrium	
1	0.35	0.52	0.14	263
2	0.26	0.51	0.23	309
3	0.34	0.48	0.18	282
4	0.36	0.45	0.18	287
Pooled	0.33	0.49	0.18	1141

$$P_k = \alpha + \beta P_{k-1} + \gamma P_{k-2} + \varepsilon_k \,, \tag{9}$$

was estimated under the assumption $E\varepsilon_k = 0$. In order to keep the time notation clear, we let $P_t^c = P_k$, where the index k denotes the ordinal index of all contracts during periods 1–14 and again in periods 15–terminal. The estimates are contained in Table 3. In all parts of all experiments the estimated model converges. The limits of the convergence process are given by the formula $\frac{\hat{\alpha}}{1-\hat{\beta}-\hat{\gamma}}$ and are listed in Table 3 beside the column that contains the theoretical competitive equilibrium. The next two columns contain the statistics that measure the distance of the limit from the competitive equilibrium price. In all cases (except experiment 2), the hypothesis that the limit equals the competitive equilibrium price can not be rejected. Two checks on the model were performed. A Durbin–Watson test fails to detect the existence of any additional seriel correlation. In addition, a robust estimation was performed as possible correction for heteroskedasticity. The t-statistics remains virtually unchanged. In summary, in all parts of all experiments (except experiment 2) the data are converging to the competitive equilibrium. In experiment 2, as the previous analysis showed, the direction of movement is toward the competitive equilibrium. ■

Result 3. *After several periods prices are "near" the competitive equilibrium.*

Support. Support for this result is provided by the data from periods before a parameter change occurs. These are periods 14 in all experiments, just before the parameter shift. In addition, relevant observations are from period 25 in all experiments. The end of the experiment was announced before the terminal period so the relevant periods for observation are before that and even though the experiments lasted different lengths; period 25 was chosen for consistency.

The data are studied in two ways. The first is a simple examination of the distance in dollar terms between the last contract price and the equilibrium. The period 14 final prices deviated from equilibrium by 0¢, 10¢, 0.5¢, and 0.1¢ in the four experiments. The near final period contracts deviated from the equilibrium by 5¢, 3¢, 2¢, and 0.1¢. Of course one is free to apply personal standards for "nearness" but 5¢ is frequently considered "close" and if it is accepted as a criterion, then in all cases but one prices converge to near the competitive equilibrium after having some time for adjustment. The exception is the first part of experiment 2 in which the subject made the mistake described previously.

The second way to study the data is by examining the mean prices. If all contracts in the relevant periods are pooled across experiments the resulting mean is insignif-

Table 3. Least square estimates of the model $P_k = \alpha + \beta P_{k-1} + \gamma P_{k-2} + \varepsilon_k$.

Experiment	Period	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	n	R^2	DW	p_t	$\frac{\hat{\alpha}}{1-\hat{\beta}-\hat{\gamma}}$	$\hat{\sigma}$	$\frac{\frac{\hat{\alpha}}{1-\hat{\beta}-\hat{\gamma}}-p_t}{\hat{\sigma}}$	Robust estimation $\hat{\sigma}$	Robust estimation $\frac{\frac{\hat{\alpha}}{1-\hat{\beta}-\hat{\gamma}}-p_t}{\hat{\sigma}}$
1	1–14	44.41 (2.85)	0 54 (6.61)	0.32 (3.91)	137	0.69	2.12	300	309	37.52	0.24	37.71	0.24
	15–26	22.20 (1.75)	0.68 (8.15)	0.28 (3.30)	134	0.91	2.11	600	551	89.81	−0.55	98.95	−0.50
2	1–14	100.61 (3.86)	0.72 (8.86)	0.06 (0.80)	155	0.59	1.98	300	462	12.62	12.85	12 39	13.09
	15–26	96.75 (3.40)	0.46 (6.28)	0.35 (4.79)	164	0.58	2.20	600	523	21.09	−3.66	22.05	−3.50
3	1–14	63.37 (2.35)	0.82 (9.37)	0.10 (1.23)	121	0.86	2.02	825	798	34.15	−0 79	31.09	−0.87
	15–26	11.92 (0.67)	0.65 (8.67)	0.33 (4.40)	164	0.90	2.04	425	528	219.41	0.47	263.24	0.39
4	1–14	61.00 (3.35)	0.78 (9.01)	0.15 (1.74)	134	0.91	2 06	825	776	46.16	−1.05	42.42	−1.14
	15–26	8.58 (1.28)	0.79 (9.99)	0.19 (2.41)	158	0.97	2.01	425	404	105.33	−0.19	68.31	−0.30

$\frac{\hat{\alpha}}{1-\hat{\beta}-\hat{\gamma}}$ = Limit of process; $\hat{\sigma}$ = standard error of $\frac{\hat{\alpha}}{1-\hat{\beta}-\hat{\gamma}}$; p_t = competitive equilibrium price; $\frac{\frac{\hat{\alpha}}{1-\hat{\beta}-\hat{\gamma}}-p_t}{\hat{\sigma}}$ = t-statistics icantly different from the equilibrium at period 14 (where $\overline{P} - p = 34.19$, $n = 31$, $t = 0.54$) and at period 25 (where $\overline{P} - p = 14.97$, $n = 32$, $t = 0.21$). ■

Previous empirical work has found a relationship between the number of bids and asks in a period and the change in average price the next period. No theory exists to explain this phenomenon. However, given the substantially different nature of the overlapping generations markets, an investigation is warranted. As will be seen in the next result, the behavior exists in the overlapping generations economies.

Result 4. *The magnitude of the difference between the number of bids and the number of asks in period t is a predictor of the change in average price between periods t and $t+1$.*

Support. Let us introduce the notation:

$$\begin{aligned} \overline{P}_t &= \text{The average price of contracts in period } t, \\ B_t &= \text{The number of bids in period } t, \text{ and} \\ A_t &= \text{The number of asks in period } t. \end{aligned}$$

The following model was estimated

$$\overline{P}_{t+1} - \overline{P}_t = \alpha + \beta(B_t - A_t) + \varepsilon_t ,$$

where $\varepsilon_t \sim N(0, \sigma^2)$. Support for the result is inferred if $\hat{\beta} > 0$. The estimates for α and β for each of the four experiments can be found in Table 4. As can be seen, the coefficient β is positive and significant at the 0.05 level for experiments 1, 2, and 3. In experiment 4 the hypothesis of no relationship cannot be rejected at the conventional levels of significance. The pooled data support the result. Thus, support for this result is established. ■

Table 4. Least square estimates of the model $\overline{P}_{t+1} - \overline{P}_t = \alpha + \beta(B_t - A_t) + \varepsilon_t$ (t statistics in parentheses)

Experiment Number	Coefficient α	Coefficient β	Number of Observations	R^2	SER	ρ[a]
1	5.18 (0.55)	2.16 (2.43)	25	0.26	58.20	−0.66
2	−2 39 (−0 43)	0.93 (1.97)	26	0.10	40.75	−0.71
3	−0 82 (−0.11)	1 36 (2 40)	28	0.17	59.38	−0.54
4	2.56 (0.18)	1.27 (1 24)	28	0.30	46.87	0.38
Pooled Without AR1	2.47 (0 45)	1.16 (2.66)	107	0.06	55.82	2.72 DW
Pooled With AR1	0.70 (0 18)	1 72 (5.03)	107	0.14	53.58	−0 36

[a] The model is corrected for AR1

The dynamics of the markets exhibit another interesting property that has been noticed as a property of non-overlapping generations markets. The final price movement in a period tends to point the direction of change in average price in the following period. The next result identifies that phenomenon as a property of overlapping generations economies.

Result 5. *In experiments* 1, 2, *and* 4 *the change in price between the last contract price in a period and the next-to-last contract price is positively related to the next period change in average price.*

Support. Let

$$\overline{P}_t = \text{The average price in period } t, \text{ and}$$
$$P_t^\ell = \text{The price of the last contract in period } t,$$

and estimate the model $\overline{P}_{t+1} - \overline{P}_t = \alpha + \beta(P_t^\ell - P_t^{\ell-1})$, where the estimated coefficients α and β are in Table 5. Experiment 3 is an exception. In all other experiments $\hat{\alpha} = 0$ and $\hat{\beta} > 0$ (weakly in experiment 1). The pooled data support the result. The observed tendency exists as is stated in the result. ■

Table 5. Coefficient estimates and t statistics for the model $\overline{P}_{t+1} - \overline{P}_t = \alpha + \beta(P_t^\ell - P_t^{\ell-1}) + \varepsilon_t$

Experiment	α	β	n	R^2	DW
1	21.18 (1 68)	0 44 (1.82)	25	0.13	2.94
2	5 30 (0 73)	0 43 (2 24)	28	0.16	2.43
3	−12.95 (−1 09)	−0.40 (−1.25)	26	0.06	2.70
4	−0.97 (−0.11)	0.80 (3.61)	28	0.33	1.40
Pooled	−0 11 (−0.02)	0.30 (2 29)	107	0.05	2.33

The above two results establish two important connections between the non-overlapping generations economies and the overlapping generations case. The next result explores the speed of adjustment that was discussed in Sect. 7. As was stated there, the speed of adjustment is known to be related to excess demand in the non-overlapping case. The next result was from an attempt to explore the possibility that similar phenomena exist in the overlapping generations case when the stationary model is replaced by the Generalized Adjustment Model. The basic question posed is whether prices seem to respond to the existence of instantaneous excess demand or whether there is no relationship (or perhaps a negative relationship) as suggested by the competitive equilibrium time path.

Result 6. *The change in average price in a period over that of the previous period is positively related to the instantaneous excess demand that existed at the beginning of the period.*

Support. Many features of the environment at the time series help to make operational the concepts used in the result. The equation

$$\overline{P}_t - \overline{P}_{t-1} = \alpha + \beta\big[D_{t-1} - b_{t-1}^{t-1} + D_t - (S_{t-1} - s_{t-1}^{t-1} + S_t)\big] + \varepsilon_t\,,$$

where $\varepsilon_t \sim N(0, \sigma^2)$, was estimated. In all experiments the shapes of the demand and supply functions are vertical in a neighborhood of the prices in which transactions take place and therefore are insensitive to price in that neighborhood. Thus, D_t and S_t depend only upon whether or not a shift of parameters had occurred and therefore can be treated as period dependent constants. With this in mind, the magnitude $D_{t-1} - b_{t-1}^{t-1}$ is the number of demand units carried over by generation $t-1$ into period t. The magnitude will be called *the demand of the old*. The magnitude $S_{t-1} - s_{t-1}^{t-1}$ is the supply carried over from generation $t-1$ into period t. It will be called *the supply of the old*. The instantaneous excess demand at period t is thus

$$Z_t = \big(D_{t-1} - b_{t-1}^{t-1} + D_t\big) - \big(S_{t-1} - s_{t-1}^{t-1} + S_t\big)\,,$$

and can be used as the independent variable in the model.

Support for the model is interpreted as $\hat{\alpha} = 0$ and $\hat{\beta} > 0$. The regression results are in Table 6. The estimates in experiments 2, 3, and 4 support the result. Experiment 1 differs from the other three and has α significantly positive and β insignificantly positive. The reasons for the exceptional behavior of experiment 1 are not apparent. Nevertheless, the pooled data yield strong support for the result as stated. The generalized adjustment model is supported. Furthermore, the strong positive estimate of β is directly contrary from the property of the dynamics identified in Sect. 7 that instantaneous excess demand be negatively related to price changes. ■

The above result induces a type of paradox. How can markets converge to the competitive equilibrium steady states while the dynamic properties of the competitive equilibrium price changes are completely absent? Furthermore, how can a variable (such as instantaneous excess demand) that seems so *ad hoc* and lacking in expectation properties be so powerful representing the data? The next two results and a conjecture provide some intuition about how the patterns are related to expectation.

Table 6. Coefficient estimates of $\overline{P}_{t+1} - \overline{P}_t = \alpha + \beta Z_t + \varepsilon_t$

Experiment	α	β	n	R^2	ρ
1	19.69 (2 38)	0 83 (1 02)	25	0.08	−0 59
2	3 55 (0 82)	1 00 (2.24)	28	0.10	−0.74
3	3 14 (0 45)	2 34 (3.45)	28	0.27	−0.53
4	2 53 (0.15)	2.97 (1 89)	28	0.35	0.50
Pooled	6 17 (1 60)	1 94 (4.95)	109	0.14	−0.30

In the overlapping generations environment agents have the freedom to choose the time pattern of their activities over their life. They can choose to consume/supply when they are young or they can wait until they are old. To the extent that buyers have delayed their purchases until they are old a demand from the old generation can be identified. Accordingly, **old demand** can be defined as $D_{t-1}(P_t) - b_{t-1}^{t-1}$, where b_{t-1}^{t-1} are the purchases of generation $t-1$ during period $t-1$ when they are young. That is, old demand is the residual demand that was carried over from the previous period to the current period. Similarly, **old supply** is exactly $S_{t-1}(P_t) - s_{t-1}^{t-1}$.

We can now define a **demographic squeeze** as a relationship between old demand and old supply. In particular, the maximum demographic squeeze occurs when either $D_{t-1}(P_t) - b_{t-1}^{t-1} = s_{t-1}^{t-1} = 0$ or when $b_{t-1}^{t-1} = S_{t-1}(P_t) - s_{t-1}^{t-1} = 0$. That is, the demographic squeeze occurs when one side of the market acts when it is young and the other side does not. Under such circumstances a situation occurs in which both the young and the old on one side of the market find themselves together facing only a young generation on the other side of the market. Put another way, if the degree of the squeeze is defined to be the number

$$\left[D_{t-1}(P_t) - b_{t-1}^{t-1}\right] - \left[S_{t-1}(P_t) - s_{t-1}^{t-1}\right],$$

then this number differs from the instantaneous excess demand only by the amount $D_t(P_t) - S_t(P_t)$.

Result 7 shows the first of two systematic tendencies. A demographic squeeze is occuring at the time of price adjustments even though its quantitative relationship to price change is open to speculation. Given the relationship with instantaneous excess demand, Result 7 is understandable in a statistical sense. As we shall see, when it is incorporated with Result 8, a behavioral conjecture begins to emerge.

Result 7. *During the periods of price changes, after the demand shift, a demographic squeeze develops in all experiments. Increasing prices (experiments 1 and 2) are associated with a growing old demand and shrinking old supply. Decreasing prices (experiments 3 and 4) are associated with a shrinking old demand and growing old supply.*

Support. Figures 10 through 13 are designed to permit one to see the relevant comparisons. The top panel contains the time series of prices. The middle panel contains a time series graph of units of potential old demand/supply, i.e., $D_{t-1} - b_{t-1}^{t-1}$ and $S_{t-1} - s_{t-1}^{t-1}$. The extremes are also shown in the panel. As can be seen in Figs. 10 and 11, the potential demand of the old increases after period 14 during the periods of increasing prices and the potential supply of the old shrinks. Similarly, in Figs. 12 and 13, the periods of decreasing prices after the parameter shift are associated with the opposite dynamics.

In order to make support for the result precise, the following two regressions were run:

$$D_t - b_t^t = \alpha_b + \beta_b t + \varepsilon_t\,, \quad t \in \{14, 15, 16, 17, 18\};$$
$$S_t - s_t^t = \alpha_s + \beta_s t + \varepsilon_t\,, \quad \varepsilon_t \sim N(0, \sigma^2)\,.$$

The results are in Table 7. As can be seen, $\hat{\beta}_b > 0$, $\hat{\beta}_s < 0$ (except for $\hat{\beta}_s$ in experiment 2) are significant for experiments 1 and 2, and $\hat{\beta}_b < 0$, $\hat{\beta}_s > 0$ but the significance is less impressive in experiments 3 and 4. The pooled data help increase the size of n. The demographic squeeze is as described in the result. ■

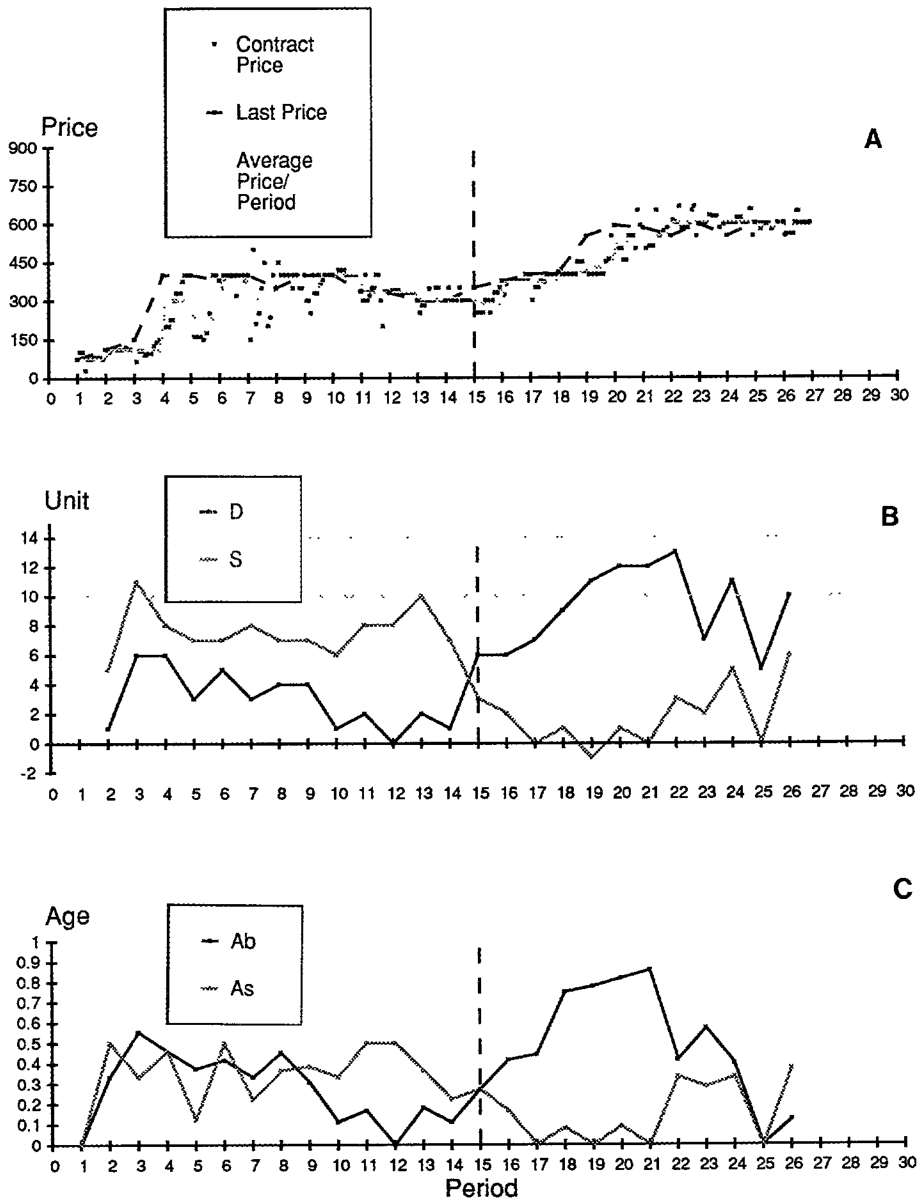

Fig. 10A–C. Prices, buyer and seller carry forward and average ages; experiment #1 (12/01/89); **A** Prices; **B** carried forward demand and supply; **C** average age of buyers and sellers

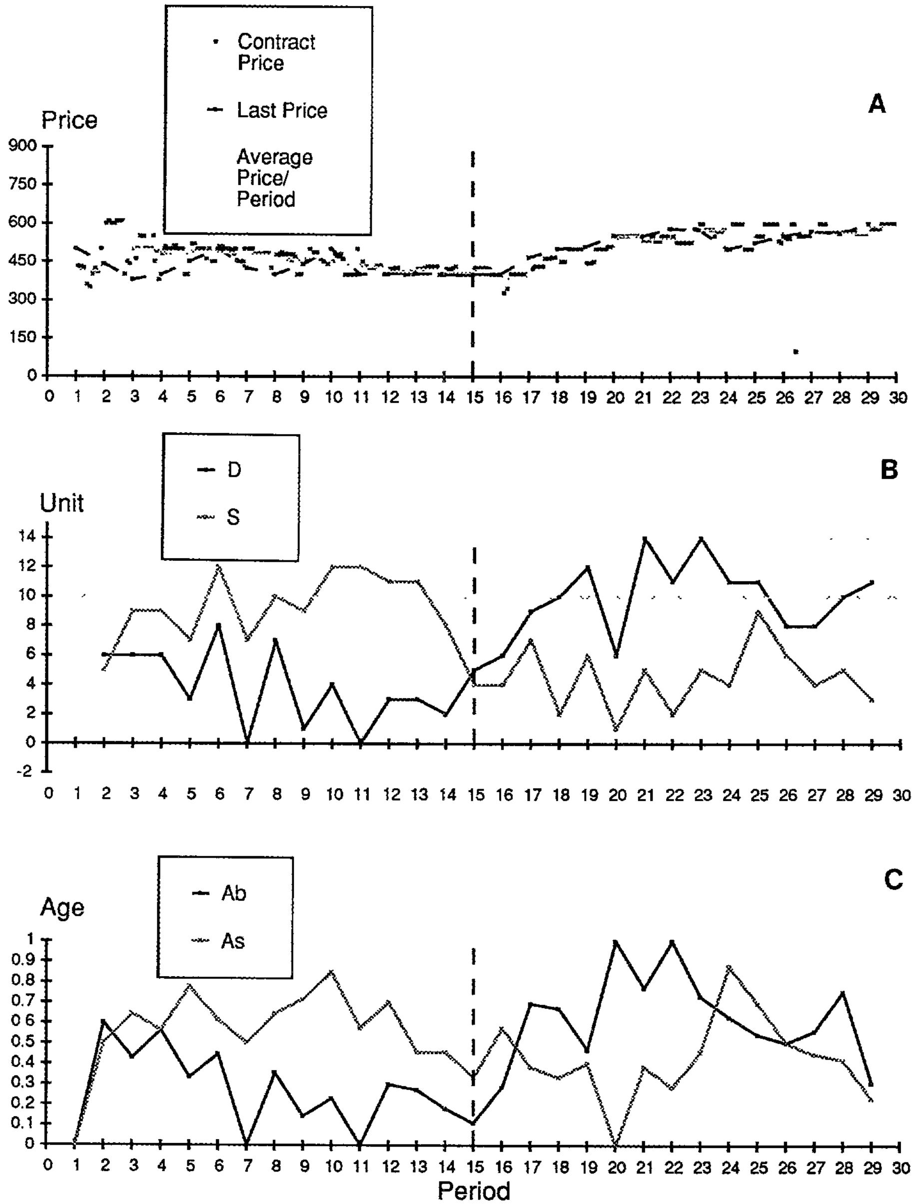

Fig. 11A–C. Prices, buyers and seller forward and average ages; experiment #2 (01/11/90). **A** Prices; **B** carried forward demand and supply; **C** average age of buyers and sellers

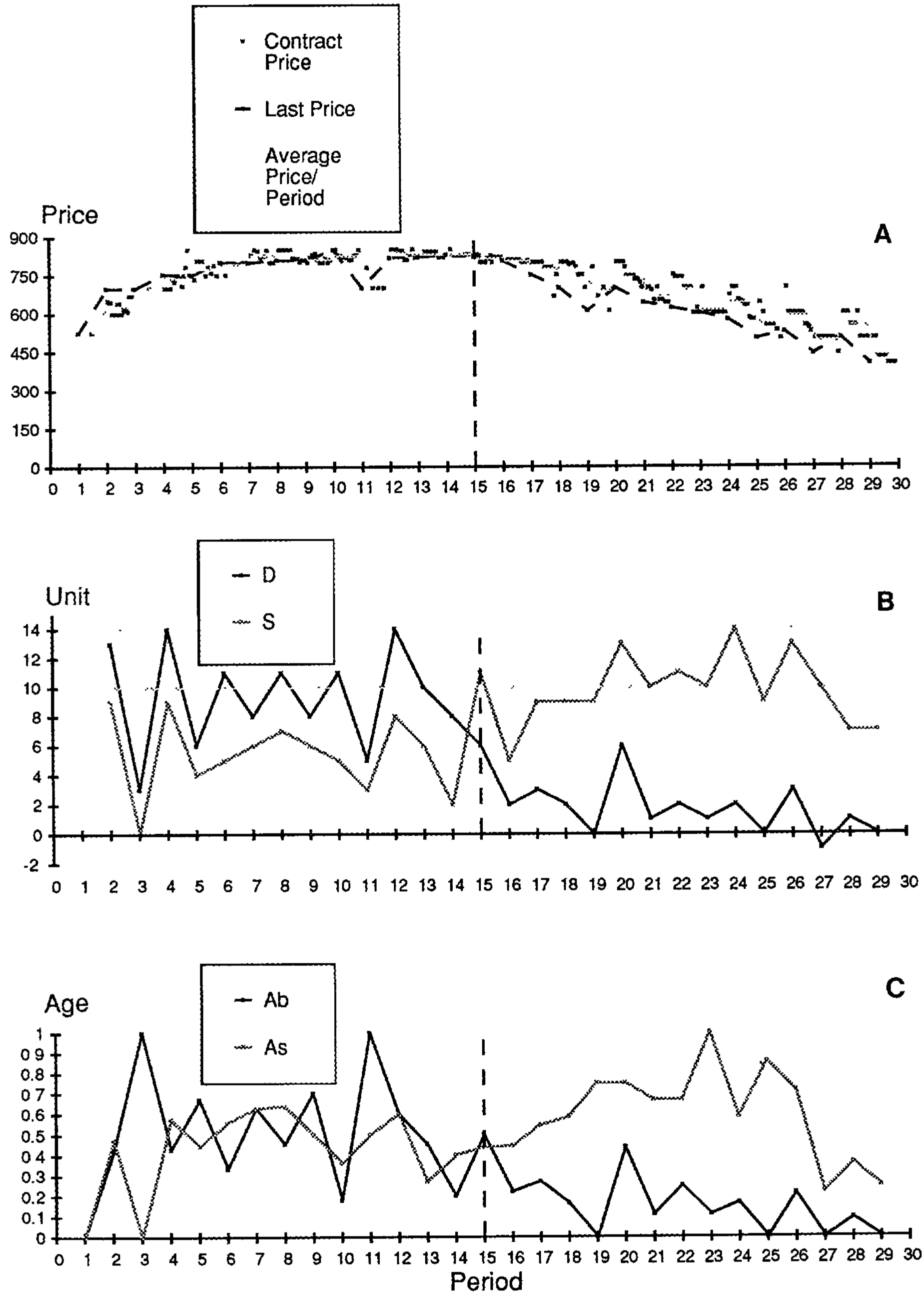

Fig. 12A–C. Prices, buyer and seller carry forward and average ages; experiment #3 (01/30/90). **A** Prices; **B** carried forward demand and supply; **C** average age of buyers and sellers

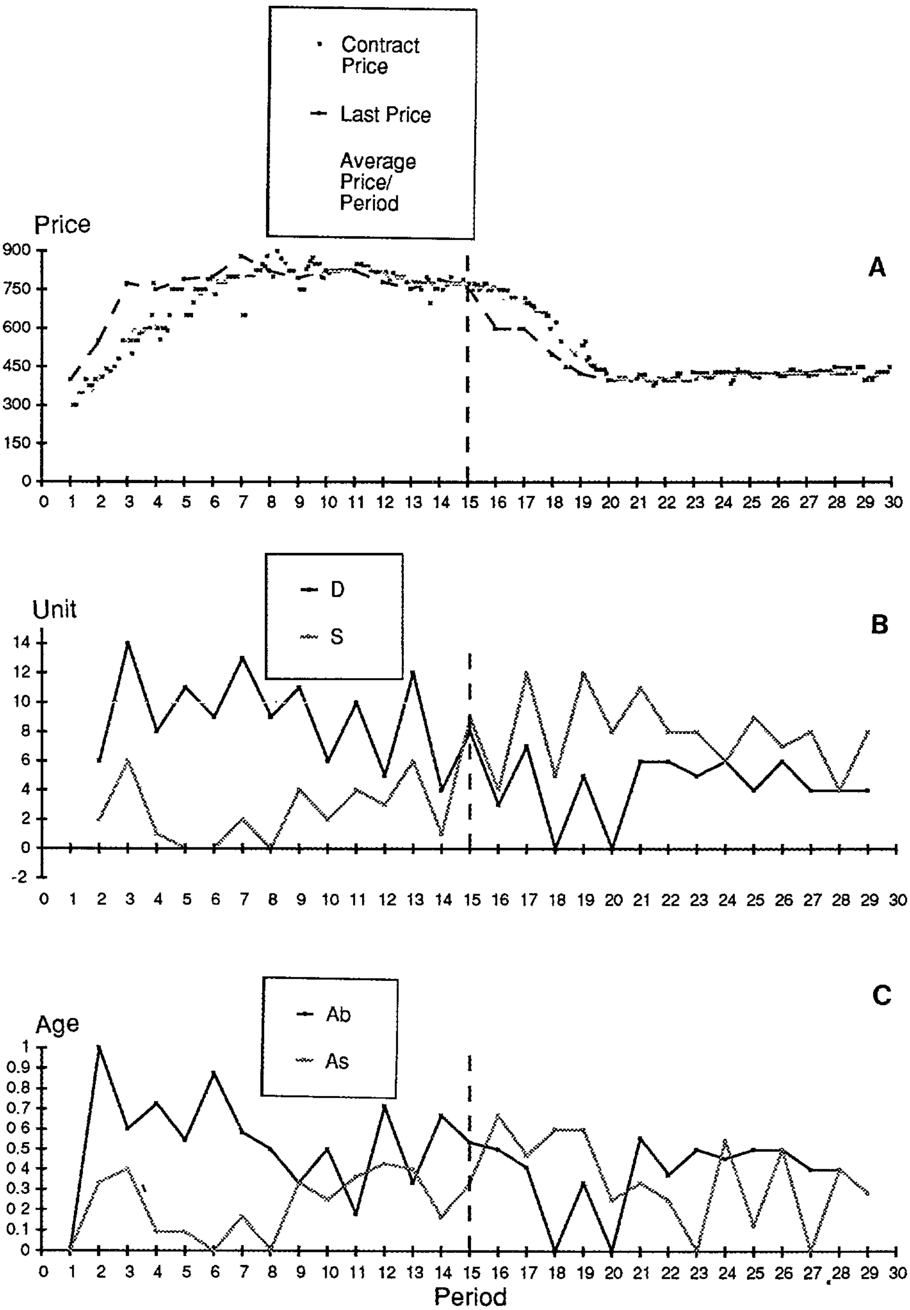

Fig. 13A–C. Prices, buyer and seller carry forward and average ages; experiment #4 (03/01/90). **A** Prices; **B** carried forward demand and supply; **C** average age of buyers and sellers

Table 7. $D^t - b_t^t = \alpha_b + \beta_b t + \varepsilon_t$ and $S^t - s_t^t = \alpha_s + \beta_s t + \varepsilon_t$

Experiment	α_b	β_b	R^2	DW	α_s	β_s	R^2	DW	n
1	−21 4 (−3.00)	1 70 (3.83)	0.83	2.43	26 6 (3 50)	−1.50 (−3.17)	0.77	1.94	5
2	25 6 (−7.96)	2 00 (10 00)	0.97	3.33	19 4 (1.66)	−0 90 (−1 24)	0.34	2.67	5
3	28 20 (3 83)	−1.50 (−3.27)	0.78	2.06	−12 00 (−0.66)	1.20 (1 06)	0.27	3.17	5
4	18.80 (1 11)	−0.90 (−0.86)	0.20	3.08	11 40 (−0 49)	−1.10 (0.76)	0.16	3.15	5
Pooled 1&2	23.5 (−6 37)	1.05 (8 05)	0.89	2.17	23.00 (2.91)	1.20 (−2.44)	0.43	1.54	10
Pooled 3&4	23 5 (2.88)	−1.20 (−2 36)	0.41	2.99	−11 70 (−0 91)	1.15 (1.43)	0.20	3.20	10

The demographic squeeze is closely related to another phenomenon that will be called the *generation gap*. Let the "age" of the young agents be 0 and the "age" of old agents be 1. With the zero and one measures of age given, the average age of a buyer of a unit in a given period t is $A_b = \frac{b_t^{t-1}}{b_t^{t-1}+b_t^t}$ and the average age of a seller of a unit is $A_s = \frac{s_t^{t-1}}{s_t^{t-1}+s_t^t}$. The **generation gap** G_g is the difference, i.e.,

$$G_g = A_b - A_s = \frac{b_t^{t-1}}{b_t^{t-1} + b_t^t} - \frac{s_t^{t-1}}{s_t^{t-1} + s_t^t}.$$

The variables A_b and A_s measure the aggressiveness with which an age group takes action. The generation gap appears if one age group is aggressive on one side of the market and the other age group is aggressive on the other. Since the maximum age is 1 and the minimum age is 0 the gap can range from -1 to 1. The next result summarizes a major dynamic property of the generation gap in these markets.

Result 8. *During the periods immediately after the parameter shift, a generation gap appears. The gap is positive when price is increasing (experiments 1 and 2) and it is negative when price is decreasing (experiments 3 and 4). Buyers (sellers) tend to be old (young) when prices are increasing and the opposite occurs when prices are decreasing.*

Support. The bottom panels in Figs. 10 through 13 contain graphs of the time series of A_b and A_s. In both experiments 1 and 2 the variable A_b increases and the variable A_s decreases after period 15. The opposite occurs in experiments 3 and 4.

In order to make the price relationships, the data from periods 14 through 18 were used to fit the regression $A_b - A_s = \alpha + \beta t + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$. The regression results are in Table 8. The increase in the generation gap in experiments 1 and 2 is captured by $\hat{\beta} > 0$ in both cases. The decrease in the generation gap in experiments 3 and 4 is captured by $\hat{\beta} < 0$ in both cases. ■

Table 8. OLS estimates for the equation $A_b - A_s = \alpha + \beta t + \varepsilon_t$

Experiment	α	β	R^2	DW	n
1	−2.95 (−15 30)	0.20 (16.66)	0.98	2.53	5
2	−2 82 (−2 98)	0 17 (2.96)	0.74	2.60	5
3	1.02 (1 40)	−7.69 (−1 69)	0.49	2.63	5
4	3 91 (4 92)	−0 25 (−4 97)	0.89	3.13	5
Pooled 1&2	−2 88 (−4 11)	0.19 (4.29)	0.70	1.05	10
Pooled 3&4	2 46 (3 22)	−0.16 (−3.38)	0.59	1.54	10

Results 7 and 8 together suggest a conjecture about the reason that instantaneous excess demand has a powerful relationship with price changes that was identified in Result 6. The conjecture is that agents' expectations are lagged during the initial periods after the parameter change. It is listed separately for clarity.

Conjecture. *Expectations begin by following a lagged property. After several periods of price movements the price movements tend to be accurately anticipated.*

The intuition for the formulation of the conjecture seems to be along the following lines. Agents treat the market like an impersonal random process. The markets operated for many periods of stationary parameters and the change was implemented in a manner that was substantially undetectable by a single individual. After the upward (for example) demand shift the prices experienced a natural upward pressure. Buyers thinking that the prices were temporarily high delayed purchases until they were old. Sellers, thinking the prices were a "good deal" rushed to sell while they were young. Thus, the demographic squeeze identified in Result 7 begins to occur. Once old, the buyers begin to find themselves in competition with the young generation of buyers for a supply that is only available from young sellers. Faced with the prospect of dying without having consumed or paying more to outbid the young competitors, they do the latter. Consequently, the generation gap identified in Result 8 can be identified. Since both the demographic squeeze and the generation gap disappear after several periods, the expectations may have become more forwardlooking with experience. The opposite dynamic occurs in experiments 3 and 4 in which prices decrease after the parameter shift.

8. Concluding remarks

Experimentalists have been unable to explore the rich implications of the competitive equilibrium model in overlapping generations environments because of practical complications and related computational difficulties. The practical difficulties stem from the infinite (long-run) features of standard models which are difficult to associate with the operational features of experiments. This paper helps reduce the problem by focusing on a special class of finite (short-run) environments. In these environments assumptions of stationarity help "tie down" the competitive equilibria at the ends of time paths. In addition, a single (opposing) shift of parameters occurs. The combination of the two sufficiently narrow the problem for computational techniques to be identified. Thus, a rich new class of environments is made accessible to the application of experimental methods.

Four experiments were conducted. The details of the experimental environments chosen reflect a decision to maintain as much continuity as is possible with non-overlapping generations experimental environments. The overlapping structure was the primary focus, so the other features of the markets that were created for experimental purposes were features that have been present in many other experiments. In particular, agents were not told the parameters of other agents nor were they at any time publically made aware of a potential state change. The existence of very little public information about parameters is thought to be favorable to the predictive accuracy of competitive models but more importantly the behavioral implications of public information about parameters has not been studied carefully. Complicating the market with both overlapping generations structure *and* public information about preferences was thought to be unwise.

A second feature that some might see as troublesome is the fact that agents were reincarnated. Again, this choice reflected both practical and continuity considerations. At the practical level without reincarnation the number of subjects needed would make the study impossible under existing laboratory conditions. In addition, a continuous introduction of new agents in each period could easily interject a type of "noise" typical of the first period of any experiment when some sort of learning/convergence takes place. The desire was to avoid such complications. The problem of learning/convergence might be overcome by giving agents longer lives, more than two generations. However, the theory has not been worked out for that case. Furthermore, as is implicit in the demographic squeeze result, convergence in a multigeneration environment might be very slow and thus necessitate experiments that are much longer than current laboratory technology and procedures can handle.

The major result of the paper is that a dynamic form of the law of supply and demand appears to be operative. Actual prices converge to the stationary equilibrium prices of the competitive model. A convergence process or some type of "disequilibrium" phenomenon found in non-overlapping markets exists in the overlapping case as well. In addition, the quantitative relationships among price changes on one hand and bids, asks and closing prices on the other hand, that are found in the non-overlapping case are also found existing in the overlapping case.

Some of the most interesting properties of the market dynamics exist in the demographics. The behavior of age dynamics strongly suggests the relevance of lagged expectations models although none were investigated here. Moreover, we conjecture that the lagged expectations yield to more forward looking expectations after a very few periods. However, the possibility of lagged expectations leads naturally to speculations about the possibility of observing cycles in other environments. Clearly, this is an interesting possibility to pursue.

These ideas about expectation phenomena are intuitive. This interest is then followed by a natural curiosity about the possible role that future markets (and other instruments that are known to have information content) have to play in changing those expectations. Of course, all of this can be explored experimentally as theory and time permit.

References

1. Aliprantis, C.D., Brown, D.J., Burkinshaw, O.: Equilibria in exchange economies with a countable number of agents. *J. Math. Anal Appl* **142**, 250–299 (1989)
2. Aliprantis, C.D., Brown, J.D., Burkinshaw, O.: *Existence and optimality of competitive equilibria.* Berlin, Heidelberg, New York: Springer 1989
3. Daniels, B.P., Plott, C.R.: Inflation and expectation in experimental markets. In: Tistg, R., Albers, W., Selton, R. (Eds.) *Bounded rational behavior in experimental games and markets*, pp. 198–218. Berlin, Heidelberg, New York: Springer 1988
4. Geanakoplos, J.: The overlapping generations model of general equilibrium. In: *The new Palgrave*, pp. 767–779. New York, London: Macmillan 1987
5. Johnson, A., Lee, H.Y., Plott, C.R.: Multiple unit double auction users manual. Social Science working paper #650, California Institute of Technology, June 1988
6. King, R.R., Smith, V.L., Williams, A.W.: The robustness of bubbles and crashes in experimental stock markets. University of Arizona working paper, 1989
7. Lim, S., Prescott, E.C., Sunder, S.: A stationary solution to the overlapping generations model of fiat money: experimental evidence. University of Minnesota working paper, 1988
8. Marimon, R., Sunder, S.: Rational expectations vs adaptive evidence in a hyperinflationary world: experimental evidence. University of Minnesota working paper, July, 1988
9. Marimon, R., Sunder, S : Indeterminacy of equilibria in a hyperinflationary world: Experimental evidence. Carnegie-Mellon University working paper, January, 1990
10. Plott, C.R : Rational choice in experimental markets. *J. Business* **59**, 301–327 (1986)
11. Plott, C.R.: An updated review of industrial organization: applications of experimental methods. In: Schmalensee, R., Willig, R.D. (Eds.) *Handbook of industrial organization, Vol. II*, pp. 1009–1176. New York: Elsevier Science Publishers 1989
12. Plott, C.R., George, G.: Marshallian vs Walrasian stability in an experimental market. Social Science working paper #683. California Institute of Technology, October, 1988
13. Plott, C.R., Gray, P.: The multiple unit double auction. *J Econ. Behav. Organiz.* **13**, 245–258 (1990)
14. Samuelson, P.A.: An exact consumption-loan model of interest with or without the social contrivance of money. *J. Polit Econ.* **66**, 467–482 (1958)
15. Smith, V.L., Suchanek, G., Williams, A.W.: Bubbles, crashes and endogenous expectations in experimental spot asset markets. *Econometrica* **56**, 1119–1151 (1988)
16. Wilson, C.A.: Equilibrium in dynamic models with an infinity of agents. *J. Econ. Theory* **24**, 95–111 (1981)

Appendix

Existence and uniqueness of equilibria

The objective of this section is to establish that every short-run OLG model has (essentially) a unique sequence of competitive equilibrium prices. This is a basic result for our work. The proof will be presented by establishing several lemmas and theorems that are of some independent interest in their own right.

Lemma 1. *Let $(p_1, p_2, \ldots)$ be a sequence of equilibrium prices in an OLG model and assume that agents in two consecutive periods t and $t+1$ have identical demand and supply curves, say as shown in Fig. 4A.*

1. *If $p_t > p_{t+1}$, then $p_{t+1} \geq p_0$; and*
2. *If $p_t < p_{t+1}$, then $p_{t+1} \leq p_0$.*

Proof . We shall prove (1) and leave the identical arguments of (2) for the reader. So, assume $p_t > p_{t+1}$. For simplicity, let us write $D_t(p) = D_{t+1}(p) = D(p)$ and $S_t(p) = S_{t+1}(p) = S(p)$.

Start by observing that the buyers' maximization behavior implies

$$b_t^t = 0 \quad \text{and} \quad b_{t+1}^t = D(p_{t+1}).$$

Similarly, the sellers' maximization actions yield

$$s_{t+1}^t = 0 \quad \text{and} \quad s_t^t = S(p_t).$$

Therefore, the clearance condition at $t+1$ ($b_{t+1}^t + b_{t+1}^{t+1} = s_{t+1}^t + s_{t+1}^{t+1}$) becomes

$$D(p_{t+1}) + b_{t+1}^{t+1} = s_{t+1}^{t+1}.$$

From the inequalities $D(p_{t+1}) \le D(p_{t+1}) + b_{t+1}^{t+1}$ and $s_{t+1}^{t+1} \le S\big(\max\{p_{t+1}, p_{t+2}\}\big)$, we obtain

$$D(p_{t+1}) \le s_{t+1}^{t+1} \le S\big(\max\{p_{t+1}, p_{t+2}\}\big). \tag{1}$$

Next, notice that $p_{t+2} > p_{t+1}$ cannot hold. Indeed, if $p_{t+2} > p_{t+1}$ is true, then we must have $s_{t+1}^{t+1} = 0$. Consequently, from (1), we see that $D(p_{t+1}) \le s_{t+1}^{t+1} = 0$, which is a contradiction. Hence, $p_{t+2} \le p_{t+1}$. The latter conclusion implies $S\big(\max\{p_{t+1}, p_{t+2}\}\big) = S(p_{t+1})$, and so from (1), we infer that $D(p_{t+1}) \le S(p_{t+1})$, or $D(p_{t+1}) - S(p_{t+1}) > 0$. Now a glance at Fig. 4A shows that $p_{t+1} \ge p_0$ must hold, and the proof is finished. ■

The next lemma informs us that if in a sequence of equilibrium prices two consecutive prices are equal and the demand and supply curves remain identical, then subsequent prices do not change.

Lemma 2. *Let $(p_1, p_2, \ldots)$ be a sequence of equilibrium prices in an OLG model and assume that agents in three consecutive periods $t-1$, t, and $t+1$ have identical demand and supply curves, say as shown in Fig. 4A. If $p_{t-1} = p_t$, then $p_{t+1} = p_t$.*

Proof . Let $D(p)$ and $S(p)$ denote the common demand and supply curves, and assume by way of contradiction that $p_t > p_{t+1}$. Consequently by part (1) of Lemma 1, we get $p_{t+1} \ge p_0$, and so

$$p_t > p_0. \tag{2}$$

The profit maximization assumptions yield

$$b_t^t = 0 \quad \text{and} \quad s_{t+1}^t = 0,$$

and so $s_t^t = s_t^t + s_{t+1}^t = S\big(\max\{p_t, p_{t+1}\}\big) = S(p_t)$. Therefore, the clearance condition at t ($b_t^{t-1} + b_t^t = s_t^{t-1} + s_t^t$) becomes

$$b_t^{t-1} = s_t^{t-1} + S(p_t).$$

Now from the inequalities

$$S(p_t) \le s_t^{t-1} + S(p_t) = b_t^{t-1} \le b_t^{t-1} + b_{t-1}^{t-1} = D\big(\min\{p_{t-1}, p_t\}\big) = D(p_t),$$

we see that $D(p_t) - S(p_t) \ge 0$, which implies (by a glance at Fig. 4A) $p_t \le p_0$. However, the latter conclusion contradicts (2), and so the inequality $p_t > p_{t+1}$

cannot happen. Similarly, $p_{t+1} > p_t$ is false, and so $p_{t+1} = p_t$ must be true, as claimed. ■

The next lemma tells us that if the demand and supply curves in three consecutive periods are the same, then in these periods three consecutive decreases (or increases) in prices cannot occur.

Lemma 3. *Let $(p_1, p_2, \ldots)$ be a sequence of equilibrium prices in an OLG model and assume that agents in three consecutive periods t, $t+1$, and $t+2$ have identical demand and supply curves, say as shown in Fig. 4A. Then the string of inequalities $p_t > p_{t+1} > p_{t+2}$ (or $p_t < p_{t+1} < p_{t+2}$) cannot happen.*

Proof. Assume $p_t > p_{t+1} > p_{t+2}$. By Lemma 1, we know that $p_{t+2} \geq p_0$. Therefore,

$$p_{t+1} > p_0 \,. \tag{3}$$

The profit maximization assumptions imply

$$b_t^t = b_{t+1}^{t+1} = 0 \quad \text{and} \quad s_{t+1}^t = s_{t+2}^{t+1} = 0 \,.$$

Consequently,

$$b_{t+1}^t = b_{t+1}^t + b_t^t = D\big(\min\{p_t, p_{t+1}\}\big) = D(p_{t+1})$$

and

$$s_{t+1}^{t+1} = s_{t+1}^{t+1} + s_{t+2}^{t+1} = S\big(\max\{p_{t+1}, p_{t+2}\}\big) = S(p_{t+1}) \,.$$

Now the clearance condition at $t+1$ ($b_{t+1}^t + b_{t+1}^{t+1} = s_{t+1}^t + s_{t+1}^{t+1}$) becomes $D(p_{t+1}) = S(p_{t+1})$, from which it follows that $p_{t+1} = p_0$, contrary to (3). The "increasing string" can be established in a similar fashion, and the proof of the lemma is complete. ■

Lemma 4. *If in an OLG model the new agents in $r+2$ consecutive periods $\ell, \ell+1, \ldots, \ell+r, \ell+r+1$ have identical demand and supply functions (as in Fig. 4A), then every sequence $(p_1, p_2, \ldots)$ of equilibrium prices satisfies*

$$p_{\ell+1} = p_{\ell+2} = \cdots = p_{\ell+r} = p_{\ell+r+1} \,.$$

Moreover, if p is this constant value, then

$$\big|D(p) - S(p)\big| \leq \tfrac{\max\{D(p), S(p)\}}{r} \,.$$

In particular, when D and S are bounded, $\lim_{r\to\infty} p = p_0$ (and so if the new agents have identical demand and supply functions at each period $t \geq \ell$, then $p = p_0$).

Proof. If $p_\ell = p_{\ell+1}$, then (by Lemma 2) it follows that

$$p_\ell = p_{\ell+1} = \cdots = p_{\ell+r} = p_{\ell+r+1} \,.$$

So, assume that $p_\ell \neq p_{\ell+1}$. Without less of generality, we can suppose that $p_\ell > p_{\ell+1}$. By Lemma 1, we see that

$$p_\ell > p_{\ell+1} \geq p_0 \,.$$

Now (by Lemma 3) the inequality $p_{\ell+1} > p_{\ell+2}$ cannot happen. On the other hand, if $p_{\ell+1} < p_{\ell+2}$, then (by Lemma 1) we infer that $p_{\ell+1} < p_{\ell+2} \le p_0$, which contradicts $p_{\ell+1} \ge p_0$. Hence, $p_{\ell+1} = p_{\ell+2}$ must hold, and the conclusion follows from Lemma 2.

Next, note that the agents of generations $\ell+1, \ldots, \ell+r$ have a net total of excess demand $r|D(p) - S(p)|$. This net total excess demand cannot exceed the total demand or supply provided by the new agents of generations ℓ and $\ell+r+1$; a maximum total of $2\max\{D(p), S(p)\}$. So, we have $r|D(p) - S(p)| \le 2\max\{D(p), S(p)\}$, or

$$\left|D(p) - S(p)\right| \le \tfrac{2\max\{D(p), S(p)\}}{r},$$

as claimed, and the proof of the lemma is finished. ■

Lemma 5. *Assume that in periods $\ell, \ell+1, \ldots, \ell+r-1$ of an OLG model the new agents have identical demand and supply functions $D(p)$ and $S(p)$ and the new agents of generations $\ell+r, \ell+r+1, \ldots, \ell+r+s$ have identical demand and supply functions $D^*(p)$ and $S^*(p)$ that satisfy the opposing shift property (as shown in Fig. 1). Also, let $(p_1, p_2, \ldots)$ be a sequence of equilibrium prices such that the constant prices*

$$p = p_{\ell+1} = \cdots = p_{\ell+r-1} \quad \textit{and} \quad p^* = p_{\ell+r+1} = \cdots = p_{\ell+r+s}$$

satisfy $p < p^*$. Then, we have

$$p_0 \le p\,.$$

Proof. For simplicity, we shall denote the switch period by k, i.e., $k = \ell+r$. Assume by way of contradiction that

$$p < p_0\,. \tag{4}$$

We distinguish four cases.

Case I: $p_k < p_{k-1} < p_{k+1} = p^*$.

In this case, notice that $s_k^{k-1} = 0$, $s_k^k = 0$, $b_k^{k-1} = D(p_k)$, and $b_k^k = D^*(p_k)$. So, the clearance condition at period k yields

$$D(p_k) + D^*(p_k) = 0 + 0 = 0\,,$$

which is absurd.

Case II: $p = p_{k-1} = p_k < p_{k+1}$.

In this case, let us consider the equation of demand and supply from periods $\ell+1$ up to (and including) period k. To do this, let D_ℓ (resp. S_ℓ) denote the demand (resp. the supply) by the young buyers (resp. the young sellers) of generation ℓ that is transferred to generation $\ell+1$. Using our hypothesis and an easy calculation, we see that

$$D_\ell + \sum_{i=\ell+1}^{k-1} D(p) + D^*(p) = S_\ell + \sum_{i=\ell+1}^{k-1} S(p)\,.$$

Now from $D(p) \le D^*(p)$ and $S_\ell \le S(p)$, we see that

$$\sum_{i=\ell+1}^{k} D(p) \le D_\ell + \sum_{i=\ell+1}^{k-1} D(p) + D^*(p) \quad \text{and} \quad S_\ell + \sum_{i=\ell+1}^{k-1} S(p) \le \sum_{i=\ell+1}^{k} S(p)\,.$$

So, $\sum_{i=\ell+1}^{k} D(p) \le \sum_{i=\ell+1}^{k} S(p)$, or $D(p) \le S(p)$. The latter implies $p \ge p_0$, which contradicts (4).

Case III: $p = p_{k-1} < p_k < p_{k+1}$.

Again, the equation of demand and supply from periods $\ell + 1$ up to (and including) period $k-1$ yields

$$D_\ell + \sum_{i=\ell+1}^{k-1} D(p) = S_\ell + \sum_{i=\ell+1}^{k-2} S(p) \le \sum_{i=\ell+1}^{k-1} S(p).$$

It follows that $\sum_{i=\ell+1}^{k-1} D(p) \le \sum_{i=\ell+1}^{k-1} S(p)$. Hence, $D(p) \le S(p)$, and so $p \ge p_0$, contrary to (4).

Case IV: $p = p_{k-1} < p_{k+1} \le p_k$.

Once more, the equation of demand and supply from periods $\ell + 1$ up to (and including) period $k-1$ yields

$$D_\ell + \sum_{i=\ell+1}^{k-1} D(p) = S_\ell + \sum_{i=\ell+1}^{k-2} S(p),$$

from which it follows again that $p \ge p_0$, a contradiction. ■

And now we are ready to state the most important properties of the sequences of equilibria prices for the short-run OLG models.

Theorem 6. *Assume that in periods $1, 2, \ldots, k-1$ of a short-run OLG model the new agents have identical demand and supply functions $D(p)$ and $S(p)$ and the new agents of generations $k, k+1, \ldots, k+m$ have identical demand and supply functions $D^*(p)$ and $S^*(p)$ that satisfy the opposing shift property (as shown in Fig. 1). Let $(p_1, \ldots, p_k, \ldots, p_{k+m})$ be a (finite) sequence of equilibrium prices. We choose the parameters k and m large enough so that the constant prices*

$$p = p_1 = \cdots = p_{k-1} \quad \textit{and} \quad p^* = p_{k+1} = \cdots = p_{k+m}$$

satisfy $p < p^*$. Then, we have

$$p_0 \le p < p^* \le p_0^*.$$

Proof. By Lemma 5, we already know that $p_0 \le p$. To establish the other inequality, assume by way of contradiction that

$$p_0^* < p^*. \tag{5}$$

Here, we distinguish three cases.

Case I: $p_k < p_{k-1} < p_{k+1} = p^*$.

In this case, notice that $s_k^{k-1} = 0$, $s_k^k = 0$, $b_k^{k-1} = D(p_k)$, and $b_k^k = D^*(p_k)$. So, the clearance condition at period k yields

$$D(p_k) + D^*(p_k) = 0 + 0 = 0,$$

which is absurd.

Case II: $p = p_{k-1} \leq p_k < p_{k+1} = p^*$.

In this case, the equation of demand and supply from periods $k+1$ up to (and including) period $k+m$ becomes

$$\sum_{i=k+1}^{k+m} D^*(p^*) = \sum_{i=k}^{k+m} S^*(p^*),$$

or $(m-1)D^*(p^*) = mS^*(p^*) \geq (m-1)S^*(p^*)$. It follows that $D^*(p^*) \geq S^*(p^*)$, from which it follows that $p^* \leq p_0^*$, which contradicts (5).

Case III: $p = p_{k-1} < p^* = p_{k+1} \leq p_k$.

Once more, the equation of demand and supply from periods k up to (and including) period $k+m$ yields

$$\sum_{i=k}^{k+m} D^*(p^*) = S(p_k) + S^*(p_k) + \sum_{i=k+1}^{k+m} S^*(p^*).$$

Taking into account that $S^*(p_k) \geq S^*(p^*)$, we see that

$$\sum_{i=k}^{k+m} D^*(p^*) \geq S^*(p^*) + \sum_{i=k+1}^{k+m} S^*(p^*) = \sum_{i=k}^{k+m} S^*(p^*),$$

and so $D^*(p^*) \geq S^*(p^*)$, which implies $p^* \leq p_0^*$, contrary to (5). ■

The next theorem allows us to compute the path of equilibrium prices and establishes their uniqueness.

Theorem 7. (Uniqueness of Equilibrium) *Assume that in periods $1, 2, \ldots, k-1$ of a short-run OLG model the new agents have identical demand and supply functions $D(p)$ and $S(p)$ and the new agents of generations $k, k+1, \ldots, k+m$ have identical demand and supply functions $D^*(p)$ and $S^*(p)$ that satisfy the opposing shift property (as shown in Fig. 1). Let $(p_1, \ldots, p_k, \ldots, p_{k+m})$ be a (finite) sequence of equilibrium prices. We choose the parameters k and m large enough so that the constant prices*

$$p = p_1 = \cdots = p_{k-1} \quad \textit{and} \quad p^* = p_{k+1} = \cdots = p_{k+m}$$

satisfy $p < p^*$. Then:

1. The price p_k satisfies $p = p_{k-1} \leq p_k \leq p_{k+1} = p^*$.

2. If $p_{k-1} < p_k = p_{k+1}$, then the prices p and p^* satisfy the system of equations

$$\tfrac{D(p)}{S(p)} = \tfrac{k-2}{k-1} \quad \textit{and} \quad mD^*(p^*) = S(p^*) + mS^*(p^*).$$

3. If $p_{k-1} = p_k < p_{k+1}$, then the prices p and p^* satisfy the system of equations

$$\tfrac{D^*(p^*)}{S^*(p^*)} = \tfrac{m+1}{m} \quad \textit{and} \quad (k-1)D(p) + D^*(p^*) = (k-1)S(p).$$

4. If $p_{k-1} < p_k < p_{k+1}$, then the prices p_k, p, and p^* satisfy the system of equations

$$D^*(p_k) = S(p_k), \quad \tfrac{D(p)}{S(p)} = \tfrac{k-2}{k-1}, \quad \textit{and} \quad \tfrac{D^*(p^*)}{S^*(p^*)} = \tfrac{m}{m-1}.$$

In this case, the price p_k is the price q shown in Fig. 1.

Proof . (1) If $p_k < p_{k-1} < p_{k+1}$ holds true, then $b_k^{k-1} + b_k^k = s_k^{k-1} + s_k^k$ (the clearance condition at period k) yields

$$D(p_k) + D^*(p_k) = 0 + 0 = 0\,,$$

which is absurd. Similarly, if $p_{k-1} < p_{k+1} < p_k$, then the clearance condition at period k yields

$$0 + 0 = S(p_k) + S^*(p_k)\,,$$

which is again absurd. So, $p_{k-1} \leq p_k \leq p_{k+1}$ holds true.

(2) Assume $p = p_{k-1} < p_k = p_{k+1} = p^*$. The equality of demand and supply from period 1 up to (and including) period $k-1$ yields $\sum_{i=1}^{k-1} D(p) = \sum_{i=1}^{k-2} S(p)$, or $(k-1)D(p) = (k-2)S(p)$. Similarly, the equality of demand and supply from period k up to period $k+m$ yields $\sum_{i=k}^{k+m} D^*(p^*) = S(p^*) + \sum_{i=k}^{k+m} S^*(p^*)$, or $mD^*(p^*) = S(p^*) + mS^*(p^*)$.

(3) Assume $p = p_{k-1} = p_k < p_{k+1} = p^*$. The equality of demand and supply from period 1 up to (and including) period k yields $\sum_{i=1}^{k-1} D(p) + D^*(p) = \sum_{i=1}^{k-1} S(p)$, or $(k-1)D(p) + D^*(p) = (k-1)S(p)$. Similarly, the equality of demand and supply from period $k+1$ up to period $k+m$ yields $\sum_{i=k+1}^{k+m} D^*(p^*) = S(p^*) + \sum_{i=k+1}^{k+m} S^*(p^*)$, or $mD^*(p^*) = (m+1)S^*(p^*)$.

(4) Assume $p = p_{k-1} < p_k < p_{k+1} = p^*$. The clearance condition $b_k^{k-1} + b_k^k = s_k^{k-1} + s_k^k$ at period k yields $0 + D^*(p_k) = S(p_k) + 0$, or $D^*(p_k) = S(p_k)$. The other equalities follow in a similar manner as in the preceding cases. ■

The final result of this section demonstrates the existence of a sequence of competitive equilibrium prices in our short-run OLG model. We shall establish this result for one case only. (The identical arguments for the other cases are left for the reader.)

Theorem 8. (Existence of Equilibrium) *Assume that in periods $1, 2, \ldots, k-1$ of a short-run OLG model the new agents have identical demand and supply functions $D(p)$ and $S(p)$ and the new agents of generations $k, k+1, \ldots, k+m$ have identical demand and supply functions $D^*(p)$ and $S^*(p)$ that satisfy the opposing shift property (as shown in Fig. 1). Let $(p_1, \ldots, p_k, \ldots, p_{k+m})$ be a (finite) sequence of equilibrium prices. We choose the parameters k and m large enough so that the constant prices*

$$p = p_1 = \cdots = p_{k-1} \quad \textit{and} \quad p^* = p_{k+1} = \cdots = p_{k+m}$$

satisfy $p < p^*$. If the prices p, p_k and p^* satisfy the system of equations

$$D^*(p_k) = S(p_k), \quad \tfrac{D(p)}{S(p)} = \tfrac{k-2}{k-1}, \quad \textit{and} \quad \tfrac{D^*(p^*)}{S^*(p^*)} = \tfrac{m}{m-1}\,,$$

then the sequence of prices $(p_1, \ldots, p_{k-1}, p_k, p_{k+1}, \ldots, p_{k+m})$ can be supported by trades as an equilibrium sequence of prices.

Proof . For simplicity, let us write $S = S(p)$, $D = D(p)$, $S^* = S^*(p^*)$, and $D^* = D^*(p^*)$. Then an easy computation shows that the sequence of trade allocations

$$T_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} (k-2)(S-D) & (k-2)D-(k-3)S \\ S & 0 \end{bmatrix},$$
$$T_3 = \begin{bmatrix} (k-3)(S-D) & (k-3)D-(k-4)S \\ S & 0 \end{bmatrix}, \ldots, T_{k-2} = \begin{bmatrix} 2(S-D) & 2D-S \\ S & 0 \end{bmatrix},$$
$$T_{k-1} = \begin{bmatrix} S-D & D \\ S & 0 \end{bmatrix}, \quad T_k = \begin{bmatrix} 0 & D^*(p_k) \\ S(p_k) & 0 \end{bmatrix}, \quad T_{k+1} = \begin{bmatrix} 0 & D^* \\ S^* & D^*-S^* \end{bmatrix},$$
$$T_{k+2} = \begin{bmatrix} 0 & D^* \\ 2S^*-D^* & 2(D^*-S^*) \end{bmatrix}, \ldots, T_{k+m-1} = \begin{bmatrix} 0 & D^* \\ (m-1)S^*-(m-2)D^* & (m-1)(D^*-S^*) \end{bmatrix},$$
$$T_{k+m} = \begin{bmatrix} 0 & D^* \\ mS^*-(m-1)D^* & m(D^*-S^*) \end{bmatrix},$$

is supported by the sequence of prices $(p_1, \ldots, p_{k-1}, p_k, p_{k+1}, \ldots, p_{k+m})$.

Notice that the conditions $\frac{D(p)}{S(p)} = \frac{k-2}{k-1}$ and $\frac{D^*(p^*)}{S^*(p^*)} = \frac{m}{m-1}$ imply

$$(k-2)(S-D) = D, \quad (m-1)(D^*-S^*) = S^*, \quad \text{and } m(D^*-S^*) = D^*.$$

Consequently, if we choose some $\varepsilon > 0$ such that $\varepsilon < \min\{D, D^*, S, S^*\}$, then replacing T_1, T_2, T_{k+m-1}, and T_{k+m} by

$$T_1 = \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}, \quad T_2 = \begin{bmatrix} D-\varepsilon & (k-2)D-(k-3)S \\ S-\varepsilon & 0 \end{bmatrix},$$
$$T_{k+m-1} = \begin{bmatrix} 0 & D^*-\varepsilon \\ (m-1)S^*-(m-2)D^* & S^*-\varepsilon \end{bmatrix}, \text{ and } T_{k+m} = \begin{bmatrix} \varepsilon & D^*-\varepsilon \\ \varepsilon & D^*-\varepsilon \end{bmatrix},$$

we get another trade allocation. The sequence of prices $(p_1, \ldots, p_{k-1}, p_k, p_{k+1}, \ldots, p_{k+m})$ supports this allocation. ■

This article was processed by the author
using the Springer-Verlag TEX EconThe macro package 1991.