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On the Behavioral Foundations of the Law of Supply and Demand: Human Convergence and Robot Randomness*

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Abstract

This research builds on the work of D.K. Gode and Shyam Sunder who demonstrated the existence of a strong relationship between market institutions and the ability of markets to seek equilibrium—even when the agents themselves have limited intelligence and behave with substantial randomness. The question posed is whether or not market institutions account for the operation of the law of supply and demand in markets populated by humans with no role required of human rationality. Are institutions responsible for the operations of the law of supply and demand or are behavioral principles also at work? Experiments with humans and simulations with robots both conducted in conditions in which major institutional and structural aids to convergence were removed, produced clear answers. Human markets converge, while robot markets do not. The structural and institutional features certainly facilitate convergence under conditions of substantial irrationality, but they are not necessary for convergence in markets in which agents have the rationality of humans.

Keywords: experiments, rationality, equilibration, robots

JEL Classification: C9, D5

1. Introduction

In a seminal series of papers,[1] Gode and Sunder (1993b, 1997a, 1997b) have explored the relationship between limited rationality, market institutions and the general equilibration

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of markets to the competitive equilibrium. Their fundamental discovery is that within the classical double auction market institution only the weakest elements of rationality need to be present for markets to exhibit high allocative efficiency and price convergence. While Gode and Sunder place more emphasis on allocative efficiency[2] than on price convergence, the apparent price convergence increases the agreement between their simulation results and observed price convergence in single isolated periods of double auction markets with humans.[3] Their 'Zero Intelligence' [ZI] agents, governed by completely random choice and constrained only by a budget constraint, are coordinated by market forces to the competitive equilibrium. The results are closely related to the results by Becker (1962) that the budget constraint alone, in the presence of randomly behaving agents, assures that demand curves will be downward sloping.

Such results stimulate natural questions about the foundations of economics and the most fundamental laws of supply and demand. Is no more intelligence necessary for the aggregate operation of these laws than is present in agents whose individual behavior is limited only by their budget constraint? Is nothing else implied by the consistency or the price convergence observed in economic experiments? Is only the randomness of individual behavior responsible for the law of supply and demand or are deeper principles of behavior in operation? The experiments reported in this paper are designed to explore these questions.

While the questions above are motivated by the work of Gode and Sunder, they are not the questions that Gode and Sunder posed. Our questions are different. Gode and Sunder were keenly aware of the relationship between market institutions and individual rationality and their experiments exposed that relationship. Within their environment market dynamics tend to follow a path that leads trading prices to the competitive equilibrium price. This particular type of dynamics was first postulated by Alfred Marshall and later incorporated into theories of the dynamic behavior of the double auction (e.g., Easley and Ledyard, 1993). This property receives even greater emphasis in Cason and Friedman (1993). The sequence of trades along the Marshallian path is a particular pairing of traders such that the last trade is necessarily at the equilibrium. It is easy to see that the nature of the Marshallian dynamic, the non-tatonnement Marshallian path, is operating the Gode and Sunder framework. Since this dynamic is operating, convergence to the competitive equilibrium and the predictions of the law of supply and demand necessarily follow.

The issue of path has certainly not been lost to economic theory. Indeed, Walras invented the concept of tatonnement to illustrate how convergence to equilibrium could occur without the intervening influences of a trading path. The fear was that disequilibrium trades could change the equilibrium. Similarly, early experiments (Chamberlin, 1948[4] and Smith, 1962[5]) were clearly concerned about the possibility that trades, especially trades involving extra-marginal units, could change the equilibrium by shifting the intersection of what they call the "moving" supply and demand. Chamberlin (1948) argued that one consequence of such a moving (or *instantaneous*) model is that predictions of price based on initial demand and supply would be impossible without also having knowledge of the trading path. Smith (1962), while aware of the moving model, discovered that initial supply and demand is sufficient to characterize the equilibrium market prices that eventually occur. Because his (and later) experiments involve a series of market periods where the initial supply and demand conditions are repeated, the market can be thought of as "learning" or involving "price discovery" over time, with the trading path adjusting to one which is more and more consistent with the single price equilibrium predictions of initial supply and demand. In contrast, Gode and Sunder, through numerical simulations, suggest that the predictions of initial supply and demand might be accurate only because of the nature of probable trading paths induced by random behavior within the rules of market institutions.

In summary, the conjunction of Gode and Sunder together with the Ledyard and Easley model of the convergence path suggest that the accuracy of the demand and supply model, as observed first by Smith (1962), is due primarily to the tendency for the Marshallian path to emerge. Clearly the Marshallian path is sufficient for convergence. The question posed here is whether or not it is also necessary. The experiments reported in this paper explore markets in which path in the traditional sense plays no role even though the system is non-tatonnement, and asks if demand and supply still have predictive power.

The approach taken here is to design an experimental environment that works against the Marshallian dynamics. We will ask on the one hand if markets populated by humans converge to the competitive equilibrium predicted by the law of supply and demand within the new environment. On the other hand, we will ask if robots with limited intelligence will exhibit the convergence process. The answer to the first question will be yes. Humans will converge. The answer to the second question will be no. The low intelligence robots will not converge. When the path for convergence identified by Gode and Sunder is not operative and when the market contains humans, the law of supply and demand will be observed. Thus, humans bring some crucial feature to the convergence process that is not present in the Gode and Sunder ZI robots. Of course, what elements of human behavior are necessary for this convergence process is an open question.

In posing these questions, we will invent a new framework and environment for the study of markets. In contrast to standard laboratory environments involving fixed supply and demand, these markets are characterized by continuously refreshed supply and demand (CRSD). The environment we study bears some superficial similarity to the steady-state decentralized bargaining markets studied by Rubinstein and Wolinsky (1985, 1990), however, we maintain crucial details of the double-auction price discovery mechanism in our framework and appeal to notions of competitive equilibrium in our models, features abandoned by Rubinstein and Wolinsky—who instead studied a combination of matching and two-person bargaining.[6] Both CRSD and the R-W Steady-state markets are characterized by supply and demand that is replenished as agents trade. We maintain that at least two different representations of the law of supply and demand are possible in this new environment—an *Instantaneous* representation and a *Velocity-based* representation. Natural opportunities for exploring these new representations of the law of supply and demand will arise in the process of reevaluating the contribution of human rationality towards determining market equilibrium.

The remainder of this paper is organized as follows. Section 2 will introduce some notions of Marshallian path dynamics and its importance for price convergence in previous double auction experiments. These ideas apply regardless of whether a market is populated by ZI robots or humans. In Section 3, we will construct an environment that works against the operation of the Marshallian path, if indeed a Marshallian path can even be defined in this environment. In Section 4, we will explore models that might be appropriate for the new environment. Section 5 describes the procedures used in the laboratory experiments and ZI simulations. Section 6 reports the results of laboratory experiments involving humans and computational experiments involving the ZI robots. Section 7 reports conclusions.

2. Marshallian price dynamics

2.1. The Marshallian path

The *Marshallian path* is simply a sequence of trades from left to right along the supply and demand curves. For example, in figure 1 the Marshallian path theory predicts the following sequence of trades: (trade 1) buyer with value 140/seller with cost 30, (trade 2) buyer with value 125/seller with cost 35, (trade 3) buyer with value 110/seller with cost 40, (trade 4) buyer with value 95/seller with cost 45, (trade 5) buyer with value 80/seller with cost 50, (trade 6) buyer with value 65/seller with cost 55. No further trades are possible because the next buyer has a value [50] less than the seller's cost [60]. Trade prices can vary anywhere between a buyer's unit value and the seller's unit cost. Thus, the initial possible range of prices is quite wide [30–140], but the possible range of prices is forced closer to

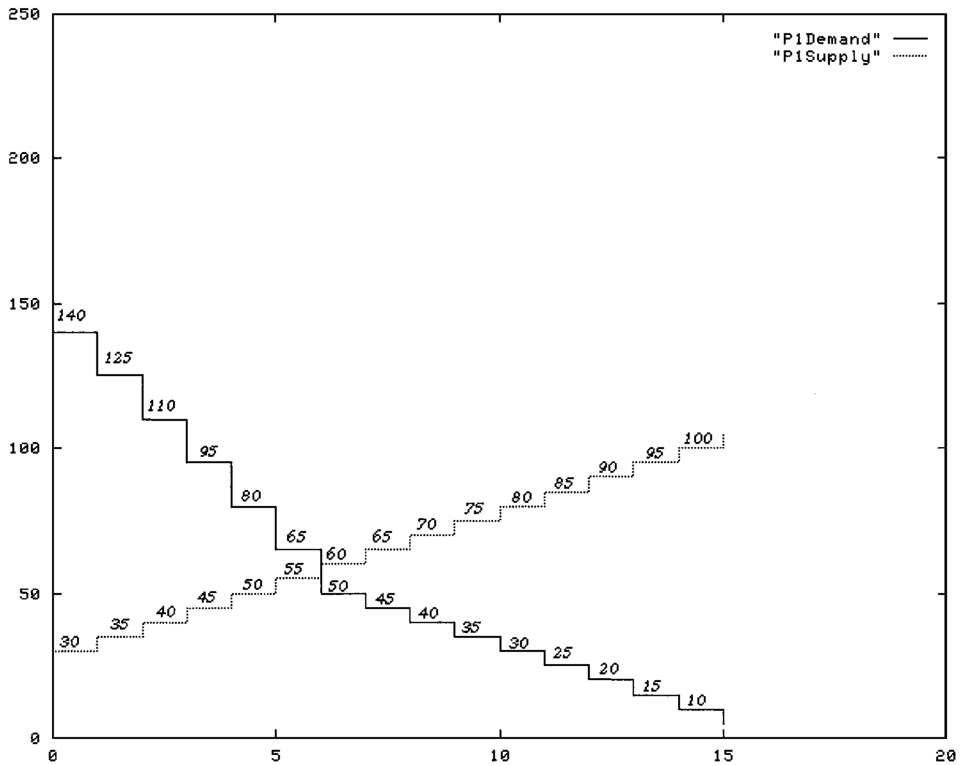


Figure 1. Sample supply/demand environment.

the equilibrium as trading progresses with the final trade [55–65] constrained to be near the competitive equilibrium [$55 < P < 60$].

In order for the Marshallian path to have empirical support it must be modified by some notion of randomness. The sequence of trading is generally very noisy in comparison to any exact ordering of trading partners.

As a practical model, aspects of the Marshallian path correspond well with stylized facts observed in both Sunder's ZI simulations and in laboratory data with human traders: (1) highly profitable trades tend to occur before less profitable trades (see Cason and Friedman, 1993, p. 277); (2) variance in prices tends to decrease as trading progresses (Smith, 1962); (3) trading price for the last unit is near the CE price [generally observed]; (4) the total units traded is equal to the CE quantity [approximately observed]; (5) the final bid and ask in a period are close to the extra-marginal redemption values and costs (Jamison and Plott, 1997).

2.2. *Review of ZI robot behavior (The Gode and Sunder phenomena)*

A brief replication of the Gode and Sunder results will help motivate the questions we pose. The potential that trading can occur *noisily* along a Marshallian path, while receiving little formal attention,[7] is essentially underlying price convergence in the ZI robot demonstrations. The purpose of this section is to briefly review the results on ZI robots and to show the robots' tendency to trade noisily along the Marshallian path.

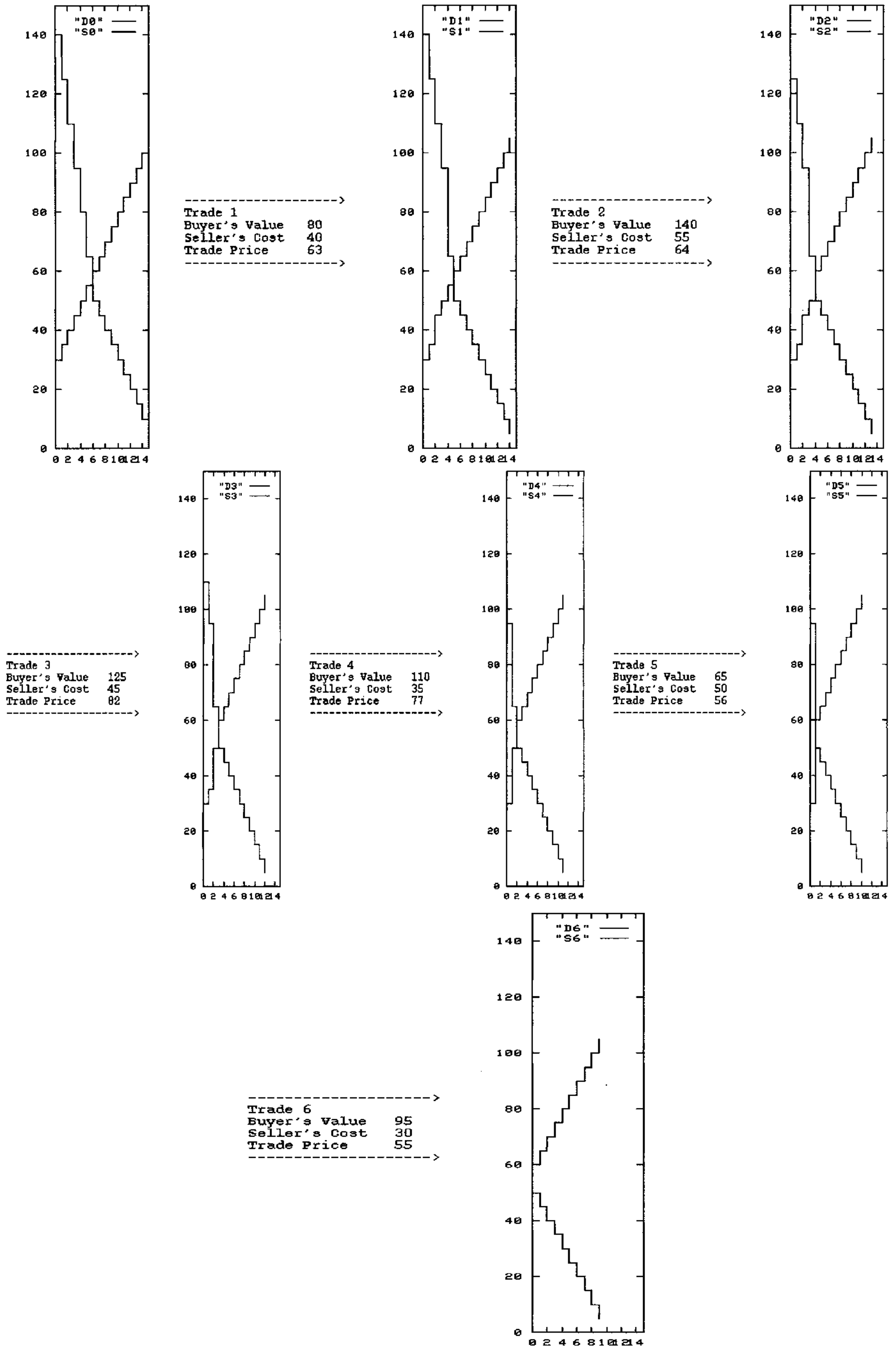
In Gode and Sunder (1993a, 1993b), ZI robots submit random bids and asks drawn from a uniform distribution with support equal to an agent's budget constraint. A buyer's bids are distributed U[0,v], where v is the buyer's value for a unit. A seller's ask is distributed U[c,H] where c is the seller's cost for a unit and H is an upper limit of trading. With the ZI robots, a pre-defined upper limit to trading is necessary as a uniform distribution over the seller's actual budget constraint; U[c, ∞) would be ill-defined. Typically, H is set at least as high as the highest buyer's value.

Under the standard double auction rules, a trade occurs when a new bid is made that is greater than a pre-existing ask, or when a new ask is made that is less than a pre-existing bid. The trading price is equal to that of the pre-existing bid/ask, whose acceptance is triggered automatically by the new entry.

Two important features of ZI robot trading immediately follow: (1) ZIs continue trading until gains from trade are exhausted, (2) while trades are random, the probability that the high value buyer trades with the low cost seller is higher than any other pairing.

An example of a trading sequence from ZI robots is shown in figure 2. In this example, the supply and demand curves of remaining traders are shown before and after each trade. Individual bids and asks of the ZI robots are not shown. In this example we see that the initial trades are far from equilibrium but the final trade occurs at a price (55) near the competitive equilibrium ($55 < P < 60$).

Like the Marshallian path theory, early transaction prices can be far from the equilibrium while later transaction prices are generally forced closer to the equilibrium by the absence of high surplus buyers and sellers. There is a possibility that early trades can involve extra-marginal buyers or sellers, and so ZI trading need not be 100% efficient as would be predicted



Efficiency = Total Trading Surplus / Maximum Possible Surplus = 360/360 = 100%

Figure 2. Example ZI robot trading period.

under the Marshallian path. Still, extramarginal trades are not frequent. ZI trading is a kind of noisy traversal of a Marshallian path in the following sense: although the buyers and sellers are picked more or less at random, at each moment in time the current high value buyer and current low cost seller have the highest probability of trading.

The consistency of such patterns led Gode and Sunder to the conclusion that markets populated by ZI traders converge to competitive equilibrium. By repeating the example of figure 2, we can examine the basis for these claims.

Figure 3 shows the price distributions of initial and final trades obtained in 1000 ZI trading periods using the environment of figure 1. Here we can clearly see that the initial trades are rather widely distributed in price compared to the final trades. The final trades, while not exactly clustered around the equilibrium of [55–60], are much closer to the equilibrium than the initial trades. Thus, there is an appearance of convergence.

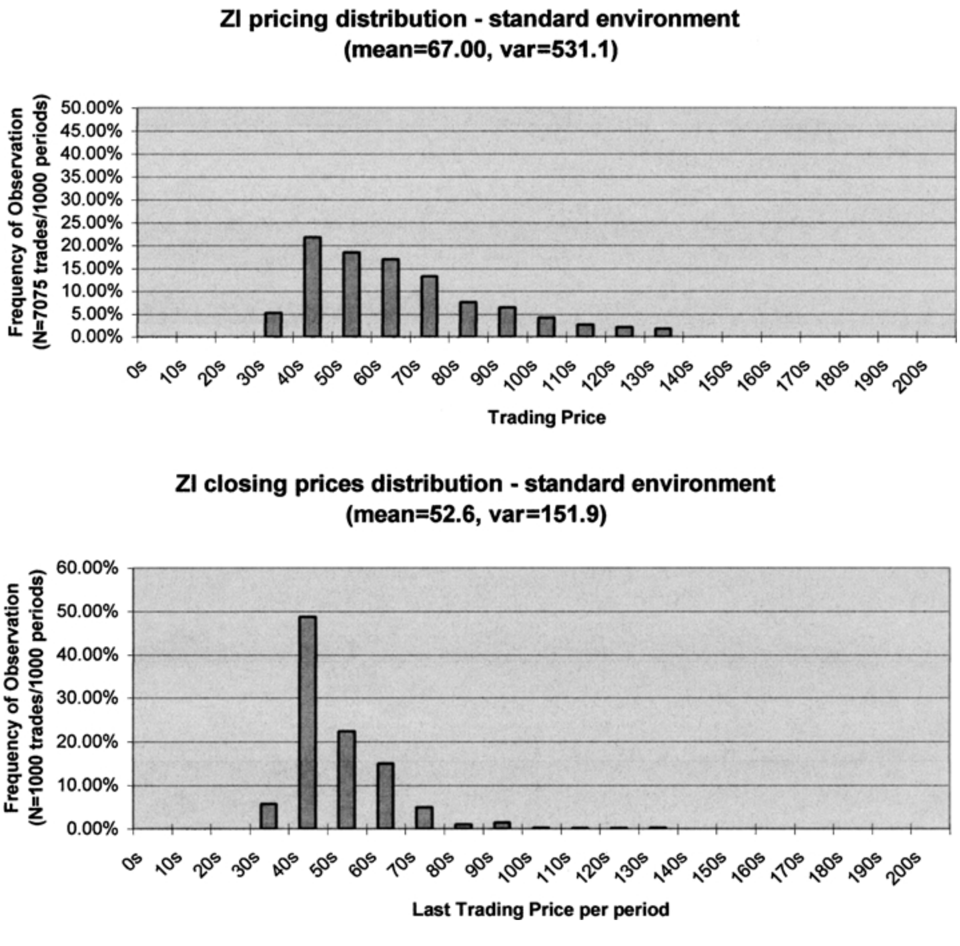


Figure 3. Price distributions within markets populated by ZI traders.

If convergence of market prices to competitive equilibrium is due simply to the existence of a Marshallian path, then it might be possible to construct an environment where a classical Marshallian path does not exist in a classical sense. Constructing such an environment is the primary topic of our next section.

3. Continuously refreshed supply and demand environment

The purpose of this section is to describe the parameters used in the experiments of Section 5. In particular, we seek to construct a new kind of supply/demand environment together with the supporting methodology necessary to conduct laboratory research.

Continuously refreshing the supply and demand parameters leads to an environment where the supply and demand curves do not shrink back to the left as trading progresses. Of course, the classical Marshallian path is removed as a process for price convergence.

3.1. Intuition

A bit of intuition regarding the new CRSD environment in terms of more familiar markets may be useful.[8] Twenty-four hour electronic world markets already exist, or are forthcoming, for several popular commodities and securities. Some form of steady, unending supply and demand flows may be more descriptive of these markets than a standard model involving a market carried out over a series of periods, with each period consisting of definite opening, trading, and closing phases.

Other practical examples may exist in unbalanced matching environments such as housing or job search: Osborne and Rubinstein (1990) note that filling one housing vacancy is likely to create another vacancy as the new occupants move away and sell their old home. The same would seem to be true of a job market.

The goal here, however, is not to argue for the practical applicability of these approaches in describing field markets. The goal is to help the reader to understand the CRSD environment that we implemented in the laboratory. In contrast to a model of an existing market, our Continuously Refreshed Supply and Demand laboratory market is simply one where the unending supply and demand flows take a particular form that is theoretically convenient for studying both human rationality and alternative specifications of the law of supply and demand. These details will be made clear in the remainder of the paper.

3.2. Previous methodology

A bit of explanation about the history of our methodology is useful. In Chamberlin (1948) and subsequent early research,[9] buyers had cards that told them the value of purchasing a single unit. The card could only be used once. Similarly sellers had cards, usable only for sales of a single unit, which explained a unit's cost. Later experiments (Plott and Smith, 1978) expanded the continuous auction methodology to multiple units. Giving buyers and sellers sheets of paper on which redemption values and costs were listed facilitated accounting. These sheets are typically called *redemption value sheets* for buyers and *cost sheets* for sellers. The cards/sheets are private information. Profits of a buyer or seller are simply the difference between the price obtained in the market and the value or cost on the sheet.

The arbitrage of redemption values/costs against the market is the source of agents' profits in traditional experiments. The experimenter set the redemption values and costs privately and observed the resulting, public, market behavior. That part of the methodology is retained.

The new methodology involves two elements: a Private Markets methodology and a Refreshing methodology. The Private Markets involve a move of the redemption value and cost sheets onto the computer in the form of a private market, where they can then be continuously refreshed in a particular way.

3.3. *Private markets methodology*

In our experiments the arbitrage opportunity takes the form of public and private markets instead of a market and cost/redemption value sheets. Each participant sat at a PC running a specialized market program prepared in Java. The program divided the participant's screen into two sections—a *private* market and a *public* market. The public market was public in the sense that it provides a means for a participant to post offers to buy or sell that can be seen and acted upon by other participants. The private markets, however, are different for each participant.

The private markets contain the equivalent of redemption value and cost sheets. That is, in the "private markets" buyers and sellers receive non-negotiable offers from the experimenter. In the private markets, participants could not make counter offers or negotiate in the private markets in any way. The offers from the experimenter are "private" in the sense that they can only be seen by or executed by a particular participant. The private markets serve as an electronic replacement for the redemption value and cost sheets used in traditional experiments, but the function is essentially the same.

The division of subjects into roles as buyers or sellers is operationalized with private markets in the same way that it would be with value and cost sheets. For instance, a buyer receives buy offers from the experimenter in the private market in our new experiments, just as a buyer would receive buy offers in the form of redemption values from the experimenter in a traditional experiment. A seller receives sell offers from the experimenter in the private market, just as a seller would receive sell offers in the form of cost sheets from the experimenter in a traditional experiment.

Offers placed in the private market by the experimenter expired after two minutes, but participants were also informed that new offers could appear on the screen at any time. Thus, if no new orders were distributed, the environment would have been similar to a traditional environment with two-minute periods.

The public market showed buy and sell offers from the other participants. Participants could negotiate with each other in the public market under price improvement rules standard to most double auction experiments: buy offers must go up, sell offers must go down—and after a trade any remaining offers are cleared from the market.

3.4. *Continuously refreshed redemption values and costs*

When a unit was traded in a private market by exercising a private market offer, or when the two minutes expired, the offer was immediately recycled to another participant. For example, if buyer #3 used a private market offer (a redemption value) from the experimenter, this same offer would immediately be made to the *next* buyer (e.g., buyer #4). Similarly, offers to sell (costs) were recycled to the next seller. Subjects had no knowledge at all about this refreshing.[10] Subjects knew only that new orders could appear in their private markets at any time.

Refreshing the private offers in this way keeps the instantaneous supply and demand curves constant at every moment in time. If an offer is used or expires, it does not vanish from the pool of supply and demand. Instead, it is recycled to someone else. Thus, the opportunities of gains from trade are never exhausted. The market demand and supply functions as represented by redemption values and costs are always constant-independent of the patterns of trade.

Table 1. Supply and demand parameters maintained in the experiments.

Experiment	Interval/environment	Private market orders	Instantaneous competitive equilibrium
1	1/1	Buy (140, 125, 110, 95, 80, 65, 50, 45, 40, 35, 30, 25, 20, 15, 10, 5)	$55 \leq P \leq 60$
	(tr#1-793)	Sell (30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105)	
	2/2	Buy (210, 205, 200, 195, 190, 185, 180, 175, 170, 165, 160, 155, 150, 145, 140, 135)	$180 \leq P \leq 185$
	(tr#820-end)	Sell (100, 115, 130, 145, 160, 175, 190, 195, 200, 205, 210, 215, 220, 225, 230, 235)	(see figure 4)
2	3/3	Buy (200, 175, 150, 125, 100, 75, 50, 45, 40, 35, 30, 25, 20, 15, 10, 5)	$55 \leq P \leq 60$
	(tr# 1-466)	Sell (30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105)	
	4/4	Buy (210, 205, 200, 195, 190, 185, 180, 175, 170, 165, 160, 155, 150, 145, 140, 135)	$180 \leq P \leq 185$
	(tr#504-end)	Sell (40, 65, 90, 115, 140, 165, 190, 195, 200, 205, 210, 215, 220, 225, 230, 235)	(see figure 5)
3	5/5	Buy (225, 200, 175, 150, 125, 100, 75, 70, 65, 60, 55, 50, 45, 40, 35, 30)	$80 \leq P \leq 85$
	(tr#1-162)	Sell (55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130)	
	6/6	Buy (235, 230, 225, 220, 215, 210, 205, 200, 195, 190, 185, 180, 175, 170, 165, 160)	$205 \leq P \leq 210$
	(tr#176-313)	Sell (65, 90, 115, 140, 165, 190, 215, 220, 225, 230, 235, 240, 245, 250, 255, 260)	
	7/5	Buy (225, 200, 175, 150, 125, 100, 75, 70, 65, 60, 55, 50, 45, 40, 35, 30)	$80 \leq P \leq 85$
	(tr#348-end)	Sell (55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130)	(see figure 6)
		Note: Same as period 5	

Note: The V-CE or velocity based competitive equilibrium is not provided in this table, as it is not known ex-ante. The V-CE is an ex-post model that depends on the velocity at which the various private market units traded.

In practical terms, the experimenter has a finite cash budget and finite time to conduct the experiments. Eventually the experiment must be terminated. To avoid any end-of-experiment effect, participants were given only a general idea of how long the experiment would last (e.g., 2–3 hours).

3.5. *Experimental parameters*

Table 1 and figures 4–6 show the offers put into the private markets in each of the three experiments that were run. The Buy offers from the experimenter create an induced demand curve and the sell offers create an induced supply curve, allowing the calculation of competitive equilibrium prices in the usual way. Each experiment consists of continuous trading under two or three different economic environments.[11] Unlike most experiments where a trading period has a start and end announced by the experimenter whereby subjects might be signaled that the environment is about to change, changes in environment in our experiment are unannounced. Because of these differences we will refer to *trading intervals* rather than trading periods.

Notice that many of the environments have apparently identical competitive equilibria but different slopes in the induced supply and demand curves. We chose the values shown in an attempt to separate various interpretations of the law of supply and demand. This will be more fully explained after covering models of the operations of the law in the next section.

4. Models in the new environment

The purpose of this section is to describe some qualitative models about how markets populated by either ZI robots or Human traders might behave given the continuously refreshed

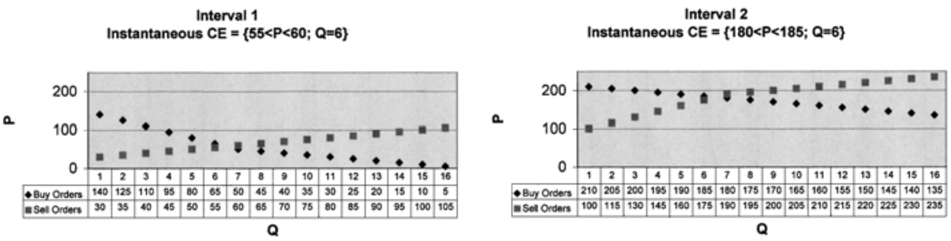


Figure 4. Parameters for experiment 1.

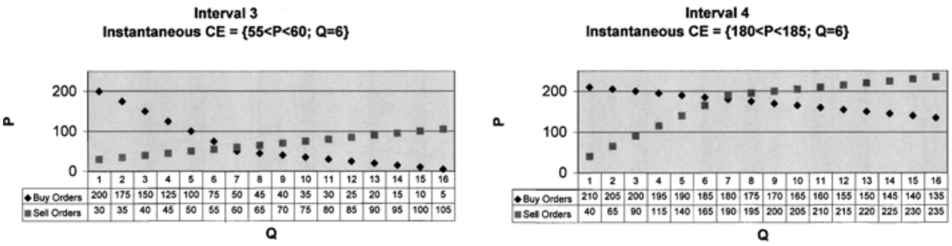


Figure 5. Parameters for experiment 2.

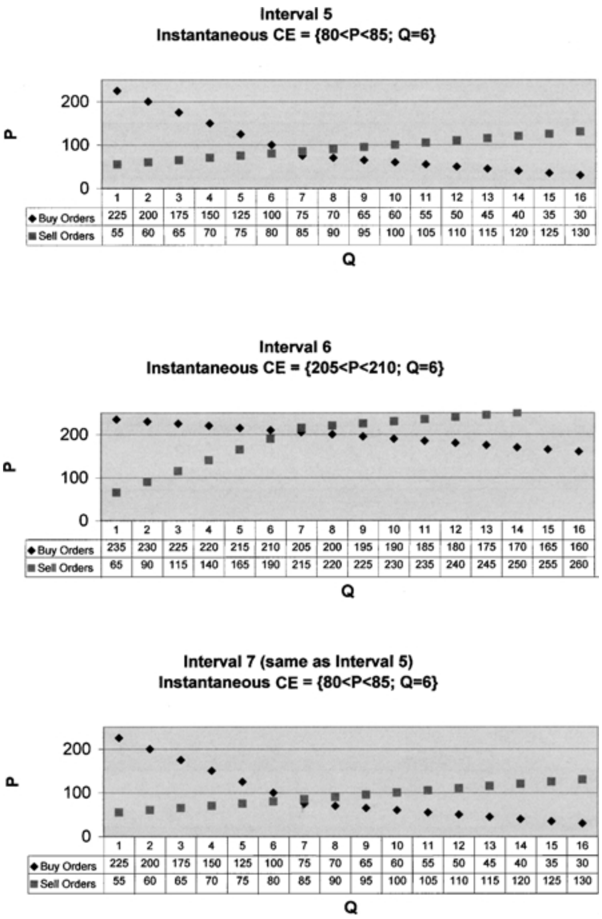


Figure 6. Parameters for experiment 3.

supply and demand environments of Section 3. As pointed out by Cason and Friedman (1993), Easley and Ledyard (1993) and others, there is no fully accepted model of double auction market dynamics. As no fully worked out and acceptable models exist, what follows must by necessity be quite rough. The discussion here will attempt to answer three questions: (1) What concept of convergence has been used? (2) What are some models of price convergence and non-convergence applicable to the continuously refreshed experimental environments? (3) What are the hypothesized relationships between markets populated by ZI robots vs. markets populated by humans?

4.1. Price convergence: Definition

Typically in double auction data one observes a trend of prices, usually moving towards some notion of competitive equilibrium. This trend may be described as "price convergence," but across the literature there is no universally adopted measurement for describing if, when or how this convergence is or is not taking place. For purposes of making definitions operational this convergence will be characterized by three properties, which are given below:

4.1.1. Properties of price convergence (relative to some theoretical equilibrium price P_{eq}).

1. Initial prices are further from the equilibrium than final prices.
2. Variance of prices decreases over time
3. If a parameter change moves P_{eq}, the prices move towards the new equilibrium.

These properties of convergence require a non-stochastic equilibrium concept, i.e., a single price or range of prices. The next section explores what equilibrium concepts are appropriate to the experimental environments under consideration.

4.2. Competitive equilibrium models

Three equilibrium concepts can be identified with the literature. Two of these are appropriate to environments with continuously refreshed supply and demand. We will call these models the *Traditional Supply and Demand, the Instantaneous Competitive Equilibrium* (I-CE) and *Velocity based Competitive Equilibrium* (V-CE) models.[12] Different supply and demand curves are used in the two models, as stated below:

4.2.1. T-CE model. *Traditional supply and demand*—the supply and demand curves are computed from the redemption values and costs that existed at the beginning of a period. Clearly the concept of a period is needed. Since periods do not exist in the environment we study, this model is listed only for completeness.

4.2.2. I-CE model. *Instantaneous supply and demand*—The instantaneous supply and demand curves are computed from the private market orders that exist in the market at an instant. In a traditional environment these curves change after each trade. In an overlapping generations environment the I-CE has been developed and studied by Aliprantis and Plott (1992). In a continuously refreshed environment the instantaneous supply and demand curves are stationary. The intersection of instantaneous supply and demand curves determines the competitive equilibrium.

4.2.3. V-CE model. *Velocity-based (ex-post) supply and demand*—the supply and demand curves are adjusted (ex-post) to take account of the number of times a particular supply or demand unit appeared in the private market of some participant (velocity).[13] The intersection of adjusted supply and adjusted demand determines the equilibrium. This model cannot be determined ex-ante, because the velocities are only known ex-post.

4.3. Experimental design and velocity effects

In the environments we study, the private market offers in some pairs of environments have been purposefully chosen to yield identical I-CE equilibria. For instance, Intervals 1 and 3 have an I-CE of 55–60, and Intervals 2 and 4 have an I-CE of 180–185. However, while the I-CEs are identical, the private market offers are quite different. For instance, Interval 3 has a steeper slope on the buy side than Interval 1 (200, 175, 150, ... vs. 140, 125, 110 ...). Interval 4 has a steeper slope on the sell side than Interval 2. In addition to the steeper slopes, there are also greater gains from trade on the initial units.

Velocity effects could cause the V-CE model to be sensitive to the differences in slopes and surpluses of initial units in the private market offers, whereas the I-CE model is only concerned with the intersection. If units circulate at different speeds, the shape of the V-CE model curves will change and the equilibrium of the model could be changed as a result.

Consider two environments that have the same I-CE. Suppose one environment has a steeper slope in the buyers' private offers than the other environment. Intervals 1 and 3 defined above have this property with Interval 3 having the steeper slope in buyers' induced values. Suppose further that the steeper buyers' surpluses in Interval 3 induce differences in trading velocities among the buyers in Interval 3 vis-à-vis the buyers in Interval 1. Ex-ante, one might expect trading velocity to increase with increasing profit, which would pull equilibrium prices higher in Interval 3. Via a similar argument, one might expect trading velocity differences on the sell-side of the market in Intervals 2 and 4 to pull equilibrium prices lower in Interval 4.

4.4. Classification of trades as inframarginal or extramarginal

The classification of realized trades as inframarginal or extramarginal depends on the existence of an equilibrium model. In this paper we have two equilibrium models (I-CE and V-CE), so some care is needed, but otherwise the original meaning is preserved. First we review the standard use of these terms with reference to the standard supply/demand environment of figure 1, and then explain how these classification terms will apply with respect to I-CE and V-CE.

The standard use of the term *inframarginal* refers to a trade of units where both the buyer's marginal value and the seller's marginal cost appear to the left of the intersection of supply and demand, whereas the term *extramarginal* trade refers to a trade where either the buyer or the seller had a unit to the right of the intersection of supply and demand.

For example, in figure 1 the intersection of supply and demand gives a theoretical CE price of 55–60. A realized trade (which need not be at the CE price) between a buyer with value 125 and a seller with cost 50 is inframarginal because both the buyer's marginal value on D (unit #2 of D) and the seller's marginal cost on S (unit #5) lie to the left of the CE intersection ($P = 55$–60, $Q = 6$). However, a trade between a buyer with value 125 and a seller with cost 110 is extramarginal, because the seller's marginal cost is higher than the CE price of 55–60 and would thus appear to the right of the CE intersection. Similarly, a trade between a buyer with a value of 40 and a seller with a cost of 30 would also be inframarginal, because the buyer's value of 40 is unit #9 of D, and lies below and to the right of the CE intersection. This is standard usage.

A simple, standard result follows. If the realized market price is constant at the CE price, and if traders follow their budget constraint ($C < P < V$), then all the trades will be inframarginal and none will be extramarginal. Positive surplus extramarginal trades typically require price to vary away from CE. This feature will be useful both to distinguish models and to describe efficient behavior.

Now we turn to the problem of having two CE models, the I-CE and V-CE. The method for dealing with this is quite simple: we simply make note of whether the I-CE or V-CE was used to classify a trade as inframarginal or extramarginal. As the I-CE and V-CE are likely to produce different CE prices and intersections, some trades classified as extramarginal under one model may be inframarginal under the other model.

4.5. *Efficiency of allocation*

Because of problems defining the 'maximum possible gains from trade' in a market involving continuously refreshed supply and demand, standard definitions of efficiency of allocation are difficult to apply.[14] Here we briefly discuss the potential of examining some definitions of inefficiency that are more readily applied.

Two types of inefficiencies are easy to observe: extramarginal trades, and inframarginal expirations. Extramarginal trades always have lower surplus than inframarginal trades, and so one might use the number of extramarginal trades as a measure of one type of inefficiency. Similarly, inframarginal units should not be allowed to expire untraded, as this represents profit that was allowed to disappear unclaimed by a trader.[15]

The biggest problem with either of these methods of identifying inefficiency is that they are CE model-dependent—that is, they depend on whether we identify inframarginal vs. extramarginal trades via the I-CE or via the V-CE. In the standard environment, the definition of the maximum possible gains from trade is model-independent, but in the CRSD environment we do not have this concept to fall back on.

Therefore, until a more model-independent notion of efficiency or inefficiency can be devised, our conclusions regarding efficiency or inefficiency must remain somewhat cautious and limited.

4.6. *Non-convergence models*

Given the properties of price convergence above, many stationary stochastic models of prices will be non-convergence models. Three models are considered:

IID random. Prices are independent, identically distributed random draws from some stationary random distribution.
Martingale. Prices drift with constant variance per unit trade. Differences in prices from one trade to the next are normally distributed with mean 0 and finite, constant variance.
Other. Prices do not converge, but are not IID random or Martingale in nature.

4.7. *ZI behavior under continuously refreshed supply and demand*

From our previous discussion of the operation of the ZI robots in Section 2, it would appear that there is no mathematical mechanism for price convergence in the ZI-populated markets when demand and supply are continuously refreshed. The absence of a Marshallian path means that a squeeze between willingness to accept and willingness to pay never occurs. Instead of drying up, demand and supply is continuously replenished.

Under the ZI robot algorithm, the trading price P_{ZI} can be thought of as a random variable whose distribution is dependent upon the instantaneous supply and demand curves and whose support is limited to the range of possible voluntary trades. In previous traditional market experiments the instantaneous supply and demand curves are shrinking in such a way that the support of P_{ZI} shrinks and prices appear to converge.

Thus, with continuously refreshed supply and demand, the instantaneous supply and demand curves are held constant, and so P_{ZI} must give independent and identical (IID) distributed random draws. There can be no price-convergence in ZI-populated markets in an environment with continuously refreshed supply and demand, according to this model.

Similarly, as the partners in each trade are effectively determined by an IID random draw from a difficult to calculate, but stationary, distribution, the inframarginal and extramarginal properties of trading will remain stationary in a ZI-populated CRSD market.[16] It will never be the case that some CE model will become better and better at separating the inframarginal and extramarginal sets of private orders being executed, because the probability distribution over which private orders (redemption values, costs) will be involved in the next trade is the same as that of the previous trade. This probability distribution of trading partners is independent of both the price of the previous trade and the identities of the previous traders.

As ZIs are not expected to exhibit price convergence, the primary issue in our experiments is whether or not we will see equilibration in markets populated by humans.

5. Experimental procedures

Details regarding the special methodologies and parameters used in these experiments can be found in Section 2.

5.1. Laboratory experiments

Participants in the experiments were Caltech undergraduates recruited via an announcement on a web-based bulletin board. Each experiment involved 16 participants and lasted 2–3 hours. Trading was continuous, with 1 or 2 unannounced parameter changes and no announcements as to when the experiment would be terminated. Participants received cash payments in proportion to trading profits.[17]

Software for the experiments was written in Java and ran inside a Netscape browser. Even though our software was web-capable, the experiments were conducted in the standard, controlled manner—with groups of participants assembling at the laboratory to listen to instructions, ask questions, and take part in the experiment.

Each participant's screen was divided into two sections, with the private market on the left side and the public market on the right side. Each market displayed the current buy and sell orders against a grid of prices. Entering buy and sell offers into the public market was accomplished by using the mouse to click at the relevant price. The large 21″ computer screens used in the laboratory made precision pricing of orders easy. The interface was very efficient and resulted in a much higher trading volume than could be expected with MUDA or similar double auction software.

5.2. ZI simulations

A Monte Carlo study of trades using the ZI robot algorithm was performed for comparison to the human-subject experiments. The software was written in version 5 of the PERL language, which provides a built-in random number generator. Since most random number generators are, in fact, deterministic and since some are not sufficiently 'random,' the generator was tested. This pre-simulation testing of the random number generator revealed no flaws either in distribution or serial correlation (independence of draws).

The ZI algorithm was run on a Linux-based workstation for several hours. Data were obtained for approximately 1.5 million trades[18] for each of the 6 experimental environments. In addition, we ran 1,000 period replications of environment 1 for the non-refreshed, standard supply and demand case. This allowed a comparison with previous ZI results in order to check procedures and to generate figures 2 and 3 used in our review of ZI trading behavior given in Section 2.

6. Experimental results

Figures 7 to 9 provide, for each of the three experiments, a side-by-side comparison of the predictions of the instantaneous CE model (left pane), the trading prices observed in the experiment (center pane), and the velocity-corrected CE model calculated ex-post (right pane).

In the center pane, the two solid lines show the instantaneous CE at various points in the experiment.[19] Two symbols are used for the trades. Diamonds represent 'buys' or acceptances of a sell offer by a buyer, and pluses represent 'sells' or acceptances of a buy offer by a seller.

Table 2 provides mean prices, price variance for each experiment as well as a count of extramarginal trades for each model. These data are reported by experiment, by interval, and by every 100 trades within an interval. For completeness, transition intervals are also identified and reported but do not play a role in the analysis presented here. The table shows that within each interval, price variance tends to decrease as trades occur.

The most striking feature of figures 7–9 is that prices appear to converge. These tendencies are summarized as Result 1.

Result 1. *Price convergence can occur in human-populated markets within continuously refreshed supply and demand environments.*

Support. Each of the three convergence properties of Section 4 must be shown in operation.

1. *Initial prices are further from the equilibrium than final prices.* From figure 7 we see that for interval 1, initial prices are near 100 and are further from either the V-CE or I-CE

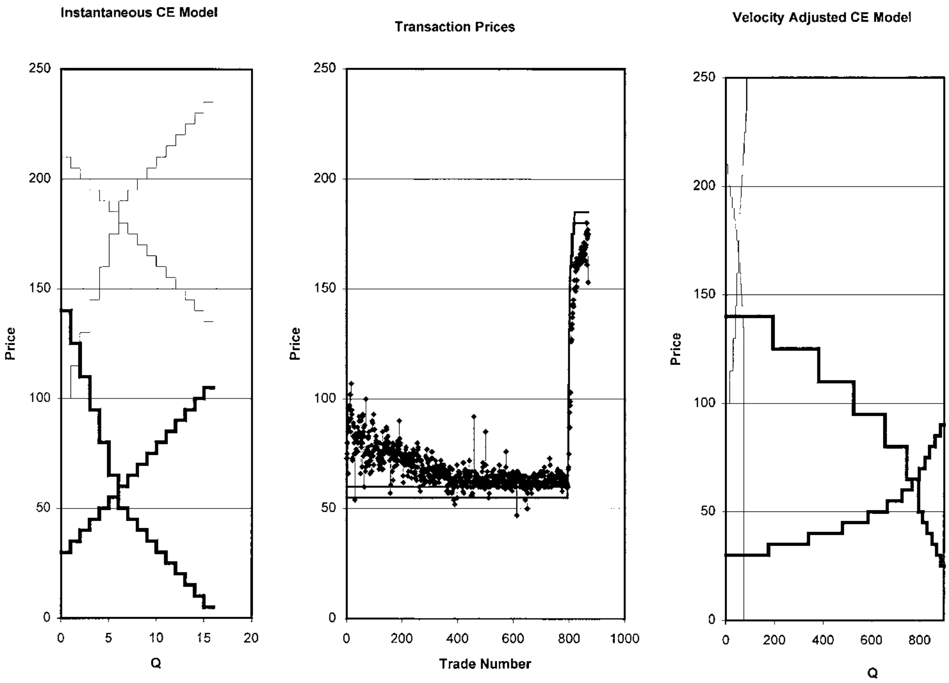


Figure 7. Models and transactions price data for experiment 1.

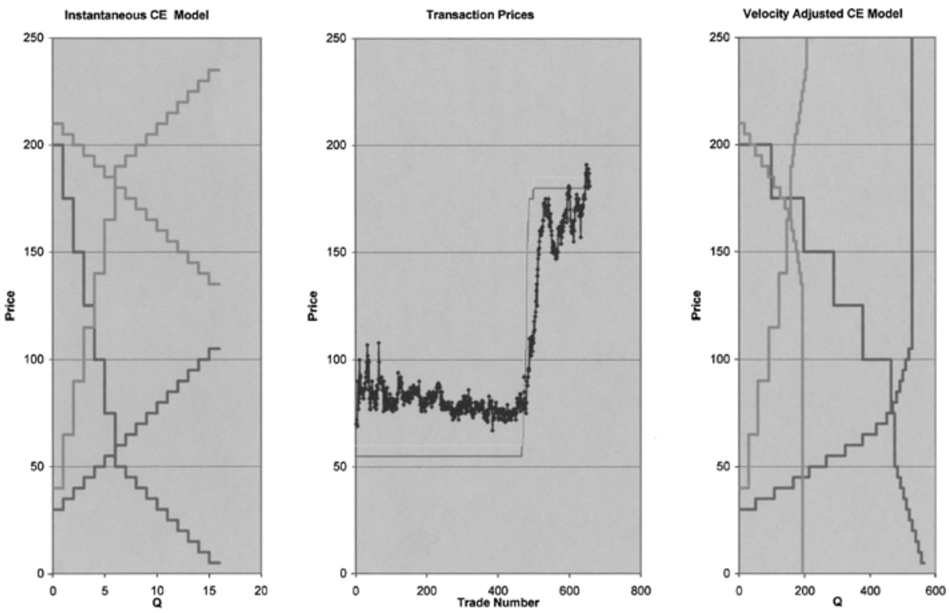


Figure 8. Models and transactions price data for experiment 2.

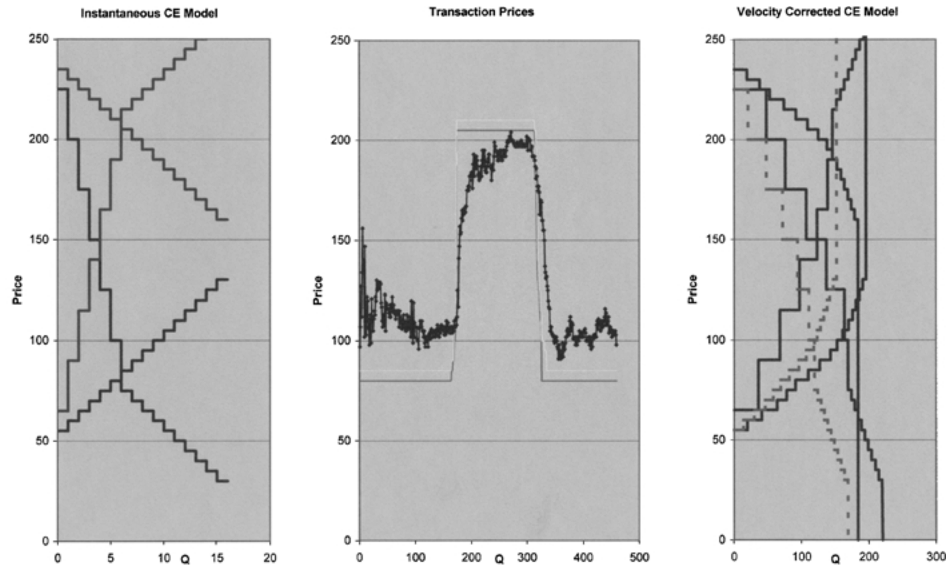


Figure 9. Models and transaction price data for experiment 3.

than the final prices, that are near 60–65. Table 2 also supports this same observation as the first 100 trades of interval 1 have an average price of 81.2 while the final 93 trades have an average price of 63.4. Similarly, for Interval 2 in the latter part of figure 7, prices move toward the equilibrium from below. Interval 3 prices start high and move towards V-CE/I-CE equilibria—from table 2 we see that the first 100 trades have an average price of 85.9, and the final 66 trades have an average price of 75.9. Interval 4, in the latter half of figure 8, shows convergence from below. In figure 9, Intervals 5 and 7 show some evidence of convergence from above, while Interval 6 shows convergence from below.

2. *Variance of prices decreases over time.* While some notion of variance can be seen from the dispersion of points in the figures, Table 2 is more reliable, with the transaction price variance tabulated for groups of 100 trades. For Interval 1, the variance of the first 100 trades is 85.5, followed by 26.3, 21.5, 18.1, 22.3, 8.9, 9.6 for each successive 100 trades, and finally, for the last 93 trades, the transaction price variance is 6.5. Over Interval 1 the variance decreases by a factor of 13:1, and with 2 minor exceptions, ($18.1 \rightarrow 22.3$, $8.9 \rightarrow 9.6$), the variance is strictly decreasing with time. Similar patterns are seen in Interval 3 (decrease from 55.0 to 8.7), Interval 4 (decrease from 167 to 77), Interval 5 (decrease from 97.1 to 9.3), and Interval 6 (decrease from 245.1 to 5.7).
3. *If a parameter change moves P_{eq}, the prices move towards the new equilibrium.* Intervals 2, 4, 6, and 7 involved a parameter shift from a previous trading interval. In each transition, Table 2 shows a sudden increase in transaction price variance, while the figures show the price moving toward the new equilibrium.

The next result addresses the question of which model is more accurate. The Instantaneous-CE (I-CE) model seems to predict the trend of prices, though not the exact

Table 2. Averages and variances of prices observed in experiments (humans).

Experiment/ interval/trades	Number of transactions	Mean (transaction price)	Var (transaction price)	I-CE Model Extramarginal trades	V-CE Model Extramarginal trades
Experiment 1	868	74.8	637.6		
Interval 1: trades 1–793	793	68.0	68.8	$P_{\text{I-CE}} = 60$ Extm. Trades 36 (4.5%)	$P_{\text{V-CE}} = 65$ Extm. Trades 19 (2.4%)
Detail					
1–100	100	81.2	85.5	17 (17%)	8 (8%)
101–200	100	75.8	26.3	12 (12%)	8 (8%)
201–300	100	69.8	21.5	4 (4%)	1 (1%)
301–400	100	64.8	18.1	1 (1%)	1 (1%)
401–500	100	63.7	22.3	0 (0%)	0 (0%)
501–600	100	63.2	8.9	1 (1%)	0 (0%)
601–700	100	61.9	9.6	0 (0%)	0 (0%)
701–793	93	63.4	6.5	1 (1%)	1 (1%)
Transition 12: trades 794–819	26	112.8	1036.4		
Interval 2: trades 820–868	49	164.7	40.4	$P_{\text{I-CE}} = 180$ Extm. Trades 10 (20%)	$P_{\text{V-CE}} = 170$ Extm. Trades 3 (6%)
Experiment 2	657	100.9	1304.9		
Interval 3: trades 1–466	466	80.5	32.5	$P_{\text{I-CE}} = 60$ Extm. Trades 91 (19.5%)	$P_{\text{V-CE}} = 75$ Extm. Trades 9 (1.9%)
Detail					
1–100	100	85.1	55.0	16 (16%)	3 (3%)
101–200	100	83.0	13.8	23 (23%)	2 (2%)
201–300	100	79.8	15.9	19 (19%)	2 (2%)
301–400	100	77.3	9.6	20 (20%)	2 (2%)
401–466	66	75.9	8.7	15 (23%)	0 (0%)
Transition 34: trades 467–503	37	94.2	176.3		
Interval 4: trades 504–657	154	164.1	174.1	$P_{\text{I-CE}} = 180$ Extm. Trades 33 (21.4%)	$P_{\text{V-CE}} = 165$ Extm. Trades 8 (5.2%)
Detail					
504–603	100	159.5	167.3	23 (23%)	8 (8%)
604–657	54	172.5	77.7	10 (19%)	0 (0%)
Experiment 3	459	134.3	1491.6		
Interval 5: trades 1–162	162	110.5	87.6	$P_{\text{I-CE}} = 85$ Extm. Trades 32 (19.8%)	$P_{\text{V-CE}} = 105$ Extm. Trades 5 (3.1%)

(*Continued on next page*.)

Table 2. (*Continued*).

Experiment/ interval/trades	Number of transactions	Mean (transaction price)	Var (transaction price)	I-CE Model Extramarginal trades	V-CE Model Extramarginal trades
Detail					
1–100	100	114.3	97.1	27 (27%)	5 (5%)
101–162	62	104.2	9.3	15 (24%)	0 (0%)
Interval 6: trades 176–313	138	187.6	216.0	$P_{\text{I-CE}} = 205$ Extm. Trades 18 (13%)	$P_{\text{V-CE}} = 190$ Extm. Trades 1 (0.7%)
Detail					
176–275	100	183.9	245.1	14 (14%)	1 (1%)
276–313	38	197.4	5.7	4 (11%)	0 (0%)
Interval 7: trades 348–459	112	102.9	24.8	$P_{\text{I-CE}} = 85$ Extm. Trades 22 (19.6%)	$P_{\text{V-CE}}100$ Extm. Trades 0 (0%)

price to which prices converge. While we cannot say that prices converge to the instantaneous CE model, a convergence phenomenon does appear to be present as the prices do track changes in the parameters in a similar manner to the instantaneous CE.

The right panes of figures 7–9 provide the alternative model—the Velocity Based Competitive Equilibrium (V-CE) model—that better approximates a convergence process. The visual impression is stated below as our next result.

Result 2. *The Velocity Based Competitive Equilibrium* (*V-CE*) *Model best corresponds to the observed pattern of prices in the human-populated markets.*

Support. The models defined in Section 4 are considered for completeness. Non-convergence models can be rejected as Result 1 shows that price variance appears to be decreasing. The I-CE model, based on Instantaneous supply and demand, and the V-CE model, based on a velocity adjustment calculated ex-post, can be compared in figures 7–9. In particular, note that the I-CE model predicts identical outcomes for intervals 1 and 3 [$55 \leq P \leq 60$], intervals 2 and 4 [$180 \leq P \leq 185$], and intervals 5 and 7 [$80 \leq P \leq 85$]. From Table 2 we see that Intervals 2 and 4 and intervals 5 and 7 have almost identical final price levels, but in interval 3 prices are observed to converge to a much higher level [75.9 for the last 100 trades] than in interval 1 [63.4 for the last 100 trades]. Looking to figures 7 and 8 for Intervals 1 and 3, we see that both Intervals 1 and 3 fit the V-CE model very well. Thus, the V-CE model accounts for a difference in observed prices that the I-CE model did not capture.

Result 3. *The Velocity Based Competitive Equilibrium* (*V-CE*) *Model appears to correspond to a market-clearing price for the human-populated markets.*

Support. Here, only the I-CE and V-CE models are candidates for determining a market clearing price. From Table 2 we see that the I-CE model results in a much higher classification

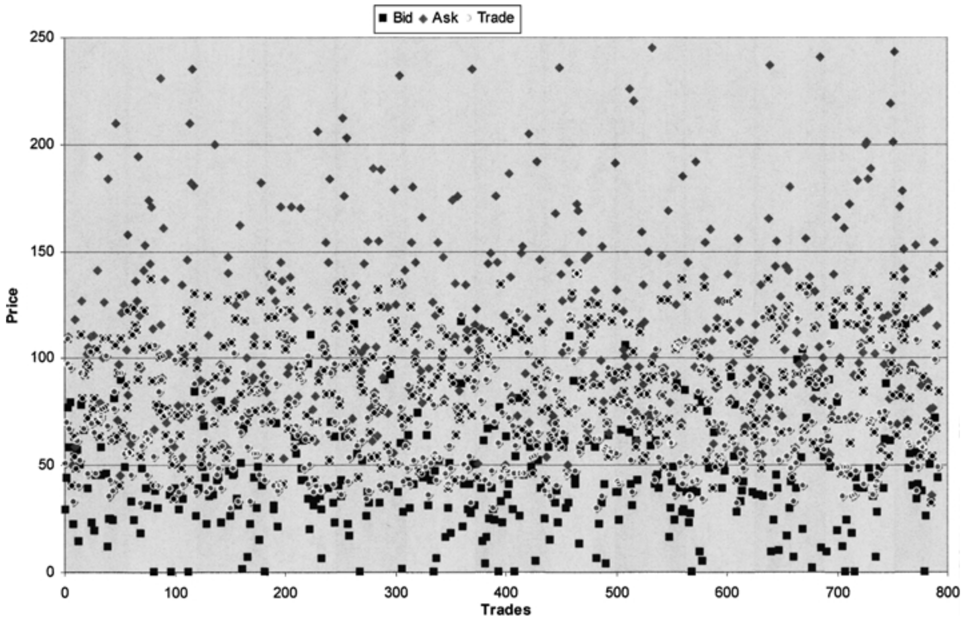


Figure 10. Highest bids, lowest asks, and transaction prices [ZI robots, environment 1].

of extramarginal trades than the V-CE model. Furthermore, the proportion of extramarginal trades decreases to near 0% as trading progresses if we take the market clearing price to be the V-CE price. If we take the market clearing price to be the I-CE price, the proportion of extramarginal trades decreases in some cases, but remains steady at a level that is generally 15% or higher.

Turning to examination of markets populated by the ZI robots, figure 10 shows the price sequence of 1,000 ZI trades with the environment 1 continuously refreshed environment. Price variance is constant and prices are independent draws from a complicated random distribution. We are left with Result 3.

Result 4. *Price Convergence does not occur in markets populated by ZI traders with a continuously refreshed supply and demand environment.*

Support. The simulation produced exactly what the model predicts: trades have IID random prices, and price variances do not decrease over time but instead remain constant.[20] The constant variances and other market properties that were observed are reported in Table 3, and are generally quite high in comparison with even the initial variance in the markets populated by humans. Since prices are not decreasing in variance, by definition price convergence does not occur.

A summary of the ZI trading data is reported in Table 3. Results 5 and 6 will compare and contrast the markets populated by the ZI robots and markets populated by the humans. While there are a number of differences, there also will remain a puzzling similarity:

Table 3. Price data from ZI markets.

ZI parameters	Data collected (trades)	Mean (P)	Var (P)	Instantaneous CE Ex-Ante	Extramarginal trades relative to I-CE	Velocity-adjusted CE Ex-Post	Extramarginal trades relative to V-CE
S/D not refreshed	7075 trades	67.0 (all trades)	531	Initial CE	N/A	N/A	N/A
Environment 1	1000 periods	52.6 (final)	152	$55 \leq P \leq 60$			
S/D continuously refreshed							
Environment 1	1.56 million	78.31	675	$55 \leq P \leq 60$	337293 (29%)	80	200563 (17%)
Environment 2	1.41 million	176.82	486	$180 \leq P \leq 185$	226436 (21%)	175	182971 (17%)
Environment 3	1.77 million	95.04	1657	$55 \leq P \leq 60$	477182 (36%)	90	215731 (16%)
Environment 4	1.57 million	169.58	964	$180 \leq P \leq 185$	306738 (26%)	165	173025 (15%)
Environment 5	1.73 million	118.16	1650	$80 \leq P \leq 85$	456793 (35%)	110	248233 (19%)
Environment 6	1.57 million	193.47	1017	$205 \leq P \leq 210$	310508 (26%)	190	175513 (15%)

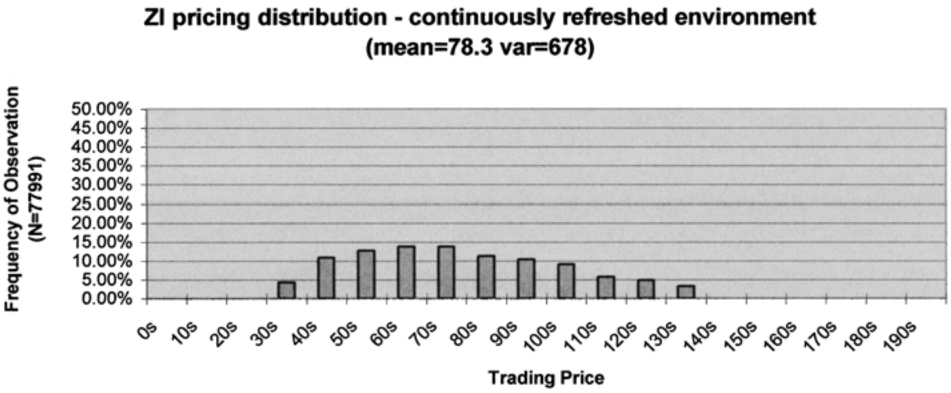


Figure 11. ZI price distribution with continuously refreshed supply/demand.

although price convergence does not occur in the markets populated by ZI robots, the mean prices correspond well to V-CE equilibrium prices.

In the continuously refreshed case, markets populated by ZI robots exhibit neither price convergence nor pricing distributions similar to human markets. It is possible that there are significant differences between the behavior of markets populated by ZI-robots and markets populated by humans under conditions of ordinary supply and demand as well. The differences may be harder to detect in practice, but figures 10 and 11 suggest that they might exist.[21]

A comparison of the distribution of prices between the markets populated by the ZI traders and the markets populated by human traders is shown as figure 12. In these graphs, the "Observed" data are from the humans and the "ZI" data are from the robot simulations.

Each row of the graph provides this data for one of our three experiments, with the data for each interval shown sequentially in panes from left to right.

Result 5. *In environments with continuously refreshed supply and demand, markets populated by humans differ from markets populated by ZI robots as follows:*

(a) *transaction prices in markets populated by humans tend to converge whereas in markets populated by ZI robots transaction prices do not converge*
(b) *relative to the V-CE price, extramarginal trades decrease over time in markets populated by humans, whereas in markets populated by ZI robots the ratio of extramarginal trades remains constant*
(c) *the distribution of transaction prices is more tightly peaked in markets populated by humans and does not exhibit the stepped artifacts of ZI robot pricing distributions.*
(d) *peaks of the observed distribution of transaction prices are in different locations in the two types of markets*

Support. (a) is merely a restatement of Results 1 and 4. (b) is a restatement of Results 3 and 4. Also, it is worth noting that the levels of extramarginal trades (relative to the

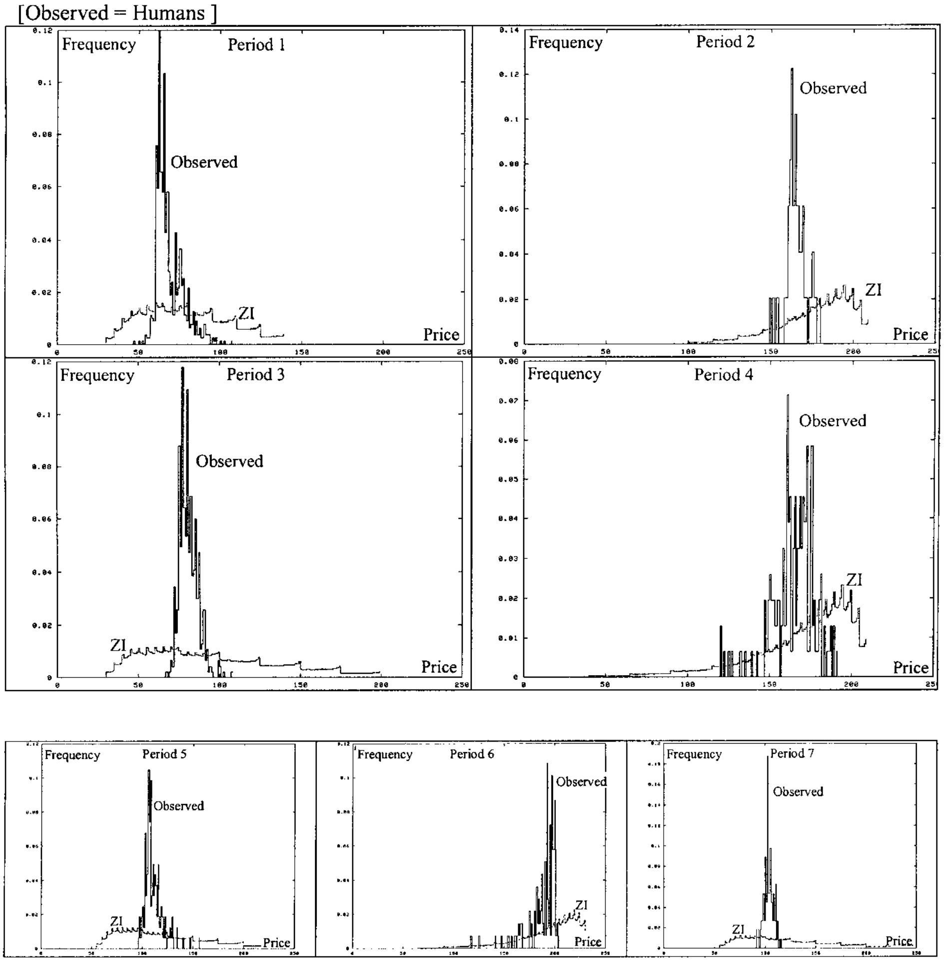


Figure 12. Comparison of price distributions: ZI markets vs. human markets.

V-CE price) reported in Table 3 for the ZI-robots are constant in the range 15%–19%, which is generally above the level of extramarginal trade observed even *at the beginning* of the human markets (∼8–10%) as reported in Table 2. The difference in tightness of the transaction price distribution can be clearly seen in any of the trading intervals in figure 12. For example, in Intervals 1, 2, and 3, given in the 3 larger panes of figure 12, the density for markets populated by humans peaks at about 10–12% franc. Given the same instantaneous supply and demand parameters, the transaction price distribution for markets populated by ZI-robots have peak densities of 1–2%/franc. (d) the difference in location of the peak of transaction price distribution can be seen in any of the trading intervals in figure 12 with the exception of Interval 1. For example, in Interval 2 the peak (or mode) of the distribution occurs at a price of about 170 for markets populated by humans, but at a price of about 190 for markets populated by ZIs.

As can be seen, the mean transaction prices of ZI robots in environments 1, 3, and 5 are far from the instantaneous CE implied by the parameters. While environments 2, 4, and 6 show mean prices near the I-CE, the variance is constant and quite high as a proportion of price. In contrast, the mean transaction prices correspond well with the V-CE calculated for the robot trading. Given the lack of price convergence for the ZI robot markets, the existence of a correspondence between the mean prices and the V-CE is a bit puzzling.

The similarities between the markets populated by humans or ZI robots, related to the predictive abilities of the V-CE model in both cases are stated below as Result 6.

Result 6. *In environments with continuously refreshed supply and demand, the V-CE model predicts mean transaction prices in markets populated by Humans as well as markets populated by ZI robots.*

Support. From Result 3 we know the V-CE model is appropriate to mean prices generated in human markets. It remains to be shown that V-CE corresponds to mean prices observed in the robot simulations. Table 3 shows that the V-CE model also applies to the mean prices exhibited by the robots. In Environment 1, the I-CE price range is $55 < P < 60$, the V-CE price is 80, and the actual mean is 78.3. Similarly, in Environment 2, V-CE (175) compares more favorably to mean price (176.82) than the I-CE ($180 < P < 185$). While in each trading environment the mean transaction price of ZI robots corresponded more closely to the V-CE price than to the I-CE price, the biggest differences between V-CE and I-CE are seen in Environment 3 (I-CE (55, 60), V-CE 90, mean $[P] = 95.04$) and Environment 5 (I-CE (80, 85), V-CE 110, mean $[P] = 118.16$.

7. Concluding remarks

The principal result is that the Marshallian path does not account for the observed accuracy of the law of supply and demand as a price discovery process in markets populated by humans. An immediate corollary is that some aspect of rationality in addition to that imposed by the budget constraint and trading institutions is operating. While the Marshallian path can be called the "cause" of price convergence in markets populated by ZI robots it is not the "cause" of price convergence in markets populated by humans. While it might be helpful in some environments, it is not essential.

The continuously refreshed environment offers a new paradigm for studying the type of market adjustments that the concept of tatonnement was designed to study. Thus the environment holds the potential for theory development and testing. In this context the volume adjusted competitive model suggests itself as a step towards an improved theory for how markets adjust. The flows or speeds with which demand and supply units are received in the market are natural parameters with which to adjust the classical models.

The experiments also help to focus on the fact that supply and demand equilibria correspond more to some form of time aggregation (where velocity can be important) than to an instantaneous model (where velocity is not important). More work is clearly needed to understand the dynamics of price convergence in markets. The V-CE model is in some ways too simple and unsatisfying. There are many alternatives, and also too many unknowns (what determines velocity? Why did we seem to observe a velocity effect in Interval 1 vs. Interval 3 but not Interval 2 vs. Interval 4?). One could consider models where there is a fixed time window into the past, or where various moving averages of instantaneous demands are taken. Various forms of differential equations might also be considered.

The experiments help define the context in which the use of robots might be most valuable. Humans are known to be more complex than ZI robots in their information processing capabilities. ZI robots are designed *not* to learn, imitate, adapt, or otherwise react in a strategic manner. It is this simplicity of ZI robots that is appealing in conducting simulations, as otherwise the simulations would depend on a number of initial parameters (beliefs, adaptation speeds, length of memory for use in histories, etc.) that are conveniently absent. However, this simplicity means that the ZI behavior will not robustly predict the behavior of humans in all possible market environments, as human behavior involves the complexities that are purposefully omitted from ZI robots. The results reported here demonstrate that the path of convergence requires more intelligence than the Gode and Sunder ZI framework postulates.

Notes

1. Gode and Sunder (1993) introduce the ZI algorithm, with the theme of their paper being that random behavior subject to market and individual budget constraints can yield efficient outcomes—the only trader rationality that is required is the ability to abide by budget constraints. Gode and Sunder (1993b, 1997a, 1997b) use the ZI algorithm to test various institutional rules and budget constraints within the double auction framework to determine which rules are most responsible for market efficiency. Gode and Sunder (1997a) use the ZI algorithm to test the implications of non-binding price-ceilings in markets.
2. The efficiency emphasis is clear from their titles: 'What makes markets allocationally *efficient*?', 'Allocative efficiency of markets with zero-intelligence traders,' 'Lower bounds for efficiency of surplus extraction . . . '
3. There are some differences among authors as to the definition of convergence. Gode and Sunder (1993a, p. 29) associate convergence with the final price in the market being closer to the predictions of initial supply and demand that early prices: 'By the end of a period, the price series in budget constrained ZI trader markets converges to the equilibrium level almost as precisely as the price series from human trader markets does.' Convergence can have a more restricted meaning when learning is possible over a series of repeated market periods—for example Smith (1962) discovered all the trades in a period may occur at the CE price given sufficient repetition of that period. Gjerstad and Dickhaut (1998; ft. 5) use this stricter notion in evaluating their model: 'We argue that prices in a stable market environment converge, if, after several periods, the mean deviation of all trades from equilibrium is small.' The ZI robots cannot exhibit this type of convergence because they do not learn from previous periods—instead repeating similar stochastic behavior at the start of each new period. Other definitions of convergence, related to time series within and across periods have been based on the Ashenfelter-El Gamal model developed in Noussair et al. (1996).
4. Chamberlin, p. 102, suggests "Information during the market as to the equilibrium price would help establish a trend in that direction, but information as to actual prices may do the opposite, in so far as they are divergent from equilibrium and are falsely interpreted to be near it."
5. Smith, footnote 6, points out that 'Whenever a buyer and seller make a contract and drop out of the market, the demand and supply schedules are shifted to the left in a manner depending upon the buyer's and seller's positions in the schedules. Hence, the supply and demand functions continually alter as the trading process occur . . . This means that the intra-trading period schedules are not independent of the transactions taking place.'

6. Rubinstein and Wolinsky are primarily concerned with markets that serve as meeting places where buyers and sellers may identify each other, wander off, and bargain among themselves. If agreement is not reached, the market can be used to find another potential partner. The role of formal institutions within the market, such as the public announcement of bid-ask offers and the bid-ask improvement rule, are not explored (though they are mentioned briefly in the textbook by Osborne and Rubinstein). We deviate from the Rubinstein and Wolinsky steady-state framework by incorporating these institutional features of the double-auction and also by having the higher degree of heterogeneity of agents typical of earlier double auction experiments.
7. Gjerstad and Shachat (1996, p. 14), and John Ledyard, in correspondence, have made the observation that the ZI algorithm and Marshallian path are really special cases of the "B-process" described for a general exchange economy in Hurwicz et al. (1975a, 1975b). In the B-process, agents continually choose points in their upper-contour sets and suggest trades until competitive equilibrium is reached. The ZI algorithm differs only by not allowing resale.
8. The main contribution to our intuition for designing this market environment was not a field example, but instead came from earlier experimental literature. Macro/Micro Experiments with multiple markets involving circular flow among the markets have been previously reported in the literature (Lian and Plott, 1998). In these markets, price ratios converge to certain CE predictions despite the possible absence of a Marshallian dynamic, and to some extent because—to the subjects—the market seems infinitely lived and constantly replenished. The environment studied in this paper was at least partially inspired by the circular flow model and, to the best of our knowledge, represents the first example of an experimental study of continuously refreshed supply and demand in a single market.
9. See Plott (1982) or Smith (1982) for a review of the results and methodologies in use from the 1960s–early 1980s.
10. Implications of revealing this information are unclear since any Pareto improvements in decisions would involve both coordination and public goods problems. Our purpose was to study markets in which the set of redemption values and costs in a market are unchanged by trading without becoming involved with behavior that might be motivated by attempts to optimize payoffs by coordination over time.
11. Changes in private market offers from one environment to another are done without stopping the trading or alerting the subjects, but are distributed in a continuous fashion to create a smoother "transition region."
12. Smith (1962) makes a distinction between the "initial" supply and demand as the basis of equilibria as opposed to instantaneous supply and demand. His primary reason for using the initial supply and demand is that it would be too cumbersome to recalculate the instantaneous supply and demand after each trade. He suggests that the differences between initial and instantaneous equilibria are likely to be small when trading is efficient. However, his concern indicates that the relevant equilibrium is by no means obvious.
13. The idea of a flow or a rate of demanders and suppliers appearing in a market is frequently found in the classical literature. For example Marshall makes an attempt to deal with it by postulating a long run analysis as opposed to a short run analysis.
14. The standard definition of efficiency of allocation in markets with finite supply and demand is a ratio of observed gains from trade to maximum possible: the denominator—the maximum possible gains from trade—is the area between the initial supply and demand curves. The numerator—the observed gains from trade—is a sum of gains from trade determined from each pair of private market orders (gains from trade = redemption value-cost) that are executed as a result of a public market trade. In most reported experiments within the classical environment, observed efficiencies are generally well above 90%. In a CRSD environment, the maximum possible gains from trade could be infinite—as recycling is instantaneous, an infinite number of positive surplus trades could occur in any time interval. If one accepts a maximum rate of trade, then the maximum possible surplus would involve the highest surplus pairing (highest value, lowest cost) at each trade. Neither of these definitions is appealing from a practical perspective, easily and regularly producing "efficiencies" close to 0%.
15. Recall that human traders had 2 minutes to trade a private market unit before it expired and was refreshed back into the market via the screen of the next trader. Computerized ZI traders had no such time limit.
16. We need to be a little cautious here due to the ex-post, aggregative nature of the V-CE model. If we have ZI data for 1 million trades, and calculate a single V-CE model from these trades ex-post, then the ratio of inframarginal or extramarginal trades in subsamples with respect to this V-CE should be IID.

17. These payments were, on average, roughly US$30 per participant.
18. Since there is no natural stopping point in this environment, we could have kept going or terminated the programs earlier. This size does allow us to generate distributions for statistics such as finite-sample price means and variance.
19. Note that the dashed lines also track the I-CE in the region where one period's private values are being changed to another, such as the transition region of trades 793–820 that separates period 1 from period 2 in experiment 1.
20. This should not be surprising as both the model and the simulation are essentially mathematical in nature.
21. Figure 3 shows the price distribution for 1000 repetitions of a interval 1 ZI market with non-refreshed supply and demand. Figure 11 shows the price distribution from 1.5 million trades from a interval 1 ZI market with continuously refreshed supply and demand. In figure 3, the upper pane shows the distribution of prices for all trades in 1000 repetitions of the standard environment. The second pane shows the distribution of prices for the final trades in these 1000 repetitions. Notice that while the variance decreases at the final trade of each interval, the distribution is somewhat skewed and has a large mass in the 40s—well below the CE of $55 < P < 60$. Thus, convergence in the standard environment is not necessarily to the CE although the decrease in variance suggests some sort of convergence is occurring. Because the distribution in figure 11 is even flatter than that of the overall trades in figure 3, and is peaked above the CE, it is clear that price convergence does not occur with ZI robots when there is continuously refreshed supply and demand even though it does occur to a limited extent in the non-refreshed environment.

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