THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **Year: 2024**

Instructions

1. This paper consists of Section A and B.

2. Answer all questions in section A and any four questions in section B.

1. (a) Equal squares which are as large as possible are drawn on a rectangular board measuring 54 cm by 78 cm. Find the size of each square.

(b) In a class of 40 students, 17 students are boys and the rest are girls. Determine:

(i) the percentage of girls and boys.

(ii) the number of boys in decimal.

(a) Find the GCD of 54 and 78:

$54 = 2 \times 3^3$

$78 = 2 \times 3 \times 13$

GCD = $2 \times 3 = 6$

Answer: 6 cm

(b)(i) Total students = 40

Boys = 17

Girls = 40 - 17 = 23

Percentage of boys = (17 / 40) x 100 = 42.5%

Percentage of girls = (23 / 40) x 100 = 57.5%

Answer: Boys: 42.5%, Girls: 57.5%

(b)(ii) Number of boys = 17

2. (a) Express $(5 + \sqrt{7}) / (3 + \sqrt{7})$ in the form $a + b\sqrt{c}$, where a, b, and c are integers.

(b) A line is defined by a logarithmic equation 3 log_3 x + 4 = log_3 24. Without using a mathematical table or calculator, find the value of x.

(a) Rationalize:

$(5 + \sqrt{7}) / (3 + \sqrt{7}) \times (3 - \sqrt{7}) / (3 - \sqrt{7})$

Numerator: $(5 + \sqrt{7})(3 - \sqrt{7}) = 15 - 5\sqrt{7} + 3\sqrt{7} - 7 = 8 - 2\sqrt{7}$

Denominator: $9 - 7 = 2$

$(8 - 2\sqrt{7}) / 2 = 4 - \sqrt{7}$

$a = 4, b = -1, c = 7$

Answer: $4 - \sqrt{7}$

(b) $3 \log\_3 x + 4 = \log\_3 24$ $3 \log\_3 x = \log\_3 24 - 4$ $\log\_3 x^3 = \log\_3$

$24 - \log\_3 81$ $\log\_3 x^3 = \log\_3 (24 / 81) = \log\_3 (8 / 27)$ $x^3 = 8 /$

27 $x = (8 / 27)^\wedge(1/3) = (2^3 / 3^3)^\wedge(1/3) = 2 / 3$

Answer: $x = 2/3$

3. (a) In a school of 100 students, 45 students prefer Music subject, 40 students prefer Theatre Arts subject, and 5 students prefer both subjects. Find the number of students who prefer none of the two subjects by using formula.

(b) The following table shows the number of tables in twenty offices from a certain company:

(i) two tables.

(ii) at least five tables.

(a) $n(M \cup T) = n(M) + n(T) - n(M \cap T)$

$= 45 + 40 - 5 = 80$

None $= 100 - 80 = 20$

Answer: 20 students

(b) Total offices = 20

(i) Offices with 2 tables = 5

P(two tables) = 5 / 20 = 1/4

(ii) Offices with at least 5 tables = 1 + 2 = 3

P(at least five tables) = 3 / 20

Answer: (i) 1/4, (ii) 3/20

4. (a) A quadrilateral has the vertices, A(6,-4), B(8,4), P(6,1), and Q(7,5). Determine whether or not AB is parallel to PQ.

(b) The points P and Q have position vectors a and b, respectively. If $a = 2i - 7j$ and $2a + 3b = 13i - 2j$, find the components of vector b.

(a) Slope of AB:

$(4 - (-4)) / (8 - 6) = 8 / 2 = 4$ Slope of PQ:

$(5 - 1) / (7 - 6) = 4 / 1 = 4$

Slopes are equal, so AB is parallel to PQ.

Answer: AB is parallel to PQ

(b) $a = 2i - 7j$

$2a = 4i - 14j$

$2a + 3b = 13i - 2j$

$4i - 14j + 3b = 13i - 2j$

$3b = (13i - 2j) - (4i - 14j) = 9i + 12j$

$b = (9/3)i + (12/3)j = 3i + 4j$

Answer: $b = 3i + 4j$

5. (a) The area of a regular six-sided plot of land inscribed in a circular track of radius r is 720 m^2. Find the value of r by rounding off the answer to the nearest:

(i) tens.

(ii) tenths.

(b) A firm reserved two triangular plots for the construction of more offices as shown in the following diagram:

(i) Prove that the triangles AEB and CDE are similar.

(ii) Using similarity properties, find the length of CD in metres.

(a) Area of regular hexagon = $(3\sqrt{3} / 2) r^2$

$(3\sqrt{3} / 2) r^2 = 720$

$r^2 = 720 \times 2 / (3\sqrt{3})$

$\sqrt{3} \approx 1.732$ $r^2 \approx 720 \times 2 / (3 \times$

$1.732) \approx 277.136$ $r \approx \sqrt{277.136} \approx$

16.647 (i) Nearest tens: 20

(ii) Nearest tenths: 16.6

Answer: (i) 20 m, (ii) 16.6 m

6. (a) On one rainy day, it was observed that 850 millilitres of water were collected in a tank every minute. If it rained continuously from 8:10 a.m. to 11:52 a.m., calculate in litres, the amount of water collected.

(b) The time t in seconds that is used by Mary to go back home from school varies inversely with her average speed v in metres per second. She gets back home in 30 minutes at an average speed of 10 m/s.

(i) Write an equation expressing t in terms of v.

(ii) If she wants to get back home in 15 minutes, what will be her average speed?

(a) Time from 8:10 a.m. to 11:52 a.m.:

8:10 a.m. to 11:10 a.m. = 3 hours

11:10 a.m. to 11:52 a.m. = 42 minutes

Total = $3 \times 60 + 42 = 222$ minutes

Water = 850 ml/min x 222 min = 188700 ml = 188.7 litres

Answer: 188.7 litres

(b)(i) $t = k / v$

30 min = $30 \times 60 = 1800$ s, $v = 10$ m/s

$1800 = k / 10$

$k = 18000$

$t = 18000 / v$

Answer: t = 18000 / v

(b)(ii) t = 15 min = 15 x 60 = 900 s

900 = 18000 / v v =

18000 / 900 = 20 m/s

7. (a) If the sum and difference of ages of Amina and Bakari are in the ratio 5:4, what would be the ratio of their respective ages?

(b) On 1st July, 2021, a small scale trader started a business with capital in cash of 1,000,000 shillings. Extract a trial balance from this cash account.

(a) Amina's age = A, Bakari's age = B

(A + B) / (A - B) = 5 / 4

4(A + B) = 5(A - B)

4A + 4B = 5A - 5B

A = 9B

A : B = 9 : 1

Answer: 9 : 1

(b) Cash Account:

Dr: Capital: 1000000, Sales: 500000 + 310000 = 810000

Total Dr: 1810000

Cr: Purchases: 700000 + 400000 + 235000 = 1335000, Rent: 110000, Wages: 55000, Balance c/d: 310000

Total Cr: 1810000

Trial Balance:

Account	Dr	Cr
Cash	310000	
Capital		1000000
Purchases	1335000	

Rent	110000	
Wages	55000	
Sales		810000
Total	1810000	1810000

8. (a) An entrepreneur borrowed a loan from a bank under the condition that he is required to repay the loan on monthly installment basis. The entrepreneur repaid 20000, 22000, 24000 shillings for the first, second, and third installments respectively until the loan was fully repaid.

(a) If the final monthly installment was 114000 shillings, how long did the entrepreneur take to repay the whole loan?

(b) Calculate the total amount of loan borrowed.

(a) AP: $a_1 = 20000$, $d = 2000$, $a_n = 114000$

$a_n = a_1 + (n-1)d$

$114000 = 20000 + (n-1) \times 2000$

$94000 = (n-1) \times 2000$

$n - 1 = 47 \quad n = 48$

Answer: 48 months

(b) Sum of AP:

$S_n = (n/2)(a_1 + a_n)$

$S_{48} = (48/2)(20000 + 114000)$

$= 24 \times 134000 = 3216000$

Answer: 3216000 shillings

9. (a) By applying your knowledge of trigonometry show that $4 \sin 75 = \sqrt{6} + \sqrt{2}$.

(b) An electricity post AB is supported by two pieces of wire, AC and AD, on the top at point A and the ground at points C and D. If the length of AC is 10 m, the distance between points C and D is 4 m, and the height of the post is 6 m, find the length of wire AD, leaving your answer in surd form.

(a) $\sin 75 = \sin (45 + 30)$ $\sin (45 + 30) = \sin 45$

$x \cos 30 + \cos 45 \ x \sin 30$

$= (\sqrt{2} / 2) \ x \ (\sqrt{3} / 2) + (\sqrt{2} / 2) \ x \ (1 / 2)$

$= \sqrt{6} / 4 + \sqrt{2} / 4 = (\sqrt{6} + \sqrt{2}) / 4$ 4

$\sin 75 = 4 \ x \ (\sqrt{6} + \sqrt{2}) / 4 = \sqrt{6} + \sqrt{2}$ (b)

Triangle ABC (right at B):

AB = 6 m, AC = 10 m

$BC = \sqrt{(10^2 - 6^2)} = \sqrt{(100 - 36)} = \sqrt{64} = 8$ m

CD = 4 m, assume D beyond C

BD = 8 + 4 = 12 m

Triangle ABD (right at B):

$AD = \sqrt{(AB^2 + BD^2)}$

$= \sqrt{(6^2 + 12^2)} = \sqrt{(36 + 144)} = \sqrt{180} = 6\sqrt{5}$

10. (a) A candidate obtained a minimum average of 53 marks in two Mathematics tests. If she scored 54 marks in the first test, what is the least possible mark she would have scored in the second test?

(b) You are asked to make a fence for a garden using a wire which is 56 metres long. The fence should be rectangular in shape with an area of 171 square metres. What will be the dimensions of the fence enclosing the garden? (a) Average >= 53

$(54 + x) / 2 >= 53$

$54 + x >= 106$ x

$>= 52$

Least mark = 52

Answer: 52 marks

(b) Perimeter: $2(l + w) = 56$

$l + w = 28$

Area: l x w = 171 w = 28

- 1 1 * (28 - l) = 171 l^2 -

28l + 171 = 0 l = [28 ±

√(784 - 684)] / 2

= [28 ± √100] / 2

= [28 ± 10] / 2 l

= 19 or l = 9

If l = 19, w = 28 - 19 = 9

11. (a) In the terminal examination of a certain school, the scores of students in Geography subject were grouped as shown in the following table:

Scores	65-69	70-74	75-79	80-84	85-89	90-94	95-99
Cumulative Frequency	10	22	43	49	58	62	66

Using the information given in the table:

(i) find the mean score correct to 2 decimal places, given an assumed mean of 77.

(ii) draw the cumulative frequency curve (ogive) of the scores.

(iii) calculate the mode of the scores correct to 3 decimal places.

(b) In the following figure, PT is a tangent to a circle whose centre is at point O and the radius is 5 cm. If PC is 8 cm, find the length of PT.

(a)(i) Frequencies:

65-69: 10

70-74: 22 - 10 = 12

75-79: 43 - 22 = 21

80-84: 49 - 43 = 6

85-89: 58 - 49 = 9

90-94: 62 - 58 = 4

95-99: 66 - 62 = 4

Assumed mean = 77, class width = 5

Midpoints: 67, 72, 77, 82, 87, 92, 97   d

= (x - 77) / 5

Class	f	x	d	fd
65-69	10	67	-2	-20
70-74	12	72	-1	-12
75-79	21	77	0	0
80-84	6	82	1	6
85-89	9	87	2	18
90-94	4	92	3	12
95-99	4	97	4	16

Total f = 66, $\Sigma(fd) = 20$

Mean = 77 + (20 / 66) x 5

≈ 77 + 1.515 ≈ 78.52

Answer: 78.52

(a)(ii) Ogive:

Points: (65, 0), (70, 10), (75, 22), (80, 43), (85, 49), (90, 58), (95, 62), (100, 66)

X-axis: Scores (65 to 100)

Y-axis: Cumulative frequency (0 to 70)

Connect with smooth curve

Answer: As described

12. (a)(iii) Modal class: 75-79 ($f_m = 21$)

Mode ≈ 75 + ((21 - 12) / ((21 - 12) + (21 - 6))) x 5

= 75 + (9 / (9 + 15)) x 5

$= 75 + (9 / 24) \times 5$

$\approx 75 + 1.875 = 76.875$

Answer: 76.875

(b) PT tangent, $OC \perp PT$, radius = 5 cm, PC = 8 cm

Triangle OPT:

$PT^2 + 5^2 = 8^2$

$PT^2 = 64 - 25 = 39$

$PT = \sqrt{39}$

Answer: $\sqrt{39}$ cm

13. (a) (i) The surface area of a sphere is 113.04 cm². Find its diameter (Use $\pi = 3.14$).

(ii) 75360 litres of water are poured into a cylindrical tank of inside diameter of 40 cm. Calculate the height of the water level.

(b) Find the distance in nautical miles between each of the following pairs of points:

(i) A(18°N, 12°E) and B(65°N, 12°E).

(ii) C(31°S, 76°W) and D(22°N, 76°W).

(a)(i) $4\pi r^2 = 113.04$ 4 x

$3.14 \times r^2 = 113.04$ $r^2 =$

$113.04 / (4 \times 3.14) \approx 9$

$r = 3$

Diameter $= 2 \times 3 = 6$ cm

Answer: 6 cm

(a)(ii) Volume = 75360 litres = 75360000 cm³

Radius = 40 / 2 = 20 cm

$V = \pi r^2 h$

$75360000 = 3.14 \times 20^2 \times h$ h

$= 75360000 / (3.14 \times 400)$

$\approx 75360000 / 1256 \approx 60000$ cm = 600 m

Answer: 600 m

(b)(i) Latitude difference = 65°N - 18°N = 47°

Distance = 47 x 60 = 2820 nautical miles

Answer: 2820 nautical miles

(b)(ii) Latitude difference = 31°S to 22°N = 31 + 22 = 53°

Distance = 53 x 60 = 3180 nautical miles

Answer: 3180 nautical miles

14. (a) Roza and Juma were asked to write the examples of a 2 × 2 matrix and they came up with (2 3; 5 6) and (-6 2; 4 3) respectively.

(i) Find the sum of twice the Roza's matrix and thrice the Juma's matrix.

(ii) Show that the difference between the determinants of these matrices is ±1.

(b) If a farm measuring 72 m by 88 m is enlarged by a scale factor of 4, what will be the area of the enlarged farm?

(a)(i) Roza: (2 3; 5 -6)

2 x Roza = (4 6; 10 -12)

Juma: (-6 2; 4 3)

3 x Juma = (-18 6; 12 9)

Sum = (4 - 18, 6 + 6; 10 + 12, -12 + 9)

= (-14, 12; 22, -3)

Answer: (-14 12; 22 -3)

(a)(ii) Det(Roza) = 2 x (-6) - 3 x 5 = -12 - 15 = -27

Det(Juma) = (-6) x 3 - 2 x 4 = -18 - 8 = -26

Difference = -27 - (-26) = -1 or

-26 - (-27) = 1

Answer: ±1

(b) Area = 72 x 88 = 6336 m^2

Scale factor = 4

Area scale factor = 4^2 = 16

New area = 6336 x 16 = 101376 m^2

Answer: 101376 m^2

15. (a) During an inter-school quiz competition each correct answer y was awarded a number of points x related by the function $y = 3x^2$.

(i) How many points would a competitor collect by answering 8 questions correctly?

(ii) Find the domain and range of the inverse of the given function.

(b) A cook wishes to mix two types of foods, I and II in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B, and 8 units of vitamin C. Formulate the linear programming model to minimize the cost and find the minimum cost.

(a)(i) $y = 3x^2$ $y(8) = 3 \times 8^2$

= 3 x 64 = 192

Answer: 192 points

(a)(ii) $y = 3x^2$ x

$= \sqrt{(y / 3)}$

Domain of inverse: y >= 0

Range of inverse: x >= 0

Answer: Domain: $[0, \infty)$, Range: $[0, \infty)$

(b)(i) x = kg of Food I, y = kg of Food II

Minimize: C = 6000x + 10000y

Constraints:

x + 2y >= 10 2x

+ 2y >= 12 x +

y >= 6   3x + y

>= 8   x >= 0,

y >= 0

Answer: Minimize C = 6000x + 10000y, subject to x + 2y >= 10, x + y >= 6, 3x + y >= 8, x >= 0, y >= 0

(b)(ii) Vertices: x + y = 6, x + 2y = 10:

y = 4, x = 2

C = 6000 x 2 + 10000 x 4 = 12000 + 40000 = 52000

x + y = 6, 3x + y = 8:

x = 1, y = 5

C = 6000 x 1 + 10000 x 5 = 6000 + 50000 = 56000

Minimum at (2, 4)

Answer: 52000 Tshs