

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041

BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours

Year: 2018

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

1. (a). If $m = 0.27$ and $n = 0.15$, find the fraction n/m in its simplest form.

(b). Find the GCF of 210, 357 and 252.

(a) $m = 0.27$

$$100m = 27.27$$

$$100m - m = 27.27 - 0.27$$

$$99m = 27 \quad m =$$

$$27/99 = 3/11 \quad n =$$

$$0.15$$

$$100n = 15.15$$

$$100n - n = 15.15 - 0.15$$

$$99n = 15 \quad n = 15/99 = 5/33 \quad n/m = (5/33) / (3/11)$$

$$= 5/33 \times 11/3 = 55/99 = 5/9$$

Answer: $5/9$

(b) Factorize:

$$210 = 2 \times 3 \times 5 \times 7$$

$$357 = 3 \times 7 \times 17$$

$$252 = 2^2 \times 3^2 \times 7$$

$$\text{GCF} = 3 \times 7 = 21$$

Answer: 21

2. (a). Evaluate $\log_{10} 40,500$ given that $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$ and $\log_{10} 5 = 0.6990$.

(b). Find the values of x and y if $3^{(x+2)} / 5^{(2y-5)} = 2025$.

(a) $40500 = 405 \times 100 = 5 \times 81 \times 100 = 5 \times 3^4 \times 10^2$ $\log_{10} 40500 = \log_{10} (5 \times$

$$3^4 \times 10^2) = \log_{10} 5 + 4 \log_{10} 3 + 2 \log_{10} 10$$

$$= 0.6990 + 4 \times 0.4771 + 2 \times 1$$

$$= 0.6990 + 1.9084 + 2 = 4.6074$$

Answer: 4.6074

$$(b) 2025 = 3^4 \times 5^2$$

$$3^{(x+2)} / 5^{(2y-5)} = 3^4 \times 5^2$$

$$3^{(x+2)} \times 5^{(5-2y)} = 3^4 \times 5^2$$

$$\text{Equate exponents: } x + 2 = 4$$

$$\rightarrow x = 2$$

$$5 - 2y = 2 \rightarrow -2y = -3 \rightarrow y = 3/2$$

Answer: $x = 2, y = 3/2$

3. (a). In a school of 60 teachers, some drink Fanta and some drink Coca-Cola. If 46 drink Fanta, 18 drink Coca-Cola and 14 drink both Coca-Cola and Fanta. How many teachers drink neither Fanta nor Coca-Cola? (Use Venn diagram)

(b). Use the figure below to answer the following questions:

(i). Write the expression for the total area of rectangles A and B.

(ii). If the total area of rectangles A and B is 98 square centimeters, find the value of x.

(a) Fanta (F): 46, Coca-Cola (C): 18, $F \cap C$: 14

$$F \text{ only} = 46 - 14 = 32$$

$$C \text{ only} = 18 - 14 = 4$$

$$F \cup C = 32 + 14 + 4 = 50$$

$$\text{Neither} = 60 - 50 = 10$$

Answer: 10 teachers

(b)

(i) Area A = $(x + 2)(2x)$, Area B = $(x + 1)(x + 14)$

$$\text{Total area} = (x + 2)(2x) + (x + 1)(x + 14)$$

(ii) Given, $(x + 2)(2x) + (x + 1)(x + 14) = 98$

$$x = 2.6 \text{ cm}$$

4. (a). If $a = 2x \mathbf{i} + 3 \mathbf{j}$, $b = (x^2 + y) \mathbf{i} + 4y \mathbf{j}$ and $v = (8/3) \mathbf{i} + (15/12) \mathbf{j}$. Find x and y given that $v = (1/4) a + (1/3) b$.

(b). Find the point of intersection of the lines $x - 2y = -5$ and $2x + 7y - 34 = 0$.

$$(a) v = (1/4)(2x \mathbf{i} + 3 \mathbf{j}) + (1/3)((x^2 + y) \mathbf{i} + 4y \mathbf{j})$$

$$= (x/2 + (x^2 + y)/3) \mathbf{i} + (3/4 + 4y/3) \mathbf{j}$$

$$\text{Equate to } v = (8/3) \mathbf{i} + (15/12) \mathbf{j} = (8/3) \mathbf{i} + (5/4) \mathbf{j}$$

$$\mathbf{i}: x/2 + (x^2 + y)/3 = 8/3 \quad \mathbf{j}: 3/4 + 4y/3 = 5/4 \quad \mathbf{j}: 4y/3$$

$$= 5/4 - 3/4 = 1/2 \quad y = 3/8 \quad \mathbf{i}: x/2 + (x^2 + 3/8)/3 =$$

$$8/3$$

$$3x + 2x^2 + 3/4 = 16 \quad 2x^2$$

$$+ 3x - 61/4 = 0 \quad x = [-3$$

$$\pm \sqrt{(9 + 122)} / 4 \quad x = [-3$$

$$\pm 11] / 4 \quad x = 2 \text{ or } x = -$$

$$7/2 \quad y = 3/8$$

$$\text{Answer: } (2, 3/8) \text{ or } (-7/2, 3/8)$$

(b) $x - 2y = -5 \rightarrow x = 2y - 5$

$$2x + 7y - 34 = 0$$

$$2(2y - 5) + 7y - 34 = 0$$

$$4y - 10 + 7y - 34 = 0$$

$$11y = 44 \quad y$$

$$= 4$$

$$x = 2(4) - 5 = 3$$

$$\text{Answer: } (3, 4)$$

5. In the following figure, a regular hexagon is inscribed in a circle. If the perimeter of the hexagon is 42 cm, find:

(a) The radius of the circle.

(b) The area of the circle and the regular polygon.

(c) The area of the shaded region.

(a) Perimeter = 42 cm

Side of hexagon = $42/6 = 7$ cm

In regular hexagon, radius of circumcircle = side length

Radius = 7 cm

Answer: 7 cm

(b) Circle area = $\pi r^2 = \pi \times 7^2 = 49\pi$ cm²

Hexagon area = $(3\sqrt{3}/2) \times \text{side}^2 = (3\sqrt{3}/2) \times 7^2 = 147\sqrt{3}/2 \approx 127.306$ cm²

Answer: Circle: 49π cm², Hexagon: $147\sqrt{3}/2$ cm²

(c) Shaded = Circle - Hexagon = $49\pi - 147\sqrt{3}/2 \approx 153.938 - 127.306 = 26.632$

Answer: 26.6 cm²

6. (a) Mukasa received Ushs 1,000,000 from his sister in Uganda. How much did he get in Tanzanian currency (Tshs) if one Ugandan shilling was equivalent to 0.65 Tanzanian shilling?

(b) The energy (E) stored in an elastic band varies as the square of the extension (x). When the elastic band is extended by 4 cm, the energy stored is 240 joules. What is the energy stored when the extension is 6 cm? What is the extension when the stored energy is 60 joules?

(a) $1,000,000 \times 0.65 = 650,000$

Answer: 650,000 Tshs

(b) $E \propto x^2$

$$E = kx^2$$

$$240 = k \times 4^2$$

$$= 240/16 = 15$$

$$E = 15x^2$$

For $x = 6$: $E = 15 \times 6^2 = 15 \times 36 = 540$

For $E = 60$: $60 = 15x^2$ $x^2 = 4$ $x = 2$

Answer: 540 joules, 2 cm

7. (a) Three relatives shared Tshs 140,000 so that the first one got twice as much as the second, and the second got twice as much as the third. How much money did the first relative get ADAM?

(b) Kitwana paid Tshs 900,000 for a desktop computer and sold it the following year for Tshs 720,000. Find:

(i) The loss made,

(ii) The percentage loss.

(a) Third = x

Second = $2x$

First = $4x$

$$4x + 2x + x = 140,000$$

$$7x = 140,000 \quad x =$$

$$20,000$$

$$\text{First} = 4 \times 20,000 = 80,000$$

Answer: 80,000 Tshs

$$(b)(i) \text{ Loss} = 900,000 - 720,000 = 180,000$$

Answer: 180,000 Tshs

$$(b)(ii) \% \text{ loss} = (180,000 / 900,000) \times 100 = 20\%$$

Answer: 20%

8. (a) If an arithmetic progression has A_1 as the first term and d as the common difference, (i)

write the second, third, fourth and fifth terms.

(ii) Establish the formula for the sum of the first five terms of the arithmetic progression by using the results in part (i).

(b) The first and second terms of a geometric progression are 3 and 9 respectively.

(i) Find the third, fourth and fifth terms.

(ii) Verify that the sum of the first 5 terms is given by $S_n = G_1 (r^n - 1)/(r - 1)$ by using the results in part (i).

(a)(i) $A_1, A_1 + d, A_1 + 2d, A_1 + 3d, A_1 + 4d$

Answer: $A_1 + d, A_1 + 2d, A_1 + 3d, A_1 + 4d$

(a)(ii) $\text{Sum} = A_1 + (A_1 + d) + (A_1 + 2d) + (A_1 + 3d) + (A_1 + 4d)$

$$= 5A_1 + (0 + 1 + 2 + 3 + 4)d$$

$$= 5A_1 + 10d$$

$$= \frac{5}{2} (2A_1 + 4d)$$

Answer: $S_5 = \frac{5}{2} (2A_1 + 4d)$

(b)(i) $r = 9/3 = 3$

$$\text{Third} = 9 \times 3 = 27$$

$$\text{Fourth} = 27 \times 3 = 81$$

$$\text{Fifth} = 81 \times 3 = 243$$

Answer: 27, 81, 243

(b)(ii) $\text{Sum} = 3 + 9 + 27 + 81 + 243 = 363$

$$S_n = 3 (3^5 - 1)/(3 - 1) = 3 \times (243 - 1)/2 = 3 \times 242/2 = 363$$

Answer: Verified, $S_5 = 363$

9. (a) Find the distance PR in the following figure if the lines PR and RQ are perpendicular.

(b) A flagpole is 5 meters high. Find to the nearest cm, the length of its shadow when the elevation of the sun is 60° .

(a) [No figure; cannot solve]

Answer: Cannot solve without figure

$$(b) \tan 60^\circ = \sqrt{3} = 5/\text{shadow}$$

$$\text{Shadow} = 5/\sqrt{3} \approx 2.886$$

Answer: 289 cm

10. (a) Use factorization method to solve the quadratic equation $x^2 - 9x + 14 = 0$.

(b) Find the values of x that satisfies the equation $1350/x - 1350/(x+3) = 5$.

$$(a) x^2 - 9x + 14 = (x - 7)(x - 2) = 0 \quad x = 7, x = 2$$

Answer: $x = 7, 2$

$$(b) 1350/x - 1350/(x+3) = 5$$

$$1350(x+3 - x)/(x(x+3)) = 5$$

$$1350 \times 3 = 5x(x+3)$$

$$4050 = 5x^2 + 15x \quad 5x^2 + 15x - 4050 = 0 \quad x^2 +$$

$$3x - 810 = 0 \quad x = [-3 \pm \sqrt{(9 + 3240)}] / 2 = [-3$$

$$\pm \sqrt{3249}] / 2$$

$$\sqrt{3249} \approx 57$$

$$x = (54/2) = 27 \text{ or } x = (-60/2) = -30$$

Answer: $x = 27, -30$

11. A farmer needs to buy up to 25 cows for a new herd. He can buy either brown cows at 50,000/= each or black cows at 80,000/= each and he can spend a total of not more than 1,580,000/=. He must have at least 9 cows of each type. On selling the cows he will make a profit of 5,000/= on each brown cow and 6,000/= on each black cow. How many of each type he should buy to maximize profit?

Let x = brown cows, y = black cows

$$\text{Maximize: } P = 5000x + 6000y$$

Constraints:

$$x \geq 9, y \geq 9$$

$$x + y \leq 25$$

$$50000x + 80000y \leq 1,580,000 \rightarrow 5x + 8y \leq 158 \quad \text{Vertices:}$$

$$(9,9): P = 5000(9) + 6000(9) = 45,000 + 54,000 = 99,000$$

$$(9,16): 5(9) + 8y = 158 \rightarrow 8y = 113 \rightarrow y \approx 14.125, \text{ use } y = 14 \text{ P}$$

$$= 5000(9) + 6000(14) = 45,000 + 84,000 = 129,000$$

$$(16,9): P = 5000(16) + 6000(9) = 80,000 + 54,000 = 134,000$$

Maximum at (16,9)

Answer: 16 brown, 9 black

12. The scores of 45 pupils in a Civics test were recorded as follows:

30, 65, 50, 62, 40, 35, 64, 32, 28, 59, 60, 82, 24, 35, 63, 68, 46, 48, 73, 92, 54, 46, 63, 75, 58, 43, 71, 72, 27, 28, 61, 71, 36, 64, 80, 61, 64, 76, 64, 35, 76, 73, 70, 64, 46

(a) Construct a frequency distribution table of the given data, taking equal class intervals 21-40, 41-60, ...

(b) Calculate the mean score.

(c) Draw the cumulative frequency curve and use it to estimate the median.

(a) 21-40: 24, 27, 28, 28, 30, 32, 35, 35, 36 \rightarrow 9

41-60: 43, 46, 46, 46, 48, 50, 54, 58, 59, 60 \rightarrow 10

61-80: 61, 61, 62, 63, 63, 64, 64, 64, 64, 65, 68, 70, 71, 71, 72, 73, 73, 75, 76, 76, 80 \rightarrow 21

81-100: 82, 92 \rightarrow 2 Table:

Class	Frequency
21 – 40	9
41 – 60	10
61 – 80	21
81 – 100	2

(b) Midpoints: 30.5, 50.5, 70.5, 90.5

$$\text{Sum} = (30.5 \times 9) + (50.5 \times 10) + (70.5 \times 21) + (90.5 \times 2)$$

$$= 274.5 + 505 + 1480.5 + 181 = 2441$$

$$\text{Mean} = 2441 / 45 \approx 54.244$$

Answer: 54.2

(c) Cumulative frequency:

21-40: 9

41-60: 19

61-80: 40

81-100: 42

Median: $45/2 = 22.5$ th term, in 61-80

$$\text{Median} \approx 61 + (22.5 - 19)/21 \times 20 \approx 63.33$$

Answer: 63.3

13. (a) In the following cuboid, $AB = 5$ cm, $BC = 12$ cm and $BG = 10$ cm. Calculate:

(i) The length of AH (give your answer correct to one decimal place).

(ii) The angle CAH.

(b) In the following figure A, B, C and D lie on the circle; O is the centre of the circle, BD is its diameter and PAT is the tangent of the circle at A.

If angle $ABD = 59^\circ$, $CDB = 35^\circ$, find ACD , ADB , DAT and CAO .

(a)(i) In cuboid, AH is diagonal

$$AH^2 = AB^2 + BC^2 + BG^2 = 5^2 + 12^2 + 10^2 = 25 + 144 + 100 = 269$$

$$AH = \sqrt{269} \approx 16.401$$

Answer: 16.4 cm

(a)(ii) In triangle CAH:

$$CA = BG = 10 \text{ cm}, AH = 16.4 \text{ cm}, CH = AB = 5 \text{ cm}$$

$$\cos(\angle CAH) = (CA^2 + AH^2 - CH^2) / (2 \times CA \times AH)$$

$$= (100 + 269 - 25) / (2 \times 10 \times 16.4) = 344 / 328 \approx 1.0488$$

(b) BD is diameter, so $\angle BAD = 90^\circ$, $\angle BCD = 90^\circ$

$\angle ABD = 59^\circ$, $\angle CDB = 35^\circ$

In triangle ABD:

$\angle ADB = 180^\circ - 90^\circ - 59^\circ = 31^\circ$

In triangle BCD:

$\angle BCD = 90^\circ$, $\angle CDB = 35^\circ$, $\angle CBD = 55^\circ$

$\angle ACD = \angle CBD = 55^\circ$ (angles in same segment)

PAT tangent at A, so $\angle DAT = 90^\circ$

In triangle CAD, need $\angle CAO$, but insufficient data

Answer: $\angle ACD = 55^\circ$, $\angle ADB = 31^\circ$, $\angle DAT = 90^\circ$, $\angle CAO$ cannot be determined

14. Mwanne commenced business on 1st April, 2015 with capital in cash 200,000/= April 2 bought goods for cash 100,000/= 3 bought goods for cash 300,000/= 4 purchased shelves for cash 230,000/= 5 sold goods for cash 400,000/= 9 paid wages for cash 50,000/= 12 purchased goods for cash 70,000/= 13 sold goods for cash 600,000/= 16 paid rent for cash 100,000/= 20 bought goods for cash 60,000/= 25 sold goods for cash 300,000/= 27 paid salary for cash 70,000/=

Prepare the following:

(a) Cash account,

(b) Trial balance.

(a) Cash Account:

Dr:

01/04 Capital: 200,000

05/04 Sales: 400,000

13/04 Sales: 600,000

25/04 Sales: 300,000

Total: 1,500,000 Cr:

02/04 Goods: 100,000

03/04 Goods: 300,000

04/04 Shelves: 230,000

09/04 Wages: 50,000

12/04 Goods: 70,000

16/04 Rent: 100,000

20/04 Goods: 60,000

27/04 Salary: 70,000

Total: 980,000

Balance c/d: 520,000 (b)

Trial Balance:

Account	Dr	Cr
Capital		200,000
Purchases	530,000	
Sales		1,300,000
Shelves	230,000	
Wages	50,000	
Rent	100,000	
Salary	70,000	
Cash	520,000	
Total	1,500,000	1,500,000

15. (a) Find the point $P(x, y)$ if $(2 \ 3; 4 \ -1)(x, y) = (-23; -11)$.

(b) A translation T maps point $P(x, y)$ in part (a) into $(3, 2)$. Find where it takes the point $(7, 4)$.

(c) Find the image of the point obtained in part (b) under a rotation of 90° followed by another rotation of 180° anticlockwise.

$$(a) (2x + 3y; 4x - y) = (-23; -11)$$

$$2x + 3y = -23 \quad (1)$$

$$4x - y = -11 \quad (2)$$

$$(2): y = 4x + 11$$

$$(1): 2x + 3(4x + 11) = -23$$

$$2x + 12x + 33 = -23$$

$$14x = -56 \quad x = -4$$

$$y = 4(-4) + 11 = -5$$

Answer: $(-4, -5)$

$$(b) T: (-4, -5) \rightarrow (3, 2)$$

$$\text{Translation vector: } (3 - (-4), 2 - (-5)) = (7, 7)$$

$$T(7, 4) = (7 + 7, 4 + 7) = (14, 11)$$

Answer: $(14, 11)$

$$(c) \text{ Point: } (14, 11)$$

$$90^\circ \text{ anticlockwise: } (x, y) \rightarrow (-y, x) \rightarrow (-11, 14)$$

$$180^\circ \text{ anticlockwise: } (-x, -y) \rightarrow (-(-11), -14) = (11, -14)$$

Answer: $(11, -14)$

16. (a) A bag contains 6 white shirts and 3 blue shirts. Three shirts are picked at random one after another with replacement. Determine the probability that:

(i) All three shirts are blue in colour,

(ii) Two shirts are white and one shirt is blue,

(iii) One shirt is white and two shirts are blue.

(b) The function f is defined by $f(x) = \begin{cases} -2 & \text{if } x < -1; \\ 0 & \text{if } x = -1; \\ x + 2 & \text{if } x \geq -1 \end{cases}$

(i) Sketch the graph of f .

(ii) Use the graph to determine the domain and range of f .

(a)(i) $P(\text{blue}) = 3/9 = 1/3$

$P(\text{all blue}) = (1/3)^3 = 1/27$

Answer: $1/27$

(a)(ii) $P(\text{white}) = 6/9 = 2/3$

Sequences: WWB, WBW, BWW

$P = 3 \times (2/3)^2 \times (1/3) = 3 \times 4/9 \times 1/3 = 4/9$

Answer: $4/9$

(a)(iii) Sequences: WBB, BWB, BBW

$P = 3 \times (2/3) \times (1/3)^2 = 3 \times 2/3 \times 1/9 = 2/9$

Answer: $2/9$

(b)(i) $x < -1$: $y = -2$, open at $(-1, -2)$ $x =$

-1 : $y = 0$, point $(-1, 0)$ $x \geq -1$: $y = x + 2$,

line from $(-1, 1)$ closed

(b)(ii) Domain: All real numbers

Range: $y = -2$ for $x < -1$, $y = 0$ at $x = -1$, $y \geq 1$ for $x > -1$

Answer: Domain: \mathbb{R} , Range: $\{-2, 0\} \cup [1, \infty)$