THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **Year: 2022**

Instructions

1. This paper consists of Section A and B.

2. Answer all questions in section A and any four questions in section B.

1. (a) Find the percentage of numbers which are multiples of 5 from the set $\{1, 2, 3, ..., 52\}$. Write the answer correct to one decimal place.

(b) (i) Arrange the following fractions in ascending order: 1/2, 2/9, 3/8, 1/12, 2/5.

(ii) Simplify the expression $(7 \times 10^4) / 0.000035$, hence write the answer in standard form.

(a) Multiples of 5: 5, 10, ..., 50

Number of terms: 50 / 5 = 10

Total numbers: 52

Percentage = $(10 / 52) \times 100 \approx 19.2308$

Answer: 19.2%

(b)(i) LCM of denominators 2, 9, 8, 12, 5 = 360

1/2 = 180/360

2/9 = 80/360

3/8 = 135/360

1/12 = 30/360

2/5 = 144/360

Order: 30, 80, 135, 144, 180

Answer: 1/12, 2/9, 3/8, 2/5, 1/2

(b)(ii) $(7 \times 10^4) / 0.000035 = (7 \times 10^4) / (3.5 \times 10^{-5})$

$= (7 / 3.5) \times 10^{4+5} = 2 \times 10^9$

Answer: 2×10^9

2. (a) Find the value of x if 8^(x-1) = 16.

(b) (i) Simplify the expression $\log_a \sqrt{a} + \log_a (a^2)$.

(ii) Rationalise the denominator of the expression $(5 + \sqrt{2}) / (\sqrt{6} - \sqrt{2})$.

(a) 8^(x-1) = 16

$8 = 2^3$, $16 = 2^4$

$(2^3)^{(x-1)} = 2^4$

$2^{(3x-3)} = 2^4$

$3x - 3 = 4$

$3x = 7$ x

$= 7/3$

Answer: $x = 7/3$

(b)(i) $\log_a \sqrt{a} + \log_a (a^2) = \log_a (a^{(1/2)}) + \log_a (a^2)$

$= (1/2) \log_a a + 2 \log_a a$

$= 1/2 + 2 = 5/2$

Answer: 5/2

(b)(ii) $(5 + \sqrt{2}) / (\sqrt{6} - \sqrt{2})$

$= (5 + \sqrt{2})(\sqrt{6} + \sqrt{2}) / (\sqrt{6} - \sqrt{2})(\sqrt{6} + \sqrt{2})$

$= (5\sqrt{6} + 5\sqrt{2} + \sqrt{12} + 2) / (6 - 2)$

$= (5\sqrt{6} + 5\sqrt{2} + 2\sqrt{3} + 2) / 4$

Answer: $(5\sqrt{6} + 5\sqrt{2} + 2\sqrt{3} + 2) / 4$

3. (a) (i) If P = {all multiples of 5 less than 35} and Q = {all odd numbers between 14 and 30}, find $P \cap Q$.

(ii) In a village of 50 farmers, 25 grow cashew nut, and 16 grow both cashew nut and maize. If 10 farmers grow neither cashew nut nor maize, find the number of farmers who grow maize only. Do not use Venn diagram.

(b) A farmer was given three seeds to germinate in a nursery. The probability that a seed will germinate is 1/3. Using a tree diagram, find the probability that at least two seeds will germinate.

(a)(i) P = {5, 10, 15, 20, 25, 30}

Q = {15, 17, 19, 21, 23, 25, 27, 29}

$P \cap Q = \{15, 25\}$

Answer: $\{15, 25\}$

(a)(ii) Total = 50, Cashew (C) = 25, $C \cap M = 16$, Neither = 10

C only + $C \cap M$ + M only + Neither = 50

(C only + $C \cap M$) + M only + Neither = 50

25 + M only + 10 = 50

M only = 15

Answer: 15 farmers

(b) P(germinate) = 1/3, P(not germinate) = 2/3

Outcomes (GGG, GGN, GNG, NGG, GNN, NGN, NNG, NNN):

At least 2: GGG, GGN, GNG, NGG

$P(GGG) = (1/3)^3 = 1/27$

$P(GGN) = P(GNG) = P(NGG) = (1/3)^2 \times (2/3) = 2/27$

P(at least 2) $= 1/27 + 3 \times (2/27) = 1/27 + 6/27 = 7/27$

4. (a) If a = (4,3), b = (-4,1), and c = (2,5), determine which of the vectors a + 2b and 3a + c is longer than the other.

(b) If a line passing through the point (4,2) is perpendicular to another line whose equation is $2x + 3y + 14 = 0$, find the equation of the line.

(a) $a + 2b = (4,3) + 2(-4,1) = (4 - 8, 3 + 2) = (-4, 5)$

Magnitude $= \sqrt{(-4)^2 + 5^2} = \sqrt{16 + 25} = \sqrt{41}$

$3a + c = 3(4,3) + (2,5) = (12,9) + (2,5) = (14, 14)$

Magnitude $= \sqrt{14^2 + 14^2} = \sqrt{196 + 196} = \sqrt{392} = 14\sqrt{2}$

$\sqrt{41} \approx 6.4$, $14\sqrt{2} \approx 19.8$

Answer: 3a + c is longer

(b) $2x + 3y + 14 = 0$ $3y = -2x - 14$ $y = (-2/3)x - 14/3$ Slope = -

2/3

Perpendicular slope = 3/2

Line through (4,2):

$y - 2 = (3/2)(x - 4)$

$y - 2 = (3/2)x - 6$ y

$= (3/2)x - 4$

Answer: $y = (3/2)x - 4$

5. (a) The sides of a triangle are 4 cm, 5 cm, and 6 cm. If the longest side of a similar triangle is 18 cm, find the lengths of the other sides.

(b) (i) The perimeter of a regular hexagon inscribed in a circle is 72 cm. Find the radius of the circle.

(ii) The area of triangle ABC is 70 cm^2. If AB = 14 cm and AC = 20 cm, find the angle BAC.

(a) Original: 4, 5, 6 (longest 6)

Similar: longest = 18

Ratio = 18 / 6 = 3

Other sides: $4 \times 3 = 12$, $5 \times 3 = 15$

Answer: 12 cm, 15 cm

(b)(i) Perimeter = 72 cm

Side = 72 / 6 = 12 cm

Radius = side = 12 cm

Answer: 12 cm

(b)(ii) Area = $(1/2) \times AB \times AC \times \sin(\angle BAC)$

$70 = (1/2) \times 14 \times 20 \times \sin(\angle BAC)$

$70 = 140 \times \sin(\angle BAC)$ $\sin(\angle$

$BAC) = 70 / 140 = 1/2$

$\angle BAC = 30°$

6. (a) Anna walks 24 km every day. Find, in metres, the distance she walks in 2 days.

(b) (i) A dealer sells mattresses whose cost price (C) is directly proportional to the selling price (s). If the selling price and the cost price of one mattress are Tsh. 20,000 and Tsh. 18,000, respectively, find the constant of proportionality.

(ii) By using the answer obtained in part (b)(i), determine the equation that relates the cost price and the selling price.

(a) 24 km = 24,000 m

2 days = $2 \times 24{,}000 = 48{,}000$ m

Answer: 48,000 m

(b)(i) $C = k\ s$

$18{,}000 = k \times 20{,}000$ $k =$

$18{,}000 / 20{,}000 = 0.9$

Answer: $k = 0.9$

(b)(ii) $C = 0.9\ s$

Answer: $C = 0.9s$

7. (a) A damaged table that costs Tshs. 20,000 was sold at a loss of 15%. Find the loss made and the selling price.

(b) Extract a Trial Balance from the following Mabala's cash account.

(a) Loss = 15% of 20,000 = $0.15 \times 20{,}000 = 3{,}000$

Selling price = 20,000 - 3,000 = 17,000

Answer: Loss: 3,000 Tshs, Selling price: 17,000 Tshs (b)

Cash Account:

Dr: Capital: 100,000, Sales: 43,000 + 47,000 = 90,000

Total Dr: 190,000

Cr: Purchases: 80,000, Telephone bills: 28,000, Balance c/d: 82,000

Total Cr: 190,000

Trial Balance:

Account	Dr	Cr
Cash	82000	
Capital		100000
Purchases	80000	
Telephone bills	28000	
Sales		90000
Total	190000	190000

8. (a) The fifth and eleventh terms of an arithmetic progression are 8 and -34 respectively. Find the sum of the first ten terms.

(b) A school wishes to invest Tshs. 100,000,000 in a bank which pays an interest rate of 2% compounded annually.

(i) Find the total amount of money that will be accumulated after two years.

(ii) Calculate the interest after two years.

(a) $a_5 = a + 4d = 8$

$a_{11} = a + 10d = -34$

Subtract: $6d = -42$

$d = -7$ $a + 4(-7) =$

8

$a - 28 = 8$ $a = 36$

$S_{10} = (10/2)\ [2a + (10-1)d]$

$= 5 [2 \times 36 + 9 \times (-7)]$

$= 5 [72 - 63] = 5 \times 9 = 45$

(b)(i) $A = P (1 + r/100)^n$

$= 100,000,000 (1 + 2/100)^2$

$= 100,000,000 \times (1.02)^2$

$= 100,000,000 \times 1.0404 = 104,040,000$

Answer: 104,040,000 Tshs

(b)(ii) Interest = 104,040,000 - 100,000,000 = 4,040,000

Answer: 4,040,000 Tshs

9. (a) (i) Find the value of sin 690° / cos 690° without using mathematical table.

(ii) A rectangular garden ABCD is 400 m long and 300 m wide. The seedlings are to be planted along the diagonal BD at equal intervals of 1.25 m. Find the number of seedlings that were planted.

(b) From the top of a tower which is 50 m high, the angle of depression of a car parked on the ground is 30°. How far is the car from the base of the tower? Leave the answer in surd form.

(a)(i) $\sin 690° / \cos 690°$ $690° = 360° + 330°$ $\sin 690° =$

$\sin 330° = \sin (360° - 30°) = -\sin 30° = -1/2$ $\cos 690° =$

$\cos 330° = \cos (360° - 30°) = \cos 30° = \sqrt{3}/2$

$(-1/2) / (\sqrt{3}/2) = -1 / \sqrt{3} = -\sqrt{3} / 3$

Answer: $-\sqrt{3} / 3$

(a)(ii) Diagonal BD $= \sqrt{(400^2 + 300^2)}$

$= \sqrt{(160000 + 90000)} = \sqrt{250000} = 500$ m

Number of seedlings = 500 / 1.25 = 400

Answer: 400 seedlings

(b) $\tan 30° = 50 / d$ d

$= 50 / (1/\sqrt{3}) = 50\sqrt{3}$

Answer: $50\sqrt{3}$ m

10. (a) Express the equation $2t^{(-10)} - 3t^{(-5)} + 1 = 0$ in terms of x where $x = 1/t^5$.

(b) From the equation you obtained in part (a), find the value(s) of x that satisfy it by using the quadratic formula.

(a) $x = 1/t^5 = t^{(-5)}$ t^(-

10) $= (t^{(-5)})^2 = x^2$ $2x^2$ -

$3x + 1 = 0$

Answer: $2x^2 - 3x + 1 = 0$

(b) $2x^2 - 3x + 1 = 0$ x =

$[3 \pm \sqrt{(9 - 8)}] / 4 = [3 \pm$

$1] / 4$ $x = 1$ or $x = 1/2$

Answer: $x = 1, 1/2$

11. (a) Find the central angle (in degrees) made by an arc of length 22 cm in a circle whose radius is 63 cm. Use $\pi = 22/7$.

(b) In the following figure, prove that the angles x and y are supplementary given that a and b are the angles at the centre of the circle.

(c) In the following figure, AE = 8 cm, BE = 3 cm and CE = 4 cm. Find the length of DE.

(a) Arc length $= r\theta$ $22 = 63 \times \theta$ $\theta = 22 / 63 = 2/9$ radians

Degrees $= (2/9) \times (180/\pi) = (2/9) \times (180 \times 7/22)$

$= (2 \times 180 \times 7) / (9 \times 22) = 140 / 11 \approx 12.73°$

Answer: 12.7°

(b) [No figure; assume x, y are angles subtended by arcs at circumference, a, b at center]

Angles at center: $a + b = 360°$

Angles at circumference: $x = a/2$, $y = b/2$

$x + y = (a + b)/2 = 360/2 = 180°$

Answer: $x + y = 180°$, supplementary

12. (a) A bus leaves town A(3°S, 39°E) at a constant speed of 40 km/h. How many hours will the bus take to reach town B(12°S, 39°E)? Use $\pi = 3.14$ and radius of the Earth, $R = 6{,}400$ km.

(b) A box has a rectangular base UVXY with plane PQRS being vertically above UVXY.

If UV = 4.2 cm, VX = 2 cm and XR = 2.5 cm, find; (i) the

length of VR and UR, correct to one decimal place.

(ii) the angle between the diagonal UR and the base UVXY.

(a) Latitude difference = 12° - 3° = 9° $\theta = 9 \times$

$\pi/180 = 9 \times 3.14 / 180 \approx 0.1571$ radians

Distance = $6400 \times 0.1571 \approx 1005.44$ km

Time = $1005.44 / 40 \approx 25.136$

Answer: 25.1 hours

(b)(i) $VR = \sqrt{(VX^2 + XR^2)} = \sqrt{(2^2 + 2.5^2)} = \sqrt{(4 + 6.25)} = \sqrt{10.25} \approx 3.20$ cm

$UR = \sqrt{(UV^2 + VX^2 + XR^2)} = \sqrt{(4.2^2 + 2^2 + 2.5^2)}$

$= \sqrt{(17.64 + 4 + 6.25)} = \sqrt{27.89} \approx 5.28$ cm

Answer: VR = 3.2 cm, UR = 5.3 cm

(b)(ii) Angle between UR and base UVXY:

Normal to UVXY: (0, 0, 1) UR vector: (-4.2, -2, 2.5) $\cos\theta = (0$

$\times (-4.2) + 0 \times (-2) + 1 \times 2.5) / (\sqrt{(4.2^2 + 2^2 + 2.5^2)} \times 1)$

$= 2.5 / \sqrt{27.89} \approx 2.5 / 5.28 \approx 0.4735$ θ

$\approx \cos^{-1}(0.4735) \approx 61.8°$

Answer: 61.8°

13. (a) Find the values of x, y, z and w in the following matrix equation:

$(x\ 4;\ 4\ y)(-5\ -7;\ 2\ z) = (38\ 46;\ -10\ w)$

(b) By using the matrix method, find the image of the point $(3,-2)$ after a reflection in the line $y = -x$ followed by another reflection in the line $x = 0$.

(c) A translation takes point $(5,5)$ to the point $(-7,-7)$. If it takes point (x, y) to $(-4,-4)$, find the values of x and y.

(a) $(x(-5) + 4\times 2,\ x(-7) + 4z;\ 4(-5) + y\times 2,\ 4(-7) + yz) = (38,\ 46;\ -10,\ w)$

$-5x + 8 = 38 \rightarrow -5x = 30 \rightarrow x = -6$

$-7x + 4z = 46 \rightarrow -7(-6) + 4z = 46 \rightarrow 42 + 4z = 46 \rightarrow 4z = 4 \rightarrow z = 1$

$-20 + 2y = -10 \rightarrow 2y = 10 \rightarrow y = 5$

$-28 + yz = w \rightarrow -28 + 5 \times 1 = w \rightarrow w = -23$

Answer: $x = -6$, $y = 5$, $z = 1$, $w = -23$

(b) Reflection in $y = -x$: $(x, y) \rightarrow (-y, -x)$, Matrix = $(0\ -1;\ -1\ 0)$

Reflection in $x = 0$ (y-axis): $(x, y) \rightarrow (-x, y)$, Matrix = $(-1\ 0;\ 0\ 1)$

Combined: $(-1\ 0;\ 0\ 1)(0\ -1;\ -1\ 0) = (0\ 1;\ -1\ 0)$

$(0\ 1;\ -1\ 0)(3;\ -2) = (-2;\ -3)$

Answer: $(-2, -3)$

(c) Translation: $(5,5) \rightarrow (-7,-7)$

Vector = $(-7 - 5, -7 - 5) = (-12, -12)$

$(x, y) \rightarrow (-4, -4)$ x -

$12 = -4 \rightarrow x = 8$ y -

$12 = -4 \rightarrow y = 8$

Answer: $x = 8$, $y = 8$

14. (a) Given the function $f(x) = 1 / (x - 2)$, find;

(i) the domain and range.

(ii) $f^{-1}(1/3)$.

(b) Antony wishes to buy black shirts and white shirts. He intends to buy at most five black shirts. A black shirt costs Tsh. 24,000 while a white shirt costs Tsh. 30,000 and he is planning to spend up to Tsh. 180,000 for buying shirts.

(i) How many shirts of each kind should be bought so as to have maximum number of shirts?

(ii) Find the greatest number of shirts that should be bought.

(a)(i) $f(x) = 1 / (x - 2)$

Domain: $x \neq 2$, $\mathbb{R} \setminus \{2\}$

Range: $y \neq 0$, $\mathbb{R} \setminus \{0\}$

Answer: Domain: $\mathbb{R} \setminus \{2\}$, Range: $\mathbb{R} \setminus \{0\}$

(a)(ii) $y = 1 / (x - 2)$ $x - 2 = 1 / y$

$x = 1 / y + 2$ $f^{-1}(y) = 1 / y + 2$

$f^{-1}(1/3) = 1 / (1/3) + 2 = 3 + 2 = 5$

Answer: 5

(b)(i) Let x = black, y = white

Maximize: $x + y$

Constraints:

$x \leq 5$

$24000x + 30000y \leq 180,000 \rightarrow 4x + 5y \leq 30$

$x \geq 0$, $y \geq 0$ Vertices:

(0,0): 0

(0,6): 6

(5,0): 5

(5,2): $4 \times 5 + 5 \times 2 = 30$; $5 + 2 = 7$

Maximum at (5,2)

Answer: 5 black, 2 white

(b)(ii) Greatest number = $5 + 2 = 7$

Answer: 7 shirts