THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **Year: 2023**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

1. (a) (i) Arrange the given numbers in ascending order of magnitude: 0.6, 3/5, 20% of 13/4.

(ii) Evaluate the expression $13 - 2 \times 3 + 14 \div (2 + 5)$.

(b) By listing the multiples of 2, 3, and 5, find the L.C.M of these numbers.

(a)(i) 0.6 = 0.6

3/5 = 0.6

20% of 13/4 = $0.2 \times 13/4 = 0.65$

Compare: $0.6 = 3/5 < 0.65$

Answer: 0.6, 3/5, 0.65

(a)(ii) $13 - 2 \times 3 + 14 \div (2 + 5)$

$= 13 - 6 + 14 \div 7$

$= 13 - 6 + 2$

$= 7 + 2 = 9$

Answer: 9

(b) Multiples:

2: 2, 4, 6, 8, 10, 12, ...

3: 3, 6, 9, 12, 15, ...

5: 5, 10, 15, 20, 30, ...

LCM = 30

Answer: 30

2. (a) Find the values of x and y that satisfy the equations $\{5^{(x - 2y)} = 25, 3^{(2x)} \div 3^y = 3^2\}$.

(b) Solve the equation $4 + 3 \log_3 x = \log_3 24$.

(a) First equation: $5^{(x - 2y)} = 25 = 5^2$ x - 2y

= 2 (1)

Second equation: $3^{(2x)} \div 3^y = 3^{(2x - y)} = 3^2$

$2x - y = 2$ (2)

Solve (2): $y = 2x - 2$

Substitute in (1): $x - 2(2x - 2) = 2$

$x - 4x + 4 = 2$

$-3x = -2$ $x = 2/3$ $y = 2(2/3) - 2 =$

$4/3 - 2 = -2/3$ Answer: $x = 2/3$, y

$= -2/3$

(b) $4 + 3 \log_3 x = \log_3 24$ $3 \log_3 x =$

$\log_3 24 - 4$ $\log_3 x^3 = \log_3 24 - \log_3 81$

$\log_3 x^3 = \log_3 (24 / 81) = \log_3 (8 / 27)$

$x^3 = 8 / 27$ $x = (8 / 27)^\wedge(1/3) = 2 / 3$

Answer: $x = 2/3$

3. (a) Given the universal set $\xi = \{15, 30, 45, 60, 75\}$ and the subsets $A = \{15, 45\}$ and $B = \{30, 60\}$, find $(A \cup B)'$ and hence represent this information by using a Venn diagram.

(b) (i) In a class of 50 students, 35 are boys and 15 are girls. If a student is chosen at random, what is the probability that he is a boy?

(ii) Jonika has two shirts, blue and red. He also has three trousers, black, green, and yellow. By using a tree diagram, find the probability that he will put on a blue shirt and black trouser.

(a) $A \cup B = \{15, 30, 45, 60\}$

$(A \cup B)' = \xi \setminus (A \cup B) = \{75\}$

Venn diagram:

ξ circle containing A (15, 45), B (30, 60), and 75 outside both

Answer: $(A \cup B)' = \{75\}$

(b)(i) $P(\text{boy}) = 35 / 50 = 7 / 10$

Answer: 7/10

(b)(ii) Shirts: Blue (1/2), Red (1/2)

Trousers: Black (1/3), Green (1/3), Yellow (1/3)

P(blue shirt, black trouser) = $(1/2) \times (1/3) = 1/6$

Answer: 1/6

4. (a) Show that the triangle whose vertices are A(4,-4), B(-6,-2), and C(2,6) is an isosceles.

(b) A man walks 4 km from village P to village Q and then 3 km to village R. If village Q is N60°E of village P; and village R is N30°W of village Q.

(i) represent this information on a well labeled diagram.

(ii) find the resultant displacement of the man from P to R.

(a) Distances:

$AB = \sqrt{(4 - (-6))^2 + (-4 - (-2))^2} = \sqrt{10^2 + (-2)^2} = \sqrt{104} = 2\sqrt{26}$

$BC = \sqrt{(-6 - 2)^2 + (-2 - 6)^2} = \sqrt{(-8)^2 + (-8)^2} = \sqrt{128} = 8\sqrt{2}$

$CA = \sqrt{(2 - 4)^2 + (6 - (-4))^2} = \sqrt{(-2)^2 + 10^2} = \sqrt{104} = 2\sqrt{26}$

AB = CA, so isosceles

Answer: Isosceles (AB = CA)

(b)(i)

P at origin, Q at N60°E (4 km), R at N30°W from Q (3 km)

(b)(ii) PQ vector: $(4 \cos 60°, 4 \sin 60°) = (2, 2\sqrt{3})$

QR vector: $(3 \cos 150°, 3 \sin 150°) = (-3\sqrt{3}/2, 3/2)$

$PR = PQ + QR = (2 - 3\sqrt{3}/2, 2\sqrt{3} + 3/2)$

Magnitude $= \sqrt{(2 - 3\sqrt{3}/2)^2 + (2\sqrt{3} + 3/2)^2}$

$\approx \sqrt{19.25} \approx 4.39$ km

Answer: 4.39 km

5. (a) In the following figure, RPQ = PQR and SPQ = SQP. Find the size of the angle RPS.

(b) A rectangular field is 72 m long and 40 m wide. If a triangular field with a base of 60 m has an area which is equal to the area of the rectangular field, find the height of the triangular field.

(a)

RPQ = PQR, SPQ = SQP implies PQ is axis of symmetry

$\angle RPQ = \angle PQR$, $\angle SPQ = \angle SQP$

In triangle PQS, $\angle SPQ + \angle SQP = 180° - \angle QPS$

In triangle PQR, $\angle RPQ + \angle PQR = 180° - \angle QRP$

Without figure, assume $\angle RPS = 180°$ (straight line)

Answer: 180° (assumed)

(b) Rectangle area = $72 \times 40 = 2880$ m²

Triangle area = $(1/2) \times 60 \times h = 2880$

$30h = 2880$ h

= 96 m

Answer: 96 m

6. (a) Anna walks 24 km every day. Compute in metres, the distance she walks in 2 days.

(b) A dealer sells mattresses whose buying price is directly proportional to the selling price. If the selling price and the buying price of one mattress are Tshs 20,000 and Tshs 18,000, respectively, find;

(i) the equation that relates the buying price and the selling price.

(ii) the new selling price when the buying price is increased by 15%.

(a) 24 km = 24,000 m

2 days = $2 \times 24{,}000 = 48{,}000$ m

Answer: 48,000 m

(b)(i) $B = k S$

$18{,}000 = k \times 20{,}000$

$k = 18{,}000 / 20{,}000 = 0.9$

$B = 0.9\ S$

Answer: $B = 0.9\ S$

(b)(ii) New $B = 18{,}000 \times 1.15 = 20{,}700$

$20{,}700 = 0.9\ S$

$S = 20{,}700 / 0.9 = 23{,}000$

7. (a) Ally and Jane shared 64,000 shillings in the ratio 3:5 respectively. Find the difference between their shares.

(b) Mr. Mrisho recorded the transactions of his business in February, 2022 in a cash account. Using the given cash account, extract the trial balance as at 28th February, 2022.

(a) Total parts = 3 + 5 = 8

Ally = (3/8) × 64,000 = 24,000

Jane = (5/8) × 64,000 = 40,000

Difference = 40,000 - 24,000 = 16,000

Answer: 16,000 Tshs

(b) Cash Account:

Dr: Capital: 1,500,000, Sales: 1,200,000 + 800,000 = 2,000,000

Total Dr: 3,500,000

Cr: Purchases: 1,000,000 + 1,400,000 = 2,400,000, Transport: 200,000, Balance c/d: 900,000 Total

Cr: 3,500,000

Trial Balance:

Account	Dr	Cr
Cash	900000	
Capital		1500000
Purchases	2400000	
Transport	200000	

Sales		2000000
Total	3500000	3500000

8. (a) Write down the first four terms of a sequence whose general term is n(2n - 1). Briefly explain whether it is an arithmetic progression or a geometric progression.

(b) The sum of the first eleven terms of an arithmetic progression is 517. If its first term is 7, find the sum of the fourth and ninth terms. (a) $T_n = n(2n - 1) = 2n^2 - n$ n = 1: 2 - 1 = 1 n = 2: 8 - 2 = 6 n = 3: 18 - 3 = 15 n = 4: 32 - 4 = 28

Terms: 1, 6, 15, 28

Differences: 5, 9, 13 (not constant)

Ratios: 6/1, 15/6, 28/15 (not constant)

Neither AP nor GP

Answer: 1, 6, 15, 28; neither AP nor GP

(b) $S_{11} = 517$, a = 7

$S_n = (n/2) [2a + (n-1)d]$

$517 = (11/2) [2 \times 7 + 10d]$

$517 = (11/2) (14 + 10d)$

94 = 14 + 10d 10d

= 80 d = 8

$a_4 = 7 + 3 \times 8 = 31$

$a_9 = 7 + 8 \times 8 = 71$

Sum = 31 + 71 =

102

Answer: 102

9. (a) A rectangular plot of land is 40 metres long. If the length of its diagonal is 50 metres, how wide is the plot?

(b) (i) Given that A is an acute angle for which 13 cos A - 5 = 0, find the value of tan A without using mathematical table.

(ii) A triangular pond ABC is such that AB = 8 m, AC = 5 m and BAC = 60°. Determine the length of BC.

(a) Length = 40 m, Diagonal = 50 m

$\text{Width}^2 + 40^2 = 50^2$

$\text{Width}^2 + 1600 = 2500$

$\text{Width}^2 = 900$

Width = 30 m

Answer: 30 m

(b)(i) $13 \cos A - 5 = 0$ $\cos A = 5 / 13$ $\sin^2 A = 1$

$- (5/13)^2 = 1 - 25/169 = 144/169$ $\sin A = 12 / 13$

$\tan A = \sin A / \cos A = (12/13) / (5/13) = 12 / 5$

Answer: 12/5

(b)(ii) Cosine rule: $BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos(BAC)$

$= 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 60°$

$= 64 + 25 - 80 \times (1/2)$

$= 89 - 40 = 49$

$BC = \sqrt{49} = 7$ m

Answer: 7 m

10. (a) If $x = -3$ and $x = 1/3$ are solutions of the equation $ax^2 + bx + c = 0$ where a, b, and c are integers, determine the values of a, b, and c.

(b) Solve the inequality $10 - x \leq 3(x + 10)$, where x is an integer. Hence, state the first four values of x that satisfy the given inequality. (a) Roots: $x = -3$, $x = 1/3$

Sum of roots = $-3 + 1/3 = -8/3$

Product of roots = $-3 \times 1/3 = -1$

Equation: x^2 - (sum)x + product = 0

$x^2 - (-8/3)x - 1 = 0$ $3x^2 + 8x - 3 = 0$

$a = 3, b = 8, c = -3$

Answer: $a = 3, b = 8, c = -3$

(b) $10 - x \leq 3(x + 10)$

$10 - x \leq 3x + 30$

$-4x \leq 20$ $x \geq -5$

Integer x: x = -5, -4, -3, -2, ...

First four: -5, -4, -3, -2

Answer: -5, -4, -3, -2

11. The given frequency distribution table shows the scores of 30 students in a Mathematics test.

Class interval	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	2	4	7	9	5	3

(a) Calculate the median score, correct to two decimal places.

(b) Find the mean score, correct to four significant figures.

(c) Draw a histogram and use it to estimate the mode.

(a) Cumulative frequency:

40-49: 2

50-59: 6

60-69: 13

70-79: 22

80-89: 27

90-99: 30

Median: $30/2 = 15$th term, in 70-79

Median $\approx 70 + (15 - 13)/9 \times 10 \approx 70 + 2.222 \approx 72.22$ Answer:

72.22

(b) Midpoints: 44.5, 54.5, 64.5, 74.5, 84.5, 94.5

Sum $= (44.5 \times 2) + (54.5 \times 4) + (64.5 \times 7) + (74.5 \times 9) + (84.5 \times 5) + (94.5 \times 3)$

$= 89 + 218 + 451.5 + 670.5 + 422.5 + 283.5 = 2135$

Mean $= 2135 / 30 \approx 71.1667$

Answer: 71.17

(c)

Histogram: Highest frequency at 70-79

Mode $\approx 70 + (9 - 7)/((9 - 7) + (9 - 5)) \times 10$

$= 70 + 2/6 \times 10 \approx 73.33$

Answer: 73.3 (estimated)

12. (a) The following figure represents a square box ABCDEFGH whose sides are 8 cm each.

(i) Determine the total surface area of the box.

(ii) Calculate the angle between the line segment AF and the plane ABCD, giving your answer to the nearest degree.

(b) Boeing 787 Dream Liner flying at 500 km/h leaves Julius Nyerere International Airport in Tanzania (7°S, 45°E) at 8:00 am. When will it arrive at Addis Ababa in Ethiopia (9°N, 45°E)? (Use the substitution $2\pi R / 360° = 112$).

(a)(i) Surface area $= 6 \times 8^2 = 6 \times 64 = 384$ cm²

Answer: 384 cm²

(a)(ii) AF $= \sqrt{(8^2 + 8^2 + 8^2)} = \sqrt{192} = 8\sqrt{3}$ cm

Normal to ABCD: (0, 0, 1) AF vector: (8, 8, 8) cos

$\theta = (0 \times 8 + 0 \times 8 + 1 \times 8) / (\sqrt{(8^2 + 8^2 + 8^2)} \times 1)$

$= 8 / (8\sqrt{3}) = 1 / \sqrt{3}$ $\theta \approx$

$\cos^{-1}(1/\sqrt{3}) \approx 54.74°$

Answer: 55°

(b) Latitude difference = 9°N - 7°S = 16°

Distance = $16 \times 112 = 1792$ km

Time = 1792 / 500 = 3.584 hours ≈ 3h 35m

8:00 am + 3h 35m ≈ 11:35 am

Answer: 11:35 am

13. (a) Find the values of x and y given that [x 4; 3 y] + [3x 5; -3 7] = [8 9; 0 9].

(b) In a multiple choice test, 2 marks were awarded for each correct answer, 1 mark was deducted from each incorrect answer and 0 for not writing an answer. Anna answered 49 questions and scored 62 marks out of 100.

(i) Represent this information in a matrix form; letting x be the number of correct answers and y be the number of incorrect answers.

(ii) Using the inverse matrix method, determine the number of questions that Anna answered correctly.

(c) A triangle has its vertices at the points A(1,3), B(2,5), and C(4,1). If the triangle is rotated through 180° anticlockwise about the origin;

(i) find the coordinates of the points A', B', and C' which are the images of A, B, and C respectively.

(ii) draw the triangles ABC and A'B'C' on the same set of axes.

(a) (x + 3x, 4 + 5; 3 + (-3), y + 7) = (8, 9; 0, 9)

$4x = 8 \rightarrow x = 2$

4 + 5 = 9 (true) 3

\- 3 = 0 (true) y +

$7 = 9 \rightarrow y = 2$

Answer: x = 2, y = 2

(b)(i) x (correct) + y (incorrect) = 49

$2x - y = 62$

Matrix: (1 1; 2 -1)(x; y) = (49; 62)

Answer: As shown

(b)(ii) A = (1 1; 2 -1), det(A) = -1 - 2 = -3

Inverse = (-1/3)(-1 -1; -2 1) = (1/3, 1/3; 2/3, -1/3)

(x; y) = (1/3 1/3; 2/3 -1/3)(49; 62) x = (1/3)(49 +

62) = 111/3 = 37 y = (2/3 × 49 - 1/3 × 62) = (98 -

62)/3 = 36/3 = 12

Answer: 37 correct

(c)(i) 180° anticlockwise: $(x, y) \rightarrow (-x, -y)$

$A(1,3) \rightarrow A'(-1,-3)$

$B(2,5) \rightarrow B'(-2,-5)$

$C(4,1) \rightarrow C'(-4,-1)$

Answer: A'(-1,-3), B'(-2,-5), C'(-4,-1)

(c)(ii)

Plot ABC: A(1,3), B(2,5), C(4,1)

Plot A'B'C': A'(-1,-3), B'(-2,-5), C'(-4,-1)

14. (a) A function is defined on the set of integers as $f(x) = \{-2 \text{ if } 0 < x \leq 5,\ x + 1 \text{ if } -6 \leq x < 0\}$

(i) Find the values of f(4) and f(-5).

(ii) State the domain and range of f(x).

(b) The Air Tanzania Company wants to buy two types of airplanes, A and B. Type A requires 6 dam² of parking space, type B requires 2 dam² of parking space while the company has 60 dam² of parking space available. Also the company has 480 billion shillings and the cost for buying airplanes of type A and type B are 20 billion shillings and 30 billion shillings respectively. Find the greatest number of airplanes the company can buy.

(a)(i) $f(4)$: $0 < 4 \leq 5$, $f(4) = -2$ $f(-5)$:

$-6 \leq -5 < 0$, $f(-5) = -5 + 1 = -4$

Answer: $f(4) = -2$, $f(-5) = -4$

(a)(ii) Domain: Integers from -6 to 5 = $\{-6, -5, ..., 4, 5\}$

For $-6 \leq x < 0$: $f(x) = x + 1$, range = $\{-5, -4, -3, -2, -1\}$

For $0 < x \leq 5$: $f(x) = -2$, range = $\{-2\}$

Range: $\{-5, -4, -3, -2, -1\}$

Answer: Domain: $\{-6, -5, ..., 5\}$, Range: $\{-5, -4, -3, -2, -1\}$

(b) Let x = type A, y = type B

Maximize: $x + y$

Constraints:

$6x + 2y \leq 60 \rightarrow 3x + y \leq 30$

$20x + 30y \leq 480 \rightarrow 2x + 3y \leq 48$

$x \geq 0$, $y \geq 0$ Vertices:

(0,0): 0

(0,16): $2 \times 0 + 3 \times 16 = 48$; $0 + 16 = 16$

(10,0): $3 \times 10 + 0 = 30$; $10 + 0 = 10$

(9,3): Solve $3x + y = 30$, $2x + 3y = 48$

$y = 30 - 3x$, $2x + 3(30 - 3x) = 48$

$2x + 90 - 9x = 48$ $-7x = -42$ $x = 6$, $y =$

$30 - 3 \times 6 = 12$ (not vertex)

Try (9,3): $3 \times 9 + 3 = 30$, $2 \times 9 + 3 \times 3 = 27 \leq 48$

$9 + 3 = 12$

Maximum at (0,16)

Answer: 16 airplanes (0 A, 16 B)