THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **Year: 2021**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

1. (a) Find the lowest common multiple of the numbers 15, 35, and 40.

(b) Find the approximate value of the expression $(0.0695 \times 19812) / 6.8125$ by rounding off each number to one significant figure.

(a) $15 = 3 \times 5$

$35 = 5 \times 7$

$40 = 2^3 \times 5$

LCM $= 2^3 \times 3 \times 5 \times 7 = 8 \times 3 \times 5 \times 7 = 840$

Answer: 840

(b) $0.0695 \approx 0.07$

$19812 \approx 20000$

$6.8125 \approx 7$

$(0.07 \times 20000) / 7 = 1400 / 7 = 200$

Answer: 200

2. (a) Express the equation $3^{(2y-1)} + 2 \times 3^{(y-1)} = 1$ in terms of P given that $P = 3^y$.

(b) Determine the value of y in the equation $\log_{10}(3y + 2) - 1 = \log_{10}(y - 4)$.

(a) $P = 3^y$

$3^{(2y-1)} = 3^{(2y)} \times 3^{(-1)} = (3^y)^2 / 3 = P^2 / 3$

$3^{(y-1)} = 3^y \times 3^{(-1)} = P / 3$

Equation: $(P^2 / 3) + 2 (P / 3) = 1$

$P^2 / 3 + 2P / 3 = 1$

$(P^2 + 2P) / 3 = 1$

$P^2 + 2P = 3$

$P^2 + 2P - 3 = 0$

Answer: $P^2 + 2P - 3 = 0$

(b) $\log_{10}(3y + 2) - 1 = \log_{10}(y - 4)$

$\log_{10}(3y + 2) - \log_{10} 10 = \log_{10}(y - 4)$ $\log_{10}((3y$

$+ 2) / 10) = \log_{10}(y - 4)$

$(3y + 2) / 10 = y - 4$

$3y + 2 = 10y - 40$

$42 = 7y$ $y = 6$

Answer: $y = 6$

3. In a class of 45 students, 30 study Chemistry, 20 study Physics and 5 study neither of the two subjects.

(a) Represent this information in a well labeled Venn diagram.

(b) Using part (a) find:

(i) the number of students studying both subjects.

(ii) the probability that a student selected at random from the class will be studying Chemistry only.

(a) Total = 45, Neither = 5

Chemistry (C) = 30, Physics (P) = 20

C only + $C \cap P = 30$

P only + $C \cap P = 20$

C only + P only + $C \cap P$ + Neither = 45

C only + P only + $C \cap P + 5 = 45$

C only + P only + $C \cap P = 40$

Let $C \cap P = x$

C only = $30 - x$, P only = $20 - x$

$(30 - x) + (20 - x) + x + 5 = 45$

$50 - x = 40$ x

$= 10$

C only = 20, P only = 10, $C \cap P = 10$

Venn diagram: [Describe]

Chemistry circle: 20 (only), 10 (both)

Physics circle: 10 (only), 10 (both)

Outside: 5

(b)(i) Both = $C \cap P = 10$

Answer: 10 students

(b)(ii) P(Chemistry only) = 20 / 45 = 4/9

Answer: 4/9

4. (a) An engineer is in the process of constructing two straight roads, R_1 and R_2 which will meet at the right angles. If R_1 will be represented by the equation $2x - 3y - 4 = 0$ and R_2 will pass through the point (4,-2), find the equation representing R_2 in the form $ax + by + c = 0$.

(b) A boat crosses a river at a velocity of 30 km/h southwards. Water in the river flows at 5 km/h due East. By using the knowledge of vectors, calculate the resultant velocity of the boat. Give the answer correct to 2 decimal places.

(a) R_1: $2x - 3y - 4 = 0$

$3y = 2x - 4$ $y =$

$(2/3)x - 4/3$

Slope = 2/3

R_2 perpendicular, slope = -3/2

R_2 through (4, -2): $y -$

$(-2) = (-3/2)(x - 4)$ y

$+ 2 = (-3/2)x + 6$ $y =$

$(-3/2)x + 4$

$3x + 2y - 8 = 0$

Answer: $3x + 2y - 8 = 0$

(b) Boat: (0, -30), River: (5, 0)

Resultant: (5, -30)

Velocity = $\sqrt{(5^2 + (-30)^2)} = \sqrt{(25 + 900)} = \sqrt{925} = 5\sqrt{37}$

$\approx 5 \times 6.082 = 30.41$

Answer: 30.41 km/h

5. (a) The lengths of two similar rectangles are 6 cm and 8 cm. If the area of the small rectangle is 73.8 cm^2, find the area of the large rectangle.

(b) The exterior and interior angles of a regular polygon are in the ratio 2:4 respectively. Find the number of sides of the polygon.

(a) Linear ratio = 8/6 = 4/3

Area ratio = $(4/3)^2 = 16/9$

Small area = 73.8 cm^2

Large area = $73.8 \times (16/9) \approx 131.2$

Answer: 131.2 cm^2

(b) Exterior : Interior = 2 : 4 = 1 : 2

Exterior = 180 - Interior

Interior = 2 × Exterior

Exterior = 180 - 2 × Exterior

3 × Exterior = 180

Exterior = 60°

Number of sides = 360 / 60 = 6

Answer: 6 sides

6. (a) A piece of length 7.42 m is cut off from a string that is 13.5 m long. If the remaining part of the string is divided into equal pieces of length 32 cm, how many pieces are there?

(b) The mass (M) which can be supported by a beam varies directly with the breadth (b) and inversely with the length (l). If a beam of breadth 2 m and length 15 m can support a mass of 200 kg, what mass can be supported by a beam which is 3 m broad and 20 m long?

(a) Remaining = 13.5 - 7.42 = 6.08 m = 608 cm

Pieces = 608 / 32 = 19

Answer: 19 pieces

(b) $M = k b / l$ $200 = k \times 2 / 15$ $k = 200 \times 15 / 2 = 1500$

$M = 1500 \times 3 / 20 = 225$

Answer: 225 kg

7. (a) What do the following terms mean as used in Accounts?

(i) Trading account

(ii) Profit and loss account

(iii) Balance sheet

(iv) Cash account

(b) A car which its buying price was shs 12,500,000 was sold at a loss of 20 percent. Find the loss made and selling price.

(a)(i) Trading account: Shows gross profit/loss from buying and selling goods.

(ii) Profit and loss account: Shows net profit/loss after expenses.

(iii) Balance sheet: Snapshot of assets, liabilities, and capital at a point in time.

(iv) Cash account: Records cash transactions (receipts and payments).

Answer: As defined

(b) Loss = 20% of 12,500,000 = 0.2 × 12,500,000 = 2,500,000

Selling price = 12,500,000 - 2,500,000 = 10,000,000

Answer: Loss: 2,500,000 shs, Selling price: 10,000,000 shs

8. (a) A farmer wants to plant 6 mango seedlings in a row at a fixed interval of 7 metres. Determine the length of the row.

(b) If the fifth and the sixth terms of a Geometric Progression (G.P.) are 162 and 486 respectively, find the common ratio and the first term of the G.P.

(a) 6 seedlings, 5 intervals of 7 m

Length = $5 \times 7 = 35$ m

Answer: 35 m

(b) $a_5 = ar^4 = 162$ $a_6 = ar^5 = 486$ $r = 486 / 162 = 3$ $ar^4 = 162$ a

$\times 3^4 = 162$ $a \times 81 = 162$ $a = 2$

Answer: $r = 3$, $a = 2$

9. (a) Show that $\cos(90° + \theta) = -\sin \theta$.

(b) In triangle UVW, UV = 3 cm and UW = 5 cm. If the angle formed between the two sides is 60°, find VW correct to two decimal places.

(a) $\cos(90° + \theta) = \cos 90° \cos \theta - \sin 90° \sin \theta$

$= 0 \times \cos \theta - 1 \times \sin \theta = -\sin \theta$

Answer: $\cos(90° + \theta) = -\sin \theta$

(b) Cosine rule: $VW^2 = UV^2 + UW^2 - 2 \times UV \times UW \times \cos(60°)$

$= 3^2 + 5^2 - 2 \times 3 \times 5 \times (1/2)$

$= 9 + 25 - 15 = 19$

$VW = \sqrt{19} \approx 4.36$

10. (a) A trapezium has the area of $2x^2 - 8x + 6$ square units. If the parallel sides are $(2x + 3)$ units and $(2x - 7)$ units long, find its height.

(b) The difference between two positive numbers is 7. If their product is 30, find the numbers.

(a) Area = (1/2) × (sum of parallel sides) × height

$2x^2 - 8x + 6 = (1/2) \times (2x + 3 + 2x - 7) \times h$

$2x^2 - 8x + 6 = (1/2) \times (4x - 4) \times h$

$2x^2 - 8x + 6 = 2x - 2$ h =

$(2x^2 - 8x + 6) / (2x - 2)$

$= (2(x^2 - 4x + 3)) / (2(x - 1))$

$= (x - 3)(x - 1) / (x - 1) = x - 3$

Answer: h = x - 3 units (b) Let

numbers be y, y + 7 y(y + 7) =

30 $y^2 + 7y - 30 = 0$ y = [-7 ±

√(49 + 120)] / 2 = [-7 ± √169] /

2 = [-7 ± 13] / 2 y = 3 or y = -

10

Positive: y = 3, y + 7 = 10

11. The following are the marks obtained by 40 students in one of the Basic Mathematics examinations:

48, 47, 57, 56, 71, 62, 46, 45, 50, 76, 58, 66, 48, 32, 89, 60, 42, 47, 54, 67, 64, 49, 37, 64, 67, 44, 45, 45, 42, 34, 47, 44, 73, 44, 58, 43, 54, 35, 54, 52

(a) Prepare a frequency distribution using the information: number of classes = 8; size of each class = 8 and the lower limit of the first class interval = 32.

(b) Use the frequency distribution obtained in part (a) to find the actual mean when the assumed mean is 83.5.

(c) Calculate the difference between the actual mean and the median of this distribution. Hence, comment on the difference obtained.

(a) Classes: 32-39, 40-47, 48-55, 56-63, 64-71, 72-79, 80-87, 88-95

32-39: 32, 34, 35, 37 → 4

40-47: 42, 42, 43, 44, 44, 44, 45, 45, 46, 47, 47, 47 → 12

48-55: 48, 48, 49, 50, 52, 54, 54, 54 → 8

56-63: 56, 57, 58, 58, 60, 62 → 6

64-71: 64, 64, 66, 67, 67, 71 → 6

72-79: 73, 76 → 2

80-87: None → 0

88-95: 89 → 1

Table:

Class	Frequency
32-39	4
40-47	12
48-55	8
56-63	6
64-71	6
72-79	2
80-87	0
88-95	1

(b) Assumed mean = 83.5

Midpoints: 35.5, 43.5, 51.5, 59.5, 67.5, 75.5, 83.5, 91.5 d = (x - 83.5) / 8

Class	f	x	d	f × d
32-39	4	35.5	-6	-24
40-47	12	43.5	-5	-60
48-55	8	51.5	-4	-32
56-63	6	59.5	-3	-18
64-71	6	67.5	-2	-12
72-79	2	75.5	-1	-2

80-87	0	83.5	0	0
88-95	1	91.5	1	1
	Total f = 40		Σ(f × d) = 147	

Mean = 83.5 + (-147 / 40) × 8

= 83.5 - 29.4 = 54.1

Answer: 54.1

(c) Cumulative frequency:

32-39: 4

40-47: 16

48-55: 24

56-63: 30

64-71: 36

72-79: 38

80-87: 38

88-95: 39

Median: 40/2 = 20th term, in 48-55

Median ≈ 48 + (20 - 16)/8 × 8 = 48 + 4 = 52

Difference = 54.1 - 52 = 2.1

Comment: Small positive difference indicates slight positive skew.

Answer: Difference = 2.1, slight positive skew

12. The following rectangular block is 8 cm long, 6 cm wide and 5 cm high.

(a) Name the angles formed between line AG with the planes ABCD and BCFG.

(b) Calculate:

(i) the length of AC.

(ii) the length of AF.

(iii) the size of angle CAF.

(c) A ship sails from point A(10°S, 30°W) to point B(11°N, 30°W) at a speed of 900 km/h. If it leaves point A at 10:00 am, when will it arrive at B? (Radius of the Earth (R) = 6400 km).

(a) AG with plane ABCD: $\angle AGD$

AG with plane BCFG: $\angle AGF$

Answer: $\angle AGD$, $\angle AGF$

(b)(i) $AC = \sqrt{(8^2 + 6^2)} = \sqrt{(64 + 36)} = \sqrt{100} = 10$ cm

Answer: 10 cm

(b)(ii) $AF = \sqrt{(8^2 + 6^2 + 5^2)} = \sqrt{(64 + 36 + 25)} = \sqrt{125} = 5\sqrt{5} \approx 11.18$ cm

Answer: 11.18 cm

(b)(iii) In triangle CAF:

CA = 10 cm, AF = $5\sqrt{5}$ cm, CF = 5 cm cos($\angle$

CAF) = $(CA^2 + AF^2 - CF^2) / (2 \times CA \times AF)$

$= (100 + 125 - 25) / (2 \times 10 \times 5\sqrt{5}) = 200 / (100\sqrt{5}) = 2 / \sqrt{5}$

$\angle CAF \approx \cos^{-1}(2/\sqrt{5}) \approx 26.57°$

Answer: 26.6°

(c) Latitude difference = 11°N - 10°S = 21° θ

$= 21 \times \pi/180 \approx 0.3665$ radians

Distance = $6400 \times 0.3665 \approx 2345.6$ km

Time = $2345.6 / 900 \approx 2.606$ hours ≈ 2h 36m

10:00 am + 2h 36m = 12:36 pm

Answer: 12:36 pm

13. (a) Solve the following system of simultaneous equations by using the matrix method.

$\{2x - y = 5, 3x + 2y = 4\}$

(b) Triangle ABC has the vertices A(1,1), B(2,4) and C(5,3). Find the vertices of its image under the transformation matrix T = (2 1; 1 3).

(c) Find the image of the point A(4,2) after a rotation about the origin through 120° anticlockwise.

(a) (2 -1; 3 2)(x; y) = (5; 4)

A = (2 -1; 3 2), det(A) = $2 \times 2 - (-1) \times 3 = 7$

Inverse = (1/7)(2 1; -3 2)

(x; y) = (1/7)(2 1; -3 2)(5; 4)

= (1/7)(14; -7) = (2; -1)

Answer: $x = 2$, $y = -1$

(b) T(A) = (2 1; 1 3)(1; 1) = (3; 4)

T(B) = (2 1; 1 3)(2; 4) = (8; 14)

T(C) = (2 1; 1 3)(5; 3) = (13; 14)

Answer: A'(3,4), B'(8,14), C'(13,14)

(c) Rotation 120°: $(x, y) \rightarrow (x \cos 120° - y \sin 120°, x \sin 120° + y \cos 120°)$ $\cos 120° = -1/2$, $\sin 120° = \sqrt{3}/2$

$(4, 2) \rightarrow (4(-1/2) - 2(\sqrt{3}/2), 4(\sqrt{3}/2) + 2(-1/2))$

$= (-2 - \sqrt{3}, 2\sqrt{3} - 1)$

Answer: $(-2 - \sqrt{3}, 2\sqrt{3} - 1)$

14. (a) Jennifer makes two types of garments, Batiki and Kitenge. Batiki requires 2.5 metres of material while Kitenge requires 2 metres of material. The business uses up to 400 metres of materials daily for the production of both types of garments but produces at most 80 metres of Batiki and at least 60 metres of Kitenge daily. Taking x to represent the number of Batiki and y the number of Kitenge produced daily; (i) write down the inequalities satisfying the given information.

(ii) find the number of each type of garments the business can produce in order to get the maximum income if the income is given by $f(x, y) = 300x + 200y$.

(b) What is the importance of studying linear programming? Give 2 points.

(a)(i) Material: $2.5x + 2y \leq 400$

Batiki: $x \leq 80$

Kitenge: $y \geq 60$ x

≥ 0, $y \geq 0$

Answer: $2.5x + 2y \leq 400$, $x \leq 80$, $y \geq 60$, $x \geq 0$, $y \geq 0$

(a)(ii) Maximize: $f(x, y) = 300x + 200y$

Vertices:

(0,60): $f = 200 \times 60 = 12000$

(0,200): $2y = 400$, $y = 200$; $f = 200 \times 200 = 40000$

(80,60): $f = 300 \times 80 + 200 \times 60 = 24000 + 12000 = 36000$

(80,100): $2.5 \times 80 + 2y = 400$, $2y = 200$, $y = 100$ $f = 300 \times$

$80 + 200 \times 100 = 24000 + 20000 = 44000$

Maximum at (80,100) Answer:

80 Batiki, 100 Kitenge (b)

Importance:

Optimizes resource allocation.

Aids decision-making in business and operations.