A Simplified Sweat Capital Model

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I want to start with a simplified sweat capital model. The key features that I would like are

1. Two types of capital
2. Occupation choice

Hopefully the model retains all the key economics and is easy to solve and code up. Let me start with the dynamic programs of owners and workers.

1 Time investment

1.1 Dynamic programs

Owners

$$V^b\left(a,\kappa,s\right)=\max_{c,a'\geq 0,\kappa'}u(c)-g(e)+\beta\sum_{s}\Pi(s'|s)V\left(a',\kappa',s'\right)$$

$$c+a'=y^b\left(\kappa,s\right)+Ra$$
$$\kappa'=\left(1-\delta_\kappa\right)\kappa+e$$

Workers

$$V^w\left(a,\kappa,s\right)=\max_{c,a'\geq 0}u(c)+\beta\sum_{s}\Pi(s'|s)V\left(a',\kappa',s'\right)$$

$$c+a'=y^w\left(s\right)+Ra$$
$$\kappa'=\lambda\kappa$$

Occupational choice

$$V\left(a,\kappa,s\right)=\int\max_{d}\left(d\left[V^b\left(a,\kappa,s\right)+\eta\right]+(1-d)\left[V^w\left(a,\kappa,s\right)\right]\right)d\Gamma\left(\eta\right)$$

1.2 Application of Endogenous grid

The optimality of owners and workers is given by

$$u_c(c) - \lambda = \beta \sum_s \Pi(s'|s) V_a(a', \kappa', s')$$

$$\lambda a' = 0, \quad \lambda \geq 0, \quad a' \geq 0$$

$$g_e(e) = \beta \sum_s \Pi(s'|s) V_\kappa(a', \kappa', s')$$

- Assume that we have a guess for V and via that we obtain $\{V_a, V_\kappa\}$
- We can back out functions $\{C(a', \kappa', s), E(a', \kappa', s)\}_{A' \times K' \times S}$ assuming the EE holds with equality
- Using $\{C(a', \kappa', s), E(a', \kappa', s)\}_{A' \times K' \times S}$ we can back out $\{A(a', \kappa', s), K(a', \kappa', s)\}_{A' \times K' \times S}$ as the associated levels of assets and sweat capital today
- Now we need to interpolate. Suppose $A(0, \kappa', s) > 0$, under some conditions [?] should be that the borrowing constraint binds.
  - let $\{(A(0, \kappa', s), K(0, \kappa', s))\}_{\kappa'}$ be the cutoffs. Suppose they are well-behaved, that is they nicely segment $A \times K$ space into a region $R^- \subseteq A \times K$ where the constraint binds and a region R^+ where it doesn't.
  - For $(a, \kappa) \in R^-$ we can use $C^{min}(a, \kappa, s) = y^b(\kappa, s) + Ra$. We can add these points
  - Elsewere we use $\{C(a', \kappa', s), A(a', \kappa', s), K(a', \kappa', s)\}$ to interpolate
- For $E(a', \kappa', s)$ we can just use $\{E(a', \kappa', s), A(a', \kappa', s), K(a', \kappa', s)\}$
- Using the interpolations we can obtain V^b
- Something analogous should give us V^w
- Then the discrete choice is closed form and gives us d as well as the new guess for V

2 Money investment

2.1 Dynamic programs

Owners

$$V^b(a, \kappa, s) = \max_{c, a' \geq 0, \kappa'} u(c) + \beta \sum_s \Pi(s'|s) V(a', \kappa', s')$$

$$c + a' = y^b(\kappa, s) + Ra - e$$

$$\kappa' = (1 - \delta_\kappa)\kappa + g(e)$$

Workers

$$V^w(a,\kappa,s) = \max_{c,a'\geq 0} u(c) + \beta \sum_s \Pi(s'|s) V(a',\kappa',s')$$

$$c + a' = y^w(s) + Ra$$
$$\kappa' = \lambda\kappa$$

Occupational choice

$$V(a,\kappa,s) = \int \max_d \left(d\left[V^b(a,\kappa,s) + \eta\right] + (1-d)\left[V^w(a,\kappa,s)\right]\right) d\Gamma(\eta)$$

2.2 Application of Endogenous grid

The optimality of owners and workers is given by

$$u_c(c) - \lambda = \beta \sum_s \Pi(s'|s) V_a(a',\kappa',s')$$

$$\lambda a' = 0, \quad \lambda \geq 0, \quad a' \geq 0$$
$$\frac{u_c(c)}{g_e(e)} = \underbrace{\beta \sum_s \Pi(s'|s) V_\kappa(a',\kappa',s')}_{\chi}$$

- Assume that we have a guess for V and via that we obtain $\{V_a, V_\kappa\}$
- There exists functions $C^{min}(a,\kappa',\chi), E^{min}(a,\kappa',\chi), K^{min}(a,\kappa',\chi)$ that satisfy
$$\frac{u_c(c)}{g_e(e)} = \chi$$
$$c = y^b(\kappa,s) + Ra - e$$
$$\kappa' = (1-\delta_\kappa)\kappa + g(e)$$
- We can back out functions $\{C(a',\kappa',s), E(a',\kappa',s)\}_{A'\times K'\times S}$ assuming the EE holds with equality
- Using $\{C(a',\kappa',s), E(a',\kappa',s)\}_{A'\times K'\times S}$ we can back out $\{A(a',\kappa',s), K(a',\kappa',s)\}_{A'\times K'\times S}$ as the associated levels of assets and sweat capital today
- Now we need to interpolate. Suppose $A(0,\kappa',s) > 0$, under some conditions [?] should be that the borrowing constraint binds.

- let $\{(A(0,\kappa',s), K(0,\kappa',s))\}_{\kappa'}$ be the cutoffs. Suppose they are well-behaved, that is they nicely segment $A \times K$ space into a region $R^- \subseteq A \times K$ where the constraint binds and a region R^+ where it doesn't.
- For $(a,\kappa) \in R^-$ we can use $\left\{C^{min}(a,\kappa',s), K^{min}(a,\kappa',\chi)\right\}_{a<A(a',\kappa',s)}$. We can add these points
- Elsewere we use $\{C(a',\kappa',s), A(a',\kappa',s), K(a',\kappa',s)\}$ to interpolate
- Same for $E(a',\kappa',s)$

* Using the interpolations we can obtain V^b
* Something analogous should give us V^w
* Then the discrete choice is closed form and gives us d as well as the new guess for V