BEGS: Quantitative Results

September 2013

Numerical exercise

Solve a $N = 5$ agent economy with realistic level and movements in wage dispersion across booms and recessions

- Long run dynamics: Study settings that differ in covariance of interest rates and output
- Transient dynamics: Study outcomes in recessions that are accompanied by higher inequality

Aggregate shocks affect,

1. Wages:
$$\log \theta_i = \epsilon[1 + (.9 - i)m]$$
2. Payoffs:
$$P = 1 + \chi\epsilon$$

Calibration

Parameter	Value	Description
$\{\bar{\theta}_i\}$	{1 , 1.4, 2.1, 3.24, 4.9}	Wages dispersion for {10,25,50,75,90} percentiles
γ	2	Average Frisch elasticity of labor supply of 0.5
β	0.98	Average (annual) risk free interest rate of 2%
m	$\frac{1.5}{.8}$	Changes in dispersion
χ	-0.06	covariance between holding period returns and labor productivity%
σ_e	0.03	vol of labor productivity
g	.13 %	Average pre-transfer expenditure- output ratio of 12 %

Table : Benchmark calibration

The Pareo weights and initial distribution of wealth is chosen to match

Calibration: Interest rates

Let $q_t^{(n)}$ be the log price of a nominal bond of maturity n. We can define the real holding period returns $r_{t,t+1}^{(n)}$ as follows

$$r_{t,t+1}^{(n)} = q_{t+1}^{(n-1)} - q_t^{(n)} - \pi_{t+1}$$

With the transfromation $y_t^{(n)} : -\frac{1}{n} q_t^{(n)}$ we can express $r_{t,t+1}^{(n)}$ as follows:

$$r_{t,t+1}^{(n)} = \underbrace{y_t^{(n)}}_{\text{Ex-ante part}} - (n-1) \Bigg[\underbrace{\left(y_{t+1}^{(n)} - y_t^{(n)} \right)}_{\text{Interest rate risk given } n} + \underbrace{\left(y_{t+1}^{(n-1)} - y_{t+1}^{(n)} \right)}_{\text{Term structure risk}} \Bigg] - \underbrace{\pi_{t+1}}_{\text{Inflation risk}}$$

Calibration: Interest rates

- In the model the holding period returns are given by $\log\left[\frac{P_{t+1}}{Q_t}\right]$ and $Q_t = \frac{\beta \mathbb{E}_t u_{c,t+1} P_{t+1}}{u_{c,t}}$.
- P_{t+1} allows us to captures ex-post fluctuations in returns to the government's debt portfolio coming from maturity and inflation.
- Since ϵ_t is i.i.d over time in our calibration $\chi = \frac{\sigma_r}{\sigma_\epsilon} Corr(r, \epsilon)$

Using data on labor productivity ϵ_t and $\{q_t^n\}_n$ we can compute the correlation table as follows:

Maturity (n)	2yr	3yr	4yr	5yr
$Corr(\epsilon_{t+1}, r_{t,t+1}^{(n)})$	-0.11	-0.093	-0.083	-0.072
$Corr(\epsilon_{t+1}, r_{t,t+1}^{(n)} - ny_t^{(n)})$	0.00	-0.0463	-0.080	-0.091
$Corr(\epsilon_{t+1}, y_t^{(n)} - \pi_{t+1})$	-0.097	-0.086	-0.080	-0.073

Table

Further $Corr(\epsilon_{t+1}, \pi_{t+1}) = 0.068$ and for 3 month real tbill returns $Corr(\epsilon_{t+1}, y_t^{1qtr} - \pi_{t+1}) = -0.11$

Long run

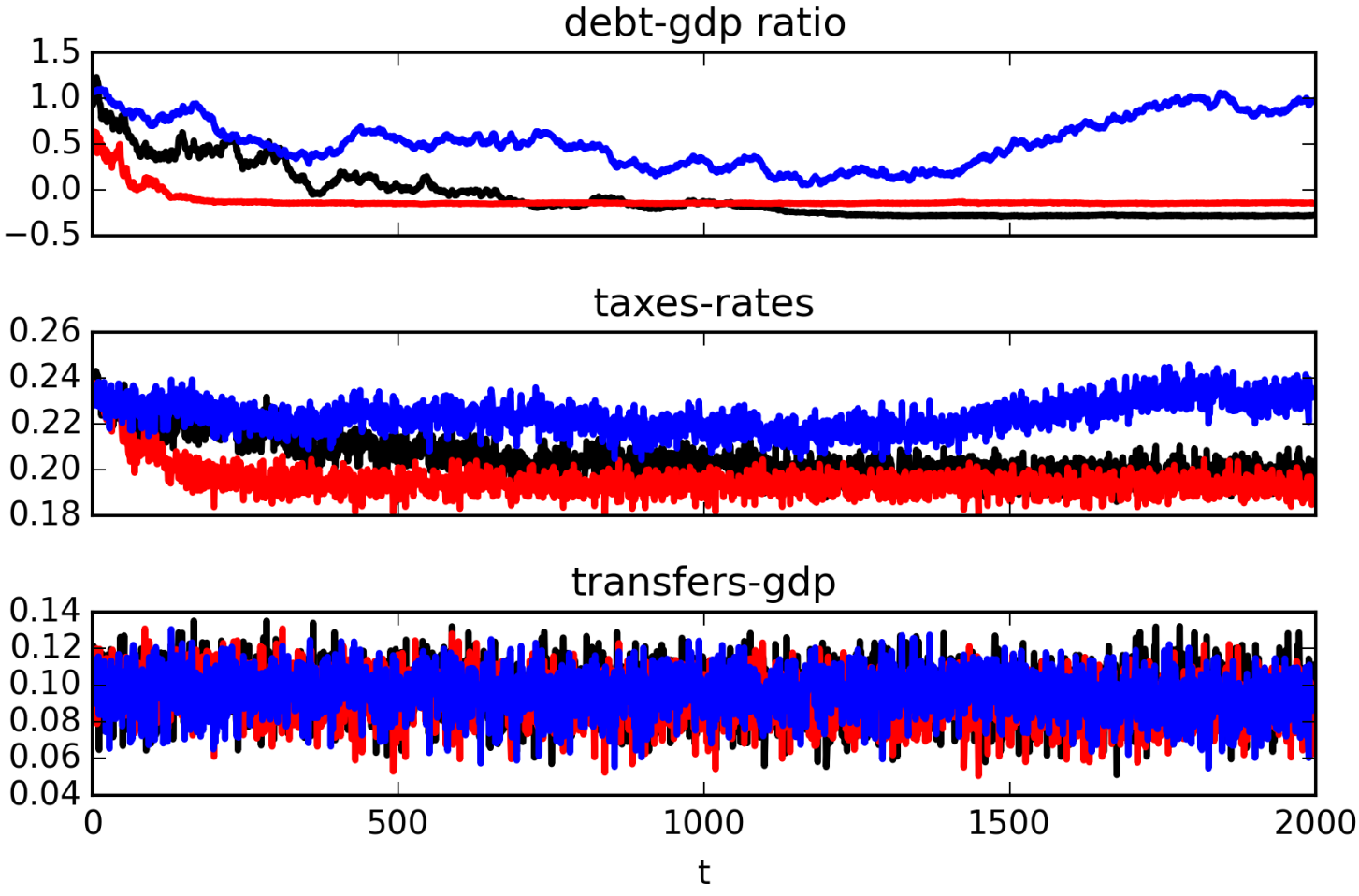

Figure : The red, black and blue lines plot simulations for a common sequence of shocks for values of $\chi = -1.5, 0, 1.5$ respectively

Long run: Speed of convergence

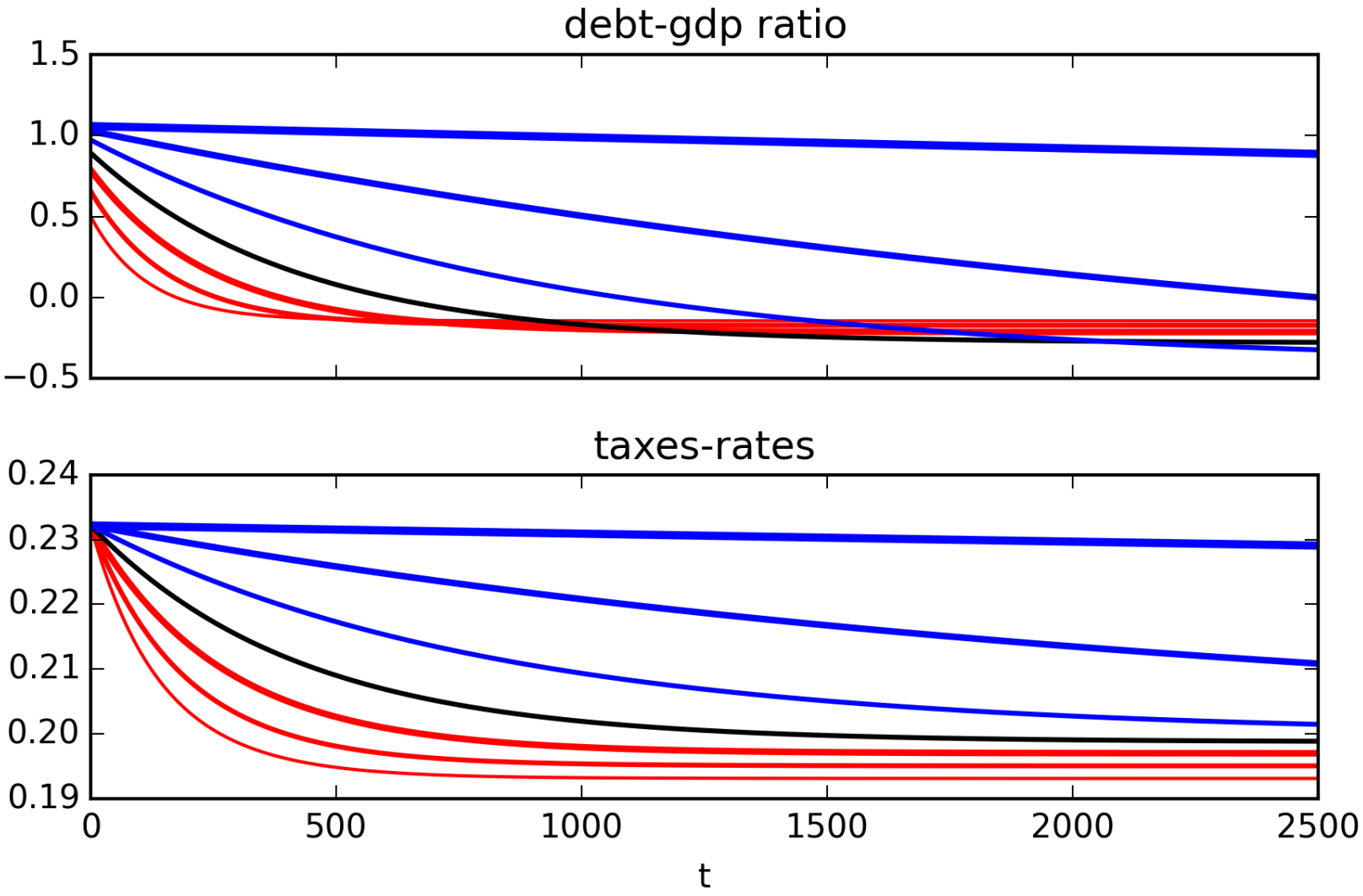

Figure : The plot shows conditional mean paths for different values of χ. The red (blue) lines have $\chi < 0$ ($\chi > 0$). The thicker lines represent larger values

Spreading of taxes

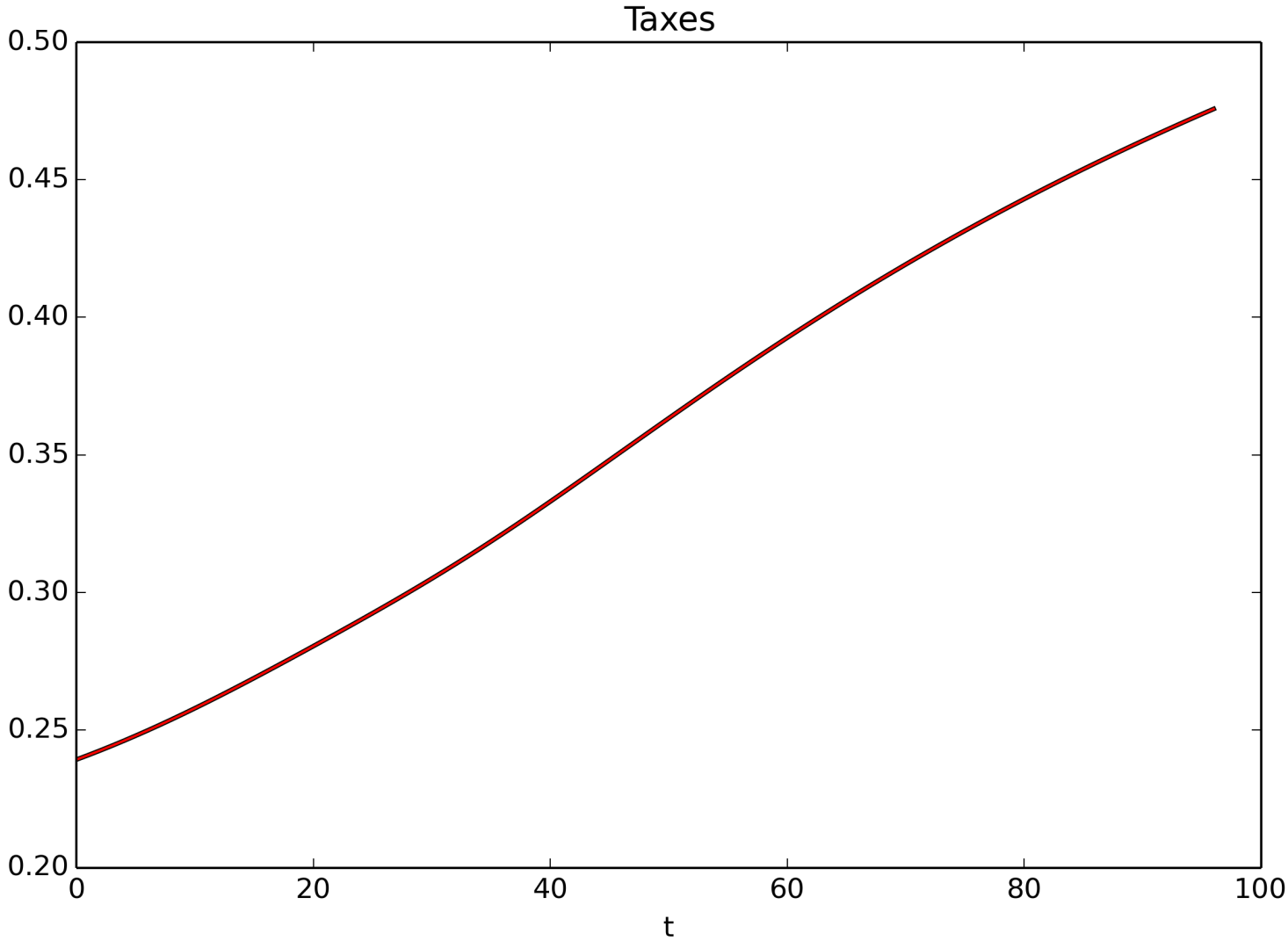

Figure : Taxes for a sequence of -1 s.d shocks to aggregate productivity

Short run

Lets denote consecutive period of negative (positive) one s.d ϵ shocks a "recession" (boom)

- Simulate a recession that is followed by no further shocks
- Decompose responses into TFP component and inequality component:

$$\textbf{Baseline: } \log \theta_i = \epsilon[1 + (.9 - i)m]$$

  - Only TFP:

$$\log \theta_i = \epsilon$$

  - Only Ineq:

$$\log \theta_i = \epsilon[(.9 - i)m]$$

Recessions with higher inequality

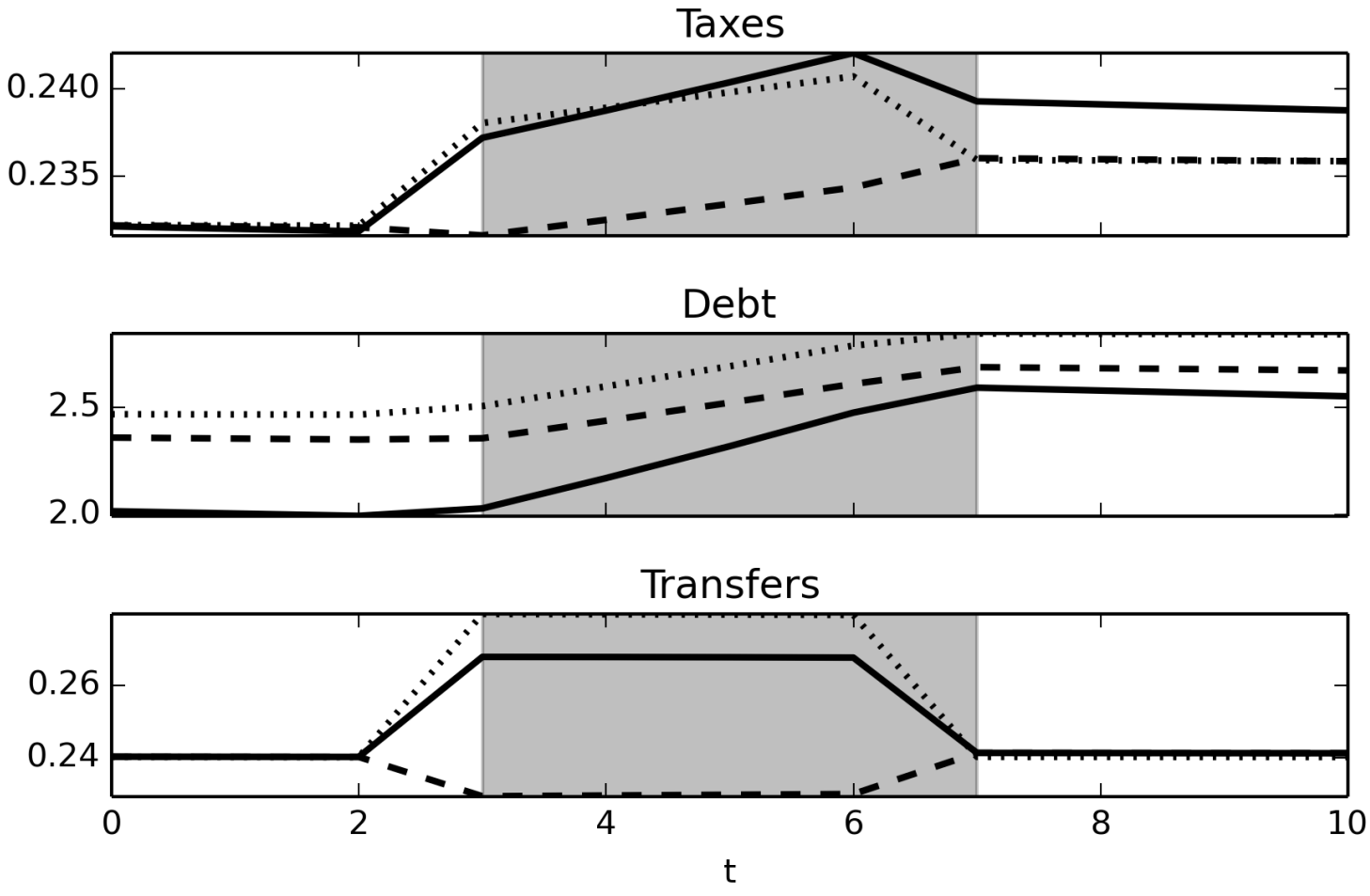

Figure : The bold line is the total response. The dashed (dotted) line reflects the only TFP (inequality) effect. The shaded region is the recession

Tfp and Tfp+Ineq recessions: Sample moments

Moments	Tfp	Tfp+Ineq
vol. of taxes	0.003	0.006
vol. of transfers	0.01	0.02
autocorr. in taxes	0.93	0.66
autocorr. in transfers	0.17	0.18
corr. of taxes with tfp	0.15	-0.63
corr. of transfers with tfp	0.99	-0.98

Table : These are sample moments averaged acrosss simulations of 100 periods

Redistribution in recessions

- TFP : Relative inequality is unchanged and planner redistributes by lowering tax-rates on impact.
- Only Ineq : Earnings gap increases by factor m. The planner mainly redistributes mainly through higher transfers and taxes.
- TFP + Ineq: For both tax rates and transfers are higher. Relative to the Tfp case the volatility of taxes and trasnfers is twice as much.