Taxes, Debts, and Redistribution

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3 Questions

1. How costly are government debts?
2. How do concerns for redistribution affect tax smoothing motives?
3. Quantify how should tax, transfer and debt policies respond to aggregate shocks that change inequality?

Point of Departure: Representative agent models

Debt levels:

- Higher levels of debt are generally "more" distortionary.

Tax smoothing:

- LS: With complete markets tax rates should be smooth.
- Werning: Extends the LS insights to heterogeneous agents by establishing an aggregation result.
- AMSS: With a risk free bond, tax rates are eventually smooth and all aggregate shocks are financed using transfers.

Implicit and explicit redistribution

- Representative agent models implicitly model redistribution as a non-negativity constraint on lump sum transfers.
- Hardwires a discontinuity in the costs of fluctuating transfers around zero.
  1. The Ramsey planner either uses state-contingent securities to hedge aggregate shocks.

     or

  2. Accumulates a war chest of assets big enough to finance expenditures using returns on assets plus only positive transfers.

We begin with explicit redistribution motives and let the government set transfers optimally.

Key ingredients

- **Heterogeneity:** Agents are heterogeneous in productivities and assets
- **Instruments:** A tax system that is affine in labor income
- **Markets:** All agents trade a *single* security whose payoffs can depend on aggregate shocks

Answers to the 3 questions

1. **Invariance of debt level:**
   Absent borrowing constraints, Ricardian equivalence holds.
   Borrowing constraints weakly increase welfare
2. **Invariant distribution of tax rates:**
   - Has wide support
   - The mean and variance of tax rates are driven by market incompleteness. The "covariances" that matter are
     - Between returns on the asset and aggregate shocks (Public sector)
     - Between consumption and (pre-tax) earnings (Private sector)
3. **Business cycle dynamics:** (Preliminary) With countercyclical payoffs, tax rate and transfers increases with aggregate TFP.

Environment

- **Uncertainty**: Markov aggregate shocks s_t
- **Demography**: Continuum of infinitely lived agents plus a benevolent planner
- **Technology**: Output $\int \theta_{i,t} l_{i,t} di$ is linear in labor supplies.
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, l_{i,t}\right)$$

- **Preferences** (Planner): Given Pareto weights $\{\omega_i\}$

$$\mathbb{E}_0 \int \omega_i \sum_{t=0}^{\infty} \beta^t U_t^i \left(c_{i,t}, l_{i,t}\right) di$$

- **Asset markets**: A "risky" bond with payoffs $P_t = \mathbb{P}(s_t | s_{t-1})$

Environment, II

- **Affine Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_{i,t} l_{i,t}$$

- **Budget constraints** Let $R_{t-1,t} = \frac{P_t}{q_{t-1}}$
  - Agent i: $c_{i,t} + b_{i,t} = (1 - \tau_t)\,\theta_{i,t} l_{i,t} + R_{t-1,t} b_{i,t-1} + T_t$
  - Government: $g_t + B_t + T_t = \tau_t \int \theta_{i,t} l_{i,t} di + R_{t-1,t} B_{t-1}$

- **Market Clearing**
  - Goods: $\int c_{i,t} di + g_t = \int \theta_{i,t} l_{i,t} di$
  - Assets: $\int b_{i,t} di + B_t = 0$

- **Initial conditions**: Distribution of assets, productivities $(\Psi_0(b_{i,-1}, \theta_{i,-1}), B_{-1}, s_{-1})$

Ramsey Problem

Definition

Allocation, price system, government policy: Standard

Definition

Competitive equilibrium: Given $(\Psi_0(b_{i,-1}, \theta_{i,-1})_i, B_{-1}, s_{-1})$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$ all allocations are chosen optimally, markets clear

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given $(\Psi_0(b_{i,-1}, \theta_{i,-1}), B_{-1}, s_{-1})$

Recursive formulation

Ricardian Equivalence

- **Result**: A large set of transfers and asset profiles support the same competitive allocation
  Taking away a unit of all agents' assets and increasing transfers by a unit leaves budget sets unchanged

- **Implication:** Ceteris paribus, an economy with higher level of initial government debt but same relative holdings has the same welfare

- **Corollary:** Exogenous borrowing constraints of the form $b_{it} > \underline{b}_i$ are not restrictive

  If some borrowing constraints bind, the planner can change transfers to slacken all of them

Thus, Ricardian equivalence holds with distortionary taxes and ad hoc borrowing limits

Active channels

1. Varying labor taxes imposes dead weight losses.
2. Effects of taxes on redistribution depends on cross sectional heterogeneity of consumption and earnings.
3. Fluctuating transfers is costly because of concerns for redistribution.
4. The benefits of fluctuating transfers depend of limits to fiscal hedging.

Simple Example: 2 Agent QL economy

Consider a "seemingly" AMSS economy

1. Two classes of agent with constant productivities $\theta_1 > 0, \theta_2 = 0$
2. Preferences given by $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$
3. Pareto weights $\{\omega, 1-\omega\}$
4. Only i.i.d aggregate shocks to $g(s)$

In this example,

- Costs of transfers will come from Pareto weights
- Hedging motive will be controlled using payoffs $P(s)$

Case 1: Risk free bond

Normalize the assets of the unproductive agent to zero and let b_t denote the debt issued by the government.

Theorem

Let $\omega > \bar{\omega}$ and suppose $P(s) = 1$, $\lim_t \tau_t = -\infty$, $\lim_t b_t = -\infty$ a.s

Corollary

Suppose we augment our problem with a constraint $b_t \geq \underline{b}$. Then with risk-free debt there is an invariant distribution ψ. Morever, for any $\hat{b} \in (\underline{b}, \bar{b}_n)$, $\psi\left(\left[\underline{b}, \hat{b}\right]\right) > 0$ and $\psi\left(\left[\hat{b}, \bar{b}_n\right]\right) > 0$..

The invariant distribution of taxes is wide as fluctuations in transfers are costly

Case 2: Perfect hedging

Theorem

Let $\omega > \bar{\omega}$ and suppose $\overline{b} < \overline{b}_n$ and payoffs satisfy

$$P(s) = 1 - \frac{\beta}{\overline{b}}(g(s) - \mathbb{E}g)$$

then for all b_{-1}, $b_t \to \overline{b}$ a.s

Corollary

The invariant distribution of b (and also tax rates) is degenerate with $\overline{b} = -\beta \frac{\mathrm{var}[g(s)]}{\mathrm{cov}[P(s), g(s)]}$

The limiting allocation corresponds to a complete market Ramsey problem

Case 3: Imperfect hedging

Decompose payoffs

$$P(s) = \hat{P}(s) + \bar{P}(s)$$

where $\overline{P}(s) = 1 - \frac{\beta}{\overline{b}}(g(s) - \mathbb{E}g)$ and $\hat{P}(s)$ is orthogonal to $g(s)$.

Theorem

For $\omega > \bar{\omega}$, the ergodic distribution of debt of the policy rules linearized around $(\overline{b}, \overline{P}(s))$ will have mean $\overline{b}$ and and coefficient of variation

$$\frac{\sigma_b}{\overline{b}} \leq \sqrt{\frac{\mathrm{var}(\hat{P})}{\mathrm{var}(\bar{P})}}$$

The speed of convergence to the ergodic distribution given by

$$\frac{\mathbb{E}_{t-1}(b_t - \overline{b})}{(b_{t-1} - \overline{b})} = \frac{1}{1 + |\mathrm{cov}(P(s), g(s))|}$$

More generally

1. **Risk aversion**
   - Endogenous component to covariance between payoffs and expenditure needs.
   - Costs of transfers come from spreads in marginal utilities.

2. **Idiosyncratic risk**
   - Joint distribution of consumption and pre-tax earnings drives the mean level of taxes
   - The spread of taxes is mainly a determined by public sector's fiscal hedging abilities

Numerical example

Features:

- Continuum of agents
- Pre tax wage earnings with transitory and persistent component
- IID aggregate TFP shocks

Calibration:

- For now innovation to aggregate and idiosyncratic wages are orthogonal
- Initial distribution of wages and earnings calibrated to broadly match Daz-Gimnez et al

Study responses to aggregate shocks

Findings

1. As before tax rates can drift in the long run.
2. In the short run what matters is how needs for redistribution change over business cycle.
   - The heterogeneity in wealth effect drives how aggregate shocks affect the distribution of labor earnings
   - The cylicality of interest rate affect needs and costs of increasing transfers

Bond Economy

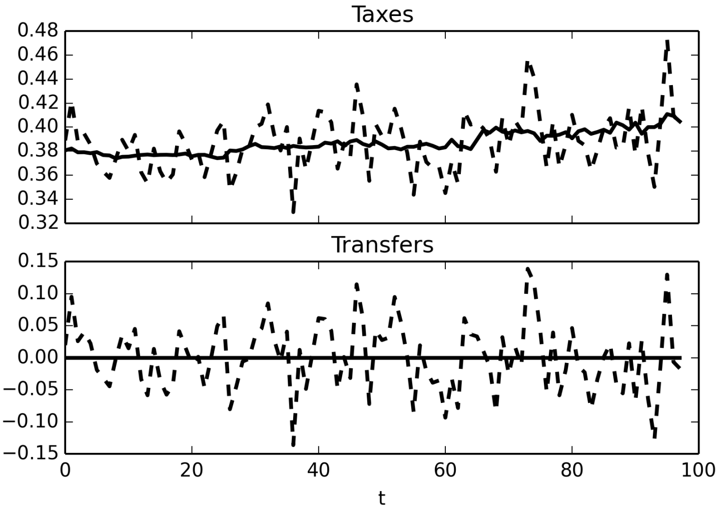


Figure: This plots a simulation for taxes and transfers for 100 periods. The dotted (bold) line is the economy with (without) i.i.d aggregate shocks

Bond Economy

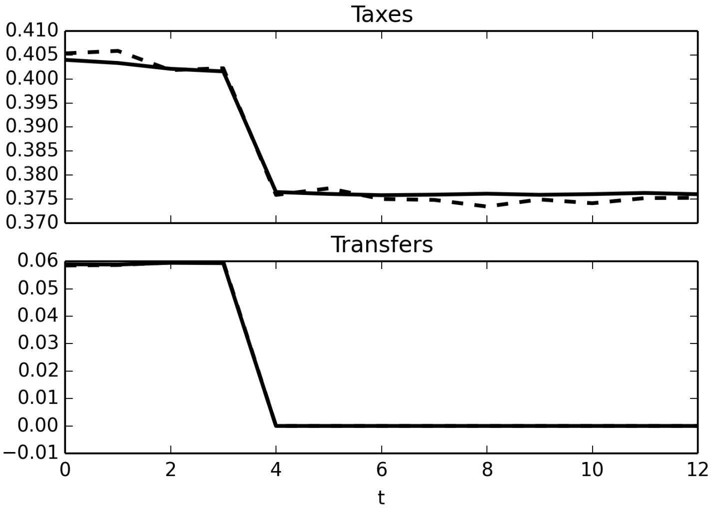


Figure: This plots a "impulse response" (5 period of high aggregate TFP followed by no aggregate shocks) for taxes, transfers. The dotted (bold) line is the economy with (without) idiosyncratic risk

Changes in inequality: Bond Economy

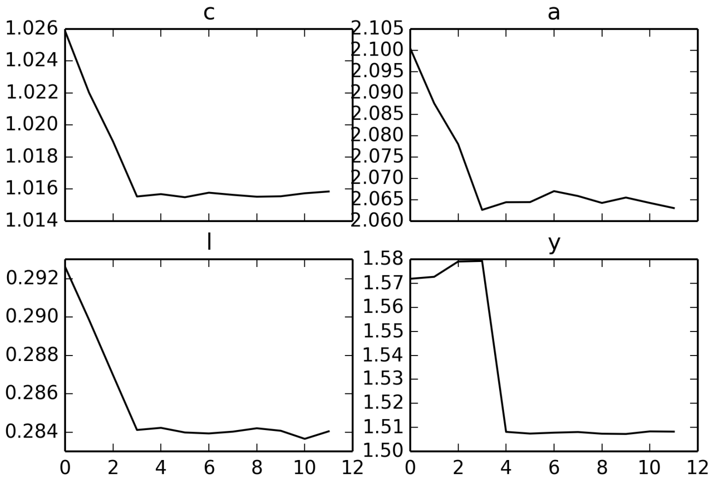
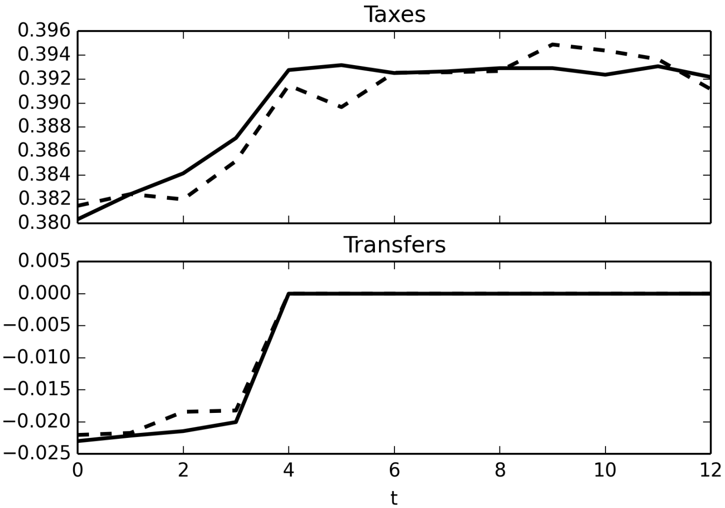


Figure: Path for (cross-sectional) coefficient of variation after 5 periods of "High" TFP followed by no shocks

Procyclical payoffs



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Changes in inequality: Procyclical payoffs

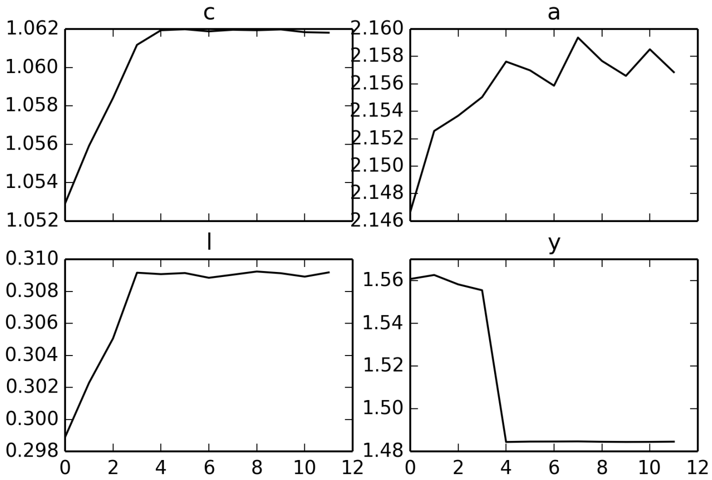

Figure: Path for (cross-sectional) coefficient of variation after 5 periods of "High" TFP followed by no shocks

Conclusions/Next Steps

- Size of the debt alone does not matter
- Optimal tax and transfer scheme balance
  1. welfare losses from fluctuating taxes
  2. welfare losses from fluctuating transfers
- Limits to fiscal hedging are key for dynamics of taxes and transfers

Next Step:

- Calibrate a richer model for idiosyncratic risk that allows for higher moments to vary systematically over business cycle
- More richer taxation scheme

Ramsey problem: Recursive formulation

Split into two parts

1. $\mathbf{t \geq 1}$: Ex-ante continuation problem with state variables $\{\Gamma_{t-1}(x_{i,t-1}, m_{i,t-1}, \theta_{i,t-1}), s_{t-1}\}$, where
   - Scaled assets: $x_{i,t-1} = u_{c,i,t-1} b_{i,t}$
   - Scaled "market weights": $m_{i,t-1} \propto \frac{1}{u_{c,i,t-1}}$
2. $\mathbf{t = 0}$: Ex-post initial problem with state variables $(\Psi_0(b_{i,-1}), s_0)$

Bellman Equation for $t \geq 1$

$$V(\Gamma_{\text{-}}, s_{\text{-}}) = \max_{\substack{c_i(s), l_i(s), x_i(s), m_i(s) \\ \tau(s), T(s), \alpha_1, \alpha_2(s)}} \sum_s \Pr(s|s_{\text{-}}) \left[\int \omega_i u(c_i(s), l_i(s)) di + \beta V(\Gamma(s), s) \right]$$

where the maximization is subject to

$$\frac{x_{i,\text{-}} u_{i,c}(s) P(s|s_{\text{-}})}{\beta \mathbb{E}^i_{s_{\text{-}}} u_{i,c}(s) P(s|s_{\text{-}})} = u_{i,c}(s)[c_i(s) - T(s)] + u_{i,l}(s) l_i(s) + x_i(s)$$

$$\alpha^1 = m_{i,\text{-}} \mathbb{E}^i_{s_{\text{-}}} u_{i,c}(s)$$

$$\alpha^2(s) = m_i(s) u_{i,c}(s)$$

$$-u_{i,l}(s) = [(1 - \tau(s)] u_{i,c}(s) \theta_i(s)$$

$$\int m_i(s) di = 1$$

$$\int l_i(s) \theta_i(s) di = \int c_i(s) di + g(s)$$

Bellman equation for $t = 0$

$$V_0\left(\Psi_0(b_{i,-1}), s_0\right) = \max_{\substack{c_{i,0}, l_{i,0}, x_{i,0}, m_{i,0} \\ \tau_0, T_0}} \int \omega_i U^i(s_0) + \beta V\left(\Gamma_0, s_0\right)$$

where the maximization is subject to

$$b_{i,-1} u_{i,c,0} = u_{i,c,0}[c_{i,0} - T_0] + u_{i,l,0} l_{i,0} + x_{i,0}$$

$$-u_{i,l,0} = (1 - \tau_0) u_{i,c,0} \theta_{i,0}$$

$$\int m_{i,0} di = 1$$

$$\int l_{i,0} \theta_{i,0} di = \int c_{i,0} di + g_0$$

back