Taxes, Debts, and Redistribution

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September 2013

Motivation

- How costly are high levels of government debt? What determines welfare cost of debt?
- Should the gov't try to reduce its initial high debt? If so, how quickly?
- How should tax rates, transfers, and government debt respond to aggregate shocks, especially if markets are incomplete?

Motivation

- Analysis with complete markets is well known:
  - Tax rates (approximately) constant
  - Arrow securities used to finance all expenditure needs

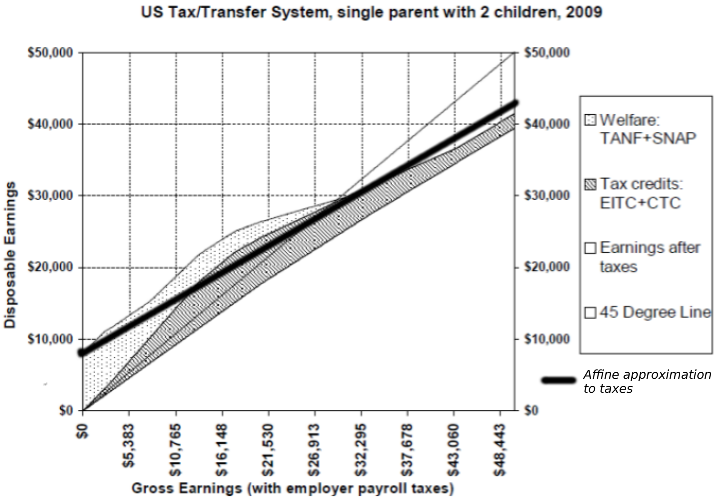
Our focus: Markets less than fully complete

Key ingredients

- **Heterogeneity:** Agents are heterogeneous in productivities and assets
- **Instruments:** A tax system that is linear in labor income and an intercept that is uniform across agents
- **Markets:** All agents trade a *single* security whose payoff might depend on aggregate shocks

Characterize optimal tax rate, transfers and asset purchases

US taxes: Affine taxes



Findings

- **Welfare cost** of debt is determined by distribution of asset positions across agents
- **Ergodic distribution** of debts and taxes, in particular mean, variance and speed of convergence depend on
  - correlation of returns on the traded asset with govt's needs for revenue, and
  - govt's concern for redistribution
- Analytical results for quasilinear preferences and some extensions to more general preferences
- **Optimal responses over business cycle**
  - For short run responses, nature of shock matters
  - In recessions with high inequality: big increase in transfers and debt, moderate increase in tax rates

Insights

- **Ricardian logic:** Increasing all agents' assets and reducing transfers keeps budget sets unaltered
- **Policy tradeoffs:** Use tax rates or transfers to hedge aggregate shocks?
- What mechanisms drives **long run debt and tax rates**?
  - If interest rate co-moves with revenue needs: issue positive debt
  - Larger the correlation: lower the magnitude debt and higher is the speed of convergence
  - More redistributive governments: larger transfers and less incentives to accumulate assets
- In the US data,
  - Correlation of interest rates and business cycles is small
  - In recent recessions, low income agents faced much larger drops in income than high income agents
- Normative predictions on which policy instrument to use can be very different if these considerations are ignored

Related literature

- Representative agent incomplete market economies
  - Barro (1974, 1979), Aiyagari et al (2002), Faraglia-Marcet-Scott (2012), Farhi (2010), etc
- Representative agent complete market economies
  - Lucas-Stokey (1983), Chari-Kehoe (1999), etc
- Heterogeneous agents with complete markets
  - Werning (2007), Azzimonti-Francisco-Krusell (2008)

Environment

- **Uncertainty**: Markov aggregate shocks s_t
- **Demography**: N types of infinitely lived agents (mass n_i) plus a benevolent planner
- **Technology**: Output $\sum_i n_i \theta_{i,t} l_{i,t}$ is linear in labor supplies.
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, l_{i,t}\right)$$

- **Preferences** (Planner): Given Pareto weights $\{\omega_i\}$

$$\mathbb{E}_0 \sum_i \omega_i \sum_{t=0}^{\infty} \beta^t U_t^i \left(c_{i,t}, l_{i,t}\right)$$

- **Asset markets**: A risky bond with payoffs $P_t = \mathbb{P}(s_t | s_{t-1})$

Environment, II

- **Affine Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_{i,t} l_{i,t}$$

- **Budget constraints** Let $R_{t-1,t} = \frac{P_t}{q_{t-1}}$
  - Agent i: $c_{i,t} + b_{i,t} = (1 - \tau_t)\,\theta_{i,t} l_{i,t} + R_{t-1,t} b_{i,t-1} + T_t$
  - Government: $g_t + B_t + T_t = \tau_t \sum_i n_i \theta_{i,t} l_{i,t} + R_{t-1,t} B_{t-1}$

- **Market Clearing**
  - Goods: $\sum_i n_i c_{i,t} + g_t = \sum_i n_i \theta_{i,t} l_{i,t}$
  - Assets: $\sum_i n_i b_{i,t} + B_t = 0$

- **Initial conditions**: $(\{b_{i,-1}, B_{-1}\}_i, s_{-1})$

Ramsey Problem

Definition

Allocation, price system, government policy: Standard

Definition

Competitive equilibrium: Given $(\{b_{i,-1}\}_i, B_{-1}, s_{-1})$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$, a competitive equilibrium is an allocation and price system such that households are optimizing and markets clear

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given $(\{b_{i,-1}\}_i, B_{-1}, s_{-1})$

Ricardian Equivalence

Result: A **large set** of transfers and asset profiles support the same competitive equilibrium allocation
Notation: $\tilde{b}_{i,t} = b_{i,t} - b_{1,t}$: **relative assets** of Agent i

Theorem

Given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$*, let* $\{\{c_{i,t}, l_{i,t}, b_{i,t}\}_i, B_t, R_t\}_t$ *and* $\{\tau_t, T_t\}_t$ *be a competitive equilibrium.*
For any bounded sequences $\left\{\hat{b}_{i,t}\right\}_{i,t\geq -1}$ *that satisfy*

$$\hat{b}_{i,t} - \hat{b}_{1,t} = \tilde{b}_{i,t} \text{ for all } t \geq -1, i \geq 2,$$

there exist sequences $\left\{\hat{T}_t\right\}_t$ *and* $\left\{\hat{B}_t\right\}_{t\geq -1}$ *such that* $\left\{\left\{c_{i,t}, l_{i,t}, \hat{b}_{i,t}\right\}_i, \hat{B}_t, R_t\right\}_t$ *and* $\left\{\tau_t, \hat{T}_t\right\}_t$ *constitute a competitive equilibrium given* $\left(\left\{\hat{b}_{i,-1}\right\}_i, \hat{B}_{-1}\right)$.

Ricardian Equivalence: Implications

- Present value of tax revenues and gov't debt is pinned down but not period-by-period transfers
- Can set $b_{i,t} = 0$ for any t, i or government without loss of generality
- Generally, more equally spread debt promised (implicit Social Security promises, debt in Japan) are less distortionary than debt skewed towards highly productive agents or foreigners (debt in Greece)
- **Extension:** Welfare is weakly higher with exogenous borrowing constraints of the form $b_{i,t} > \underline{b}_i$

More details

Characterization of optimal policy: Road map

- Active channels:
  1. Limited hedging ability
  2. Concerns for redistribution
- Analytical results:
  1. Quasi Linear preferences :$u(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$
  2. IID aggregate shocks
- 2 step build up:
  1. Assume first that there is one agent and no ability to use T
  2. Use results to characterize outcomes in the more general settings with heterogeneous agents and no restriction on transfers
- Reasoning:
  1. Allows us to disentangle hedging and redistribution motives
  2. Informative about cases where the government does not care enough about redistribution

Single agent quasi-linear economy with $T \equiv 0$

- Decompose the set of payoffs

$$\mathcal{P}^* = \left\{ P(s) : P(s) = 1 + \frac{\beta}{B^*}(g(s) - \mathbb{E}g) \text{ for some } B^* \in [\overline{B}, \underline{B}] \right\}$$

- Let $V(B_-)$ be the maximum ex-ante value the government can achieve with assets B_-.

$$V(B_-) = \max_{c(s), l(s), B(s)} \sum_s \pi(s) \left\{ c(s) - \frac{l(s)^{1+\gamma}}{1+\gamma} + \beta V(B(s)) \right\}$$

subject to

$$c(s) - B(s) = l(s)^{1+\gamma} - \beta^{-1} P(s) B_-$$

$$c(s) + g(s) \leq \theta l(s)$$

$$\underline{B} \leq B(s) \leq \bar{B}$$

Invariant distribution

Theorem

1. *Suppose* $P \notin \mathcal{P}^*$, *there is an invariant distribution of government assets such that*

$$\forall \epsilon > 0, \quad \Pr\{B_t < \underline{B} + \epsilon \text{ and } B_t > \overline{B} - \epsilon \quad i.o\} = 1$$

2. *Suppose* $P(s) - P(s') > \beta \frac{g(s) - g(s')}{\underline{B}} \quad \forall s, s'$, *then for large enough government assets (or debt) there is a drift towards the interior region. In particular the value function* $V(B)$ *is strictly concave and there exists* $B_1 < B_2$ *such that*

$$\mathbb{E}V'(B(s)) > V'(B_-) \quad B_- > B_2$$

and

$$\mathbb{E}V'(B(s)) < V'(B_-) \quad B_- < B_1$$

3. *Suppose* $P(s) \in \mathcal{P}^*$, *then the long run government assets converge to a degenerate steady state*

$$\lim_t B_t = B^* \quad a.s \quad \forall B_{-1}$$

Perfect spanning

- For $P(s) \in \mathcal{P}^*$, we can replicate complete markets perfectly asymptotically
- Target assets
$$B^* = \beta \frac{\mathrm{var}(g(s))}{\mathrm{cov}(P(s), g(s))}$$
- Tax rate is constant in long run and inversely related to B^*.
- Use this to construct an approximation for the ergodic distribution of debt and taxes of an economy with $P(s)$ "close" enough to $\mathcal{P}^*$. In particular split $P(s)$
$$P(s) = \hat{P}(s) + P^*(s)$$
where $P^*(s) \in \mathcal{P}^*$ and $\hat{P}(s)$ is orthogonal to $g(s)$. More details

Imperfect spanning

Theorem

The ergodic distribution of debt (under the first order approximation of dynamics near $P^*(s)$) has the following properties,

- **Mean:** *The ergodic mean is B^* which corresponds to the steady state level of govt. assets of an economy with payoff vector $P^*(s)$*
- **Variance:** *The coefficient of variation of assets satisfies*

$$\frac{\sigma(B)}{\mathbb{E}(B)} = \sqrt{\frac{\mathrm{var}(P(s)) - |\mathrm{cov}(g(s), P(s))|}{(1 + |\mathrm{cov}(g(s), P(s))|)|\mathrm{cov}(g(s), P(s))|}} \leq \sqrt{\frac{\mathrm{var}(\hat{P}(s))}{\mathrm{var}(P^*(s))}}$$

- **Convergence rate:** *The speed of convergence to the ergodic distribution is*

$$\frac{\mathbb{E}_{t-1}(B_t - B^*)}{(B_{t-1} - B^*)} = \frac{1}{1 + |\mathrm{cov}(P(s), g(s))|}$$

Ergodic distribution

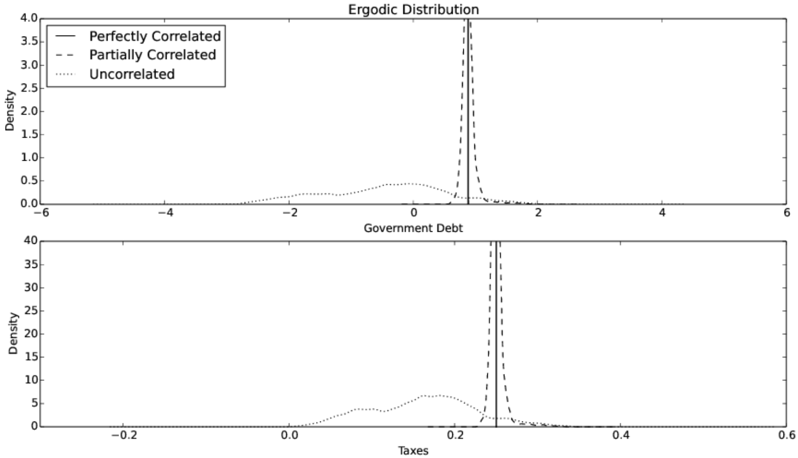


Figure : Ergodic distribution for debt and taxes in the representative agent quasilinear economy for three choices $P(s)$.

Summary and next steps

So far: In a single agent - quasilinear - no transfers economy we saw that,

- Target level of assets maximizes spanning
  - taxes are constant when perfect spanning is achieved
- When markets are imperfect, can be far away from the target
  - invariant distribution of taxes also has large support
- Speed of moving to the target debt level depends on covariance of asset payoff and shocks
  - low covariance $\Longrightarrow$ slow speed

Next: A version with heterogeneous agents and no restrictions on transfers

- This adds a new instrument to hedge shocks but welfare cost of using transfers is endogenous
- The single agent results are informative about cases where these costs are large

Heterogeneous agents

Suppose we have 2 agents

- Quasi-linear preferences as before
- Productivities: $\theta_1 > \theta_2 = 0$
- Pareto weights, mass of Agent 1 and 2: $\omega, 1 - \omega$ and $n, 1 - n$ respectively
- Non-negative consumption: $c_2 \geq 0$

Normalize $b_{2,t} = 0$, thus $B_t = -n_1 b_{1,t}$ are interpreted to be government assets

Heterogeneous Agents

Theorem

Let ω, n be the Pareto weight and mass of the productive agent with $n < \frac{\gamma}{1+\gamma}$. The optimal tax, transfer and asset policies $\{\tau_t, T_t, B_t\}$ are characterized as follows,

1. *For $\omega \geq n\left(\frac{1+\gamma}{\gamma}\right)$ we have $T_t = 0$ and the optimal policy is same as in our representative agent economy studied*
2. *For $\omega < n\left(\frac{1+\gamma}{\gamma}\right)$, suppose we assume that $P(s) \notin \mathcal{P}^*$ and $\min_s\{P(s)\} > \beta$. There exists $\mathcal{B}(\omega)$ and $\tau^*(\omega)$ with $\mathcal{B}'(\omega) > 0$ such that*

    2.1 $B_- > \mathcal{B}(\omega)$

    $$T_t > 0, \quad \tau_t = \tau^*(\omega), \text{ and } B_t = B_- \quad \forall t$$

    2.2 $B_- \leq \mathcal{B}(\omega)$

    $$T_t > 0 \text{ i.o.}, \quad \lim_t \tau_t = \tau^*(\omega) \text{ and } \lim_t B_t = \mathcal{B}(\omega) \quad a.s$$

Concerns for redistribution

- Balancing costs of fluctuations in tax rates and transfer
  - fluctuations in taxes is costly: deadweight loss
  - fluctuations in transfers is costly: deviations from target level of redistribution
- For large ω transfers are costly as the planner gives resources to unproductive agents
- For low ω, transfers are used:
  - For low initial debt, interior solution: All shocks hedged by transfers
  - For high debt, accumulate assets until costs of transfers are equalized to costs of collecting labor taxes
- The more redistributory the planner is:
  - bigger average tax rates and transfers
  - less need to accumulate assets for precautionary reasons

Risk aversion

- With risk aversion: for a (generic) set of parameters there is asset allocation replicating complete market economy
  - arguments harder since "real" interest rates $\mathbb{E}_t U'(c_{t+1}) R(s_{t+1}) / U'(c_t)$ is endogenous
- Same general flavor as quasi-linear economy
  - cost of fluctuations in transfers comes from cost of fluctuation in $U_c \iff$ similar to multiplier on constraint $c \geq 0$ in quasi-linear case
  - If real payoffs are positively correlated with g : accumulate assets
  - If real payoffs are (sufficiently) negatively correlated with g : accumulate debt
  - absolute amount of asset/debt is decreasing in redistributive objective

More details

Numerical exercise

Solve $N = 5$ agent economy with realistic level and movements in wage dispersion across booms and recessions

- Long run dynamics: Study settings that differ in covariance of interest rates and output
- Transient dynamics: Study outcomes in recessions that are accompanied by higher inequality

Aggregate shocks affect,

1. Wages:
$$\log \theta_i = \epsilon[1 + (.9 - d(i))m]$$
2. Payoffs:
$$P = 1 + \chi\epsilon$$

Calibrating *m*: Inequality over business cycles

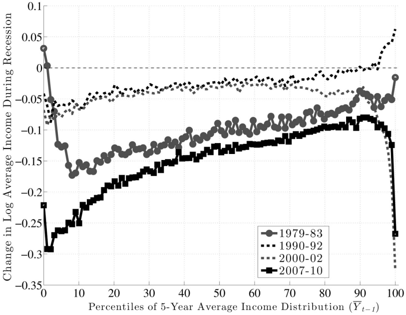


Figure : Change in log average earnings during recessions, prime-age males from Guvenen et all [2014]

Calibrating χ: Ex post variation in Payoffs

Let $q_t^{(n)}$ be the log price of a nominal bond of maturity n. We can define the real holding period returns $r_{t,t+1}^{(n)}$ as follows

$$r_{t,t+1}^{(n)} = q_{t+1}^{(n-1)} - q_t^{(n)} - \pi_{t+1}$$

With the transformation $y_t^{(n)} : -\frac{1}{n} q_t^{(n)}$ we can express $r_{t,t+1}^{(n)}$ as follows:

$$r_{t,t+1}^{(n)} = \underbrace{y_t^{(n)}}_{\text{Ex-ante part}} -(n-1)\left[\underbrace{\left(y_{t+1}^{(n)} - y_t^{(n)}\right)}_{\text{Interest rate risk given } n} + \underbrace{\left(y_{t+1}^{(n-1)} - y_{t+1}^{(n)}\right)}_{\text{Term structure risk}}\right] - \underbrace{\pi_{t+1}}_{\text{Inflation risk}}$$

Interest rates and TFP

- In the model the holding period returns are given by $\log\left[\frac{P_{t+1}}{q_t^1}\right]$ and $q_t^1 = \frac{\beta \mathbb{E}_t u_{c,t+1} P_{t+1}}{u_{c,t}}$.
- P_{t+1} allows us to captures ex-post fluctuations in returns to the government's debt portfolio coming from maturity and inflation.
- Since ϵ_t is i.i.d over time in our calibration $\chi = \frac{\sigma_r}{\sigma_\epsilon} Corr(r, \epsilon)$

Using data on labor productivity ϵ_t and $\{q_t^n\}_n$:

Maturity (n)	2yr	3yr	4yr	5yr
$Corr(\epsilon_{t+1}, r_{t,t+1}^{(n)})$	-0.11	-0.093	-0.083	-0.072
$Corr(\epsilon_{t+1}, r_{t,t+1}^{(n)} - ny_t^{(n)})$	0.00	-0.0463	-0.080	-0.091
$Corr(\epsilon_{t+1}, y_t^{(n)} - \pi_{t+1})$	-0.097	-0.086	-0.080	-0.073
$\frac{\sigma(r_{t+1}^n)}{\sigma(\epsilon_{t+1})}$	0.820	0.835	0.843	0.845

Table

Calibration

Parameter	Value	Description
$\{\bar{\theta}_i\}$	$\{1$, 1.4, 2.1, 3.24, 4.9$\}$	Wages dispersion for $\{$10,25,50,75,90$\}$ percentiles
γ	2	Average Frisch elasticity of labor supply of 0.5
β	0.98	Average (annual) risk free interest rate of 2%
m	$\frac{1.5}{.8}$	Changes in dispersion
χ	-0.06	covariance between holding period returns and labor productivity
σ_e	0.03	vol of labor productivity
g	.13 %	Average pre-transfer expenditure- output ratio of 12 %

Table : Benchmark calibration

The Pareto weights and initial distribution of wealth are chosen to match an average tax rate of 20%, and debt to gdp ratio of 100%, transfers to gdp ratio of 10%, and deciles of US wealth distribution

Long run

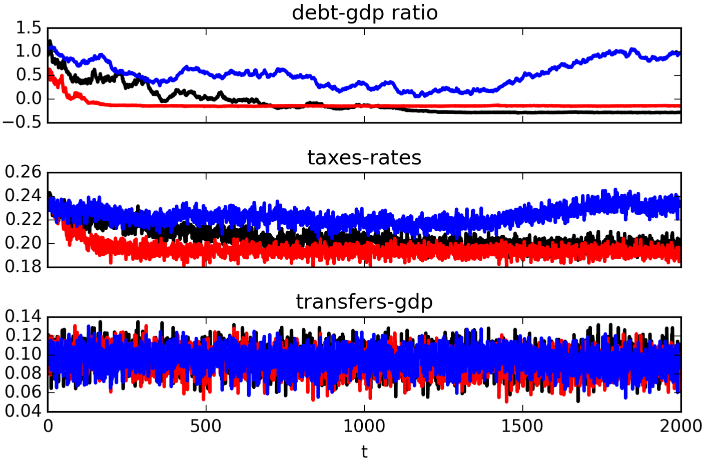


Figure : The red, black and blue lines plot simulations for a common sequence of shocks for values of $\chi = -1.0, 0, 1.0$ respectively

Long run: Speed of convergence

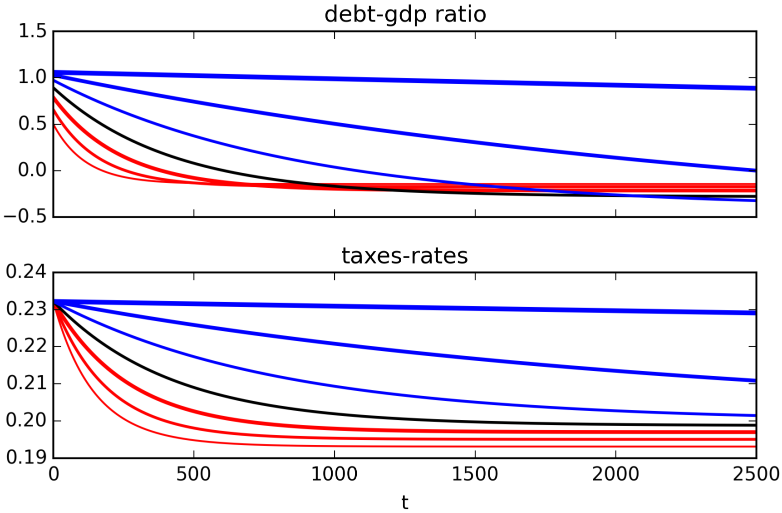


Figure : The plot shows conditional mean paths for different values of χ. The red (blue) lines have $\chi < 0$ ($\chi > 0$). The thicker lines represent larger values

Spreading of tax rates

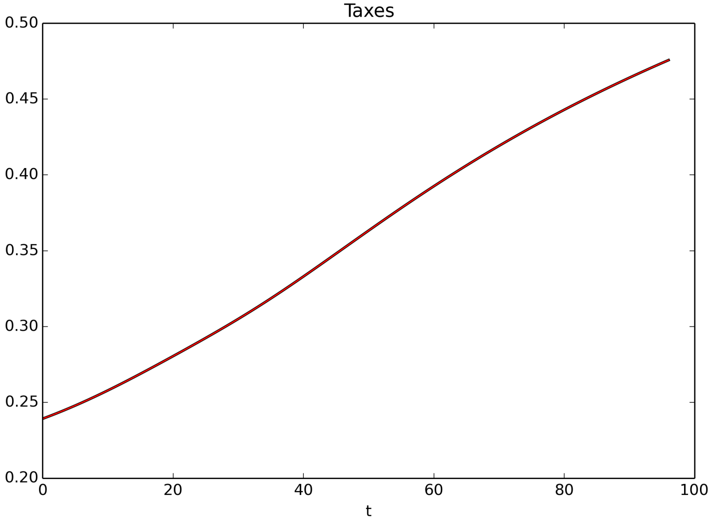


Figure : Tax rate for a sequence of -1 s.d shocks to aggregate productivity

Short run

Let us denote consecutive period of negative (positive) one s.d ϵ shocks a "recession" (boom)

- Engineer a recession of four periods from $t = 3$. Before and after this recession, the economy receives $\epsilon_t = 0$.
- Decompose responses into TFP component and inequality component:

$$\textbf{Baseline:}\ \log\theta_i = \epsilon[1 + (.9 - d(i))m]$$

  - Only TFP:
$$\log\theta_i = \epsilon$$
  - Only Ineq:
$$\log\theta_i = \epsilon[(.9 - d(i))m]$$

Recessions with higher inequality

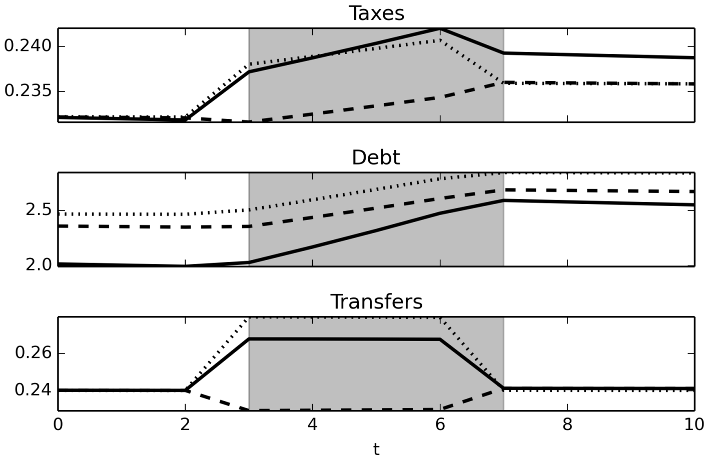


Figure : The bold line is the total response. The dashed (dotted) line reflects the only TFP (inequality) effect. The shaded region is the recession

Tfp and Tfp+Ineq recessions: Sample moments

Moments	Tfp	Tfp+Ineq
vol. of tax rates	0.003	0.006
vol. of transfers	0.01	0.02
autocorr. in tax rates	0.93	0.66
autocorr. in transfers	0.17	0.18
corr. of taxes with tfp	0.15	-0.63
corr. of transfers with tfp	0.99	-0.98

Table : These are sample moments averaged acrosss simulations of 100 periods

Conclusion

Credit limits

Impose $b_{i,t} \geq \underline{b}_i$. and extend the definition of competitive eqb. in the obvious way. Now we have

Theorem

Given an initial asset distribution $\left(\{b_{i,-1}\}_i, B_{-1}\right)$*, let* $\{c_{i,t}, l_{i,t}\}_{i,t}$ *and* $\{R_t\}_t$ *be a competitive equilibrium allocation and interest rate sequence in an economy without exogenous borrowing constraints. Then for any exogenous constraints* $\{\underline{b}_i\}_i$*, there is a government tax policy* $\{\tau_t, T_t\}_t$ *such that* $\{c_{i,t}, l_{i,t}\}_{i,t}$ *is a competitive equilibrium allocation in an economy with exogenous borrowing constraints* $\left(\{b_{i,-1}, \underline{b}_i\}_i, B_{-1}\right)$ *and* $\{\tau_t, T_t\}_t$.

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Ergodic distribution: Linear approximation

- For a given $P(s), g(s)$, we can compress the equilibrium conditions to two functions $b(\mu_{-})$ and a law of motion $\mu(s|\mu_{-})$
- Instead of approximating near a deterministic steady state we,
  - explicitly recognize that policy rules depend on payoffs: $\mu(s|\mu_{-}, \{P(s)\}_s)$ and $b(\mu_{-}, \{P(s)\}_s)$
  - take the first order expansion with respect to both μ_{-} and $\{P(s)\}$ around the vector $(\bar{\mu}, \{\bar{P}(s)\}_s)$ where $\bar{P}(s) \in \mathcal{P}^*$:
- The choice of $\bar{P}(s)$ is pinned down by

$$\min_{\tilde{P} \in \mathcal{P}^*} \sum_s \pi(s)(P(s) - \tilde{P}(s))^2.$$

- The law of motion approximated by

$$\mu_t - \mu^* = (\mu_{t-1} - \mu^*)B(s_t) + C(s_t)$$

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More details on cases with risk aversion

- With risk aversion $\|S\| = 2$ is necessary for a steady state to exist
- Existence: Consider an economy consisting of two types of households with only one productive agent and i.i.d binary shocks to his productivity

Theorem

Suppose $u(c, l) = \ln c - \frac{1}{2} l^2$ *and* $g < \theta(s)$ *for all* s*. Let* $x = U_c^2(s)\left[b_2(s) - b_1(s)\right]$

1. **Countercyclical interest rates.** *If* $P(s_H) = P(s_L)$*, then there exists a steady state* $\left(x^{SS}, \rho^{SS}\right)$ *such that* $x^{SS} > 0,\ R^{SS}(s_H) < R^{SS}(s_L)$*.*
2. **Procyclical interest rates.** *There exists a pair* $\{P(s_H), P(s_L)\}$ *such that there exists a steady state with* $x^{SS} < 0$ *and* $R^{SS}(s_H) > R^{SS}(s_L)$*.*

In both cases, tax rates $\tau(s) = \tau^{SS}$ *and ratio of consumption* $\frac{c_1}{c_2}$ *are independent of the realized state.*

- We then develop a test for local stability as in the quasilinear case.

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