Taxes, Debts, and Redistribution

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Motivation

- How costly are high levels of government debt? What determines welfare cost of debt?
- Should the gov't try to reduce its initial high debt? If so, how quickly?
- How should tax~~es~~ rates, transfers, and government debt respond to aggregate shocks, especially if markets are incomplete?

Motivation

- Analysis with complete markets is well known:
  - Tax~~es~~ rate (approximately) constant
  - Arrow securities used to finance all expenditure needs

Our focus: Markets less than fully complete

Key ingredients

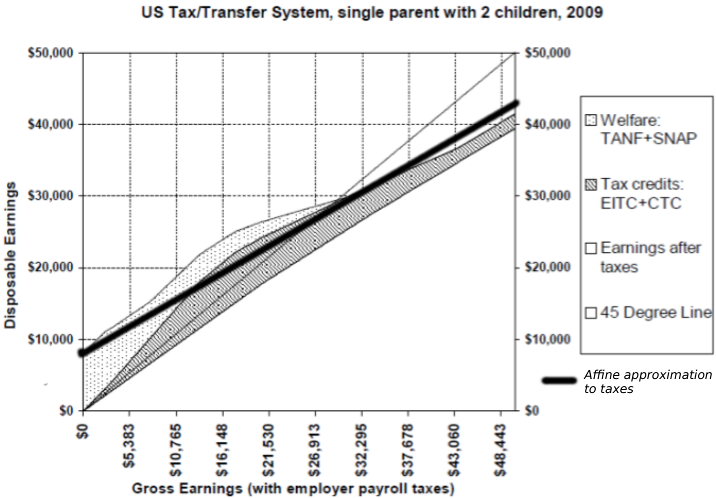
- **Heterogeneity:** Agents are heterogeneous in productivities and assets
- **Instruments:** A tax system that is linear in labor income and an intercept that is uniform across agents

  might
- **Markets:** All agents trade a *single* security whose payoff~~s~~ can depend on aggregate shocks

Characterize optimal tax~~es~~, transfers and asset purchases

rate

US taxes: Affine taxes



Findings

- **Welfare cost** of debt is determined by distribution of ~~relative~~ [illegible] asset~~s~~ positions across agents s
- **Ergodic distribution** of debt~~(s)~~ and taxes~~, in particular mean, variance~~ and speed of convergence depend~~s~~ on~~,~~ to it
  - Correlation of returns on the traded asset with govt's needs for revenue
  - Government's concern~~s~~ for redistribution
- Analytical results for quasilinear ~~economies~~ preferences and some extensions to more general preferences
- **Optimal responses over business cycle**
  - For short run responses, nature of shock matters
  - In recessions with high inequality: big increase in transfers and debt, moderate increase in tax~~es~~ rate

Insights

- **Ricardian logic:** Increasing all agents' assets and reducing transfers keeps budget sets unaltered
- **Policy tradeoffs:** ~~Fluctuate~~ Use tax~~es~~ rate or transfers to hedge aggregate shocks?
- What mechanisms drives **long run debt and tax~~es~~** rate?
  - If interest rate~~s~~ co-moves with revenue needs: issue positive debt
  - Larger the correlation: lower the magnitude debt and higher is the speed of convergence
  - More redistributive governments: larger transfers and less incentives to accumulate assets

    US
- In the data,
  - Correlation of interest rates and business cycles is small
  - In recent recessions, low income agents faced much larger drops in income than high income agents
- Optimal responses that do not condition on these facts can lead to very different policy implications

govt expend. shocks

productivity shocks

rewrite this

Related literature

- Representative agent incomplete market economies
  - Barro (1974, 1979), Aiyagari et al (2002), Faraglia-Marcet-Scott (2012), Farhi (2010), etc
- Representative agent complete market economies
  - Lucas-Stokey (1983), Chari-Kehoe (1999), etc
- Heterogeneous agents with complete markets
  - Werning (2007), Azzimonti-Francisco-Krusell (2008)

Environment

types of

- **Uncertainty**: Markov aggregate shock~~s~~ s_t
- **Demography**: N infinitely lived agents (mass n_i) plus a benevolent planner
- **Technology**: Output $\sum_i n_i \theta_{i,t} l_{i,t}$ is linear in labor supplies.
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, l_{i,t} \right)$$

- **Preferences** (Planner): Given Pareto weights $\{\omega_i\}$

$$\mathbb{E}_0 \sum_i \omega_i \sum_{t=0}^{\infty} \beta^t U_t^i \left(c_{i,t}, l_{i,t} \right)$$

- **Asset markets**: A risky bond with payoffs $P_t = \mathbb{P}(s_t | s_{t-1})$

Environment, II

- **Affine Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_{i,t} l_{i,t}$$

- **Budget constraints** Let $R_{t-1,t} = \frac{P_t}{q_{t-1}}$
  - Agent i: $c_{i,t} + b_{i,t} = (1 - \tau_t)\,\theta_{i,t} l_{i,t} + R_{t-1,t} b_{i,t-1} + T_t$
  - Government: $g_t + B_t + T_t = \tau_t \sum_i n_i \theta_{i,t} l_{i,t} \cancel{di} + R_{t-1,t} B_{t-1}$

- **Market Clearing**
  - Goods: $\sum_i n_i c_{i,t} + g_t = \sum_i n_i \theta_{i,t} l_{i,t} \cancel{di}$
  - Assets: $\sum_i n_i b_{i,t} + B_t = 0$

- **Initial conditions**: $(\{b_{i,-1}, B_{-1}\}_i, s_{-1})$

Ramsey Problem

and $\{R_t\}$,

privately

Definition

Allocation, price system, government policy: Standard

Definition

Competitive equilibrium: Given $(\{b_{i,-1}\}_i, B_{-1}, s_{-1})$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$ all allocations are chosen optimally, markets clear

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given $(\{b_{i,-1}\}_i, B_{-1}, s_{-1})$

Ricardian Equivalence

equilibrium

Result: A **large set** of transfers and asset profiles support the same competitive allocation ✓

Notation: $\tilde{b}_{i,t} = b_{i,t} - b_{1,t}$ ~~be~~ the **relative assets** of Agent i

Theorem

Given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$*, let* $\{\{c_{i,t}, l_{i,t}, b_{i,t}\}_i, B_t, R_t\}_t$ *and* $\{\tau_t, T_t\}_t$ *be a competitive equilibrium.*

For any bounded sequences $\left\{\hat{b}_{i,t}\right\}_{i,t\geq -1}$ *that satisfy*

$$\hat{b}_{i,t} - \hat{b}_{1,t} = \tilde{b}_{i,t} \text{ for all } t \geq -1, i \geq 2,$$

there exist sequences $\left\{\hat{T}_t\right\}_t$ *and* $\left\{\hat{B}_t\right\}_{t\geq -1}$ *such that* $\left\{\left\{c_{i,t}, l_{i,t}, \hat{b}_{i,t}\right\}_i, \hat{B}_t, R_t\right\}_t$ *and* $\left\{\tau_t, \hat{T}_t\right\}_t$ *constitute a competitive equilibrium given* $\left(\left\{\hat{b}_{i,-1}\right\}_i, \hat{B}_{-1}\right)$.

Ricardian Equivalence: Implications

here you mean "tax collections"; elsewhere "tax rate"

are

- Present value of taxes and gov't debt ~~is~~ pinned down but not period-by-period transfers

$B_i = 0$

- Can set $b_{i,t} = 0$ for any t, i or ~~government~~ without loss of generality
- Generally, more equally spread debt promised (implicit Social Security promises, debt in Japan) are less distortionary than debt skewed towards high ly productive ~~guys~~ agents or foreigners (debt in Greece)
- **Extension:** Exogenous borrowing constraints of the form $b_{i,t} > \underline{b}_i$ ~~are not restrictive~~

More details

actually they are in a sense - they help the planner because of that

Characterization of optimal policy: Road map

- Active channels:
  1. Limited hedging abilities
  2. Concerns for redistribution
- Analytical results:
  1. Quasi Linear preferences :$u(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$
  2. IID aggregate shocks
- 2 step build up:
  1. Assume first that there is one agent and no ability to use T
  2. Use results to characterize outcomes in the more general settings with heterogeneous agents and no restriction on transfers
- Reasoning:
  1. Allows us to disentangle hedging and redistribution motives
  2. Informative about cases where the government does not care enough about redistribution

     much

Quasi-linear economy , $T = 0$

- Decompose the set of payoffs

$$\mathcal{P}^* = \left\{ P(s) : P(s) = 1 + \frac{\beta}{B^*}(g(s) - \mathbb{E}g) \text{ for some } B^* \in [\overline{B}, \underline{B}] \right\}$$

- Let $V(B_-)$ be the maximum ex-ante value the government can achieve with assets B_-.

$$V(B_-) = \max_{c(s),l(s),B(s)} \sum_s \pi(s) \left\{ c(s) - \frac{l(s)^{1+\gamma}}{1+\gamma} + \beta V(B(s)) \right\}$$

subject to

$$c(s) - B(s) = l(s)^{1+\gamma} - \beta^{-1} P(s) B_-$$

$$c(s) + g(s) \leq \theta l(s)$$

$$\underline{B} \leq B(s) \leq \bar{B}$$

Invariant distribution

Theorem

1. *Suppose $P \notin \mathcal{P}^*$, there is an invariant distribution of government* assets *such that*

$$\forall \epsilon > 0, \quad \Pr\{B_t < \underline{B} + \epsilon \text{ and } B_t > \overline{B} - \epsilon \quad i.o\} = 1$$

2. *Suppose $P(s) - P(s') > \beta \frac{g(s) - g(s')}{\underline{B}} \quad \forall s, s'$, then for large enough* govt *assets (or debt) there is a drift towards the interior region. In particular the value function $V(B)$ is strictly concave and there exists $B_1 < B_2$ such that*

$$\mathbb{E}V'(B(s)) > V'(B_-) \quad B_- > B_2$$

and

$$\mathbb{E}V'(B(s)) < V'(B_-) \quad B_- < B_1$$

govt

3. *Suppose $P(s) \in \mathcal{P}^*$, then ~~the long run~~ assets converge to a degenerate steady state*

$$\lim_t B_t = B^* \quad a.s \quad \forall B_{-1}$$

Perfect spanning

- For $P(s) \in \mathcal{P}^*$, ~~Then~~ we can replicate complete markets ~~perfectly~~ asymptotically
- Target assets

$$B^* = \beta \frac{\mathrm{var}(g(s))}{\mathrm{cov}(P(s), g(s))}$$

- Tax~~es~~ rate ~~are~~ is constant in long run, $\lim_{t\to\infty} \tau_t \to \tau^*$
- Use this to construct an approximation for the ergodic distribution of debt and taxes of an economy with $P(s)$ "close" enough to $\mathcal{P}^*$. In particular split $P(s)$

$$P(s) = \hat{P}(s) + P^*(s)$$

where $P^*(s) \in \mathcal{P}^*$ and $\hat{P}(s)$ is orthogonal to $g(s)$. More details

Imperfect spanning

Theorem

The ergodic distribution of debt (~~using the~~ under a first order approximation of dynamics near $P^*(s)$) has the following properties,

- **Mean:** *The ergodic mean is B^* which corresponds to the steady state level of debt of an economy with payoff vector $P^*(s)$*
- **Variance:** *The coefficient of variation* of debt *~~is given by~~* satisfies

$$\frac{\sigma(B)}{\mathbb{E}(B)} = \sqrt{\frac{\mathrm{var}(P(s)) - |\mathrm{cov}(g(s), P(s))|}{(1 + |\mathrm{cov}(g(s), P(s))|)|\mathrm{cov}(g(s), P(s))|}} \leq \sqrt{\frac{\mathrm{var}(\hat{P}(s))}{\mathrm{var}(P^*(s))}}$$

- **Convergence rate:** *The speed of convergence to the ergodic distribution ~~described by~~* is

$$\frac{\mathbb{E}_{t-1}(B_t - B^*)}{(B_{t-1} - B^*)} = \frac{1}{1 + |\mathrm{cov}(P(s), g(s))|}$$

Ergodic distribution

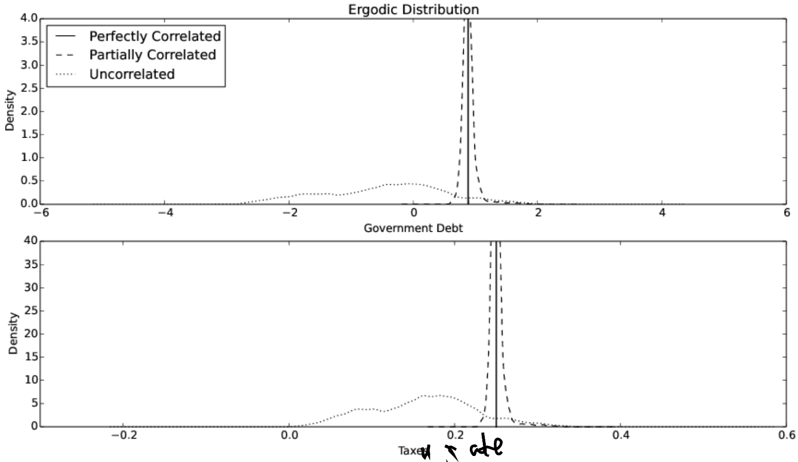


Figure : Ergodic distribution for debt and taxes rate in the representative agent quasilinear economy for three ~~choices~~ $P(s)$.

Summary

- Target level of assets maximizes spanning
  - tax~~es~~ rate ~~are~~ is constant when perfect spanning is achieved
- When markets are imperfect, can be far away from the target
  - invariant distribution of tax~~es~~ rate also has large support
- Speed of moving to the target debt level depends on covariance of asset payoff and shocks
  - low covariance $\Longrightarrow$ slow speed

Heterogeneous agents

Pareto

Suppose we have 2 agents

- Quasi-linear preferences ~~as before~~
- Productivities: $\theta_1 > \theta_2 = 0$
- Parteo weights, mass of Agent 1 and 2: $\omega, 1-\omega$ and $n, 1-n$ respectively

  0.5
- Non-negative consumption: $c_2 \geq 0$

$c_1 \geq 0$ too?

Normalize $b_{2,t} = 0$, thus $B_t = -n_1 b_{1,t}$ are interpreted to be government assets

Heterogeneous Agents

Theorem

Let ω, n be the Pareto weight and mass of the productive agent with $n < \frac{\gamma}{1+\gamma}$. The optimal tax, transfer and asset policies $\{\tau_t, T_t, B_t\}$ are characterized as follows,

1. *For $\omega \geq n\left(\frac{1+\gamma}{\gamma}\right)$ we have $T_t = 0$ and the optimal policy is same as in ~~a~~ our representative agent economy ~~studied~~*
2. *For $\omega < n\left(\frac{1+\gamma}{\gamma}\right)$, suppose we ~~further~~ assume that $P(s) \notin \mathcal{P}^*$ and $\min_s\{P(s)\} > \beta$, there exists $\mathcal{B}(\omega)$ and $\tau^*(\omega)$ with $\mathcal{B}'(\omega) > 0$ such that*
   - 2.1 $B_- > \mathcal{B}(\omega)$ (immediate)

     $$T_t > 0, \quad \tau_t = \tau^*(\omega), \text{ and } B_t = B_- \quad \forall t$$

   - 2.2 $B_- \leq \mathcal{B}(\omega)$ (eventual)

     $$T_t > 0 \text{ i.o.}, \quad \lim_t \tau_t = \tau^*(\omega) \text{ and } \lim_t B_t = \mathcal{B}(\omega) \quad a.s$$

Concerns for redistribution

- Balancing costs of fluctuations in tax~~es~~ rate and transfer
  - fluctuations in tax~~es~~ rate is costly: deadweight loss
  - fluctuations in transfer~~s~~ is costly: deviations from target level of redistribution
- For large ω transfers are costly as the planner gives resources to unproductive agents
- For low ω, transfers are used:
  - For low initial debt, interior solution: All shocks hedged by transfers
  - For high debt, accumulate assets until costs of transfers are equalized to costs of ~~using~~ collecting labor taxes
- The more redistributory the planner is:
  - bigger average tax rates and transfers
  - less need to accumulate assets for precautionary reasons

Risk aversion

- With risk aversion: for a (generic) set of parameters there is asset allocation replicating complete market economy
  - arguments harder since "real" interest rates $\mathbb{E}_t U'(c_{t+1}) R(s_{t+1}) / U'(c_t)$ is endogenous
- Same general flavor as quasi-linear economy
  - cost of fluctuations in transfers comes from cost of fluctuation in $U_c \iff$ similar to multiplier on constraint $c \geq 0$ in quasi-linear case
  - if real payoffs are positively correlated with g : accumulate assets
  - if real payoffs are (sufficiently) negatively correlated with g : accumulate debt
  - absolute amount of asset/debt is decreasing in redistribut~~ory~~ ive objective

More details

Numerical exercise

Solve a $N = 5$ agent economy with realistic level and movements in wage dispersion across booms and recessions

- Long run dynamics: Study settings that differ in covariance of interest rates and output
- Transient dynamics: Study outcomes in recessions that are accompanied by higher inequality

Aggregate shocks affect,

1. Wages:
$$\log \theta_i = \epsilon[1 + (.9 - d(i))m]$$
2. Payoffs:
$$P = 1 + \chi\epsilon$$

Calibrating *m*: Inequality over business cycles

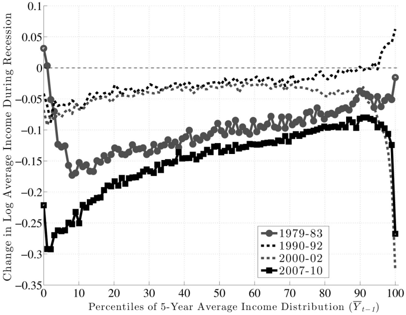


Figure : Change in log average earnings during recessions, prime-age males from Guvenen et all [2014]

Calibrating χ: Ex post variation in Payoffs

Let $q_t^{(n)}$ be the log price of a nominal bond of maturity n. We can define the real holding period returns $r_{t,t+1}^{(n)}$ as follows

$$r_{t,t+1}^{(n)} = q_{t+1}^{(n-1)} - q_t^{(n)} - \pi_{t+1}$$

With the transfromation $y_t^{(n)} : -\frac{1}{n} q_t^{(n)}$ we can express $r_{t,t+1}^{(n)}$ as follows:

$$r_{t,t+1}^{(n)} = \underbrace{y_t^{(n)}}_{\text{Ex-ante part}} -(n-1) \left[\underbrace{\left(y_{t+1}^{(n)} - y_t^{(n)} \right)}_{\text{Interest rate risk given } n} + \underbrace{\left(y_{t+1}^{(n-1)} - y_{t+1}^{(n)} \right)}_{\text{Term structure risk}} \right] - \underbrace{\pi_{t+1}}_{\text{Inflation risk}}$$

Interest rates and TFP

- In the model, the holding period returns are given by $\log\left[\frac{P_{t+1}}{q_t^1}\right]$ and $q_t^1 = \frac{\beta \mathbb{E}_t u_{c,t+1} P_{t+1}}{u_{c,t}}$.
- P_{t+1} allows us to captures ex-post fluctuations in returns to the government's debt portfolio coming from maturity and inflation.
- Since ϵ_t is i.i.d over time in our calibration $\chi = \frac{\sigma_r}{\sigma_\epsilon} Corr(r, \epsilon)$

Using data on labor productivity ϵ_t and $\{q_t^n\}_n$:

Maturity (n)	2yr	3yr	4yr	5yr
$Corr(\epsilon_{t+1}, r_{t,t+1}^{(n)})$	-0.11	-0.093	-0.083	-0.072
$Corr(\epsilon_{t+1}, r_{t,t+1}^{(n)} - ny_t^{(n)})$	0.00	-0.0463	-0.080	-0.091
$Corr(\epsilon_{t+1}, y_t^{(n)} - \pi_{t+1})$	-0.097	-0.086	-0.080	-0.073
$\frac{\sigma(r_{t+1}^n)}{\sigma(\epsilon_{t+1})}$	0.820	0.835	0.843	0.845

Table

Calibration

Parameter	Value	Description
$\{\bar{\theta}_i\}$	$\{1\ ,\ 1.4,\ 2.1,\ 3.24,\ 4.9\}$	Wages dispersion for {10,25,50,75,90} percentiles
γ	2	Average Frisch elasticity of labor supply of 0.5
β	0.98	Average (annual) risk free interest rate of 2%
m	$\frac{1.5}{.8}$	Changes in dispersion
χ	-0.06	covariance between holding period returns and labor productivity
σ_e	0.03	vol of labor productivity
g	.13 %	Average pre-transfer expenditure- output ratio of 12 %

Table : Benchmark calibration

Pareto   are

The Pareo weights and initial distribution of wealth ~~is~~ chosen to match an average tax rate of 20% and debt to gdp ratio of 100% and transfers to gdp ratio of 10% and deciles of US wealth distribution

Long run

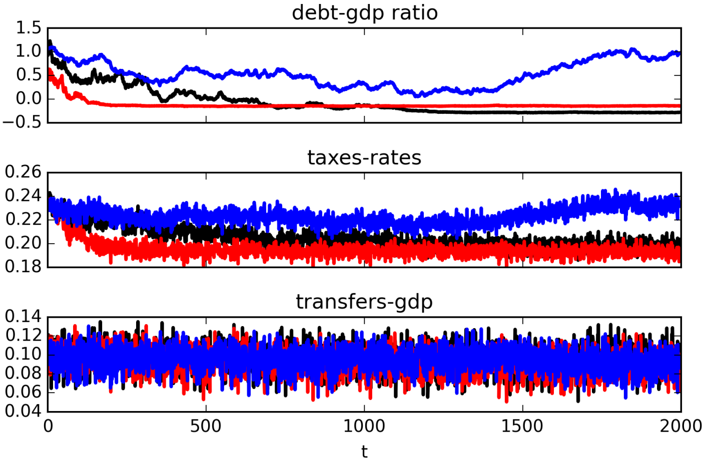


Figure : The red, black and blue lines plot simulations for a common sequence of shocks for values of $\chi = -1.0, 0, 1.0$ respectively

Long run: Speed of convergence

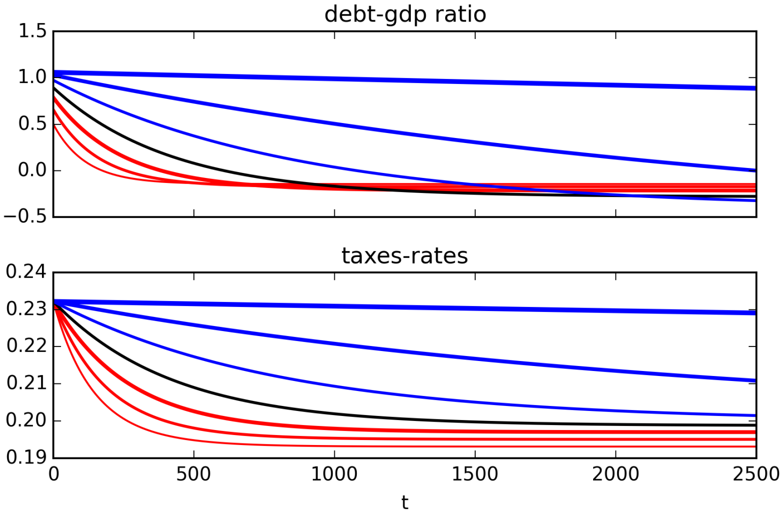
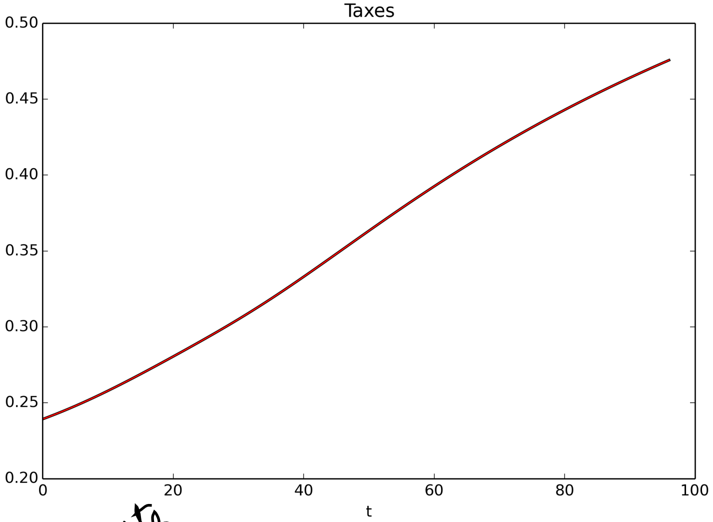


Figure : The plot shows conditional mean paths for different values of χ. The red (blue) lines have $\chi < 0$ ($\chi > 0$). The thicker lines represent larger values

Spreading of taxes rate



rates

Figure : Taxes for a sequence of -1 s.d shocks to aggregate productivity

Short run

Let's

Lets denote consecutive period of negative (positive) one s.d ϵ shocks a “recession” (boom)

- Engineer a recession of four periods from $t = 3$. Before and after this recession, the economy receives $\epsilon_t = 0$.
- Decompose responses into TFP component and inequality component:

$$\textbf{Baseline: } \log\theta_i = \epsilon[1 + (.9 - d(i))m]$$

  - Only TFP:

$$\log\theta_i = \epsilon$$

  - Only Ineq:

$$\log\theta_i = \epsilon[(.9 - d(i))m]$$

Recessions with higher inequality

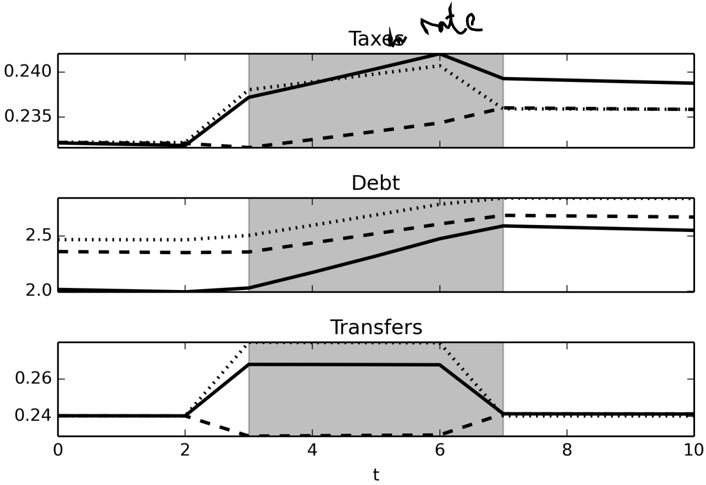


Figure : The bold line is the total response. The dashed (dotted) line reflects the only TFP (inequality) effect. The shaded region is the recession

Tfp and Tfp+Ineq recessions: Sample moments

Moments	Tfp	Tfp+Ineq
vol. of tax~~es~~ rate	0.003	0.006
vol. of transfers	0.01	0.02
autocorr. in tax~~es~~ rate	0.93	0.66
autocorr. in transfers	0.17	0.18
corr. of taxes with tfp	0.15	-0.63
corr. of transfers with tfp	0.99	-0.98

Table : These are sample moments averaged acrosss simulations of 100 periods

Conclusion

Credit limits

Impose $b_{i,t} \geq \underline{b}_i$. and extend the definition of competitive eqb. in the obvious way. Now we have that:

Theorem

Given an initial asset distribution $\left(\{b_{i,-1}\}_i, B_{-1}\right)$*, let* $\{c_{i,t}, l_{i,t}\}_{i,t}$ *and* $\{R_t\}_t$ *be a competitive equilibrium allocation and interest rate sequence in an economy without exogenous borrowing constraints. Then for any exogenous constraints* $\{\underline{b}_i\}_i$*, there is a government tax policy* $\{\tau_t, T_t\}_t$ *such that* $\{c_{i,t}, l_{i,t}\}_{i,t}$ *is a competitive equilibrium allocation in an economy with exogenous borrowing constraints* $\left(\{b_{i,-1}, \underline{b}_i\}_i, B_{-1}\right)$ *and* $\{\tau_t, T_t\}_t$.

back

Ergodic distribution: Linear approximation

- For a given $P(s), g(s)$, we can compress the equilibrium conditions to ~~look for~~ two functions $b(\mu_-)$ and a law of motion $\mu(s|\mu_-)$
- Instead of approximating near a deterministic steady state we,
  - explicitly recognize that policy rules depend on payoffs: $\mu(s|\mu_-, \{P(s)\}_s)$ and $b(\mu_-, \{P(s)\}_s)$
  - take the first order expansion with respect to both μ_- and $\{P(s)\}$ around the vector $(\bar{\mu}, \{\bar{P}(s)\}_s)$ where $\bar{P}(s) \in \mathcal{P}^*$:
- The choice of $\bar{P}(s)$ is pinned down by

$$\min_{\tilde{P} \in \mathcal{P}^*} \sum_s \pi(s)(P(s) - \tilde{P}(s))^2.$$

- The law of motion is ~~described as~~ approximated by

$$\mu_t - \mu^* = (\mu_{t-1} - \mu^*)B(s_t) + C(s_t)$$

back

More details on cases with risk aversion

- With risk aversion $\|S\| = 2$ is necessary for a steady state to exist
- Existence: Consider an economy consisting of two types of households with only one productive agent and i.i.d binary shocks to his productivity

Theorem

Suppose $u(c, l) = \ln c - \frac{1}{2} l^2$ *and* $g < \theta(s)$ *for all* s*. Let* $x = U_c^2(s)\left[b_2(s) - b_1(s)\right]$

1. **Countercyclical interest rates.** *If* $P(s_H) = P(s_L)$*, then there exists a steady state* $\left(x^{SS}, \rho^{SS}\right)$ *such that* $x^{SS} > 0,\ R^{SS}(s_H) < R^{SS}(s_L)$.
2. **Procyclical interest rates.** *There exists a pair* $\{P(s_H), P(s_L)\}$ *such that there exists a steady state with* $x^{SS} < 0$ *and* $R^{SS}(s_H) > R^{SS}(s_L)$.

In both cases, taxes $\tau(s) = \tau^{SS}$ *and ratio of consumption* $\frac{c_1}{c_2}$ *are independent of the realized state.*

- We then develop a test for local stability as in the quasilinear case.

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