using

Consider a Ramsey planner ~~with~~ who decides how to finance government expenditure shocks using proportionate labor taxes, transfers and borrowing in a risk free debt market. This note highlights how the dynamics of government's asset holdings depend on restrictions which possibly alter the fiscal hedging abilities transfers.

Under an affine tax system as studied here, restrictions on transfers additionally affect the planner's ability to redistribute across agents. We first begin with a representative agent economy in section 1 that abstracts from this feature and then come back to it section 2 where we have multiple agents.

1 Representative agent

The setup has a representative agent who values consumption and leisure with quasi linear preferences of the form $u(c,l) = c - \frac{l^{1+\gamma}}{1+\gamma}$ and has access to a risk free bond market. Let b_t be his assets at time t. Using the primal approach, we substitute tax rates and interest rates appearing in the budget constraint of the agent using the corresponding first order conditions and describe the optimal Ramsey allocation as a solution to the following problem: Given b_{-1},

$$W(b_{-1}) = \max_{\{c_t,l_t,b_t,T_t\}_t,} \mathbb{E}_0 \sum_t \beta^t \left[c_t - \frac{l_t^{1+\gamma}}{1+\gamma}\right] \tag{1}$$

give background on how we got $l_t^{1+\gamma}$

subject to the following for $t \geq 0$

$$c_t + b_t = l_t^{1+\gamma} + \beta^{-1} b_{t-1} + T_t, \quad t \geq 0 \tag{2}$$

$$c_t + g_t \leq \theta l_t \tag{3}$$

$$\underline{b} \leq b_t \leq \overline{b} \tag{4}$$

The limits on government debt $\underline{b}$ and $\overline{b}$ are arbitrary at this point. They can correspond to natural debt limits which are a function of the government policies.

The optimal allocation depends on what restrictions we impose on transfers. Theorem 1 describes outcomes under three cases that progressively restrict transfers T_t.

Theorem 1 *For the representative agent with quasilinear utility,*

1. ***[First Best]:*** *If T_t are unrestricted*

$$\tau_t = 0, l_t = \theta^{1/\gamma}, c_t = \theta^{1+\frac{1}{\gamma}} - g_t, T_t = b_{-1}(\beta^{-1} - 1) - g_t, b_t = b_{-1}$$

For the next two cases we can show that labor taxes, $\tau_t = \tau(s_t|b_{t-1})$ with $\frac{\partial \tau(s_t|b_{t-1})}{\partial b} > 0$. Suppose $\overline{b}$ is the natural debt limit for the government,

$$\lim_{b_- \to \overline{b}} \tau(s|b_-) = \frac{\gamma}{1+\gamma} \text{ and } \lim_{b_- \to -\infty} \tau(s|b_-) = -\infty$$

move to below

2. ***[AMSS]:*** *If $T_t \geq 0$ and $\underline{b} < \frac{-\max_s g(s)}{\beta^{-1}-1}$*

$$\lim_t \tau_t = 0, \lim_t l_t = \theta^{\frac{1}{\gamma}}, \lim_t c_t = \theta^{1+\frac{1}{\gamma}} - g_t, \lim_t b_t = \frac{-\max_s g(s)}{\beta^{-1}-1}, T_t > 0 \quad i.o$$

3. ***[BEGS]:****If* $T_t \equiv 0$*, there is an invariant distribution of government assets such that*

others
learning

$$\forall \epsilon > 0, \quad \Pr\{b_t < \underline{b} + \epsilon \quad or \; b_t > \bar{b} - \epsilon \quad i.o\} = 1$$

Further as $\underline{b} \to -\infty$,

$$\lim_t b_t = -\infty, \lim_t \tau_t = -\infty$$

on this impl

It is apparent that outcomes can differ from the first best with no distortions because of the binding implementability constraint 2. What is crucial is how the the multiplier and its dynamics change because of the restrictions we impose on T_t. This multiplier can be interpreted as the marginal costs of raising revenues through taxes and its ~~tightness~~ value depends on the initial debt holdings of the government.

The first case underscores an observation that absent any restriction on T_t, the constraint 2 is effectively slack (the multiplier on this constraint is zero). The planner thus implements the first best allocation from date $t = 0$.

The second case with non negative transfers is the one studied by AMSS. Here the multiplier is a martingale and the non negativity constraint imposes an upper bound of zero. Standard martingale convergence arguments imply that eventually this constraint will be slack. The dynamics of asset holdings are different because the government cannot hedge the shocks completely with transfers and thus incompleteness of markets is binding. This induces a precautionary motive which leads it to accumulate assets until the revenues from the asset are enough to finance the worst possible expenditure shock. At this point the excess revenues associated with other shocks are returned to the household in costless manner using positive transfers.

The last case eliminates transfers by setting T_t to zero at all histories. Effectively the government looses its ability ever to completely smooth labor taxes and their associated distortions. For any given level of initial assets consider a policy that has tax rates constant across all shocks that can occur in the next period. This will imply fluctuations in the net of interest deficit , and absent transfers translate into fluctuations in asset holdings. Thus depending on the shock the government will have more or less assets in the next period. Since the tightness of the implementability constraint depends on the asset holdings with which the government enters in any period, marginal costs of raising resources through taxes are *not* constant. This contradicts the policy to smooth taxes. The optimal plan features fluctuating debt/taxes and depending on the sequences of shocks can have wide variations in their levels. The ergodic distribution of debt visits all neighbourhoods of the bounds we impose with probability one. Since taxes are related to debt, they vary accordingly.

reasoning contradiction ??

2 Two QL agents

To study how restrictions on transfers interfere with the planner's ability to redistribute across different agents we modify the previous setting to include a positive mass of unproductive agents with quasilinear preferences. A Ricardian equivalence property allows to normalize the asset holdings of the unproductive agents to zero and analogous to the previous section, the (optimal) (Ramsey) allocation solves the following problem:

$$W(b_{-1}) = \max_{\{c_{1,t}, c_{2,t}, l_{1,t}, b_{1,t}\}_t} \mathbb{E}_0 \sum_t \beta^t \left[\omega \left(c_{1,t} - \frac{l_{1,t}^{1+\gamma}}{1+\gamma} \right) + (1-\omega) c_{2,t} \right] \tag{5}$$

$b_{1,-1}$?

$b_{1,-1}$

$c_{1t} + b_{1t} = l_{1t}^{1+b} + \beta^{-1} b_{1t-1} + T_t$

$c_{2t} = T_t$

subject to the following two constraints for $t \geq 0$

$$c_{1,t} - c_{2,t} + b_{1,t} = l_{1,t}^{1+\gamma} + \beta^{-1} b_{1,t-1} \cancel{+ T_t} \tag{6}$$

$$nc_{1,t} + (1-n)c_{2_t} \leq n\theta l_{1,t} \tag{7}$$

$$\underline{b} \leq b_t \leq \bar{b} \tag{8}$$

The allocation depends on constraints on $c_{2,t}$. Notice that given that we normalize the assets of agent 2 to zero, $T_t = c_{2,t}$

Theorem 2 • *If $c_{2,t}$ is unconstrained,*

1. *The problem is concave only if $\omega < n\left(\frac{1+\gamma}{\gamma}\right)$. In this case the supremum $W(b_{-1})$ is finite and the (unique) optimal debt holdings are given by $b_t = b_{-1}$. The allocation:*

both groups pay transfers

$$l_{1,t} = \left(\frac{n\theta_1}{\omega - (\omega - n)(1+\gamma)}\right)^{\frac{1}{\gamma}}$$

0

$$c_{2,t} = nl_{1,t}(\theta_1 - l_{1,t}^{\gamma}) - n(\beta^{-1} - 1)b_{-1} - g_t$$

T_t

0

$$c_{1,t} = (1-n)(\beta^{-1} - 1)b_{-1} - g_t + l_{1,t}^{1+\gamma} - n(\theta_1 - l_{1,t}^{\gamma})$$

2. *For $\omega \geq n\left(\frac{1+\gamma}{\gamma}\right)$ maximizing a*n *objective function that is convex function over an unbounded choice set implies no solution. However we can construct a limiting argument. Suppose $\tau_t \geq \underline{\tau}$. The optimal policy will have a corner solution for taxes with $\tau_t = \underline{\tau}, b_t = b_{-1}, T_t = \underline{\tau} n\theta_1 l_1(\underline{\tau}) - (\beta^{-1} - 1)b_{-1} - g_t)$ for all $t \geq 0$ and as $\underline{\tau} \to -\infty$, $W(b_{-1})$ approaches ∞.*

$W(b_{-1})$

say what $W(b_{-1})$ is now

- *If $c_{2,t} \geq 0$*

τ_t same

$b_t = b_{1t}$

1. *For $\omega \geq n\left(\frac{1+\gamma}{\gamma}\right)$, $T_t = 0$. The optimal policy $\{\tau_t, b_t\}$ is same* the *as in the representative agent economy without transfers.*
2. *For $\omega < n\left(\frac{1+\gamma}{\gamma}\right)$ there exist*s *a $\mathcal{B}(\omega)$ satisfying $\mathcal{B}'(\omega) < 0$ and a $\tau^*(\omega)$ such that*
   (a) *If $b_{-1_-} \leq \mathcal{B}(\omega)$*

$$T_t > 0, \quad \tau_t = \tau^*(\omega), \text{ and } b_t = b_{-1} \quad \forall t \geq 0$$

$b_{1t}, b_{1,-1}$

   (b) *If $b_{-1} > \mathcal{B}(\omega)$ then*

$$T_t > 0 \text{ i.o.}, \quad \lim_t \tau_t = \tau^*(\omega) \text{ and } \lim_t B_t = \mathcal{B}(\omega) \quad a.s$$

With multiple agents using transfers to hedge aggregate shocks also implies giving resources to agents that the planner may not care enough about. When ω, the Pareto weight of the productive agent is low(relative to the mass n) welfare costs of transfers: due their inferior re distributional feature are also low.

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to

Across economies

in

In the first case when consumption of agent 2 can be ~~not~~ negative, for low enough ω allows the planner to hedge all shocks with transfers and keep asset holdings, tax rates constant. Tax rates are decreasing ω and approach $-\infty$ when ω reaches the threshold.

Now imposing non-negativity constaint on $c_{2,t}$ makes this solution infeasible especially for high values of initial government debt where desired transfers would have been otherwise negative. In this case the optimal policy gradually reduces until debt or builds up assets until it reaches a threshold where it can finance resulting fluctuations in the net-of interest deficit entirely with positive transfers and smooth tax rates. This threshold level of debt is higher for more redistributive planners. Such planners (with lower ω) collect higher are revenues from labor taxes and consequently the returns on asset holdings needed to finance expenditure shocks are smaller. economies with

Good

For large enough ω, transfers are too costly for any level of initial debt and hence the non-negativity constraint implies that they are equal to zero at all dates. Once can immediately see that having an unproductive agent with zero consumption is isopmorhic to a represenative agent economy where transfers are not allowed. This case was studied in the previous section and we can import the results about tax, debt dynamics from there.

Thus we have a complete characterization of dynamics of taxes and debt when transfers can be potentially restricted.

As a remark we describe the case when the unproductive agent is strictly risk averse. In this if $\underline{b} = \infty$, we have that

- If $n > \frac{\gamma}{1+\gamma}$
$$\tau_t = \tau^*(n,\gamma), T_t = \infty, b_t \to -\infty$$
and
- If $n < \frac{\gamma}{1+\gamma}$
$$\tau_t = -\infty, T_t = T^* < \infty, b_t \to -\infty$$

Thus government assets diverge irrespective of Pareto weights. Adding a finite lower bound to b_t will give us an invariant distribution whose properties are not easy to characterize analytically.