Taxes, Debts, and Redistributions

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3 Questions

1. How costly are government debts?
2. How do concerns for redistribution affect tax smoothing motives?
3. How should tax, transfer, debt policies respond to aggregate shocks that might also change inequality?

Representative agent models (I)

Representative agent models

- Higher levels of debt are "more" distortionary.
- LS: With complete markets tax rates should be smooth.
- Werning: Extends the LS insights to heterogeneous agents by establishing an aggregation result.
- AMSS: With a risk free bond tax rates are eventually smooth and all aggregate shocks are financed using transfers.

Representative agent models (II)

- Redistribution is implicitly modeled as a non-negativity constraint on lump sum transfers.
- Hardwires a discontinuity in the costs of fluctuating transfers around zero.
  1. The Ramsey planner either uses state-contingent securities to hedge aggregate shocks.

     or

  2. Accumulates a war chest of assets big enough to finance expenditures using returns on assets plus only positive transfers.

We begin with explicit redistribution motives and let the government set transfers optimally.

Key ingredients

- **Heterogeneity:** Agents are heterogeneous in

  a) Productivities
  b) Pareto weights, and
  c) Assets.

- **Instruments:** Affine tax system

- **Markets:** All agents trade a *single* security whose payoffs can depend on aggregate shocks

Answers to the 3 questions

1. **Invariance of debt level:**
   Absent borrowing constraints, Ricardian equivalence holds.
   Borrowing constraints only increase welfare
2. **Invariant distribution of tax rates:**
   - Has wide support
   - The mean and variance of tax rates are driven by market incompleteness. The two "covariances" that matter are
     - Between returns on the asset and aggregate shocks (Public sector)
     - Between consumption and (pre-tax) labor earnings (Private sector)
3. **Business cycle dynamics:** (Preliminary) Recessions that are accompanied by higher inequality call for increase in both taxes and transfers

Environment

- **Uncertainty**: Markov aggregate shocks s_t
- **Demography**: Continuum of infinitely lived agents plus a benevolent planner
- **Technology**: Output $\int \theta_{i,t} l_{i,t} di$ is linear in labor supplies.
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, l_{i,t}\right)$$

- **Preferences** (Planner): Given Pareto weights $\{\omega_i\}$

$$\mathbb{E}_0 \int \omega_i \sum_{t=0}^{\infty} \beta^t U_t^i \left(c_{i,t}, l_{i,t}\right) di$$

- **Asset markets**: A "risky" bond with payoffs $\mathbb{P} = P(s|s_{-})$

Environment, II

- **Affine Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_{i,t} l_{i,t}$$

- **Budget constraints** Let $R_{t-1,t} = \frac{P_t}{q_{t-1}}$
  - Agent i: $c_{i,t} + b_{i,t} = (1-\tau_t)\,\theta_{i,t} l_{i,t} + R_{t-1,t} b_{i,t-1} + T_t$
  - Government: $g_t + B_t + T_t = \tau_t \int \theta_{i,t} l_{i,t} di + R_{t-1,t} B_{t-1}$

- **Market Clearing**
  - Goods: $\int c_{i,t} di + g_t = \int \theta_{i,t} l_{i,t} di$
  - Assets: $\int b_{i,t} di + B_t = 0$

- **Initial conditions**: Distribution of assets $(\Psi_0(b_{i,-1}), B_{-1}, s_{-1})$

Ramsey Problem

Definition
Allocation, price system, government policy: Standard

Definition
Competitive equilibrium: Given $(\Psi_0(b_{i,-1})_i, B_{-1}, s_{-1})$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$ all allocations are chosen optimally, markets clear [1]

Definition
Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given $(\Psi_0(b_{i,-1}), B_{-1}, s_{-1})$

[1] Usually, we impose only "natural" debt limits.

Ramsey problem: Recursive formulation

Split into two parts

1. $\mathbf{t \geq 1}$: Ex-ante continuation problem with state variables $\{\Gamma_{t-1}(x_{i,t-1}, m_{i,t-1}, \theta_{i,t-1}), s_{t-1}\}$, where
   - Scaled assets: $x_{i,t-1} = u_{c,i,t-1} b_{i,t}$
   - Scaled "market weights": $m_{i,t-1} \propto \frac{1}{u_{c,i,t-1}}$
2. $\mathbf{t = 0}$: Ex-post initial problem with state variables $(\Psi_0(b_{i,-1}), s_0)$

Bellman Equation for $t \geq 1$

$$V(\Gamma_{\_}, s_{\_}) = \max_{\substack{c_i(s), l_i(s), x_i(s), m_i(s) \\ \tau(s), T(s), \alpha_1, \alpha_2(s)}} \sum_s \Pr(s|s_{\_}) \left[\int \omega_i u(c_i(s), l_i(s)) di + \beta V(\Gamma(s), s) \right]$$

where the maximization is subject to

$$\frac{x_{i,\_} u_{i,c}(s) P(s|s_{\_})}{\beta \mathbb{E}^i_{s_{\_}} u_{i,c}(s) P(s|s_{\_})} = u_{i,c}(s)[c_i(s) - T(s)] + u_{i,l}(s) l_i(s) + x_i(s)$$

$$\alpha^1 = m_{i,\_} \mathbb{E}^i_{s_{\_}} u_{i,c}(s)$$

$$\alpha^2(s) = m_i(s) u_{i,c}(s)$$

$$-u_{i,l}(s) = [(1 - \tau(s)] u_{i,c}(s) \theta_i(s)$$

$$\int m_i(s) di = 1$$

$$\int l_i(s) \theta_i(s) di = \int c_i(s) di + g(s)$$

Bellman equation for $t = 0$

$$V_0\left(\Psi_0(b_{i,-1}), s_0\right) = \max_{\substack{c_{i,0}, l_{i,0}, x_{i,0}, m_{i,0} \\ \tau_0, T_0}} \int \omega_i U^i(s_0) + \beta V\left(\Gamma_0, s_0\right)$$

where the maximization is subject to

$$b_{i,-1} u_{i,c,0} = u_{i,c,0}[c_{i,0} - T_0] + u_{i,l,0} l_{i,0} + x_{i,0}$$

$$- u_{i,l,0} = (1 - \tau_0) u_{i,c,0} \theta_{i,0}$$

$$\int m_{i,0} di = 1$$

$$\int l_{i,0} \theta_{i,0} di = \int c_{i,0} di + g_0$$

A review of LS and AMSS

- Both impose $T_t \geq 0$
- The multiplier on the implementability constraint μ_t,
  - LS: Constant

    $$\mu_t = \mu_0$$

    With CES preferences tax rates are smooth.
  - AMSS: Positive risk adjusted martingale

    $$\mu_t = \mathbb{E}_t u_{c,t+1} \mu_{t+1}$$

    With QL preferences μ_t converges to zero and tax rates converge to zero
- This makes the constraint on transfers either always slack or eventually slack.

Ricardian Equivalence

- **Result**: A large set of transfers and asset profiles support the same competitive allocation
  Taking away a unit of all agents' assets and increasing transfers by a unit leaves budget sets unchanged

- **Implication:** Ceteris paribus, an economy with higher level of initial government debt but same relative holdings has the same welfare

- **Corollary:** Exogenous borrowing constraints of the form $b_{it} > \underline{b}_i$ are not restrictive

  If some borrowing constraints bind, the planner can change transfers to slacken all of them

Thus, Ricardian equivalence holds with distortionary taxes and ad hoc borrowing limits

Active channels

1. Varying labor taxes imposes dead weight losses.
2. Fluctuating transfers is costly because of concerns for redistribution.
3. Effects of taxes on redistribution depends on cross sectional heterogeneity of consumption and earnings.
4. The benefits of fluctuating transfers depend of limits to fiscal hedging.

Understanding the channels

1. Study a two type QL economy: This will shut down idiosyncratic risk
   - Costs of transfers will come from Pareto weights
   - Hedging motive will be controlled using payoffs $P(s)$
2. More generally
   - Costs of transfers will come from spreads in marginal utilities. A key determinant will be the nature of idiosyncratic risk.
   - Hedging motives will depend on covariance of asset returns with aggregate shocks
3. Study a calibrated economy

Simple Example: 2 Agent QL economy

Consider a "seemingly" AMSS economy

1. Two classes of agent with constant productivities $\theta_1 = 0, \theta_2 > 0$
2. Preferences given by $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$
3. Pareto weights $\{\omega, 1-\omega\}$
4. Only i.i.d aggregate shocks to $g(s)$

Normalization: Let the assets of the productive agent be denoted by b

Case 1: Risk free bond

Theorem

Let $\omega > \bar{\omega}$ and suppose $P(s) = 1$, $\lim_t \tau_t = -\infty$, $\lim_t b_t = -\infty$ a.s

Corollary

Suppose we augment our problem with a constraint $b_t \geq \underline{b}$. Then with risk-free debt there is an invariant distribution ψ. Morever, for any $\hat{b} \in \left(\underline{b}, \bar{b}_n\right)$, $\psi\left(\left[\underline{b}, \hat{b}\right]\right) > 0$ and $\psi\left(\left[\hat{b}, \bar{b}_n\right]\right) > 0$..

The invariant distribution of taxes is wide as fluctuations in transfers are costly

Case 2: Perfect hedging

Theorem

Let $\omega > \bar{\omega}$ and suppose $\overline{b} < \overline{b}_n$ and payoffs satisfy

$$P(s) = 1 - \frac{\beta}{\overline{b}}(g(s) - \mathbb{E}g)$$

then for all b_{-1}, $b_t \to \overline{b}$ a.s

Corollary

Suppose we augment our problem with a constraint $b_t \geq \underline{b} < \overline{b}$. Then the invariant distribution of b (and also tax rates) is degenerate with $\overline{b} = -\beta \frac{\mathrm{var}[g(s)]}{\mathrm{cov}[P(s), g(s)]}$

The limiting allocation corresponds to a complete market Ramsey problem

Case 3: Imperfect hedging

Decompose payoffs

$$P(s) = \hat{P}(s) + \bar{P}(s)$$

where $\overline{P}(s) = 1 - \frac{\beta}{\overline{b}}(g(s) - \mathbb{E}g)$ and $\hat{P}(s)$ is orthogonal to $g(s)$.

Theorem

For $\omega > \bar{\omega}$, the ergodic distribution of debt of the policy rules linearized around $(\overline{b}, \overline{P}(s))$ will have mean $\overline{b}$ and and coefficient of variation

$$\frac{\sigma_b}{\overline{b}} \leq \sqrt{\frac{\mathrm{var}(P(s)) - |\mathrm{cov}(g(s), P(s))|}{|\mathrm{cov}(g(s), P(s))|}}$$

The speed of convergence to the ergodic distribution given by

$$\frac{\mathbb{E}_{t-1}(b_t - \overline{b})}{(b_{t-1} - \overline{b})} = \frac{1}{1 + |\mathrm{cov}(P(s), g(s))|}$$

Adding risk aversion

Modifications:

- Endogenous component to covariance between payoffs and expenditure needs.
- Costs of transfers come from spreads in marginal utilities.

Implications:

1. Inequality distortions both call for a negative correlation between productivities and net assets.
2. This is exacerbated with countercyclical returns on fiscal assets

Inequality distortions

TFP: Adjust tax rate τ or transfers T, both are costly

Suppose $b = 0$

1. Present value of earnings of productive agent are higher
2. A reduction in transfers hurts the low productivity agent more

Then

b is same as increasing the **debt** of the productive agent

This drives the after-tax, after-interest incomes of both agents closer together

Interest rate fluctuations

- **Countercyclical interest rates:**

  ↓ TFP: If the tax rate τ is left unchanged, the government faces a shortfall of revenues.

  1. Reminder: b is **debt** of the government
  2. By holding positive assets the govt. can use higher interest income to offset some revenue losses from its tax on labor in recessions
  3. This force is present in representative agent economies with endogenous fluctuations in interest rates

  ↓ The government reduces debt b

Adding idiosyncratic risk

- Eliminates negative correlation between pre-tax earnings and assets.
- Joint distribution of consumption and pre-tax earnings drives the mean level of taxes
- The spread of the ergodic distribution is mainly a determined by public sector's fiscal hedging abilities

Decomposing the tax rate

Define $w_{i,t} = u^i_{c,t}[\omega^i - \mu^i_t(1+\gamma)]$ and $\bar{w}_t = \int u^i_{c,t}\omega^i di$ and $\bar{w}_{i,t} = \frac{g_{i,t}}{\bar{w}_t}$.

$$\frac{1}{1-\tau_t} = \underbrace{\frac{\hat{Y}_t}{Y_t}}_{\substack{\text{Effectiveness}\\\text{of}\\\text{Taxes}}} \underbrace{\frac{\bar{w}_t}{\lambda_t}}_{\substack{\text{Benefits}\\\text{of}\\\text{Transfers}}}$$

Where $Y_t = \int y^i_t di$ and $\hat{Y}_t = \int y^i_t \bar{w}_{i,t} di$

Two stark cases

1. Start with a representative agent economy with no aggregate shocks.
2. Consider two process for idiosyncratic risk
   - IID shocks
   - Persistent (close to unit root) shocks

How to taxes evolve as the distribution fans out?

Tax rates in the IID economy

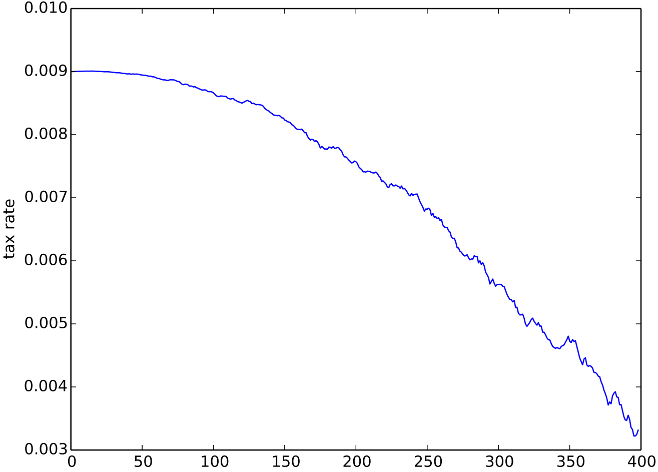

Figure: Taxes in the iid economy.

Tax rates in the close to unit root economy

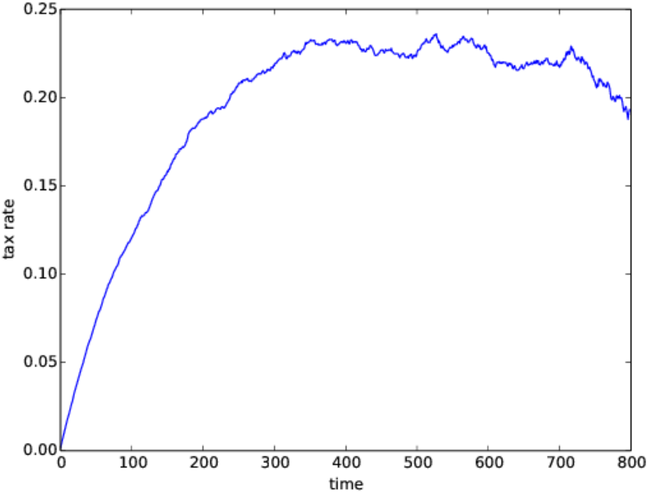

Figure: Taxes in the close to unit root economy.

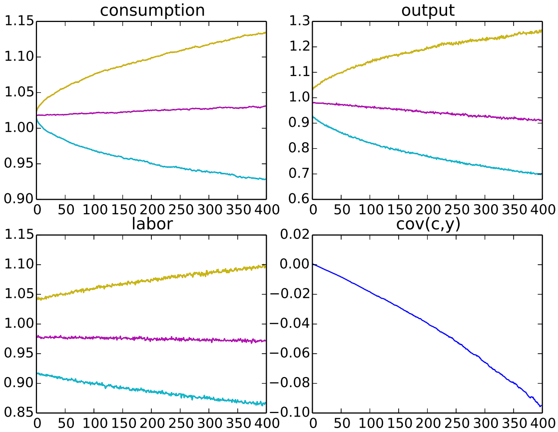

Figure: Inequality in the iid economy. The figure plots the quantiles for consumption, pre-tax labor earnings, labor and the covariance between consumption and pre-tax labor earnings

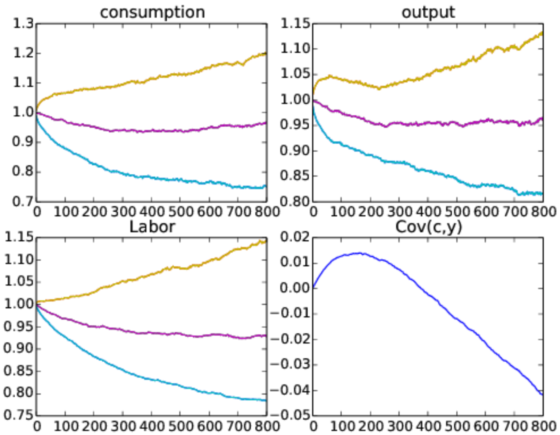

Figure: Inequality in the unit root economy. The figure plots the quantiles for consumption, pre-tax labor earnings, labor and the covariance between consumption and pre-tax labor earnings

Tax rates in the close to unit root economy with aggregate shocks

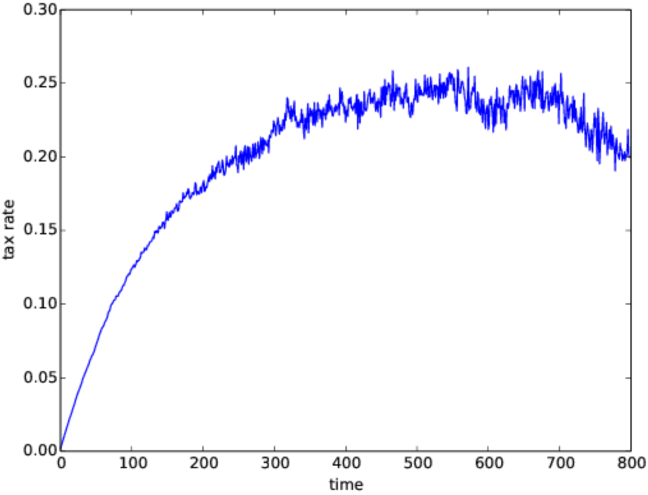

Figure: Taxes in the close to unit root economy with aggregate shocks

Calibrated Example

Take a 2-shock 2-type economy with preferences
$U(c, l) = \psi \log(c) + (1 - \psi) \log(1 - l)$ and with TFP shocks $\theta_i(s)$.

- Pick baseline parameters to match some low frequency moments
- Calibrate outcome fluctuations to match three US recessions (i.e., 1991-92, 2001-02 and 2008-10):
  1. The left tail of the cross-section distribution of labor income falls more than right tail
  2. Short term interest rates fall
  3. Booms last longer than recessions

Calibration

Parameter	Value	Description	Target
ψ	0.6994	Frisch elasticity of labor supply	0.5
$\bar{\theta}_1$	4	Log 90-10 wage ratio (Autor et al.)	4
$\bar{\theta}_2$	1	Normalize to 1	1
β	0.98	Average (annual) risk free interest rate	2%
α_1	0.69	Marginal tax rate in the economy with no shocks	20%
g	12%	Average pre-transfer expenditure- output ratio	12 %
$\frac{\hat{\theta}_2}{\hat{\theta}_1}$	2.5	Relative drop in wage income of 10th percentile	2.5
$\hat{\theta}_1$	1.2%	Average output loss	3%
$\hat{\beta}(s)$	1.96%	Difference in real interest rates between booms and recession	1.96%
$P(r\|r)$	0.63	Duration of recessions	2.33 years
$P(b\|b)$	0.84	Duration of booms	7 years

Table: Benchmark calibration

Initial conditions chosen to make debt to GDP ratio be 60% [2]

[2] We use the same normalization as before i.e, the low productive agent has zero assets

Results: Some variants

We study perturbations of the Benchmark calibration

1. Countercyclical interest rates
2. Acyclical interest rates by adjusting payoffs
3. A case with no inequality where all agents' productivities (TFP shock) fall in parallel

Short Run

To understand the short run responses

- Solve the time 0 problem with identical initial conditions across different settings.
- Use optimal policies to compute fluctuations of different components in the government budget constraint as we transition between booms to recessions

Results: Short run

	Δg	ΔB	ΔT	$\Delta[\tau\theta_1 l_1]$	$\Delta[\tau\theta_2 l_2]$	ΔY	$\Delta\tau$
Benchmark	0.0000	-1.1561	0.6871	-0.1593	-0.3096	-2.8536	0.3732
Acyclical Interest Rates	0.0000	-1.1126	0.6591	-0.1497	-0.3038	-2.8613	0.3879
Countercyclical Interest Rates	0.0000	-1.0794	0.6387	-0.1415	-0.2992	-2.8677	0.3997
No Inequality	0.0000	-0.1380	**-0.5459**	-0.5635	-0.1204	-2.6294	0.0622

Table: The tables summarizes the changes in the different components of the government budget as the economy transits from "boom" to "recession". All numbers except τ are normalized by un-distorted GDP and reported in percentages.

1. For each variable z in the table we report $\Delta z \equiv \left(z\left(s_l|x_0, m_0, s_0\right) - z\left(s_h|x_0, m_0, s_0\right)\right)/\bar{Y}$ where $\bar{Y}$ is average undistorted GDP in percentages
2. Predetermined variables like repayment on existing debt drop out

$$\Delta[g] + \Delta[T] + \Delta[B] = \Delta[\tau\theta_1 l_1] + \Delta[\tau\theta_2 l_2]$$

Conclusions

- Concerns for redistribution imply costs of fluctuating transfers
- This affects the prescriptions of optimal policy for smoothing tax rates
- Market incompleteness is a key determinant of the invariant distribution of taxes and debt
- **Next step**: Calibrate a "realistic" idiosyncratic risk and aggregate risk process to learn about the quantitative implications of the model.