Optimal Taxation with Incomplete Markets

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Abstract

KEYWORDS:

1 Introduction

2 Environment

Markov aggregate shocks $s_t \in \mathcal{S}$; $S \times S$ stochastic matrix Π; $g_t = g(s_t); \theta_t = \theta(s_t)$ An infinitely lived representative agent plus a benevolent planner

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c(s^t), l(s^t)\right)$$

Technology: Output $y_t = \theta_t l_t$

A single possibly risky asset $S \times S$ matrix $\mathbb{P}$ with time t payoff being

$$p_t = \mathbb{P}(s_t|s_{t-1})$$

Agent i's tax bill

$$-T_t + \tau_t \theta_t l_t, \; T_t \geq 0$$

q_t is price of asset. The household's time t budget constraint is

$$c_t + b_t = (1 - \tau_t)\,\theta_t l_t + \frac{p_t}{q_{t-1}} b_{t-1} + T_t$$

and the government's is

$$g_t + B_t + T_t = \tau_t \theta_t l_t + \frac{p_t}{q_{t-1}} B_{t-1}$$

Market clearing for goods is

$$c_t + g_t = \theta_t l_t$$

and for assets

$$b_t + B_t = 0$$

Initial assets satisfy $b_{-1} = -B_{-1}$ and an initial state s_{-1} is given.

Definition 2.1. ***Allocation, price system, government policy***

Definition 2.2. ***Competitive equilibrium****: Given $(b_{-1} = -B_{-1}, s_{-1})$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$, all allocations are individually rational, markets clear* [1]

Definition 2.3. ***Optimal competitive equilibrium****: A welfare-maximizing competitive equilibrium for a given (b_{-1}, B_{-1}, s_{-1})*

1. **Primal approach**: To eliminate tax rates and prices, use household's first order conditions:

$$U_{c,t} q_t = \beta \mathbb{E}_t p_{t+1} U_{c,t+1}$$

$$(1 - \tau_t)\theta_t U_{c,t} = -U_{l,t}$$

2. **Implementability constraints**: Derive by iterating the household's budget equation forward at every history

   $\Rightarrow$ for $t \geq 1$, these impose *measurability restrictions* on Ramsey allocations

3. The $t \geq 1$ **measurability constraints** contribute the only difference from Lucas-Stokey's Ramsey problem.

4. **Transfers:** We temporarily restrict transfers $T_t = 0\ \forall t$. This is convenient for our analytical results. We eventually show that this assumption is not restrictive.

[1] Usually, we impose only "natural" debt limits.

2.1 Ramsey problem (Lucas-Stokey)

$$\max_{\{c_t,l_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Implementability constraint**

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

2.2 Ramsey problem (BEGS)

$$\max_{\{c_t,l_t,b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Lucas-Stokey implementability constraint**

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

(c) **Measurability constraints**

$$\frac{b_{t-1} U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1} p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \text{ for } t \geq 1$$

2.3 Roadmap, analytic strategy

- Ramsey allocation – especially asymptotic properties – varies with **asset returns** that reflect
  - Prices $\{q_t(s^t | B_{-1}, s_{-1})\}_t$
  - Payoffs $\mathbb{P}$

- To focus on the exogenous $\mathbb{P}$ part of returns, we first study quasi-linear preferences that pin down $q_t = \beta \mathbb{E}_t \mathbb{P}(s_{t+1}|s_t)$
- Activate risk aversion and fluctuating q_t later

2.4 Analysis with quasi-linear preferences

Quasilinear preferences $U(c,l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

To characterize **long-run** debt and taxes, we construct and then invert mapping $\mathbb{P}^*(b)$

- Given **arbitrary** initial govt. assets b_{-1}, what is an **optimal** asset payoff matrix $\mathbb{P}^* = \mathbb{P}^*(b_{-1})$?
- Under a Ramsey plan for an **arbitrary** payoff matrix $\mathbb{P}$, when would $b_t \to b^*$, where

$$\mathbb{P} = \mathbb{P}^*(b^*) \text{ or } b^* = \mathbb{P}^{*-1}(\mathbb{P})?$$

- We first reverse engineer an optimal $\mathbb{P}^*(b_{-1})$ from a Lucas-Stokey Ramsey allocation
- In a binary IID world, we identify a big set of $\mathbb{P}$'s that imply that b_t under a Ramsey plan converges to b^* that solves

$$\mathbb{P} = \mathbb{P}^*(b^*)$$

- For more general shock structures, we numerically verify an ergodic set of b_t's hovering around $\tilde{b}$. The optimal $\mathbb{P}^*$ associated with $\tilde{b}$ seems close to $\mathbb{P}$:

$$\mathbb{P} \approx \mathbb{P}^*(\tilde{b})$$

2.5 Optimal asset payoff matrix $\mathbb{P}^*$

1. Given b_{-1}, compute a Lucas-Stokey Ramsey allocation
2. Notice that the measurability constraints are invariant to scaling of p_t by a constant k_{t-1} that can depend on s^{t-1}.

3. From this class we select a p_t that imposes the normalization $\mathbb{E}_{t-1} U_{c,t} p_t = 1$

$$p_t = \frac{\beta}{U_{c,t-1} b_{t-1} U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right)$$

4. By construction, p_t disarms the time $t \geq 1$ measurability constraints.

5. Using the fact that the Lucas-Stokey allocation is stationary, we can construct the optimal payoff matrix

$$\mathbb{P}^*(s_t, s_{t-1} | b_{-1}) = p_t$$

2.6 Quasilinear preferences $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

Given initial assets b_{-1}, let $\mu(b_{-1})$ be the Lagrange multiplier on the Lucas-Stokey implementability constraint

1. **Multiplier → Tax rate:**

$$\tau(\mu) = \frac{\gamma \mu}{(1+\gamma)\mu - 1}$$

2. **Tax rate → net of interest surplus:**

$$S(s, \tau) = \theta(s)^{\frac{\gamma}{1+\gamma}} (1-\tau)^{\frac{1}{\gamma}} \tau - g(s)$$

3. **Surplus → optimal payoff structure:**

$$\mathbb{P}^*(s, s_- | b_{-1}) = (1-\beta) \frac{S(s, \tau)}{\mathbb{E}_{s_-} S(s, \tau)} + \beta$$

2.7 Initial holdings influence optimal asset payoff structure

Denote state s as "adverse" if it has "high" govt. expenditures or "low " TFP; formally, s is "adverse" if

$$g(s) \mathbb{E}_{s_-} \theta^{\frac{\gamma}{1+\gamma}} - \theta(s)^{\frac{\gamma}{1+\gamma}} \mathbb{E}_{s_-} g > 0$$

Properties of optimal payoff matrix $\mathbb{P}$

- With positive initial govt. assets: want an asset that pays *more* in "adverse" states
- With negative initial govt. assets: want an asset that pays *less* in "adverse" states

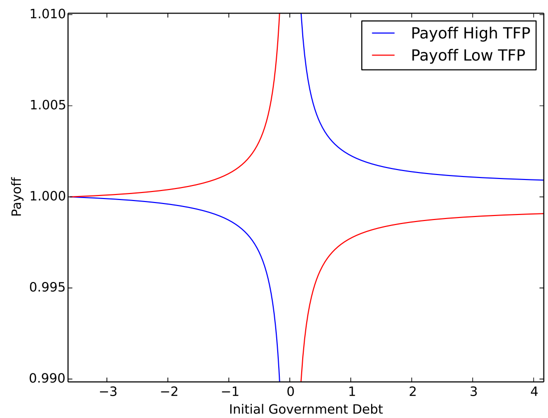

Figure 1: Optimal asset payoff structure as a function of initial government debt when TFP follows a 2 shock i.i.d process

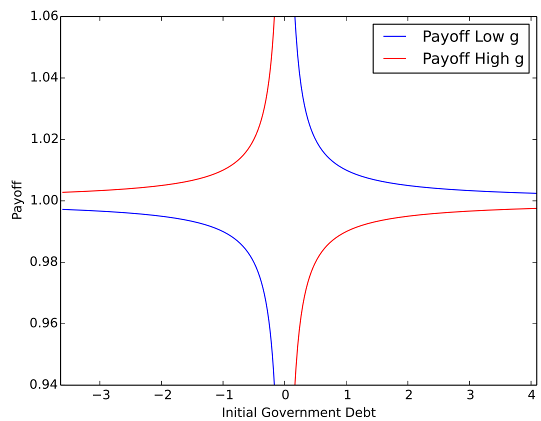

Figure 2: Optimal asset payoff structure as a function of initial government debt when government expenditures follow 2 shock i.i.d process

2.8 Inverting the $\mathbb{P}^*$ mapping

1. **Exogenous payoff structure:** Suppose $\mathbb{P} \neq \mathbb{P}^*(b_{-1})$
2. **Steady States:** A steady state is a government debt b^* such that

$$b_t = b^* \text{ implies } b_{t+\tau} = b^* \quad \forall \tau > 0$$

3. **Characterization:** Given an asset payoff structure $\mathbb{P}$
   - Does a steady state exist? Is it unique?
   - Value of b^*?
   - For what *initial government debts* b_{-1} does b_t converge to b^*?

2.9 Existence and $\mathbb{P}^{*-1}$

When shocks are i.i.d and take two values

1. $\mathbb{P}(s_-, s)$ is independent of s_- (so $\mathbb{P}$ can be a vector)
2. Under the normalization $q_t = \beta$, $\mathbb{E}\mathbb{P}(s) = 1$. Payoffs are then determined by a scalar $\boldsymbol{p}$.
   - $\boldsymbol{p}$ is the asset's payoff in the "good" state s
   - A risk-free bond is a security for which $\boldsymbol{p} = 1$
3. A steady state is obtained by inverting the optimal payoff mapping p^*

$$b^* \text{ satisfies } \boldsymbol{p} = \boldsymbol{p}^*(b^*) \text{ or } p^{*-1}(p) = b^*$$

   One equation in one unknown b^*

2.10 Existence regions in $\boldsymbol{p}$ space

The payoff $\boldsymbol{p}$ in good state $\in (0, \infty)$.

We categorize a set of economies with different asset payoffs into 3 regions via thresholds $\alpha_2 \geq \alpha_1 \geq 1$

- Low enough $\boldsymbol{p}(\leq \alpha_1)$: government holds assets in steady state
- High enough $\boldsymbol{p}(\geq \alpha_2)$: government issues debt in steady state
- Intermediate $\boldsymbol{p}(\alpha_1 > \boldsymbol{p} > \alpha_2)$: steady state does not exist

2.11 Thresholds: $\alpha_1 < \alpha_2$

- With only government expenditure shocks

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$

- With only TFP shocks

$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

  and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

2.12 Convergence

- Our analysis verifies existence of a steady state in a 2-state i.i.d. economy.
- To study long-run properties of a Ramsey allocation, we want to know whether steady state is stable
- **Risk-adjusted martingale:**

  The Lagrange multiplier μ_t on the implementability constraint satisfies

$$\mu_t = \mathbb{E}_t p_{t+1}\mu_{t+1}$$

  or

$$\mathbb{E}_t\mu_{t+1} = \mu_t - Cov_t(p_{t+1}, \mu_{t+1})$$

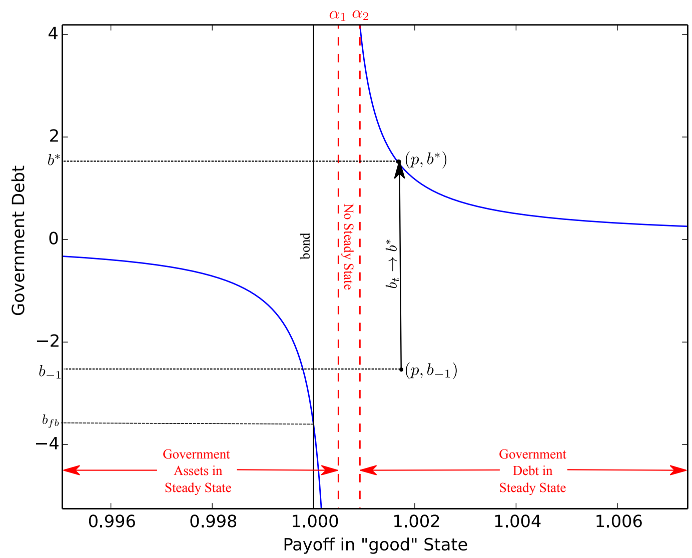

Figure 3: Existence regions in $\boldsymbol{p}$ space

- **Stability criterion:** Away from a steady state, is the drift of μ_t big enough?

2.13 Characterizing convergence under quasi-linearity, iid, and $S = 2$

- Reminder: $\boldsymbol{p}$ is the payoff in the "good" state.

- We partition the "$\boldsymbol{p}$ space" into stable and unstable regions

Theorem 2.4. *Let b^* denote steady state govt. debt and b_{fb} be govt. debt that supports the first-best allocation with complete markets. Then*

1. ***Low p**: If $\mathbf{p} \leq \min(\alpha_1, 1)$ then $b_{fb} < b^* < 0$ and $b_t \to b^*$ with probability 1.*

2. ***High p**: If $\mathbf{p} \geq \alpha_2$ then $0 < b^*$ and $b_t \to b^*$ with probability 1.*

For the intermediate region where b^* either does not exist or is unstable, there is a tendency to run up debt

Stability regions

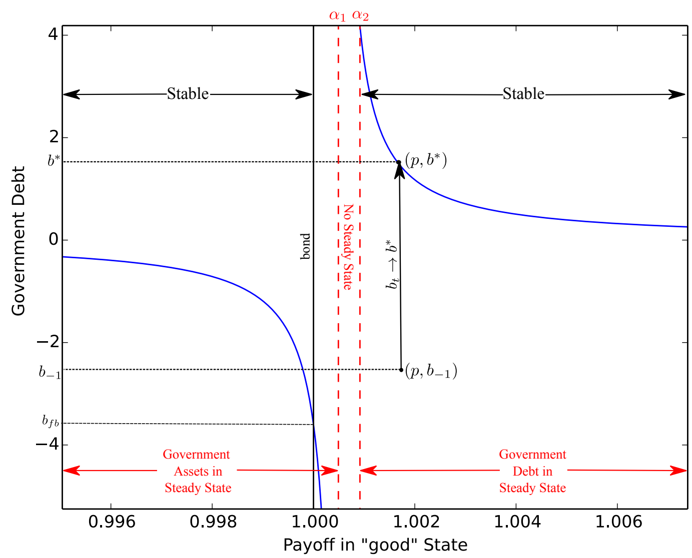

Figure 4: Stability regions

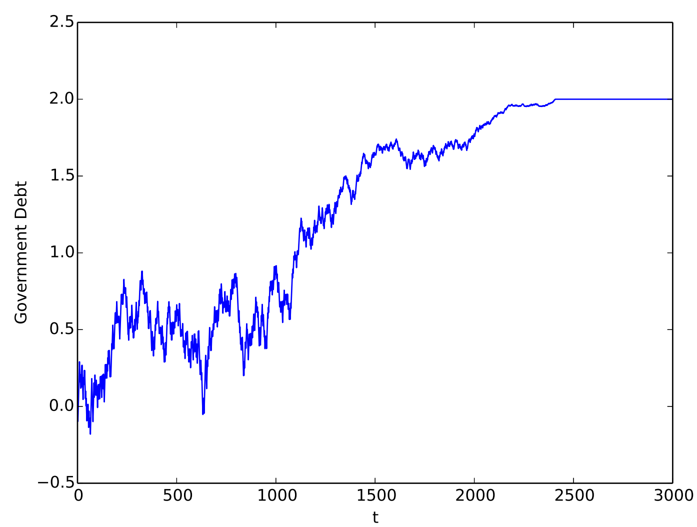

Figure 5: A sample path with $\boldsymbol{p} > 1$

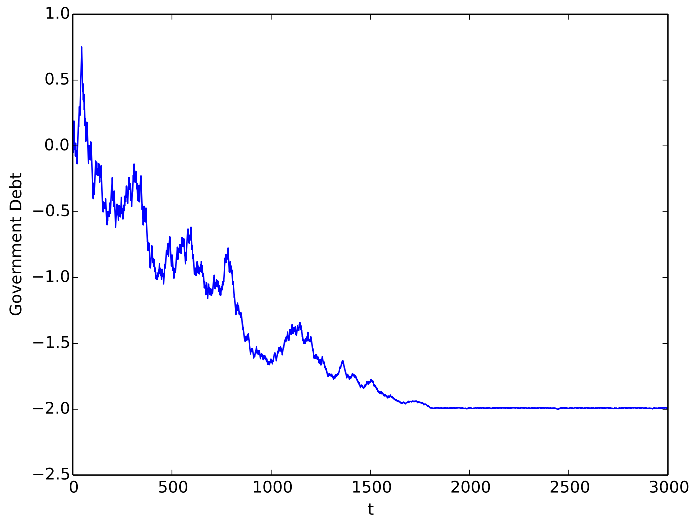

Figure 6: A sample path with $\boldsymbol{p} < 1$

2.14 Intuition for Convergence

- The Ramsey policy with incomplete markets smooths the welfare costs of distorting labor taxes by manipulating debt positions
- With a risk-free bond, the marginal cost of raising funds μ_t is a martingale. Changes in debt levels help smooth tax distortions across time.
- If the payoff matrix of the asset differs across states, then by generating state contingent revenues, the level of government debt smooths tax distortions across states.
- The steady state b^* is a unique debt level that provides enough "state contingency" completely to overcome missing assets markets
- When issuing debt, the government takes takes this benefit into account by distorting the martingale and either accumulating or decumulating debt.
- Although this is achieved by raising taxes, locally the welfare costs of taxes are second order and dominated by the gains from coming closer to b^*, which are first order in terms of welfare.

2.15 Outcomes with quasi-linear preferences

Outcomes:

1. Often $b_t \to b^*$ when the aggregate state follows a 2-state i.i.d. process

2. The level and sign of b^* depend on the **exogenous payoff structure** $\mathbb{P}$

3. The limiting allocation corresponds to a complete market Ramsey allocation for initial govt. debt b^*

2.16 Turning on risk-aversion

Modifications:

- Another source of return fluctuations – the risk-free interest rate

- Marginal utility adjusted debt encodes history dependence

- With binary i.i.d shock shock process, $x_t = u_{c,t} b_t$ converges

- Long-run properties of x_t depend on equilibrium returns $R_{t,t+1} = \frac{\mathbb{P}(s_t, s_{t+1})}{q_t(s^t)}$. Now q_t varies in interesting ways

2.17 Roadmap, II

Two subproblems

1. $t = 0$ Bellman equation in value function $W(b_{-1}, s_0)$

2. $t \geq 1$ Bellman equation in value function $V(x, s_{\text{-}})$

Seek steady states x^* such that $x_t \to x^*$

2.18 A Recursive Formulation

1. Commitment implies that government actions at $t \geq 1$ are constrained by the public's anticipations about them at $s < t$

2. This contributes additional state variables like marginal utility of consumption

3. Scaling the budget constraint by marginal utility makes Ramsey problem recursive in $x = U_c b$

$$\frac{x_{t-1} p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} = U_{c,t} c_t + U_{l,t} l_t + x_t$$

2.19 Bellman equation for $t \geq 1$ (*ex ante*)

$$V(x, s_{-}) = \max_{c(s), l(s), x'(s)} \sum_s \Pi(s, s_{-}) \Big(U(c(s), l(s)) + \beta V(x'(s), s) \Big)$$

subject to $x'(s) \in [\underline{x}, \overline{x}]$

$$\begin{aligned} \frac{x \mathbb{P}(s) U_c(s)}{\beta \mathbb{E}_{s_{-}} \mathbb{P} U c} &= U_c(s) c(s) + U_l(s) l(s) + x'(s) \\ c(s) + g(s) &= \theta(s) l(s) \end{aligned}$$

2.20 Time 0 Bellman equation (*ex post*)

Given an initial debt b_{-1}, state s_0, and continuation value function $V(x, s_{-})$

$$W(b_{-1}, s_0) = \max_{c_0, l_0, x_0} U(c, l) + \beta V(x_0, s0)$$

subject to time zero implementability constraint

$$U_c(c_0, l_0) c + U_l(c_0, l_0) l_0 + x_0 = U_c(c_0, l_0) b_{-1}$$

and resource constraint

$$c_0 + g(s_0) = \theta(s_0) l_0$$

and

$$x_0 \in [\underline{x}, \overline{x}]$$