Notes on Calibration

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Abstract

0.1 Calibration

We calibrated the model in two steps. The baseline parameters governing preferences and technology were picked such that the static [1] version of the economy. In the second step, the magnitude of fluctuations is pinned down by the average peak-trough spreads observed in the past three recessions.

We first discuss calibration of $\left(\psi, \alpha, \bar{\theta}_i, \bar{g}, \bar{\beta}\right)$. Although these parameters jointly determine the relevant moments it is more intuitive to explain which moment in the data mainly drives each parameter. We normalize $\bar{\theta}_2 = 1$ and pick $\bar{\theta}_1$ to match log wage ratio of 90 wage percentile to 10 wage percentile of 4 from Autor et al. The discount factor $\bar{\beta}$ set to match the (annual) interest rate 2% .The parameter ψ is picked to match Frisch elasticity of labor supply equal to 0.5. In the model $\bar{g}$ corresponds to non-transfer government expenditures. This level of g varied in the post WWII period from 12% in 1950 to 17% in 2010 . We set the level $\bar{g}$ at 12% of GDP. [2]Finally, we set Pareto weights α to match the average marginal tax rate in the US of about 20% (Chari et al, 1993).

Next we turn to the business cycle facts. We calibrate $\{e_{i,\theta}, e_b, \Pr\left(s|s_-\right)\}$ to match the following four facts about booms and recessions: log income of both 10th and 90th percentile falls in the recessions but 10th percentile falls by more, inflation adjusted interest rate on government debt is generally lower in recessions and both booms are more persistent than recession The average spread in labor productivity is calibrated to match an average 3% loss in output seen in the last three recessions. The inequality shock is designed to match the facts documented in Fatih et all that the fall in earnings of the 10-percentile is about 2.5 times of 90-percentile. For the purpose of comparison, we also calibrate a drop in government expenditure to get a similar drop in output as the previous two cases. The discount factor shocks match the average boom-recession difference of about 1.96% in the real risk-free interest rate (3 month T bill rate -inflation rate) seen in the last three recessions. The trasnition matrix is calibrated to get match the average duration of booms vs recesions. Note that all three shocks are perfectly correlated - A recession is a period when TFP falls, inequality rises, and interest rates are lower. The initial level of government debt is chosen to be 60% to match the ratio of federal debt held by public in the beginning of 2010

The details of calibration are summarized in Table 1

0.2 Results

We discuss separately the long run and the short run implications for the optimal policy. In particular we will study the economy (**"Benchmark"**) with calibration discussed above and discuss a few variants that turnoff certain features.

1. **Constant Interest Rates** :In the first variant, we recalibrate the discount factor shocks such that the risk free rate is uncorrelated with output.

[1] Formally, an equilibrium in an economy where all shocks are equal to their mean value

[2] ANMOL:This is obtained from NIPA. It includes depreciation on assets owned by the government. Do we want to recompute with something like 15% ?

Parameter	Target	
ψ	Frisch elasticity of labor supply	0.6994
$\bar{\theta}_1$	log 90-10 wage ratio (Autor et all)	4.0552
$\bar{\theta}_2$	Normalize to 1	1
$\hat{\theta}_1$	Business cycle fluctuations in the earnings of 90th percentile	1.2 %
$\hat{\theta}_2$	Business cycle fluctuations in the earnings of 10th percentile	3%
β	Average (annual) risk free interest rate	0.98
$hat\beta(s)$	Business cycle fluctuations	.02
α_1	marginal tax rate in the economy with no shocks	0.69
g	average pre-transfer expenditure- output ratio	12 %
$P(r\|r)$	Recession-Boom episodes	0.63
$P(b\|b)$	Recession-Boom episodes	0.84

2. **Constant Discount Factors** : Here we shut off the discount factor shocks by setting $\hat{\beta}\left(s\right)=0$.. Note that under this assumption, interest rates in the model are countercyclical
3. **No Inequality** : This variant modifies the "Benchmark" by setting $\hat{\beta}\left(s\right)=0$ and $\hat{\theta}_1(s)=\hat{\theta}_2(s)=3\%$ This corresponds to a case when the only source of business cycle fluctuations is TFP shock which affects all agents equally. This case more closely matches the experiments in the RBC literature such as Chari et al (1993).
4. **Expenditure Shocks** :The last variant compares the optimal responses to the shock to government expenditures. In this experiment we set $\hat{\theta}\left(s\right)=\hat{\beta}\left(s\right)=0$ and choose $\hat{g}\left(s\right)$ to produce a drop in output of a similar magnitude to that in the first three experiments. This compares to the studies of the response to government shocks of AMSS and Faraglia et al

Long run Figure 1 plots dynamics of assets and taxes . All experiments start with government debt to GDP ratio of 60%. Several features emerge from this figure. First, as we discussed in Section **??**, over time economy tends to a particular region.

- In line with Section **??**, we converge to some long run region. Also, when there are no discount factor shocks, this region has assets. As alluded before the optimal policy manipulates the net asset positions to correct the two key constraints imposed on the policy - absence of agent specific transfers (*Affine Taxes*) and absence of state contingent assets (*Risk-free Debt*). Starting from a point when the relative assets of the unproductive agent (or the government if we use the previous normalization) are low, extracting resources through lower transfers exacerbates inequality. This is costly since the government has to use higher taxes in future to redistribute. On the margin the government has incentives to accumulate assets. On the other hand interest rate fluctuations interact with net asset position to generate state contingent flows. If interest rate are high when the government needs additional revenues, accumulating assets relaxes the the restriction imposed by absence of state contingent assets. Thus with countercyclical interest rates, these forces reinforce each other long run asset positions are positive.
- In data, however interest rates are generally lower in recessions. This flips the spanning argument and for large enough fluctuations, the government may accu-

mulate assets. In Figure 1, the black line represents the benchmark with discount factor shocks aimed to replicate the pro-cylical fluctuations in interest rates. We see for a very long time, the economy starting from debt maintains this level.[3] The blue and the red lines are respectively lower and zero discount factor spreads and we clearly see a trend towards more assets.

- We also see that convergence is very slow. It takes about 3,000 years for the economy that starts with 60% debt-GDP ratio to pay-off all that debt before even starting accumulating assets with persistent shocks. With discount factor shocks, it will either take even longer to repay the debt, or actually be indebted even after 5,000 years.

Two lessons from this is the importance of interest rate fluctuations for long run and low frequency properties of optimal allocations

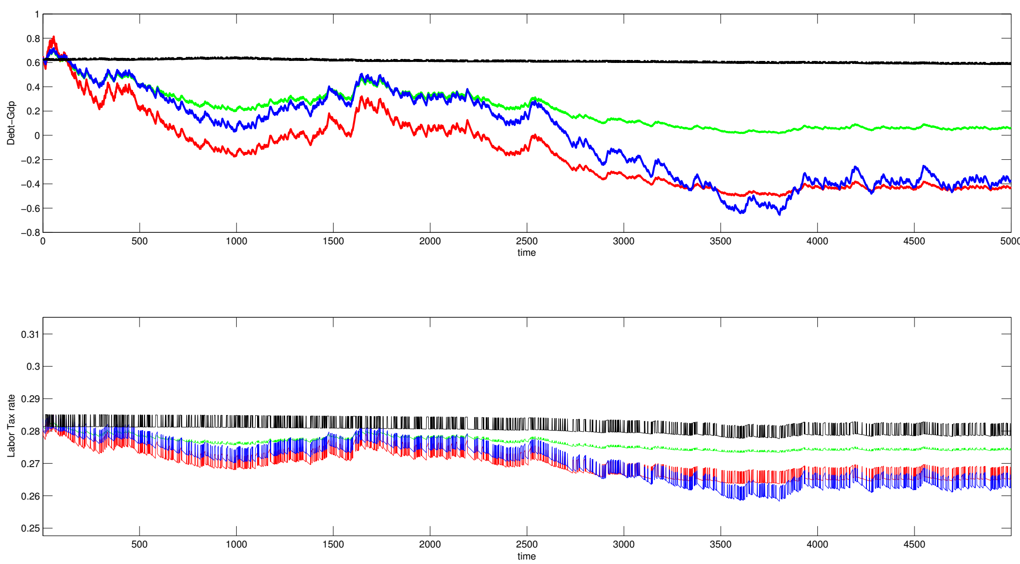

Figure 1: *This plots a long sample path for debt-to-gdp ratio and labor taxes across various settings for the same sequence of shocks. The black line is the "Benchmark", the blue line is the "Constant Interest Rates" , the red line is the "Constant discount factor" and the green line is the "No inequality" scenario.*

Short run The analysis of the previous section showed that there exists a very low frequency component in the optimal policy. In this section we focus on the business cycle frequency optimal response. In our setting this can be broken down in two sub-parts. The magnitude of change as we switch from "booms" to "recession" and the dynamics during the periods these states persist.

First solve the time 0 problem (with same initial conditions across different settings) . This pins down the initial state (the exogenous state is such that we are in an outset

[3] Inscyclical the fact that with large discount factor shocks there exists regions low volatility and the government *not* accumulating assets, these are typically unstable. This is explained by the two forces highlighted before now work in opposite direction and the net effect depends on the relative strengths

	Δg	ΔB	ΔT	$\Delta[\tau\theta_1 l_1]$	$\Delta[\tau\theta_2 l_2]$	ΔY	$\Delta\tau$
Benchmark	0.0000	-1.1561	0.6871	-0.1593	-0.3096	-2.8536	0.3732
Constant Interest Rates	0.0000	-1.1126	0.6591	-0.1497	-0.3038	-2.8613	0.3879
Constant Discount Factor	0.0000	-1.0794	0.6387	-0.1415	-0.2992	-2.8677	0.3997
No Inequality	0.0000	-0.1380	-0.5459	-0.5635	-0.1204	-2.6294	0.0622
GShocks	-7.5037	2.9137	2.8612	-1.3759	-0.3530	-2.3443	-1.1598

Table 1: *The tables shows the short run policy mix for different types of shocks. All numbers are normalized by un-distorted gdp except* τ

of a recession) for the recursive problem. We then use the policy rules to compute the across state fluctuations of different components in the government budget constraint. These responses are summarized in table 1. For each variable z in the table we report in the form $\Delta z \equiv \left(z\left(s_l|x_0,\rho_0,s_0\right) - z\left(s_h|x_0,\rho_0,s_0\right)\right)/\bar{Y}$ where $\bar{Y}$ is average undistorted-GDP [4]. We review below the main findings

- Source of shocks is very important. We have three different types of shocks producing drops in GDP with very different results, both qualitatively and quantitatively. The most stark observations is the difference between only TFP shocks and TFP + inequality shocks for optimal transfers. The model predicts positive jump in transfers when recessions are accompanied with higher inequality as against a negative with just productivity shocks.
- In terms of magnitudes, in the benchmark the optimal policy draws down assets to make up for the lost tax revenue (40%)and higher transfers (60%). With only TFP shocks, about (75%) of the low tax revenues are made up by *reducing* transfers. With expenditure shock, the government finances about 40% of the higher expenditure by reducing transfers, 40% by extra borrowing and the rest by revenue from taxes.
- Discount factor shocks have a minor effect on impact and matter more for the transient dynamics resulting in very different long run properties.

Lastly figure 2 and 3 show how the transient dynamics for prolonged booms (or recessions) are different with and without discount factor shocks. The four panels have taxes, transfers, debt and interest rate movements for a path of 25 years. The bold line in figure 2 is the benchmark and in figure . 3 refers to the variant with uncorrelated interest rates. The dotted line in both figures is the variant with no-discount factor shocks and the shaded regions are periods with low output. We see that in a prolonged booms, the government accumulates assets, lowers taxes when there are no discount factor shocks.

[4] Note that predetermined variables like repayment on existing debt drop out of the accounting and we have

$$\Delta[g] + \Delta[T] + \Delta[B] = \Delta[\tau\theta_1 l_1] + \Delta[\tau\theta_2 l_2]$$

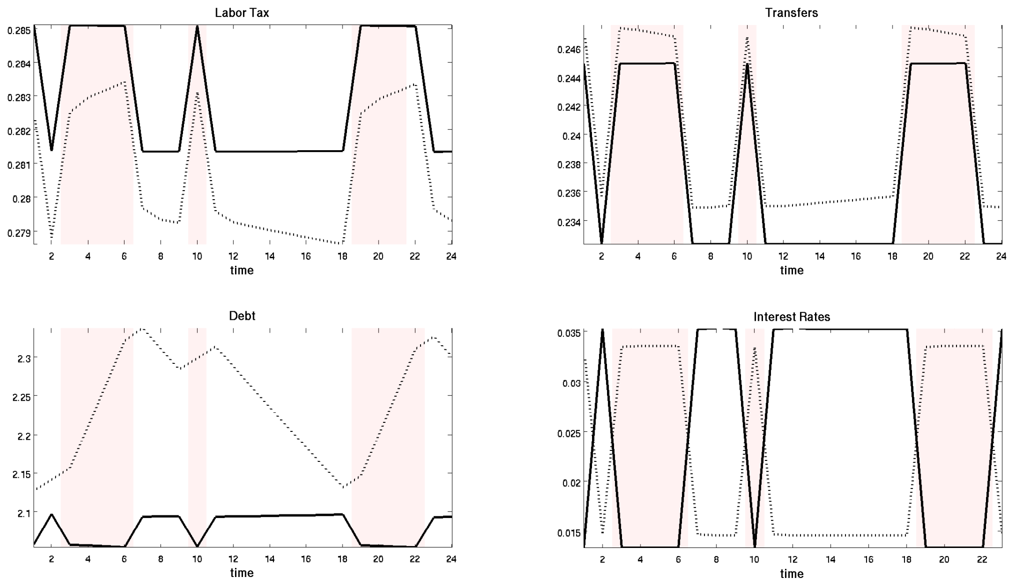

Figure 2: *This plots a typical sample path taxes, transfers,debt and interest rates. The bold (dotted) lines are with (without) large discount factor shocks and the shaded regions are recessions*

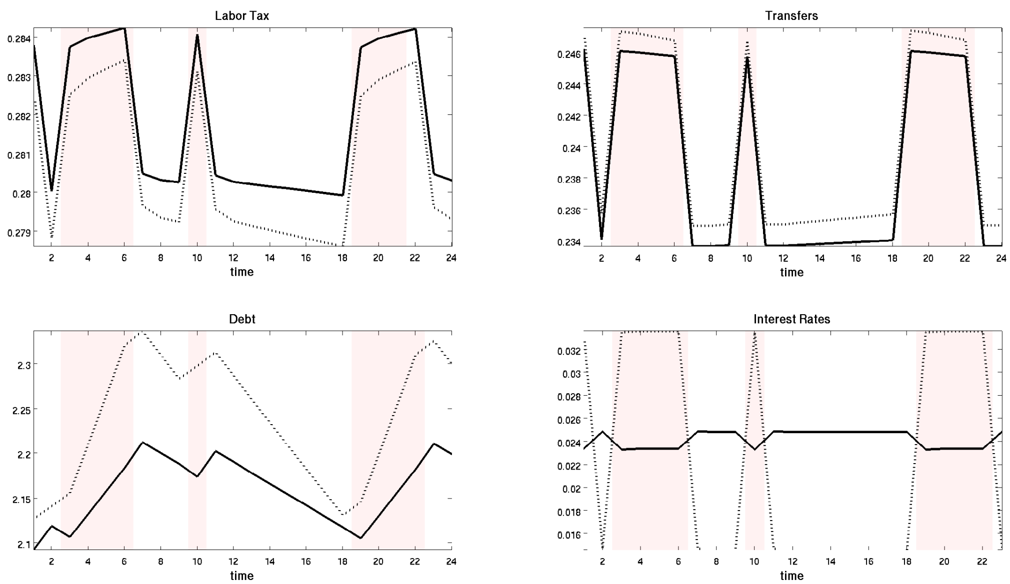

Figure 3: *This plots a typical sample path taxes, transfers,debt and interest rates. The bold (dotted) lines are with (without) small discount factor shocks and the shaded regions are recessions*