Optimal fiscal policy with incomplete asset markets

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Optimal taxation under commitment and a representative agent

- **Incomplete markets**
  $\rightarrow$ assets with alternative exogenous payoff patterns
- **Linear tax schedules**
  $\rightarrow$ Proportional tax on labor earnings (maybe *nonnegative* transfers)
- **Aggregate shocks**
  $\rightarrow$ To productivities, government expenditures, etc.

Questions

1. **Tax rate**: Can government adjust assets to smooth costs of distorting tax
2. **Government debt**: Why do different governments issue different amounts of debt?
   + Lucas Stokey (1984): Inherited from initial condition
   + AMSS (2002): Govt. accumulates assets sufficient to finance activities using interest revenues
3. **Extreme assumptions:** LS – complete markets; AMSS — a risk-free bond only

Our analysis

1. Cases intermediate between LS and AMSS
   + We restrict government to trade a single asset only
   + We exogenously restrict payoffs of this single asset
     $\Longrightarrow$ E.g., bonds that pay less during adverse times
2. Asset **levels** can help smooth tax distortions **across states**
3. This differs from the usual role of debt in previous incomplete markets economies where **changes** in debt levels help smooth tax distortions **over time**

Environment

- **Uncertainty**: Markov aggregate shocks s_t
- **Demography**: Infinitely lived representative agent plus a benevolent planner
- **Technology**: Aggregate output $y_t = \theta_t l_t$ is linear in labor supply
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c(s^t), l(s^t)\right)$$

Environment, II

- **Asset markets**: Private sector has complete markets; Government trades are restricted:
  - A unit of government debt pays off p_s in state s
- **Linear Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_t l_t, \;\; T_t \geq 0$$

- **Budget constraints**
  - Agents: $c_t + q_t b_t = (1 - \tau_t)\, \theta_t l_t + p_t b_{t-1} + T_t,$
  - Government: $g_t + q_t B_t + T_t = \tau_t \theta_t l_t + p_t B_{t-1},$
- **Market Clearing**
  - Goods: $c_t + g_t = \theta_t l_t$
  - Assets: $b_t + B_t = 0$
- **Initial conditions**: Assets b_{-1}, B_{-1} and s_{-1}

Ramsey Problem

Definition
Allocation, price system, government policy: Standard

Definition
Competitive equilibrium: Given (b_{-1}, B_{-1}) and $\{\tau_t, T_t\}_{t=0}^{\infty}$ all allocations are chosen optimally, markets clear [1]

Definition
Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given (b_{-1}, B_{-1})

[1] Usually, we impose only "natural" debt limits.

Roadmap

1. Given initial assets, characterize the **optimal asset payoff structure**:
   ⇒ What single asset would government choose to issue?
   We reverse engineer payoffs from a Lucas-Stokey complete market allocation
2. Study a Ramsey problem when the asset payoff structure is not optimal
3. Quasilinear preferences
   - With general preferences the government asset returns have both exogenous component (p_s) and endogenous component $\frac{1}{q_t}$.
   - To remove the endogenous component, we initially restrict ourselves to quasilinear preferences.

Sequential problem

$$\max_{\{c_t, l_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Implementability constraints**

$$\frac{b_{t-1} U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1} p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \text{ for } t \geq 1$$

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

Sequential problem: Remarks

1. **Primal approach**: To eliminate tax rates and prices, use consumer's first order conditions
2. **Implementability constraints**: Derive by iterating the consumer's budget equation forward at every history
   $\Rightarrow$With incomplete market economies these impose a *measurability restrictions* on allocations
3. **Transfers:** We temporarily restrict transfers $T_t = 0 \ \forall t$. This is convenient for our analytical results. We eventually show that this assumption is not restrictive.

Optimal asset payoff structures

1. Compute a Lucas Stokey allocation given b_{-1}
   Find an optimal allocation ignoring $t \geq 1$ measurability restrictions.
2. Reverse engineer payoffs

$$p_t = \frac{\beta}{U_{c,t} b_{t-1}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right)$$

   In effect, the Ramsey planner chooses an asset payoff structure p_t so that the measurability constraints for $t \geq 1$ are automatically satisfied
3. Since allocations are history independent

$$p_t = p^*(s_t | s_{t-1})$$

Quasilinear preferences: $U(c,l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

Given initial assets b, let $\mu(b)$ be the Lagrange multiplier on the the implementability constraint

1. **Multiplier $\rightarrow$ Tax rate:**

$$\tau(\mu) = \frac{\gamma\mu}{(1+\gamma)\mu - 1}$$

2. **Tax rate $\rightarrow$ Surplus:**

$$S(s,\tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1+\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

3. **Surplus $\rightarrow$ Payoff structure:**

$$p^*(s|s_-) = (1-\beta)\frac{S(s,\tau)}{\mathbb{E}_{s_-}S(s,\tau)} + \beta$$

Initial holdings influence optimal asset payoff structure

How does the optimal payoff structure vary with initial conditions

1. **Initial assets:**
   - With positive initial assets: want a payoff structure that pays more in “adverse” states
   - With negative initial assets: want a payoff structure that pay less in “adverse” states
2. **Nature of shocks:** when do we approximate a risk-free bond?
   - With i.i.d shocks to government expenditures: when initial assets approach infinity
   - With i.i.d shocks to TFP: When assets are sufficient to support the first best

Optimal Payoff Structure: Expenditure shocks

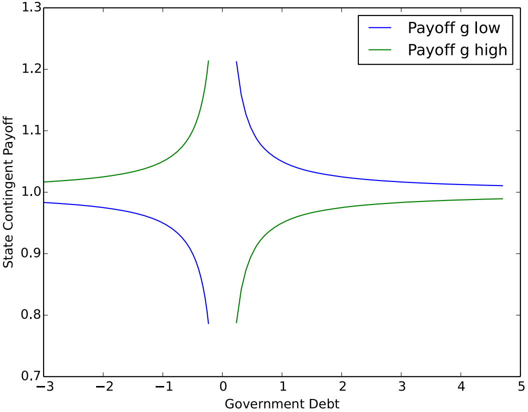


Figure: Optimal asset payoff structure as a function of initial government debt when government expenditures follow 2 shock i.i.d process

Optimal Payoff Structure: TFP shocks

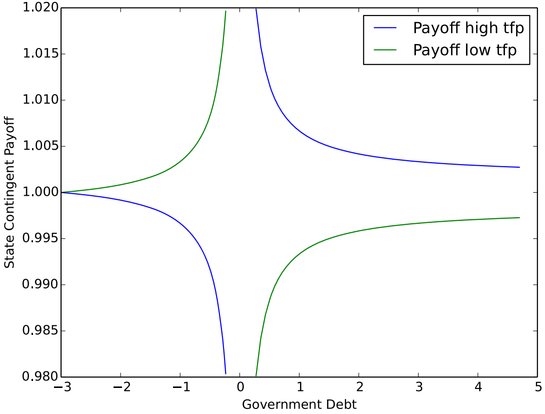


Figure: Optimal asset payoff structure as a function of initial government debt when TFP follows a 2 shock i.i.d process

Incomplete markets:

- **Exogenous payoff structure:** Suppose $p \neq p^*(b_{-1})$
- **Steady States:** *Do government assets asymptotically approximate a steady state for which the exogenous asset payoff structure is optimal?*
- **Characterization:** Given an asset payoff structure p, with a 2 state i.i.d process for the aggregate state
  - When does a steady state exist? is it unique?
  - What is the level of government debt or assets in a steady state?
  - For what levels of *initial government debt* does convergence to a steady state occur?
- **Extensions:** To richer aggregate stochastic process and preferences exhibiting risk aversion

Link of steady state outcomes to exogenous asset payoffs

1. Finding a steady state amounts to finding a complete markets optimal allocation whose optimal payoff structure matches p_t
2. We explore different asset payoff structures
   - Suppose $s = 1$ is the "good" state (low expenditure or high TFP)
   - Index incomplete market economies by $p_1 = p(s = 1)$. [2]
3. **Outcomes in 3 Regions:**
   - Low enough p_1: government holds assets in steady state
   - High enough p_1: government issues debt in steady state
   - Intermediate p_1: steady states do not exist

   α_1, α_2 are two thresholds that split the "p_1 space"

[2] We get p_2 by using the normalization that $\mathbb{E}p(s) = 1$. A risk-free bond sets $p_1 = 1$

Thresholds: $\alpha_1 < \alpha_2$

- With only government expenditure shocks

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$

- With only TFP shocks

$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

Convergence

- Our analysis verifies the existence of a steady state in a 2-state i.i.d. economy.
- In order to study long-run properties of an optimal allocation with incomplete markets we need to determine whether these steady states are stable
- **Risk adjusted martingale:**
  Under an optimal policy, the Lagrange multiplier on the implementability constraint μ_t satisfies

  $$\mu_t = \mathbb{E}_t p_{t+1} \mu_{t+1}$$

  or

  $$\mathbb{E}_t \mu_{t+1} = \mu_t - Cov_t(p_{t+1}, \mu_{t+1})$$

  μ_t follows a risk adjusted martingale.
- **Stability:** Is the drift μ_t big enough away from a steady state?

Characterizing Convergence

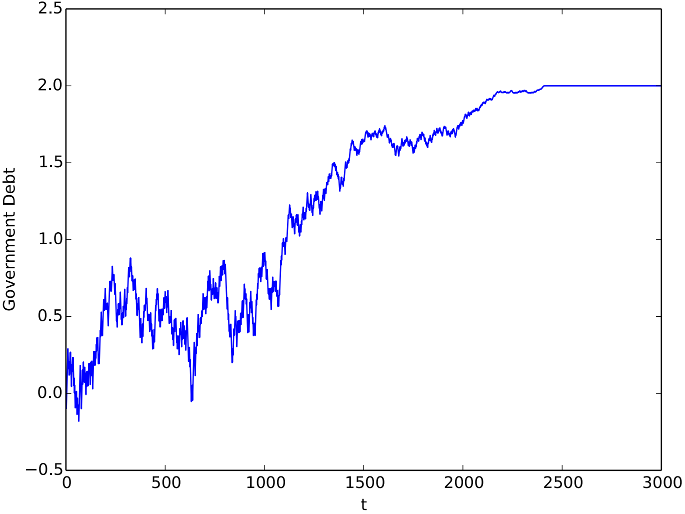
- Reminder: p_1 is the payoff in the "good" state.
- As with existence, we can partition the "p_1 space" into different regions
- Let b_{fb}, μ_{fb} be the debt level and associated Lagrange multiplier where the government can implement first-best via access to complete markets

Theorem

Let b^* denote the steady state level of debt. Then $\exists \phi_1 < \phi_2$ such that

1. **Low** p_1*: If $p_1 \leq \min(\phi_1, 1)$ then the steady state is stable with $b_{fb} < b^* < 0$ and $b_t \to b^*$ with probability 1.*
2. **High** p_1*: If $p_1 \geq \phi_2$ then the steady state is stable with $0 < b^*$ and $b_t \to b^*$ with probability 1.*
3. **Intermediate** p_1**:**
    - *If $1 < p_1 < \phi_1$ then a steady state exists with $b^* < b_{fb}$, but it is unstable with $\mu_t > \mathbb{E}_t \mu_{t+1}$ for $\mu < \mu_{fb}$.*
    - *If $\phi_1 \leq p_1 < \phi_2$ then a steady state does not exist and $\mu_t > \mathbb{E}_t \mu_{t+1}$*

$p_1 > 1$



Government Debt
t
2.5
2.0
1.5
1.0
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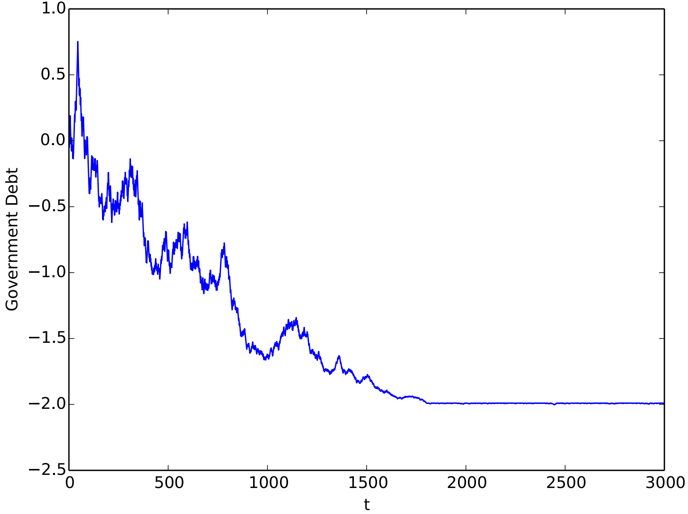
0
500
1000
1500
2000
2500
3000

$p_1 < 1$


1.0
0.5
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Government Debt
0
500
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1500
2000
2500
3000
t

Incomplete markets with risk aversion

Quasilinear preferences:

1. With quasilinear preferences, we showed that $b_t \to b^*$ when the aggregate state followed a 2-state i.i.d. process
2. The level and sign of b^* is a function of the **exogenous payoff structure** $p(s)$

Risk aversion:

- Marginal utility adjusted assets: $x_t = u_{c,t} b_t$ encodes history dependence
- Long-run properties of x_t depend on returns $R_{t,t+1} = \frac{p(s_{t+1})}{q_t}$
- To focus on the **endogenous component** of payoffs, set $p(s) = 1$

Roadmap, II

- Show that $x_t = U_{c,t} b_t$ is sole state variable
- Split the Ramsey problem in two
  1. $t = 0$ Bellman equation in $W(b_{-1}, s_0)$ Anmol and David XXXXXX: I altered the notation
  2. $t \geq 1$ Bellman equation in $V(x, s_{\_})$
- Analyze steady states x^* such that $x_t \to x^*$

Interpretation: x^* corresponds to an initial condition when the optimal portfolio in a LS economy is a risk-free bond

A Recursive Formulation

1. Commitment implies that government actions at $t \geq 1$ are constrained by anticipations about them at $s < t$
2. This contributes additional state variables like marginal utility of consumption
3. Scaling the budget constraint by marginal utility makes it recursive in product $x = U_c b$

$$\frac{x_{t-1} p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} = U_{c,t} c_t + U_{l,t} l_t + x_t$$

Bellman equation for $t \geq 1$

$$V(x, s_-) = \max_{c(s), l(s), x'(s)} \sum_s \pi(s, s_-) \Big(U(c(s), l(s)) + \beta V(x'(s), s) \Big)$$

subject to $x'(s) \in [\underline{x}, \overline{x}]$

$$\frac{x p(s) U_c(s)}{\beta \mathbb{E} p U c} = U_c(s) c(s) + U_l(s) l(s) + x'(s)$$

$$c(s) + g(s) = \theta(s) l(s)$$

Time 0 problem

Given an initial debt b_0, state s_0, and continuation value function $V(x, s_-)$ Anmol and David XXXXX: I altered the below by adding the W function; but there is a problem with the timing of the b argument. See earlier slide where W is defined.

$$W(b_{-1}, s_0) = \max_{c,l,x'} U(c,l) + \beta V(x', s0)$$

subject to time zero implementability constraint

$$U_c(c,l)c + U_l(c,l)l + x' \geq U_c(c,l)b_0$$

and resource constraint

$$c + g(s_0) = \theta l$$

and

$$x' \in [\underline{x}, \overline{x}]$$

Revisiting steady states with risk aversion

Let $x'(s; x, s_{-})$ be an optimal law of motion for the state variable for the $t \geq 1$ recursive problem.

Definition

A steady state x^* satisfies $x^* = x'(s; x^*, s_{-})$ for all s, s_{-}

Thus, a steady state is a node at which a continuation allocation and tax rate has no further history dependence.

Existence

Let x_{fb} be a value of the state x from which the government can institute first best from that period onwards. Assume a CRRA utility specification $U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$. Finally, let c^{fb} and l^{fb} be consumption and leisure under first best.

Proposition

For a 2 state i.i.d. process, let s_g and s_b denoting the states with high and low consumption at first best. Suppose that

$$\frac{g(s_g)}{1 - \frac{\beta\mathbb{E}[(c^{fb})^{-\sigma}]}{c^{fb}(s_g)}} > \frac{g(s_b)}{1 - \frac{\beta\mathbb{E}[(c^{fb})^{-\sigma}]}{c^{fb}(s_b)}}$$

Then there exists a multiplier μ and complete markets allocations c^μ, l^μ such that

$$\underline{x} < \frac{U_{c_\mu}(s)c_\mu(s) + U_{l_\mu}(s)l_\mu(s)}{\frac{U_{c_\mu}(s)}{\beta\mathbb{E}[U_{c_\mu}]} - 1} = x^* < 0$$

Stability

1. In this setup, interest rates are aligned with marginal utility of consumption; they are low in “good” states (high tfp or low expenditure)
2. The government holds claims against the private sector in the steady state. This is similar to the quasilinear case when p_1 was low.
3. The incomplete markets Ramsey allocation converges to a Lucas-Stokey allocation for all initial conditions

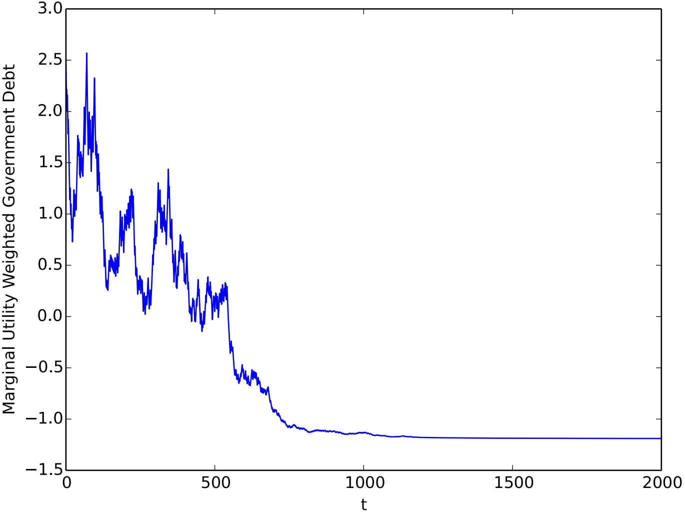
Proposition

Let $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ solve the incomplete markets Ramsey problem. Then $x_t(s^{t-1}) \to x^*$ as $t \to \infty$ with probability 1 for all initial conditions

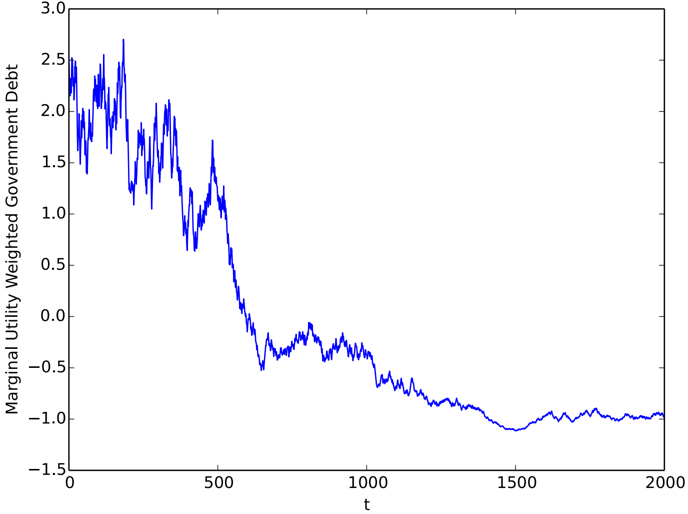
Quasilinear vs. risk aversion

- With quasilinear preferences, with a risk-free bond, the interest rate is always $1/\beta$.
- With risk averse preferences, interest rates are higher in period of high government expenditure.
- Thus, while the government has greater expenses in high government expenditure states, the higher interest rate means the government can accumulate less and still cover future government expenditures.
- After the government has accumulated enough assets, it is actually better off in periods of high government expenditure than in periods with low government expenditures (since its claims to consumption are worth more).
- By holding assets, the government is able to reallocate resources across states, something it is not able to do in the quasilinear case.

2 State i.i.d. process with Risk Aversion



$S > 2$ states

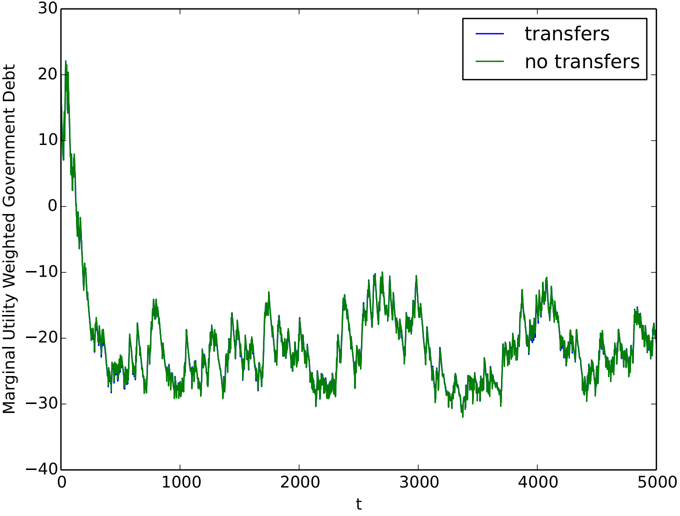


3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Marginal Utility Weighted Government Debt
0
500
1000
1500
2000
t

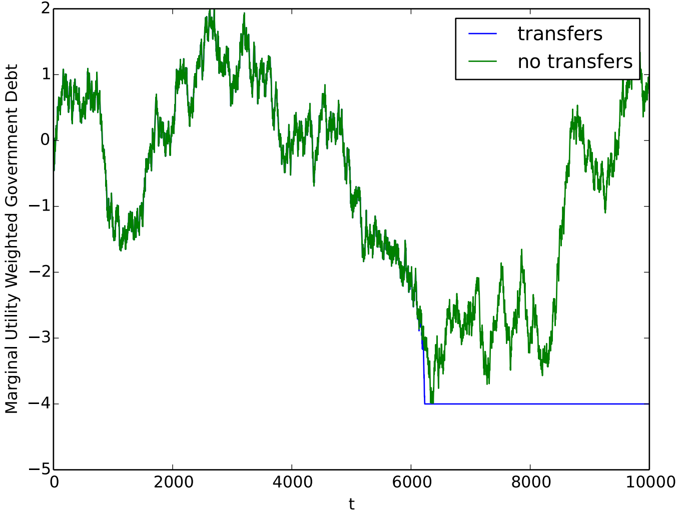
Transfers

- That the government can use its assets to smooth tax rate distortions carries over even when the government has access to lump sum transfers
- Access to nonnegative transfers makes first-best level of assets trivially a “steady state”.
- With lump sum transfers, in cases where the steady state exists and is stable, if the initial debt of the government exceeds its steady state, the economy converges with probability 1 to the steady state.

AMSS calibration with and without transfers



Quasilinear Preferences and Risk Free Bond with and without Transfers



Concluding remarks

- With market incompleteness, the asset payoff structure has big implications for the government's long run debt
- If the asset payoff structure offers greater returns in good states of the world then bad, then the Ramsey government asymptotically accumulates debt Anmol and David XXXXXX: does 'accumulate debt' mean 'run up debt' or 'accumulate assets'?
- With risk aversion, cyclical properties of interest rates affects government debt asymptotically
- Access to nonnegative transfers play little role in determining the evolution of the system. Rather the key force is the government's ability to use its debt position to reallocate resources across states
- **Future Research**: How does the type of market incompleteness effect long run wealth distributions with heterogeneous agents