Optimal Fiscal policies in some economies with incomplete asset markets

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Optimal taxation under commitment and a representative agent

- **Incomplete markets**
  → assets with various exogenous payoff patterns
- **Linear tax schedules**
  → A proportional tax on labor earnings (maybe plus *nonnegative* transfers)
- **Aggregate shocks**
  → To productivities, government expenditures, etc.

Questions

1. **Ramsey taxation**: Can the government use assets to smooth costs of distorting tax
2. **Optimal debt level**: Why do different governments issue different amounts of debt?
   - Lucas Stokey (1984): Initial conditions
   - AMSS (2002): Accumulate assets sufficient to finance activities using interest revenues
3. **Extreme assumptions:** LS – complete markets; AMSS — a risk-free bond only

Our analysis

1. We study intermediate cases
   + Government is restricted to trade a single asset only
   + We exogenously restrict payoffs of this single asset
     $\Longrightarrow$ E.g: bonds that pay less during adverse times
2. Though asset markets are incomplete but **level of assets** help smooth tax distortions **across states**
3. That differs from the usual role of debt in previous models of incomplete markets economies where **changes in debt levels** help smooth tax distortions **over time**

Environment

- **Uncertainty**: Markov aggregate shocks s_t
- **Demography**: Infinitely lived representative agent plus a benevolent planner
- **Technology**: Aggregate output $y_t = \theta_t l_t$ is linear in labor supply
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c(s^t), l(s^t)\right)$$

Environment, II

- **Asset markets**: Private sector has complete markets; Government trades are restricted:
  - A unit of government debt is restricted to payoff p_s in state s
- **Linear Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_t l_t, \;\; T_t \geq 0$$

- **Budget constraints**
  - Agents: $c_t + q_t b_t = (1 - \tau_t)\, \theta_t l_t + p_t b_{t-1} + T_t,$
  - Government: $g_t + q_t B_t + T_t = \tau_t \theta_t l_t + p_t B_{t-1},$
- **Market Clearing**
  - Goods: $c_t + g_t = \theta_t l_t$
  - Assets: $b_t + B_t = 0$
- **Initial conditions**: Assets b_{-1}, B_{-1} and s_{-1}

Ramsey Problem

Definition

Allocation, price system, government policy: Standard

Definition

Competitive equilibrium: Given (b_{-1}, B_{-1}) and $\{\tau_t, T_t\}_{t=0}^{\infty}$ all allocations are chosen optimally, markets clear [1]

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given (b_{-1}, B_{-1})

[1] Usually, we impose only "natural" debt limits.

Roadmap

1. Given initial assets, characterize the **optimal payoff structure**:
   ⇒ What single asset would government choose to issue?
   We reverse engineer payoffs from a Lucas Stokey complete market allocation
2. Study a Ramsey problem when the asset payoff structure is not optimal
3. Quasilinear preferences
   - With general preferences the government asset returns have both exogenous component (p_s) and endogenous component $\frac{1}{q_t}$.
   - To remove the endogenous component, we initially restrict ourselves to quasilinear preferences.

Sequential problem

$$\max_{\{c_t, l_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Implementability constraints**

$$\frac{b_{t-1} U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1} p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \text{ for } t \geq 1$$

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

Sequential problem: Remarks

1. **Primal approach**: To eliminate tax rates and prices, use consumer's first order conditions
2. **Implementability constraints**: Derive by iterating the consumer's budget equation forward at every history
   $\Rightarrow$With incomplete market economies they impose a *measurability restrictions* on allocations
3. **Transfers:** We restrict $T_t = 0$ for all t. This is convenient for our analytical results. We will show later, numerically, that this assumption is not restrictive.

Optimal asset payoff structures

1. Compute a Lucas Stokey allocation given b_{-1}
   This solves for the optimal allocation ignoring $t \geq 1$ measurability restrictions.
2. Reverse engineer payoffs

$$p_t = \frac{\beta}{U_{c,t} b_{t-1}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right)$$

   In effect, the Ramsey planner chooses an asset payoff structure p_t so that the measurability constraints for $t \geq 1$ are automatically satisfied
3. Since allocations are history independent

$$p_t = p(s_t | s_{t-1})$$

Quasilinear preferences: $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

Given initial assets b, let $\mu(b)$ be the Lagrange multiplier on the the implementability constraint

1. **Multiplier $\rightarrow$ Tax rate:**

$$\tau(\mu) = \frac{\gamma\mu}{(1+\gamma)\mu - 1}$$

2. **Tax rate $\rightarrow$ Surplus:**

$$S(s, \tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1+\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

3. **Surplus $\rightarrow$ Payoff structure:**

$$p(s|s_{-}) = (1-\beta)\frac{S(s,\tau)}{\mathbb{E}_{s_{-}} S(s,\tau)} + \beta$$

Graphs

a graph that explains the main parts of the theorem
- payoff structures and taxes as a function of initial debt

Initial holdings influence optimal asset payoff structure

How does the optimal payoff structure vary with initial conditions

1. **Initial assets:**
   - With positive initial assets: Want assets that pay more in "adverse" states
   - With negative initial assets: Want assets that pay less in "adverse" states
   -
2. **Nature of shocks:** When do we approximate a "risk-free" bond?
   - With i.i.d shocks to government expenditures: As assets approach infinity
   - With i.i.d shocks to TFP: As allocation tends to first best

Incomplete markets:

- **Exogenous payoff structure:** Suppose $p \neq p^*(b_{-1})$
- **Steady States:** *Optimal policy approaches a level of assets ("steady sate") that eventually makes the exogenous payoff structure optimal*
- **Characterization:** Given an asset payoff structure, with a 2 state i.i.d process for the aggregate state
  - When does a steady state exist? is it unique?
  - What is the level of government debt of assets in a steady state?
  - For what levels of *initial government debt* does convergence to a steady state occur?
- **Extensions:** To preferences exhibiting risk aversion and more complex aggregate processes

Exogenous payoffs to steady states

1. Finding a steady state amounts to finding a complete markets optimal allocation whose optimal payoff structure matches p_t
2. We order economies with different asset payoff structures as follows
   - Suppose $s = 2$ is the adverse state (high expenditure or low TFP)
   - Index incomplete market economies by $p_1 = p(s = 1)$. [2]
   - **Anmol and David XXXXXX: Here should say what p_2 is too.**
3. **3 Regions:**
   - Low p_1: Government issues debt in steady state
   - High p_1: Government holds assets in steady state
   - Intermediate p_1: steady states do not exist

   Let α_1, α_2 be the two thresholds that split the "p_1 space"

[2] A risk free bond corresponds to $p_1 = 1$

Thresholds: $\alpha_1 < \alpha_2$

For either a pure TFP shock or a pure government expenditure shock, we compute

- With nly government expenditure shocks

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$

- With only TFP shocks

$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

Convergence

- Our analysis verifies the existence of a steady state in a 2-state i.i.d. economy.
- In order to study long run properties of the optimal allocation in incomplete markets we need to determine whether these steady states are stable
- **Risk adjusted martingale:**
  Under an optimal policy, the Lagrange multiplier on the implementability μ_t constraint satisfies

  $$\mu_t = \mathbb{E}_t p_{t+1} \mu_{t+1}$$

  or

  $$\mathbb{E}_t \mu_{t+1} = \mu_t - \mathbb{C}\rtimes\gtrapprox_t p_{t+1} \mu_{t+1}$$

  μ_t *follows a risk adjusted martingale.*
- **Stability:** Is there a sufficiently big drift μ_t away from a steady state?

Result: Characterizing Convergence

- Reminder: p_1 is the payoff in the "good" state.
- As with existence, we can partition the "p_1 space" into different regions
- Let b_{fb}, μ_{fb} be the debt level and associated Lagrange multiplier where the government can implement first best via access to complete markets

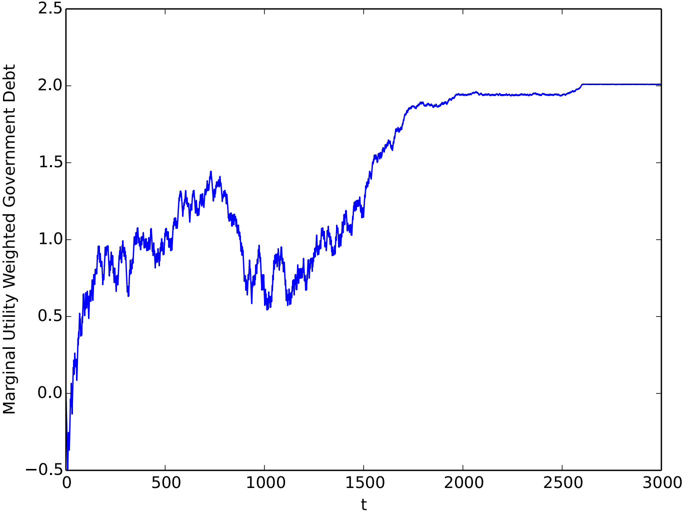
Theorem

Let b^* denote the steady state level of debt. Then $\exists \alpha_1 < \alpha_2$ such that

1. **Low** p_1*: If $p_1 \leq \min(\alpha_1, 1)$ then the steady state is stable with $b_{fb} < b^* < 0$ and $b_t \to b^*$ with probability 1.*
2. **High** p_1*: If $p_1 \geq \alpha_2$ then the steady state is stable with $0 < b^*$ and $b_t \to b^*$ with probability 1.*
3. **Intermediate** p_1**:**
    - *If $1 < p_1 < \alpha_1$ then a steady state exists with $b^* < b_{fb}$ but it is unstable with $\mu_t > \mathbb{E}_t \mu_{t+1}$ for $\mu < \mu_{fb}$.*
    - *If $\alpha_1 \leq p_1 < \alpha_2$ then a steady state does not exist and*

Anmol and David XXXXX: on the next two slides, might you want to adjust the vertical label just to say government debt, since preferences are quasi linear?

$p_1 > 1$



Marginal Utility Weighted Government Debt
2.5
2.0
1.5
1.0
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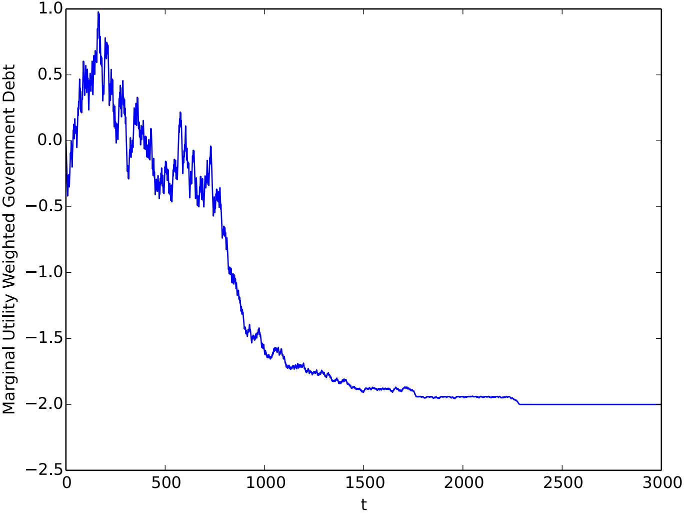
0
500
1000
1500
2000
2500
3000
t

$p_1 < 1$



Marginal Utility Weighted Government Debt
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1.0
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0
500
1000
1500
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3000

Incomplete markets with risk aversion

Quasilinear preferences:

1. With quasilinear preferences, we showed that $b_t \to b^*$ when the aggregate state followed a 2 -state i.i.d. process
2. The level of and sign b^* is a function of the **exogenous payoff structure**: $p(s)$

Risk aversion:

- Marginal utility adjusted assets: $x_t = u_{c,t} b_t$ encodes history dependence
- Long run properties of x_t depend on returns: $R_{t,t+1} = \frac{p(s_{t+1})}{q_t}$
- To focus on the **endogenous component** of payoffs, set: $p(s) = 1$

Roadmap, II

- Show that $x_t = U_{c,t} b_t$ is sole state variable for optimal policies
- Split the Ramsey problem into two Bellman equations
  1. $V(b_{-1}, s_0)$: time $t = 0$ Bellman equation
  2. $V(x, s_{\_})$: time $t \geq 1$ Bellman equation
- Analyze steady states: x^* such that $x_t \to x^*$

Interpretation: x^* corresponds to an initial condition when the optimal portfolio in a LS economy is a risk free bond

A Recursive Formulation

1. Under commitment, government actions at $t \geq 1$ are constrained by anticipations about them at $s < t$
2. This contributes additional state variables like marginal utility of consumption in addition to debt
3. However scaling the budget equation by marginal utilities makes it recursive in product $x = U_c b$

$$\frac{x_{t-1} p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} = U_{c,t} c_t + U_{l,t} l_t + x_t$$

Bellman equation for $t \geq 1$

$$V(x, s_-) = \max_{c(s), l(s), x'(s)} \sum_s \pi(s, s_-)\Big(U(c(s), l(s)) + \beta V(x'(s), s)\Big)$$

subject to $x'(s) \in [\underline{x}, \overline{x}]$

$$\frac{xp(s)U_c(s)}{\beta \mathbb{E} pUc} = U_c(s)c(s) + U_l(s)l(s) + x'(s)$$

$$c(s) + g(s) = \theta(s)l(s)$$

Time 0 problem

Given an initial debt b_0, state s_0, and continuation value function $V(x, s\_)$

$$\max_{c,l,x'} U(c,l) + \beta V(x', s0)$$

subject to time zero implementability constraint

$$U_c(c,l)c + U_l(c,l)l + x' \geq U_c(c,l)b_0$$

and resource constraint

$$c + g(s_0) = \theta l$$

and

$$x' \in [\underline{x}, \overline{x}]$$

Revisiting steady states with risk aversion

Anmol and David XXXXX: I exterminated some ρs in this slide. Let $\Psi(s; x, s_{\_})$ be an optimal law of motion for the state variable for the $t \geq 1$ recursive problem, i.e.,

$$\Psi(s; x, s_{\_}) = \left(x'(s), (s)\right)$$

attains $t \geq 1$ value function given state $(x, s_{\_})$

Definition

A steady state (x^*) satisfies $(x^*) = \Psi(s; x^*, s_{-})$ for all $s, s_{\_}$

A steady state is a node at which a continuation allocation and tax rate has no further history dependence.

Existence

Let x_{fb} be a value of the state x from which the government can institute first best from that period onwards. Assume a CRRA utility specification $U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$. Finally, let c^{fb} and l^{fb} be consumption and leisure under first best.

Proposition

For a 2 state i.i.d. process, let s_g and s_b denoting the states with high and low consumption at first best. Suppose that

$$\frac{g(s_g)}{1 - \frac{\beta \mathbb{E}[(c^{fb})^{-\sigma}]}{c^{fb}(s_g)}} > \frac{g(s_b)}{1 - \frac{\beta \mathbb{E}[(c^{fb})^{-\sigma}]}{c^{fb}(s_b)}}$$

Then there exists a multiplier μ and complete markets allocations c^{μ}, l^{μ} such that

$$\underline{x} < \frac{U_{c_\mu}(s) c_\mu(s) + U_{l_\mu}(s) l_\mu(s)}{\frac{U_{c_\mu}(s)}{\beta \mathbb{E}[U_{c_\mu}]} - 1} = x^* < 0$$

Stability

1. The steady state is independent of assets
2. The incomplete markets Ramsey allocation converges to a Lucas Stokey allocation for all initial conditions

Anmol and David XXXXXX: something is off or missing in the following statement, especially following 'with ...'.

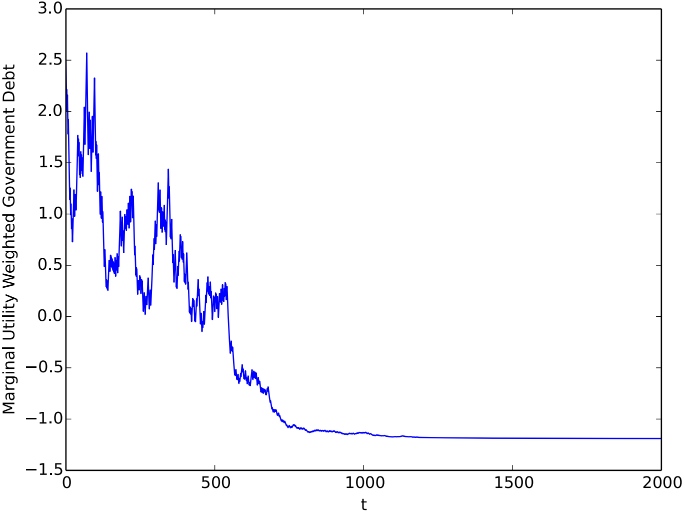
Proposition

Let $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ *solve the incomplete markets Ramsey problem with. Then* $x_t(s^{t-1}) \to x^*$ *as* $t \to \infty$ *with probability 1 for all initial conditions*

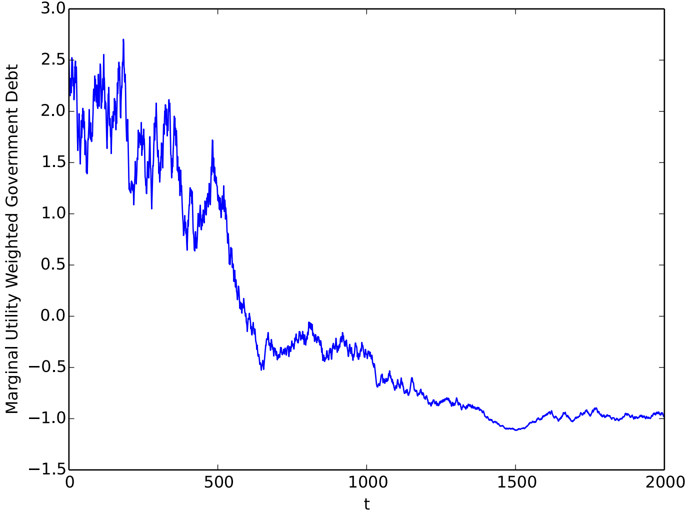
Quasilinear vs. risk aversion

- With quasilinear preferences, with a risk-free bond, the interest rate is always $1/\beta$.
- With risk averse preferences, interest rates are higher in period of high government expenditure.
- Thus, while the government has greater expenses in high government expenditure states, the higher interest rate means the govt. has to accumulate less to cover future government expenditures.
- After the government has accumulated enough assets, it is actually better off in periods of high government expenditure than in periods with low government expenditures (since its claims to consumption are worth more).
- By holding assets, the government is able to reallocate resources across states, something it is not able to do in the quasilinear case.

2 State i.i.d. process with Risk Aversion



$S > 2$ states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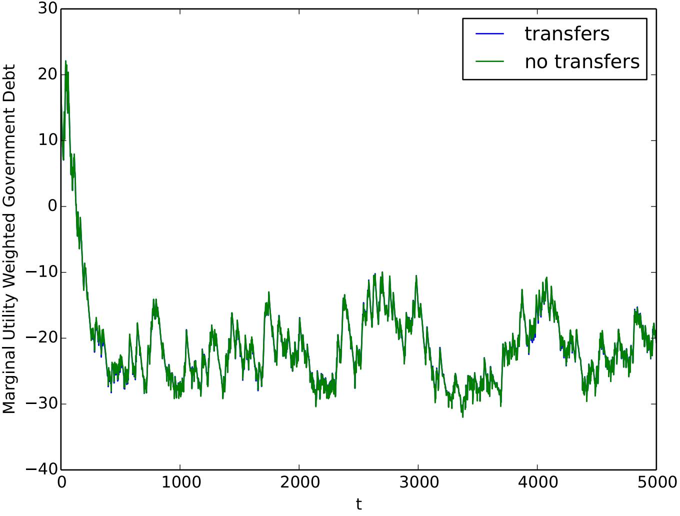
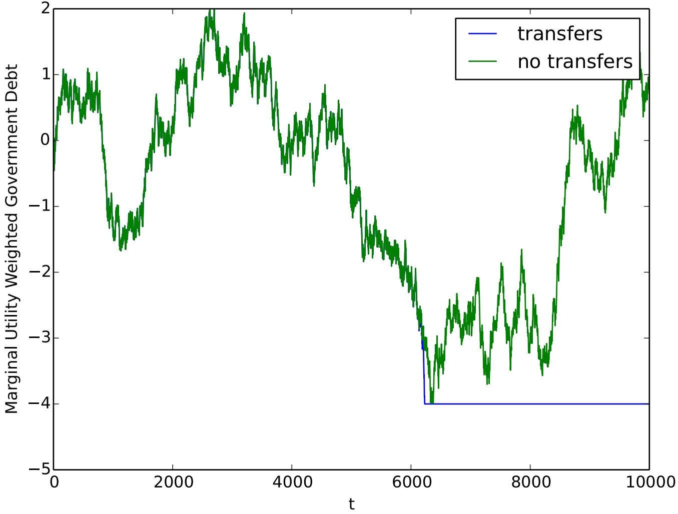
Transfers

- That the government can use its assets to smooth tax rate distortions carries over even when the government has access to lump sum transfers
- Access to nonnegative transfers makes first best level of assets trivially a "steady state". .
- Even with lump sum transfers, in cases where the steady state exists and is stable, if the initial debt of the government is above its steady state, the economy converges with probability 1 to the steady state.

AMSS calibration with and without transfers



Quasilinear Preferences and Risk Free Bond with and without Transfers



Conclusion

- With market incompleteness, the payoff structure of the government asset has big implications for the long run debt position of the government
- If the asset payoff structure offers greater returns in good states of the world then bad, then the Ramsey government accumulates debt in the long run.
- **Anmol and David XXXXX: I don't understand this one!** With risk aversion, the payoff structure of the time-consistent savings variable is what determines the long run debt position of the government.
- Access to nonnegative transfers play little role in determining the evolution of the system. Rather the key force is the government's ability to use its debt position to reallocate resources across states.
- **Future Research**: How does the type of market incompleteness effect long run wealth distributions with heterogeneous agents