# Taxes, Debts, and Redistribution

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# What do we do ?

We study optimal taxation under commitment with

- **Heterogeneous agents**
  $\rightarrow$ Different productivities
- **Incomplete markets**
  $\rightarrow$ All trade a risk-free bond
- **Affine tax schedules**
  $\rightarrow$ Government levies a proportional tax on labor earnings
  $+$ lump sum (tax or transfer)
- **Aggregate shocks**
  $\rightarrow$ To productivities, government expenditure etc.

# What are we after?

1. How costly are government debts?
2. What are the long run properties of optimal government policies and equilibrium allocations?
3. How should government policy respond to aggregate shocks?

# Environment

- **Uncertainty**: Markov aggregate shocks $s_t$
- **Demography**: $I$ types of infinitely lived agents (of mass $\pi_i$) plus a benevolent planner
- **Technology**: Output is linear in labor supply. Agents differ in their productivities $\{\theta_i(s_t)\}_{i,t}$
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \bar{\beta}_t U^i \left( c_i(s^t), l_i(s^t) \right)$$

  where $\bar{\beta}_t = \left[ \Pi_{j=0}^{t-1} \beta(s_j) \right]$
- **Preferences** (Planner): Given Pareto weights $\{\alpha_i\}$

$$\mathbb{E}_0 \sum_{i=1}^{I} \pi_i \alpha_i \sum_{t=0}^{\infty} \bar{\beta}_t U_t^i \left( c_{i,t}, l_{i,t} \right)$$

- **Asset markets**: A risk-free bond only

# Environment, II

- **Affine Taxes**: Agent $i$'s tax bill

$$-T_t + \tau_t \theta_{i,t} l_{i,t}$$

- **Budget constraints**
  - Agent $i$: $c_{i,t} + b_{i,t} = (1 - \tau_t)\, \theta_{i,t} l_{i,t} + R_{t-1} b_{i,t-1} + T_t$
  - Government: $g_t + B_t + T_t = \tau_t \sum_{i=1}^{I} \pi_i \theta_{i,t} l_{i,t} + R_{t-1} B_{t-1}$

- **Market Clearing**
  - Goods: $\sum_{i=1}^{I} \pi_i c_i(s^t) + g\,(s_t) = \sum_{i=1}^{I} \pi_i \theta_i\,(s_t)\, l_i(s^t)$
  - Assets: $\sum_{i=1}^{I} \pi_i b_{i,t} + B_t = 0$

- **Initial conditions**: Distribution of assets $\{b_{i,-1}\}_i$ and $B_{-1}$

# Ramsey Problem

### Definition

**Allocation, price system, government policy**: Standard

### Definition

**Competitive equilibrium**: Given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$ all allocations are chosen optimally, markets clear

### Definition

**Optimal competitive equilibrium**: A welfare-maximizing competitive equilibrium for a given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$

# Contrast with AMSS

Representative agent with linear taxes

- Higher levels of debt are distortionary
- With incomplete markets, the optimal government policy is to accumulate assets

# Redistribution and optimal transfers

- Representative agent models impose restrictions on transfers
  - These are motivated only implicitly by concerns about redistribution: Poor people can't afford lump sum taxes
  - These constraints *almost* always bind (e.g., Lucas Stokey, AMSS) and drive long run debt dynamics
- We begin with explicit redistribution motives but leave transfers to be determined optimally

*Prescriptions for optimal tax-transfers differ substantially with explicitly modeled redistribution concerns*

# Main working parts

- **Welfare Criterion**: Benevolent planner with explicit redistribution motives
- **Instruments**: Transfers and a tax on labor income
- **Restrictions**:
  1. The tax on labor income is linear in wage earnings
  2. Transfers are unrestricted in sign and magnitude but cannot be conditioned on identities
  3. There are endogenous welfare costs to varying tax rates and transfers over times and across states
- **Trade-offs**:
  1. Dead weight losses are associated with varying labor taxes
  2. With explicit redistribution motives come costs of fluctuating transfers. Withdrawing a unit of consumption good affects rich and poor people differently

# Key forces

Optimal policy balance these trade-offs

- Because transfers are unrestricted, the level of government debt is not distortionary.
- What matters is govt. debt is distributed across agents
- Higher correlations of wages and assets are more distortionary
- Since welfare costs depend on the distribution of assets, optimal policy is affected by and affects the distribution of net assets
  - **Absence of agent specific transfers**: This prompts the govt. to engineer a negative correlations between net assets and labor earnings
  - **Absence of state contingent securities**: This prompts the govt. to exploit endogenous fluctuations in the interest rate

# Ricardian Equivalence

- *Result*: A **large set** of transfers and asset profiles support the same competitive allocation
- *Logic* : Taking away a unit of all agents' assets and increasing transfers by a unit leaves budget sets unchanged

## Theorem

*Given* $\left(\{b_{i,-1}\}_i, B_{-1}\right)$, *let* $\left\{\{c_{i,t}, l_{i,t}, b_{i,t}\}_i, B_t, R_t\right\}_t$ *and* $\{\tau_t, T_t\}_t$ *be a competitive equilibrium. For any bounded sequences* $\left\{\hat{b}_{i,t}\right\}_{i,t\geq -1}$ *that satisfy*

$$\hat{b}_{i,t} - \hat{b}_{1,t} = \tilde{b}_{i,t} \equiv b_{i,t} - b_{1,t} \text{ for all } t \geq -1, i \geq 2,$$

*there exist sequences* $\left\{\hat{T}_t\right\}_t$ *and* $\left\{\hat{B}_t\right\}_{t\geq -1}$ *satisfying the market clearing such that* $\left\{\left\{c_{i,t}, l_{i,t}, \hat{b}_{i,t}\right\}_i, \hat{B}_t, R_t\right\}_t$ *and* $\left\{\tau_t, \hat{T}_t\right\}_t$ *constitute a competitive equilibrium given* $\left(\left\{\hat{b}_{i,-1}\right\}_i, \hat{B}_{-1}\right)$.

# Ricardian Equivalence: Implications

- No precautionary motive: WLOG we can normalize government assets $B_t$ to zero
- Exogenous borrowing constraints are not restrictive

**Theorem**

*For every competitive equilibrium in an economy without exogenous borrowing constraints there is a government tax policy such the same allocation and interest rate sequence is part of a competitive equilibrium in an economy with exogenous borrowing constraints of the form* $b_{i,t} > \underline{b}_i$

*Thus, Ricardian equivalence holds with distortionary taxes and ad hoc borrowing limits*

# Optimal allocations: Primal Approach

Focus on interior equilibria. Take first-order necessary conditions for the consumer's problem are

1. Eliminate tax rate $\tau_t$:

$$(1-\tau_t)\,\theta_{i,t} U^i_{c,t} = -U^i_{l,t},$$

2. Eliminate risk free interest rate $R_t$:

$$U^i_{c,t} = \beta_t R_t \mathbb{E}_t U^i_{c,t+1}.$$

3. Eliminate transfers $T_t$:

$$(c_{i,t} - c_{1,t}) + \tilde{b}_{i,t} = -\frac{U^i_{l,t}}{U^i_{c,t}} l_{i,t} + \frac{U^1_{l,t}}{U^1_{c,t}} l_{1,t} + \frac{U^i_{c,t-1}}{\beta_{t-1}\mathbb{E}_{t-1} U^i_{c,t}} \tilde{b}_{i,t-1}\ \forall i \geq 2, t.$$

This yields "implementability constraints"

**Notation**: $\tilde{b}_{i,t} = b_{i,t} - b_{1,t}$, called the "net assets" of agent $i$

# Optimal Allocations: Sequential Formulation

Denote $Z_t^i = U_{c,t}^i c_{i,t} + U_{l,t}^i l_{i,t} - \frac{U_{c,t}^i}{U_{c,t}^1}\left[U_{c,t}^1 c_{1,t} + U_{l,t}^1 l_{1,t}\right]$. The optimal policy solves,

$$\max_{c_{i,t}, l_{i,t}, \tilde{b}_{i,t}} \mathbb{E}_0 \sum_{i=1}^{I} \pi_i \alpha_i \sum_{t=0}^{\infty} \bar{\beta}_t U_t^i \left(c_{i,t}, l_{i,t}\right),$$

subject to

$$\tilde{b}_{t-1} \frac{U_{c,t-1}^i}{\beta_{t-1}} = \left(\frac{\mathbb{E}_{t-1} U_{c,t}^i}{U_{c,t}^i}\right) \mathbb{E}_t \sum_{k=t}^{\infty} \left[\prod_{j=t}^{k-1} \beta_j\right] Z_k^i \quad \forall t \geq 1$$

$$\tilde{b}_{-1} = \mathbb{E}_{-1} \sum_{k=0}^{\infty} \left[\prod_{j=0}^{k-1} \beta_j\right] Z_k^i$$

$$\frac{\mathbb{E}_{t-1} U_{c,t}^i}{U_{c,t-1}^i} = \frac{\mathbb{E}_{t-1} U_{c,t}^j}{U_{c,t-1}^j}$$

$$\sum_{i=1}^{I} \pi_i c_i(s^t) + g\left(s_t\right) = \sum_{i=1}^{I} \pi_i \theta_i \left(s_t\right) l_i(s^t),$$

$$\frac{U_{l,t}^i}{\theta_{i,t} U_{c,t}^i} = \frac{U_{l,t}^1}{\theta_{1,t} U_{c,t}^1}$$

$$\tilde{b}_{t-1} \frac{U_{c,t-1}^i}{\beta_{t-1}} \text{ is bounded}$$

# Ramsey Problem: Recursive Formulation

Split into two parts

1. $\mathbf{t \geq 1}$:Ex-ante continuation problem with state variables $(x, \boldsymbol{\rho}, s_{-})$

$$x = \beta^{-1}\left(U^2_{c,t-1}\tilde{b}_{2,t-1}, ..., U^I_{c,t-1}\tilde{b}_{I,t-1}\right)$$

$$\boldsymbol{\rho} = \left(U^2_{c,t-1}/U^1_{c,t-1}, ..., U^I_{c,t-1}/U^1_{c,t-1}\right)$$

2. $\mathbf{t = 0}$: Ex-post initial problem with state variables $(\tilde{b}_{\mathbf{-1}}, s_0)$

# Ramsey Problem at $t \geq 1$

$$V(x, \boldsymbol{\rho}, s_-) = \max_{c_i(s), l_i(s), x'(s), \rho'(s)} \sum_s \Pr(s|s_-) \left( \left[ \sum_i \pi_i \alpha_i U^i(s) \right] + \beta(s) V(x'(s), \boldsymbol{\rho}'(s), s) \right)$$

where the maximization is subject to

$$U^i_c(s) \left[ c_i(s) - c_1(s) \right] + U^i_c(s) \left( \frac{U^i_l(s)}{U^i_c(s)} l_i(s) - \frac{U^1_l(s)}{U^1_c(s)} l_1(s) \right) + \beta(s) x'_i(s) = \frac{x U^i_c(s)}{\mathbb{E}_{s_-} U^i_c} \text{ for all } s, i \geq 2$$

$$\frac{\mathbb{E}_{s_-} U^i_c}{\mathbb{E}_{s_-} U^1_c} = \rho_i \text{ for all } i \geq 2$$

$$\frac{U^i_l(s)}{\theta_i(s) U^i_c(s)} = \frac{U^1_l(s)}{\theta_1(s) U^1_c(s)} \text{ for all } s, i \geq 2$$

$$\sum_i \pi_i c_i(s) + g(s) = \sum_i \pi_i \theta_i(s) l_i(s) \quad \forall s$$

$$\rho'_i(s) = \frac{U^i_c(s)}{U^1_c(s)} \text{ for all } s, i \geq 2$$

$$\underline{x}_i(s; x, \boldsymbol{\rho}, s_-) \leq x_i(s) \leq \bar{x}_i(s; x, \boldsymbol{\rho}, s_-)$$

# Ramsey Problem at $t = 0$

$$V_0\left(\{\tilde{b}_{i,-1}\}_{i=2}^{I}, s_0\right) = \max_{c_{i,0}, l_{i,0}, x_0, \rho_0} \sum_i \pi_i \alpha_i U^i(c_{i,0}, l_{i,0}) + \beta(s_0) V\left(x_0, \rho_0, s_0\right)$$

where the maximization is subject to

$$U^i_{c,0}\left[c_{i,0} - c_{1,0}\right] + U^i_{c,0}\left(\frac{U^i_{l,0}}{U^i_{c,0}} l_{i,0} - \frac{U^1_{l,0}}{U^1_{c,0}} l_{1,0}\right) + \beta(s_0) x_{i,0} = U^i_{c,0} \tilde{b}_{i,-1} \text{ for all } i \geq 2$$

$$\frac{U^i_{l,0}}{\theta_{i,0} U^i_{c,0}} = \frac{U^1_{l,0}}{\theta_{1,0} U^{1,0}_c} \text{ for all } i \geq 2$$

$$\sum_i \pi_i c_{i,0} + g_0 = \sum_i \pi_i \theta_{i,0} l_{i,0}$$

$$\rho_{i,0} = \frac{U^i_{c,0}}{U^1_{c,0}} \text{ for all } i \geq 2$$

# Steady States

Let $\Psi(s; x, \boldsymbol{\rho}, s_{-})$ be an optimal law of motion for the state variables for the $t \geq 1$ recursive problem, i.e.,

$$\Psi(s; x, \boldsymbol{\rho}, s_{-}) = \left(x'(s), \rho'(s)\right)$$

attains $t \geq 1$ value function given state $(x, \boldsymbol{\rho}, s_{-})$

## Definition

A steady state $\left(x^{SS}, \boldsymbol{\rho}^{SS}\right)$ satisfies
$\left(x^{SS}, \boldsymbol{\rho}^{SS}\right) = \Psi\left(s; x^{SS}, \boldsymbol{\rho}^{SS}, s_{-}\right)$ for all $s, s_{-}$

*A steady state is a node at which the continuation allocation and tax schedule has no further history dependence.*

# Existence

- Quasi Linear: An SS exists for a wide range of parameters and shocks. Further, the economy reaches a steady state in one period. Output and taxes are constant thereafter.
- For general preferences ,a degenerate SS exists if shocks are IID and take two values. The economy converges to this SS starting from all initial conditions.
- Outside the binary - IID case, there exists an ergodic region in which $(x, \rho)$ is no longer constant, but fluctuations tend to be markedly reduced relative to the transient fluctuations that occur during an approach to a SS

# Intuition

- Consider 2 agents with $\theta_1(s) > \theta_2 = 0$.
- The state variable $x$ is marginal utility scaled relative assets of an unproductive agent: $U_c^2(s)[b_2(s) - b_1(s)]$.
- One can normalize $b_2(s) = 0$ so that $x$ can be interpreted as scaled assets held by the government

Two main forces determine the dynamics of the tax rate, transfers, and assets:

- **Fluctuations in inequality** as measured by spreads in marginal utilities
- **Fluctuations in the interest rates**

For quasi linear preferences both forces are absent

# Inequality distortions

Anmol XXXXX: may we please discuss this slide? It needs some clarification. Start with a spread in discount factors set to equalize interest rates across states, i.e., $R(s_l) = R(s_h)$. Then SS $x > 0$

TFP ($\theta_1$): Adjust tax rate $\tau$ or transfers $T$

Suppose $x = 0$ or $b_2(s) = b_1(s)$,

Then reductions in transfers hurt the low productivity agent

more

*A fall in transfers that increases inequality gives rise to a cost not present in representative agent economies. This gives the planner an incentive to reduce the costs of inequality distortions by ...*

$x$

Reducing the relative asset holdings of the productive agent eventually drives the after-tax, after-interest incomes of both agents closer together

# Interest rates fluctuations

1. Suppose discount factors are constant. Anmol XXXX: *What simple example?* In our simple example, interest rates are countercyclical and we still have $x > 0$. Consider again
   - $\Downarrow$ TFP

     If the tax rate $\tau$ is left unchanged, the government faces a shortfall of revenues. The optimal policy recommends

     $\Uparrow$ $x$. This achieves that same thing as having the government accumulate assets[1]
   - The govt. can use higher interest income to offset some revenue losses from its tax on labor
   - This force is present in representative agent economies with endogenous fluctuations in interest rates
2. If discount factor spreads are large enough to generate pro-cyclical interest rates, it can emerge that SS $x < 0$

[1] Normalize $b_2(s) = 0$

# Comparative statics with Pareto weights

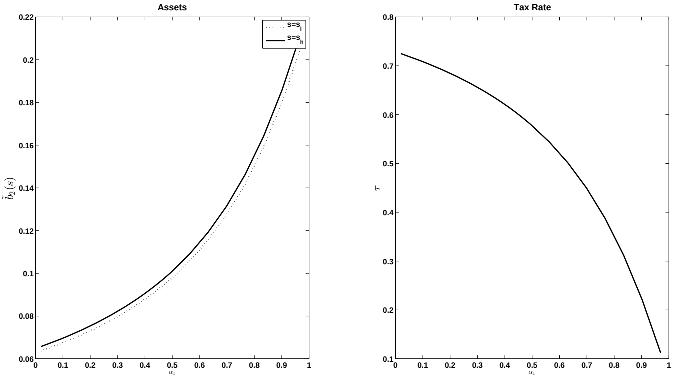


Figure: Steady state govt. assets: $\tilde{b}_2(s) = \frac{\beta x^{SS}}{U_c^2(s)}$ and taxes: $\tau^{SS}$ as a function of a (high skilled) agent 1's Pareto weight

# Numerical Example

Use a calibrated version of the economy to

- Approximate magnitudes of these forces and
- Study optimal policy responses at business cycle frequencies when an economy is possibly far away from a steady state

# Numerical Example: Calibration

Take a 2-shock 2-type economy with preferences $U(c, l) = \psi \log(c) + (1 - \psi) \log(1 - l)$ and allow $\theta_i(s), \beta(s), g(s)$ to depend on $s$.

- Pick baseline parameters to match some low frequency moments
- Calibrate outcome fluctuations to match recent US recessions (i.e., 1991-92, 2001-02 and 2008-10):
  1. The left tail of the cross-section distribution of labor income falls by more than right tail
  2. Short term interest rates fall
  3. Recessions last longer than booms

# Calibration

| Parameter | Value | Description | Target |
|---|---|---|---|
| $\psi$ | 0.6994 | Frisch elasticity of labor supply | 0.5 |
| $\bar{\theta}_1$ | 4 | Log 90-10 wage ratio (Autor et al.) | 4 |
| $\bar{\theta}_2$ | 1 | Normalize to 1 | 1 |
| $\beta$ | 0.98 | Average (annual) risk free interest rate | 2% |
| $\alpha_1$ | 0.69 | Marginal tax rate in the economy with no shocks | 20% |
| $g$ | 12% | Average pre-transfer expenditure- output ratio | 12 % |
| $\frac{\hat{\theta}_2}{\hat{\theta}_1}$ | 2.5 | Relative drop in wage income of 10th percentile | 2.5 |
| $\hat{\theta}_1$ | 1.2% | Average output loss | 3% |
| $\hat{\beta}(s)$ | 1.96% | Difference in real interest rates between booms and recession | 1.96% |
| $P(r\|r)$ | 0.63 | Duration of recessions | 2.33 years |
| $P(b\|b)$ | 0.84 | Duration of booms | 7 years |

Table: Benchmark calibration

Initial conditions chosen to make debt to GDP ratio be 60%

# Results: Some variants

We study the following perturbations of the Benchmark calibration

1. Acyclical interest rates: Smaller spread in discount factor shocks
2. Countercyclical interest rates: No discount factor shocks
3. No inequality: Equal fall in all agents' productivities (TFP shock) and no discount factor shocks
4. Government expenditure shocks: A fall in $g$ that produces a comparable fall in output

# Results: Long run

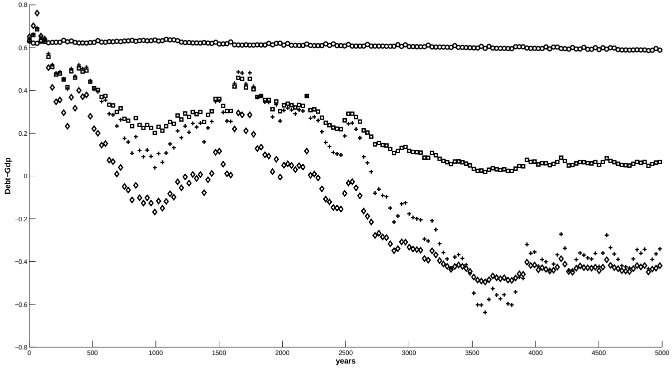


Figure: Govt. debt for several economies: benchmark (o), acyclical interest rates (+), countercyclical interest rates ($\diamond$) and no inequality shocks ($\square$)

# Observations

- Long run tendency to converge to some ergodic set. But convergence is very slow more details on speed of convergence.
- With low discount factor shocks, outcomes approach positive govt. assets
- With high discount factor shocks that produce procyclical real interest rates, there is no tendency to reduce govt. debt even after 5000 years

# Short Run

To understand the short run responses,

- We set the exogenous state $s_0$ so that we are at the outset of a recession
- Solve the time 0 problem with identical initial conditions across different settings. This pins down the initial state vector $x_0, \rho_0$ that appears in our time 0 Bellman equation
- We then use the policy rules to compute fluctuations of different components in the government budget constraint across states

For each variable $z$ in the table we report in the form $\Delta z \equiv \left(z\left(s_l | x_0, \rho_0, s_0\right) - z\left(s_h | x_0, \rho_0, s_0\right)\right) / \bar{Y}$ where $\bar{Y}$ is average undistorted GDP in percentages

# Results: Short run

| | $\Delta g$ | $\Delta B$ | $\Delta T$ | $\Delta[\tau\theta_1 l_1]$ | $\Delta[\tau\theta_2 l_2]$ | $\Delta Y$ | $\Delta\tau$ |
|---|---|---|---|---|---|---|---|
| **Benchmark** | 0.0000 | -1.1561 | 0.6871 | -0.1593 | -0.3096 | -2.8536 | 0.3732 |
| **Acyclical Interest Rates** | 0.0000 | -1.1126 | 0.6591 | -0.1497 | -0.3038 | -2.8613 | 0.3879 |
| **Countercyclical Interest Rates** | 0.0000 | -1.0794 | 0.6387 | -0.1415 | -0.2992 | -2.8677 | 0.3997 |
| **No Inequality** | 0.0000 | -0.1380 | **-0.5459** | -0.5635 | -0.1204 | -2.6294 | 0.0622 |
| **Expenditure Shocks** | -7.5037 | 2.9137 | 2.8612 | -1.3759 | -0.3530 | -2.3443 | -1.1598 |

Table: The tables summarizes the changes in the different components of the government budget as the economy transits from "boom" to "recession". All numbers except $\tau$ are normalized by un-distorted GDP and reported in percentages.

Note that predetermined variables like repayment on existing debt drop out of the accounting and we have

$$\Delta[g] + \Delta[T] + \Delta[B] = \Delta[\tau\theta_1 l_1] + \Delta[\tau\theta_2 l_2]$$

# Conclusions

- Size of government debt alone is irrelevant $\Longrightarrow$ need to know the distribution of net assets
- Optimal tax and transfer scheme balance
  1. welfare losses from fluctuating taxes
  2. welfare losses from fluctuating transfers
- Since welfare costs depend on the how debt is distributed, the planner has incentives to move net assets over time
- With incomplete markets, interest rate fluctuations are a key determinant of long-run correlations between productivities and net assets
- Ignoring heterogeneity produces misleading results about the size and direction of short run optimal policy responses

# Speed of convergence (I)

Suppose we are in the binary-IID world where steady states are deterministic.

- The optimal policy induces two risk adjusted martingales $\{\boldsymbol{\mu}_t, \boldsymbol{\rho}_t\}$.
- One can represent the optimal allocation recursively in terms of $\{\boldsymbol{\mu}(s^{t-1}), \boldsymbol{\rho}(s^{t-1})\}$ and $s_t$.
- Why $(\boldsymbol{\mu}, \boldsymbol{\rho})$ instead of $(x, \boldsymbol{\rho})$?
- Linearize optimal policies for each $s_t$ around the degenerate steady state.
- Study the eigenvalues of the conditional mean and variance dynamics (these are deterministic linear systems)

# Speed of convergence (II)

Let $\hat{\Psi}_t = \begin{bmatrix} \boldsymbol{\mu}_t - \boldsymbol{\mu}^{SS} \\ \boldsymbol{\rho}_t - \boldsymbol{\rho}^{SS} \end{bmatrix}$. Then

$$\hat{\Psi}_{t+1} = B(s_{t+1})\hat{\Psi}_t$$

This linearized system has coefficients that are functions of the shock $s$.

## Proposition

*If the (real part) of eigenvalues of $\mathbb{E}B(s)$ are less than 1, the system converges to zero in mean. Further for large $t$, the conditional variance of $\hat{\Psi}$, denoted by $\Sigma_{\Psi,t}$, follows a deterministic process governed by*

$$vec(\Sigma_{\Psi,t}) = \hat{B}vec(\Sigma_{\Psi,t-1}),$$

*where $\hat{B}$ is a square matrix of dimension $(2N-2)^2$. In addition, if the (real parts) of eigenvalues of $\hat{B}$ are all less than 1, the system converges in probability.*

*The eigenvalues (in particular the largest one) are instructive not only for whether the system is locally stable but also for how quickly the steady state is reached*

# Speed of convergence: Size of shocks and risk aversion

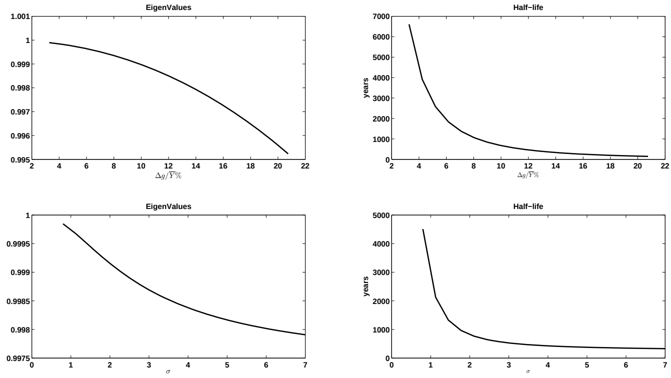


Figure: The top (bottom) panel plots the dominant eigenvalue of $\hat{B}$ and the associated half life as we increase the spread between the expenditure levels (risk aversion).Anmol XXXXX: it would be great if font sizes on axes and titles could be increased.

Back to main.