Taxes, Debts, and Redistributions with Aggregate Shocks

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What do we do?

We study optimal taxation under commitment with

- **Heterogeneous agents**
  $\rightarrow$ Different productivities
- **Incomplete markets**
  $\rightarrow$ All trade a risk-free bond
- **Affine tax schedules**
  $\rightarrow$ Government levies a proportional tax on labor earnings
  $+$ lump sum (tax or transfer)
- **Aggregate shocks**
  $\rightarrow$ To productivities, government expenditure etc.

What are we after?

1. How costly are government debts?
2. What are the long run properties of optimal government policies and equilibrium allocations?
3. How should government policies respond to aggregate shocks?

Environment

- **Uncertainty**: Markov aggregate shocks s_t
- **Demography**: I types of infinitely lived agents (of mass π_i) plus a benevolent planner
- **Technology**: Output $\sum_{i=1}^{I} \theta_i l_{i,t}$ is linear in labor supplies. Productivities $\{\theta_i(s_t)\}_{i,t}$ differ across i.
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \bar{\beta}_t U^i \left(c_i(s^t), l_i(s^t) \right)$$

  where $\bar{\beta}_t = \beta(s_{t-1})\bar{\beta}_{t-1}$, $\beta(s) \in (0,1)$ and $\beta(s_0) = 1$
- **Preferences** (Planner): Given Pareto weights $\{\alpha_i\}$

$$\mathbb{E}_0 \sum_{i=1}^{I} \pi_i \alpha_i \sum_{t=0}^{\infty} \bar{\beta}_t U_t^i \left(c_{i,t}, l_{i,t} \right)$$

- **Asset markets**: A risk-free bond only

Environment, II

- **Affine Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_{i,t} l_{i,t}$$

- **Budget constraints**
  - Agent i: $c_{i,t} + b_{i,t} = (1 - \tau_t)\, \theta_{i,t} l_{i,t} + R_{t-1} b_{i,t-1} + T_t$
  - Government: $g_t + B_t + T_t = \tau_t \sum_{i=1}^{I} \pi_i \theta_{i,t} l_{i,t} + R_{t-1} B_{t-1}$

- **Market Clearing**
  - Goods: $\sum_{i=1}^{I} \pi_i c_i(s^t) + g\,(s_t) = \sum_{i=1}^{I} \pi_i \theta_i\,(s_t)\, l_i(s^t)$
  - Assets: $\sum_{i=1}^{I} \pi_i b_{i,t} + B_t = 0$

- **Initial conditions**: Distribution of assets $\{b_{i,-1}\}_i$ and B_{-1}

Ramsey Problem

Definition

Allocation, price system, government policy: Standard

Definition

Competitive equilibrium: Given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$ all allocations are chosen optimally, markets clear [1]

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$

[1] Usually, we impose only "natural" debt limits.

Contrast with representative agent models

Representative agent with linear taxes

- Higher levels of debt are distortionary
- With incomplete markets (as in AMSS), the optimal government policy is to accumulate assets

Does AMSS Confirm Reinhart-Rogoff Fears?

- Higher govt. debts are more distorting, but ...
- Because it is distortionary, asymptotically govt. doesn't issue debt

Things change in our environment

Redistribution and optimal transfers

- Representative agent models impose restrictions on transfers
  - These are motivated only implicitly by concerns about redistribution: poor people can't afford lump sum taxes
  - These constraints *almost* always bind (e.g., Lucas Stokey, AMSS) and drive long run debt dynamics
- We begin with explicit redistribution motives and let the government set transfers optimally

Prescriptions for optimal tax-transfers differ substantially with explicitly modeled redistribution concerns

Main working parts

- **Welfare Criterion**: Benevolent planner with explicit redistribution motives
- **Instruments**: Transfers and a flat tax on labor income
- **Restrictions**:
  1. The tax on labor income is linear in wage earnings
  2. Transfers are unrestricted in sign and magnitude but not conditioned on agents' identities
  3. Incomplete markets
- **Trade-offs**:
  1. Varying labor taxes imposes dead weight losses
  2. With explicit redistribution motives come costs of fluctuating transfers. Withdrawing a unit of consumption affects rich and poor people differently

Key forces

Optimal policies balance these trade-offs

- Because transfers are unrestricted, the level of government debt is not distortionary.
- What matters is how govt. debt is distributed across agents
- Higher correlations of wages and assets are more distortionary
- Since welfare costs depend on the distribution of assets, optimal policy is affected by and affects the distribution of net assets
  - **Absence of agent specific transfers**: This prompts the govt. to engineer a negative correlation between net assets and labor earnings
  - **Absence of state contingent securities**: This prompts the govt. to exploit endogenous fluctuations in the interest rate

Ricardian Equivalence

- *Result*: A **large set** of transfers and asset profiles support the same competitive allocation
- *Logic*: Taking away a unit of all agents' assets and increasing transfers by a unit leaves budget sets unchanged

Theorem

Given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$, *let* $\left\{\{c_{i,t}, l_{i,t}, b_{i,t}\}_i, B_t, R_t\right\}_t$ *and* $\{\tau_t, T_t\}_t$ *be a competitive equilibrium. For any bounded sequences* $\left\{\hat{b}_{i,t}\right\}_{i,t\geq -1}$ *that satisfy*

$$\hat{b}_{i,t} - \hat{b}_{1,t} = \tilde{b}_{i,t} \equiv b_{i,t} - b_{1,t} \text{ for all } t \geq -1, i \geq 2,$$

there exist sequences $\left\{\hat{T}_t\right\}_t$ *and* $\left\{\hat{B}_t\right\}_{t\geq -1}$ *satisfying market clearing such that* $\left\{\left\{c_{i,t}, l_{i,t}, \hat{b}_{i,t}\right\}_i, \hat{B}_t, R_t\right\}_t$ *and* $\left\{\tau_t, \hat{T}_t\right\}_t$ *constitute a competitive equilibrium given* $\left(\left\{\hat{b}_{i,-1}\right\}_i, \hat{B}_{-1}\right)$.

Ricardian Equivalence: Implications

- No precautionary motive: WLOG we can normalize government assets B_t to zero
- Exogenous borrowing constraints are not restrictive

Theorem

For every competitive equilibrium in an economy without exogenous borrowing constraints there is a government tax policy such the same allocation and interest rate sequence is part of a competitive equilibrium in an economy with exogenous borrowing constraints of the form $b_{i,t} > \underline{b}_i$

Thus, Ricardian equivalence holds with distortionary taxes and ad hoc borrowing limits

Optimal allocations: Primal approach

Focus on interior equilibria. Take first-order necessary conditions for the consumer's problem are

1. Eliminate tax rate τ_t:

$$(1-\tau_t)\,\theta_{i,t} U^i_{c,t} = -U^i_{l,t},$$

2. Eliminate risk free interest rate R_t:

$$U^i_{c,t} = \beta_t R_t \mathbb{E}_t U^i_{c,t+1}.$$

3. Eliminate transfers T_t:

$$(c_{i,t} - c_{1,t}) + \tilde{b}_{i,t} = -\frac{U^i_{l,t}}{U^i_{c,t}} l_{i,t} + \frac{U^1_{l,t}}{U^1_{c,t}} l_{1,t} + \frac{U^i_{c,t-1}}{\beta_{t-1}\mathbb{E}_{t-1} U^i_{c,t}} \tilde{b}_{i,t-1} \; \forall i \geq 2, t.$$

   This yields "implementability constraints"

Notation: $\tilde{b}_{i,t} = b_{i,t} - b_{1,t}$, called the "net assets" of agent i

Optimal allocations: Sequential formulation

Denote $Z_t^i = U_{c,t}^i c_{i,t} + U_{l,t}^i l_{i,t} - \frac{U_{c,t}^i}{U_{c,t}^1}\left[U_{c,t}^1 c_{1,t} + U_{l,t}^1 l_{1,t}\right]$. The optimal policy solves,

$$\max_{c_{i,t}, l_{i,t}, \tilde{b}_{i,t}} \mathbb{E}_0 \sum_{i=1}^{I} \pi_i \alpha_i \sum_{t=0}^{\infty} \bar{\beta}_t U_t^i\left(c_{i,t}, l_{i,t}\right),$$

subject to

$$\tilde{b}_{t-1}\frac{U_{c,t-1}^i}{\beta_{t-1}} = \left(\frac{\mathbb{E}_{t-1} U_{c,t}^i}{U_{c,t}^i}\right)\mathbb{E}_t \sum_{k=t}^{\infty}\left[\prod_{j=t}^{k-1}\beta_j\right] Z_k^i \quad \forall t \geq 1$$

$$\tilde{b}_{-1} = \mathbb{E}_{-1}\sum_{k=0}^{\infty}\left[\prod_{j=0}^{k-1}\beta_j\right] Z_k^i$$

$$\frac{\mathbb{E}_{t-1} U_{c,t}^i}{U_{c,t-1}^i} = \frac{\mathbb{E}_{t-1} U_{c,t}^j}{U_{c,t-1}^j}$$

$$\sum_{i=1}^{I}\pi_i c_i(s^t) + g\left(s_t\right) = \sum_{i=1}^{I}\pi_i \theta_i\left(s_t\right) l_i(s^t),$$

$$\frac{U_{l,t}^i}{\theta_{i,t} U_{c,t}^i} = \frac{U_{l,t}^1}{\theta_{1,t} U_{c,t}^1}$$

$$\tilde{b}_{t-1}\frac{U_{c,t-1}^i}{\beta_{t-1}} \text{ is bounded}$$

Ramsey problem: Recursive formulation

Split into two parts

1. $\mathbf{t \geq 1}$: Ex-ante continuation problem with state variables $(x, \boldsymbol{\rho}, s_{-})$

$$x = \beta^{-1} \left(U^2_{c,t-1} \tilde{b}_{2,t-1}, ..., U^I_{c,t-1} \tilde{b}_{I,t-1} \right)$$

$$\boldsymbol{\rho} = \left(U^2_{c,t-1} / U^1_{c,t-1}, ..., U^I_{c,t-1} / U^1_{c,t-1} \right)$$

2. $\mathbf{t = 0}$: Ex-post initial problem with state variables $(\tilde{b}_{\mathbf{-1}}, s_0)$

Bellman Equation for $t \geq 1$

$$V(x, \boldsymbol{\rho}, s_{-}) = \max_{c_i(s), l_i(s), x'(s), \rho'(s)} \sum_s \Pr(s|s_{-}) \left(\left[\sum_i \pi_i \alpha_i U^i(s) \right] + \beta(s) V(x'(s), \boldsymbol{\rho}'(s), s) \right)$$

where the maximization is subject to

$$U_c^i(s)\left[c_i(s) - c_1(s)\right] + U_c^i(s) \left(\frac{U_l^i(s)}{U_c^i(s)} l_i(s) - \frac{U_l^1(s)}{U_c^1(s)} l_1(s) \right) + \beta(s) x_i'(s) = \frac{x U_c^i(s)}{\mathbb{E}_{s_{-}} U_c^i} \text{ for all } s, i \geq 2$$

$$\frac{\mathbb{E}_{s_{-}} U_c^i}{\mathbb{E}_{s_{-}} U_c^1} = \rho_i \text{ for all } i \geq 2$$

$$\frac{U_l^i(s)}{\theta_i(s) U_c^i(s)} = \frac{U_l^1(s)}{\theta_1(s) U_c^1(s)} \text{ for all } s, i \geq 2$$

$$\sum_i \pi_i c_i(s) + g(s) = \sum_i \pi_i \theta_i(s) l_i(s) \quad \forall s$$

$$\rho_i'(s) = \frac{U_c^i(s)}{U_c^1(s)} \text{ for all } s, i \geq 2$$

$$\underline{x}_i(s; x, \boldsymbol{\rho}, s_{-}) \leq x_i(s) \leq \bar{x}_i(s; x, \boldsymbol{\rho}, s_{-})$$

Bellman equation for $t = 0$

$$V_0\left(\{\tilde{b}_{i,-1}\}_{i=2}^{I}, s_0\right) = \max_{c_{i,0}, l_{i,0}, x_0, \rho_0} \sum_i \pi_i \alpha_i U^i(c_{i,0}, l_{i,0}) + \beta(s_0) V\left(x_0, \rho_0, s_0\right)$$

where the maximization is subject to

$$U^i_{c,0}\left[c_{i,0} - c_{1,0}\right] + U^i_{c,0}\left(\frac{U^i_{l,0}}{U^i_{c,0}} l_{i,0} - \frac{U^1_{l,0}}{U^1_{c,0}} l_{1,0}\right) + \beta(s_0) x_{i,0} = U^i_{c,0}\tilde{b}_{i,-1} \text{ for all } i \geq 2$$

$$\frac{U^i_{l,0}}{\theta_{i,0} U^i_{c,0}} = \frac{U^1_{l,0}}{\theta_{1,0} U^{1,0}_c} \text{ for all } i \geq 2$$

$$\sum_i \pi_i c_{i,0} + g_0 = \sum_i \pi_i \theta_{i,0} l_{i,0}$$

$$\rho_{i,0} = \frac{U^i_{c,0}}{U^1_{c,0}} \text{ for all } i \geq 2$$

Steady States (SS)

Let $\Psi\left(s; x, \boldsymbol{\rho}, s_{-}\right)$ be an optimal law of motion for the state variables for the $t \geq 1$ recursive problem, i.e.,

$$\Psi\left(s; x, \boldsymbol{\rho}, s_{-}\right) = \left(x'\left(s\right), \rho'\left(s\right)\right)$$

attains $t \geq 1$ value function given state $\left(x, \boldsymbol{\rho}, s_{-}\right)$

Definition

A steady state $\left(x^{SS}, \boldsymbol{\rho}^{SS}\right)$ satisfies $\left(x^{SS}, \boldsymbol{\rho}^{SS}\right) = \Psi\left(s; x^{SS}, \boldsymbol{\rho}^{SS}, s_{-}\right)$ for all s, s_{-}

A steady state is a node at which the continuation allocation and tax schedule has no further history dependence.

Existence

- Quasi-linear preferences: An SS exists for a wide range of parameters and shocks. Further, the economy reaches a steady state in one period. Output, tax rates, and net assets are constant thereafter.
- For general preferences, an SS exists if shocks are IID and take two values. The economy converges to this SS starting from any initial condition.
- Beyond the binary - IID case, there exists an ergodic region in which $(x, \boldsymbol{\rho})$ is no longer constant, but fluctuations are markedly reduced relative to the transient fluctuations that occur during an approach to a SS

Intuition: A two agent example

- Consider I=2 with $\theta_1(s) > \theta_2 = 0$.
- The state variable x is marginal utility scaled relative assets of an unproductive agent: $U_c^2(s)[b_2(s) - b_1(s)]$.
- One can normalize $b_1(s) = 0$ so that x can be interpreted as scaled assets held by the government

Two main forces determine the dynamics of the tax rate, transfers, and assets:

- **Fluctuations in inequality** measured by spreads in marginal utilities
- **Fluctuations in interest rate**

For quasi linear preferences both forces are absent

Inequality distortions

Start with a spread in discount factors set to equalize interest rates across states, i.e., $R(s_l) = R(s_h)$. Then SS $x > 0$

$\Downarrow$ TFP (θ_1): Adjust tax rate τ or transfers T, both are costly

Suppose $x = 0$ or $b_2(s) = b_1(s)$,

$\Downarrow$ Then reductions in transfers hurt the low productivity agent more

A fall in transfers that increases inequality gives rise to a cost not present in representative agent economies. This gives the planner an incentive to reduce the costs of inequality distortions by ...

$\Uparrow$ x

Reducing the relative asset holdings of the productive agent eventually drives the after-tax, after-interest incomes of both agents closer together

Interest rate fluctuations

1. Suppose discount factors are constant (this would imply countercyclical interest rates for TFP shocks). We still have $x > 0$. Consider again
   - $\Downarrow$ TFP

     If the tax rate τ is left unchanged, the government faces a shortfall of revenues. The optimal policy recommends

     $\Uparrow$ x. This achieves that same thing as having the government accumulate assets[2]
   - The govt. can use higher interest income to offset some revenue losses from its tax on labor
   - This force is present in representative agent economies with endogenous fluctuations in interest rates
2. If discount factor spreads are large enough to generate pro-cyclical interest rates, it can emerge that SS $x < 0$

[2] Normalizing $b_2(s) = 0$ implies $B(s) = -b_1(s) = \tilde{b}_2(s)$

Comparative statics with Pareto weights

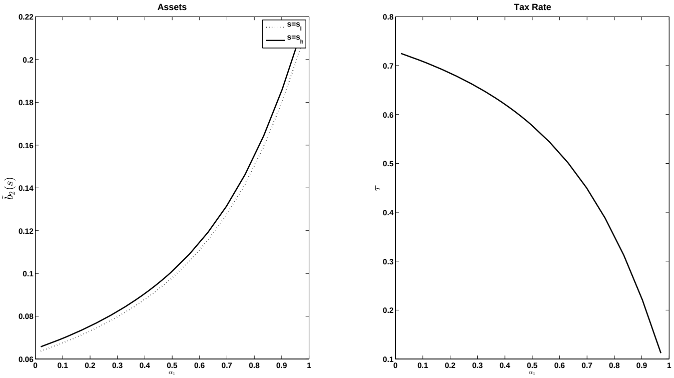

Figure: Steady state govt. assets: $\tilde{b}_2(s) = \frac{\beta x^{SS}}{U_c^2(s)}$ and taxes: τ^{SS} as a function of a (high skilled) agent 1's Pareto weight

Remarks on SS

- What matters for our second force is the comovement of the interest rate with fundamental shocks.
- For 2 agents, $x = U_c^2[b_2 - b_1]$, so the sign of x equals the sign of the difference between the financial wealths of agents with different productivity levels.
- For more than 2 agents, we have similar mechanics. In particular
  1. Inequality distortions call for a negative correlation between productivities and (scaled) net assets
  2. However, if the interest rate is sufficiently low in a recession, the government may want to hold debt. By borrowing more from agents with higher productivities, the govt. can the reduce welfare costs of lowering transfer in adverse times. This force can flip the sign of the correlation between productivities and net assets to be positive

Numerical Example

Use a calibrated version of the economy to

- Approximate magnitudes of these forces and
- Study optimal policy responses at business cycle frequencies when an economy is possibly far away from a steady state

Numerical Example: Calibration

Take a 2-shock 2-type economy with preferences $U(c, l) = \psi \log(c) + (1 - \psi) \log(1 - l)$ and allow $\theta_i(s), \beta(s), g(s)$ to depend on s.

- Pick baseline parameters to match some low frequency moments
- Calibrate outcome fluctuations to match three US recessions (i.e., 1991-92, 2001-02 and 2008-10):
  1. The left tail of the cross-section distribution of labor income falls more than right tail
  2. Short term interest rates fall
  3. Booms last longer than recessions

Calibration

Parameter	Value	Description	Target
ψ	0.6994	Frisch elasticity of labor supply	0.5
$\bar{\theta}_1$	4	Log 90-10 wage ratio (Autor et al.)	4
$\bar{\theta}_2$	1	Normalize to 1	1
β	0.98	Average (annual) risk free interest rate	2%
α_1	0.69	Marginal tax rate in the economy with no shocks	20%
g	12%	Average pre-transfer expenditure- output ratio	12 %
$\frac{\hat{\theta}_2}{\hat{\theta}_1}$	2.5	Relative drop in wage income of 10th percentile	2.5
$\hat{\theta}_1$	1.2%	Average output loss	3%
$\hat{\beta}(s)$	1.96%	Difference in real interest rates between booms and recession	1.96%
$P(r\|r)$	0.63	Duration of recessions	2.33 years
$P(b\|b)$	0.84	Duration of booms	7 years

Table: Benchmark calibration

Initial conditions chosen to make debt to GDP ratio be 60%

Results: Some variants

We study perturbations of the Benchmark calibration

1. Acyclical interest rates: Smaller spread in discount factor shocks
2. Countercyclical interest rates: No discount factor shocks
3. No inequality: Equal fall in all agents' productivities (TFP shock) and no discount factor shocks
4. Government expenditure shocks: A fall in g that produces a comparable fall in output

Results: Long run

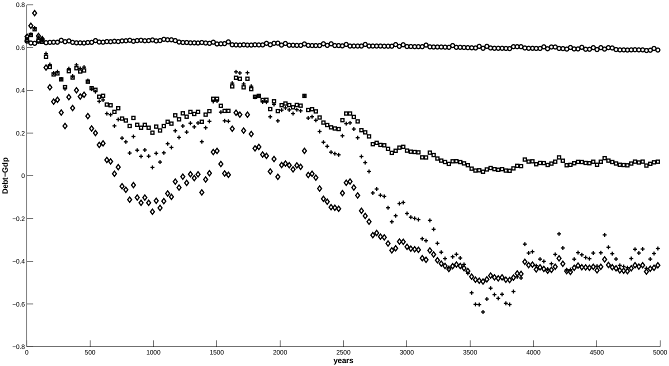

Figure: Govt. debt for several economies: benchmark (o), acyclical interest rates (+), countercyclical interest rates ($\diamond$) and no inequality shocks ($\square$)

Observations

- Long run tendency to converge to some ergodic set. But convergence is very slow more details on speed of convergence.
- With low discount factor shocks, outcomes approach positive govt. assets
- With high discount factor shocks that produce procyclical real interest rates, there is no tendency to reduce govt. debt even after 5000 years

Short Run

To understand the short run responses

- We set the exogenous state s_0 to put as at the onset of a recession
- We solve the time 0 problem with identical initial conditions across different settings. This pins down the initial state vector x_0, ρ_0 that appears in our time 0 Bellman equation
- We use optimal policies to compute fluctuations of different components in the government budget constraint across states

For each variable z in the table we report $\Delta z \equiv \left(z\left(s_l | x_0, \rho_0, s_0\right) - z\left(s_h | x_0, \rho_0, s_0\right)\right) / \bar{Y}$ where $\bar{Y}$ is average undistorted GDP in percentages

Results: Short run

	Δg	ΔB	ΔT	$\Delta[\tau\theta_1 l_1]$	$\Delta[\tau\theta_2 l_2]$	ΔY	$\Delta\tau$
Benchmark	0.0000	-1.1561	0.6871	-0.1593	-0.3096	-2.8536	0.3732
Acyclical Interest Rates	0.0000	-1.1126	0.6591	-0.1497	-0.3038	-2.8613	0.3879
Countercyclical Interest Rates	0.0000	-1.0794	0.6387	-0.1415	-0.2992	-2.8677	0.3997
No Inequality	0.0000	-0.1380	**-0.5459**	-0.5635	-0.1204	-2.6294	0.0622
Expenditure Shocks	-7.5037	2.9137	2.8612	-1.3759	-0.3530	-2.3443	-1.1598

Table: The tables summarizes the changes in the different components of the government budget as the economy transits from "boom" to "recession". All numbers except τ are normalized by un-distorted GDP and reported in percentages.

Note that predetermined variables like repayment on existing debt drop out of the accounting and we have

$$\Delta[g] + \Delta[T] + \Delta[B] = \Delta[\tau\theta_1 l_1] + \Delta[\tau\theta_2 l_2]$$

Conclusions

- Size of government debt alone is irrelevant $\Longrightarrow$ need to know the distribution of net assets
- Optimal tax and transfer scheme balance
  1. welfare losses from fluctuating taxes
  2. welfare losses from fluctuating transfers
- Since welfare costs depend on the how debt is distributed, the planner has incentives to move net assets over time
- With incomplete markets, interest rate fluctuations are a key determinant of long-run correlations between productivities and net assets
- Ignoring heterogeneity produces misleading results about the size and direction of short run optimal policy responses

Speed of convergence (I)

Suppose we are in the binary-IID world where steady states are deterministic.

- The optimal policy induces two *risk adjusted* martingales
  1. Multiplier on the implementability constraint : $\boldsymbol{\mu}_t$
  2. The ratio of marginal utilities: $\boldsymbol{\rho}_t$
- One can represent the optimal allocation recursively in terms of $\{\boldsymbol{\mu}(s^{t-1}), \boldsymbol{\rho}(s^{t-1})\}$ and s_t.
- Why $(\boldsymbol{\mu}, \boldsymbol{\rho})$ instead of $(x, \boldsymbol{\rho})$?
- Linearize optimal policies for each s_t around the constant steady state.
- Study the eigenvalues of the conditional mean and variance dynamics (these are deterministic linear systems)

Speed of convergence (II)

Let $\hat{\Psi}_t = \begin{bmatrix} \boldsymbol{\mu}_t - \boldsymbol{\mu}^{SS} \\ \boldsymbol{\rho}_t - \boldsymbol{\rho}^{SS} \end{bmatrix}$. Then

$$\hat{\Psi}_{t+1} = B(s_{t+1})\hat{\Psi}_t$$

This linearized system has coefficients that are functions of the shock s.

Proposition

If the (real part) of eigenvalues of $\mathbb{E}B(s)$ are less than 1, the system converges to zero in mean. Further for large t, the conditional variance of $\hat{\Psi}$, denoted by $\Sigma_{\Psi,t}$, follows a deterministic process governed by

$$vec(\Sigma_{\Psi,t}) = \hat{B} vec(\Sigma_{\Psi,t-1}),$$

where $\hat{B}$ is a square matrix of dimension $(2N-2)^2$. In addition, if the (real parts) of eigenvalues of $\hat{B}$ are all less than 1, the system converges in probability.

The eigenvalues (in particular the largest one) are instructive not only for whether the system is locally stable but also for how quickly the steady state is reached

Speed of convergence: Size of shocks and risk aversion

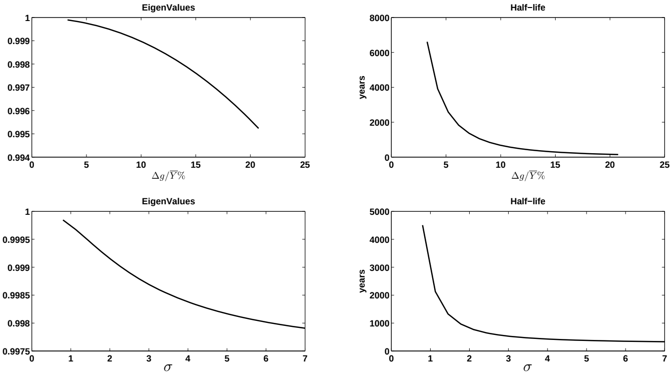

Figure: The top (bottom) panel plots the dominant eigenvalue of $\hat{B}$ and the associated half life as we increase the spread between the expenditure levels (risk aversion).

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