# Optimal fiscal policy with incomplete asset markets

Anmol Bhandari, David Evans, Mikhail Golosov, Thomas J. Sargent

October 2013

# Optimal taxation under commitment and a representative agent

- **Incomplete markets**
  $\rightarrow$ assets with alternative exogenous payoff patterns
- **Linear tax schedules**
  $\rightarrow$ Proportional tax on labor earnings (maybe *nonnegative* transfers)
- **Aggregate shocks**
  $\rightarrow$ To productivities, government expenditures, etc.

# Questions

1. **Tax rate**: How should government accumulate or decumulate assets to smooth tax distortions
2. **Government debt**: Why do different governments issue different amounts of debt? Difference answers under polar assumptions: LS – complete markets; AMSS — a risk-free bond only
   - Lucas Stokey (1984): Inherited from initial condition
   - AMSS (2002): Govt. accumulates assets sufficient to finance activities using interest revenues

# Our analysis

1. **Asset structure**
   + We restrict government to trade a single asset only
   + We exogenously restrict asset payoffs
     $\Longrightarrow$ E.g., bonds that pay less during adverse times
2. **Forces**
   - Asset **levels** can help smooth tax distortions **across states**
   - This differs from the role of debt in previous incomplete markets economies where **changes** in debt levels help smooth tax distortions **over time**

# Environment

- **Uncertainty**: Markov aggregate shocks $s_t \in \mathcal{S}$
- **Demography**: Infinitely lived representative agent plus a benevolent planner
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c(s^t), l(s^t)\right)$$

- **Technology**: Aggregate output $y_t = \theta_t l_t$

# Environment, II

- **Asset markets**:
  - The government trades are restricted to an single asset
  - The payoff of a unit of government debt is given by a matrix $p_t$ parameterized by a $|\mathcal{S}|^2$ matrix $\mathbb{P}$

$$p_t = \mathbb{P}(s_t|s_{t-1})$$

- **Linear Taxes**: Agent $i$'s tax bill

$$-T_t + \tau_t \theta_t l_t, \;\; T_t \geq 0$$

- **Budget constraints** Let $q_t$ be the price of the asset,
  - Agents: $c_t + q_t b_t = (1 - \tau_t)\, \theta_t l_t + p_t b_{t-1} + T_t$,
  - Government: $g_t + q_t B_t + T_t = \tau_t \theta_t l_t + p_t B_{t-1}$,

- **Market Clearing**
  - Goods: $c_t + g_t = \theta_t l_t$
  - Assets: $b_t + B_t = 0$

- **Initial conditions**: Assets $b_{-1}, B_{-1}$ and $s_{-1}$

# Ramsey Problem

Definition

**Allocation, price system, government policy**

Definition

**Competitive equilibrium**: Given $(b_{-1}, B_{-1}, s_{-1})$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$, all allocations are individually rational, markets clear [1]

Definition

**Optimal competitive equilibrium**: A welfare-maximizing competitive equilibrium for a given $(b_{-1}, B_{-1}, s_{-1})$

[1] Usually, we impose only "natural" debt limits.

# Ramsey problem

1. **Primal approach**: To eliminate tax rates and prices, use consumer's first order conditions
2. **Implementability constraints**: Derive by iterating the consumer's budget equation forward at every history
   $\Rightarrow$With incomplete market economies these impose *measurability restrictions* on Ramsey allocations
3. **Transfers:** We temporarily restrict transfers $T_t = 0\ \forall t$. This is convenient for our analytical results. We eventually show that this assumption is not restrictive.

# Sequential Ramsey problem

$$\max_{\{c_t, l_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Implementability constraints**

$$\frac{b_{t-1} U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1} p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left( U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \text{ for } t \geq 1$$

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( U_{c,t} c_t + U_{l,t} l_t \right)$$

# Roadmap, the questions

1. The properties of the optimal allocation are a function of **returns on the debt**
   - Prices: $\{q_t(s^t|B_{-1}, s_{-1})\}_t$
   - Payoffs: $\mathbb{P}$

   *To focus on the exogenous part of return, we first study preferences quasi-linear in consumption where $q_t = \beta$ and then economies with risk aversion*

2. To characterize the **long run** level of debt and associated taxes we split the analysis into two parts
   - Given arbitrary initial assets, what would be an **optimal** asset payoff matrix $\mathbb{P}^*(b)$ ?
   - In an economy with an arbitrary payoff matrix $\mathbb{P}$ when would $b_t \to b^*$ ?

$$\mathbb{P}(b^*) = \mathbb{P}^*(b^*)$$

# Roadmap, the answers

- In a binary IID world we can classify $\mathbb{P}$'s where the debt level under the optimal policy converges to $b^*$
- For more general shock structures we numerically verify that the ergodic set of debt is concentrated around $b^*$

## Quasilinear preferences $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

Given initial assets $b$, let $\mu(b)$ be the Lagrange multiplier on implementability constraint at $t = 0$

1. **Multiplier $\rightarrow$ Tax rate:**

$$\tau(\mu) = \frac{\gamma\mu}{(1+\gamma)\mu - 1}$$

2. **Tax rate $\rightarrow$ Surplus:**

$$S(s, \tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1+\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

3. **Surplus $\rightarrow$ payoff structure:**

$$\mathbb{P}^*(s|s_-) = (1-\beta)\frac{S(s,\tau)}{\mathbb{E}_{s_-}S(s,\tau)} + \beta$$

and

$$\beta^{-1}\mathbb{P}^*(s|s_-) = \frac{S(s,\tau)}{b} + 1$$

# Initial holdings influence optimal asset payoff structure

Denote state $s$ as “adverse” if it has “high” expenditure or “low ” TFP, formally,

$$g(s)\mathbb{E}_{s\_}\theta^{\frac{\gamma}{1+\gamma}} - \theta(s)^{\frac{\gamma}{1+\gamma}}\mathbb{E}_{s\_}g > 0$$

The optimal payoff matrix in general has the following properties,

- With positive initial assets: want a payoff structure that pays *more* in “adverse” states
- With negative initial assets: want a payoff structure that pay *less* in “adverse” states

# Optimal Payoff Structure: TFP shocks

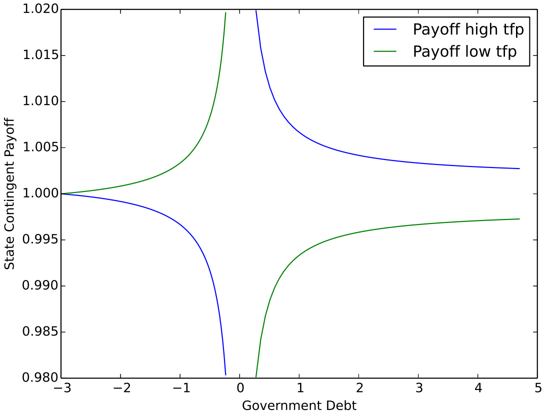



Figure: Optimal asset payoff structure as a function of initial government debt when TFP follows a 2 shock i.i.d process

# Optimal Payoff Structure: Expenditure shocks

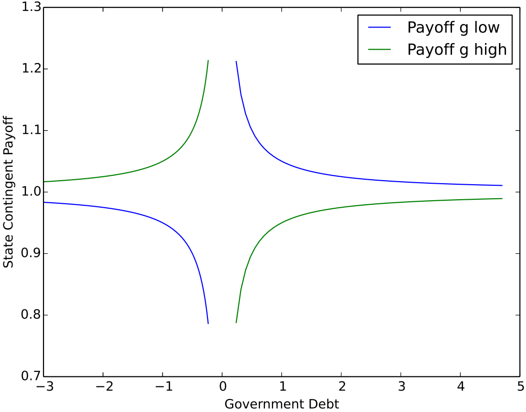
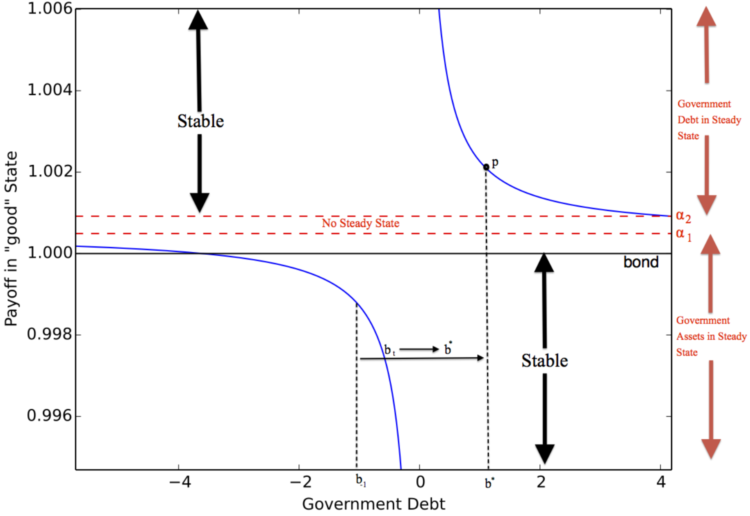



Figure: Optimal asset payoff structure as a function of initial government debt when government expenditures follow 2 shock i.i.d process

# show $b \to b^*$



1.006
1.004
1.002
1.000
0.998
0.996
−4
−2
0
2
4
b-1
b*
Government Debt
Payoff in "good" State
Stable
Stable
No Steady State
p
bond
bt
b*
α2
α1
Government Debt in Steady State
Government Assets in Steady State

# Incomplete markets

1. **Exogenous payoff structure:** Suppose $\mathbb{P} \neq \mathbb{P}^*(b_{-1})$
2. **Steady States:** A steady state is a debt level for the government $b^*$ such that

$$b_t = b^* \text{ implies } b_{t+s} = b^* \quad s > 0$$

3. **Characterization:** Given an asset payoff structure $\mathbb{P}$
   - Does a steady state exist? is it unique?
   - What is the level of government debt or assets in a steady state?
   - For what levels of *initial government debt* does convergence to a steady state occur?

# Existence

A steady state is obtained by inverting the optimal payoff structure i.e

$$b^* : \mathbb{P}(b^*) = \mathbb{P}^*(b^*) \tag{2}$$

When shocks are i.i.d and take two values

1. $\mathbb{P}(s|s_-)$ is independent of $s_-$
2. We can normalize $\mathbb{E}\mathbb{P}(s) = 1$ and w.l.o.g, denote payoffs by a scalar $p$.
   - $p$ is the payoff in the "good" state $s$
   - A risk free bond is a security when $p = 1$
3. The steady restriction (2) solves one equation in one unknown.

# Existence: Regions in $p$ space

The payoff $p$ in good state $\in (0, \infty)$.
We can decompose a class of economies with different payoff structuresinto 3 regions using thresholds $\alpha_2 \geq \alpha_1 \geq 1$

- Low enough $p(\leq \alpha_1)$: government holds assets in steady state
- High enough $p(\geq \alpha_2)$: government issues debt in steady state
- Intermediate $p(\alpha_1 > p > \alpha_2)$: steady states do not exist

# Thresholds: $\alpha_1 < \alpha_2$

- With only government expenditure shocks

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$

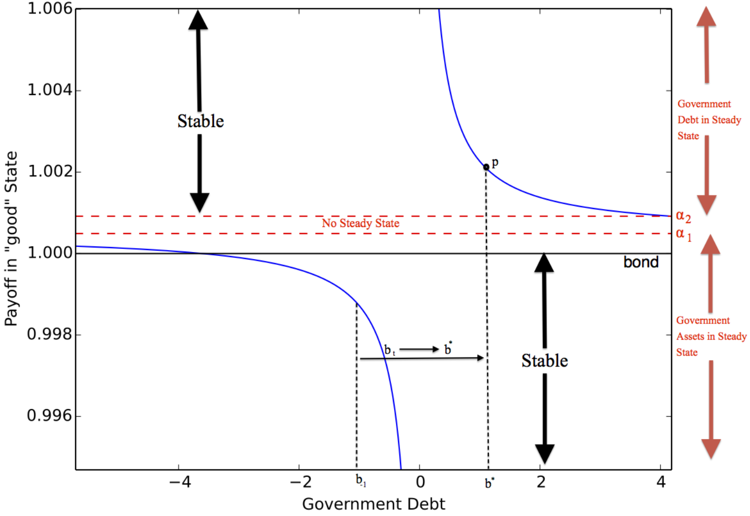
- With only TFP shocks

$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

# show the existence region



# Convergence

- Our analysis verifies the existence of a steady state in a 2-state i.i.d. economy.
- To study long-run properties of a Ramsey allocation with incomplete markets, we need to determine whether these steady states are stable
- **Risk-adjusted martingale:**
  Under an optimal policy, the Lagrange multiplier $\mu_t$ on the implementability constraint satisfies

  $$\mu_t = \mathbb{E}_t p_{t+1} \mu_{t+1}$$

  or

  $$\mathbb{E}_t \mu_{t+1} = \mu_t - Cov_t(p_{t+1}, \mu_{t+1})$$

  $\mu_t$ *follows a risk adjusted martingale.*
- **Stability:** Away from a steady state, is the drift $\mu_t$ big enough?

# Characterizing Convergence

- Reminder: $p$ is the payoff in the "good" state.
- As with existence, we can partition the "$p$ space" into stable and unstable regions

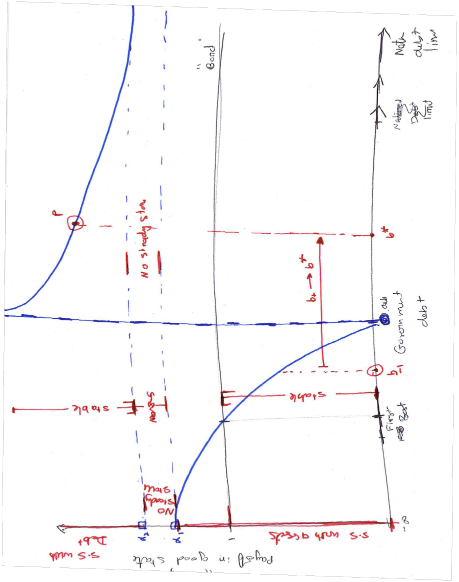
## Theorem

*Let $b^*$ denote the steady state level of debt. Then for same $\alpha_1 < \alpha_2$*

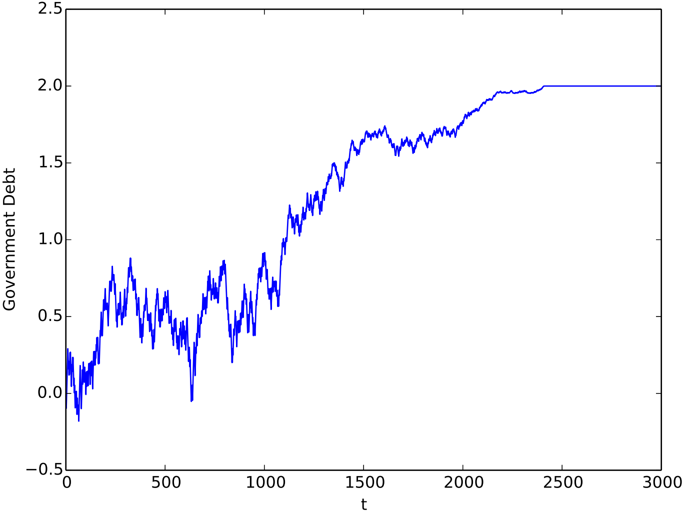
1. **Low** $p_1$*: If* $p_1 \leq \min(\alpha_1, 1)$ *then the steady state is stable with* $b_{fb} < b^* < 0$ *and* $b_t \to b^*$ *with probability 1.*
2. **High** $p_1$*: If* $p_1 \geq \alpha_2$ *then the steady state is stable with* $0 < b^*$ *and* $b_t \to b^*$ *with probability 1.*

For the intermediate region where either $b^*$ does not exist or is unstable, there is a tendency towards accumulating debt

# Show the stable region

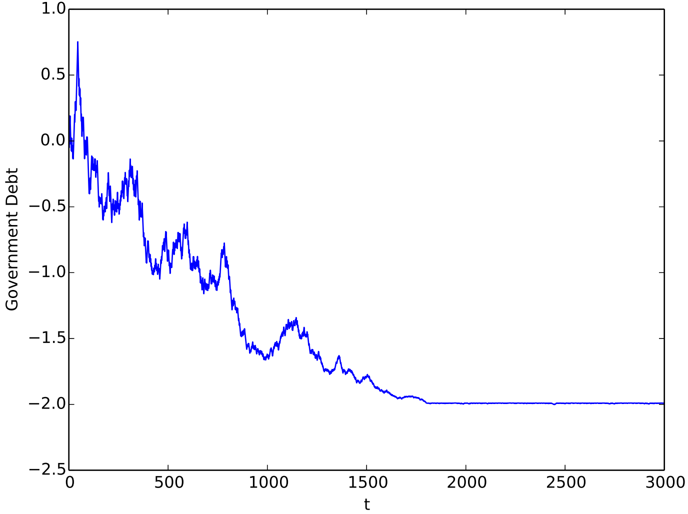


## $p_1 > 1$



2.5
2.0
1.5
1.0
0.5
0.0
−0.5
Government Debt
0
500
1000
1500
2000
2500
3000
t

## $p_1 < 1$



1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
Government Debt
0
500
1000
1500
2000
2500
3000
t

# Incomplete markets with risk aversion

**Quasilinear preferences:**

1. With quasilinear preferences, we showed that $b_t \to b^*$ when the aggregate state follows a 2-state i.i.d. process
2. The level and sign of $b^*$ is a function of the **exogenous payoff structure** $\mathbb{P}$
3. The limiting allocation corresponds to a complete market allocation

**Risk aversion:**

- This marginal utility adjusted debt will encode history dependence
- Instead of government debt $x_t = u_{c,t} b_t$ will converge with binary i.i.d shock shock process
- Long-run properties of $x_t$ depend on returns $R_{t,t+1} = \frac{\mathbb{P}(s_{t+1}|s_t)}{q_t(s^t)}$

# Roadmap, II

- Split the Ramsey problem in two
  1. $t = 0$ Bellman equation in $W(b_{-1}, s_0)$
  2. $t \geq 1$ Bellman equation in $V(x, s_{\_})$
- Analyze steady states $x^*$ such that $x_t \to x^*$

So far,

1. **Risk free bond** Existence proved only under a special case of a risk-free bond $\mathbb{P}(s|s_{\_}) = 1 \forall s, s_{\_}$
   This focuses on *endogenous* component of returns
2. **Interpretation:** $x^*$ corresponds to an initial condition when the optimal portfolio in a LS economy is a risk-free bond

# A Recursive Formulation

1. Commitment implies that government actions at $t \geq 1$ are constrained by anticipations about them at $s < t$
2. This contributes additional state variables like marginal utility of consumption
3. Scaling the budget constraint by marginal utility makes it recursive in $x = U_c b$

$$\frac{x_{t-1} p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} = U_{c,t} c_t + U_{l,t} l_t + x_t$$

# Bellman equation for $t \geq 1$

$$V(x, s_-) = \max_{c(s), l(s), x'(s)} \sum_s \pi(s, s_-) \Big( U(c(s), l(s)) + \beta V(x'(s), s) \Big)$$

subject to $x'(s) \in [\underline{x}, \overline{x}]$

$$\frac{x p(s) U_c(s)}{\beta \mathbb{E} p U c} = U_c(s) c(s) + U_l(s) l(s) + x'(s)$$

$$c(s) + g(s) = \theta(s) l(s)$$

# Time 0 Bellman equation

Given an initial debt $b_{-1}$, state $s_0$, and continuation value function $V(x, s_{-})$

$$W(b_{-1}, s_0) = \max_{c_0, l_0, x_0} U(c, l) + \beta V(x_0, s0)$$

subject to time zero implementability constraint

$$U_c(c_0, l_0)c + U_l(c_0, l_0)l_0 + x_0 = U_c(c_0, l_0)b_{-1}$$

and resource constraint

$$c_0 + g(s_0) = \theta(s_0)l_0$$

and

$$x_0 \in [\underline{x}, \overline{x}]$$

# Revisiting steady states with risk aversion

Let $x'(s; x, s_-)$ be an optimal law of motion for the state variable for the $t \geq 1$ recursive problem.

### Definition

A steady state $x^*$ satisfies $x^* = x'(s; x^*, s_-)$ for all $s, s_-$

*Thus, a steady state is a node at which continuation allocation and tax rate have no further history dependence.*

# Existence

1. For a class of economies with separable iso-elastic preferences $U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$
2. Shocks that take two values and are i.i.d

Let $x_{fb}$ be a value of the state $x$ from which the government can implement first best from that period onwards.

## Proposition

*Let $q^{fb}(s)$ be the shadow price of government debt in state $s$ and using the first best allocation. if*

$$\frac{1 - q^{fb}(s_b)}{1 - q^{fb}(s_g)} > \frac{g(s_b)}{g(s_g)}$$

*Then there exists a steady state with $x^{fb} > x^* > 0$*

# Stability

1. In this setup, interest rates are aligned with marginal utility of consumption; they are low in "good" states (high TFP or low expenditure)
2. The government holds claims against the private sector in the steady state. This is similar to the quasilinear case when $p_1$ was low.
3. The incomplete markets Ramsey allocation converges to a Lucas-Stokey Ramsey allocation for all initial conditions

## Proposition

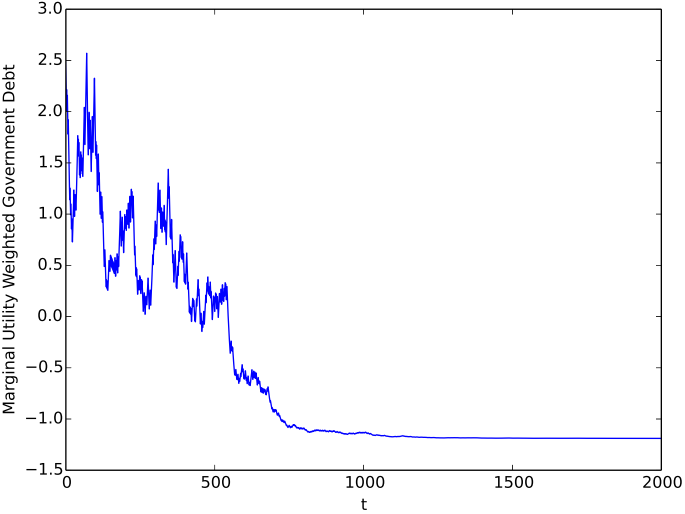
*Let $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ solve the incomplete markets Ramsey problem. Then $x_t(s^{t-1}) \to x^*$ as $t \to \infty$ with probability 1 for all initial conditions*

# Quasilinear vs. risk aversion

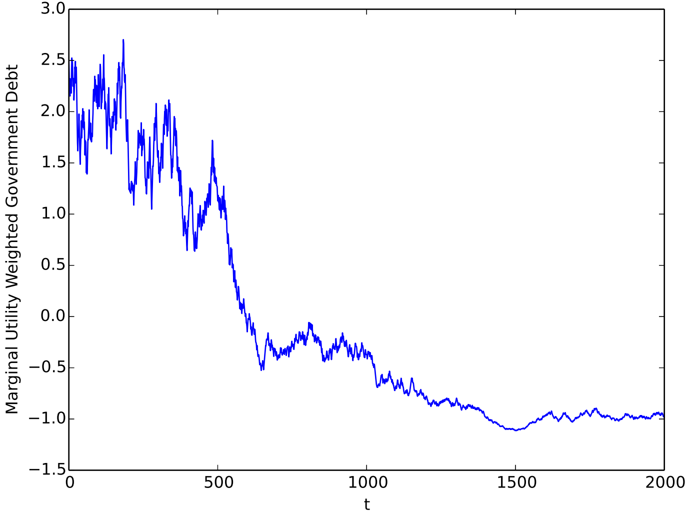
Redo this slide

- With quasilinear preferences and a risk-free bond, the interest rate is always $1/\beta$.
- With risk averse preferences, interest rate is higher in periods of high government expenditure.
- Thus, while the government has greater expenses in high government expenditure states, the higher interest rate means that the government can accumulate less and still cover future government expenditures.
- After the government has accumulated enough assets, it is actually better off in periods of high government expenditure than in periods with low government expenditures (since its claims to consumption are worth more).
- By holding assets, the government is able to reallocate resources across states, something it is not able to do in the quasilinear case.

# 2 State i.i.d. process with Risk Aversion



# $S > 2$ states

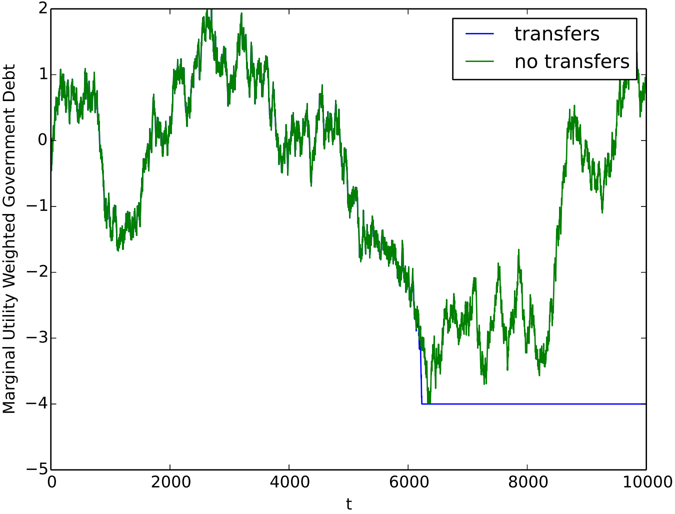


3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Marginal Utility Weighted Government Debt
0
500
1000
1500
2000
t

# Transfers

- That the government can use its assets to smooth tax rate distortions carries over to when the government has access to lump sum transfers
- Access to nonnegative transfers makes first-best level of assets trivially a “steady state”.
- With lump sum transfers, in cases where the steady state exists and is stable, if the initial debt of the government exceeds its steady state, the economy converges with probability 1 to the steady state.

# Quasilinear preferences and risk-free bond with and without transfers



Add a slide comparing stuff to Buera Nicolini, Angelotos

# Concluding remarks

- With market incompleteness, the asset payoff structure has big implications a Ramsey government's long run debt
- If the asset offers lower returns in adverse states of the world, the Ramsey government asymptotically runs up a debt to the private sector.
- With risk aversion, cyclical properties of interest rate affects government debt asymptotically
- Access to nonnegative transfers play little role in shaping outcomes. Rather, the key force is the government's ability to use its debt position to reallocate resources across states
- **Future Research**: With heterogeneous agents and unrestricted transfers, how does the type of market incompleteness affect long run wealth distributions and other outcomes?