Taxes, Debts, and Redistributions with Aggregate Shocks

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Optimal Taxation Under Commitment With

- **Heterogeneous agents**
  $\rightarrow$ Different productivities
- **Incomplete markets**
  $\rightarrow$ All trade only a risk-free bond
- **Affine tax schedules**
  $\rightarrow$ Government levies a proportional tax on labor earnings + lump sum (tax or transfer)
- **Aggregate shocks**
  $\rightarrow$ To productivities, government expenditures, discount factors

Questions

1. How costly is government debt?
2. What are the long run properties of optimal government policies and equilibrium allocations?
3. How should government policies respond to aggregate shocks?

Environment

- **Uncertainty**: Markov aggregate shocks s_t
- **Demography**: I types of infinitely lived agents (of mass π_i) plus a benevolent planner
- **Technology**: Output $\sum_{i=1}^{I} \theta_{i,t} l_{i,t}$ is linear in labor supplies. Productivities $\{\theta_i(s_t)\}_{i,t}$ differ across i.
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \bar{\beta}_t U^i \left(c_i(s^t), l_i(s^t)\right)$$

  where $\bar{\beta}_t = \left[\Pi_{j=0}^{t-1} \beta(s_j)\right]$

- **Preferences** (Planner): Given Pareto weights $\{\alpha_i\}$

$$\mathbb{E}_0 \sum_{i=1}^{I} \pi_i \alpha_i \sum_{t=0}^{\infty} \bar{\beta}_t U_t^i \left(c_{i,t}, l_{i,t}\right)$$

- **Asset Markets**: A risk-free bond only

Environment, II

- **Affine Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_{i,t} l_{i,t}$$

- **Budget Constraints**
  - Agent i: $c_{i,t} + b_{i,t} = (1 - \tau_t)\, \theta_{i,t} l_{i,t} + R_{t-1} b_{i,t-1} + T_t$
  - Government: $g_t + B_t + T_t = \tau_t \sum_{i=1}^{I} \pi_i \theta_{i,t} l_{i,t} + R_{t-1} B_{t-1}$

- **Market Clearing**
  - Goods: $\sum_{i=1}^{I} \pi_i c_{i,t} + g_t = \sum_{i=1}^{I} \pi_i \theta_{i,t} l_{i,t}$
  - Assets: $\sum_{i=1}^{I} \pi_i b_{i,t} + B_t = 0$

- **Initial Distribution of Assets** $\{b_{i,-1}\}_i$ and B_{-1}

Ramsey Problem

Definition

Allocation, price system, government policy: Standard

Definition

Competitive equilibrium: Given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$, all allocations are chosen optimally, markets clear [1]

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$

[1] Usually, we impose only "natural" debt limits.

Contrast with representative agent models

Representative agent and linear taxes

- Higher levels of government debt are distortionary
- With incomplete markets (as in AMSS), the optimal government policy is to accumulate assets

Redistribution and optimal transfers

- Representative agent models restrict transfers
  - Implicitly motivated by concerns about redistribution: poor people can't afford lump sum taxes
  - These constraints almost always bind (e.g., Lucas Stokey, AMSS) and drive long-run govt debt dynamics
- We begin with explicit redistribution motive and let the government set transfers optimally

Prescriptions for optimal tax-transfers differ substantially with explicitly modeled redistribution concerns

Main working parts

- **Welfare Criterion**: Benevolent planner with explicit redistribution motives
- **Instruments**: Transfers and a flat tax on labor income
- **Restrictions**:
  1. The tax on labor income is linear in wage earnings
  2. Transfers are unrestricted in sign and magnitude but cannot be conditioned on agents' identities
  3. Incomplete markets
- **Trade-offs**:
  1. Changes in labor taxes impose dead weight losses
  2. Explicit redistribution motives imply costs of fluctuating transfers. Withdrawing a unit of consumption affects rich and poor people differently

Key forces

- **Heterogeneity**
  1. Two sources of heterogeneity: Productivities and asset holdings
  2. Unrestricted transfers → *Level* of government debt is irrelevant; *distribution* across agents matters
     E.G, high productivity agents owning more government debt can be more distortionary
- **Responses**
  Since welfare costs depend on the distribution of assets, optimal policy is affected by and affects the distribution of net assets
  1. **Absence of agent-specific transfers**: This motivates the govt. to engineer a negative correlation between net assets and labor earnings
  2. **Absence of state contingent securities**: This motivates the govt. to exploit endogenous fluctuations in the interest rate

Ricardian Equivalence

- *Result*: A **large set** of transfers and asset profiles support the same competitive allocation
- *Logic*: Withdrawing a unit of assets from all agents and increasing transfers by a unit leaves budget sets unchanged

Notation: $\tilde{b}_{i,t} = b_{i,t} - b_{1,t}$ be the **relative assets** of agent i

Theorem

Given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$, *let* $\{\{c_{i,t}, l_{i,t}, b_{i,t}\}_i, B_t, R_t\}_t$ *and* $\{\tau_t, T_t\}_t$ *be a competitive equilibrium.*

For any bounded sequences $\left\{\hat{b}_{i,t}\right\}_{i,t\geq -1}$ *that satisfy*

$$\hat{b}_{i,t} - \hat{b}_{1,t} = \tilde{b}_{i,t} \text{ for all } t \geq -1, i \geq 2,$$

there exist sequences $\left\{\hat{T}_t\right\}_t$ *and* $\left\{\hat{B}_t\right\}_{t\geq -1}$ *such that* $\left\{\left\{c_{i,t}, l_{i,t}, \hat{b}_{i,t}\right\}_i, \hat{B}_t, R_t\right\}_t$ *and* $\left\{\tau_t, \hat{T}_t\right\}_t$ *constitute a competitive equilibrium given* $\left(\left\{\hat{b}_{i,-1}\right\}_i, \hat{B}_{-1}\right)$.

Ricardian Equivalence: Implications

- Applies to all competitive allocations, not just optimal one
- Ceteris paribus, an economy with higher level of initial government debt but same relative asset holdings has the same welfare
- Exogenous borrowing constraints of the form $b_{it} > \underline{b}_i$ are not restrictive
  Logic: If some borrowing constraints bind, the planner can change transfers to slacken *all* of them

Theorem

Take a competitive equilibrium in an economy without exogenous borrowing constraints, and suppose we impose borrowing constraints of the form $b_{i,t} > \underline{b}_i$. There is a government tax policy such the same allocation and interest rate sequence is part of a competitive equilibrium in an economy with those exogenous borrowing constraints

Thus, Ricardian equivalence holds with distortionary taxes and ad hoc borrowing limits

Optimal allocations: Primal approach

Focus on interior equilibria.

1. Eliminate tax rate τ_t:

$$(1-\tau_t)\,\theta_{i,t} U^i_{c,t} = -U^i_{l,t},$$

2. Eliminate risk free interest rate R_t:

$$U^i_{c,t} = \beta_t R_t \mathbb{E}_t U^i_{c,t+1}.$$

3. Eliminate transfers T_t:

$$(c_{i,t}-c_{1,t}) + \tilde{b}_{i,t} = -\frac{U^i_{l,t}}{U^i_{c,t}} l_{i,t} + \frac{U^1_{l,t}}{U^1_{c,t}} l_{1,t} + \frac{U^i_{c,t-1}}{\beta_{t-1}\mathbb{E}_{t-1} U^i_{c,t}} \tilde{b}_{i,t-1} \;\forall i \geq 2, t.$$

This yields "**implementability constraints**"

Optimal allocations: Sequential formulation

$$\max_{\{c_{i,t}, l_{i,t}, \tilde{b}_{i,t}\}_{i,t}} \mathbb{E}_0 \sum_{i=1}^{I} \pi_i \alpha_i \sum_{t=0}^{\infty} \bar{\beta}_t U_t^i \left(c_{i,t}, l_{i,t}\right),$$

subject to

$$\sum_{i=1}^{I} \pi_i c_{i,t} + g_t = \sum_{i=1}^{I} \pi_i \theta_{i,t} l_{i,t} \quad \text{(Feas)}$$

$$\frac{U_{l,t}^i}{\theta_{i,t} U_{c,t}^i} = \frac{U_{l,t}^1}{\theta_{1,t} U_{c,t}^1} \quad \text{(Wages)}$$

$$\frac{\mathbb{E}_{t-1} U_{c,t}^i}{\mathbb{E}_{t-1} U_{c,t}^j} = \frac{U_{c,t-1}^i}{U_{c,t-1}^j} \quad \text{(Bond)}$$

$$\tilde{b}_{i,t-1} \frac{U_{c,t-1}^i}{\beta_{t-1}} = \left(\frac{\mathbb{E}_{t-1} U_{c,t}^i}{U_{c,t}^i}\right) \mathbb{E}_t \sum_{k=t}^{\infty} \left[\prod_{j=t}^{k-1} \beta_j\right] Z_k^i \quad \text{(Meas: } t \geq 1\text{)}$$

$$\tilde{b}_{i,-1} = \mathbb{E}_{-1} \sum_{k=0}^{\infty} \left[\prod_{j=0}^{k-1} \beta_j\right] Z_k^i \quad \text{(Meas: } t = 0 \text{)}$$

$$\tilde{b}_{i,t-1} \frac{U_{c,t-1}^i}{\beta_{t-1}} \text{ is bounded}$$

where $Z_t^i = U_{c,t}^i c_{i,t} + U_{l,t}^i l_{i,t} - \frac{U_{c,t}^i}{U_{c,t}^1} \left[U_{c,t}^1 c_{1,t} + U_{l,t}^1 l_{1,t}\right]$.

Ramsey problem: Recursive formulation

Split into two parts

1. $\mathbf{t \geq 1}$: Ex-ante continuation problem with state variables $(x, \boldsymbol{\rho}, s_{-})$

$$x = \beta_{t-1}^{-1}\left(U_{c,t-1}^{2}\tilde{b}_{2,t-1}, ..., U_{c,t-1}^{I}\tilde{b}_{I,t-1}\right)$$

$$\boldsymbol{\rho} = \left(U_{c,t-1}^{2}/U_{c,t-1}^{1}, ..., U_{c,t-1}^{I}/U_{c,t-1}^{1}\right)$$

2. $\mathbf{t = 0}$: Ex-post initial problem with state variables $(\tilde{b}_{\mathbf{-1}}, s_0)$

Bellman Equation for $t \geq 1$

$$V(x, \boldsymbol{\rho}, s_{-}) = \max_{c_i(s), l_i(s), x'(s), \rho'(s)} \sum_s \Pr(s|s_{-}) \left(\left[\sum_i \pi_i \alpha_i U^i(s) \right] + \beta(s) V(x'(s), \boldsymbol{\rho}'(s), s) \right)$$

where the maximization is subject to

$$U_c^i(s)\left[c_i(s) - c_1(s)\right] + U_c^i(s)\left(\frac{U_l^i(s)}{U_c^i(s)} l_i(s) - \frac{U_l^1(s)}{U_c^1(s)} l_1(s)\right) + \beta(s) x_i'(s) = \frac{x_i U_c^i(s)}{\mathbb{E}_{s_{-}} U_c^i} \text{ for all } s, i \geq 2$$

$$\frac{\mathbb{E}_{s_{-}} U_c^i}{\mathbb{E}_{s_{-}} U_c^1} = \rho_i \text{ for all } i \geq 2$$

$$\frac{U_l^i(s)}{\theta_i(s) U_c^i(s)} = \frac{U_l^1(s)}{\theta_1(s) U_c^1(s)} \text{ for all } s, i \geq 2$$

$$\sum_i \pi_i c_i(s) + g(s) = \sum_i \pi_i \theta_i(s) l_i(s) \quad \forall s$$

$$\rho_i'(s) = \frac{U_c^i(s)}{U_c^1(s)} \text{ for all } s, i \geq 2$$

$$\underline{x}_i(s; x, \boldsymbol{\rho}, s_{-}) \leq x_i(s) \leq \bar{x}_i(s; x, \boldsymbol{\rho}, s_{-})$$

Bellman equation for $t = 0$

$$V_0\left(\{\tilde{b}_{i,-1}\}_{i=2}^{I}, s_0\right) = \max_{c_{i,0},l_{i,0},x_0,\rho_0} \sum_i \pi_i \alpha_i U^i(c_{i,0}, l_{i,0}) + \beta(s_0) V\left(x_0, \rho_0, s_0\right)$$

where the maximization is subject to

$$U^i_{c,0}\left[c_{i,0} - c_{1,0}\right] + U^i_{c,0}\left(\frac{U^i_{l,0}}{U^i_{c,0}} l_{i,0} - \frac{U^1_{l,0}}{U^1_{c,0}} l_{1,0}\right) + \beta(s_0) x_{i,0} = U^i_{c,0}\tilde{b}_{i,-1} \text{ for all } i \geq 2$$

$$\frac{U^i_{l,0}}{\theta_{i,0} U^i_{c,0}} = \frac{U^1_{l,0}}{\theta_{1,0} U^{1,0}_{c}} \text{ for all } i \geq 2$$

$$\sum_i \pi_i c_{i,0} + g_0 = \sum_i \pi_i \theta_{i,0} l_{i,0}$$

$$\rho_{i,0} = \frac{U^i_{c,0}}{U^1_{c,0}} \text{ for all } i \geq 2$$

Long Run: Steady States (SS)

Let $\Psi\left(s; x, \boldsymbol{\rho}, s_{-}\right)$ be an optimal law of motion for the state variables for the $t \geq 1$ recursive problem, i.e.,

$$\Psi\left(s; x, \boldsymbol{\rho}, s_{-}\right) = \left(x'\left(s\right), \rho'\left(s\right)\right)$$

attains $t \geq 1$ value function given state $\left(x, \boldsymbol{\rho}, s_{-}\right)$

Definition

A steady state $\left(x^{SS}, \boldsymbol{\rho}^{SS}\right)$ satisfies
$\left(x^{SS}, \boldsymbol{\rho}^{SS}\right) = \Psi\left(s; x^{SS}, \boldsymbol{\rho}^{SS}, s_{-}\right)$ for all s, s_{-}

A steady state is a node at which the continuation allocation and tax schedule has no further history dependence.

Existence

- **Quasi-linear preferences:**
  1. SS exists for a wide range of parameters and shocks
  2. The economy reaches a steady state in one period
  3. Output, tax rates are constant thereafter and levels are independent of initial conditions

  Dynamics of taxes are starkly different than AMSS
- **General preferences:**
  1. *IID shocks with two values:* SS exists and continuation allocation is independent of initial conditions
  2. *More general shocks:* There exists an ergodic region in which (x, ρ) is no longer constant, but fluctuations are markedly reduced relative to the transient fluctuations that occur during an approach to a SS

Intuition: A two-agent example

Consider I=2 with $\theta_1(s) > \theta_2 = 0$.
Two main forces determine the dynamics of the tax rate, transfers, and assets:

- **Fluctuations in inequality** measured by spreads in marginal utilities
- **Fluctuations in interest rate**

For quasi linear preferences both forces are absent

3 Cases: interest rate and relative assets

- **Normalization:** By Ricardian equivalence, we can
  1. Normalize $b_2(s) = 0$
  2. Government assets: $B(s) = -b_1(s)$
- **Interpretation:** State variable $x \equiv U_c^2 \tilde{b}_2 = U_c^2[b_2 - b_1]$. Under $b_2(s) = 0$ normalization, x is
  1. Marginal utility scaled **debt** of the productive agent
  2. Marginal utility scaled **assets** of the government
- **Results:**

Interest rates	*Discount factors*	$x = U_c^2[b_2 - b_1]$
Countercyclical	$\beta(s_l) = \beta(s_h)$	$x > 0$
Acyclical	$\beta(s_l) > \beta(s_h)$	$x > 0$
Procyclical	$\beta(s_l) >> \beta(s_h)$	$x < 0$

Inequality distortions

Consider case with acyclical interest rates

↓ TFP: Adjust tax rate τ or transfers T, both are costly

Suppose $x = 0$ (or $b_2(s) = b_1(s)$)

1. Present value of earnings of productive agent are higher
2. A reduction in transfers hurts the low productivity agent more

Then

↑ x is same as increasing the **debt** of the productive agent

This drives agents' after-tax, after-interest incomes closer together

Interest rate fluctuations

- **Countercyclical interest rates:**
  $\downarrow$ TFP: If the tax rate τ is left unchanged, the government faces a shortfall of revenues.
  1. Reminder: x is marginal utility scaled **assets** of the government
  2. By holding positive assets the govt. can use higher interest income to offset some revenue losses from its tax on labor in recessions
  3. This force is also present in representative agent economies with endogenous fluctuations in interest rate

  $\uparrow x$
- **Pro-cyclical interest rates:** If the interest rate is sufficiently low in a recession, the government may want to hold debt to free resources by lower interest payments.

  $\downarrow x$

Remarks on SS

- Stability:
  1. **Countercyclical interest rates:** Both forces push in the same direction $\rightarrow$ steady state is stable
  2. **Procyclical interest rates:** Both forces push in opposite direction $\rightarrow$ steady state is unstable
- For more than 2 agents, we have similar mechanics. In particular
  1. Inequality distortions call for a negative correlation between productivities and (scaled) net assets
  2. Procyclical interest rates may flip the sign of the correlation between productivities and net assets to be positive.
     - Low interest rate in recession prompts the government to hold debt
     - By borrowing more from agents with higher productivities, the govt. can the reduce welfare costs of lowering transfers in adverse times

Numerical Example

Use a calibrated version of the economy to

- Approximate magnitudes of these forces and
- Study optimal policy responses at business cycle frequencies when an economy is possibly far away from a steady state

Numerical Example: Calibration

Take a 2-shock 2-type economy with preferences $U(c, l) = \psi \log(c) + (1 - \psi) \log(1 - l)$ and allow $\theta_i(s), \beta(s), g(s)$ to depend on s.

- Pick baseline parameters to match some low frequency moments
- Calibrate outcome fluctuations to match three US recessions (i.e., 1991-92, 2001-02 and 2008-10):
  1. Left tail of the cross-section distribution of labor income falls more than right tail
  2. Short term interest rate falls
  3. Booms last longer than recessions

Calibration

Parameter	Value	Description	Target
ψ	0.6994	Frisch elasticity of labor supply	0.5
$\bar{\theta}_1$	4	Log 90-10 wage ratio (Autor et al.)	4
$\bar{\theta}_2$	1	Normalize to 1	1
β	0.98	Average (annual) risk free interest rate	2%
α_1	0.69	Marginal tax rate in the economy with no shocks	20%
g	12%	Average pre-transfer expenditure-output ratio	12%
$\frac{\hat{\theta}_2}{\hat{\theta}_1}$	2.5	Relative drop in wage income of 10th percentile	2.5
$\hat{\theta}_1$	1.2%	Average output loss	3%
$\hat{\beta}(s)$	1.96%	Difference in real interest rates between booms and recession	1.96%
$P(r\|r)$	0.63	Duration of recessions	2.33 years
$P(b\|b)$	0.84	Duration of booms	7 years

Table: Benchmark calibration

Initial conditions chosen to make debt to GDP ratio be 60% [2]

[2]We use the same normalization as before i.e, the low productive agent has zero assets

Results: Some variants

We study perturbations of the benchmark calibration

1. **Acyclical interest rates**: smaller spread in discount factor shocks
2. **Countercyclical interest rates**: no discount factor shocks
3. **No inequality**: equal fall in all agents' productivities (TFP shock) and no discount factor shocks

Results: Long run

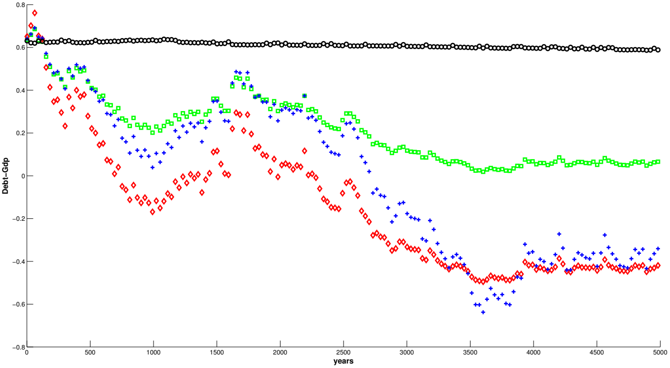

Figure: Govt. debt for several economies: benchmark (o), acyclical interest rates (+), countercyclical interest rates (◇) and no inequality shocks (□)

Observations

- Long run tendency to converge to some ergodic set. But convergence is very slow more details on speed of convergence.
- With low discount factor shocks, outcomes approach positive govt. assets
- With high discount factor shocks that produce procyclical real interest rates, there is no tendency to reduce govt. debt even after 5000 years

Short Run

To understand short run responses

- Solve the time 0 problem with identical initial conditions across different settings. This pins down the initial state vector x_0, ρ_0 that appears in our time 0 Bellman equation
- Use optimal policies to compute fluctuations of different components in the government budget constraint as we move from booms to recessions

Results: Short run

	Δg	ΔB	ΔT	$\Delta[\tau\theta_1 l_1]$	$\Delta[\tau\theta_2 l_2]$	ΔY	$\Delta\tau$
Benchmark	0.0000	-1.1561	0.6871	-0.1593	-0.3096	-2.8536	0.3732
Acyclical Interest Rates	0.0000	-1.1126	0.6591	-0.1497	-0.3038	-2.8613	0.3879
Countercyclical Interest Rates	0.0000	-1.0794	0.6387	-0.1415	-0.2992	-2.8677	0.3997
No Inequality	0.0000	-0.1380	**-0.5459**	-0.5635	-0.1204	-2.6294	0.0622

Table: The table tells changes in different components of government budget as the economy transits from "boom" to "recession". All numbers except τ are normalized by undistorted GDP and reported in percentages.

1. For each variable z in the table we report $\Delta z \equiv \left(z\left(s_l|x_0,\rho_0,s_0\right) - z\left(s_h|x_0,\rho_0,s_0\right)\right)/\bar{Y}$ where $\bar{Y}$ is average undistorted GDP in percentages
2. Predetermined variables like repayment on existing debt drop out

$$\Delta[g] + \Delta[T] + \Delta[B] = \Delta[\tau\theta_1 l_1] + \Delta[\tau\theta_2 l_2]$$

Concluding Remarks

- Size of government debt alone is irrelevant $\Longrightarrow$ need to know distribution of net assets
- Optimal tax and transfer scheme balance
  1. welfare losses from fluctuating taxes
  2. welfare losses from fluctuating transfers
- With incomplete markets, interest rate fluctuations are a key determinant of long-run correlations between productivities and net assets
- Ignoring heterogeneity produces misleading results about the size and direction of short run optimal policy responses

Speed of convergence (I)

Suppose we are in the binary-IID world where steady states are deterministic.

- The optimal policy induces two *risk adjusted* martingales
  1. Multiplier on the implementability constraint : $\boldsymbol{\mu}_t$
  2. The ratio of marginal utilities: $\boldsymbol{\rho}_t$
- One can represent the optimal allocation recursively in terms of $\{\boldsymbol{\mu}(s^{t-1}), \boldsymbol{\rho}(s^{t-1})\}$ and s_t.
- Why $(\boldsymbol{\mu}, \boldsymbol{\rho})$ instead of $(x, \boldsymbol{\rho})$?
- Linearize optimal policies for each s_t around the constant steady state.
- Study the eigenvalues of the conditional mean and variance dynamics (these are deterministic linear systems)

Speed of convergence (II)

Let $\hat{\Psi}_t = \begin{bmatrix} \boldsymbol{\mu}_t - \boldsymbol{\mu}^{SS} \\ \boldsymbol{\rho}_t - \boldsymbol{\rho}^{SS} \end{bmatrix}$. Then

$$\hat{\Psi}_{t+1} = B(s_{t+1})\hat{\Psi}_t$$

This linearized system has coefficients that are functions of the shock s.

Proposition

If the (real part) of eigenvalues of $\mathbb{E}B(s)$ are less than 1, the system converges to zero in mean. Further for large t, the conditional variance of $\hat{\Psi}$, denoted by $\Sigma_{\Psi,t}$, follows a deterministic process governed by

$$vec(\Sigma_{\Psi,t}) = \hat{B}vec(\Sigma_{\Psi,t-1}),$$

where $\hat{B}$ is a square matrix of dimension $(2N-2)^2$. In addition, if the (real parts) of eigenvalues of $\hat{B}$ are all less than 1, the system converges in probability.

The eigenvalues (in particular the largest one) are instructive not only for whether the system is locally stable but also for how quickly the steady state is reached

Speed of convergence: Size of shocks and risk aversion

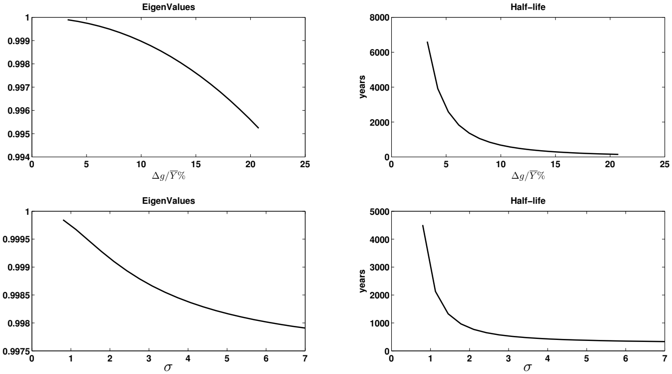

Figure: The top (bottom) panel plots the dominant eigenvalue of $\hat{B}$ and the associated half life as we increase the spread between the expenditure levels (risk aversion).

Back to main