Optimal Taxation with Incomplete Markets

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Abstract

KEYWORDS:

1 Introduction

2 Environment

We analyze economies that share the following features. Government expenditures at time t, $g_t = g(s_t)$, and a productivity shock $\theta_t = \theta(s_t)$ are both functions of a Markov shock $s_t \in \mathcal{S}$ having $S \times S$ transition matrix Π and initial condition s_{-1}. We will denote time t histories with s^t and z_t will refer to a generic random variable measurable with respect to s^t. Sometimes we will denote $z_t(s^t)$ indicate a particular realization of z_t. An infinitely lived representative consumer has preferences over allocations $\{c_t, l_t\}_{t=0}^{\infty}$ of consumption and labor supply that are ordered by

$$\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t U\left(c_t, l_t\right) \tag{1}$$ `eqn:obj`

dge Where U is the period utility function for consumption and labor. For most of the paper we will assume U separable in consumption and labor. Additional assumptions will be made as needed. Labor produces output via the linear technology

$$y_t = \theta_t l_t$$

The representative consumer's tax bill at time $t \geq 0$ is

$$-T_t + \tau_t \theta_t l_t, \; T_t \geq 0,$$

where $\tau_t(s^t,)$ is a flat rate tax on labor income and T_t is a nonnegative transfer. Often, we'll set $T_t = 0$. The government and consumer trade a single possibly risky asset whose time t payoff p_t is described by an $S \times S$ matrix $\mathbb{P}$:

$$p_t = \mathbb{P}(s_t, s_{t-1})$$

Let B_t denote the government's holdings of the asset and b_t be the consumer's holdings. Let $q_t = q_t(s^t)$ be the price of the single asset at time t. At $t \geq 0$, the household's time budget constraint is

$$c_t + b_t = (1 - \tau_t)\, \theta_t l_t + \frac{p_t}{q_{t-1}} b_{t-1} + T_t \tag{2}$$ eqn:HHbudget

and the government's is

$$g_t + B_t + T_t = \tau_t \theta_t l_t + \frac{p_t}{q_{t-1}} B_{t-1}. \tag{3}$$ eqn:Govbudget

Feasible allocations satisfy

$$c_t + g_t = \theta_t l_t, \; \forall t \geq 0 \tag{4}$$ eqn:ResFeas

Clearing in the time $t \geq 0$ market for the single asset requires

$$b_t + B_t = 0. \tag{5}$$ eqn:bondmarket

Initial assets satisfy $b_{-1} = -B_{-1}$[1] An initial value of the exogenous state s_{-1} is given. Equibrium objects including $\{c_t, l_t, \tau_t\}_{t=0}^{\infty}$ will come in the form of sequences of functions of initial government debt b_{-1} and $s^t = [s_t, s_{t-1}, \ldots, s_0, s_{-1}]$.

Borrowing from a standard boilerplate, we use the following:

[1] apb We assume that b_{-1} are obligations with acrrued interest. This is equivalent to setting $q_{-1} = 1$

Definition 2.1. *An **allocation** is a sequence $\{c_t, l_t\}_{t=0}^{\infty}$ for consumption and labor.* *dge An **asset profile** is a sequence $\{b_t, B_t\}_{t=0}^{\infty}$. A **price system** is a sequence of asset prices $\{q_t\}_{t=0}^{\infty}$. A **budget-feasible government policy** is a sequence of taxes and transfers $\{\tau_t, T_t\}_{t=0}^{\infty}$*

Definition 2.2. *Given $(b_{-1} = -B_{-1}, s_{-1})$ and a government policy, a **competitive equilibrium with distorting taxes** is a price system, dge an asset profile, a ~~budget feasible~~ government policy, and an allocation such that the allocation maximizes (1) subject to (2) dge and $\{b_t\}_{t=0}^{\infty}$ bounded given prices; and ~~the bond market clears~~ dge equations (3), (4) and (5) are satisfied.*

Definition 2.3. *Given (b_{-1}, B_{-1}, s_{-1}), a **Ramsey plan** is a welfare-maximizing competitive equilibrium with distorting taxes.*

3 Two Ramsey problems

Following Lucas and Stokey (1983) and Aiyagari et al. (2002), we use a "primal approach." To encode a government policy and price system as a restriction on an allocation, we first obtain the representative household's first order conditions[2]

⟨eqn:HHFOC⟩

$$U_{c,t} q_t = \beta \mathbb{E}_t p_{t+1} U_{c,t+1} \tag{6a}$$

eqn:Euler

$$(1 - \tau_t)\theta_t U_{c,t} = -U_{l,t} \tag{6b}$$

?eqn:lcFOC?

We substitute these into the household's budget constraint to get a difference equation that we solve forward at every history for every $t \geq 0$. That yields *implementability constraints* on a Ramsey allocation that fall into two categories: (1) the time $t = 0$ version is identical with the *single* implementability constraint imposed by Lucas and Stokey (1983); (2) the time $t \geq 1$ implementability constraints are counterparts of the additional *measurability restrictions* that Aiyagari et al. (2002) impose on a Ramsey allocation.

We first state our Ramsey problem, then Lucas and Stokey's.

rob:RamseyBEGS⟩

[2]We thus focus on interior equilibria. dge Arguments in Magill and Quinzii (1994) and Constantinides and Duffie (1996) can be used to show that any allocation c_t, l_t, b_t that satisfy equations (2),(6) and bounded $\{b_t\}$ is a solution to the consumers problem.

Problem 3.1. *The Ramsey problem is to choose an allocation and an ~~appropriately measurable~~dge bounded government debt sequence $\{b_t\}_{t=0}^{\infty}$ that attain:*

$$\max_{\{c_t,l_t,b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \tag{7}$$?eqn:Ramseyobj

subject to

$$c_t + g_t = \theta_t l_t, \; t \geq 0 \tag{8a}$$ eqn:feas

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right) \tag{8b}$$ eqn:LSimplemen

$$\frac{b_{t-1} U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1} p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \; \textit{for } t \geq 1 \tag{8c}$$ eqn:AMSSimplem

⟨prob:RamseyLS⟩ **Problem 3.2.** *Lucas and Stokey's Ramsey problem is to choose an allocation that attains*

$$\max_{\{c_t,l_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \tag{9}$$?eqn:Ramseyobj

subject to the single implementability constraint (8b) *and feasibility* (8a) *for all t, s^t.*

Remark 3.3. *Equation* (8a) *imposes feasibility, while equation* (8b) *is the single implementability constraint present in Lucas and Stokey (1983). Equations* (8c) *express additional ~~implementability constraints that comprise~~ implementability constraints at every node from time $t \geq 1$. These generalize the Aiyagari et al. (2002) measurability constraints on a Ramsey allocation to our more general payoff structure $\mathbb{P}$ for the single asset. The measurability constraints* (8c) *are cast in terms of the date, history $(t-1, s^{t-1})$ measurable state variable b_{t-1} that for $t \geq 1$ is absent from Lucas and Stokey's complete markets Ramsey problem. Evidently, Ramsey allocation for our incomplete markets economy automatically satisfies the single implementability constraint imposed by Lucas and Stokey.*

⟨rem:LSdebt⟩ **Remark 3.4.** *State-contingent, but not history-dependent, values of consumption, labor supply, and continuation government debt $\check{b}(s)$ solve the Lucas and Stokey (1983) Ramsey problem 3.2. As intermediated by the Lagrange multiplier on the implementability constraint* (8b)*, consumption, labor supply, and $\check{b}(s)$ are functions of initial government debt b_{-1} and the current state s_t, but not past history s^{t-1}.*

3.1 Motivation for quasi-linear U

usequasilinear⟩

Asymptotic properties of a Ramsey plan for our incomplete markets economy vary with asset returns $R_{t-1,t} \equiv \frac{\mathbb{P}(s_t|s_{t-1})}{q_{t-1}}$. We see that $\mathbb{P}$ affects these returns directly through the ex-post exogenous payoffs and indirectly through prices q_{t-1}. To focus exclusively on the exogenous $\mathbb{P}$ part of returns, we begin by studying an economy with quasi-linear utility function:

$$U(c,l) = c - \frac{l^{1+\gamma}}{1+\gamma}, \tag{10}$$?eqn:UQL?

which sets $U_{c,t} = 1$. Asymptotic properties of a Ramsey plan for our incomplete markets economy vary with asset returns that reflect properties of equilibrium prices $\{q_t(s^t|B_{-1}, s_{-1})\}_t$ and the exogenous asset payoff matrix $\mathbb{P}$. At an interior solution, quasi-linear preferences and the Euler equation (6a) pins down $q_t = \beta \mathbb{E}_t \mathbb{P}(s_{t+1}|s_t)$. After studying the consequences of quasi-linear utility, we shall solve for Ramsey plans for utility functions that express risk aversion with respect to consumption and so activate endogenous fluctuations in q_t.

4 Quasi-linear preferences

Throughout this section, we assume that U is quasi-linear and use an indirect three step approach to characterize the asymptotic behavior of government debt and the tax rate.
(1) **Construct an optimal payoff matrix** [apb]Do we want to define what it means to be an optimal payoff matrix (may be within a class of $\mathcal{S} \times \mathcal{S}$ matrices whose rows have conditional means of 1.

We pose the following problem:

b:PPoperator⟩?

Problem 4.1. *Given arbitrary initial government debt b_{-1}, what is an optimal asset payoff matrix?*

[dge]Let $\mathcal{P}$ be the set of all $\mathcal{S} \times \mathcal{S}$ real matrixes. Define the indirect utility function $\mathcal{W}(\mathbb{P}; b_{-1})$ as the solution to problem 3.1 for all $\mathbb{P} \in \mathcal{P}$ and initial debt b_{-1}. This induces an operator $\mathbb{P}^*$ that maps initial government debt into an optimal payoff matrix

$$\mathbb{P}^*(b_{-1}) \in \arg\max_{\mathbb{P} \in \mathcal{P}} W(\mathbb{P}; b_{-1})$$

(2) **Apply the inverse of the operator $\mathbb{P}^*$.**

For an arbitrary payoff matrix $\mathbb{P}$, let

$$\mathbb{P} = \mathbb{P}^*(b^*) \text{ or } b^* = \mathbb{P}^{*-1}(\mathbb{P})^3. \tag{11}$$ eqn:invPoperat

apbThe statement needs more discussion. It is specially vague if we dont define what we mean by "optimal" payoff matrix For initial government debt $b_{-1} = b^*$, a Ramsey plan for the incomplete markets economy has $b_t = b^*$ for all $t \geq 0$.

(3) **Long run assets**

Starting from an arbitrary initial government b_{-1} and an arbitrary payoff matrix $\mathbb{P}$, establish conditions under which $b_t \to b^*$ under a Ramsey plan.

In particular, where $S = 2$ and shocks s_t are IID, we describe a large set of $\mathbb{P}$'s for which government debt b_t under a Ramsey plan converges to b^* that solves (11). For more general shock processes, we numerically find an ergodic set of b_t's hovering around a debt level $\cancel{\tilde{b}}$ b^*;

~~The optimal $\mathbb{P}^*$ associated with $\tilde{b}$ approximates $\mathbb{P}$~~ dge(Given how important this is it is probably worth a week trying to nail this down a bit more):

apbI agree with David. I have put an adhoc proposal by defining the inverse operator more generally but we need to talk and possibly work out this part Here is how we execute these three steps.

(1) apbThe optimal payoff matrix is obtained by reverse engineering a Lucas-Stokey Ramsey allocation. for a given b_{-1} by solving problem 3.2

(2) apbNext we find a sequence $\{p_t\}_t$ that satisfies the implementability constraints imposed in (8c) Note that these implementability constraints are invariant to scaling of p_t by a constant k_{t-1} that can depend on s^{t-1}. From the equivalence class of $\{p_t\}_t$'s that satisfy (8c) at the Lucas-Stokey Ramsey allocation, select a $\{p_t\}_t$ that imposes the normalization $\mathbb{E}_{t-1}U_{c,t}p_t = 1$ and satisfies

$$p_t = \frac{\beta}{U_{c,t-1}b_{t-1}U_{c,t}}\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j}c_{t+j} + U_{l,t+j}l_{t+j}\right) \tag{12}$$ eqn:pdisarm

Note that by construction, p_t disarms the time $t \geq 1$ measurability constraints[4]. apb???

[3] apbIn cases where this inverse does not exist, we define $\mathbb{P}^{*-1}(\mathbb{P}) = \min_b \|\mathbb{P} - \mathbb{P}^*(b^*)\|$ under XXXX norm

[4] apbThe normalization of p_t reduces to a restriction of its conditional mean to unity in the quasi linear setup. Although we are studying the quasi linear preferences in this section, the exercise of obtaining the optimal payoffs sequence $\{p_t\}_t$ is general.

Do we have an expression for b_{t-1} appearing in the (12) ? Are we doing this for just QL preferences?

(3) Using the fact noted in remark 3.4 that the Lucas-Stokey Ramsey allocation is not history-dependent, construct the optimal payoff matrix as

$$\mathbb{P}^*(s_t, s_{t-1}|b_{-1}) = p_t.$$

Thus, given initial government debt b_{-1}, let $\mu(b_{-1})$ be the Lagrange multiplier on the Lucas-Stokey implementability constraint (8b) at the Lucas-Stokey Ramsey allocation.

[apb]Do we want something in the appendix to fill in the missing steps ?

The tax rate in the Ramsey allocation is $\tau(\mu) = \frac{\gamma\mu}{(1+\gamma)\mu-1}$, which implies a net-of-interest government surplus $S(s,\tau)$ that satisfies

$$S(s,\tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1-\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

The 'disarm-the-measurability-constraints' equation (12) then implies that the optimal payoff matrix is

$$\mathbb{P}^*(s, s_-|b_{-1}) = (1-\beta)\frac{S(s,\tau)}{\mathbb{E}_{s_-}S(s,\tau)} + \beta. \tag{13}$$ eqn:optPP

[tjs]Anmol and David XXXXXX: let's add some snazzy interpretation of the previous nice equation.

To appreciate how initial government debt level influences the optimal asset payoff structure via formula (13), call a state s "adverse" if it implies either "high" government expenditures or "low " TFP; formally, say that s is "adverse" if

$$g(s)\mathbb{E}_{s_-}\theta^{\frac{\gamma}{1+\gamma}} - \theta(s)^{\frac{\gamma}{1+\gamma}}\mathbb{E}_{s_-}g > 0$$

A "good" state is the opposite of an "adverse" state. [apb]XXXX We need an expression for (??) interms for b_{-1} to make sense of the statement below.

Then (13) implies that when initial government assets are positive, $\mathbb{P}^*$ pays *more* in "adverse" states, while when initial government assets are negative, $\mathbb{P}^*$ pays *less* in "adverse" states.

4.1 Analysis of $S = 2$ iid shocks

We temporarily assume that s_t is i.i.d and $S = 2$. ~~In this case, $\mathbb{P}(s_-, s)$ is independent of s_-, so that $\mathbb{P}$ can be taken to be a vector. Under the normalization $q_t = \beta$, $\mathbb{E}\mathbb{P}(s) = 1$, payoffs on the single asset are determined by a scalar $\boldsymbol{p}$, the payoff in state 1. and a risk-free bond is a security for which $\boldsymbol{p} = 1$.~~

apbXXXX The previous line is confusing since $\mathbb{P}$ is exogenous and s_t being IID has no "implication" for $\mathbb{P}$ being a vector. What we are actually doing is restricting our attention to a smaller class of payoff matrices since we know that the optimal one has similar features. I would suggest something as follows

apbIn this case note that (13) implies that the optimal payoff matrix $\mathbb{P}^*$ has the same rows. Given this we restrict our attention to $\mathbb{P}(s_-, s)$ that have payoffs independent of s_-. This reduces $\mathbb{P}$ to be a vector. Under the normalization $\mathbb{E}\mathbb{P}(s) = 1$, payoffs on the single asset are determined by a scalar $\boldsymbol{p}$, the payoff in state 1. ~~and a risk-free bond is a security for which $\boldsymbol{p} = 1$.~~

dge A risk free bond is then a security for which $\boldsymbol{p} = 1$. ~~We take $\boldsymbol{p}$ to be the asset's payoff in the "good" state s.~~ dgeWithout loss of generality we will assume that $g(1)\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}} - \theta(1)^{\frac{\gamma}{1+\gamma}}\mathbb{E}g < 0$, and thus, $\boldsymbol{p}$ is the payoff in the "good" state of the world.dge As the opitmal payoff matrix can be summarized by a signle scalar variable we can reinterperate the optimal matrix map $\mathbb{P}^*(b)$ as a single scalar function $\boldsymbol{p}^*(b)$. The steady state level of debt for an exogeneous restriction on the payoff $\boldsymbol{p}$ is then the scalar, ~~Define a scalar b^* by~~

$$b^* = \mathbb{P}^{*-1}(\boldsymbol{p}) \tag{14}$$ eq-ss

~~so that b^* satisfies $p = \mathbb{P}^*(b^*)$.~~ tjsAnmol and David XXXXXX: I have changed and slightly abused notation. Let's talk about it. apbXXXX It is confusing. A possibility is define it implicitly

$$\mathbb{P}^*(b^*) = \mathbb{P}(\boldsymbol{p}) \tag{15}$$ eq-ss

:ssexistence)? **Proposition 4.2.** ~~*Suppose that s shocks affect either g or θ. Then there*~~[5] dge*There exists* $0 \geq \alpha_2 \geq \alpha_1 \geq 1$ *such that*

a. *If* $\boldsymbol{p} \leq \alpha_1$, *then* $b^* < 0$

[5] dgeWe don't need to assume either g or θ shocks as we have allready characterized what an adverse state of the world is

b. *If* $\boldsymbol{p} \geq \alpha_2$, *then* $b^* > 0$

c. *If* $\alpha_1 > \boldsymbol{p} > \alpha_2$, *then* b^* *solving* (15) *does not exist*

Proof. [dge]Let g_1 and θ_1 denote the exogenous government expenditure and TFP in the "good" state of the world. Before starting this proof we should note some facts about the complete markets solution. ~~As~~ $\cancel{(1-\tau)^{\frac{1}{\gamma}}\tau}$ ~~is maximized at~~ $\cancel{\frac{\gamma}{1+\gamma}}$ ~~there will be an upper bound for government debt for which a solution to the complete markets problem exists:~~ $\cancel{\bar{b}}$~~. This space of solutions to the complete markets problem can be indexed by~~ $\cancel{b \in (-\infty, \bar{b}]}$~~. If we let~~ $\cancel{\tau(b)}$ ~~the mapping from initial government debt into optimal tax rate, then~~ $\cancel{\tau}$ ~~is an increasing function of~~ $\cancel{b}$ ~~and~~ $\cancel{\tau((-\infty,\bar{b}]) = (-\infty, \frac{\gamma}{1+\gamma}]}$~~. Substituting~~ $\cancel{S(s,\tau)}$ ~~into equation we obtain~~ [dge] The government surplus in any state of the world is given by

$$S(s,\tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1-\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

This surplus is therefore maximized when $(1-\tau)^{\frac{1}{\gamma}}\tau$ is maximized at $\tau = \frac{\gamma}{1+\gamma}$, moreover in the region $(-\infty, \frac{\gamma}{1+\gamma}]$ the function $S(\cdot,\tau)$ is an increasing function of τ. In an i.i.d. world then government debt with complete markets associated with a constant tax rate τ is

$$\frac{1}{1-\beta}\sum_s \Pi(s)S(s,\tau)$$

is an increasing function of tau. The maximal initial government debt sustainable in incomplete markets is then given by

$$\bar{b} = \frac{1}{1-\beta}\sum_s \Pi(s)\theta(s)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s)$$

Inverting the mapping from tax rates into government debt we obtain the function $\tau(b)$, which is the mapping from initial government debt into the optimal tax rate. $\tau(b)$ is an increasing function of b on the domain of complete markets initial debts $(-\infty, \bar{b}]$, with $\tau((-\infty,\bar{b}]) = \left(-\infty, \frac{\gamma}{1+\gamma}\right]$.

[dge]Substituting the formula for $S(s,\tau)$ into equation (13) we obtain

$$\boldsymbol{p}^*(\tau) = (1-\beta)\frac{\theta_1^{\frac{\gamma}{1+\gamma}}(1-\tau)^{\frac{1}{\gamma}}\tau - g_1}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}(1-\tau)^{\frac{1}{\gamma}}\tau - \mathbb{E}g} + \beta$$

Solving for $(1-\tau)^{\frac{1}{\gamma}}\tau$ we obtain

$$(1-\tau)^{\frac{1}{\gamma}}\tau = \frac{(\boldsymbol{p}^* - \beta)\mathbb{E}g - (1-\beta)g_1}{(\boldsymbol{p}^* - \beta)\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}} - (1-\beta)\theta_1^{\frac{\gamma}{1+\gamma}}}$$

The space of complete market optimal tax rates is $(-\infty, \frac{\gamma}{1+\gamma}]$, as $(1-\tau)^{\frac{1}{\gamma}}\tau$ is one to one on this domain and $b(\tau)$ is increasing on this domain we conclude that $\boldsymbol{p}^*(b)$ is one to one. Differentiating $\boldsymbol{p}^*(\tau)$with respect to τ quickly yields

$$\frac{d}{d\tau}\boldsymbol{p}^*(\tau) = (1-\beta)(1-\tau)^{\frac{1}{\gamma}-1}\left[\gamma - (1+\gamma)\tau\right]\frac{g_1\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}} - \theta_1^{\frac{\gamma}{1+\gamma}}\mathbb{E}g}{(\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}(1-\tau)^{\frac{1}{\gamma}}\tau - \mathbb{E}g)^2} < 0$$

implying that $\boldsymbol{p}^*(b)$ is decreasing in b. As $b=0$ implies that $\mathbb{E}S(\tau(b)) = 0$ the function function $\boldsymbol{p}^*(b)$ will have a pole at $b=0$. $\boldsymbol{p}^*(b)$ decreasing in b must therefore imply that $\lim_{b\to 0^-}\boldsymbol{p}^*(b) = -\infty$ and $\lim_{b\to 0^+}\boldsymbol{p}^*(b) = \infty$. We conclude that

$$\boldsymbol{p}^*((-\infty, \bar{b}]) = \boldsymbol{p}^*((-\infty, 0)) \cup \boldsymbol{p}^*((0, \bar{b}]) = (-\infty, \alpha_1) \cup [\alpha_2, \infty)$$

The bounds α_1 and α_2 are then computed by taking the limits of $\boldsymbol{p}^*$ as b approaches $-\infty$ and $\bar{b}$ (upper bound for complete market debt), or equivalently as τ approaches $-\infty$ and $\frac{\gamma}{1+\gamma}$ respectively. $\square$

With only government expenditure shocks, we compute

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$

With only TFP shocks, we compute

$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

Theorem 4.3. *Let b_{fb} denote the level of government debt ~~that supports~~ [dge] associated with* *the first-best allocation with complete markets.* [apb]*XXXX We can define this as the right hand side of the time 0 implementability constraint evaluated at the FB allocation right ?* *Then*

a. If $\boldsymbol{p} \leq \min(\alpha_1, 1)$*, then* $b_{fb} < b^* < 0$ *and* $b_t \to b^*$ *with probability 1.*

b. If $\boldsymbol{p} \geq \alpha_2$*, then* $0 < b^*$ *and* $b_t \to b^*$ *with probability 1.*

c. If $\cancel{\alpha_2 < \boldsymbol{p} < \min(\alpha_1, 1)}$[dge]$\min(\alpha_1, 1) < \boldsymbol{p} < \alpha_2$*,* b^* *either does not exist or is unstable.*

For $\boldsymbol{p}$ in region (c.), the government tends to run up debt over time.

Proof. [dge]~~The first order conditions governing the optimal allocation allow us to treat μ_t the multiplier on the measurability constraints as the state variable.~~ [apb]The optimal allocation permits recursive representation as functions $c_t(\mu_t), l_t(\mu_t)$ and $b_t(\mu_t)$ with a law of motion for μ_t. [dge] We will show global stability under the assumption that $\mu'(\mu, s)$ an increasing function of μ[6] The heart of the proof revolves around the twisted-martingale equation for μ:

$$\mu_t = \sum_s \Pi(s) p_s \mu'(\mu_t, s) = \mathbb{E}_t p_{t+1} \mu_{t+1}$$

We have shown that there is at most an unique μ^* such that $\mu'(\mu^*, s) = \mu^*$ for all s. For this sketch we will focus on showing global stability for $\mu < \mu^*$. The twisted-martingale equation can be decomposed as follows

$$\mu_t = \mathbb{E}_t \mu_{t+1} + Cov_t(p_{t+1}, \mu_{t+1})$$

by signing $Cov_t(p_{t+1}, \mu_{t+1})$ we are able to determine if μ_t follows a sub or super-martingale. Given that μ_t is bounded from above[7], we can conclude global convergence to the steady state if μ_t is a supermartingale. As in the statement of the theorem we will split the proof up into three cases, recall $\boldsymbol{p}$ is the payoff in state 1 (the "good" state)

1. $\boldsymbol{p} < \min\{1, \alpha_1\}$**:** Let $\bar{b}_s$ be maximal debt the government ~~good~~ could enter with and be able to pay off assuming it received shock s from this period onward, then

$$\bar{b}_s = \left(\frac{p_s}{\beta} - 1\right)^{-1} \left(\theta_s^{\frac{\gamma}{1+\gamma}} \left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}} \frac{\gamma}{1+\gamma} - g_s\right)$$

[6][dge] I'm still working on this. I'm not sure if we'll need to include it as an assumption or not

[7][dge]Given that $\mu'(\mu^*, s) = \mu^*$ and $\mu'(\mu, s)$ is increasing in μ we know that if $\mu_t < \mu^*$ then $\mu_{t+j} < \mu^*$ for all histories s^{t+j}

as the government tax revenue is maximized by setting the proportional tax to $\frac{\gamma}{1+\gamma}$. For $\boldsymbol{p} < \alpha_2$ it is possible to show that $\bar{b}_1 > \bar{b}_2$ and thus the natural debt limit is obtained by the "adverse" state. This implies that $\lim_{\mu\to-\infty} b(\mu) = \bar{b}_2$ and thus $\lim_{\mu\to-\infty} \mu'(\mu, 2) = -\infty$. In order for the period by period budget constraint

$$\frac{p_s}{\beta} b(\mu) = S(\mu'(\mu, s)) + b(\mu'(\mu, s))$$

to be satisfied for all s it must be $\lim_{\mu\to-\infty} \mu'(\mu, 1) > -\infty$ (as $\bar{b}_1 > \bar{b}_2$). Continuity of μ with the uniqueness of the steady state μ^* then implies that $\mu'(\mu, 1) > \mu'(\mu, 2)$ for all $\mu < \mu^*$. $\boldsymbol{p} < 1$ implies that $p_1 < p_2$ allowing us to conclude that $Cov_t(p_{t+1}, \mu_{t+1}) < 0$. We then have that

$$\mu_t < \mathbb{E}_t \mu_{t+1}$$

for $\mu_t < \mu^*$. As $\mu'(\mu, s)$ is increasing, continuous and $\mu'(\mu^*, s) = \mu^*$, we can iterate on the policy rules to show that if $\mu_t < \mu^*$ the for all $j > 0$ we must have $\mu_{t+j} < \mu^*$. Thus, if $\mu_t < \mu^*$ then μ_t is a supermartingale bounded from below implying that $\mu_t \to \tilde{\mu}$ for some constant $\tilde{\mu}$ with probability 1. It is then just a matter of using the continuity of $\mu'(\mu, s)$ to show that

$$\mu'(\tilde{u}, s) = \tilde{\mu}$$

implying that $\tilde{\mu} = \mu^*$, as μ^* is the unique steady state. The steady state is then globally stable as $\mu_t \to \mu^*$ with probability 1.

2. $\boldsymbol{p} \geq \alpha_2$**:** Following a similar method as in the previous case we know for $\boldsymbol{p} > \alpha_2$ that $\bar{b}_1 < \bar{b}_2$ implying that the natural debt limit is obtained using the "good" state. As in the previous case by taking limits we obtain $\lim_{\mu\to-\infty} \mu'(\mu, 1) = -\infty$ and $\lim_{\mu\to-\infty} \mu'(\mu, 2) > -\infty$. This implies that $\mu'(\mu, 1) < \mu'(\mu, 2)$ which, along with $p_1 > p_2$, implies $Cov_t(p_{t+1}, \mu_{t+1}) < 0$. As before, we then have global stability of the steady state for $\mu_t < \mu^*$.

3. $\min(\alpha_1, 1) < \boldsymbol{p} < \alpha_2$**:** In this case either there exists a steady state if $1 < \boldsymbol{p} \leq \alpha_1$ or there does not exist a steady state. In either case the analysis for the first case implies that $\mu'(\mu, 1) > \mu'(\mu, 2)$ for $\mu < \mu^*$.[8] As $\boldsymbol{p} > 1$ implies that $p_1 > p_2$, we can therefore

[8]dge In the case where there does not exists a steady state take μ^* to be ∞

conclude that $Cov_t(p_{t+1}, \mu_{t+1}) > 0$ implying that

$$\mu_t > \mathbb{E}_t \mu_{t+1}$$

We thus cannot apply the martingale convergence theorem and the steady state will locally be unstable. [apb]??? We havent defined local stability?

$\square$

4.2 Economic forces driving convergence

In summary, when the aggregate state follows a 2-state i.i.d. process, government debt b_t often converges to b^*, while the tail of the allocation equals Ramsey allocation for an economy with complete markets and initial government debt b^*. The level and sign of b^* depend on the asset payoff structure, which we have expressed in terms of a scalar $\boldsymbol{p}$ that concisely captures what in more general settings we represented with the asset payoff matrix $\mathbb{P}$.

Facing incomplete markets, the Ramsey planner recognizes that the government's debt *level* combines with the payoff structure on its debt instrument to affect the welfare costs associated with varying the distorting labor tax rate across states. When the instrument is a risk-free bond, the government's marginal cost of raising funds μ_t is a martingale. In this situation, *changes* in debt levels help smooth tax distortions across time. However, if the payoff on the debt instrument varies across states, then by affecting its state-contingent revenues, the *level* of government debt can help smooth tax distortions across states. For our two state, iid shock process, the steady state debt level b^*, when it exists, is the unique amount of government debt that provides just enough "state contingency" completely to fill the void left by missing assets markets. ~~Suppose $b_t > b^*$ and while considering to accumulate assets~~ ~~When it is away from b^* and considers issuing or accumulating debt starting~~, the Ramsey planner ~~tells the government to take~~ takes into account the ~~prospective benefits that will eventually accrue from being closer to b^*~~ [dge]additional benefits from tax smoothing as the government debt approaches b^*[9]; that puts a risk-adjustment into the martingale governing μ and leads the government either to accumulate or decumulate debt. Although accumulating government assets requires raising distorting taxes, locally the welfare costs of

[9][dge]I changed this since the Ramsey planner receives benefits as it moves towards b^* since it relaxes the wedge between implementability constraints. Personally I believe that it is local benefits rather than long run benefits driving convergence.

higher taxes are second-order and so are dominated by the welfare gains from approaching b^*, which are first-order.

5 Turning on risk-aversion

We now depart from quasi-linearity of $U(c,l)$ and thus activate an additional source of return fluctuations coming from endogenous fluctuations in prices of the asset q_t ~~namely, fluctuations in the risk-free and other interest rates~~. To obtain a recursive representation of a Ramsey plan, we employ the endogenous state variable

$$x_t = u_{c,t} b_t,$$

and study how long-run properties of x_t depend on equilibrium returns $R_{t,t+1} = \frac{\mathbb{P}(s_t,s_{t+1})}{q_t(s^t)}$. Activating risk aversion in consumption makes q_t vary in interesting ways.

Commitment to a Ramsey plan implies that government actions at $t \geq 1$ are constrained by the household's anticipations about them at $s < t$ Following Kydland and Prescott (1980), we use the marginal utility of consumption that the Ramsey planner promises to the household to account for that 'forward looking' restriction [apb]For eg., that comes from the fact that the Euler equation restricts allocations such that expected marginal utility in time t is constrained by consumption choices in time $t-1$. on the Ramsey planner. It is convenient for us that scaling the household's budget constraint by the marginal utility of consumption makes Ramsey problem recursive in $x = U_c b$. In particular, implementability constraints (8c) can be represented as

$$\frac{x_{t-1}\mathbb{P}(s_t, s_{t-1})U_{c,t}}{\beta \mathbb{E}_{t-1}\mathbb{P}U_{c,t}} = U_{c,t}c_t + U_{l,t}l_t + x_t, \ \ t \geq 1 \tag{16}$$ {?}

[tjs]Team XXXXX: check the above carefully.

RamseyBellman⟩ **Problem 5.1.** *Before the realization of the time t Markov shock s_t, let $V(x, s_{-1})$ be the* expected *continuation value of the Ramsey plan at $t \geq 1$ given promised marginal utility government debt inherited from the past $x = U_{c,t}b_t$ and time $t-1$ Markov state s_{-1}. After the realization of time 0 Markov shock s_0, let $W(b_{-1}, s_0)$ be the value of the Ramsey plan when initial government debt is b_{-1}. The (ex ante) Bellman equation for $t \geq 1$ is*

$$V(x, s_-) = \max_{c(s),l(s),x'(s)} \sum_s \Pi(s, s_-)\Big(U(c(s), l(s)) + \beta V(x'(s), s)\Big) \tag{17}$$ eqn:Bellman1

subject to $x'(s) \in [\underline{x}, \overline{x}]$ *and*

$$\frac{x\mathbb{P}(s, s_{-})U_c(s)}{\beta \mathbb{E}_{s_{-}} \mathbb{P} U c} = U_c(s)c(s) + U_l(s)l(s) + x'(s) \tag{18}$$ timetBellimple

$$c(s) + g(s) = \theta(s)l(s) \tag{19}$$ timetfeas

Equation (18) *is the implementability constraint and* (19) *is feasibility. Given an initial debt* b_{-1},*time* 0 *Markov state* s_0, *and continuation value function* $V(x, s_{-})$, *the (ex post) time* 0 *Bellman equation is*

$$W(b_{-1}, s_0) = \max_{c_0, l_0, x_0} U(c, l) + \beta V(x_0, s0) \tag{20}$$?eqn:Bellman0?

subject to time zero implementability constraint

$$U_c(c_0, l_0)c + U_l(c_0, l_0)l_0 + x_0 = U_c(c_0, l_0)b_{-1}$$

and the resource constraint

$$c_0 + g(s_0) = \theta(s_0)l_0$$

and

$$x_0 \in [\underline{x}, \overline{x}]$$

apbXXX We can have a lemma here that says that W and V that solve the recursive problem also solve the sequential problem posed earlier. This could be useful and clear doubts that many people face as to how did we make AMSS work with one state variable.

5.1 What we've done

tjsTom XXXXXX: write a subsection describing what Anmol and David have done so far with this setup, computationally and analytically

5.2 Motivation to focus on risk-free bond economy

riskfreeonly⟩?

As mentioned in section 3.1, properties of a Ramsey plan for our incomplete markets economy vary sensitively with asset returns that reflect properties of equilibrium prices $\{q_t(s^t|B_{-1}, s_{-1})\}_t$ and the exogenous asset payoff matrix $\mathbb{P}$. By studying quasi-linear preferences, we eliminated fluctuations in returns coming from prices. Here we turn the table and by studying an economy with a risk-free bond, we eliminate fluctuations in returns coming from the exogenous asset payoff matrix $\mathbb{P}$. Thus, we set $\mathbb{P}(s|s_{-}) = 1 \; \forall \; (s, s_{-})$.

Let $x'(s; x, s_{-})$ be the decision rule for x' that attains the right side of the $t \geq 1$ Bellman equation (17). A steady state x^* satisfies $x^* = x'(s; x^*, s_{-})$ for all s, s_{-}. A steady state is a node at which the continuation allocation and tax rate have no further history dependence.

rop:existenceU⟩ **Proposition 5.2.** *Assume that U is separable and iso-elastic: $U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$. Assume that*

[apb] *XXXX We are changing notation to represent good states with $s = s_g$ instead of $s = 1$?*

the Markov state s take two values is i.i.d with s_b being the "adverse" state (either low TFP or high govt. expenditures) and s_g begin the good state. Let x_{fb} be a value of ~~the state x from which a government can implement first=best with complete markets~~ [dge] marginal utility weighted debt associated with the first best allocation with complete markets. Let $q_{fb}(s)$ be the shadow price of government debt in state s at the first best allocation. If

$$\frac{1 - q_{fb}(s_b)}{1 - q_{fb}(s_g)} > \frac{g(s_b)}{g(s_g)} \geq 1 \tag{21}$$ eqn:prop52suff

then there exists a steady state with $x_{fb} > x^* > 0$

Proof. [dge] As in the quasi-linear case a steady state is associated with the continuation allocation of a complete markets allocation with some initial debt level. Equivalently, we can index these allocations with their associated multiplier on the implementability constraint: μ. Letting $S(\mu, s)$ be the marginal utility government surplus in state s with multiplier μ, a steady state will be a multiplier μ^* where the budget constraint in both states of the world is satisfied

$$\frac{S(\mu^*, s_g)}{\frac{c(\mu^*, s_g)^{-\sigma}}{\beta \mathbb{E} c(\mu^*)^{-\sigma}} - 1} = \frac{S(\mu^*, s_b)}{\frac{c(\mu^*, s_b)^{-\sigma}}{\beta \mathbb{E} c(\mu^*)^{-\sigma}} - 1}$$

By choosing μ_1 such that $S(\mu_1, s_g) = 0$ we conclude that

$$0 = \frac{S(\mu_1, s_g)}{\frac{c(\mu_1, s_g)^{-\sigma}}{\beta \mathbb{E} c(\mu_1)^{-\sigma}} - 1} > \frac{S(\mu_1, s_b)}{\frac{c(\mu_1, s_b)^{-\sigma}}{\beta \mathbb{E} c(\mu_1)^{-\sigma}} - 1}$$

This is derived directly from $S(\mu, s_g) < S(\mu, s_b)$ for all μ and $c(\mu, s_g) > c(\mu, s_b)$ for all μ.

dge Substituting out for q_{fb}, equation (21) can equivalently be written as

$$\frac{g(s_g)}{1-\frac{\beta\mathbb{E}c_{fb}^{-\sigma}}{c_{fb}(s_g)^{-\sigma}}} > \frac{g(s_b)}{1-\frac{\beta\mathbb{E}c_{fb}^{-\sigma}}{c_{fb}(s_b)^{-\sigma}}}$$

Multiplying both sides by -1 and factoring out a $\beta\mathbb{E}c_{fb}^{-\sigma}$ this equation simplifies to

$$\frac{-c_{fb}(s_g)^{-\sigma}g(s_g)}{\frac{c_{fb}(s_g)^{-\sigma}}{\beta\mathbb{E}c_{fb}^{-\sigma}}-1} < \frac{-c_{fb}(s_b)^{-\sigma}g(s_b)}{\frac{c_{fb}(s_b)^{-\sigma}}{\beta\mathbb{E}c_{fb}^{-\sigma}}-1}$$

or

$$\frac{S(0,s_g)}{\frac{c(0,s_g)^{-\sigma}}{\beta\mathbb{E}c(0)^{-\sigma}}-1} < \frac{S(0,s_b)}{\frac{c(0,s_b)^{-\sigma}}{\beta\mathbb{E}c(0)^{-\sigma}}-1}$$

The existence of μ^* then comes directly from the Intermediate Value Theorem. □

⟨p:convergenceU⟩ **Proposition 5.3.** ~~*Let $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ solve the incomplete markets Ramsey problem with $x_0 > x^*$. Then $x_t(s^{t-1}) \to x^*$ as $t \to \infty$ with probability 1 for all initial conditions.*~~ dge *There exists $\underline{x} < x^*$ and $\overline{x} > 0$ such that if $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ solves the incomplete markets Ramsey problem 5.1 with bounds $\underline{x}$ and $\overline{x}$ then $x_t(s^t) \to x^*$ as $t \to \infty$ with probability 1.*

Proof. dge We can break the proof of this proposition down into two lemmas describing the structure of the policy functions. These properties will be intuitive and can be proved in the appendix. Finally the proof will rely on the assumption of concavity of the value function,

⟨lem:c_order⟩ **Lemma 5.4.** *Consumption is ordered by the state of the world. That is there exists $\underline{x}$ and $\overline{x}$ such that for all $x \in [\underline{x}, \overline{x}]$ the policy functions for consumption satisfy $c(x, s_g) > c(x, s_b)$.*

This lemma guarantees that for the same level of marginal utility weighted government debt, consumption will be larger in "good" states of the world than "adverse" states of the world.

⟨lem:x_order⟩ **Lemma 5.5.** *There exits $\underline{x}$ and $\overline{x}$ such that the optimal government savings policy, $x'(x, s)$ satisfies*

1. *For $x \in (x^*, \overline{x}]$ we have $x'(x, s_g) < x'(x, s_b)$*
2. *For $x \in [\underline{x}, x^*)$ we have $x'(x, s_g) > x'(x, s_b)$*

Furthermore, $x'(x,\cdot)$ is increasing in x.

Property 1. states that if government debt is larger than the steady state, then the government will issue more debt in bad states of the world than good states of the world. Property 2. states that if government debt is smaller that the steady state debt then the government has accumulated enough assets[10] such that the lower interest rates in the "adverse" state of the world allow it to purchase more assets (issue less debt) than in the "good" states of the world. The last part of the lemma gaurantees that if the government enters with more debt it will pass on more debt to future periods. We can now prove global convergence, we will focus on the case where $x_t \geq x^*$ as the other case is symmetric. As $x'(x,\cdot)$ is increasing in x, we can iterate the policy functions forward to conclude that $x_{t+j} > x^*$ for all j as long as $x_t > x^*$. Letting $\mu_t = V'(x_t)$ be the multiplier on the implementability constraint and $\overline{\lambda}_t$ be the multiplier on the constraint $x_t \leq \overline{x}$ we have

$$\mu_t = \frac{1}{\mathbb{E}_t[c_{t+1}^{-\sigma}]}\mathbb{E}_t[\mu_{t+1}c_{t+1}^{-\sigma}] - \overline{\lambda}_t$$

Lemma 5.5. along with concavity of V allows us to conclude that $\mu_{t+1}(s_g) > \mu_{t+1}(s_b)$. From Lemma 5.4. we know that $c_{t+1}(s_g) > c_{t+1}(s_b)$ which implies that $Cov_t(\mu_{t+1}, c_{t+1}^{-\sigma} < 0$ and thus

$$\frac{1}{\mathbb{E}_t[c_{t+1}^{-\sigma}]}\mathbb{E}_t[\mu_{t+1}c_{t+1}^{-\sigma}] < \mathbb{E}_t[\mu_{t+1}]$$

As $\overline{\lambda}_t \geq 0$, we can conclude that

$$\mu_t < \mathbb{E}_t[\mu_{t+1}]$$

Moreover $\mu_t < V'(x^*) = \mu^*$, so μ_t is a submartingale bounded from above. Applying the martingale convergence theorem we conclude that $\mu_t \to \mu^*$ with probability 1. Continuity of the policy functions and uniqueness of the steady state in the region $[x^*, \overline{x}]$ implies that $x_t \to x^*$ with probability 1. $\square$

Remark 5.6. *In this economy, fluctuations in the risk-free interest rate come from fluctuations in marginal utility of consumption. The interest rate is low in "good" states (i.e., when TFP is high or government expenditures are low). In a steady state, the government holds claims against the private sector, an outcome that resembles those in economies with quasi-linear utility and low* $\boldsymbol{p}$. *For all admissible initial levels of government debt, an incomplete markets Ramsey allocation converges to a particular Lucas-Stokey Ramsey*

[10]Remember in the steady state the government will hold assets

allocation. [tjs] *Team xXXXXX: say a little more about the particular LS allocation and its initial debt level*

dge

Remark 5.7. *Propositions 5.2 and 5.3 can be thought of as a near converse to Lemma 3. of section 5 in Aiyagari et al. (2002). There they provide sufficient conditions for the non-convergence of the economy to a complete markets allocations. Our propositions provide sufficient conditions for the existence of the complete markets steady state and the long rung convergence to that steady state. While the results of Aiyagari et al. (2002) do hold, and for more general stochastic processes a steady state will not exist, we find numerically that the results Propositions 5.2 and 5.3 are informative as to the long run dynamics of incomplete markets economys. There will exist regions of low volatility and the economy will converge to these regions in the long run.*

[tjs]Team XXXXXX: let's add some modest self-promotion here telling just how much the results immediately above add in terms of filling in loose ends from AMSS – things they just weren't able to answer.

6 To do

1. Add above at appropriate place that until now $T_t \equiv 0$.
2. Edit section about turning on nonnegative transfers.
3. Tom to write introduction and concluding sections.
4. Fill in some notation about what objects are indexed by, e.g., s^t.
5. Fill in proofs.
6. Check flow and order.
7. Add a short appendix on how Bellman equations were solved numerically. Pat selves on back for doing so and display some policy functions – think of one or two things to do with those policy functions.
8. Think of a couple of experiments that show off the policy function calculations.

6.1 Allowing nonnegative transfers

tjsTeam XXXXX: Beware – please wear hard hat in this construction area. Add BS about AMSS and what transfers did for them. Write front end of this section – good low-skill job for Tom. Access to nonnegative transfers makes first-best level of assets trivially a "steady state." With nonnegative lump sum transfers, in cases where a steady state exists and is stable, if the initial debt of the government exceeds its steady state level, the economy converges with probability 1 to the steady state. Thus, counterpart to previous results continue to hold when initial government debt exceeds its steady state value. When initial government debt is less than a steady-state value, then tjssay something that is known or what is unknown.

7 Comparison to literature

tjsTeam – please keep your hands off this section. Tom to use parts of it in the "womanly" sections (the introduction and concluding section).

1. Angeletos (2002), Buera and Nicolini (2004)
   - Begin with a complete market Ramsey allocation
   - Ask if this can be attained with a limited collection of non-contingent debts of different maturities
2. This paper
   - Begins with an incomplete markets Ramsey allocation
   - Asks whether the long-run allocation coincides with a complete markets Ramsey allocation for some initial govt debt
3. BEGS1 studies a related problem with heterogeneous agents

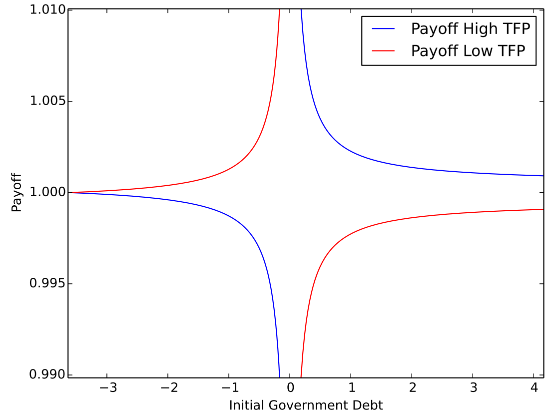

Figure 1: Optimal asset payoff structure as a function of initial government debt when TFP follows a 2 shock i.i.d process

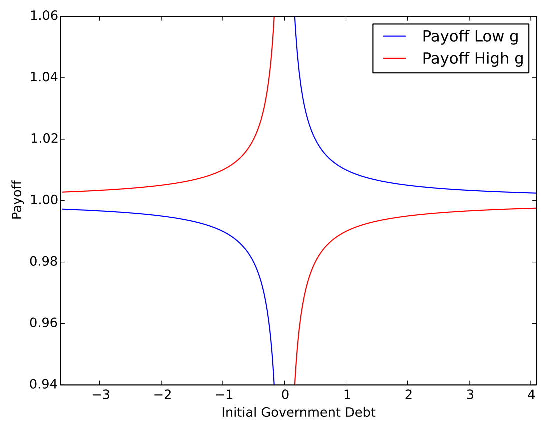

Figure 2: Optimal asset payoff structure as a function of initial government debt when government expenditures follow 2 shock i.i.d process

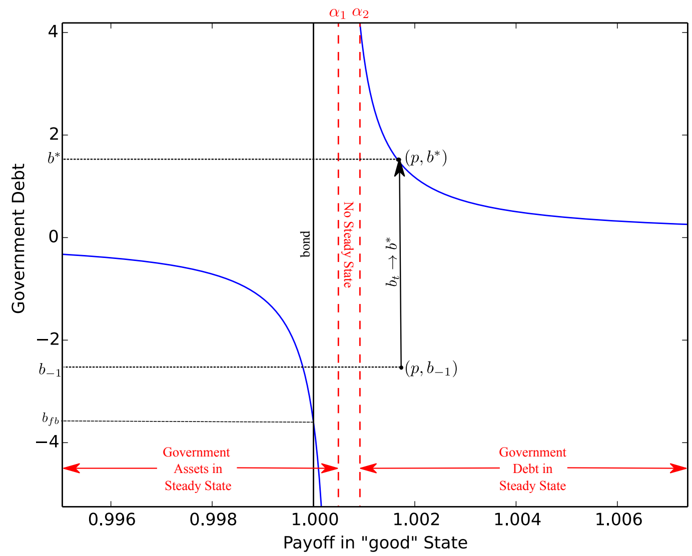

Figure 3: Existence regions in $\boldsymbol{p}$ space

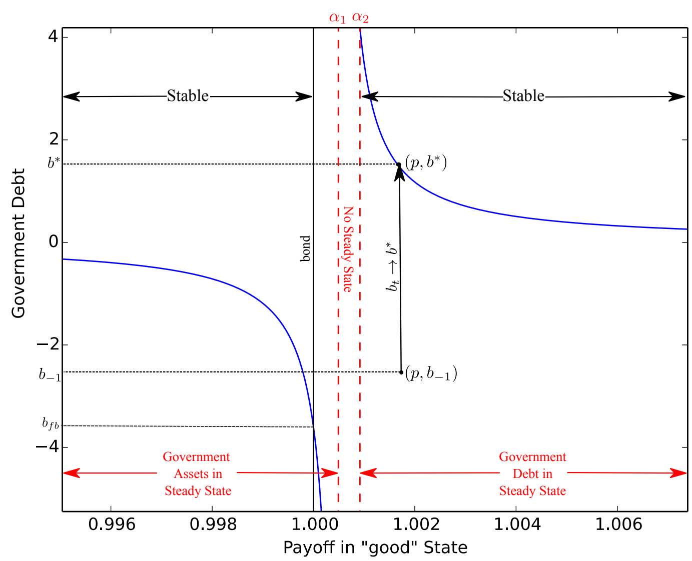

Figure 4: Stability regions

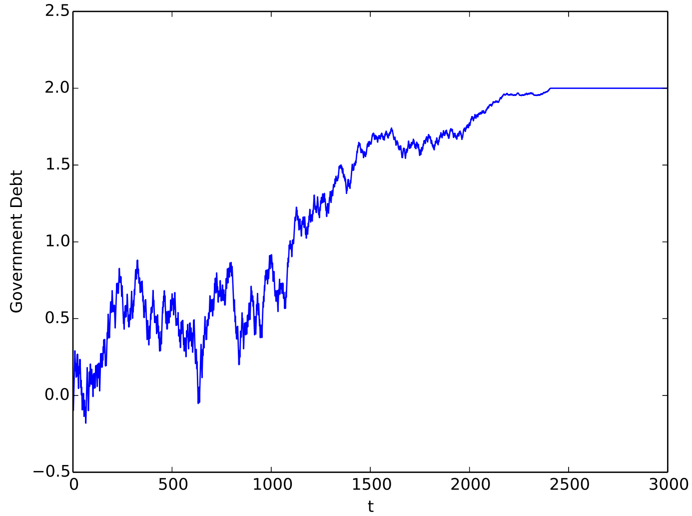

Figure 5: A sample path with $\boldsymbol{p} > 1$

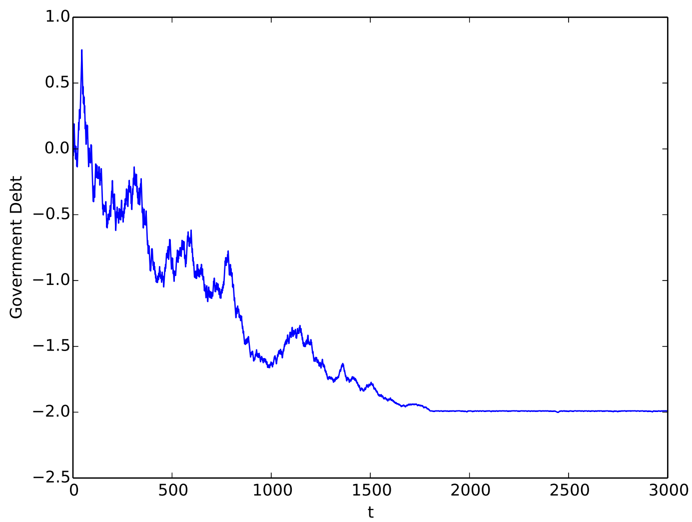

Figure 6: A sample path with $\boldsymbol{p} < 1$

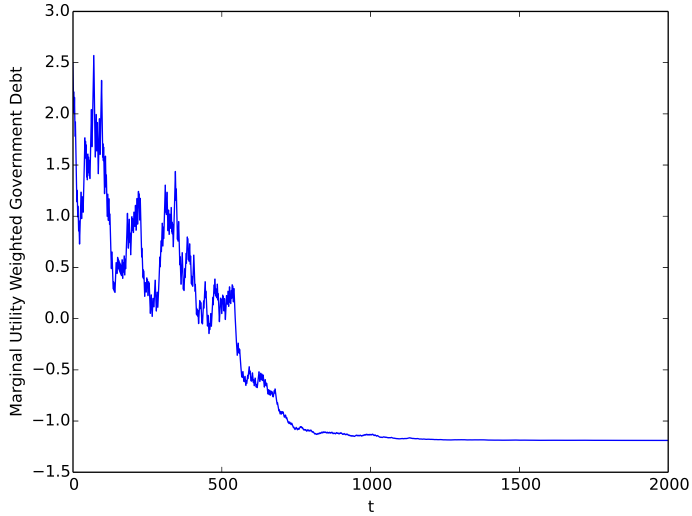

Figure 7: A sample path for 2 state i.i.d. process with risk aversion

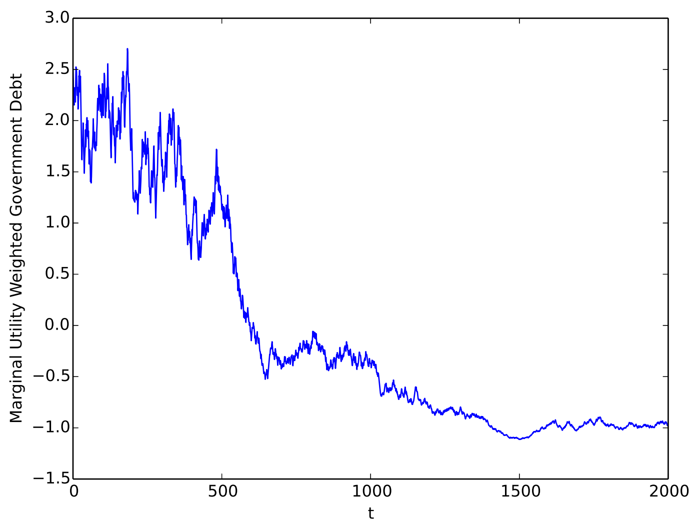

Figure 8: A sample path for economy with $S > 2$ states

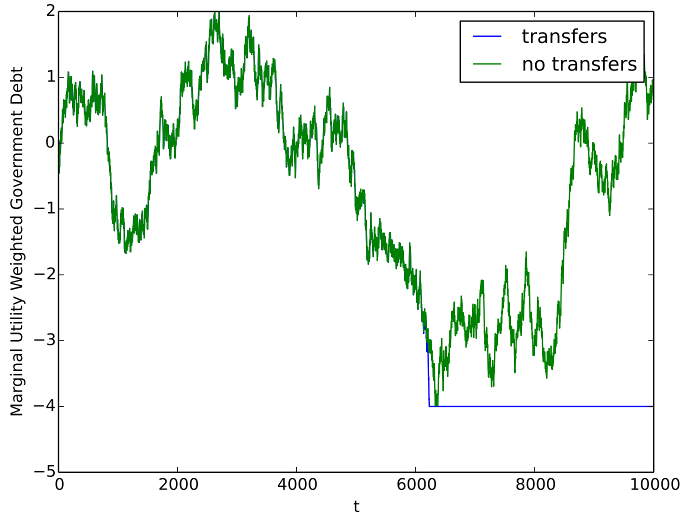

Figure 9: Quasilinear preferences and risk-free bond with and without nonnegative transfers

8 Figures

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