Optimal Taxation with Incomplete Markets

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Abstract

KEYWORDS:

1 Introduction

2 Environment

We analyze economies that share the following features. Government expenditures at time t, $g_t = g(s_t)$, and a productivity shock $\theta_t = \theta(s_t)$ are both functions of a Markov shock $s_t \in \mathcal{S}$ having $S \times S$ transition matrix Π and initial condition s_{-1}. An infinitely lived representative consumer has preferences over allocations $\{c_t(s^t), l_t(s^t)\}_{t=0}^{\infty}$ of consumption and labor supply that are ordered by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c(s^t), l(s^t)\right)$$

where U satisfies XXXXXX. Labor produces output via the linear technology

$$y_t = \theta_t l_t$$

The representative consumer's tax bill at time $t \geq 0$ is

$$-T_t + \tau_t \theta_t l_t, \ T_t \geq 0,$$

where $\tau_t(s^t, b_{-1})$ is a flat rate tax on labor income and T_t is a nonnegative transfer. Often, we'll set $T_t = 0$. The government and consumer trade a single possibly risky asset whose time t payoff p_t is described by an $S \times S$ matrix $\mathbb{P}$:

$$p_t = \mathbb{P}(s_t | s_{t-1}).$$

Let B_t denote the government's holdings of the asset and b_t be the consumer's holdings. Let $q_t = q_t(s^t; b_{-1})$ be the price of the single asset at time t. At $t \geq 0$, the household's time budget constraint is

$$c_t + b_t = (1 - \tau_t)\, \theta_t l_t + \frac{p_t}{q_{t-1}} b_{t-1} + T_t$$

and the government's is

$$g_t + B_t + T_t = \tau_t \theta_t l_t + \frac{p_t}{q_{t-1}} B_{t-1}.$$

Feasible allocations satisfy

$$c_t + g_t = \theta_t l_t, \ \forall t \geq 0$$

Clearing in the time $t \geq 0$ market for the single asset requires

$$b_t + B_t = 0.$$

Initial assets satisfy $b_{-1} = -B_{-1}$ An initial value of the exogenous state s_{-1} is given. Equibrium objects including $\{c_t, l_t, \tau_t\}_{t=0}^{\infty}$ will come in the form of sequences of functions of initial government debt b_{-1} and $s^t = [s_t, s_{t-1}, \ldots, s_0, s_{-1}]$.

Borrowing from a standard boilerplate, we use the following:

Definition 2.1. *An **allocation** is XXXXX. A **price system** is XXXXX. A **budget-feasible government policy** is XXXXX* $\{\tau_t, T_t\}_{t=0}^{\infty}$ *XXXXX*

Definition 2.2. *Given* $(b_{-1} = -B_{-1}, s_{-1})$ *and a government policy, a **competitive equilibrium with distorting taxes** is a price system, a budget-feasible government policy, and an allocation such that the allocation is individually rational and the bond market clears.*

Definition 2.3. *Given* (b_{-1}, B_{-1}, s_{-1}), *a **Ramsey plan** is a welfare-maximizing competitive equilibrium with distorting taxes.*

3 Two Ramsey problems

Following Lucas and Stokey (1983) and Aiyagari et al. (2002), we use a "primal approach." To encode a government policy and price system as a restriction on an allocation, we first obtain the representative household's first order conditions:

$$U_{c,t}q_t = \beta \mathbb{E}_t p_{t+1} U_{c,t+1}$$

$$(1-\tau_t)\theta_t U_{c,t} = -U_{l,t}$$

We substitute these into the household's budget constraint to get a difference equation that we solve forward at every history for every $t \geq 0$. That yields *implementability constraints* on a Ramsey allocation that fall into two categories: (1) the time $t = 0$ version is identical with the *single* implementability constraint imposed by Lucas and Stokey (1983); (2) the time $t \geq 1$ implementability constraints are counterparts of the additional *measurability restrictions* that Aiyagari et al. (2002) impose on a Ramsey allocation.

We first state our Ramsey problem, then Lucas and Stokey's.

b:RamseyBEGS⟩? **Problem 3.1.** *The Ramsey problem is to choose an allocation and an appropriately measurable government debt sequence* $\{b_t\}_{t=0}^{\infty}$ *that attain:*

$$\max_{\{c_t,l_t,b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \tag{2}$$

?eqn:Ramseyobj

subject to

$$c_t + g_t = \theta_t l_t, \ \ t \geq 0 \tag{3a}$$

eqn:feas

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t}c_t + U_{l,t}l_t\right) \tag{3b}$$

eqn:LSimplemen

$$\frac{b_{t-1}U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1}p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j}c_{t+j} + U_{l,t+j}l_{t+j}\right) \ \ \textit{for } t \geq 1 \tag{3c}$$

eqn:AMSSimplem

⟨prob:RamseyLS⟩ **Problem 3.2.** *Lucas and Stokey's Ramsey problem is to choose an allocation that attains*

$$\max_{\{c_t,l_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \tag{4}$$

?eqn:Ramseyobj

subject to the single implementability constraint (3b) *and feasibility* (3a) *for all* t, s^t.

Remark 3.3. *Equation* (3a) *imposes feasibility, while equation* (3b) *is the single implementability constraint present in Lucas and Stokey (1983). Equations* (3c) *express additional implementability constraints that comprise implementability constraints at every node from time* $t \geq 1$. *These generalize the Aiyagari et al. (2002) measurability constraints on a Ramsey allocation to our more general payoff structure* $\mathbb{P}$ *for the single asset. The measurability constraints* (3c) *are cast in terms of the date, history* $(t-1, s^{t-1})$ *measurable state variable* b_{t-1} *that for* $t \geq 1$ *is absent from Lucas and Stokey's complete markets Ramsey problem. Evidently, Ramsey allocation for our incomplete markets economy automatically satisfies the single implementability constraint imposed by Lucas and Stokey.*

⟨rem:LSdebt⟩ **Remark 3.4.** *State-contingent, but not history-dependent, values of consumption, labor supply, and continuation government debt* $\check{b}(s)$ *solve the Lucas and Stokey (1983) Ramsey problem 3.2. As intermediated by the Lagrange multiplier on the implementability constraint* (3b), *consumption, labor supply, and* $\check{b}(s)$ *are functions of initial government debt* b_{-1} *and the current state* s, *but not past* s*'s.*

3.1 Motivation for quasi-linear U

sequasilinear⟩

Asymptotic properties of a Ramsey plan for our incomplete markets economy vary with asset returns that reflect properties of equilibrium prices $\{q_t(s^t|B_{-1}, s_{-1})\}_t$ and the exogenous asset payoff matrix $\mathbb{P}$. To focus exclusively on the exogenous $\mathbb{P}$ part of returns, we begin by studying an economy with quasi-linear utility function:

$$U(c,l) = c - \frac{l^{1+\gamma}}{1+\gamma}. \tag{5}$$?eqn:UQL?

Asymptotic properties of a Ramsey plan for our incomplete markets economy vary with asset returns that reflect properties of equilibrium prices $\{q_t(s^t|B_{-1}, s_{-1})\}_t$ and the exogenous asset payoff matrix $\mathbb{P}$. This utility function pins down $q_t = \beta \mathbb{E}_t \mathbb{P}(s_{t+1}|s_t)$. After studying the consequences of quasi-linear utility, we shall Ramsey plans for utility functions that express risk aversion with respect to consumption and so activate fluctuating q_t.

4 Quasi-linear preferences

Throughout this section, we assume that U is quasi-linear and use an indirect three step approach to characterize the asymptotic behavior of government debt and the tax rate. (1) Construct an operator. We pose the following problem:

rob:PPoperator⟩ **Problem 4.1.** *Given arbitrary initial government debt b_{-1}, what is an optimal asset payoff matrix?*

Problem 4.1 induces an operator that maps initial government debt b_{-1} into an optimal payoff matrix $\mathbb{P}^*$ for the single asset:

$$\mathbb{P}^* = \mathbb{P}^*(b_{-1}). \tag{6}$$?eqn:PPoperato

(2) Apply the inverse of the operator $\mathbb{P}^*$. For an arbitrary payoff matrix $\mathbb{P}$, let

$$\mathbb{P} = \mathbb{P}^*(b^*) \text{ or } b^* = \mathbb{P}^{*-1}(\mathbb{P}). \tag{7}$$ eqn:invPoperat

For initial government debt $b_{-1} = b^*$, a Ramsey plan for the incomplete markets economy has $b_t = b^*$ for all $t \geq 0$. (3) Starting from an arbitrary initial government b_{-1} and an arbitrary payoff matrix $\mathbb{P}$, establish conditions under which $b_t \to b^*$ under a Ramsey plan.

In particular, where $S = 2$ and shocks s_t are IID, we describe a large set of $\mathbb{P}$'s for which government debt b_t under a Ramsey plan converges to b^* that solves (7). For more general shock processes, we numerically find an ergodic set of b_t's hovering around a debt level $\tilde{b}$; The optimal $\mathbb{P}^*$ associated with $\tilde{b}$ approximates $\mathbb{P}$:

$$\mathbb{P} \approx \mathbb{P}^*(\tilde{b})$$

Here is how we execute these three steps. (1) Given b_{-1}, solve problem 3.2 to compute a Lucas-Stokey Ramsey allocation. (2) Note that Aiyagari et al.-like measurability constraints (3c) are invariant to scaling of p_t by a constant k_{t-1} that can depend on s^{t-1}. From the equivalence class of p_t's that satisfy (3c) at the Lucas-Stokey Ramsey allocation, select a p_t that imposes the normalization $\mathbb{E}_{t-1} U_{c,t} p_t = 1$ and satisfies

$$p_t = \frac{\beta}{U_{c,t-1} b_{t-1} U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \tag{8}$$ eqn:pdisarm

Note that by construction, p_t disarms the time $t \geq 1$ measurability constraints. (4) Using the fact noted in remark 3.4 that the Lucas-Stokey Ramsey allocation is not history-dependent, construct the optimal payoff matrix as

$$\mathbb{P}^*(s_t, s_{t-1}|b_{-1}) = p_t.$$

Thus, given initial government debt b_{-1}, let $\mu(b_{-1})$ be the Lagrange multiplier on the Lucas-Stokey implementability constraint (3b) at the Lucas-Stokey Ramsey allocation. The tax rate in the Ramsey allocation is $\tau(\mu) = \frac{\gamma\mu}{(1+\gamma)\mu-1}$, which implies a net-of-interest government surplus $S(s,\tau)$ that satisfies

$$S(s,\tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1-\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

The 'disarm-the-measurability-constraints' equation (8) then implies that the optimal payoff matrix is

$$\mathbb{P}^*(s, s_{\_}|b_{-1}) = (1-\beta)\frac{S(s,\tau)}{\mathbb{E}_{s_{\_}}S(s,\tau)} + \beta. \tag{9}$$ eqn:optPP

Anmol and David XXXXXX: let's add some snazzy interpretation of the previous nice equation.

To appreciate how initial government debt level influences the optimal asset payoff structure via formula (9), call a state s "adverse" if it implies either "high" government expenditures or "low " TFP; formally, say that s is "adverse" if

$$g(s)\mathbb{E}_{s_{\_}}\theta^{\frac{\gamma}{1+\gamma}} - \theta(s)^{\frac{\gamma}{1+\gamma}}\mathbb{E}_{s_{\_}}g > 0$$

Then (9) implies that when initial government assets are positive, $\mathbb{P}^*$ pays *more* in "adverse" states, while when initial government assets are negative, $\mathbb{P}^*$ pays *less* in "adverse" states. A "good" state is the opposite of an "adverse" state.

4.1 Analysis of $S = 2$ iid shocks

We temporarily assume that s_t is i.i.d and $S = 2$. In this case, $\mathbb{P}(s_{-}, s)$ is independent of s_{-}, so that $\mathbb{P}$ can be taken to be a vector. Under the normalization $q_t = \beta$, $\mathbb{E}\mathbb{P}(s) = 1$, payoffs on the single asset are determined by a scalar $\boldsymbol{p}$ and a risk-free bond is a security for which

$\boldsymbol{p} = 1$. We take $\boldsymbol{p}$ to be the asset's payoff in the "good" state s. Define a scalar b^* by

$$b^* = \mathbb{P}^{*-1}(\boldsymbol{p}) \tag{10}$$ eq-ss

so that b^* satisfies $p = \mathbb{P}^*(b^*)$. Anmol and David XXXXXX: I have changed and slightly abused notation. Let's talk about it.

:ssexistence⟩?

Proposition 4.2. *Suppose that s shocks affect either g or θ. Then there exist $0 \geq \alpha_2 \geq \alpha_1 \geq 1$ such that*

a. If $\boldsymbol{p} \leq \alpha_1$, then $b^* < 0$

b. If $\boldsymbol{p} \geq \alpha_2$, then $b^* > 0$

c. If $\alpha_1 > \boldsymbol{p} > \alpha_2$, then b^* solving (10) does not exist

Proof. David XXXXX: please fill in and refine. With only government expenditure shocks, we compute

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$

With only TFP shocks, we compute

$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

□

Theorem 4.3. *Let b_{fb} denote the level of government debt that supports the first-best allocation with complete markets.* *Anmol and David XXXXXX: I want to clarify what the previous phrase means.* *Then*

a. If $\boldsymbol{p} \leq \min(\alpha_1, 1)$, then $b_{fb} < b^* < 0$ and $b_t \to b^*$ with probability 1.

b. If $\boldsymbol{p} \geq \alpha_2$, then $0 < b^*$ and $b_t \to b^*$ with probability 1.

c. If $\alpha_2 < \boldsymbol{p} < \min(\alpha_1, 1)$, b^* either does not exist or is unstable.

For $\boldsymbol{p}$ in region (c.), the government tends to run up debt over time.

Proof. David XXXXX: please add and refine Sketch of argument: The Lagrange multiplier μ_t on the implementability constraint (3c) satisfies $\mu_t = \mathbb{E}_t p_{t+1}\mu_{t+1}$ or $\mathbb{E}_t\mu_{t+1} = \mu_t - Cov_t(p_{t+1}, \mu_{t+1})$ and so is a risk adjusted martingale. To establish stability of b^* we require that the drift of μ_t is big enough away from a steady state. □

4.2 Economic forces driving convergence

In summary, when the aggregate state follows a 2-state i.i.d. process, government debt b_t often converges to b^*, while the tail of the allocation equals Ramsey allocation for an economy with complete markets and initial government debt b^*. The level and sign of b^* depend on the asset payoff structure, which we have expressed in terms of a scalar $\boldsymbol{p}$ that concisely captures what in more general settings we represented with the asset payoff matrix $\mathbb{P}$.

Facing incomplete markets, the Ramsey planner recognizes that the government's debt *level* combines with the payoff structure on its debt instrument to affect the welfare costs associated with varying the distorting labor tax rate across states. When the instrument is a risk-free bond, the government's marginal cost of raising funds μ_t is a martingale. In this situation, *changes* in debt levels help smooth tax distortions across time. However, if the payoff on the debt instrument varies across states, then by affecting its state-contingent revenues, the *level* of government debt can help smooth tax distortions across states. For our two state, iid shock process, the steady state debt level b^*, when it exists, is the unique amount of government debt that provides just enough "state contingency" completely to fill the void left by missing assets markets. When it is away from b^* and considers issuing or accumulating debt starting, the Ramsey planner tells the government to take takes into account the prospective benefits that will eventually accrue from being closer to b^*; that puts a risk-adjustment into the martingale governing μ and leads the government either to accumulate or decumulate debt. Although accumulating government assets requires raising distorting taxes, locally the welfare costs of higher taxes are second-order and so are dominated by the welfare gains from approaching b^*, which are first-order. Anmol and

David XXXXX: please carefully read and possibly edit my editing of your "intuition". (But please remember the Lucas tee shirt.

5 Turning on risk-aversion

We now depart from quasi-linearity of $U(c, l)$ and thus activate an additional source of return fluctuations, namely, fluctuations in the risk-free and other interest rates. To obtain a recursive representation of a Ramsey plan, we employ the endogenous state variable

$$x_t = u_{c,t} b_t,$$

and study how long-run properties of x_t depend on equilibrium returns $R_{t,t+1} = \frac{\mathbb{P}(s_t, s_{t+1})}{q_t(s^t)}$. Activating risk aversion in c makes q_t vary in interesting ways.

Commitment to a Ramsey plan implies that government actions at $t \geq 1$ are constrained by the household's anticipations about them at $s < t$ Following Kydland and Prescott (1980), we use the marginal utility of consumption that the Ramsey planner promises to the household to account for that 'forward looking' restriction on the Ramsey planner. It is convenient for us that scaling the household's budget constraint by the marginal utility of consumption makes Ramsey problem recursive in $x = U_c b$. In particular, implementability constraints (3c) can be represented as

$$\frac{x_{t-1} p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} = U_{c,t} c_t + U_{l,t} l_t + x_t, \ \ t \geq 1 \tag{11}$$ {?}

Team XXXXX: check the above carefully.

⟨amseyBellman⟩? **Problem 5.1.** *Before the realization of the time t Markov shock s_t, let $V(x, s_{-1})$ be the* expected *continuation value of the Ramsey plan at $t \geq 1$ given promised marginal utility government debt inherited from the past $x = U_c, tb_t$ and time $t-1$ Markov state s_{-1}. After the realization of time 0 Markov shock s_0, let $W(b_{-1}, s_0)$ be the value of the Ramsey plan when initial government debt is b_{-1}. The (ex ante) Bellman equation for $t \geq 1$ is*

$$V(x, s_{\_}) = \max_{c(s), l(s), x'(s)} \sum_{s} \Pi(s, s_{\_}) \Big(U(c(s), l(s)) + \beta V(x'(s), s) \Big) \tag{12}$$ eqn:Bellman1

subject to $x'(s) \in [\underline{x}, \overline{x}]$ *and*

$$\frac{x\mathbb{P}(s)U_c(s)}{\beta\mathbb{E}_{s\_}\mathbb{P}Uc} = U_c(s)c(s) + U_l(s)l(s) + x'(s) \quad (13) \quad \boxed{\texttt{timetBellimple}}$$

$$c(s) + g(s) = \theta(s)l(s) \quad (14) \quad \boxed{\texttt{timetfeas}}$$

Equation (13) *is the implementability constraint and* (14) *is feasibility. Given an initial debt* b_{-1},*time* 0 *Markov state* s_0, *and continuation value function* $V(x, s\_)$, *the (ex post) time* 0 *Bellman equation is*

$$W(b_{-1}, s_0) = \max_{c_0, l_0, x_0} U(c, l) + \beta V(x_0, s0) \quad (15) \quad ?\texttt{eqn:Bellman0}?$$

subject to time zero implementability constraint

$$U_c(c_0, l_0)c + U_l(c_0, l_0)l_0 + x_0 = U_c(c_0, l_0)b_{-1}$$

and the resource constraint

$$c_0 + g(s_0) = \theta(s_0)l_0$$

and

$$x_0 \in [\underline{x}, \overline{x}]$$

5.1 What we've done

Tom XXXXXX: write a subsection describing what Anmol and David have done so far with this setup, computationally and analytically

5.2 Motivation to focus on risk-free bond economy

riskfreeonly⟩?

As mentioned in section 3.1, properties of a Ramsey plan for our incomplete markets economy vary sensitively with asset returns that reflect properties of equilibrium prices $\{q_t(s^t|B_{-1}, s_{-1})\}_t$ and the exogenous asset payoff matrix $\mathbb{P}$. By studying quasi-linear preferences, we eliminated fluctuations in returns coming from prices. Here we turn the table and by studying an economy with a risk-free bond, we eliminate fluctuations in returns coming from the exogenous asset payoff matrix $\mathbb{P}$. Thus, we set $\mathbb{P}(s|s\_) = 1 \ \forall \ (s, s\_)$.

Let $x'(s; x, s\_)$ be the decision rule for x' that attains the right side of the $t \geq 1$ Bellman equation (12). A steady state x^* satisfies $x^* = x'(s; x^*, s_-)$ for all s, s_-. A steady state is a node at which the continuation allocation and tax rate have no further history dependence.

p:existenceU⟩?

Proposition 5.2. *Assume that U is separable and iso-elastic: $U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$. Assume that the Markov state s take two values is i.i.d with s_b being the "adverse" state (either low TFP or high govt. expenditures) and s_g begin the good state. Let x_{fb} be a value of the state x from which a government can implement first=best with complete markets. David XXXXXX: I'd like to clarify what the previous statement means. Let's talk. Let $q_{fb}(s)$ be the shadow price of government debt in state s at the first best allocation. If*

$$\frac{1 - q_{fb}(s_b)}{1 - q_{fb}(s_g)} > \frac{g(s_b)}{g(s_g)} > 1$$

then there exists a steady state with $x_{fb} > x^* > 0$

Proof. David fill in □

Proposition 5.3. *Let $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ solve the incomplete markets Ramsey problem with $x_0 > x^*$. Then $x_t(s^{t-1}) \to x^*$ as $t \to \infty$ with probability 1 for all initial conditions*

Proof. David fill in □

Remark 5.4. *In this economy, fluctuations in the risk-free interest rate come from fluctuations in marginal utility of consumption. The interest rate is low in "good" states (i.e., when TFP is high or government expenditures are low). In a steady state, the government holds claims against the private sector, an outcome that resembles those in economies with quasi-linear utility and low $\boldsymbol{p}$. For all admissible initial levels of government debt, an incomplete markets Ramsey allocation converges to a particular Lucas-Stokey Ramsey allocation. Team xXXXXX: say a little more about the particular LS allocation and its initial debt level*

Team XXXXXX: let's add some modest self-promotion here telling just how much the results immediately above add in terms of filling in loose ends from AMSS – things they just weren't able to answer.

6 To do

1. Add above at appropriate place that until now $T_t \equiv 0$.

2. Edit section about turning on nonnegative transfers.

3. Tom to write introduction and concluding sections.

4. Fill in some notation about what objects are indexed by, e.g., s^t.

5. Fill in proofs.

6. Check flow and order.

7. Add a short appendix on how Bellman equations were solved numerically. Pat selves on back for doing so and display some policy functions – think of one or two things to do with those policy functions.

8. Think of a couple of experiments that show off the policy function calculations.

6.1 Allowing nonnegative transfers

Team XXXXX: Beware – please wear hard hat in this construction area. Add BS about AMSS and what transfers did for them. Write front end of this section – good low-skill job for Tom. Access to nonnegative transfers makes first-best level of assets trivially a "steady state." With nonnegative lump sum transfers, in cases where a steady state exists and is stable, if the initial debt of the government exceeds its steady state level, the economy converges with probability 1 to the steady state. Thus, counterpart to previous results continue to hold when initial government debt exceeds its steady state value. When initial government debt is less than a steady-state value, then say something that is known or what is unknown.

7 Comparison to literature

Team – please keep your hands off this section. Tom to use parts of it in the "womanly" sections (the introduction and concluding section).

1. Angeletos (2002), Buera and Nicolini (2004)

    - Begin with a complete market Ramsey allocation
    - Ask if this can be attained with a limited collection of non-contingent debts of different maturities

2. This paper

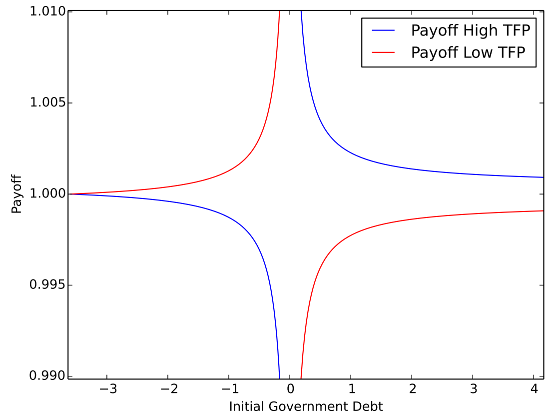

Figure 1: Optimal asset payoff structure as a function of initial government debt when TFP follows a 2 shock i.i.d process

- Begins with an incomplete markets Ramsey allocation
- Asks whether the long-run allocation coincides with a complete markets Ramsey allocation for some initial govt debt

3. BEGS1 studies a related problem with heterogeneous agents

8 Figures

References

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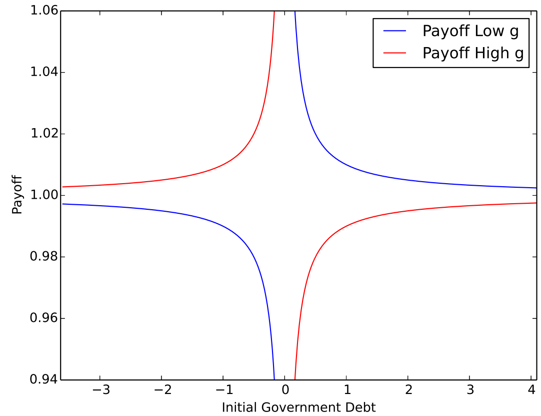

Figure 2: Optimal asset payoff structure as a function of initial government debt when government expenditures follow 2 shock i.i.d process

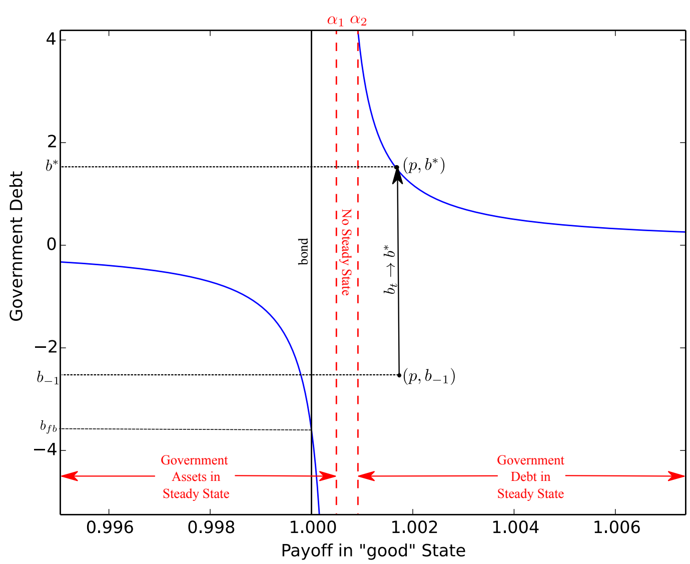

Figure 3: Existence regions in $\boldsymbol{p}$ space

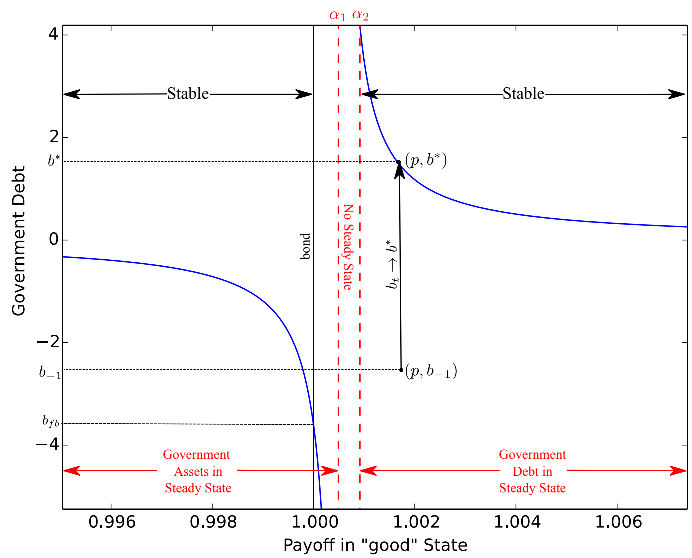

Figure 4: Stability regions

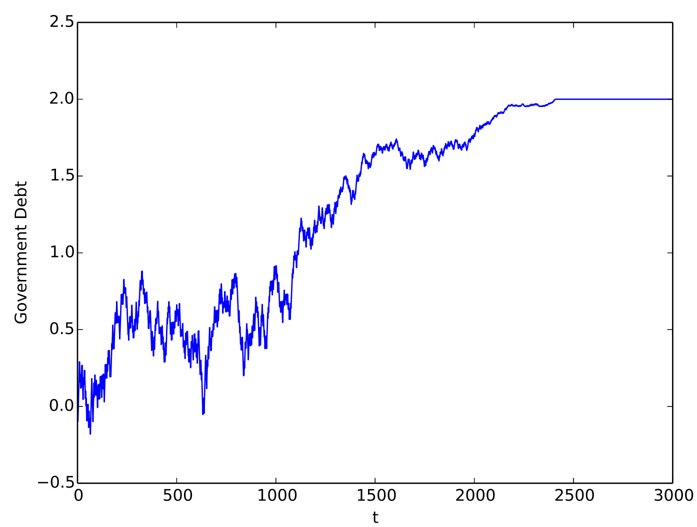

Figure 5: A sample path with $\boldsymbol{p} > 1$

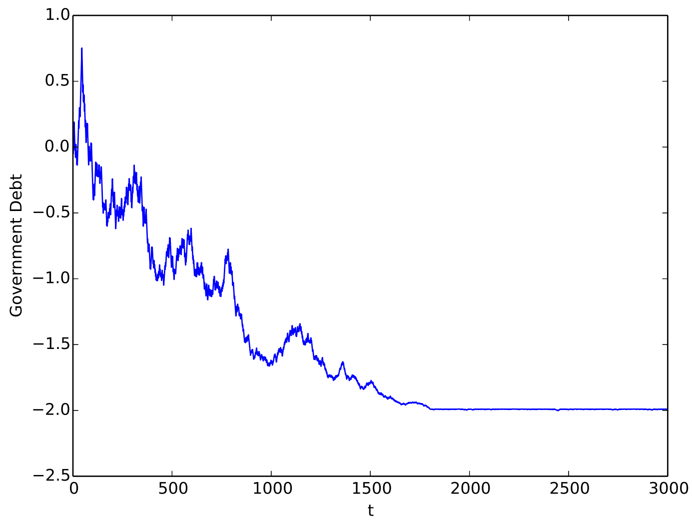

Figure 6: A sample path with $\boldsymbol{p} < 1$

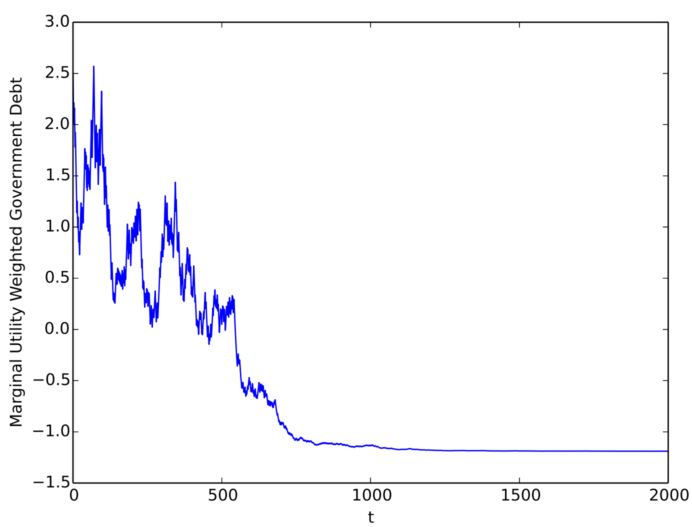

Figure 7: A sample path for 2 state i.i.d. process with risk aversion

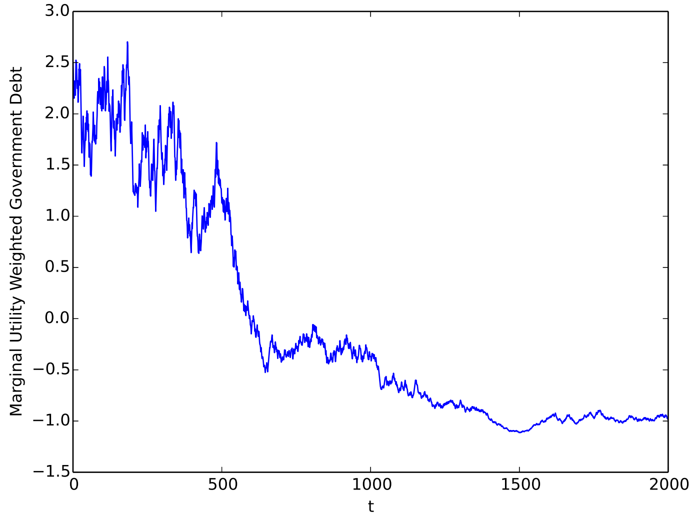

Figure 8: A sample path for economy with $S > 2$ states

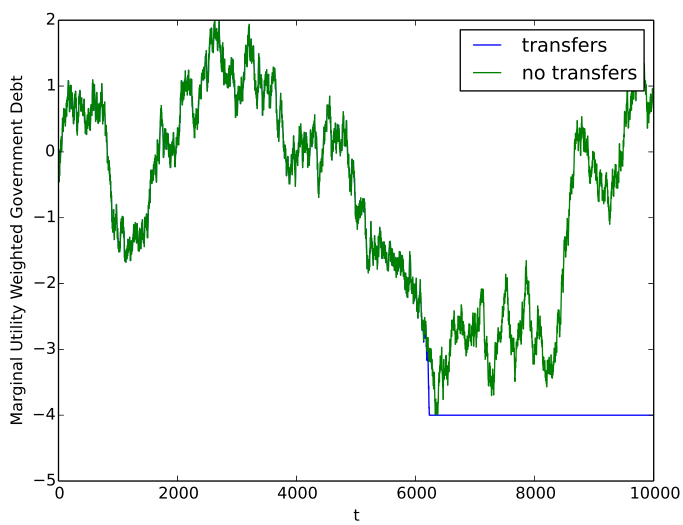

Figure 9: Quasilinear preferences and risk-free bond with and without nonnegative transfers

ydland1980 [4] Kydland, Finn E and Edward C Prescott. 1980. Dynamic optimal taxation, rational expectations and optimal control. *Journal of Economic Dynamics and Control* 2 (0):79–91.

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