Optimal fiscal policy with incomplete asset markets

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Optimal taxation under commitment and a representative agent

- **Incomplete markets**
  $\rightarrow$ assets with alternative exogenous payoff patterns
- **Linear tax schedules**
  $\rightarrow$ Proportional tax on labor earnings (maybe *nonnegative* transfers)
- **Aggregate shocks**
  $\rightarrow$ To productivities, government expenditures, etc.

Questions

1. **Tax rate**: How should government accumulate or decumulate assets to smooth tax distortions
2. **Government debt**: Why do different governments issue different amounts of debt? Difference answers under polar assumptions: LS – complete markets; AMSS — a risk-free bond only
   + Lucas Stokey (1984): Inherited from initial condition
   + AMSS (2002): Govt. accumulates assets sufficient to finance activities using interest revenues

Our analysis

1. **Asset structure**
   - A single asset only
   - We exogenously restrict asset payoffs
2. **Forces**
   - Asset **levels** can help smooth tax distortions **across states**
   - This differs from the role of debt in previous incomplete markets economies where **changes** in debt levels help smooth tax distortions **over time**

Environment

- **Uncertainty**: Markov aggregate shocks $s_t \in \mathcal{S}$; $S \times S$ stochastic matrix Π
- **Demography**: Infinitely lived representative agent plus a benevolent planner
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c(s^t), l(s^t)\right)$$

- **Technology**: Aggregate output $y_t = \theta_t l_t$

Environment, II

- **Asset markets**:
  - A single asset
  - $S \times S$ matrix $\mathbb{P}$ with time t payoff being

$$p_t = \mathbb{P}(s_t | s_{t-1})$$

- **Linear Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_t l_t, \;\; T_t \geq 0$$

- **Budget constraints** q_t is price of asset,
  - Agents: $c_t + q_t b_t = (1 - \tau_t)\, \theta_t l_t + p_t b_{t-1} + T_t,$
  - Government: $g_t + q_t B_t + T_t = \tau_t \theta_t l_t + p_t B_{t-1},$

- **Market Clearing**
  - Goods: $c_t + g_t = \theta_t l_t$
  - Assets: $b_t + B_t = 0$

- **Initial conditions**: Assets b_{-1}, B_{-1} and s_{-1}

Ramsey Problem

Definition

Allocation, price system, government policy

Definition

Competitive equilibrium: Given (b_{-1}, B_{-1}, s_{-1}) and $\{\tau_t, T_t\}_{t=0}^{\infty}$, all allocations are individually rational, markets clear [1]

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given (b_{-1}, B_{-1}, s_{-1})

[1] Usually, we impose only "natural" debt limits.

Ramsey problem

1. **Primal approach**: To eliminate tax rates and prices, use household's first order conditions
2. **Implementability constraints**: Derive by iterating the household's budget equation forward at every history
   $\Rightarrow$With incomplete market economies these impose *measurability restrictions* on Ramsey allocations
3. **Transfers:** We temporarily restrict transfers $T_t = 0 \; \forall t$. This is convenient for our analytical results. We eventually show that this assumption is not restrictive.

Ramsey problem (sequential formulation)

$$\max_{\{c_t, l_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Implementability constraints**

$$\frac{b_{t-1} U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1} p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \text{ for } t \geq 1$$

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

Roadmap, the questions

1. Properties of a Ramsey allocation vary with **asset returns** that reflect
   - Prices $\{q_t(s^t|B_{-1}, s_{-1})\}_t$
   - Payoffs $\mathbb{P}$

   To focus on the exogenous part of return, we first study preferences quasi-linear in consumption that pin down $q_t = \beta$; economies with risk aversion later

2. To characterize the **long run** level of debt and associated taxes we split the analysis into two parts
   - Given **arbitrary** initial assets, what would be an **optimal** asset payoff matrix $\mathbb{P}^*(b)$?
   - In an economy with an **arbitrary** payoff matrix $\mathbb{P}$ when would $b_t \to b^*$?

$$\mathbb{P} = \mathbb{P}^*(b^*)$$

Roadmap, the answers

- In a binary IID world, we identify $\mathbb{P}$'s for which b_t under a Ramsey policy converges to b^*
- For more general shock structures, we numerically verify an ergodic set of b_t hovering around b^*

Quasilinear preferences $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

Given initial assets b, let $\mu(b)$ be the Lagrange multiplier on implementability constraint at $t = 0$

1. **Multiplier $\rightarrow$ Tax rate:**

$$\tau(\mu) = \frac{\gamma\mu}{(1+\gamma)\mu - 1}$$

2. **Tax rate $\rightarrow$ Surplus:**

$$S(s, \tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1+\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

3. **Surplus $\rightarrow$ payoff structure:**

$$\mathbb{P}^*(s|s_-) = (1-\beta)\frac{S(s,\tau)}{\mathbb{E}_{s_-}S(s,\tau)} + \beta$$

and

$$\beta^{-1}\mathbb{P}^*(s|s_-) = \frac{S(s,\tau)}{b} + 1$$

Initial holdings influence optimal asset payoff structure

Denote state s as "adverse" if it has "high" expenditure or "low " TFP; formally, s is "adverse" if

$$g(s)\mathbb{E}_{s\_}\theta^{\frac{\gamma}{1+\gamma}} - \theta(s)^{\frac{\gamma}{1+\gamma}}\mathbb{E}_{s\_}g > 0$$

The optimal payoff matrix $\mathbb{P}$

- With positive initial assets: want a payoff structure that pays *more* in "adverse" states
- With negative initial assets: want a payoff structure that pays *less* in "adverse" states

Optimal Payoff Structure: TFP shocks

David XXXXX: please change xlabel to “Initial Government Debt”

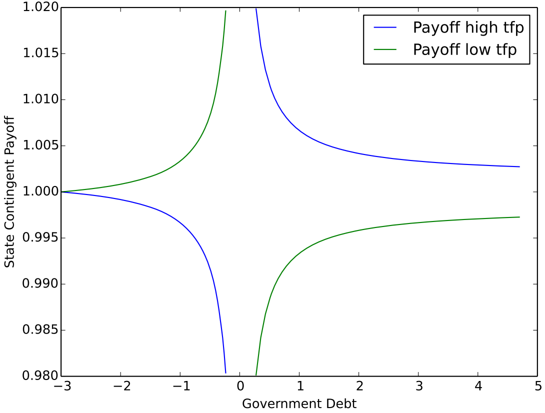

Figure: Optimal asset payoff structure as a function of initial government debt when TFP follows a 2 shock i.i.d process

Optimal Payoff Structure: Expenditure shocks

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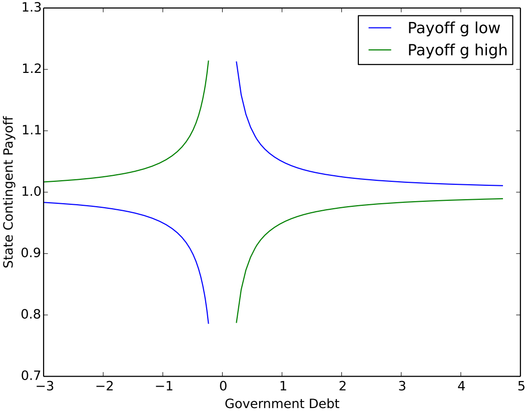


Figure: Optimal asset payoff structure as a function of initial government debt when government expenditures follow 2 shock i.i.d process

Incomplete markets

1. **Exogenous payoff structure:** Suppose $\mathbb{P} \neq \mathbb{P}^*(b_{-1})$
2. **Steady States:** A steady state is a debt level for the government b^* such that

$$b_t = b^* \text{ implies } b_{t+\tau} = b^* \quad \tau > 0$$

3. **Characterization:** Given an asset payoff structure $\mathbb{P}$
   - Does a steady state exist? is it unique?
   - Value of b^*?
   - For what levels of *initial government debt* b_{-1} does convergence to b^*?

Existence

When shocks are i.i.d and take two values

1. $\mathbb{P}(s|s_{-})$ is independent of s_{-} (so $\mathbb{P}$ can be a vector)
2. We can normalize $\mathbb{E}\mathbb{P}(s) = 1$ and w.l.o.g, denote payoffs by a scalar $\mathbf{p}$.
   - $\mathbf{p}$ is the payoff in the "good" state s
   - A risk free bond is a security when $\mathbf{p} = 1$
3. A steady state is obtained by inverting the optimal payoff structure, i.e.

$$b^* \text{ such that } \mathbf{p} = \mathbf{p}^*(b^*) \tag{2}$$

   One equation in one unknown b^*

Existence regions in **p** space

The payoff $\mathbf{p}$ in good state $\in (0, \infty)$.
We can decompose a class of economies with different payoff structures into 3 regions via thresholds $\alpha_2 \geq \alpha_1 \geq 1$

- Low enough $\mathbf{p}(\leq \alpha_1)$: government holds assets in steady state
- High enough $\mathbf{p}(\geq \alpha_2)$: government issues debt in steady state
- Intermediate $\mathbf{p}(\alpha_1 > \mathbf{p} > \alpha_2)$: steady state does not exist

Thresholds: $\alpha_1 < \alpha_2$

- With only government expenditure shocks

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$

- With only TFP shocks

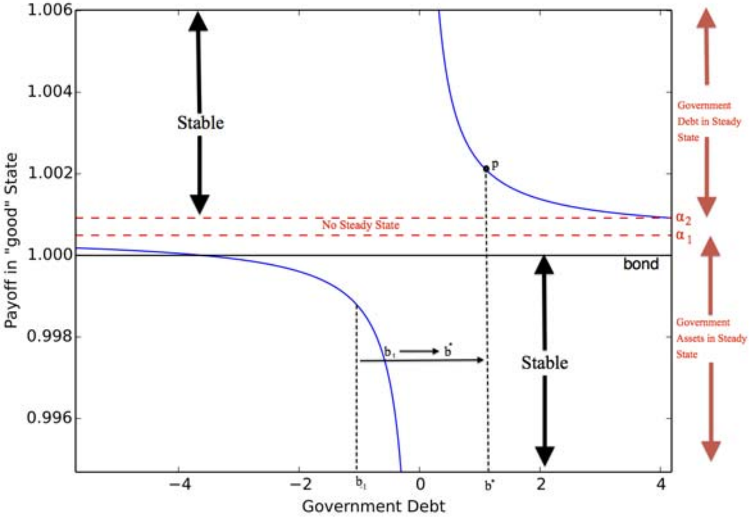
$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

Existence regions in **p** space

David XXXXX: please change xlabel to "Initial Government Debt"



Convergence

- Our analysis verifies the existence of a steady state in a 2-state i.i.d. economy.
- To study long-run properties of a Ramsey allocation with incomplete markets, we need to determine whether these steady states are stable
- **Risk-adjusted martingale:**
  Under a Ramsey policy, the Lagrange multiplier μ_t on the implementability constraint satisfies

  $$\mu_t = \mathbb{E}_t p_{t+1} \mu_{t+1}$$

  or

  $$\mathbb{E}_t \mu_{t+1} = \mu_t - Cov_t(p_{t+1}, \mu_{t+1})$$

- **Stability:** Away from a steady state, is the drift of μ_t big enough?

Characterizing Convergence

- Reminder: **p** is the payoff in the "good" state.
- As with existence, we can partition the "**p** space" into stable and unstable regions

Theorem

Let b^* denote the steady state level of debt and b_{fb} be the level of debt that can support the first best allocation with complete markets.
Then for same $\alpha_1 < \alpha_2$

1. **Low p**: *If* $\mathbf{p} \leq \min(\alpha_1, 1)$ *then the steady state is stable with* $b_{fb} < b^* < 0$ *and* $b_t \to b^*$ *with probability 1.*
2. **High p**: *If* $\mathbf{p} \geq \alpha_2$ *then the steady state is stable with* $0 < b^*$ *and* $b_t \to b^*$ *with probability 1.*

For the intermediate region where either b^* does not exist or is unstable, there is a tendency to run up debt

Stability regions

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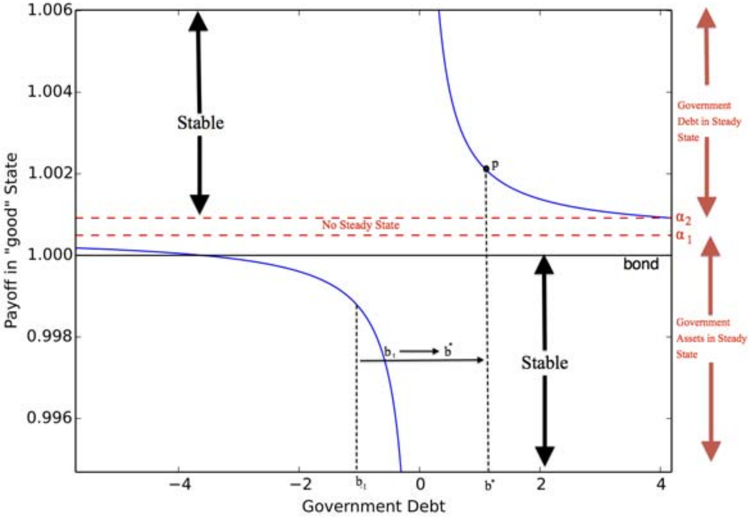
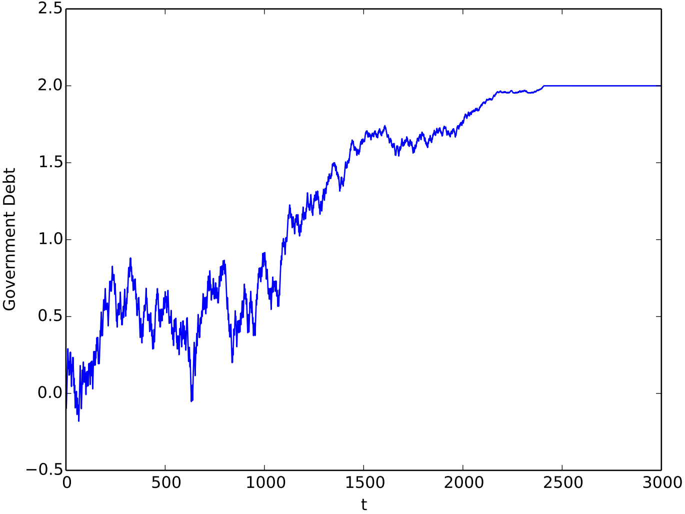
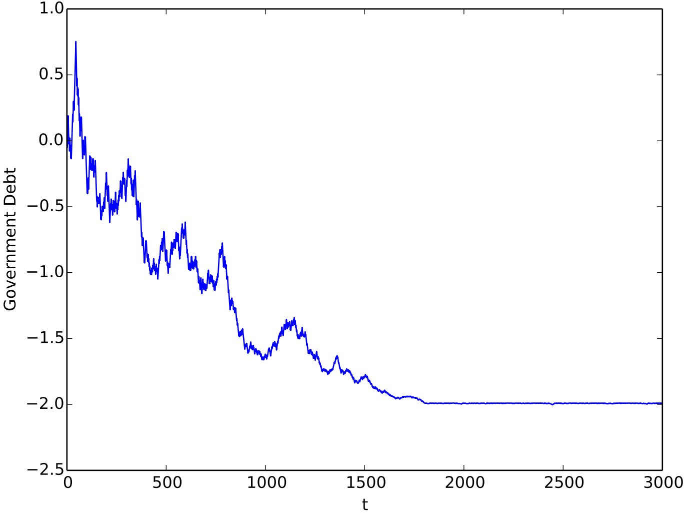


A sample path with $\mathbf{p} > 1$



A sample path with $\mathbf{p} < 1$



Incomplete markets with risk aversion

Quasilinear preferences:

1. With quasilinear preferences, we showed that $b_t \to b^*$ when the aggregate state follows a 2-state i.i.d. process
2. The level and sign of b^* depend on the **exogenous payoff structure** $\mathbb{P}$
3. The limiting allocation corresponds to a complete market Ramsey allocation

Risk aversion:

- This marginal utility adjusted debt encode history dependence
- With binary i.i.d shock shock process, instead of government debt, $x_t = u_{c,t} b_t$ converges
- Long-run properties of x_t depend on returns $R_{t,t+1} = \frac{\mathbb{P}(s_{t+1}|s_t)}{q_t(s^t)}$

Roadmap, II

- Split the Ramsey problem in two
  1. $t = 0$ Bellman equation in value function $W(b_{-1}, s_0)$
  2. $t \geq 1$ Bellman equation in value function $V(x, s_{\_})$
- Analyze steady states x^* such that $x_t \to x^*$

So far,

1. **Risk free bond** Existence proved only under a special case of a risk-free bond $\mathbb{P}(s|s_{\_}) = 1 \forall s, s_{\_}$
   This focuses on *endogenous* component of returns
2. **Interpretation:** x^* corresponds to an initial condition when the optimal portfolio in a LS economy is a risk-free bond

A Recursive Formulation

1. Commitment implies that government actions at $t \geq 1$ are constrained by anticipations about them at $s < t$
2. This contributes additional state variables like marginal utility of consumption
3. Scaling the budget constraint by marginal utility makes Ramsey problem recursive in $x = U_c b$

$$\frac{x_{t-1} p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} = U_{c,t} c_t + U_{l,t} l_t + x_t$$

Bellman equation for $t \geq 1$ (*ex ante*)

$$V(x, s_-) = \max_{c(s), l(s), x'(s)} \sum_s \Pi(s, s_-) \Big(U(c(s), l(s)) + \beta V(x'(s), s) \Big)$$

subject to $x'(s) \in [\underline{x}, \overline{x}]$

$$\frac{x \mathbb{P}(s) U_c(s)}{\beta \mathbb{E} \mathbb{P} U c} = U_c(s) c(s) + U_l(s) l(s) + x'(s)$$

$$c(s) + g(s) = \theta(s) l(s)$$

Time 0 Bellman equation (*ex post*)

Given an initial debt b_{-1}, state s_0, and continuation value function $V(x, s_-)$

$$W(b_{-1}, s_0) = \max_{c_0, l_0, x_0} U(c, l) + \beta V(x_0, s0)$$

subject to time zero implementability constraint

$$U_c(c_0, l_0)c + U_l(c_0, l_0)l_0 + x_0 = U_c(c_0, l_0)b_{-1}$$

and resource constraint

$$c_0 + g(s_0) = \theta(s_0)l_0$$

and

$$x_0 \in [\underline{x}, \overline{x}]$$

Revisiting steady states with risk aversion

Let $x'(s; x, s_{-})$ be an optimal law of motion for the state variable for the $t \geq 1$ recursive problem.

Definition

A steady state x^* satisfies $x^* = x'(s; x^*, s_{-})$ for all s, s_{-}

Thus, a steady state is a node at which continuation allocation and tax rate have no further history dependence.

Existence

1. For a class of economies with separable iso-elastic preferences $U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$
2. Shocks that take two values and are i.i.d with s_b being the "adverse" state (either low TFP or high expenditure)

Let x_{fb} be a value of the state x from which a government can implement first best with complete markets

Proposition

Let $q_{fb}(s)$ be the shadow price of government debt in state s using the first best allocation. If

$$\frac{1 - q_{fb}(s_b)}{1 - q_{fb}(s_g)} > \frac{g(s_b)}{g(s_g)} > 1$$

Then there exists a steady state with $x_{fb} > x^* > 0$

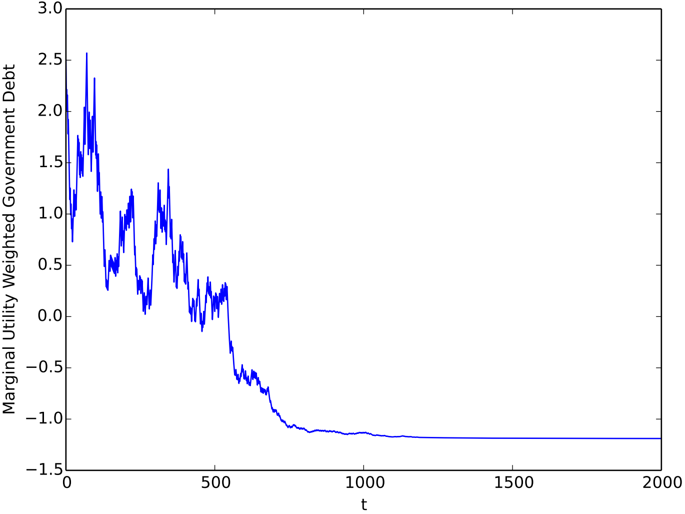
Stability

1. In this setup, interest rates are aligned with marginal utility of consumption; they are low in "good" states (high TFP or low expenditure)
2. The government holds claims against the private sector in the steady state. Similar to the quasilinear case when **p** was low.
3. The incomplete markets Ramsey allocation converges to a Lucas-Stokey Ramsey allocation for all initial conditions

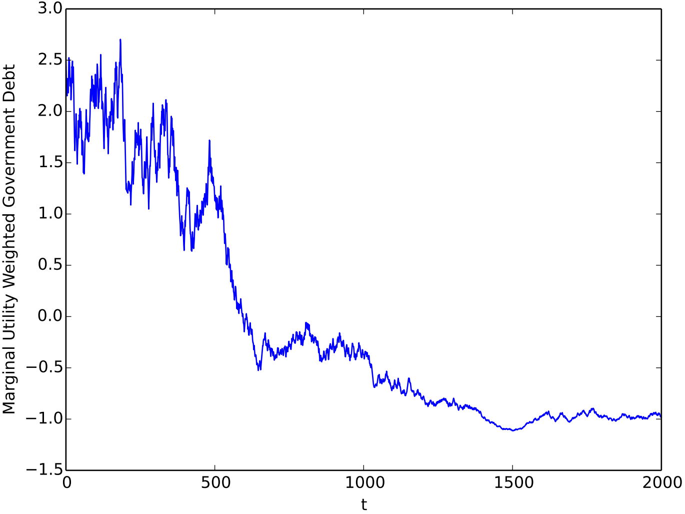
Proposition

Let $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ *solve the incomplete markets Ramsey problem. Then* $x_t(s^{t-1}) \to x^*$ *as* $t \to \infty$ *with probability 1 for all initial conditions*

A sample path for 2 state i.i.d. process with risk aversion



A sample path for economy with $S > 2$ states



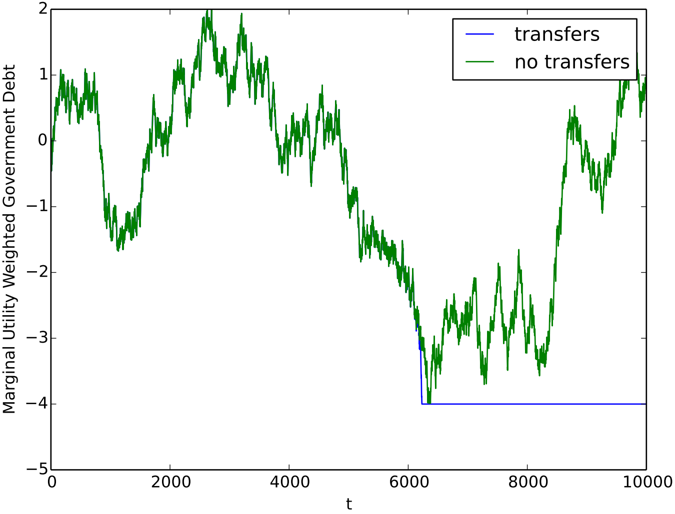
Transfers

- Access to nonnegative transfers makes first-best level of assets trivially a "steady state"
- All results hold *on one side* of steady state

Theorem

With lump sum transfers, in cases where the steady state exists and is stable, if the initial debt of the government exceeds its steady state level, the economy converges with probability 1 to the steady state.

Quasilinear preferences and risk-free bond with and without transfers



Add a slide comparing stuff to Buera Nicolini, Angelotos

Concluding remarks

- With market incompleteness, the asset payoff structure has big implications a Ramsey government's long run debt
- If the asset offers lower returns in adverse states of the world, the Ramsey government asymptotically runs up a debt to the private sector.
- With risk aversion, cyclical properties of interest rate affects government debt asymptotically
- Access to nonnegative transfers play little role in shaping outcomes. Rather, the key force is the government's ability to use its debt position to reallocate resources across states
- **Future Research**: With heterogeneous agents and unrestricted transfers, how does the type of market incompleteness affect long run wealth distributions and other outcomes?