Optimal Taxation with Incomplete Markets

Anmol Bhandari, David Evans, Mikhail Golosov, Thomas J. Sargent

October 2013

Lucas and Stokey, 1983

> . . . the option to issue state-contingent debt is
> important: tax policies that are optimal under
> uncertainty have an essential 'insurance' aspect to them.

Commitment, representative agent, no capital

- **Incomplete markets**
    - $\rightarrow$ A single, possibly risky asset
- **Linear tax schedules**
    - $\rightarrow$ Proportional tax on labor earnings (maybe plus *nonnegative* transfers)
- **Aggregate shocks**
    - $\rightarrow$ To productivities, government expenditures

Questions

1. Should a government accumulate or decumulate assets?
2. Why might different governments want to issue different amounts of debt?
3. Existing answers hinge on polar assumptions:
    + Complete markets, Lucas Stokey (1984): non history dependent debt quantities inherited from initial condition
    + A risk-free bond only, quasi-linear preferences, AMSS (2002): govt. accumulates *assets* sufficient to finance activities using interest revenues
4. Unknown: what if interest rates fluctuate?

Environment

- **Uncertainty**: Markov aggregate shocks $s_t \in \mathcal{S}$; $S \times S$ stochastic matrix Π; $g_t = g(s_t)$; $\theta_t = \theta(s_t)$
- **Demography**: Infinitely lived representative agent plus a benevolent planner
- **Preferences** (representative household)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c(s^t), l(s^t)\right)$$

- **Technology**: Aggregate output $y_t = \theta_t l_t$

Environment, II

- **Asset market**:
  - $S \times S$ matrix $\mathbb{P}$ with time t payoff being

  $$p_t = \mathbb{P}(s_t | s_{t-1})$$

- **Linear Taxes**: Agent i's tax bill

  $$-T_t + \tau_t \theta_t l_t, \ T_t \geq 0$$

- **Budget constraints** q_t is price of asset
  - Household: $c_t + q_t b_t = (1 - \tau_t) \theta_t l_t + p_t b_{t-1} + T_t$
  - Government: $g_t + q_t B_t + T_t = \tau_t \theta_t l_t + p_t B_{t-1}$

- **Market Clearing**
  - Goods: $c_t + g_t = \theta_t l_t$
  - Assets: $b_t + B_t = 0$

- **Initial conditions**: Assets b_{-1}, B_{-1} and s_{-1}

Ramsey Problem

Definition
Allocation, price system, government policy

Definition
Competitive equilibrium: Given (b_{-1}, B_{-1}, s_{-1}) and $\{\tau_t, T_t\}_{t=0}^{\infty}$, all allocations are individually rational, markets clear [1]

Definition
Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given (b_{-1}, B_{-1}, s_{-1})

---

[1]Usually, we impose only "natural" debt limits.

Ramsey problem

1. **Primal approach**: To eliminate tax rates and prices, use household's first order conditions:

$$U_{c,t} q_t = \beta \mathbb{E}_t U_{c,t+1}$$

$$(1 - \tau_t)\theta_t U_{c,t} = -U_{l,t}$$

2. **Implementability constraints**: Derive by iterating the household's budget equation forward at every history $\Rightarrow$ for $t \geq 1$, these impose *measurability restrictions* on Ramsey allocations

3. The $t \geq 1$ **measurability constraints** contribute the only difference from Lucas-Stokey's Ramsey problem.

Ramsey problem

4. **Transfers:** We temporarily restrict transfers $T_t = 0 \; \forall t$. This is convenient for our analytical results. We eventually show that this assumption is not restrictive.

Ramsey problem (Lucas-Stokey)

$$\max_{\{c_t, l_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Implementability constraint**

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

Ramsey problem (BEGS)

$$\max_{\{c_t, l_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Lucas-Stokey implementability constraint**

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

(c) **Measurability constraints**

$$\frac{b_{t-1} U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1} p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \text{ for } t \geq 1$$

Roadmap, analytic strategy

- Ramsey allocation – especially asymptotic properties – varies with **asset returns** that reflect
  - Prices $\{q_t(s^t|B_{-1}, s_{-1})\}_t$
  - Payoffs $\mathbb{P}$
- To focus on the exogenous $\mathbb{P}$ part of returns, we first study quasi-linear preferences that pin down $q_t = \beta \mathbb{E}_t \mathbb{P}(s_{t+1}|s_t)$
- Activate risk aversion and fluctuating q_t later

Battlefield

What is government debt in long-run?

	risk-free bond	risky bond
Quasi-Linear		
Risk Aversion		

Analysis with quasi-linear preferences

Quasilinear preferences $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

To characterize **long-run** debt and taxes, we construct and then invert mapping $\mathbb{P}^*(b)$

- Given **arbitrary** initial govt. assets b_{-1}, what is an **optimal** asset payoff matrix $\mathbb{P}^* = \mathbb{P}^*(b_{-1})$?

- Under a Ramsey plan for an **arbitrary** payoff matrix $\mathbb{P}$, when would $b_t \to b^*$, where

$$\mathbb{P} = \mathbb{P}^*(b^*) \text{ or } b^* = \mathbb{P}^{*-1}(\mathbb{P})?$$

Roadmap, the answers

- We first reverse engineer an optimal $\mathbb{P}^*(b_{-1})$ from a Lucas-Stokey Ramsey allocation

- In a binary IID world, we identify a big set of $\mathbb{P}$'s that imply that b_t under a Ramsey plan converges to b^* that solves

$$\mathbb{P} = \mathbb{P}^*(b^*)$$

- For more general shock structures, we numerically verify an ergodic set of b_t's hovering around $\tilde{b}$. The optimal $\mathbb{P}^*$ associated with $\tilde{b}$ seems close to $\mathbb{P}$:

$$\mathbb{P} \approx \mathbb{P}^*(\tilde{b})$$

Optimal asset payoff matrix $\mathbb{P}^*$

1. Given b_{-1}, compute a Lucas-Stokey Ramsey allocation
2. Reverse engineer payoff on single asset

$$p_t = \frac{\beta}{U_{c,t}b_{t-1}}\mathbb{E}_t\sum_{j=0}^{\infty}\beta^j\left(U_{c,t+j}c_{t+j} + U_{l,t+j}l_{t+j}\right)$$

3. By construction, the optimal payoff p_t disarms the $t \geq 1$ measurability constraints
4. Since a Lucas-Stokey Ramsey allocation is history independent,

$$p_t = \mathbb{P}^*(s_t|s_{t-1})$$

Quasilinear preferences $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

Given initial assets b_{-1}, let $\mu(b_{-1})$ be the Lagrange multiplier on the Lucas-Stokey implementability constraint

1. **Multiplier $\rightarrow$ Tax rate:**

$$\tau(\mu) = \frac{\gamma\mu}{(1+\gamma)\mu - 1}$$

2. **Tax rate $\rightarrow$ net of interest surplus:**

$$S(s, \tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1-\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

3. **Surplus $\rightarrow$ optimal payoff structure:**

$$\mathbb{P}^*(s|s_-) = (1-\beta)\frac{S(s, \tau)}{\mathbb{E}_{s_-} S(s, \tau)} + \beta$$

Initial holdings influence optimal asset payoff structure

Denote state s as "adverse" if it has "high" govt. expenditures or "low" TFP; formally, s is "adverse" if

$$g(s)\mathbb{E}_{s_-}\theta^{\frac{\gamma}{1+\gamma}} - \theta(s)^{\frac{\gamma}{1+\gamma}}\mathbb{E}_{s_-}g > 0$$

Properties of optimal payoff matrix $\mathbb{P}$

▶ With positive initial govt. assets: want an asset that pays *more* in "adverse" states

▶ With negative initial govt. assets: want an asset that pays *less* in "adverse" states

Optimal Payoff Structure: TFP shocks
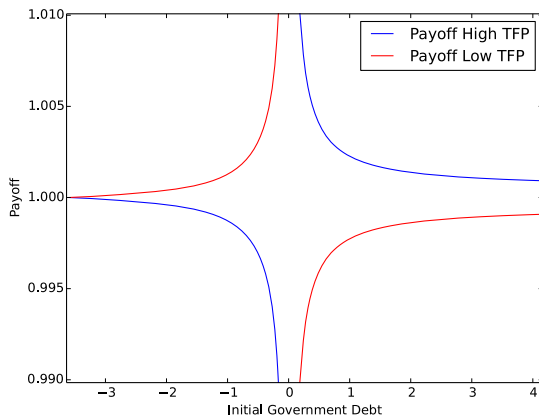
Figure: Optimal asset payoff structure as a function of initial government debt when TFP follows a 2 shock i.i.d process
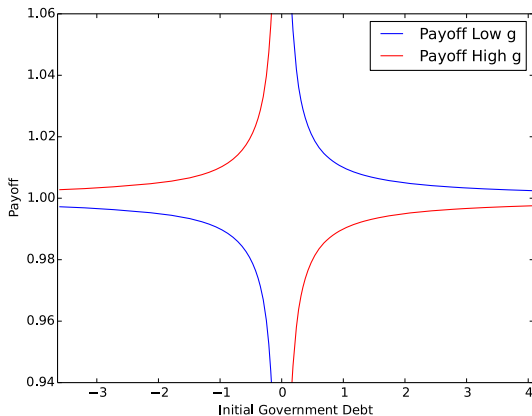
Optimal Payoff Structure: Expenditure shocks

Figure: Optimal asset payoff structure as a function of initial government debt when government expenditures follow 2 shock i.i.d process

Inverting the $\mathbb{P}^*$ mapping

1. **Exogenous payoff structure:** Suppose $\mathbb{P} \neq \mathbb{P}^*(b_{-1})$

2. **Steady States:** A steady state is a government debt b^* such that

$$b_t = b^* \text{ implies } b_{t+\tau} = b^* \quad \forall \tau > 0$$

3. **Characterization:** Given an asset payoff structure $\mathbb{P}$
   - Does a steady state exist? Is it unique?
   - Value of b^*?
   - For what *initial government debts* b_{-1} does b_t converge to b^*?

Existence and $\mathbb{P}^{*-1}$

When shocks are i.i.d and take two values

1. $\mathbb{P}(s|s_-)$ is independent of s_- (so $\mathbb{P}$ can be a vector)
2. We can normalize $\mathbb{E}\mathbb{P}(s) = 1$ and then pin down payoffs with a scalar p.
   - p is the payoff in the "good" state s
   - A risk-free bond is a security for which $p = 1$
3. A steady state is obtained by inverting the optimal payoff mapping p^*

$$b^* \text{ satisfies } p = p^*(b^*) \text{ or } p^{*-1}(p) = b^*$$

One equation in one unknown b^*

Existence regions in p space

The payoff p in good state $\in (0, \infty)$.
We can decompose a set of economies with different asset payoffs
into 3 regions via thresholds $\alpha_2 \geq \alpha_1 \geq 1$

- Low enough $p(\leq \alpha_1)$: government holds assets in steady state
- High enough $p(\geq \alpha_2)$: government issues debt in steady state
- Intermediate $p(\alpha_1 > p > \alpha_2)$: steady state does not exist

Thresholds: $\alpha_1 < \alpha_2$

- With only government expenditure shocks

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$
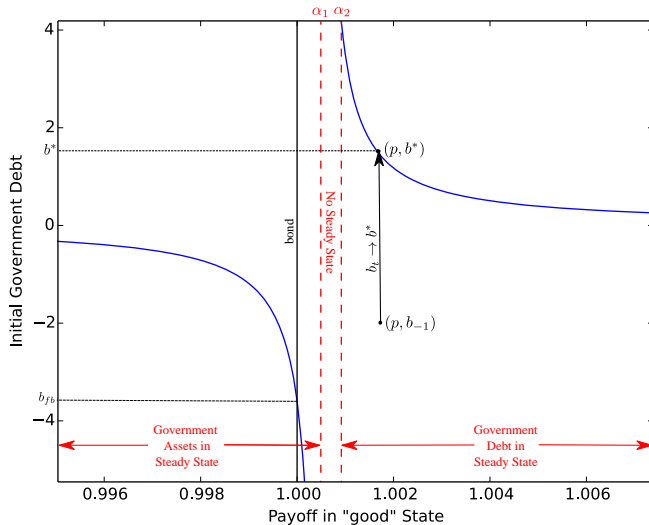
- With only TFP shocks

$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

Existence regions in p space

Convergence

- Our analysis verifies existence of a steady state in a 2-state i.i.d. economy.

- To study long-run properties of a Ramsey allocation, we want to know whether steady state is stable

- **Risk-adjusted martingale:**
  The Lagrange multiplier μ_t on the implementability constraint satisfies

  $$\mu_t = \mathbb{E}_t p_{t+1} \mu_{t+1}$$

  or

  $$\mathbb{E}_t \mu_{t+1} = \mu_t - Cov_t(p_{t+1}, \mu_{t+1})$$

- **Stability criterion:** Away from a steady state, is the drift of μ_t big enough?

Characterizing convergence under quasi-linearity, iid, and $S = 2$

- ▶ Reminder: p is the payoff in the "good" state.
- ▶ We partition the "p space" into stable and unstable regions
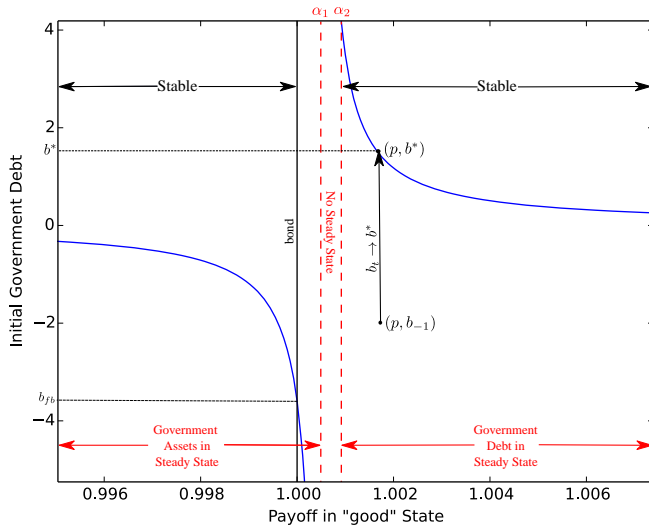
Theorem
Let b^* denote steady state govt. debt and b_{fb} be govt. debt that supports the first-best allocation with complete markets. Then
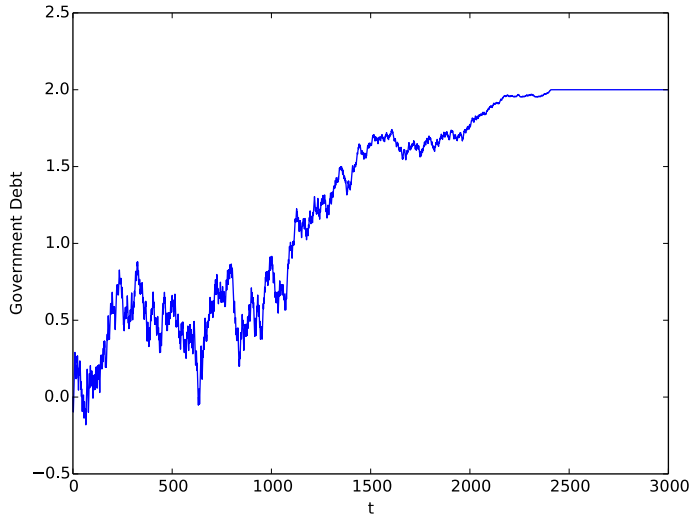
1. **Low p:** *If $p \leq \min(\alpha_1, 1)$ then $b_{fb} < b^* < 0$ and $b_t \to b^*$ with probability 1.*

2. **High p:** *If $p \geq \alpha_2$ then $0 < b^*$ and $b_t \to b^*$ with probability 1.*

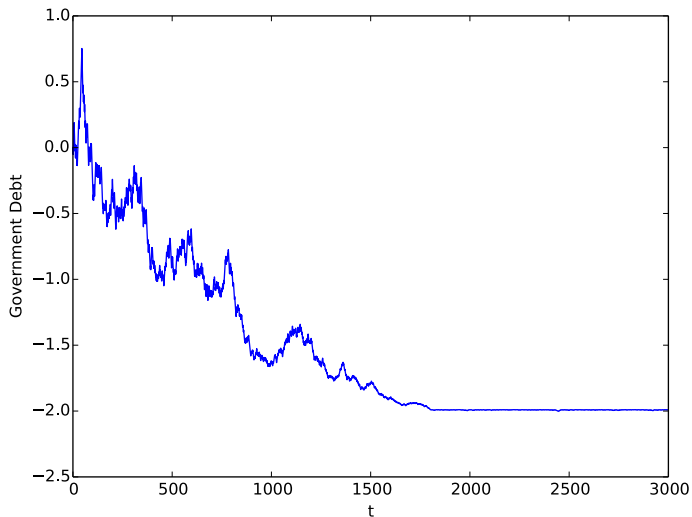For the intermediate region where b^* either does not exist or is unstable, there is a tendency to run up debt

Stability regions

A sample path with $p > 1$

A sample path with $p < 1$

Outcomes with quasi-linear preferences

Outcomes:

1. Often $b_t \to b^*$ when the aggregate state follows a 2-state i.i.d. process
2. The level and sign of b^* depend on the **exogenous payoff structure** $\mathbb{P}$
3. The limiting allocation corresponds to a complete market Ramsey allocation for initial govt. debt b^*

Turning on risk-aversion

Modifications:

- Another source of return fluctuations – the risk-free interest rate
- Marginal utility adjusted debt encodes history dependence
- With binary i.i.d shock shock process, $x_t = u_{c,t} b_t$ converges
- Long-run properties of x_t depend on equilibrium returns $R_{t,t+1} = \frac{\mathbb{P}(s_{t+1}|s_t)}{q_t(s^t)}$. Now q_t varies in interesting ways

Two subproblems

1. $t = 0$ Bellman equation in value function $W(b_{-1}, s_0)$
2. $t \geq 1$ Bellman equation in value function $V(x, s_-)$

Seek steady states x^* such that $x_t \to x^*$

A Recursive Formulation

1. Commitment implies that government actions at $t \geq 1$ are constrained by the public's anticipations about them at $s < t$
2. This contributes additional state variables like marginal utility of consumption
3. Scaling the budget constraint by marginal utility makes Ramsey problem recursive in $x = U_c b$

$$\frac{x_{t-1} p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} = U_{c,t} c_t + U_{l,t} l_t + x_t$$

Bellman equation for $t \geq 1$ (*ex ante*)

$$V(x, s_-) = \max_{c(s), l(s), x'(s)} \sum_s \Pi(s, s_-)\Big(U(c(s), l(s)) + \beta V(x'(s), s)\Big)$$

subject to $x'(s) \in [\underline{x}, \overline{x}]$

$$\frac{x\mathbb{P}(s)U_c(s)}{\beta\mathbb{E}\mathbb{P}Uc} = U_c(s)c(s) + U_l(s)l(s) + x'(s)$$

$$c(s) + g(s) = \theta(s)l(s)$$

Time 0 Bellman equation (*ex post*)

Given an initial debt b_{-1}, state s_0, and continuation value function $V(x, s_-)$

$$W(b_{-1}, s_0) = \max_{c_0, l_0, x_0} U(c, l) + \beta V(x_0, s0)$$

subject to time zero implementability constraint

$$U_c(c_0, l_0)c + U_l(c_0, l_0)l_0 + x_0 = U_c(c_0, l_0)b_{-1}$$

and resource constraint

$$c_0 + g(s_0) = \theta(s_0)l_0$$

and

$$x_0 \in [\underline{x}, \overline{x}]$$

Progress report

1. Existence proved only under a special case of a risk-free bond
   $\mathbb{P}(s|s_-) = 1 \ \forall \ (s, s_-)$
   This focuses attention on *endogenous* component of returns
   coming from $q_t(s^t)$
2. x^* is an initial condition for which the optimal asset payoff in
   a LS economy is a risk-free bond

Revisiting steady states with risk aversion

Let $x'(s; x, s_-)$ be an optimal law of motion for the state variable for the $t \geq 1$ recursive problem.

Definition

A steady state x^* satisfies $x^* = x'(s; x^*, s_-)$ for all s, s_-

Thus, a steady state is a node at which continuation allocation and tax rate have no further history dependence.

Existence

1. For a class of economies with separable iso-elastic preferences $U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$

2. Shocks that take two values and are i.i.d with s_b being the "adverse" state (either low TFP or high expenditure)

Let x_{fb} be a value of the state x from which a government can implement first=best with complete markets

Proposition

Let $q_{fb}(s)$ be the shadow price of government debt in state s using the first best allocation. If

$$\frac{1 - q_{fb}(s_b)}{1 - q_{fb}(s_g)} > \frac{g(s_b)}{g(s_g)} > 1$$

*Then there exists a steady state with $x_{fb} > x^* > 0$*

Stability

1. Here interest rates are aligned with marginal utility of consumption; they are low in "good" states (high TFP or low expenditure)
2. The government holds claims against the private sector in the steady state. Similar to the quasilinear case with low p
3. The incomplete markets Ramsey allocation converges to a Lucas-Stokey Ramsey allocation for all initial conditions
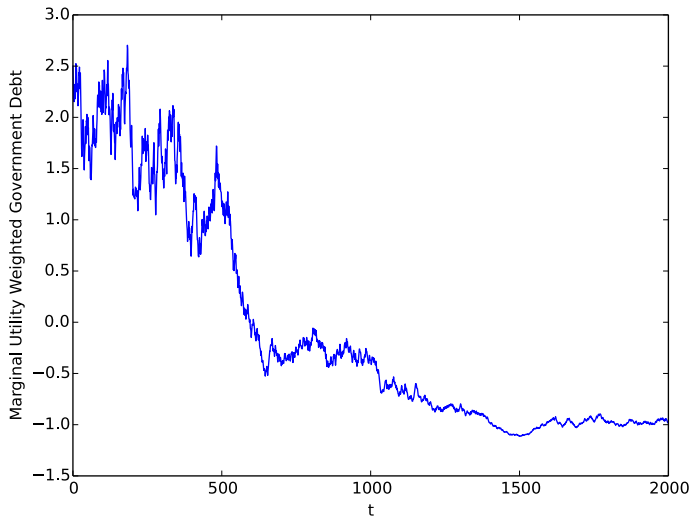
Proposition

Let $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ solve the incomplete markets Ramsey problem with $x_0 > x^*$. Then $x_t(s^{t-1}) \to x^*$ as $t \to \infty$ with probability 1 for all initial conditions

A sample path for 2 state i.i.d. process with risk aversion
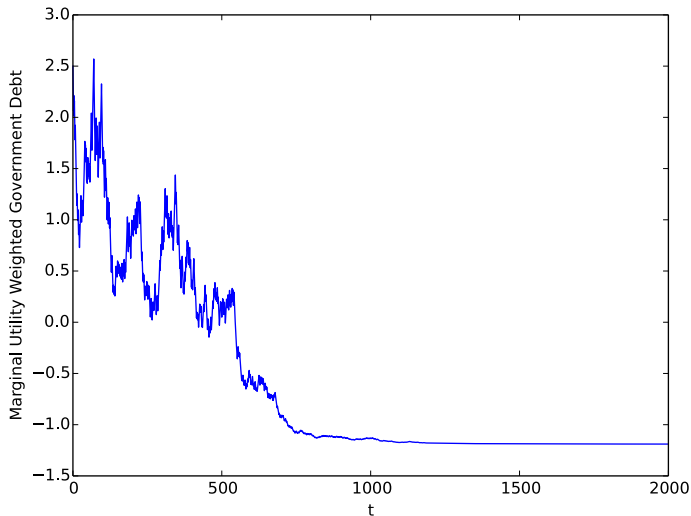
A sample path for economy with $S > 2$ states

Transfers

- Access to nonnegative transfers makes first-best level of assets trivially a "steady state"
- All results hold *on one side* of steady state

Theorem
With nonnegative lump sum transfers, in cases where a steady state exists and is stable, if the initial debt of the government exceeds its steady state level, the economy converges with probability 1 to the steady state.

Quasilinear preferences and risk-free bond with and without nonnegative transfers
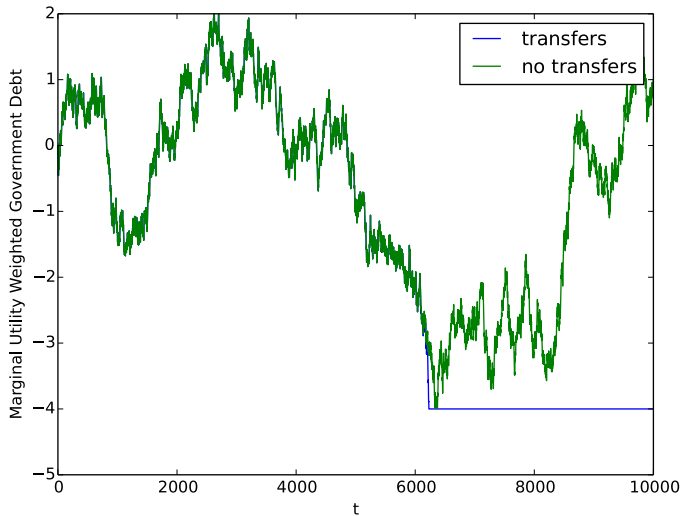
Battlefield

What is government debt in long-run?

With shocks that are IID and take two values

	risk-free bond	risky bond
Quasi-Linear	(AMSS) With $T_t \geq 0$, govt. accumulates enough assets for first best.	Partition payoff space so that govt. either a) issues or b) runs up debt eventually
Risk Aversion	Conditions under which limiting govt. assets < first best	*Conjecture:* Similar to quasi-linear outcomes

In future work we plan to study more general shocks processes

Comparison to literature

1. Angelotos (2002), Buera and Nicolini (2004)
   - Begin with a complete market Ramsey allocation
   - Ask if this can be spanned by *some* collection of non-contingents debt of different maturities?

2. This paper
   - Begin with an incomplete markets Ramsey allocation
   - Ask whether the long-run allocation coincides with *some* complete market allocation?

3. In BEGS1, we study a related problem with heterogeneous agents