Optimal fiscal policy with incomplete asset markets

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Optimal taxation under commitment and a representative agent

- **Incomplete markets**
  $\rightarrow$ assets with alternative exogenous payoff patterns
- **Linear tax schedules**
  $\rightarrow$ Proportional tax on labor earnings (maybe *nonnegative* transfers)
- **Aggregate shocks**
  $\rightarrow$ To productivities, government expenditures, etc.

Questions

1. **Tax rate**: How should government accumulate or decumulate assets to smooth tax distortions
2. **Government debt**: Why do different governments issue different amounts of debt?
   * Answers under polar assumptions: LS – complete markets; AMSS — a risk-free bond only
   + Lucas Stokey (1984): Inherited from initial condition
   + AMSS (2002): Govt. accumulates assets sufficient to finance activities using interest revenues

Our analysis

1. Cases intermediate between LS and AMSS
   + We restrict government to trade a single asset only
   + We exogenously restrict asset payoffs
     $\Longrightarrow$ E.g., bonds that pay less during adverse times
2. Asset **levels** can help smooth tax distortions **across states**
3. This differs from the role of debt in previous incomplete markets economies where **changes** in debt levels help smooth tax distortions **over time**

Environment

- **Uncertainty**: Markov aggregate shocks s_t
- **Demography**: Infinitely lived representative agent plus a benevolent planner
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c(s^t), l(s^t)\right)$$

- **Technology**: Aggregate output $y_t = \theta_t l_t$

Environment, II

- **Asset markets**: Private sector has complete markets; Government trades are restricted:
  - A unit of government debt pays off p_s in state s next period
- **Linear Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_t l_t, \;\; T_t \geq 0$$

- **Budget constraints**
  - Agents: $c_t + q_t b_t = (1 - \tau_t)\,\theta_t l_t + p_t b_{t-1} + T_t,$
  - Government: $g_t + q_t B_t + T_t = \tau_t \theta_t l_t + p_t B_{t-1},$
- **Market Clearing**
  - Goods: $c_t + g_t = \theta_t l_t$
  - Assets: $b_t + B_t = 0$
- **Initial conditions**: Assets b_{-1}, B_{-1} and s_{-1}

Ramsey Problem

Definition

Allocation, price system, government policy

Definition

Competitive equilibrium: Given (b_{-1}, B_{-1}) and $\{\tau_t, T_t\}_{t=0}^{\infty}$, all allocations are individually rational, markets clear [1]

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given (b_{-1}, B_{-1})

[1] Usually, we impose only "natural" debt limits.

Roadmap

1. Given initial assets, characterize an **optimal asset payoff structure**:
   $\Rightarrow$ What single asset would a Ramsey government issue?
   We reverse engineer payoffs from a Lucas-Stokey Ramsey plan
2. Study a Ramsey problem when the asset payoff structure is not optimal
   - Government asset returns have both exogenous component (p_s) and endogenous component $\frac{1}{q_t}$.
   - To remove the endogenous component, we initially study quasilinear preferences.

Sequential Ramsey problem

$$\max_{\{c_t, l_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Implementability constraints**

$$\frac{b_{t-1} U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1} p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \text{ for } t \geq 1$$

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

Sequential Ramsey problem

1. **Primal approach**: To eliminate tax rates and prices, use consumer's first order conditions
2. **Implementability constraints**: Derive by iterating the consumer's budget equation forward at every history
   $\Rightarrow$With incomplete market economies these impose *measurability restrictions* on Ramsey allocations
3. **Transfers:** We temporarily restrict transfers $T_t = 0 \; \forall t$. This is convenient for our analytical results. We eventually show that this assumption is not restrictive.

Optimal asset payoff structures

1. Compute a Lucas-Stokey Ramsey allocation given b_{-1}
   Find an optimal allocation ignoring $t \geq 1$ measurability constraints.
2. Reverse engineer payoff on single asset

$$p_t = \frac{\beta}{U_{c,t} b_{t-1}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right)$$

   In effect, the Ramsey planner chooses an asset payoff structure p_t so that measurability constraints for $t \geq 1$ do not bind
3. Since allocations are history independent

$$p_t = p^*(s_t | s_{t-1})$$

Quasilinear preferences $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

Given initial assets b, let $\mu(b)$ be the Lagrange multiplier on implementability constraint at $t = 0$

1. **Multiplier → Tax rate:**

$$\tau(\mu) = \frac{\gamma\mu}{(1+\gamma)\mu - 1}$$

2. **Tax rate → Surplus:**

$$S(s, \tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1+\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

3. **Surplus → payoff structure:**

$$p^*(s|s_-) = (1-\beta)\frac{S(s, \tau)}{\mathbb{E}_{s_-}S(s, \tau)} + \beta$$

Initial holdings influence optimal asset payoff structure

- With positive initial assets: want a payoff structure that pays *more* in “adverse” states
- With negative initial assets: want a payoff structure that pay *less* in “adverse” states

Influence of shock structure

When do we approximate a risk-free bond?

- With i.i.d shocks to government expenditures: when initial assets approach infinity
- With i.i.d shocks to TFP: when initial assets are sufficient to support the first best

Optimal Payoff Structure: Expenditure shocks

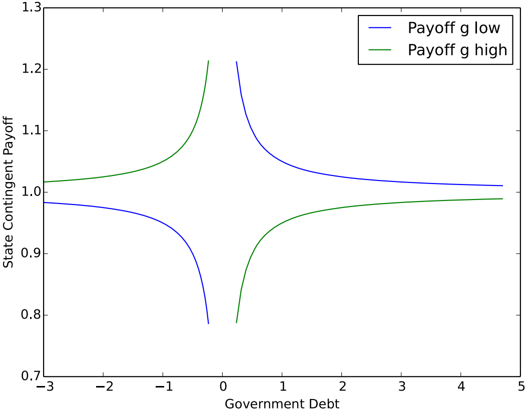


Figure: Optimal asset payoff structure as a function of initial government debt when government expenditures follow 2 shock i.i.d process

Optimal Payoff Structure: TFP shocks

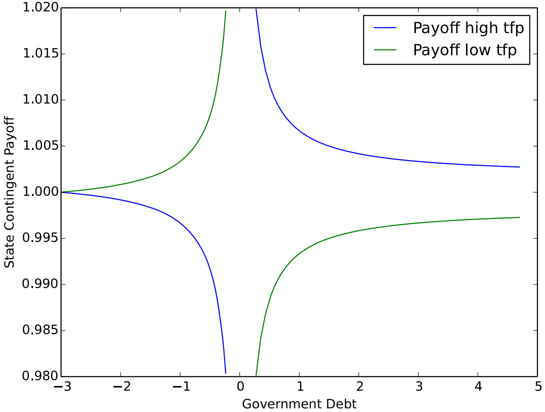


Figure: Optimal asset payoff structure as a function of initial government debt when TFP follows a 2 shock i.i.d process

Incomplete markets more generally

- **Exogenous payoff structure:** Suppose $p \neq p^*(b_{-1})$
- **Steady States:** *Do government assets asymptotically approximate a steady state for which the exogenous asset payoff structure is optimal?*
- **Characterization:** Given an asset payoff structure p, with a 2 state i.i.d process for the aggregate state
  - When does a steady state exist? is it unique?
  - What is the level of government debt or assets in a steady state?
  - For what levels of *initial government debt* does convergence to a steady state occur?
- **Extensions:** to richer aggregate stochastic process and preferences exhibiting risk aversion

Link of steady state outcomes to exogenous asset payoffs

1. Finding a steady state amounts to finding a complete markets optimal allocation and initial asset level whose optimal payoff structure matches p_t
2. We explore different asset payoff structures
   - Suppose $s = 1$ is the "good" state (low expenditure or high TFP)
   - Index incomplete market economies by $p_1 = p(s = 1)$. [2]
3. **Outcomes in 3 Regions:**
   - Low enough p_1: government holds assets in steady state
   - High enough p_1: government issues debt in steady state
   - Intermediate p_1: steady states do not exist

   Thresholds α_1, α_2 "split p_1 space"

[2] We get p_2 by using the normalization that $\mathbb{E}p(s) = 1$. A risk-free bond sets $p_1 = 1$

Thresholds: $\alpha_1 < \alpha_2$

- With only government expenditure shocks

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$

- With only TFP shocks

$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

Convergence

- Our analysis verifies the existence of a steady state in a 2-state i.i.d. economy.
- To study long-run properties of a Ramsey allocation with incomplete markets, we need to determine whether these steady states are stable
- **Risk-adjusted martingale:**
  Under an optimal policy, the Lagrange multiplier μ_t on the implementability constraint satisfies

$$\mu_t = \mathbb{E}_t p_{t+1} \mu_{t+1}$$

  or

$$\mathbb{E}_t \mu_{t+1} = \mu_t - Cov_t(p_{t+1}, \mu_{t+1})$$

  μ_t follows a risk adjusted martingale.
- **Stability:** Away from a steady state, is the drift μ_t big enough?

Characterizing Convergence

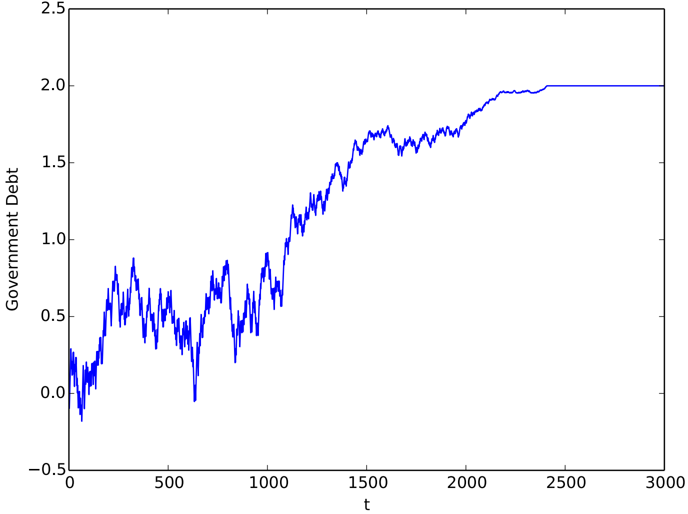
- Reminder: p_1 is the payoff in the "good" state.
- As with existence, we can partition the "p_1 space" into different regions
- Let b_{fb}, μ_{fb} be the debt level and associated Lagrange multiplier when the government can implement first-best via access to complete markets

Theorem

Let b^* denote the steady state level of debt. Then for same $\alpha_1 < \alpha_2$

1. **Low** p_1*: If $p_1 \leq \min(\alpha_1, 1)$ then the steady state is stable with $b_{fb} < b^* < 0$ and $b_t \to b^*$ with probability 1.*
2. **High** p_1*: If $p_1 \geq \alpha_2$ then the steady state is stable with $0 < b^*$ and $b_t \to b^*$ with probability 1.*
3. **Intermediate** p_1**:**
   - *If $1 < p_1 < \alpha_1$ then a steady state exists with $b^* < b_{fb}$, but it is unstable with $\mu_t > \mathbb{E}_t \mu_{t+1}$ for $\mu < \mu_{fb}$.*
   - *If $\alpha_1 \leq p_1 < \alpha_2$ then a steady state does not exist and $\mu_t > \mathbb{E}_t \mu_{t+1}$*

$p_1 > 1$



2.5
2.0
1.5
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−0.5
Government Debt
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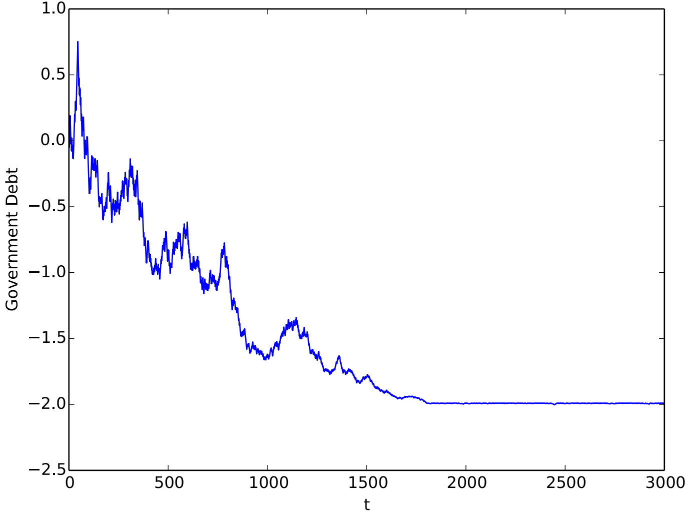
500
1000
1500
2000
2500
3000
t

$p_1 < 1$



1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
Government Debt
0
500
1000
1500
2000
2500
3000
t

Incomplete markets with risk aversion

Quasilinear preferences:

1. With quasilinear preferences, we showed that $b_t \to b^*$ when the aggregate state follows a 2-state i.i.d. process
2. The level and sign of b^* is a function of the **exogenous payoff structure** $p(s)$

Risk aversion:

- Marginal utility adjusted assets $x_t = u_{c,t} b_t$ encode history dependence
- Long-run properties of x_t depend on returns $R_{t,t+1} = \frac{p(s_{t+1})}{q_t}$

Roadmap, II

- Show that $x_t = U_{c,t} b_t$ is sole state variable
- Split the Ramsey problem in two
  1. $t = 0$ Bellman equation in $W(b_{-1}, s_0)$
  2. $t \geq 1$ Bellman equation in $V(x, s_{\_})$
- Analyze steady states x^* such that $x_t \rightarrow x^*$ (Proved only under the special case of a risk-free bond $p(s) = 1 \forall s$ – this focus on *endogenous* component of returns

Interpretation: x^* corresponds to an initial condition when the optimal portfolio in a LS economy is a risk-free bond

A Recursive Formulation

1. Commitment implies that government actions at $t \geq 1$ are constrained by anticipations about them at $s < t$
2. This contributes additional state variables like marginal utility of consumption
3. Scaling the budget constraint by marginal utility makes it recursive in $x = U_c b$

$$\frac{x_{t-1} p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} = U_{c,t} c_t + U_{l,t} l_t + x_t$$

Bellman equation for $t \geq 1$

$$V(x, s_-) = \max_{c(s), l(s), x'(s)} \sum_s \pi(s, s_-) \Big(U(c(s), l(s)) + \beta V(x'(s), s) \Big)$$

subject to $x'(s) \in [\underline{x}, \overline{x}]$

$$\frac{x p(s) U_c(s)}{\beta \mathbb{E} p U c} = U_c(s) c(s) + U_l(s) l(s) + x'(s)$$

$$c(s) + g(s) = \theta(s) l(s)$$

Time 0 Bellman equation

Given an initial debt b_{-1}, state s_0, and continuation value function $V(x, s_{\_})$

$$W(b_{-1}, s_0) = \max_{c_0, l_0, x_0} U(c, l) + \beta V(x_0, s0)$$

subject to time zero implementability constraint

$$U_c(c_0, l_0)c + U_l(c_0, l_0)l_0 + x_0 = U_c(c_0, l_0)b_{-1}$$

and resource constraint

$$c_0 + g(s_0) = \theta(s_0)l_0$$

and

$$x_0 \in [\underline{x}, \overline{x}]$$

Revisiting steady states with risk aversion

Let $x'(s; x, s_-)$ be an optimal law of motion for the state variable for the $t \geq 1$ recursive problem.

Definition

A steady state x^* satisfies $x^* = x'(s; x^*, s_-)$ for all s, s_-

Thus, a steady state is a node at which continuation allocation and tax rate have no further history dependence.

Existence

Let x_{fb} be a value of the state x from which the government can implement first best from that period onwards. Assume a CRRA utility specification $U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$. Finally, let c^{fb} and l^{fb} be consumption and leisure under first best.

Proposition

For a 2 state i.i.d. process, let s_g and s_b denoting the states with high and low consumption at first best. Suppose that

$$\frac{g(s_g)}{1 - \frac{\beta \mathbb{E}[(c^{fb})^{-\sigma}]}{c^{fb}(s_g)}} > \frac{g(s_b)}{1 - \frac{\beta \mathbb{E}[(c^{fb})^{-\sigma}]}{c^{fb}(s_b)}}$$

Then there exists a multiplier μ and complete markets allocations c^{μ}, l^{μ} such that

$$\underline{x} < \frac{U_{c_\mu}(s) c_\mu(s) + U_{l_\mu}(s) l_\mu(s)}{\frac{U_{c_\mu}(s)}{\beta \mathbb{E}[U_{c_\mu}]} - 1} = x^* < 0$$

Stability

1. In this setup, interest rates are aligned with marginal utility of consumption; they are low in "good" states (high TFP or low expenditure)
2. The government holds claims against the private sector in the steady state. This is similar to the quasilinear case when p_1 was low.
3. The incomplete markets Ramsey allocation converges to a Lucas-Stokey Ramsey allocation for all initial conditions

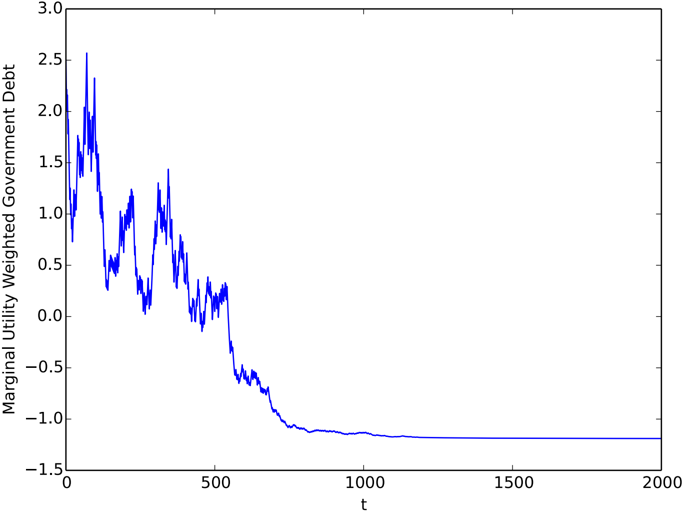
Proposition

Let $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ *solve the incomplete markets Ramsey problem. Then* $x_t(s^{t-1}) \to x^*$ *as* $t \to \infty$ *with probability 1 for all initial conditions*

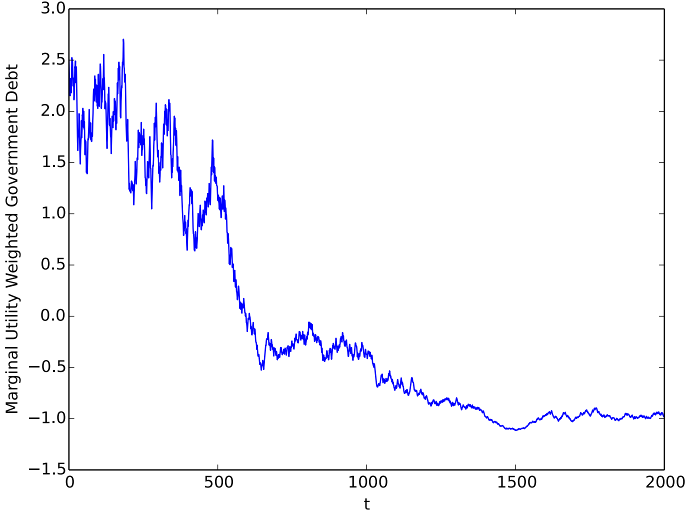
Quasilinear vs. risk aversion

- With quasilinear preferences and a risk-free bond, the interest rate is always $1/\beta$.
- With risk averse preferences, interest rate is higher in periods of high government expenditure.
- Thus, while the government has greater expenses in high government expenditure states, the higher interest rate means that the government can accumulate less and still cover future government expenditures.
- After the government has accumulated enough assets, it is actually better off in periods of high government expenditure than in periods with low government expenditures (since its claims to consumption are worth more).
- By holding assets, the government is able to reallocate resources across states, something it is not able to do in the quasilinear case.

2 State i.i.d. process with Risk Aversion



$S > 2$ states

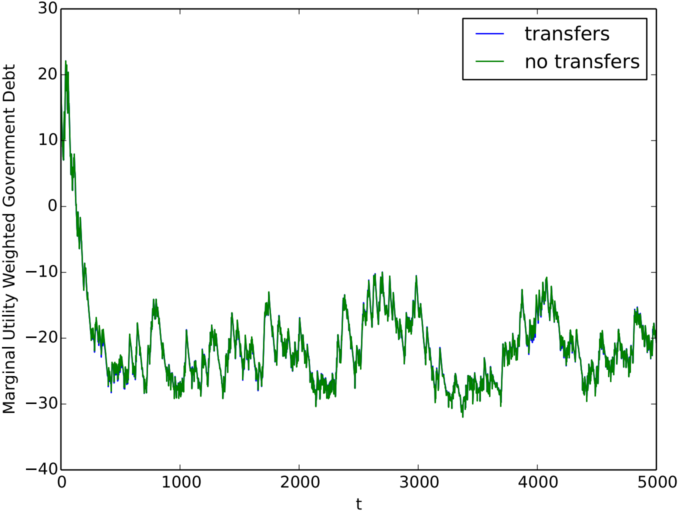


Marginal Utility Weighted Government Debt
t
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
500
1000
1500
2000

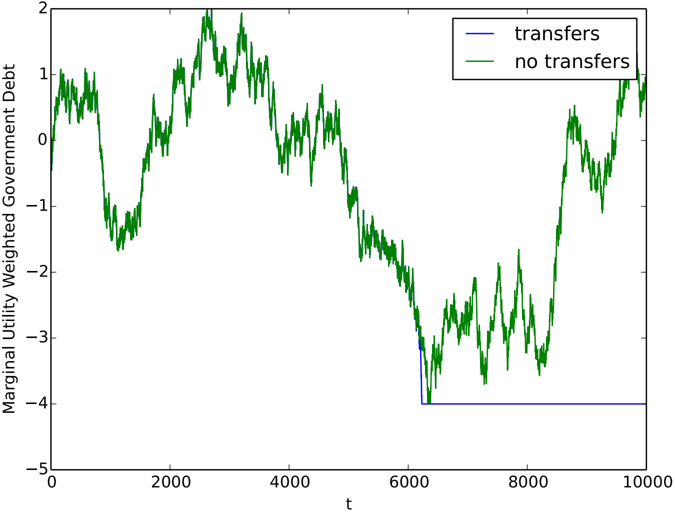
Transfers

- That the government can use its assets to smooth tax rate distortions carries over to when the government has access to lump sum transfers
- Access to nonnegative transfers makes first-best level of assets trivially a “steady state”.
- With lump sum transfers, in cases where the steady state exists and is stable, if the initial debt of the government exceeds its steady state, the economy converges with probability 1 to the steady state.

AMSS calibration with and without transfers



Quasilinear preferences and risk-free bond with and without transfers



Concluding remarks

- With market incompleteness, the asset payoff structure has big implications a Ramsey government's long run debt
- If the asset offers lower returns in adverse states of the world, the Ramsey government asymptotically runs up a debt to the private sector.
- With risk aversion, cyclical properties of interest rate affects government debt asymptotically
- Access to nonnegative transfers play little role in shaping outcomes. Rather, the key force is the government's ability to use its debt position to reallocate resources across states
- **Future Research**: With heterogeneous agents and unrestricted transfers, how does the type of market incompleteness affect long run wealth distributions and other outcomes?