Optimal fiscal policy with incomplete asset markets

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Optimal taxation under commitment and a representative agent

- **Incomplete markets**
  $\rightarrow$ A single, possibly risky asset
- **Linear tax schedules**
  $\rightarrow$ Proportional tax on labor earnings (maybe plus *nonnegative* transfers)
- **Aggregate shocks**
  $\rightarrow$ To productivities, government expenditures

Questions

1. Should a government accumulate or decumulate assets?
2. Why might different governments want to issue different amounts of debt?
3. Existing answers hinge on polar assumptions:
   + Complete markets, Lucas Stokey (1984): non history dependent debt quantities inherited from initial condition
   + A risk-free bond only, quasi-linear preferences, AMSS (2002): govt. accumulates *assets* sufficient to finance activities using interest revenues
4. Unknown: what if interest rates fluctuate?

Environment

- **Uncertainty**: Markov aggregate shocks $s_t \in \mathcal{S}$; $S \times S$ stochastic matrix Π
- **Demography**: Infinitely lived representative agent plus a benevolent planner
- **Preferences** (representative household)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c(s^t), l(s^t)\right)$$

- **Technology**: Aggregate output $y_t = \theta_t l_t$

Environment, II

- **Asset market**:
  - $S \times S$ matrix $\mathbb{P}$ with time t payoff being

$$p_t = \mathbb{P}(s_t | s_{t-1})$$

- **Linear Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_t l_t, \;\; T_t \geq 0$$

- **Budget constraints** q_t is price of asset
  - Household: $c_t + q_t b_t = (1 - \tau_t)\,\theta_t l_t + p_t b_{t-1} + T_t$
  - Government: $g_t + q_t B_t + T_t = \tau_t \theta_t l_t + p_t B_{t-1}$

- **Market Clearing**
  - Goods: $c_t + g_t = \theta_t l_t$
  - Assets: $b_t + B_t = 0$

- **Initial conditions**: Assets b_{-1}, B_{-1} and s_{-1}

Ramsey Problem

Definition

Allocation, price system, government policy

Definition

Competitive equilibrium: Given (b_{-1}, B_{-1}, s_{-1}) and $\{\tau_t, T_t\}_{t=0}^{\infty}$, all allocations are individually rational, markets clear [1]

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given (b_{-1}, B_{-1}, s_{-1})

[1] Usually, we impose only "natural" debt limits.

Ramsey problem

1. **Primal approach**: To eliminate tax rates and prices, use household's first order conditions:

$$U_{c,t} q_t = \beta \mathbb{E}_t U_{c,t+1}$$

$$(1 - \tau_t) \theta_t U_{c,t} = -U_{l,t}$$

2. **Implementability constraints**: Derive by iterating the household's budget equation forward at every history $\Rightarrow$ for $t \geq 1$, these impose *measurability restrictions* on Ramsey allocations
3. The $t \geq 1$ **measurability constraints** contribute the only difference from Lucas-Stokey's Ramsey problem.

Ramsey problem

4. **Transfers:** We temporarily restrict transfers $T_t = 0 \; \forall t$. This is convenient for our analytical results. We eventually show that this assumption is not restrictive.

Ramsey problem (Lucas-Stokey)

$$\max_{\{c_t, l_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Implementability constraint**

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

Ramsey problem (sequential formulation)

$$\max_{\{c_t, l_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

(a) **Feasibility**

$$c_t + g_t = \theta_t l_t$$

(b) **Lucas-Stokey implementability constraint**

$$b_{-1} = \frac{1}{U_{c,0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U_{c,t} c_t + U_{l,t} l_t \right)$$

(c) **Measurability constraints**

$$\frac{b_{t-1} U_{c,t-1}}{\beta} = \frac{\mathbb{E}_{t-1} p_t U_{c,t}}{p_t U_{c,t}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right) \text{ for } t \geq 1$$

Roadmap, analytic strategy

- Properties of a Ramsey allocation – especially asymptotic ones – vary with **asset returns** that reflect
  - Prices $\{q_t(s^t|B_{-1}, s_{-1})\}_t$
  - Payoffs $\mathbb{P}$
- To focus on the exogenous $\mathbb{P}$ part of return, we first study quasi-linear preferences that pin down $q_t = \beta \mathbb{E}_t \mathbb{P}(s_{t+1}|s_t)$
- Turn on risk aversion and fluctuating q_t later

Analysis with quasi-linear preferences

Quasilinear preferences $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$
To characterize **long-run** debt and taxes, we construct and then invert mapping $\mathbb{P}^*(b)$

- Given **arbitrary** initial govt. assets b_{-1}, what is an **optimal** asset payoff matrix $\mathbb{P}^* = \mathbb{P}^*(b_{-1})$?
- Under a Ramsey plan for an **arbitrary** payoff matrix $\mathbb{P}$, when would $b_t \to b^*$, where

$$\mathbb{P} = \mathbb{P}^*(b^*) \text{ or } b^* = \mathbb{P}^{*-1}(\mathbb{P})?$$

Roadmap, the answers

- We first reverse engineer an optimal $\mathbb{P}^*(b_{-1})$ from a Lucas-Stokey Ramsey allocation
- In a binary IID world, we identify a big set of $\mathbb{P}$'s that imply that b_t under a Ramsey plan converges to b^* that solves

$$\mathbb{P} = \mathbb{P}^*(b^*)$$

- For more general shock structures, we numerically verify an ergodic set of b_t's hovering around $\tilde{b}$. The optimal $\mathbb{P}^*$ associated with $\tilde{b}$ seems close to $\mathbb{P}$:

$$\mathbb{P} \approx \mathbb{P}^*(\tilde{b})$$

Optimal asset payoff matrix $\mathbb{P}^*$

1. Given b_{-1}, compute a Lucas-Stokey Ramsey allocation
2. Reverse engineer payoff on single asset

$$p_t = \frac{\beta}{U_{c,t} b_{t-1}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(U_{c,t+j} c_{t+j} + U_{l,t+j} l_{t+j} \right)$$

3. By construction, the optimal payoff p_t disarms the $t \geq 1$ measurability constraints
4. Since a Lucas-Stokey Ramsey allocation is history independent,

$$p_t = \mathbb{P}^*(s_t | s_{t-1})$$

Quasilinear preferences $U(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$

Given initial assets b_{-1}, let $\mu(b_{-1})$ be the Lagrange multiplier on the Lucas-Stokey implementability constraint

1. **Multiplier → Tax rate:**

$$\tau(\mu) = \frac{\gamma\mu}{(1+\gamma)\mu - 1}$$

2. **Tax rate → net of interest surplus:**

$$S(s, \tau) = \theta(s)^{\frac{\gamma}{1+\gamma}}(1+\tau)^{\frac{1}{\gamma}}\tau - g(s)$$

3. **Surplus → optimal payoff structure:**

$$\mathbb{P}^*(s|s_-) = (1-\beta)\frac{S(s, \tau)}{\mathbb{E}_{s_-} S(s, \tau)} + \beta$$

Initial holdings influence optimal asset payoff structure

Denote state s as "adverse" if it has "high" govt. expenditures or "low " TFP; formally, s is "adverse" if

$$g(s)\mathbb{E}_{s\_}\theta^{\frac{\gamma}{1+\gamma}} - \theta(s)^{\frac{\gamma}{1+\gamma}}\mathbb{E}_{s\_}g > 0$$

Properties of optimal payoff matrix $\mathbb{P}$

- With positive initial govt. assets: want an asset that pays *more* in "adverse" states
- With negative initial govt. assets: want an asset that pays *less* in "adverse" states

Optimal Payoff Structure: TFP shocks

David XXXXX: please relabel the y-axis simply "payoff"

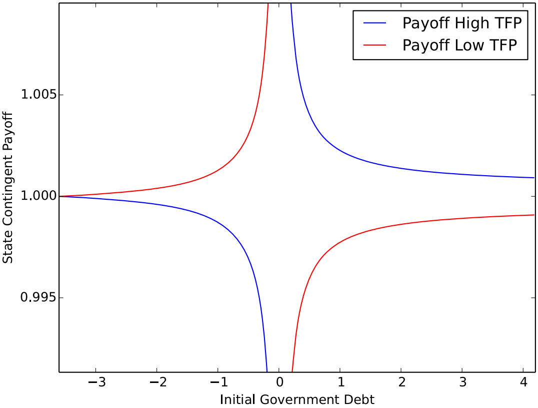


Figure: Optimal asset payoff structure as a function of initial government debt when TFP follows a 2 shock i.i.d process

Optimal Payoff Structure: Expenditure shocks

David XXXXX: please relabel the y-axis simply "payoff"

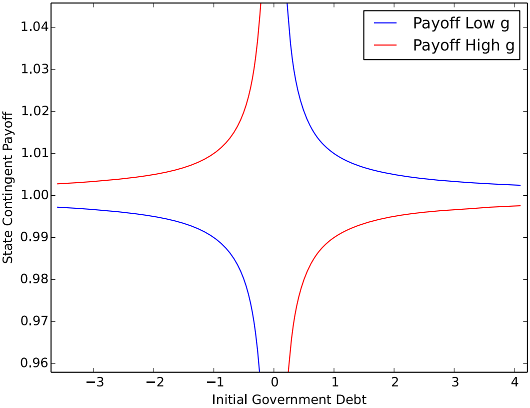


Figure: Optimal asset payoff structure as a function of initial government debt when government expenditures follow 2 shock i.i.d process

Inverting the $\mathbb{P}^*$ mapping

1. **Exogenous payoff structure:** Suppose $\mathbb{P} \neq \mathbb{P}^*(b_{-1})$
2. **Steady States:** A steady state is a government debt level b^* such that

$$b_t = b^* \text{ implies } b_{t+\tau} = b^* \quad \forall \tau > 0$$

3. **Characterization:** Given an asset payoff structure $\mathbb{P}$
   - Does a steady state exist? Is it unique?
   - Value of b^*?
   - For what levels of *initial government debt* b_{-1} does b_t converge to b^*?

Existence and $\mathbb{P}^{*-1}$

When shocks are i.i.d and take two values

1. $\mathbb{P}(s|s_-)$ is independent of s_- (so $\mathbb{P}$ can be a vector)
2. We can normalize $\mathbb{E}\mathbb{P}(s) = 1$ and then pin down payoffs with a scalar p.
   - p is the payoff in the "good" state s
   - A risk-free bond is a security for which $p = 1$
3. A steady state is obtained by inverting the optimal payoff mapping p^*

$$b^* \text{ satisfies } p = p^*(b^*) \text{ or } p^{*-1}(p) = b^*$$

   One equation in one unknown b^*

Existence regions in p space

The payoff p in good state $\in (0, \infty)$.
We can decompose a set of economies with different asset payoffs into 3 regions via thresholds $\alpha_2 \geq \alpha_1 \geq 1$

- Low enough $p(\leq \alpha_1)$: government holds assets in steady state
- High enough $p(\geq \alpha_2)$: government issues debt in steady state
- Intermediate $p(\alpha_1 > p > \alpha_2)$: steady state does not exist

Thresholds: $\alpha_1 < \alpha_2$

- With only government expenditure shocks

$$\alpha_1 = 1 \text{ and } \alpha_2 = (1-\beta)\frac{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g(s_1)}{\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - \mathbb{E}g} + \beta > 1$$

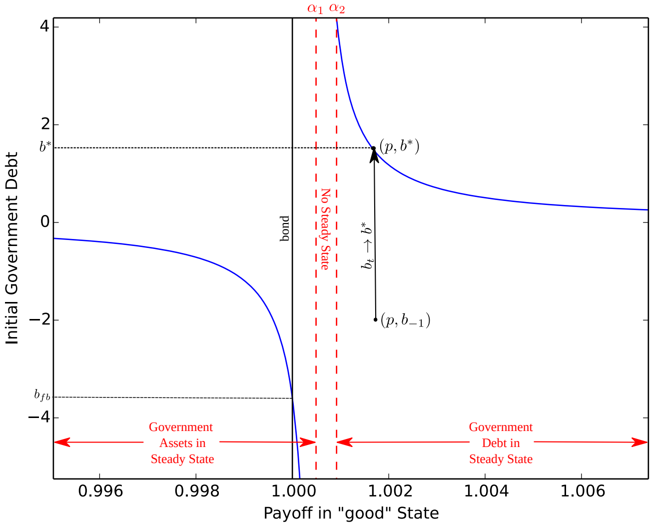
- With only TFP shocks

$$\alpha_1 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}} + \beta > 1$$

and

$$\alpha_2 = (1-\beta)\frac{\theta(s_1)^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g}{\mathbb{E}\theta^{\frac{\gamma}{1+\gamma}}\left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}}\frac{\gamma}{1+\gamma} - g} + \beta > \alpha_1$$

Existence regions in p space



Convergence

- Our analysis verifies existence of a steady state in a 2-state i.i.d. economy.
- To study long-run properties of a Ramsey allocation, we want to know whether steady state is stable
- **Risk-adjusted martingale:**
  The Lagrange multiplier μ_t on the implementability constraint satisfies

$$\mu_t = \mathbb{E}_t p_{t+1} \mu_{t+1}$$

  or

$$\mathbb{E}_t \mu_{t+1} = \mu_t - Cov_t(p_{t+1}, \mu_{t+1})$$

- **Stability criterion:** Away from a steady state, is the drift of μ_t big enough?

Characterizing Convergence under quasi-linear, iid, and $S = 2$

- Reminder: p is the payoff in the "good" state.
- We partition the "p space" into stable and unstable regions

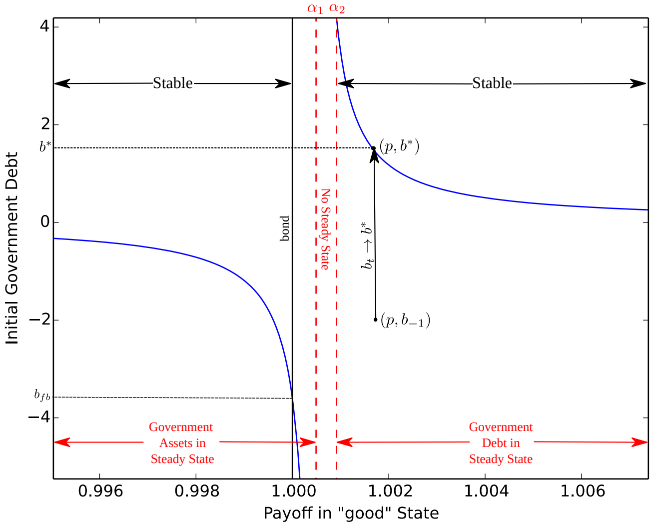
Theorem

Let b^* denote the steady state level of govt. debt and b_{fb} be the level of debt that supports the first-best allocation with complete markets. Then

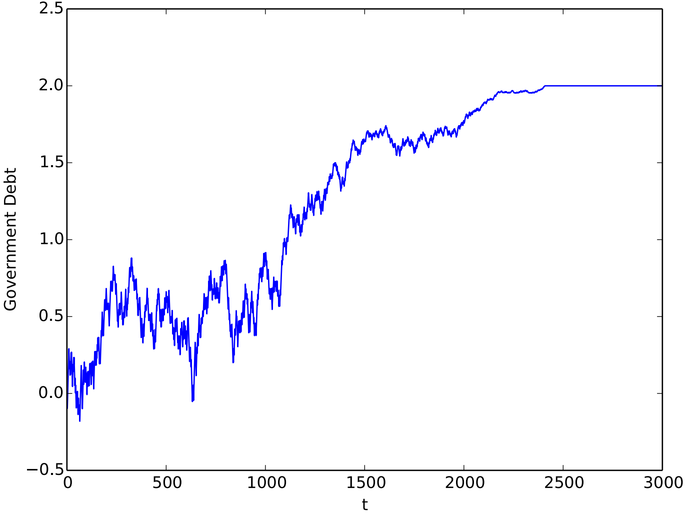
1. **Low** *p: If $p \leq \min(\alpha_1, 1)$ then $b_{fb} < b^* < 0$ and $b_t \to b^*$ with probability 1.*
2. **High** *p: If $p \geq \alpha_2$ then $0 < b^*$ and $b_t \to b^*$ with probability 1.*

For the intermediate region where b^* either does not exist or is unstable, there is a tendency to run up debt David XXXXXX: do you have graphs of simulations that illustrate this? Might we want to put one or two of these in the slides?

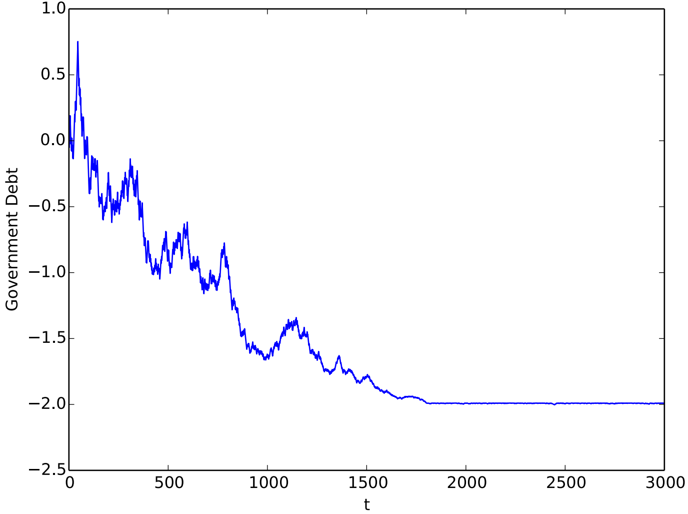
Stability regions



A sample path with $p > 1$



A sample path with $p < 1$



Outcomes with quasi-linear preferences

Outcomes:

1. Often $b_t \to b^*$ when the aggregate state follows a 2-state i.i.d. process
2. The level and sign of b^* depend on the **exogenous payoff structure** $\mathbb{P}$
3. The limiting allocation corresponds to a complete market Ramsey allocation for initial govt. debt b^*

Turning on risk-aversion

Modifications:

- Another source of return fluctuations – the risk-free interest rate
- Marginal utility adjusted debt encodes history dependence
- With binary i.i.d shock shock process, $x_t = u_{c,t} b_t$ converges
- Long-run properties of x_t depend on equilibrium returns $R_{t,t+1} = \frac{\mathbb{P}(s_{t+1}|s_t)}{q_t(s^t)}$. Now q_t varies in interesting ways

Roadmap, II

Two subproblems

1. $t = 0$ Bellman equation in value function $W(b_{-1}, s_0)$
2. $t \geq 1$ Bellman equation in value function $V(x, s_{-})$

Seek steady states x^* such that $x_t \to x^*$

A Recursive Formulation

1. Commitment implies that government actions at $t \geq 1$ are constrained by the public's anticipations about them at $s < t$
2. This contributes additional state variables like marginal utility of consumption
3. Scaling the budget constraint by marginal utility makes Ramsey problem recursive in $x = U_c b$

$$\frac{x_{t-1} p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} = U_{c,t} c_t + U_{l,t} l_t + x_t$$

Bellman equation for $t \geq 1$ (*ex ante*)

$$V(x, s_-) = \max_{c(s), l(s), x'(s)} \sum_s \Pi(s, s_-) \Big(U(c(s), l(s)) + \beta V(x'(s), s) \Big)$$

subject to $x'(s) \in [\underline{x}, \overline{x}]$

$$\frac{x \mathbb{P}(s) U_c(s)}{\beta \mathbb{E} \mathbb{P} U c} = U_c(s) c(s) + U_l(s) l(s) + x'(s)$$

$$c(s) + g(s) = \theta(s) l(s)$$

Time 0 Bellman equation (*ex post*)

Given an initial debt b_{-1}, state s_0, and continuation value function $V(x, s_{\_})$

$$W(b_{-1}, s_0) = \max_{c_0, l_0, x_0} U(c, l) + \beta V(x_0, s0)$$

subject to time zero implementability constraint

$$U_c(c_0, l_0)c + U_l(c_0, l_0)l_0 + x_0 = U_c(c_0, l_0)b_{-1}$$

and resource constraint

$$c_0 + g(s_0) = \theta(s_0)l_0$$

and

$$x_0 \in [\underline{x}, \overline{x}]$$

Progress report

1. Existence proved only under a special case of a risk-free bond
   $\mathbb{P}(s|s_{-}) = 1 \ \forall \ (s, s_{-})$
   This focuses attention on *endogenous* component of returns coming from $q_t(s^t)$
2. x^* is an initial condition for which the optimal asset payoff in a LS economy is a risk-free bond

Revisiting steady states with risk aversion

Let $x'(s; x, s_{-})$ be an optimal law of motion for the state variable for the $t \geq 1$ recursive problem.

Definition

A steady state x^* satisfies $x^* = x'(s; x^*, s_{-})$ for all s, s_{-}

Thus, a steady state is a node at which continuation allocation and tax rate have no further history dependence.

Existence

1. For a class of economies with separable iso-elastic preferences $U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$
2. Shocks that take two values and are i.i.d with s_b being the "adverse" state (either low TFP or high expenditure)

Let x_{fb} be a value of the state x from which a government can implement first=best with complete markets

Proposition

Let $q_{fb}(s)$ be the shadow price of government debt in state s using the first best allocation. If

$$\frac{1 - q_{fb}(s_b)}{1 - q_{fb}(s_g)} > \frac{g(s_b)}{g(s_g)} > 1$$

Then there exists a steady state with $x_{fb} > x^* > 0$

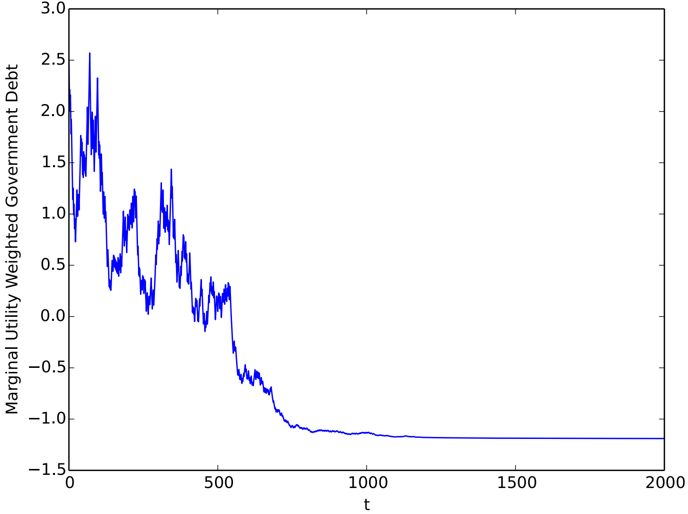
Stability

1. Here interest rates are aligned with marginal utility of consumption; they are low in “good” states (high TFP or low expenditure)
2. The government holds claims against the private sector in the steady state. Similar to the quasilinear case with low p
3. The incomplete markets Ramsey allocation converges to a Lucas-Stokey Ramsey allocation for all initial conditions

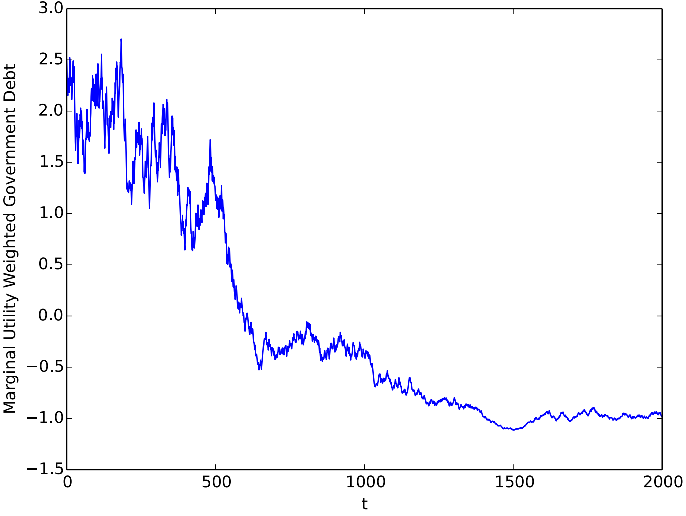
Proposition

Let $\{c_t(s^t), l_t(s^t), x_t(s^{t-1})\}$ *solve the incomplete markets Ramsey problem with* $x_0 > x^*$. *Then* $x_t(s^{t-1}) \to x^*$ *as* $t \to \infty$ *with probability 1 for all initial conditions*

A sample path for 2 state i.i.d. process with risk aversion



A sample path for economy with $S > 2$ states



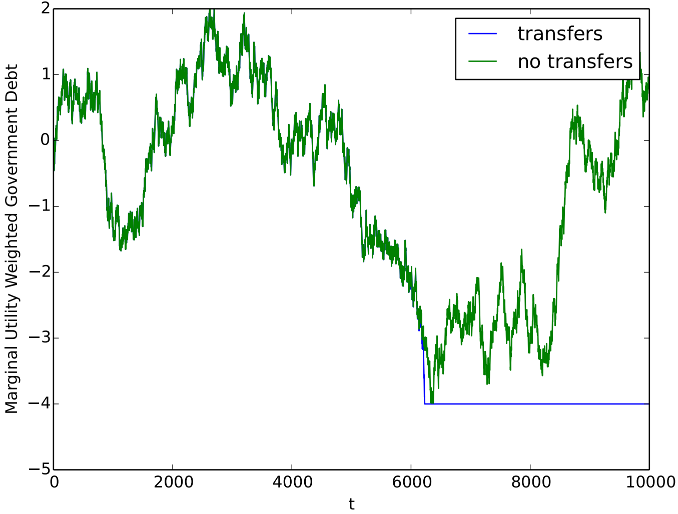
Transfers

- Access to nonnegative transfers makes first-best level of assets trivially a "steady state"
- All results hold *on one side* of steady state

Theorem

With nonnegative lump sum transfers, in cases where a steady state exists and is stable, if the initial debt of the government exceeds its steady state level, the economy converges with probability 1 to the steady state.

Quasilinear preferences and risk-free bond with and without nonnegative transfers



Battle field

	$p = 1$	$p \neq 1$
Quasi-Linear	With transfers: AMSS. Without transfers with TFP shocks, $\exists$ stable SS state at first best. Without transfers with govt. expend shocks, no SS.	$\exists$ α_1 and α_2 such that SS exists if $p_1 < \alpha_1$ or $p_1 > \alpha_2$. SS stable if $p_1 \leq 1$ or $p_1 > \alpha_2$.
Risk Aversion	$\exists$ SS in which the government holds assets. Proof of convergence to SS if initial government debt or smaller than SS assets.	Nothing proved. Conjecture that $\exists$ SS. Conjecture that $\exists$ a cutoff for p_1 determining whether eventually govt holds assets.

Add a slide comparing stuff to Buera Nicolini, Angelotos

Concluding remarks

- With market incompleteness, the asset payoff structure has big implications a Ramsey government's long run debt
- If the asset offers lower returns in adverse states of the world, the Ramsey government asymptotically runs up a debt to the private sector.
- With risk aversion, cyclical properties of interest rate affects government debt asymptotically
- Access to nonnegative transfers play little role in shaping outcomes. Rather, the key force is the government's ability to use its debt position to reallocate resources across states
- **Future Research**: With heterogeneous agents and unrestricted transfers, how does the type of market incompleteness affect long run wealth distributions and other outcomes?