LAB REPORT

8^{th} SEMESTER - JAN 20222 - MAY 2022

NONLINEAR DYNAMICS

Apoorv Badhani

Supervised by
Dr. Gunda Santosh Babu & Dr. Ritwik Das

Abstract

The most common daily phenomenons which we encounter are nonlinear in nature. Like dripping of water from a faucet, wind patterns etc. This complexity, randomness, unpredictability provides the motivation to study the inner lying factors/parameters responsible for this characteristic nature. The extent of this project was to study nonlinear dynamics in certain systems such as Classical Chua, Chua oscillator, Feigenbaum machine, Lorenz system. The idea of non linearity binds all of these. The report comprises of the results which we got when we performed these experiments. First section starts with Chua circuit, discusses it's chaotic nature and ends with results of Chua oscillator. Second section talks about Feigenbaum machine, different parts involved in the circuit, bifurcations plots obtained and calculating the Feigenbaum constant. Lastly we also constructed Lorenz circuit which simulates Lorenz equations and observing the Lorenz map/Lorenz butterfly.

Contents

1 CHUA CIRCUIT

1.1 Introduction

Chaotic systems are dynamical systems whose random states(not entirely true) of disorder and irregularities are actually governed by underlying patterns and deterministic laws that are highly sensitive to initial conditions. Chua's circuits are some of the simplest kinds of chaotic circuits. They are considered to be a classic example of true chaos due to their design and output. Three conditions that the circuit must contain are :

- At least three energy storage elements.
- At least one locally active resistor.
- At least one non-linear element.

1.2 Aim

- Construction of Chua circuit.
- Observing the behaviour of Negative Impedance Converter.
- Observing double scroll attractors at different R values.

1.3 Components

- TL082 dual opamp
- Capacitors - 11.1 nF, 5.75 nF, 21.32 nF,200 nF.
- Resistors - 22 kΩ, 2.2 kΩ, 3.3 kΩ, 220 Ω, 100kΩ.
- Inductor - 2 x 18 mH.
- DC power supply
- 2kΩ potentiometer
- Connecting wires
- Oscilloscope
- Breadboard
- Multi meter
- Banana and BNC to crocodile cables

1.4 Theory

Chua's circuit create the strange attractor known as the double scroll. These beautiful patterns are truly chaotic and can be modeled by relatively simple nonlinear equations. The equations are as follows :

$$C_1\frac{dv_{c_1}}{dt} = G(v_{c_2} - v_{c_1}) - g(v_{c_1}) \tag{1}$$

$$C_2\frac{dv_{c_2}}{dt} = G(v_{c_1} - v_{c_2}) + i_L \tag{2}$$

$$L\frac{di_L}{dt} = -v_{c_2} \tag{3}$$

Non-linearity in the circuit is introduced by adding a non-linear resistor. Non-linear resistors are

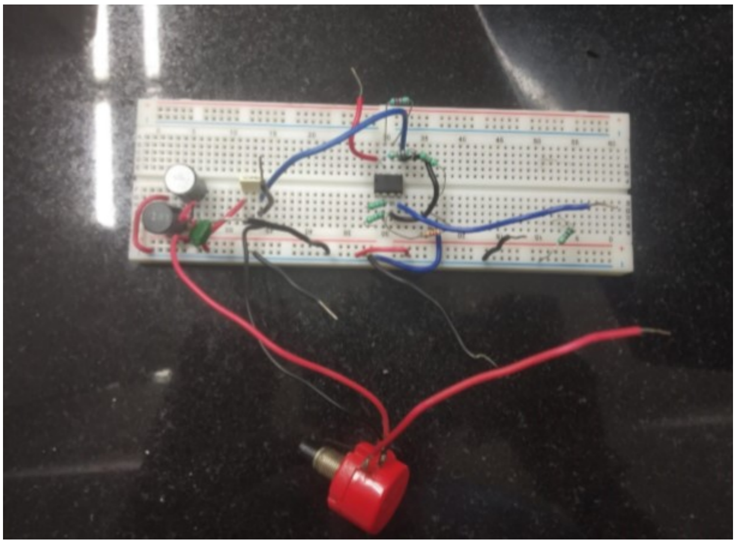

Figure 1: Chua circuit

the resistors whose voltage and current characteristics vary non-linearly. Chua diode is a type of non-linear resistor whose I-V characteristics can be described with piece wise-linear equations. This characteristic graph can be given by the function

$$g(x) = m_1x + \frac{1}{2}(m_0 - m_1)[|x + B_p| - |x - B_p|] \tag{4}$$

1.5 Observation

First the data for negative impedance converter is acquired. Here a voltage is supplied to the TL082, resistor combination. The voltage supplied and voltage across the 100 kΩ resistor is noted and attached in the following table.

The inductor, capacitor, resistor is connected to nonlinear resistor to get the bifurcations. The potentiometer is turned slowly and the corresponding values of the resistance are noted with the observed figures.

	Positive voltage	
S.No.	Input DC voltage(V)	Voltage across 100Ω resistor
1	0.1	-0.007
2	0.3	-0.025
3	0.5	-0.043
4	0.7	-0.059
5	0.9	-0.077
6	1.1	-0.093
7	1.3	-0.105
8	1.5	-0.123
9	1.7	-0.139
10	1.9	-0.147
11	2	-0.152
12	2.1	-0.156
13	2.2	-0.159
14	2.3	-0.163
15	2.5	-0.171
16	2.7	-0.178
17	3.2	-0.198
18	3.7	-0.219
19	4.2	-0.239
20	4.7	-0.26
21	5.2	-0.28
22	5.7	-0.303
23	5.8	-0.307
24	5.91	-0.312
25	6	-0.316

	Voltage reversed	
S.No.	DC Input Voltage(V)	Voltage across 100Ω resistor(V)
1	-0.124	0.007
2	-0.207	0.016
3	-0.412	0.034
4	-0.604	0.052
5	-0.81	0.07
6	-1.003	0.088
7	-1.209	0.106
8	-1.423	0.124
9	-1.608	0.141
10	-1.72	0.143
11	-1.8	0.144
12	-2	0.148
13	-2.2	0.156
14	-2.4	0.164
15	-2.6	0.173
16	-3.3	0.199
17	-3.8	0.22
18	-4.2	0.235
19	-4.82	0.262
20	-5.3	0.284
21	-5.9	0.307
22	-6.3	0.324

1.6 Results

Above table of voltage supplied and voltage measured across 100Ω is plotted using GNUplot and the slope values for two parts of the graph i.e., positive voltage and reversed polarity voltage were calculated. The plot obtained is figure 2.

The bifurcation sequences are obtained for different resistance values which are as follows:

- Period-1 limit cycle occurs at the resistance R = 2.190 kΩ.
- Period-2 limit cycle appears at resistance R = 2.171 kΩ.
- Period-3 was not very accurately determined but it exists between R = 2.162-2.168kΩ.
- Period-4 orbit occurs at the resistance R = 2.161 kΩ.
- Double scroll chua attractor appears at R = 2.048kΩ and ends at R = 2.024kΩ.
- Large limit cycle occurs at resistance R = 1.933kΩ.

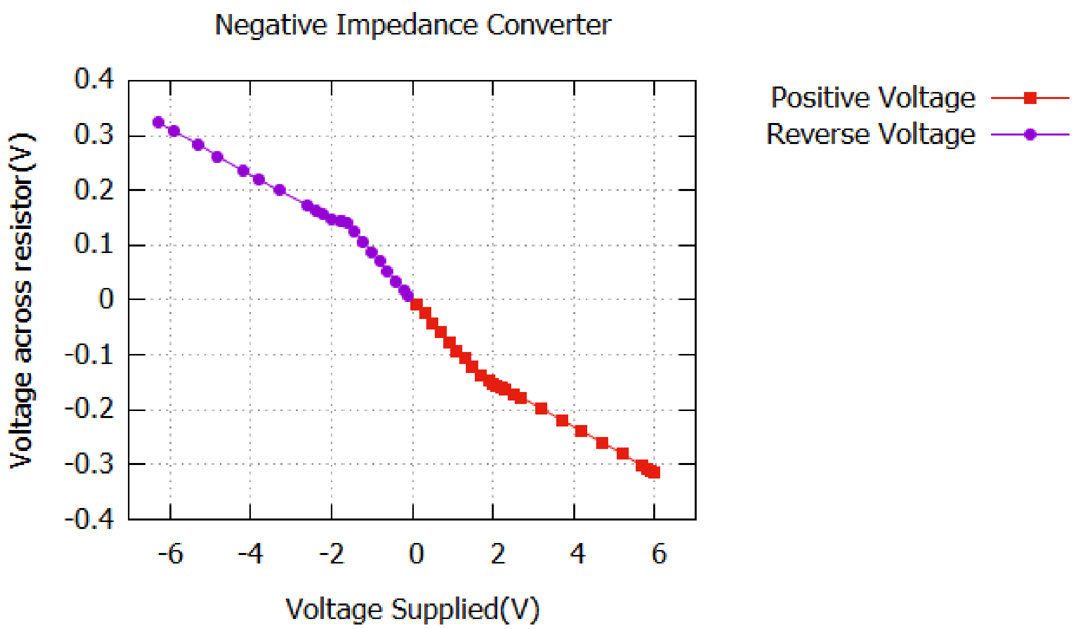

Figure 2: I-V characteristics of chua diode

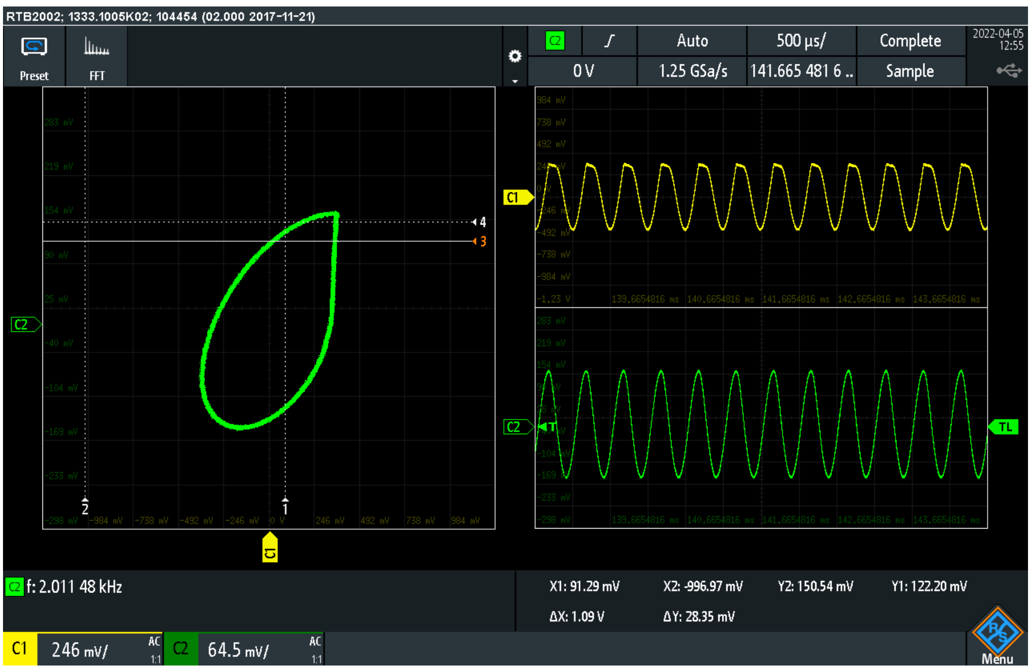

Figure 3: Period 1

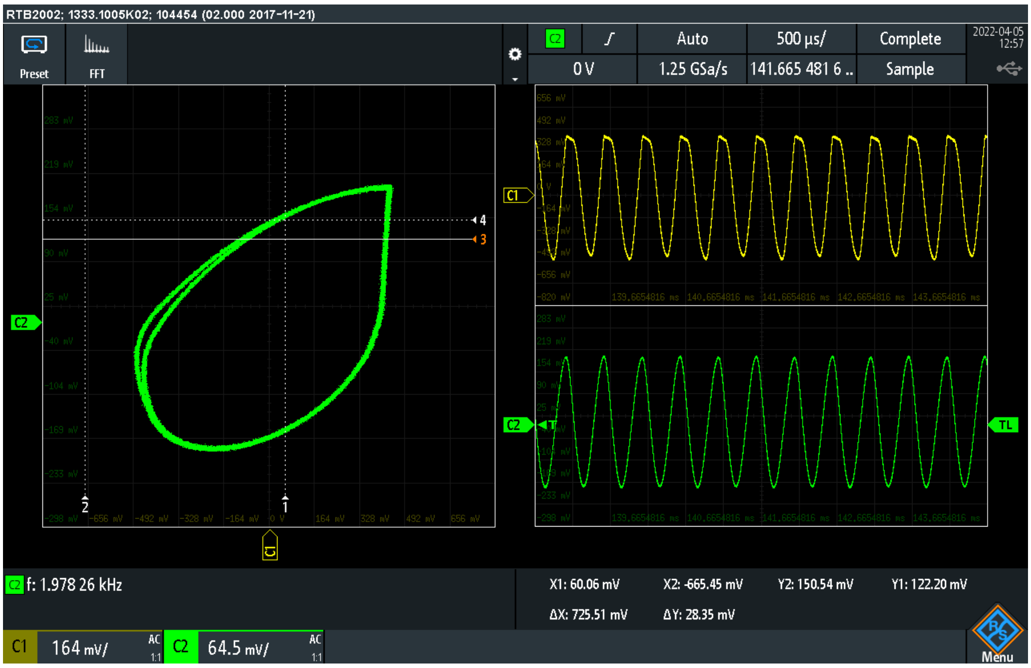

Figure 4: Period 2

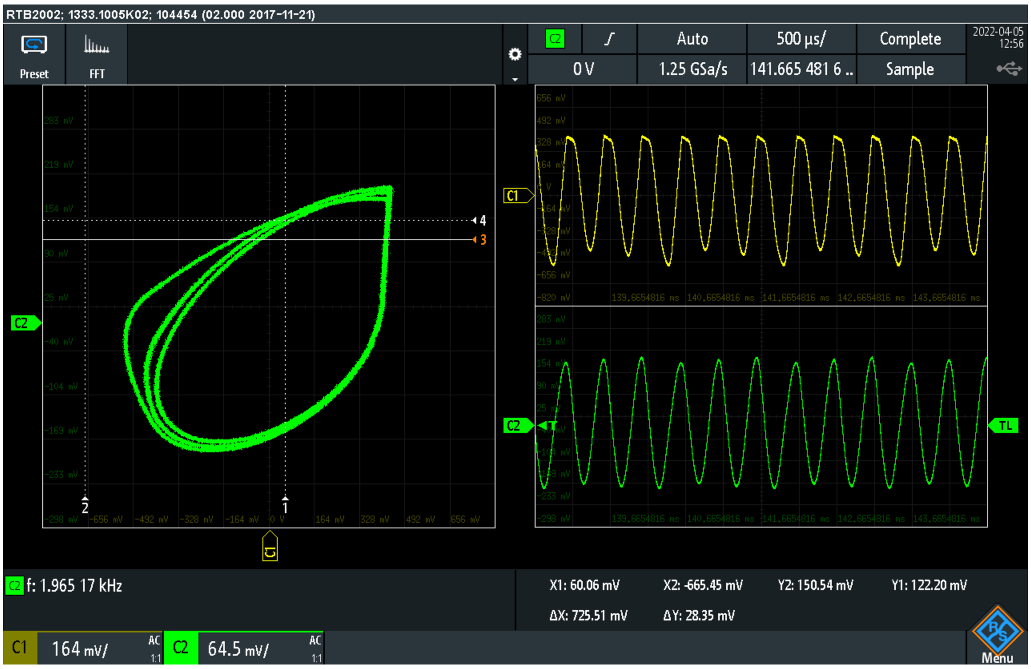

Figure 5: Period 3

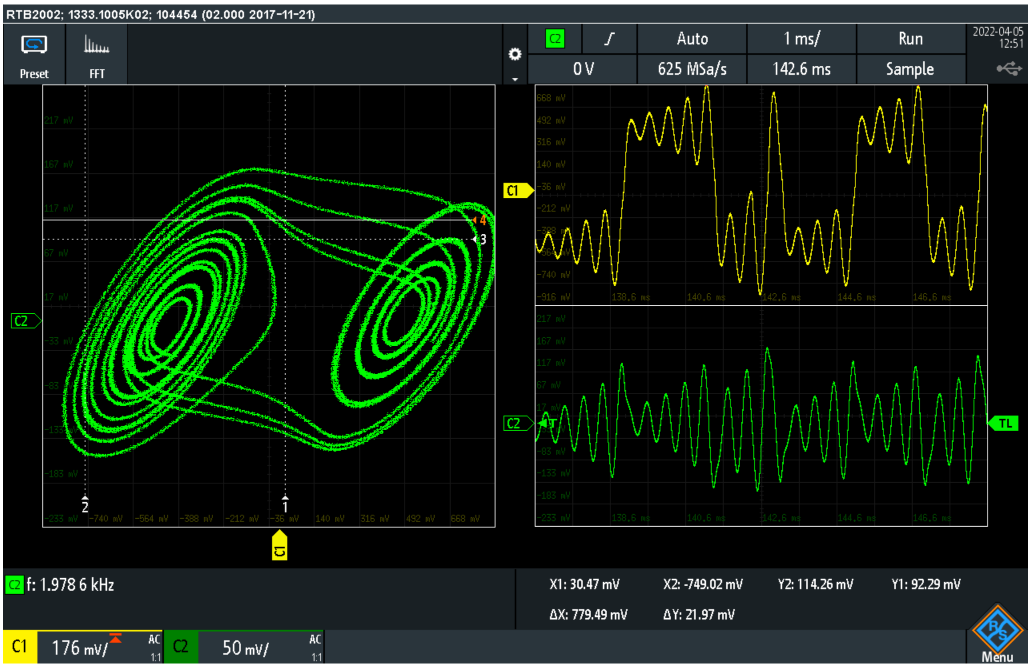

Figure 6: Double Scroll Attractor

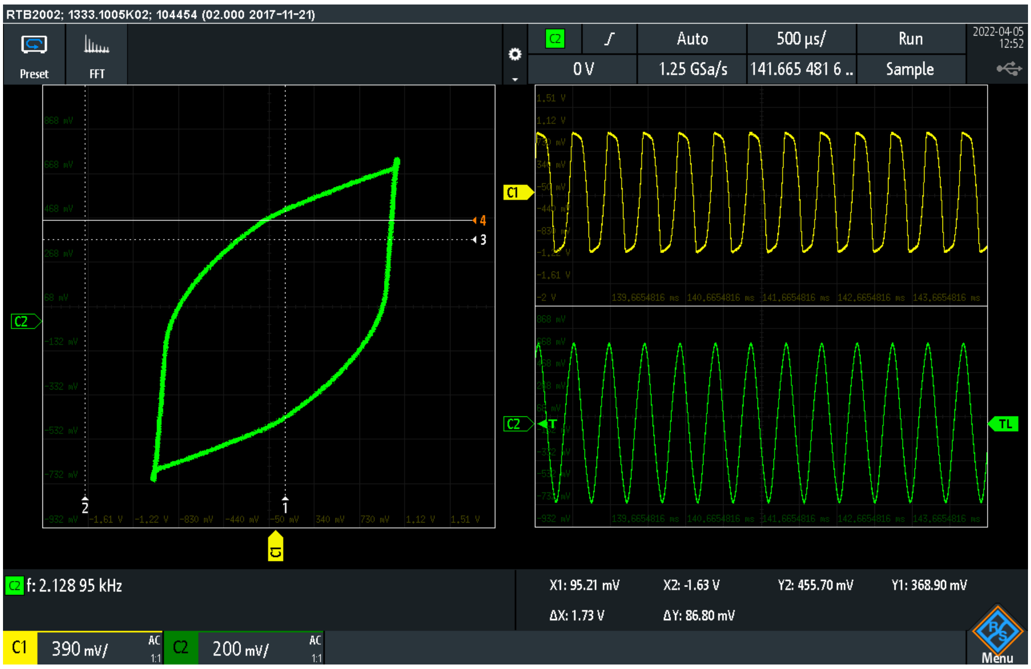

Figure 7: Large Limit cycle

1.7 Sources Of Error

- Loose connections are one of the most common sources of error in electronics.
- Error in analysis may occur due to wrong measured value.
- Wrong circuit is constructed (Human Error).

1.8 Conclusion

- Inductor of 18mH was not enough to get the results from classical chua. In our experiment we have used two inductors of 18mH = 36mH inductors. The possible reason might be that there is a power loss because of the various components in the circuit so chaos could not be sustained.
- The inductor in this experiment should have more inductance and less internal resistance. The 18mH inductor might have more internal resistance which may be why we did not observe double scroll attractor in chua . The combination of inductors work might that it has relatively more inductance and less resistance.
- Saturation value obtained from our analysis are 8.9V for positive voltage and 9V for reverse voltage which are very close to the bias voltage.
- The slopes are calculated from the data are -0.756 mA/V, -0.409 mA/V which is very close to the theoretical value calculation of -0.848 mA/V, -0.403 mA/V.
- All the possible bifurcation sequence for Chua circuit were obtained.

2 Other types of Chua

Different types of double scroll attractors can be obtained from slight variations in the chua circuit. Fundamentally the equations governing double scroll behaviour in all these variations are of the same flavour. The only major change which can be applied is changing the nonlinear resistor behaviour. Two types of these variations were performed. These are :

- Matsumoto Chua
- Chua Oscillator

2.1 Matsumoto Chua

Matsumoto chua is a chaotic attractor with a simple autonomous circuit. It is a simplified version of Chua circuit.

2.1.1 Components

- TL082 dual Opamp
- Oscilloscope
- Capacitors - 11.1 nF, 5.75 nF, 21.32 nF,200 nF.
- Resistors - 22 kΩ, 2.2 kΩ, 3.3 kΩ, 220 Ω and 30.86 Ω.
- Inductors - 12 mH,18 mH.
- DC power supply
- 2kΩ 10 turn potentiometer
- Breadboard
- Connecting wires

2.1.2 Working Equations

We have used the opamp TL082 used in the chua circuit with some different capacitance values and inductor and potentiometer are in series.

$$C_1\frac{dv_1}{dt} = \frac{(v_2 - v_1)}{R} - h(v_1) \tag{5}$$

$$C_2\frac{dv_2}{dt} = \frac{(v_1 - v_2)}{R} + i_L \tag{6}$$

$$L\frac{di_L}{dt} = -v_2 \tag{7}$$

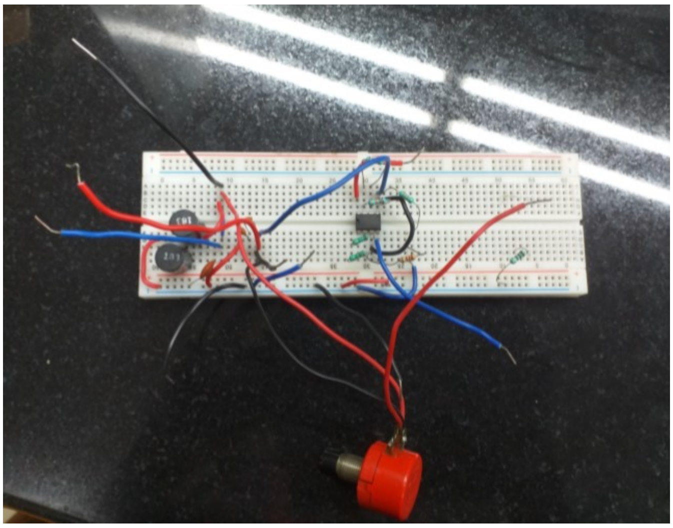

Figure 8: Matsumoto chua circuit

where C_1 and C_2 are the capacitors, v_1 and v_2 the tensions across the capacitors C_1 and C_2, R is the linear resistor, L is the inductor, i_L is the current across the inductor L, and $h(v_1)$ is the characteristic curve in the (nonlinear) diode. The function $h(v_1)$ is given by -

$$h(v_1) = m_1 v_1 + \frac{1}{2}(m_2 - m_1)(|v_1 + B_P| - |v_1 - B_P|) \tag{8}$$

where m1, m2, and BP are constants. We fixed the following values for these parameters: $1/C_1 = 9.0$, $1/C_2 = 1.0$, $1/L = 7.0$, $m_1 = -0.5$, $m_2 = -0.8$ and $B_P = 1.0$.

R is the control parameter of the model. As the parameter R changes, the Matsumoto-Chua circuit exhibits a complex and rich behavior. There are actually 4 types of attractors spiral-like attractors.

2.1.3 Results

We obtained the following plots from the Matsumoto chua circuit. We can see a pair of spiral-like attractors that merge into a "double scroll".

- Spiral chua attractor occurs at R = 1.172 kΩ.
- Double scroll attractor starts at R = 1.171 kΩ and ends at 1.144 kΩ.
- Large limit cycle occurs at R = 1.117 kΩ.

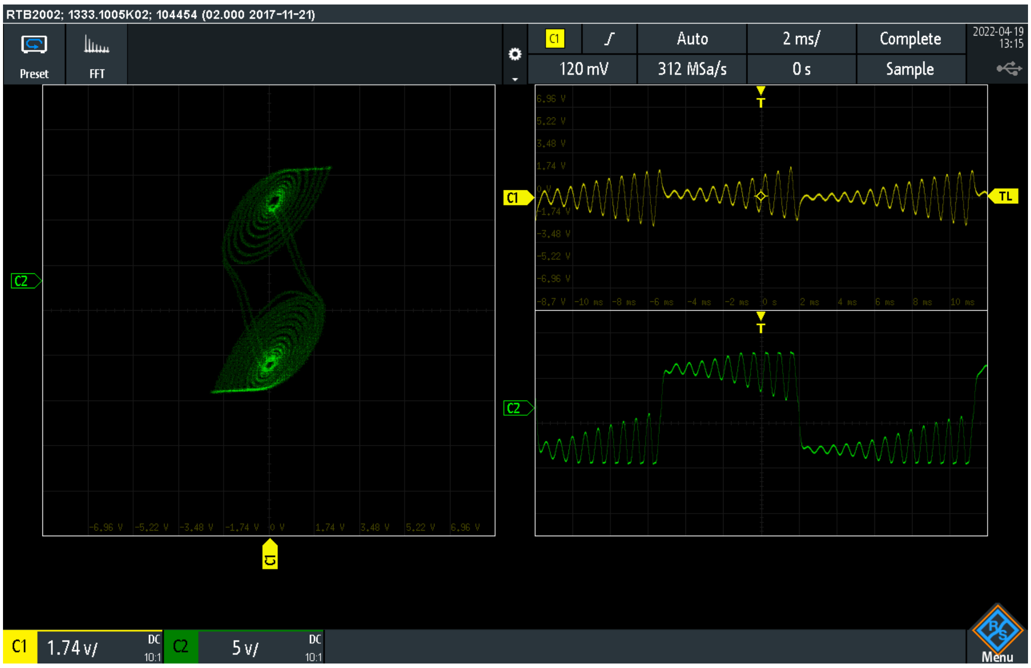

Figure 9: Spiral chua attractor

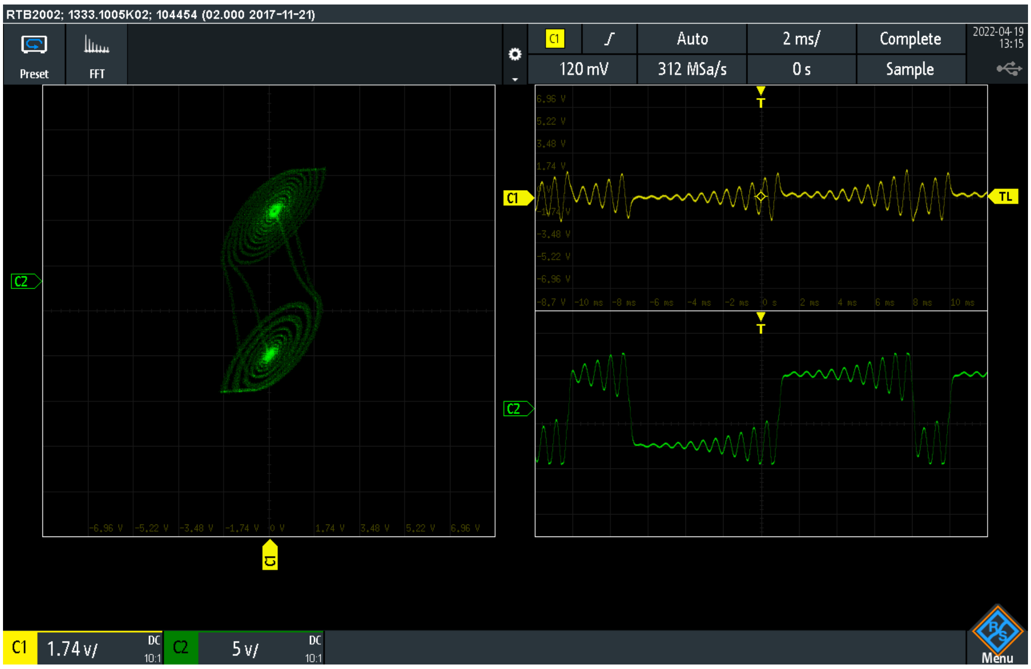

Figure 10: Intermittent state leading to double scroll

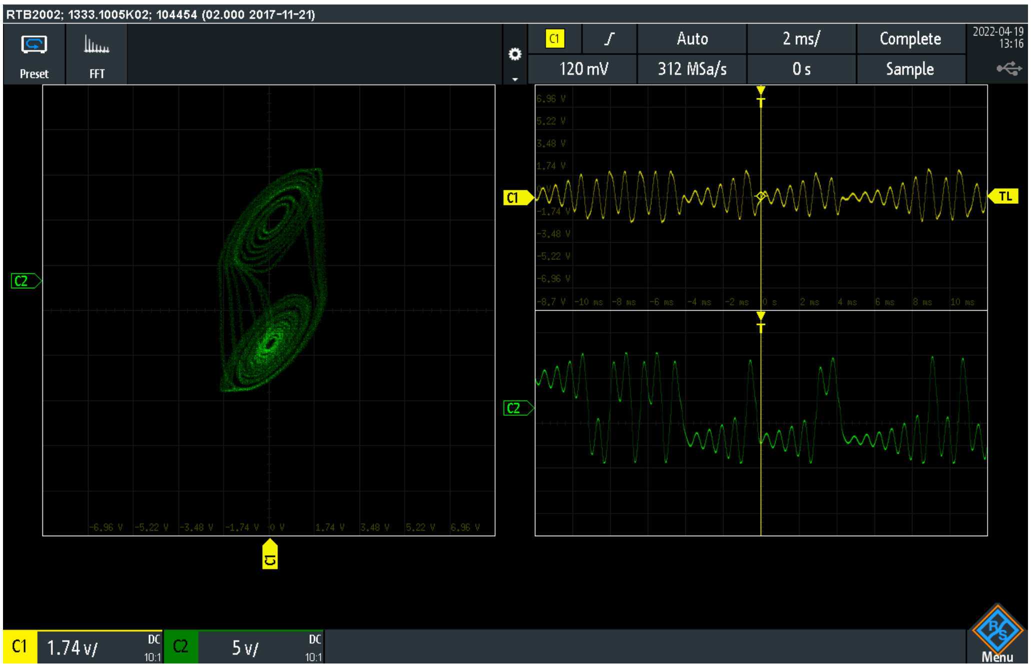

Figure 11: Matsumoto double scroll attractor

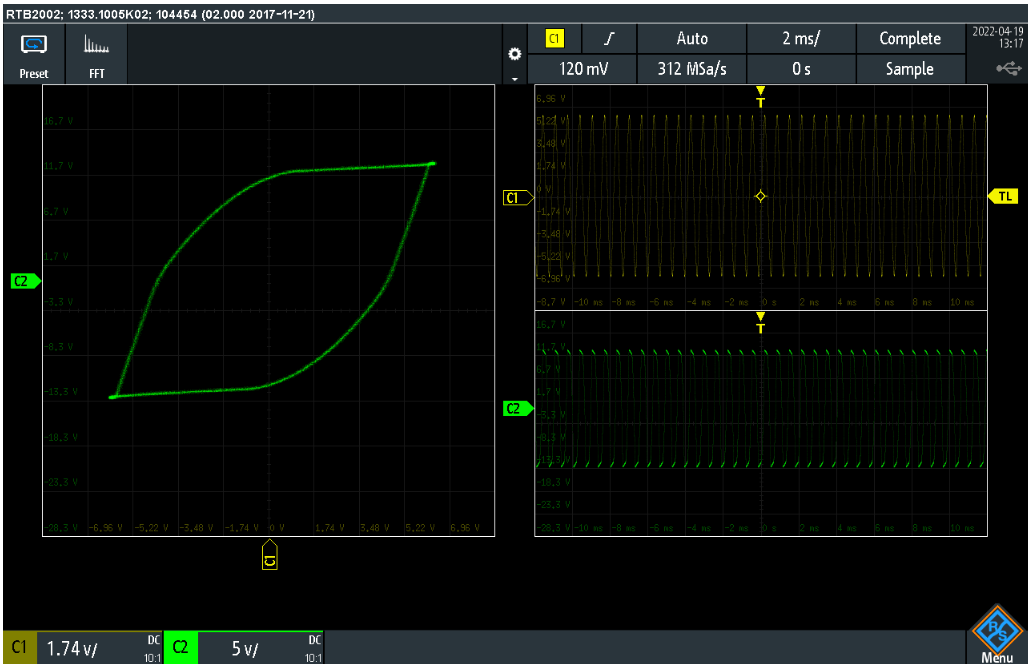

Figure 12: Large limit

2.1.4 Conclusion

- Range of parameter for Matsumoto chua circuit is (1.182 - 1.117) kΩ = 0.065 kΩ = 65Ω.
- From results we can see the instability in the periodic orbits. A minor change in the R parameter causes the system to jump from one configuration to another.
- The nonlinear resistor part is same as that in chua. So the difference in the behaviour is due to the change in components which changed the coupled differential equation.
- Matsumoto chua circuit is often used to study the homoclinic orbit which means that in phase space the voltage goes through the intersection of stable and unstable equilibrium.

2.2 Chua oscillator

Chua's oscillator is a typical Chua's diode with a 36mH inductor connected in series with 30.86 Ω resistor.

2.2.1 Components

- TL082 dual Opamp
- Oscilloscope
- Capacitors - 11.1 nF, 5.75 nF, 21.32 nF,200 nF.
- Resistors - 22 kΩ, 2.2 kΩ, 3.3 kΩ, 220 Ω and 30.86 Ω.
- Inductors - 12 mH,18 mH.
- DC power supply
- 2kΩ 10 turn potentiometer
- Breadboard
- Connecting wires

2.2.2 Working Equations

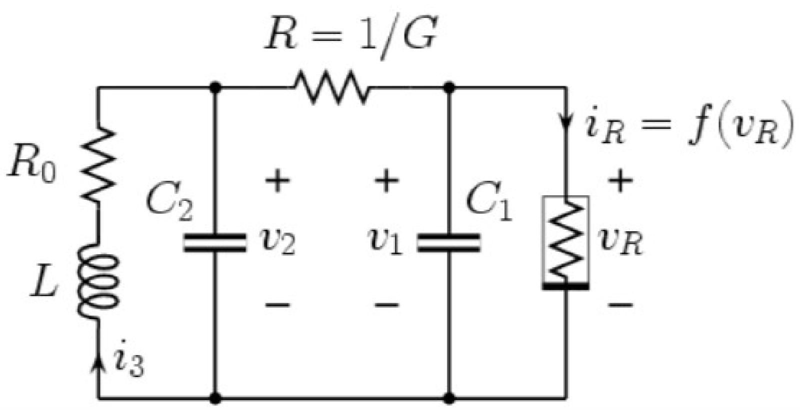

Figure 13: Chua oscillator

The equations for Chua oscillator are given by -

$$C_1 \frac{dv_1}{dt} = [G(v_2 - v_1) - f(v_1)] \tag{9}$$

$$C_2 \frac{dv_2}{dt} = [G(v_1 - v_2) + i_3] \tag{10}$$

$$L \frac{di_3}{dt} = -v_2 + R_0 i_3 \tag{11}$$

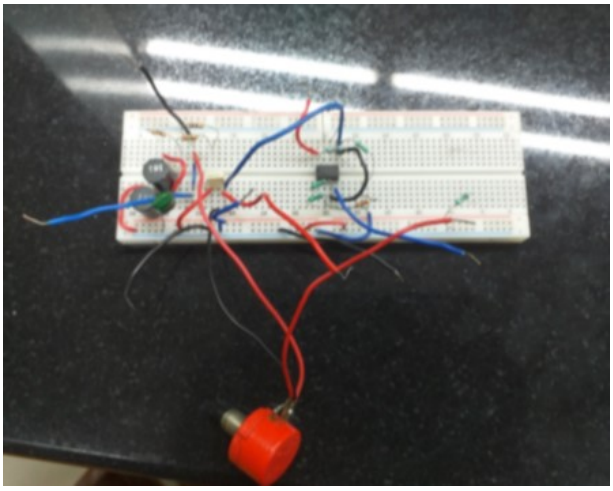

Figure 14: Our chua oscillator circuit

2.2.3 Results

- Period 1 cycle occurs at R = 1.140 kΩ.
- Period 2 cycle is observed at R = 1.132 kΩ.
- Period 4 cycle is observed at R = 1.128 kΩ.
- Double scroll attractor starts at R = 1.122 kΩ and ends at R = 1.1 kΩ.
- Large limit cycle occurs at R = 1.056 kΩ.

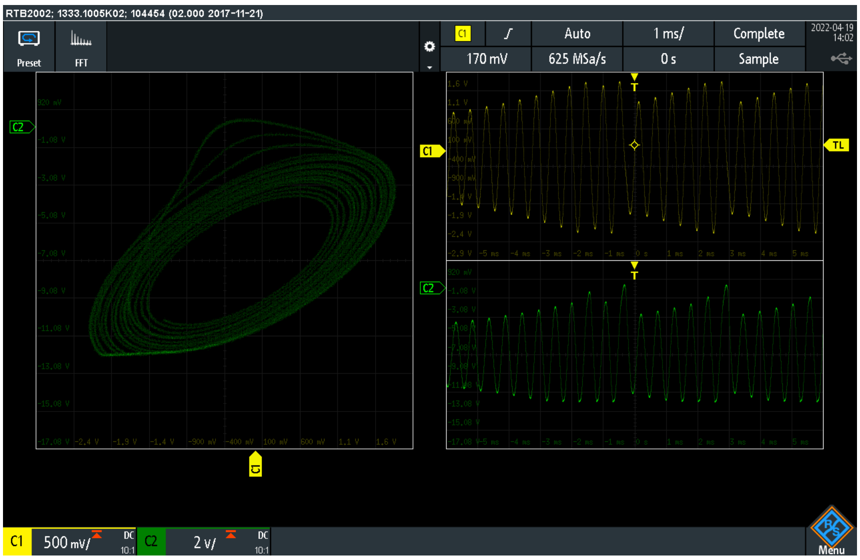

Figure 15: Spiral chua attractor at R = 1.125 kΩ

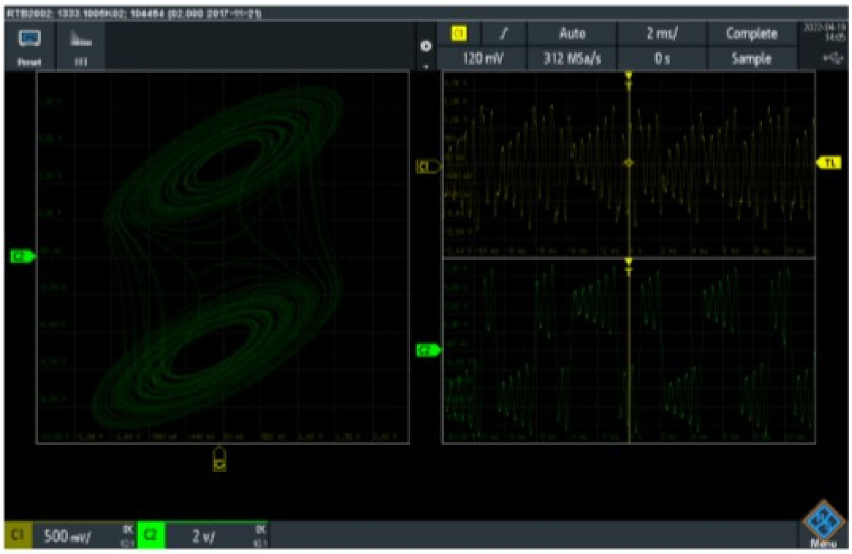

Figure 16: Chua oscillator double scroll

2.2.4 Conclusion

- The values of R_0 which is connected to the inductor is chosen in such a way as to model the loss of the real inductor.
- The R values for which bifurcations are observed are consistent with the theoretical calculations.

3 Conclusion of Chua circuits

- We studied a particular type of chaotic systems - Chua, Matsumoto Chua and Chua oscillator which are all derived from a single system and tweaked a little bit.
- After observing these systems we can draw following remarks:
  - The non linearity of the resistor is needed to generate chaos in the system.
  - (From theory) Global Unfolding theorem says that Chua's oscillator can be used to mimic the behavior of other piecewise-linear oscillators and also approximate the behavior of many others which exhibit smooth non-linearities.
- Chua's circuit is a very good model for the generation of a variety of different dynamical phenomena.

4 FEIGENBAUM MACHINE

Feigenbaum originally related the first constant to the period-doubling bifurcations in the logistic map, but also showed it to hold for all one-dimensional maps with a single quadratic maximum. As a consequence of this generality, every chaotic system that corresponds to this description will bifurcate at the same rate. In this experiment we are building a circuit to simulate logistic equation.

4.1 Aim

- Understand the working of individual parts of the circuit.
- Construct the circuit, calculate the feigenbaum constant and obtain the bifurcation map.

4.2 Components

- Breadboard
- AD633 analogue multiplier
- LF398 sample and hold
- NE555 timer
- AD741 op-amp
- 74LS08 quad AND gate
- 74LS107 dual JK flip-flop
- 10k potentiometer
- Connecting wires
- Oscilloscope
- 100k resistor
- 10k resistors
- Banana and BNC to crocodile cables
- DC power supply
- Multimeter
- 1k resistor
- 33k resistor
- 10nF capacitors
- 100nF capacitors

4.3 Working Principle

The feigenbaum machine circuit consists of 2 mini circuits. First is the clock generator part and the other part is the function generator part.

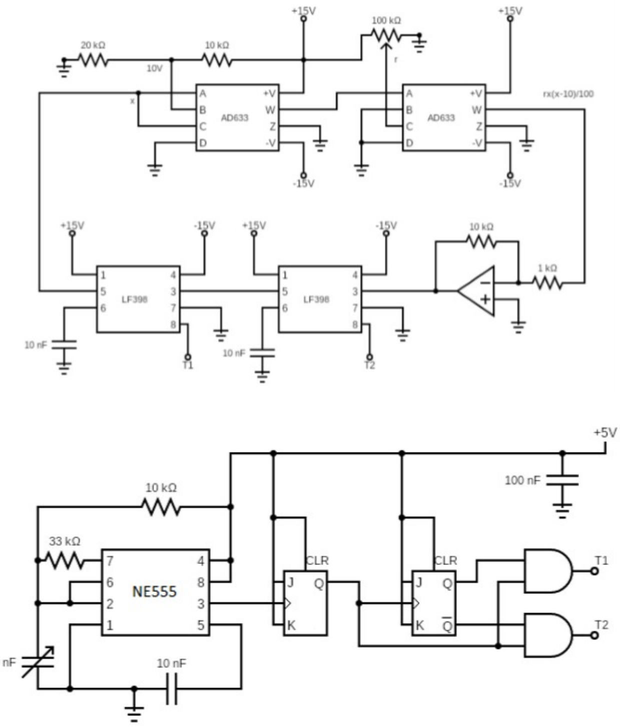

Figure 17: Circuit diagram

4.3.1 Clock Generator

It consists of NE555 timer IC, 2 JK flip flops and NAND gate. A voltage of +5V is supplied to timer IC. The resistor between pin 7, pin 2 and between pin 4, pin7 decides the duty cycle of the pulse generated which is given by -

Duty cycle $= \frac{R_2}{R_1+R_2}$

For our purpose, it is around 50%. Pulse from IC goes to JK flip flop where both J, K are in high state meaning it is in toggle mode. It's output is fed to NAND gate where the logic operation is performed. All this adds a phase to the initial pulse from the timer IC.

This output is then fed to the LF398 sample and hold circuit which only works when the frequency of the pulse matches to that of the S&H circuit. The operation performed by S&H circuit is that there is a switch inside which is responsible for it's sampling and holding operation. The time taken for each operation depends on the capacitor inside LF398.

4.3.2 Cascaded AD633

The idea of the circuit is to represent the quantity x by a voltage (v $=$ x X 10), and form the product *r*x(1-x)* with two analog multipliers. The voltage is varied by a potentiometer. This method allows a measurement of the critical values of r AD633 has 5 inputs, namely A, B, C, D

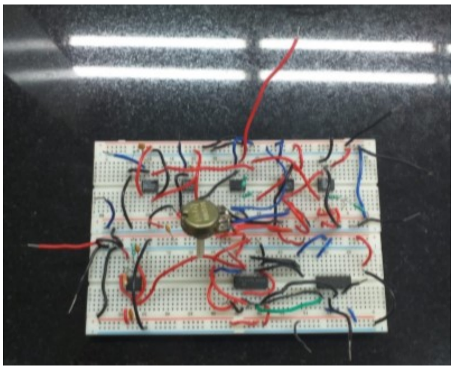

Figure 18: Our feigenbaum circuit

and offset Z. The output is denoted as W =(A B)(C D)/10 + Z. For our convenience, we work with input x between 1 and 10V. Thus -

$$x_{n+1} = \frac{rx_n(10 - x_n}{10} \tag{12}$$

For the 1^{st} multiplier, A = C = x, B = 10V, D = 0. For the 2^{nd} multiplier, A = $\frac{x(x-10)}{10}$, C = r, D = 0. The output from multipliers gives us an output of $\frac{rx(x-10)}{100}$ which is inverted and amplified through an op-amp of amplification -10. The desired output is sent to the two sample and holds.

4.4 Results

The circuit shows it's bifurcative nature around 3V. According to our analysis -

- First bifurcation occurs at 3.013V.
- Second bifurcation occurs at 3.496V.
- Third bifurcation occurs at 3.596V.

All the results starting from the output of clock generator to bifurcation plots are attached below:

We have obtained the voltage values at which these bifurcations occur. From this we can calculate of Feigenbaum constant. Here B_1, B_2, B_3 means first, second and third bifurcations.

$$\delta = \frac{V_{B_2} - V_{B_1}}{V_{B_3} - V_{B_2}} = \frac{3.496 - 3.013}{3.596 - 3.496} = 4.83 \tag{13}$$

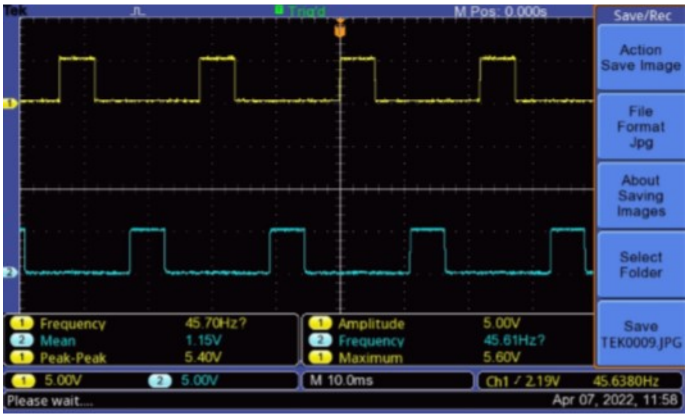

Figure 19: Clock generator output

Figure 20: Period 1

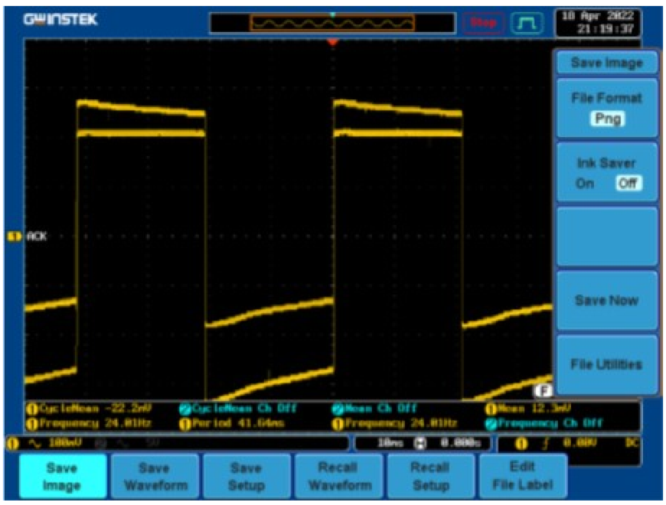

Figure 21: Period 2

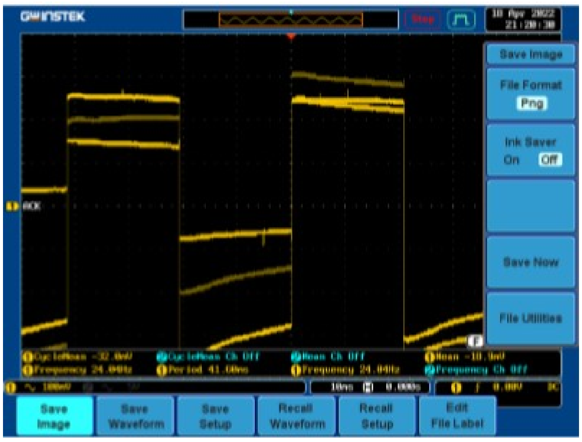

Figure 22: Period 4

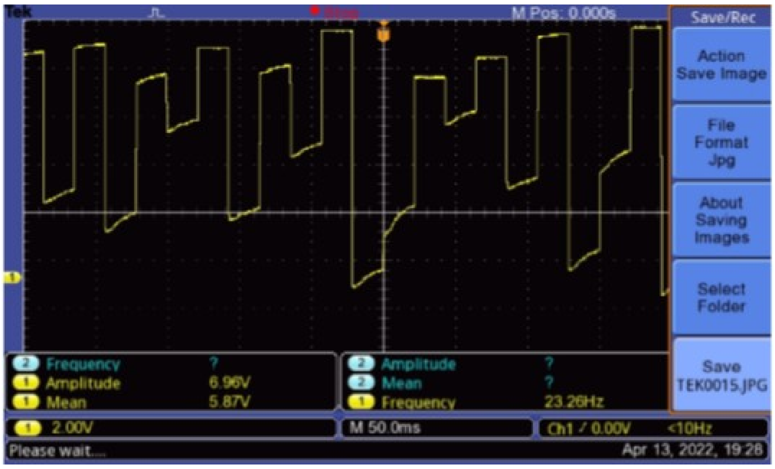

Figure 23: Period 8

Figure 24: Bifurcation map

4.5 Sources of Error

- Make the same ground for both clock generator and function circuit.
- Multipliers used may have non-zero offset.
- Loose connections might hinder the desired result.

4.6 Conclusion

- The r values for feigenbaum machine is noted and the calculated feigenbaum constant from our analysis is 4.83 which is a little off from it's theoretical value of 4.699. It is may be due to the errors in the measurement of the r value.
- The bifurcation map is obtained by operating the oscilloscope in roll-mode.

5 Lorenz Circuit

Lorenz system is a system of ordinary differential equations with chaotic solutions for certain parameter values and initial conditions. Lorenz attractor is a set of chaotic solutions of the Lorenz system.

5.1 Aim

- To observe the Lorenz butterfly, map

5.2 Components

- Breadboard
- TL082 IC
- 10k Ω, 100k Ω, 3.3k Ω, 5.6k Ω, 3.6k Ω, 3.74k Ω, 37.5k Ω
- AD633 multiplier
- Connecting wires
- Oscilloscope
- DC power supply
- Potentiometer

5.3 Working Principle

The Lorenz model is a system of differential equations given by -

$$\frac{dx}{dt} = \sigma(y - x) \tag{14}$$

$$\frac{dy}{dt} = x(\rho - z) - y \tag{15}$$

$$\frac{dz}{dt} = xy - \beta z \tag{16}$$

For $\sigma = 10$, $\beta = 8/3$, $\rho = 28$. The solution executes a trajectory, plotted in three dimensions, that winds around and around, neither predictable nor random, occupying a region known as its attractor.

5.4 Results

After constructing the circuit we get the following plots. For X-Y plot -

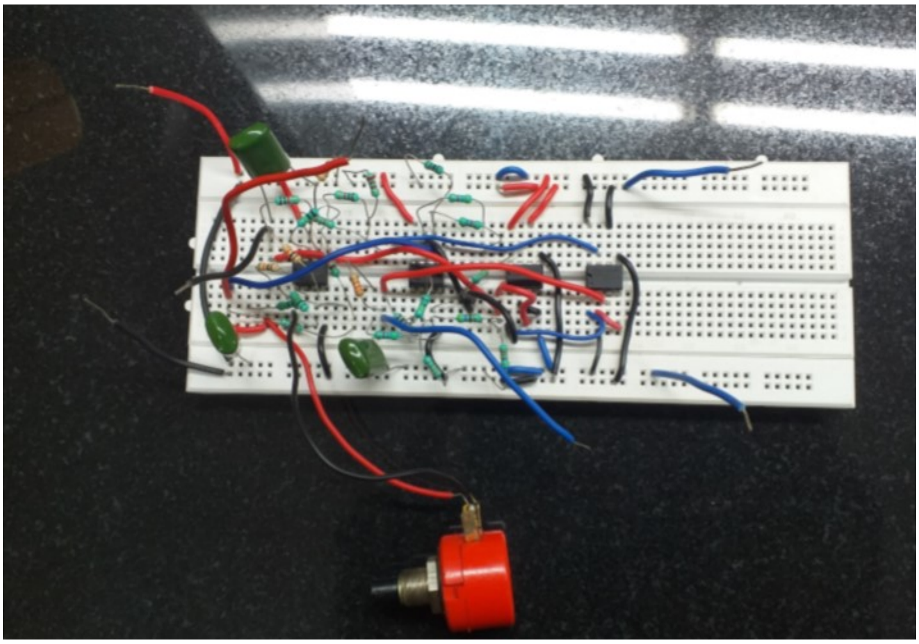

Figure 25: Lorenz circuit

- Period 1 occurs at 79.4 Ω.
- Period 2 occurs at 258.7 Ω.
- It becomes chaotic for more than 1.5 k Ω.

For Y-Z plot -

- Period 1 occurs at 62.2 Ω.
- It becomes chaotic for more than 1.2 k Ω.

For X-Z plot -

- Period 1 occurs at 62.7 Ω.
- Period 2 occurs at 168.2 Ω.
- Period 4 occurs at 253.7 Ω.
- Lorenz attractor is observed at 273.9 Ω.

5.5 Conclusion

- The aim to observe the Lorenz butterfly and Lorenz map is achieved.
- The r values of potentiometer for which periods are observed were noted.

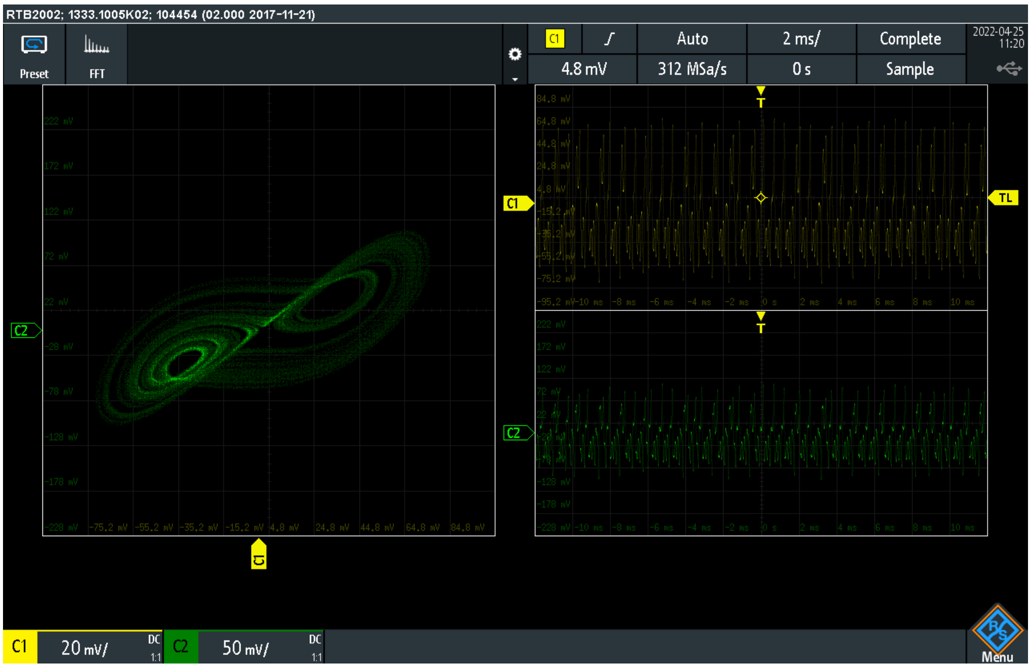

Figure 26: When X,Y are connected

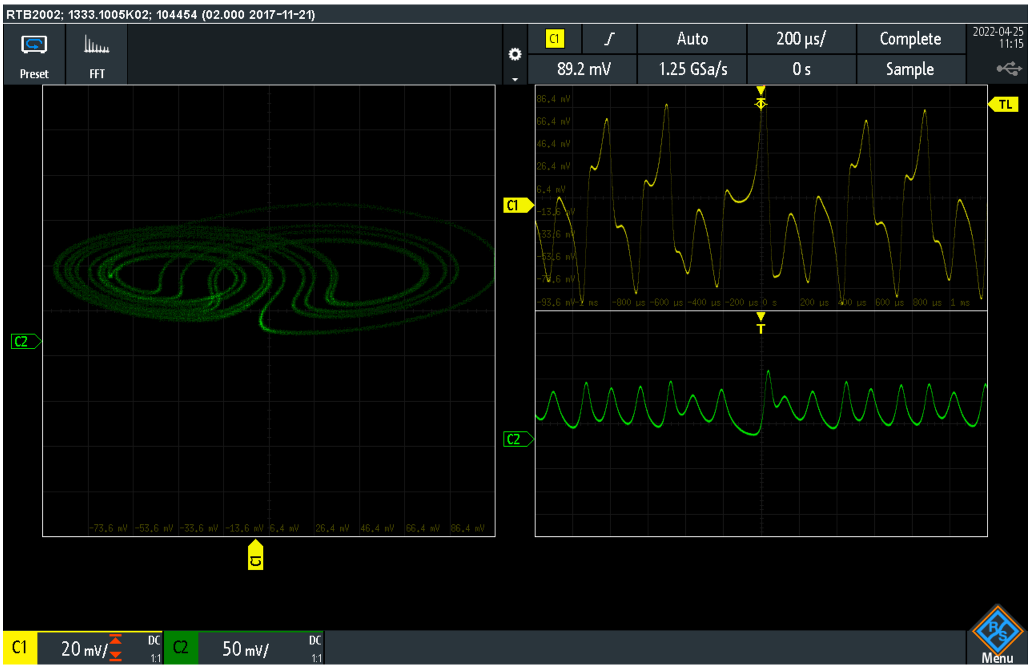

Figure 27: When Y,Z are connected

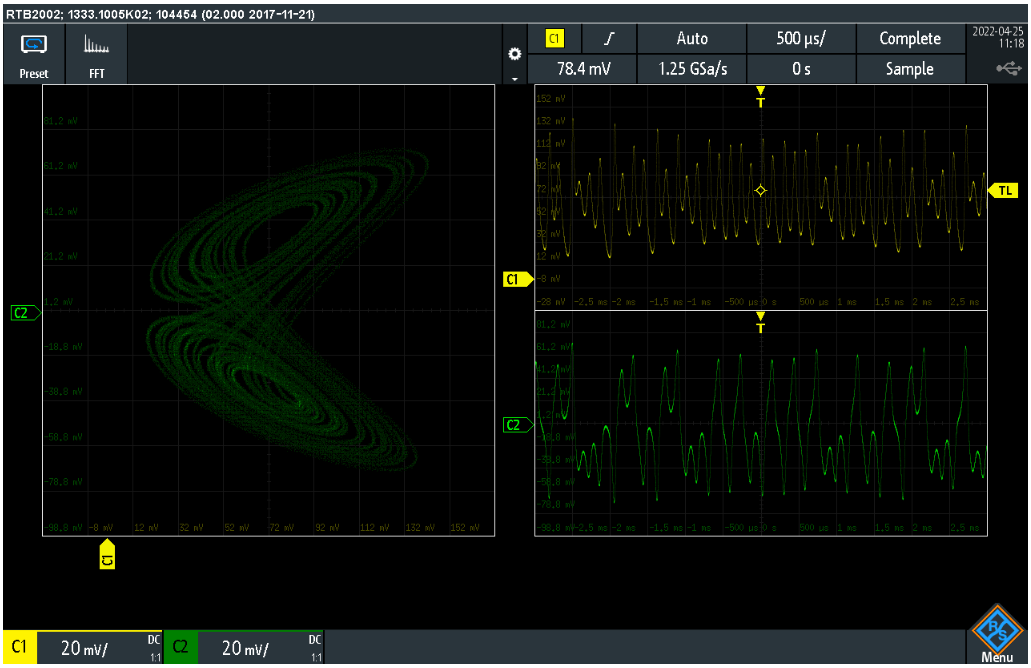

Figure 28: When X,Z are connected

References

- http://seti.harvard.edu/unusual_stuff/misc/lorenz.htm
- https://www.chaotic-circuits.com/6-lorenz-butterfly/
- https://en.wikipedia.org/wiki/Lorenz_system
- https://en.wikipedia.org/wiki/Chua%27s_circuit
- http://www.scholarpedia.org/article/Chua_circuit
- http://www.chuacircuits.com/
- https://people.eecs.berkeley.edu/ chua/papers/Pivka94.pdf
- https://www.scielo.br/j/bjp/a/qkRZjb7ZDmXdXcwn5vz5Bxg/?lang=en
- https://en.wikipedia.org/wiki/Feigenbaum_constants