EXHAUSTIVITY & HURFORD'S CONSTRAINT

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A different kind of challenge to the Neo-Gricean project comes from the so-called *Hurford's Constraint*.

HURFORD'S CONSTRAINT

Consider:

(1) a. #Seth saw a dog or an animals.
b. #Seth lives in Western Massachusetts or in Massachusetts.

One possibility is that the oddness of these sentences is an instance of a more general phenomenon:

Note that, by itself, HURFORD'S CONSTRAINT is not an *explanation* of the oddness of the sentences in (1).

(2) HURFORD'S CONSTRAINT: A sentence of the form ⌜φ or ψ⌝ is infelicitous in c if $[\![\varphi]\!]_c \subseteq [\![\psi]\!]_c$ or $[\![\psi]\!]_c \subseteq [\![\varphi]\!]_c$.

The phenomenon may be even *more* general than this:

(3) a. #If Seth lives in Western Massachusetts or in Massachusetts, he saw a lot of snow this year.
b. #Every student who owns a dog or an animal is excused from today's class.

If so, the right generalization may be:

(4) GENERALIZED HURFORD'S CONSTRAINT: A sentence containing a constituent of the form ⌜φ or ψ⌝ is infelicitous in c if $[\![\varphi]\!]_c \subseteq [\![\psi]\!]_c$ or $[\![\psi]\!]_c \subseteq [\![\varphi]\!]_c$.

Note that a possible explanation for either of these two generalizations would be in terms of Grice's maxim of BREVITY: e.g. it would be a violation of that maxim to use (1a) instead of the briefer

(5) Seth saw an animal.

which would be equivalent to (1a).

Question to think about: what notion of equivalence would we need here?

More generally:

This is a consequence of the fact that if $[\![\alpha]\!] = [\![\beta]\!]$, then $[\![\gamma[\alpha/\beta]]\!] = [\![\gamma]\!]$.

(6) a. If $[\![\varphi]\!]_c \subseteq [\![\psi]\!]_c$, then for any c, if χ is the result of replacing one or more constituents of ζ of the form ⌜φ or ψ⌝ with ψ, then $[\![\chi]\!]_c = [\![\zeta]\!]_c$.

b. If χ is the result of replacing one or more constituents of ζ of the form $\ulcorner\varphi$ or $\psi\urcorner$ with φ, then χ is briefer than ζ.
c. If $[\![\chi]\!]_c = [\![\zeta]\!]_c$ and χ is briefer than ζ, then uttering φ would result in a violation of BREVITY.
d. Therefore, GENERALIZED HURFORD'S CONSTRAINT is true.

Of course, in order to get a satisfactory explanation, we'd need a more precise formulation of BREVITY. We may return to this.

At any rate: on the face of it, HURFORD'S CONSTRAINT (and consequently, GENERALIZED HURFORD'S CONSTRAINT) has a number of counterexamples:

(7) a. Seth will bring tea or coffee or both.
b. Seth graded some or all of the papers.

Incidentally: *if* the Neo-Gricean could tell a story on which (7a) (resp. (7b)) does not generate the same implicatures as (8a) (resp. (8b)), a variant of (6) could be used to explain why the sentences in (1), but not those in (7), are odd:

(8) a. Seth will bring tea or coffee.
b. Seth graded some of the papers.

But all of the Neo-Gricean accounts we've seen thus far assign to each of the sentences in (7) the same implicatures as the corresponding sentence in (8).

After all, on the Neo-Gricean story,

$$[\![\text{Seth will bring tea and coffee}]\!]_c \subseteq [\![\text{Seth will bring tea or coffee}]\!]_c$$

and

$$[\![\text{Seth graded all of the papers}]\!]_c \subseteq [\![\text{Seth graded some of the papers}]\!]_c.$$

One straightforward way of reconciling HURFORD'S CONSTRAINT with the observation that sentences in (7) are perfectly OK:

(9) Neither argument of *or* in (7) entails the other.

The most plausible way of getting this is to deny that the first disjunct in (7a) has

(10) $\lambda w.w \in \{w' : \text{ Seth brings coffee in } w'\} \cup \{w' : \text{ Seth brings tea in } w'\}$

as its semantic value. And the most plausible way of doing *that* seems to be to assign to the first disjunct in (7a) the following semantic value:

(11) λw. Seth brings coffee or tea but not both in w.

EXHAUSTIVITY AND SCALAR IMPLICATURES

The proposal in Chierchia, Fox & Spector 2009 is a way of implementing this suggestion. The first ingredient is a so-called EXHAUSTIVITY OPERATOR, which takes as arguments a set of propositions X and a proposition p and yields a proposition:

Cf. the entry for *only* in von Fintel 1997.

(12) $[\![\text{EXH}]\!]_c(X)(p) := \lambda w.p(w) = 1 \text{ and } \forall q \in X,\ q(w) = 1 \rightarrow p \subseteq q,$

In other words, EXH takes as arguments a set of propositions X and a proposition p, and gives you the proposition that is true at a world w iff p is true at w and no member of X that is not entailed by p is true at w.

The second ingredient is an analysis of the first disjunct in (7a) as containing a silent EXH, so that the logical form of (7a) is:

(13) [[EXH R] [Seth will bring tea or coffee]] or [Seth will bring tea and coffee].

The final ingredient is a claim about what the first argument of EXH in (13) is. Remember, from our discussion of Sauerland 2004, the function $\varphi \mapsto \text{ALT}_c(\varphi)$. The suggestion here is that the first argument of EXH in (13) is a function of:

(14) $\text{ALT}_c(\mathtt{T} \vee \mathtt{C}) = \{\mathtt{T} \vee \mathtt{C}, \mathtt{T}, \mathtt{C}, \mathtt{T} \wedge \mathtt{C}\}$.

More specifically, for now we can stipulate that the first argument of EXH in the first disjunct of (7a) is

Note that we haven't yet said anything about how to generate the set in (15).

(15) $A_{\mathtt{T}\vee\mathtt{C}} := \{[\![\mathtt{T} \vee \mathtt{C}]\!]_c, [\![\mathtt{T} \wedge \mathtt{C}]\!]_c\}$,

The semantic value of the first disjunct of (13) is then:

(16) $[\![\text{EXH}]\!](A_{\mathtt{T}\vee\mathtt{C}})([\![\mathtt{T} \vee \mathtt{C}]\!]_c)$,

which is just (11).

A similar story can be told for (7b), if we assume its underlying structure is

(17) [[EXH R] [Seth graded some of the papers]] or [Seth graded all of the papers].

and if we stipulate that the first argument of EXH is

(18) $A_{\exists\mathtt{P}} = \{[\![\exists\mathtt{P}]\!], [\![\forall\mathtt{P}]\!]\}$.

For this would mean that the first disjunct of (7b) would have the same semantic value as (19a), which is given by (19b):

(19) a. Seth graded some but not all of the papers.
b. $\lambda w.([\![\exists\mathtt{P}]\!]_c(w) = 1 \text{ and } [\![\forall\mathtt{P}]\!]_c(w) = 0)$.

If we allow for the presence of silent EXH, we can recover some (all?) of the facts about embedded scalar implicatures straightforwardly. For instance, if the structure of (20a) is that in (20b), its truth conditions will be the same as those of (20c):

Of course, they would no longer be *implicatures* in the usual sense.

(20) a. Zoe believes that Seth will bring tea or coffee.
b. Zoe believes that [EXH [Seth will bring tea or coffee]].
c. Zoe believes that Seth will bring tea or coffee but not both.

Other examples require a more subtle treatment. For example, if the structure of (21a) is given by (21b), we could predict that (21a) entails (21c)

(21) a. Zoe ate the muffin or some of the candy.
b. [[EXH R_1] [Zoe ate the muffin or [[EXH R_2] [Zoe ate some of the candy]]]].
c. Zoe did not eat all of the candy.

To do so, however, we'd need to stipulate that the first argument of the matrix EXH in (21b) (i.e. the value of R_1) includes the proposition expressed by

(22) Zoe ate all of the candy.

One way to ensure that would be to claim that any scalar alternative (in the sense of Sauerland 2004) of the prejacent of EXH gives rise to a member of the first argument of EXH, and to insist that φ and ψ are scalar alternatives of $\varphi \vee \psi$. But this would lead to a contradiction pretty quickly:

(23) a. $A^{*}_{\mathtt{T}\vee\mathtt{C}} = \{[\![\mathtt{T} \vee \mathtt{C}]\!], [\![\mathtt{T}]\!], [\![\mathtt{C}]\!], [\![\mathtt{T} \wedge \mathtt{C}]\!]\}$.
b. $[\![\text{EXH}]\!](A^{*}_{\mathtt{T}\vee\mathtt{C}})([\![T \vee C]\!])(w) = 1 \Rightarrow ([\![\mathtt{T} \vee \mathtt{C}]\!](w) = 1,\ [\![\mathtt{T}]\!](w) = 0, \text{ and } [\![\mathtt{C}]\!](w) = 0)$.

Actually, we only need a proposition that is entailed by the proposition expressed by (22) which does not entail the proposition expressed by the prejacent of the matrix EXH in (21b).

The official story is more complicated. For now, we can work with this:

This is the preliminary definition used in Fox 2007.

(24) a. $\text{NW}(\varphi) := \{[\![\psi]\!] : \psi \in \text{ALT}_c(\varphi) \text{ and } [\![\varphi]\!] \nsubseteq [\![\psi]\!].\}$
b. $A_\varphi := \{p \in \text{NW}(\varphi) : \forall q \in \text{NW}([\![\varphi]\!])([\![\varphi]\!] \cap \neg p \nsubseteq q)\}$
c. $[\![\text{EXH } \varphi]\!] = [\![\text{EXH}]\!](A_\varphi)([\![\varphi]\!])$.

Note that, once we make these assumptions, we can replace our definition in (12) with

(25) $[\![\text{EXH}]\!]_c(X)(p) := \lambda w.p(w) = 1 \text{ and } \forall q \in X,\ q(w) = 0$.

In plain English: $\text{NW}(\varphi)$ is the set of propositions expressed by elements of $\text{ALT}_c(\varphi)$ that aren't weaker than $[\![\varphi]\!]$; A_φ is the set of elements of $\text{NW}(\varphi)$ whose negation is consistent, modulo φ, with the negation of any other member of $\text{NW}(\varphi)$.

Now, given these assumptions, we don't need to stipulate the presence of an embedded exhaustivity operator in (21a). For

The definition in (24) is only a first approximation. The current best approximation is even more complex:

(26) a. A subset M of a set of propositions X is *maximally consistent* iff M is consistent and whenever $M \subset Y \subseteq X$, Y is inconsistent.
b. A proposition q is *innocently excludable* with respect to X and p iff $\neg q$ is a member of all maximally consistent subsets of X that contain p.

(27) [EXH [Zoe ate the muffin or some of the candy]]

entails (21c).

Do (27) and (21b) have different truth-conditions? To begin, note that

(28) [Zoe ate the muffin] or some of the candy.

gives rise to the following set of propositions:

(29) $A_{(28)} = \{[\![\forall\mathtt{C}]\!], [\![\mathtt{M} \wedge \forall\mathtt{C}]\!], [\![\mathtt{M} \wedge \exists\mathtt{C}]\!]\}$

The working hypothesis in Chierchia, Fox & Spector 2009 is that the first argument of EXH (in c), given a prejacent φ, is the set of innocently excludable propositions with respect to $\{[\![\neg\psi]\!]_c : \psi \in \text{ALT}_c(\varphi)\}$ and $[\![\varphi]\!]_c$.

For an argument that even this hypothesis is not quite right, see Fox 2007, fn. 35.

Hence, (27) is equivalent to:

(30) $(\mathtt{M} \vee \exists\mathtt{C}) \wedge \neg\forall\mathtt{C} \wedge \neg(\mathtt{M} \wedge \forall\mathtt{C}) \wedge \neg(\mathtt{M} \wedge \exists\mathtt{C})$

To compute the semantic value of (21b), we first need to compute $A_{(31)}$, where

(31) [Zoe ate the muffin or [[EXH R_2] [Zoe ate some of the candy.]]]

is the prejacent of the matrix EXH in (21b). To do that, first note that NW(31) is the set of propositions expressed by the following sentences:

(32) $\mathtt{M}, \text{EXH}\,(\exists\mathtt{C}), \text{EXH}\,(\forall\mathtt{C}), \mathtt{M} \vee \text{EXH}\,(\forall\mathtt{C}), \mathtt{M} \wedge \text{EXH}\,(\exists\mathtt{C}), \mathtt{M} \wedge \text{EXH}\,(\forall\mathtt{C})$.

The entailment relations among them are given by diagram in figure 1, so that:

(33) $A_{(31)} = \{[\![\text{EXH}\,(\forall\mathtt{C})]\!], [\![\mathtt{M} \wedge \text{EXH}\,(\exists\mathtt{C})]\!], [\![\mathtt{M} \wedge \text{EXH}\,(\forall\mathtt{C})]\!]\}$.

And since $[\![\text{EXH}\,(\forall\mathtt{C})]\!] = [\![\forall\mathtt{C}]\!]$, we have:

(34) $A_{(31)} = \{[\![\forall\mathtt{C}]\!], [\![\mathtt{M} \wedge \text{EXH}\,(\exists\mathtt{C})]\!], [\![\mathtt{M} \wedge \forall\mathtt{C}]\!]\}$.

Feeding this as an value for R_1 in (21b) gives us:

(35) $(\mathtt{M} \vee (\exists\mathtt{C} \wedge \neg\forall\mathtt{C})) \wedge \neg\forall\mathtt{C} \wedge \neg(\mathtt{M} \wedge \forall\mathtt{C}) \wedge \neg(\mathtt{M} \wedge (\exists\mathtt{C} \wedge \neg\forall\mathtt{C}))$,

which is equivalent to:

(36) $(\mathtt{M} \vee \exists\mathtt{C}) \wedge (\mathtt{M} \vee \neg\forall\mathtt{C}) \wedge \neg\forall\mathtt{C} \wedge \neg(\mathtt{M} \wedge \forall\mathtt{C}) \wedge \neg(\mathtt{M} \wedge (\exists\mathtt{C} \wedge \neg\forall\mathtt{C}))$.

As a result, (27) and (21b) have the same truth-conditions.

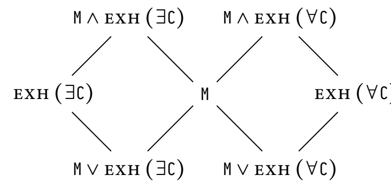

Figure 1: The elements of ALT(31). Entailment relations are represented in the usual way.

MORE EVIDENCE IN SUPPORT OF THE EXHAUSTIVITY OPERATOR

Assuming the existence of a covert exhaustivity operator: where should we expect to find it?

If HURFORD'S CONSTRAINT is right, there will be many cases where EXH is mandatory. For instance, we expect that (37a) will have the structure in (37b):

(37) a. Seth met with the first and second year students or he met with all of the students.
b. [EXH [Seth met with the first and second year students]] or he met with all of the students.

As Chierchia, Fox & Spector point out, assuming that the first argument of EXH is the set of propositions expressed by sentences of the form ⌜ Seth met with the *n*-th year students⌝, this means that the only reading of (37a) is equivalent to:

Is this an advantage of their approach? Is this really the *only* reading of (37a)?

(38) *Either* Seth met with the first and second year students, and no other students, *or* he met with to all of the students.

The claim here is somewhat subtle. With a sentence like (7a), which I repeat below for convenience, (7a) Seth will bring tea or coffee or both.

the presence of a covert exhaustivity operator has no effect on the truth conditions of the sentence, since

$$((\varphi \vee \psi) \wedge \neg(\varphi \wedge \psi)) \vee (\varphi \wedge \psi)$$

is equivalent to

$$(\varphi \vee \psi) \vee (\varphi \wedge \psi).$$

In the case of (37b), however, the presence of EXH does have an effect on the truth-conditions, since (39a), but not (37b), is compatible with (39b):

(39) a. [Seth$_i$ met with the first and second year students] or [he$_i$ met with all of the students].
b. Seth met with the first, second, and third year students, but not with the fourth year students.

If (39b) is not compatible with (37a), this suggests that there is indeed an embedded EXH.

Further evidence comes from data involving apparent violations of HURFORD'S CONSTRAINT under the scope of necessity modals:

Question: is *either* necessary to generate that implicature?

(40) a. You are required to either take dancing lessons or juggling lessons.
b. Your are required to either take dancing lessons or juggling lessons or both.

If GENERALIZED HURFORD CONSTRAINT is right, the underlying structures of (40a) and (40b) are given by (41a) and (41b), respectively:

(41) a. $\Box(\mathtt{D} \vee \mathtt{J})$.
b. $\Box((\text{EXH}\ (\mathtt{D} \vee \mathtt{J})) \vee (\mathtt{D} \wedge \mathtt{J}))$.

The starting observation is that, although (40a) and (40b) are truth-conditionally equivalent (even under the analysis in (41)), they give rise to different implicatures. In particular,

Their argument for (42) is based on a series of exchanges. The starting observation is that one can felicitously deny someone's claim just by objecting to one of its implicatures. Compare now:

(42) (40b), but not (40a), implicate that you are allowed to take both dancing lessons and juggling lessons.

(43) a. You are required to either take dancing lessons or juggling lessons.
b. No! I am required to take both.
c. #No! I am not allowed to take both.

(44) a. You are required to either take dancing lessons or juggling lessons or both.
b. No! I am required to take both.
c. No! I am not allowed to take both.

On this approach, scalar implicatures are the result of strengthening the meaning of a sentence by adding a covert EXH in matrix position. The issue then is whether (45b), but not (45a), entails (45c):

(45) a. $\text{EXH}\ (\Box(\mathtt{D} \vee \mathtt{J}))$.
b. $\text{EXH}\ (\Box((\text{EXH}\ (\mathtt{D} \vee \mathtt{J})) \vee (\mathtt{D} \wedge \mathtt{J})))$.
c. $\neg \Box \neg(\mathtt{D} \wedge \mathtt{J})$.

Let us first consider what the first argument to the matrix EXH in each of the relevant sentences is, on the assumption that

Here we are ignoring the possibility that *required* will itself generate even more scalar alternatives.

(46) $[\![\text{EXH}\ (D \vee J)]\!] = [\![(D \vee J) \wedge \neg(D \wedge J)]\!]$.

Remember that the arguments of the matrix EXH are generated by their corresponding prejacents, using the definition in (24):

(47) a. $\Box(D \vee J)$.
b. $\Box((\text{EXH}\ (D \vee J)) \vee (D \wedge J))$.

Now, $A_{(47a)}$ is just:

Note that, whereas $[\![D]\!]$ is not among the propositions whose negation is entailed by $[\![\text{EXH}\ (D \vee J)]\!]$, $[\![\Box D]\!]$ *is* among those whose negation is entailed by $[\![\text{EXH}\ (\Box(D \vee J))]\!]$. The reason is, essentially, that while $\{\neg D, \neg J, (D \vee J)\}$ is inconsistent, $\{\neg\Box D, \neg\Box J, \Box(D \vee J)\}$ is not.

(48) $\{[\![\Box(D \wedge J)]\!], [\![\Box D]\!], [\![\Box J]\!]\}$.

As a result, $[\![(45a)]\!]$ is equivalent to:

(49) $\Box(D \wedge J) \wedge \neg\Box(D) \wedge \neg\Box(J)$.

Computing $A_{(47b)}$ takes a bit of work. The set of scalar alternatives of (47b) that are not entailed by (47b) is (I think!):

(50) $\Box(\text{EXH}\ (D \vee J) \wedge D)$, $\Box(\text{EXH}\ (D \vee J) \wedge J)$, $\Box(\text{EXH}\ (D \vee J) \wedge (D \vee J))$, $\Box(\text{EXH}\ (D \vee J))$, $\Box(D \wedge J)$, $\Box D$, $\Box J$, $\Box(D \vee J)$, $\Box(\text{EXH}\ (D \wedge J) \vee D)$, $\Box(\text{EXH}\ (D \wedge J) \vee J)$.

There's a fair bit of redundancy here. In fact, NW(45b) turns out to be:

(51) $\{[\![\Box(\text{EXH}\ (D \vee J) \wedge D)]\!], [\![\Box(\text{EXH}\ (D \vee J) \wedge J)]\!], [\![\Box(\text{EXH}\ (D \vee J))]\!], [\![\Box(D \wedge J)]\!], [\![\Box D]\!], [\![\Box J]\!], [\![\Box(D \vee J)]\!]\}$.

And we can now check that $A_{(45b)}$ contains the propositions expressed by both (52a) and (52b):

Chierchia, Fox & Spector 2009 make the following strong generalization:

$$[\![\Box\varphi]\!], [\![\Box\psi]\!] \in A_{\Box(\varphi\vee\psi)}.$$

A proof of this would be nice.

(52) a. $\Box((D \vee J) \wedge \neg(D \wedge J))$
b. $\Box(D \wedge J)$.

And this in turn means that $[\![(45b)]\!]$ entails that we are not required to read both and that we are not required not to read both—as desired.

Assuming, that is, that the necessity modal distributes over conjunction.

AT LAST: DEALING WITH 'OR BOTH'

Recall a problem that neither Sauerland 2004 nor Spector 2007 had the resources to solve:

(53) a. Seth will bring tea or coffee.
b. Seth will bring tea or coffee or both.
c. Seth will not bring both tea and coffee.

On either of the two Neo-Gricean views, both (53a) and (53b) have (53c) as an implicature.

Sauerland's account does predict that different sets of sentences will generate the primary implicatures of (53a) and (53b). But the sets of *propositions* expressed by the sentences in those sets turn out to be the same. Would the presence of an embedded exhaustivity operator help?

First, note that in order to derive (53c) from (53a), the present proposal would posit an EXH in matrix position, so that (53a) has the following structure:

(54) $\textsc{exh}\,(\mathtt{T} \vee \mathtt{C})$.

As we saw above, the truth-conditions of this are given by:

(11) λw. Seth brings coffee or tea but not both in w.

Now, assuming HURFORD'S CONSTRAINT, the present proposal predicts that a covert EXH must be embedded under the second *or* in (53b):

(55) [EXH [Seth will bring tea or coffee]] or Seth will bring tea and coffee.

Without an additional EXH in matrix position, (55) will be equivalent to (53a), and thus will not entail that Seth will bring tea and coffee. But adding a matrix EXH to (55) would not make a difference.

To see why, we need to consider what the first argument of a matrix EXH in (53b) would be. Eliminating redundancies, and assuming that EXH has no effect when its prejacent is either $\mathtt{T}$, $\mathtt{C}$, or $\mathtt{T} \wedge \mathtt{C}$, we have that NW(53b) is:

(56) $\{[\![\mathtt{T} \wedge \mathtt{C}]\!], [\![\mathtt{T} \vee \mathtt{C}]\!], [\![\mathtt{T}]\!], [\![\mathtt{C}]\!], [\![\textsc{exh}\,(\mathtt{T} \vee \mathtt{C})]\!]\}$.

The crucial observation is this: the negation of each member of NW(53b) entails, modulo (53b), another member of NW(53b). As a result, $A_{(53b)}$ is empty, and thus $[\![(53b)]\!]$ does not entail (53c).

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