1 Sensitivities $\frac{\partial x}{\partial p}$

The XTK is responsible for computing the sensitivity of a change in interface location with respect to a change in design variable. Since the interface location, x_Γ , has an implicit relationship with the level set value found at each end node, chain rule is invoked.

$$\frac{\partial x_{\Gamma i}}{\partial p_j} = \frac{\partial x_{\Gamma i}}{\partial \phi_A}\frac{\partial \phi_A}{\partial p_j} + \frac{\partial x_{\Gamma i}}{\partial \phi_B}\frac{\partial \phi_B}{\partial p_j} \tag{1}$$

Where j is $1, 2, ...n_d$, n_d is the number of design variables, i is $1, .., D$ where D is the spatial dimension. $\frac{\partial x_{\Gamma i}}{\partial \phi_A}$ is the change in the interface location with respect to a change in a nodal scalar field at node A, ϕ_A. $\frac{\partial \phi_A}{\partial p_j}$ is the change in a the nodal scalar field value at node A with respect to a change in design variable, p. The second term is the same but with respect to node B.

In the XTK, usually linear basis functions are used to compute (1). These linear basis functions are given in (2).

$$N_1 = \frac{1}{2}(1 - \xi) \tag{2}$$

$$N_2 = \frac{1}{2}(1 + \xi) \tag{3}$$

The interface location in the edge local frame, ξ_Γ can be approximated using (2) as:

$$\vec{x}_\Gamma = \frac{\phi_B + \phi - 2\phi_\Gamma}{\phi_A - \phi_B} \tag{4}$$

Where ϕ_Γ is a specified threshold value. Typically, $\phi_\Gamma = 0$. In order to compute (1), the $\frac{\partial x_{\Gamma i}}{\partial \phi_A}$ and $\frac{\partial x_{\Gamma i}}{\partial \phi_A}$ terms are needed which can but found for linear functions as:

$$\frac{\partial \vec{x_\Gamma}}{\partial \phi_A} = \frac{\partial \vec{x}}{\partial \xi_\Gamma}\frac{\partial \xi_\Gamma}{\partial \phi_A} = \frac{\phi_\Gamma - \phi_B}{(\phi_A - \phi_B)^2}(\vec{x}_B - \vec{x}_A) \tag{5}$$

$$\frac{\partial \vec{x_\Gamma}}{\partial \phi_B} = \frac{\partial \vec{x}}{\partial \xi_\Gamma}\frac{\partial \xi_\Gamma}{\partial \phi_B} = \frac{\phi_A - \phi_\Gamma}{(\phi_A - \phi_B)^2}(\vec{x}_B - \vec{x}_A) \tag{6}$$

The terms $\frac{\partial \phi_A}{\partial p_j}$ and $\frac{\partial \phi_B}{\partial p_j}$ are computed analytically depending on the parameterization of ϕ.