Draw – a - Shape

Features

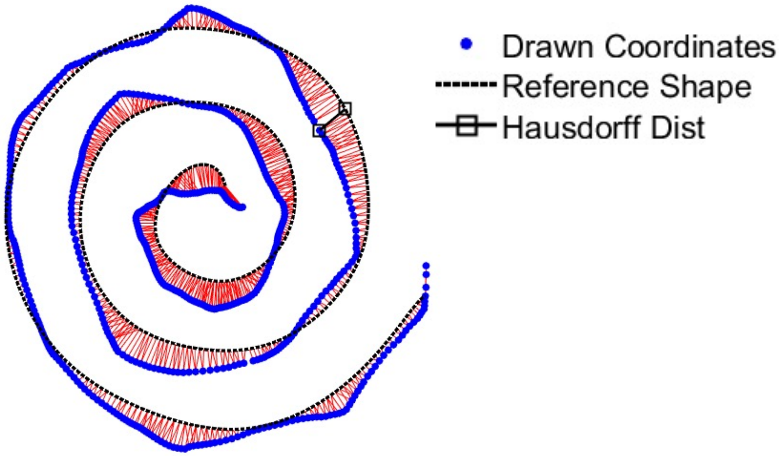
Hausdorff Distance

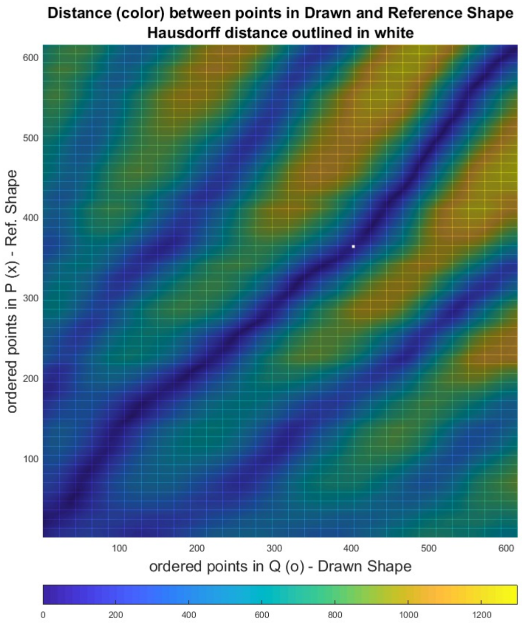
- Max distance of a set to the nearest point in the other set in metric space
- Examining the error distances doesn't give whole picture of how far apart or dissimilar shapes are from each other

Let X and Y be two-non empty subsets of a metric space (M, d). The Hausdorff distance $d_H(X, Y)$is defined as:

$$d_H(X,Y) = max\left\{\sup_{\mathrm{x}\in\mathrm{X}} \inf_{\mathrm{x}\in\mathrm{Y}} d(x,y), \sup_{y\in Y} \inf_{x\in X} d(x,y)\right\}$$

Where sup is the *supremum* and inf is the *infimum.*





Approximate Entropy ($ApEn$)

- Used to compute the irregularity of the temporal fluctuations of a time series
- A time series containing many repetitive patterns has a relatively small $ApEn$, whereas less predictable time series has a higher $ApEn$

$$ApEn(S_n, m, r) = \phi^m(r) - \phi^{m+1}(r)$$

where S_n is a signal containing n pts; m is the pattern length; r is the similarity criterion

where $$\phi^m(r) = (\mathrm{N} - \mathrm{m} + 1)^{-1} \sum_{i=1}^{N-m+1} \log C_i^m(r)$$

Where $C_i^m(r)$ is the correlation dimension: (fraction of patterns of length m that resemble the patterns of the same that begins at interval i)

$$\frac{diff[x(i), x(j)]}{N - m + 1} \leq r$$

≈ relative prevalence of repetitive patterns of length m compared to those of length m+1

- $ApEn$ estimates the log-likelihood that the next intervals after each of the patterns will differ

Image Entropy

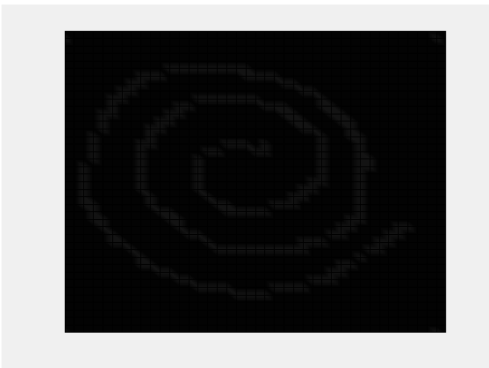
- Image entropy of heat-map transposed shape drawings (converted to grayscale) are calculated using

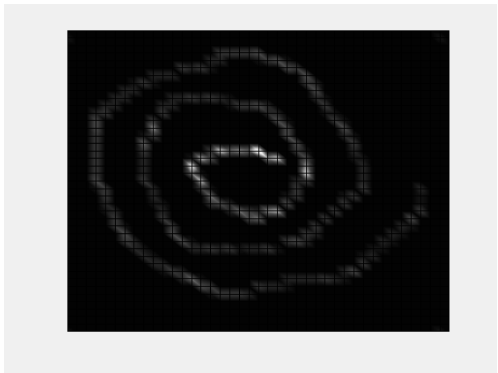
$$H = -\sum_{k} p_k \log_2(p_k)$$

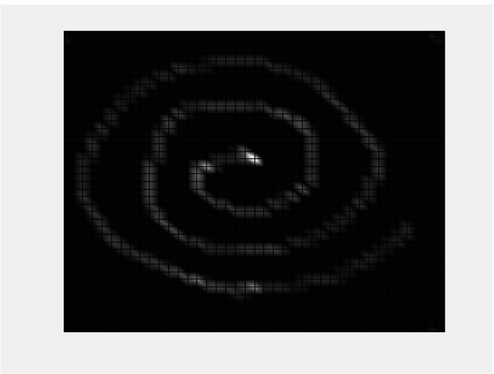
Where k is the number of gray levels and p_k is the probability associated with each gray level k.

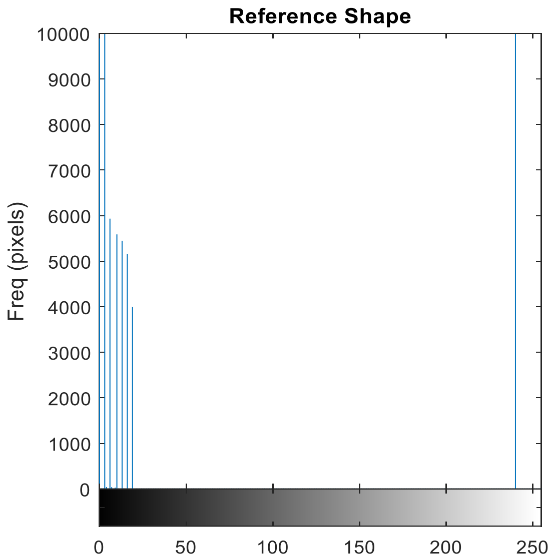
- Entropy is a common used measure of disorder in a system can be used to in image analysis for texture mapping.
- The topography of transposed pixel intensity drawings become a function of finger movements and hence entropy a measure of smooth, non-hesitant drawing.

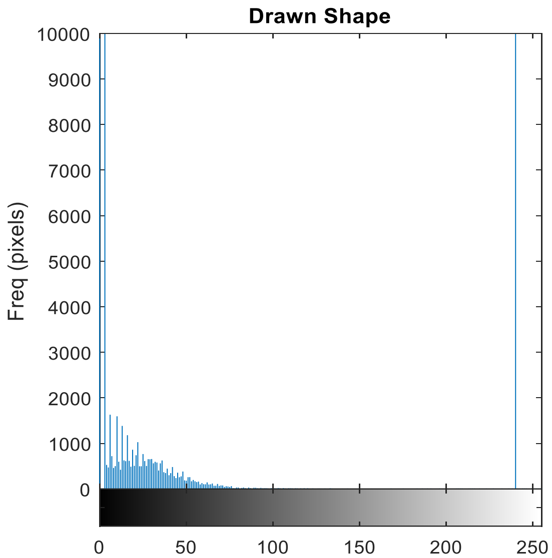
Image Entropy

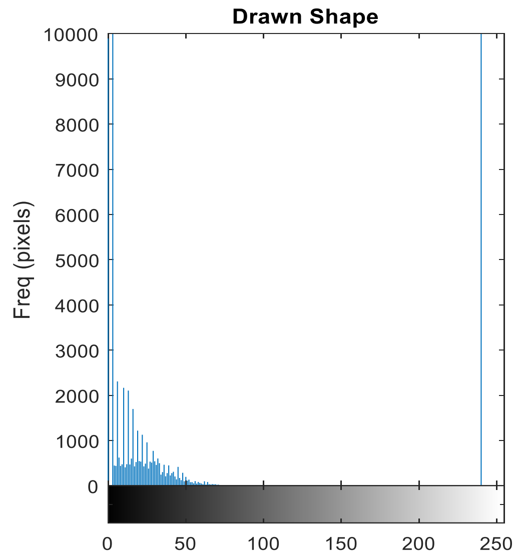












Entropy: 2.0581 2.6312 2.4953

Structural Similarity Index

The Structural Similarity (SSIM) Index quality assessment index is based on the computation of three terms, namely the luminance term, the contrast term and the structural term. The overall index is a multiplicative combination of the three terms.

$$SSIM(x,y) = [I(x,y)]^{\alpha} \cdot [C(x,y)]^{\beta} \cdot [S(x,y)]^{\gamma}$$

where

$$I(x,y) = \frac{2\mu_x\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1}, \qquad C(x,y) = \frac{2\sigma_x\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2}, \qquad S(x,y) = \frac{\sigma_{xy} + C_3}{\sigma_x\sigma_y + C_3};$$

where μ_x, μ_y , σ_x, σ_y and σ_{xy} are the local means, standard deviations and cross-covariance for images x, y. C refers to the regularization constants of the luminance, contrast, and structural terms. If α=β=1 (the default for exponents), and $C_3 = \frac{C_2}{2}$ (default selection of C_3) the index simplifies to:

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_x\sigma_y + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$$

Wang Zhou, Bovik, Alan C., Sheikh, Hamid R., and Simoncelli, Eero P. “Image Quality Assessment: From Error Visibility to Structural Similarity.” IEEE Transactions on Image Processing, Volume 13, Issue 4, pp. 600–612, April 2004

Chi-Squared Distance

- The Chi-square distance (*pChisq*) between the two histograms of heat-map transposed pixel intensity counts which can be used as a rough similarity index.
- A particular weighted Euclidean distance (modified for count data) subtracts the two histograms bin-by-bin and contributes each bin pairs equally to the distance with added normalisation:

$$\chi_{h,k} = \sqrt{\sum_{j=1}^{J} \frac{1}{C_j}(h_j - k_j)^2}$$

where h and k are the respective histogram with J bins; C_j is proportion of the j^{th} bin with respect to overall histogram.

2-D correlation coefficient

- Computes the correlation coefficient r between two images (matrices), A and B, which are the same size:

$$r = \frac{\sum_m \sum_n (A_{mn} - \bar{A})(B_{mn} - \bar{B})}{\sqrt{(\sum_m \sum_n (A_{mn} - \bar{A})^2)(\sum_m \sum_n (B_{mn} - \bar{B})^2)}}$$

Where m and n are the number of rows and columns in the images;
$\bar{A}$ & $\bar{B}$ are the mean of matrices A and B respectively.