Load the vector tools

In[1]:=
```
<< "C:\\Users\\ambik\\OneDrive - University of Louisiana
    Lafayette\\Documents\\Wolfram Mathematica\\vectorDefsMM30.m"
```

```
These Engineering Vector algorithms are copyright Alan A. Barhorst
```

In[2]:=
```
Off[ReplaceRepeated::"rrlim"]
```

Typical rotations

Generic 0 rotation (Identity)

In[3]:=
```
rot0[q_ : 1] = {{1, 0, 0}, {0, 1, 0},
                {0, 0, 1}};
MatrixForm[rot0[ ]]
```

Out[3]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Generic 1-rotation

In[4]:=
```
rot1[q_] = {{1, 0, 0}, {0, Cos[q], Sin[q]},
               {0, -Sin[q], Cos[q]}};
MatrixForm[rot1[q1[t]]]
```

Out[4]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[q_1[t]] & \mathrm{Sin}[q_1[t]] \\ 0 & -\mathrm{Sin}[q_1[t]] & \mathrm{Cos}[q_1[t]] \end{pmatrix}$$

Generic 2-rotation

In[5]:=
```
rot2[q_] = {{Cos[q], 0, -Sin[q]}, {0, 1, 0},
               {Sin[q], 0, Cos[q]}};
MatrixForm[rot2[q2[t]]]
```

Out[5]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[q_2[t]] & 0 & -\mathrm{Sin}[q_2[t]] \\ 0 & 1 & 0 \\ \mathrm{Sin}[q_2[t]] & 0 & \mathrm{Cos}[q_2[t]] \end{pmatrix}$$

Generic 3-rotation

In[6]:=
```
rot3[q_] = {{Cos[q], Sin[q], 0},
            {-Sin[q], Cos[q], 0},
            {0, 0, 1}};
MatrixForm[rot3[q3[t]]]
```

Out[6]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[q_3[t]] & \text{Sin}[q_3[t]] & 0 \\ -\text{Sin}[q_3[t]] & \text{Cos}[q_3[t]] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Define the symbols used for Unit Vectors and Unit Dyads

Define Unit Vectors and Unit Dyads for however many frames we need for the system. For this example we will use three frames of reference, with the frame **N** being the Newtonian frame. The header **unitVector** must be included. The arguments are **[frame, symbol, direction]**. So unit vector **b[1]=unitVector[B,b,1]** is the vector in the **B** frame in the **1** direction. The unitDyads are double vectors used to describe inertia properties.

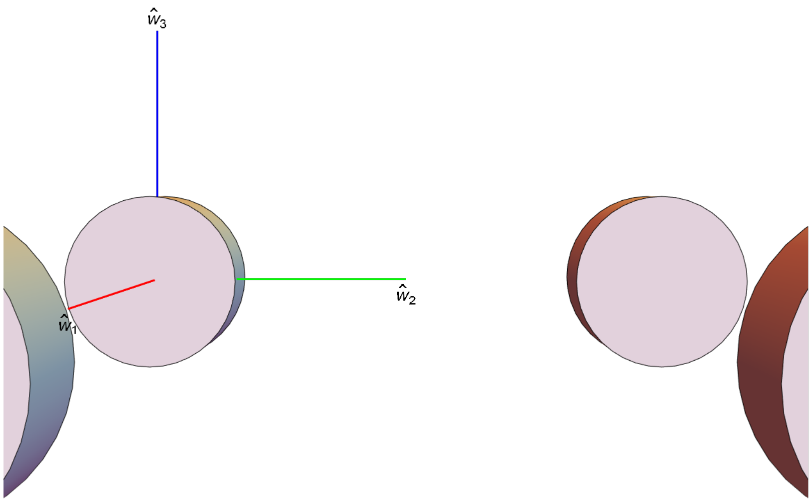
In[7]:=
```
w[x_]:=unitVector[W,w,x]
a[x_]:=unitVector[A,a,x]
b[x_]:=unitVector[B,b,x]
c[x_]:=unitVector[C,c,x]
d[x_]:=unitVector[D,d,x]
e[x_]:=unitVector[E,e,x]
f[x_]:=unitVector[F,f,x]
g[x_]:=unitVector[G,g,x]
h[x_]:=unitVector[H,h,x]
n[x_]:=unitVector[N,n,x]
ww[x_,y_]:=unitDyad[w[x],w[y]]
aa[x_,y_]:=unitDyad[a[x],a[y]]
bb[x_,y_]:=unitDyad[b[x],b[y]]
cc[x_,y_]:=unitDyad[c[x],c[y]]
dd[x_,y_]:=unitDyad[d[x],d[y]]
ee[x_,y_]:=unitDyad[e[x],e[y]]
ff[x_,y_]:=unitDyad[f[x],f[y]]
gg[x_,y_]:=unitDyad[g[x],g[y]]
hh[x_,y_]:=unitDyad[h[x],h[y]]
```

Graphical construction of robot (uses graphic primatives from *Mathematica* v6 and above)

Wheels

In[26]:=
```
vecL = 1;
wheelRadius = 1 / 3;
halfHeightWheel = 1 / 9;
wheel1Base = {0, 0, -halfHeightWheel};
wheel1Top = {0, 0, halfHeightWheel};
wheel2Base = {0, 2, -halfHeightWheel};
wheel2Top = {0, 2, halfHeightWheel};
wheel3Base = {0, 0, 2 - halfHeightWheel};
wheel3Top = {0, 0, 2 + halfHeightWheel};
wheel4Base = {0, 2, 2 - halfHeightWheel};
wheel4Top = {0, 2, 2 + halfHeightWheel};

wheelsGraphicF = {Rotate[Cylinder[{{wheel1Base, wheel1Top}, {wheel2Base, wheel2Top},
       {wheel3Base, wheel3Top}, {wheel4Base, wheel4Top}}, wheelRadius], Pi / 2, {0, 1, 0}],
   Text[ŵ1, {vecL, 0, 0}, {0, 1}], Text[ŵ2, {0, vecL, 0}, {0, 1}],
   Text[ŵ3, {0, 0, vecL}, {0, -1}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
Show[Graphics3D[wheelsGraphicF], ViewPoint -> {1, 0, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]

wheelsGraphic = {Rotate[Cylinder[{{wheel1Base, wheel1Top}, {wheel2Base, wheel2Top},
       {wheel3Base, wheel3Top}, {wheel4Base, wheel4Top}}, wheelRadius], Pi / 2, {0, 1, 0}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Out[38]=



Base platform

Draw the base platform from regular polygons

In[40]:=
```
vecL = 2;
```

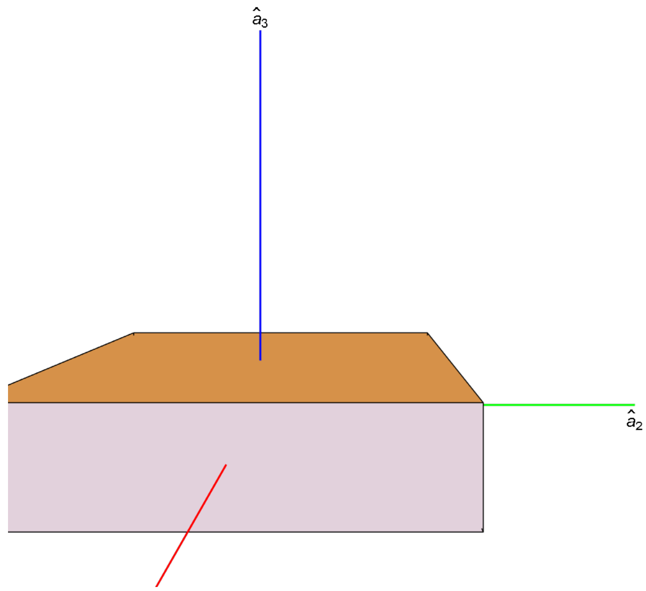
In[41]:=
```
widthBase = 2; depthBase = 2; heightBase = 1 / 2;
```

In[42]:=
```
baseShape = Cuboid[
   {-widthBase / 2, -depthBase / 2, -heightBase / 2},
   {widthBase / 2, depthBase / 2, heightBase / 2}];
```

In[43]:=
```
baseGraphicF = {baseShape,
   {Text[â₁, {vecL, 0, 0}, {0, 1}],
    Text[â₂, {0, vecL, 0}, {0, 1}], Text[â₃, {0, 0, vecL}, {0, -1}],
       {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}}};
```

In[44]:=
```
Show[Graphics3D[baseGraphicF], ViewPoint -> {1, 0, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[44]=



In[45]:=
```
baseGraphic = {baseShape,
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Riser cylinder

Draw the riser as a cylinder

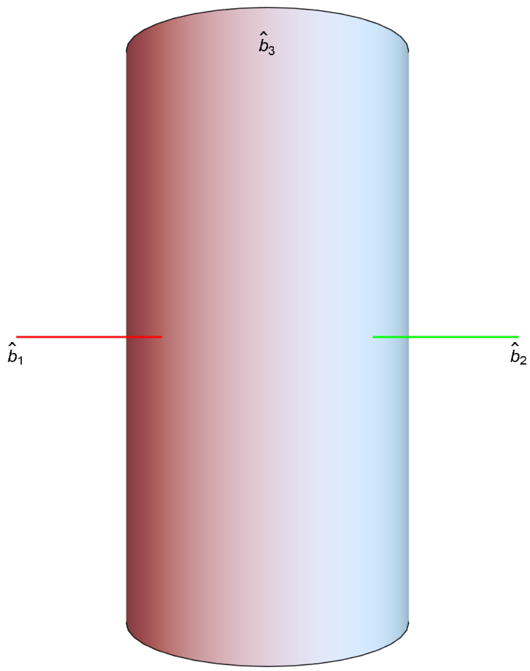
In[46]:=
```
vecL = 1;
```

In[47]:=
```
halfHeightRiser = 1;
riserBase = {0, 0, -halfHeightRiser};
riserTop = {0, 0, halfHeightRiser};
riserRadius = 1 / 2;
```

In[51]:=
```
riserGraphicF = {Cylinder[{riserBase, riserTop}, riserRadius],
        {AbsoluteThickness[1], {Text[b̂1, {vecL, 0, 0}, {0, 1}],
     Text[b̂2, {0, vecL, 0}, {0, 1}], Text[b̂3, {0, 0, vecL}, {0, -1}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
     {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
     {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}}}};
```

In[52]:=
```
Show[Graphics3D[riserGraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[52]=



In[53]:=
```
riserGraphic = {Cylinder[{riserBase, riserTop}, riserRadius],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Shoulder cylinder

Draw the shoulder

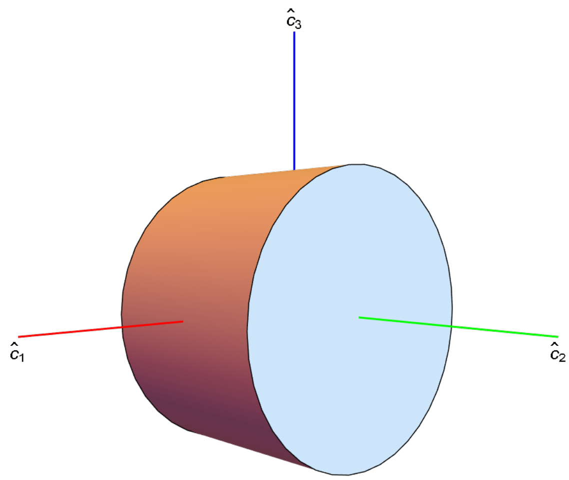
In[54]:=
```
vecL = 1;
```

In[55]:=
```
halfHeightShoulder = 1 / 6 + 1 / 6;
shoulderBase = {0, 0, -halfHeightShoulder};
shoulderTop = {0, 0, halfHeightShoulder};
shoulderRadius = 1 / 2;
```

In[59]:=
```
shoulderGraphicF =
  {Rotate[Cylinder[{shoulderBase, shoulderTop}, shoulderRadius], Pi / 2, {1, 0, 0}],
   Text[ĉ1, {vecL, 0, 0}, {0, 1}], Text[ĉ2, {0, vecL, 0}, {0, 1}],
   Text[ĉ3, {0, 0, vecL}, {0, -1}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

In[60]:=
```
Show[Graphics3D[shoulderGraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[60]=



In[61]:=
```
shoulderGraphic =
  {Rotate[Cylinder[{shoulderBase, shoulderTop}, shoulderRadius], Pi / 2, {1, 0, 0}],
      {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Arm segment 1

Draw the first arm from polygons

In[62]:=
```
vecL = 2;
```

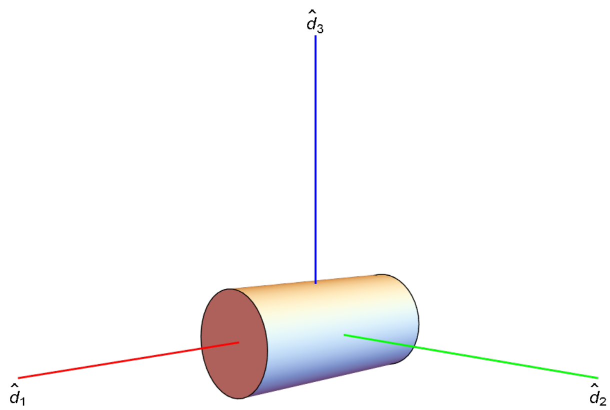
In[63]:=
```
lengthArm1 = 0.7;
arm1Radius = halfHeightShoulder;
depthArm1 = arm1Radius;
heightArm1 = arm1Radius;
```

In[67]:=
```
arm1Shape =
  Rotate[Cylinder[{{0, 0, lengthArm1}, {0, 0, -lengthArm1}}, arm1Radius], Pi / 2, {0, 1, 0}];
```

In[68]:=
```
arm1GraphicF = {arm1Shape,
   {Text[d̂₁, {vecL, 0, 0}, {0, 1}],
    Text[d̂₂, {0, vecL, 0}, {0, 1}], Text[d̂₃, {0, 0, vecL}, {0, -1}],
       {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}}};
```

In[69]:=
```
Show[Graphics3D[arm1GraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[69]=



In[70]:=
```
arm1Graphic = {arm1Shape,
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Arm segment 2

Draw the second arm from polygons

In[71]:=
```
vecL = 2;
```

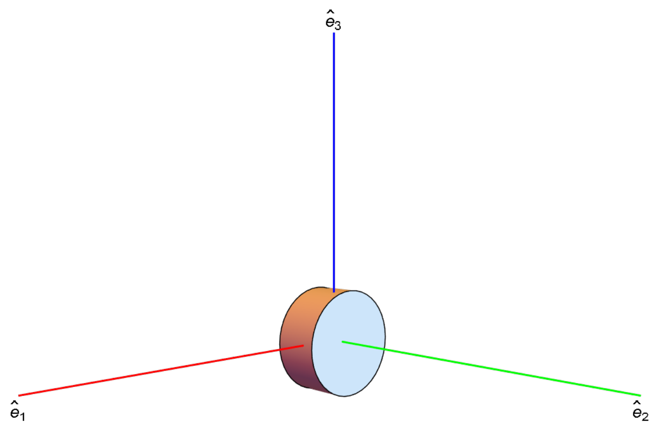
In[72]:=
```
lengthArm2 = 0.15;
arm2Radius = arm1Radius;
depthArm2 = arm2Radius;
heightArm2 = arm2Radius;
```

In[76]:=
```
arm2Shape =
  Rotate[Cylinder[{{0, 0, lengthArm2}, {0, 0, -lengthArm2}}, arm2Radius], Pi / 2, {1, 0, 0}];
```

In[77]:=
```
arm2GraphicF = {arm2Shape,
    {Text[ê1, {vecL, 0, 0}, {0, 1}],
     Text[ê2, {0, vecL, 0}, {0, 1}], Text[ê3, {0, 0, vecL}, {0, -1}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
     {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
     {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}}};
```

In[78]:=
```
Show[Graphics3D[arm2GraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[78]=



In[79]:=
```
arm2Graphic = {arm2Shape,
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Arm segment 3

Draw the third arm as a cylinder

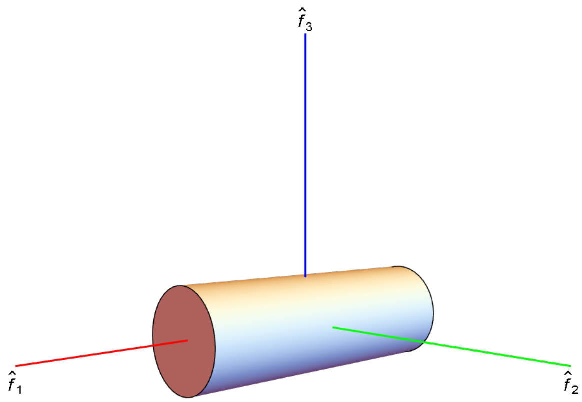
In[80]:=
```
vecL = 1;
```

In[81]:=
```
halfHeightArm3 = 1 / 2;
arm3Base = {0, 0, -halfHeightArm3};
arm3Top = {0, 0, halfHeightArm3};
arm3Radius = 1 / 6;
```

In[85]:=
```
arm3GraphicF = {Rotate[Cylinder[{arm3Base, arm3Top}, arm3Radius], Pi / 2, {0, 1, 0}],
   Text[f̂1, {vecL, 0, 0}, {0, 1}],
   Text[f̂2, {0, vecL, 0}, {0, 1}], Text[f̂3, {0, 0, vecL}, {0, -1}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

In[86]:=
```
Show[Graphics3D[arm3GraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[86]=



In[87]:=
```
arm3Graphic = {Rotate[Cylinder[{arm3Base, arm3Top}, arm3Radius], Pi / 2, {0, 1, 0}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Wrist1

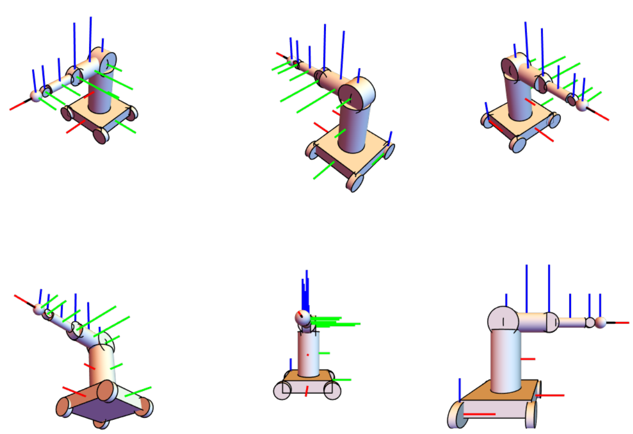
Wrist2 and pointer

Entire robot

In[103]:=
```
robotGraphic =
  {Translate[wheelsGraphic, {-1, -1, halfHeightWheel}],
   (*Base graphic*)
   Translate[baseGraphic, {0, 0, 1 / 2 heightBase}],

   (*Riser graphic*)
   Translate[riserGraphic, {0, 0, heightBase + halfHeightRiser}],

   (*Shoulder graphic*)
   Translate[shoulderGraphic,
    {0, 0, heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Arm1 graphic*)
   Translate[arm1Graphic,
    {lengthArm1, 0, heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Arm2 graphic*)
   Translate[arm2Graphic, {2 * lengthArm1 + 1 / 2 lengthArm2 ,
     0, heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Arm3 graphic*)
   Translate[arm3Graphic, {2 lengthArm1 + 2 * lengthArm2 + halfHeightArm3,
     0, heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Wrist1 graphic*)
   Translate[wrist1Graphic,
    {2 lengthArm1 + 2 * lengthArm2 + 2 halfHeightArm3 + wrist1Radius,
     0, heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Wrist2 graphic*)
   Translate[wrist2Graphic,
    { 2 lengthArm1 + 2 * lengthArm2 + 2 halfHeightArm3 + 2 * wrist1Radius + halfHeightWrist2,
     0, heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}]};
```

In[104]:=
```
Show[
 GraphicsGrid[{{Graphics3D[robotGraphic, ViewPoint → {1, 1, 1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphic, ViewPoint → {-1, 1, 1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphic, ViewPoint → {1, -1, 1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
   {Graphics3D[robotGraphic, ViewPoint → {1, 1, -1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphic, ViewPoint → {1, 0, 0}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphic, ViewPoint → {0, -1, 0}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

Out[104]=



Rotations

Lets assume the robot has a moving base, shoulder, and three link arm, with wrist1 and 2. It has a 3-2-2-1-2-1 rotation sequence. Starting from the Newtonian frame N we have a 0-rotation to A, then a 0-rotation to B, then a 3-rotation to C, then a 2-rotation to D, then a 2-rotation to E, then a 1-rotation to F, then a 2-rotation to G, then a 1-rotation to H and the tool pointer

```
In[105]:= rotW = rot3[q1[t]];
          WtoN = rotW.{n[1], n[2], n[3]}
          TranWtoN[x_] := x //. {w[1] → WtoN[[1]], w[2] → WtoN[[2]], w[3] → WtoN[[3]]}
```

Out[106]= $\left\{\mathrm{Cos}[q_1[t]]\,\hat{n}_1+\mathrm{Sin}[q_1[t]]\,\hat{n}_2,\ -\mathrm{Sin}[q_1[t]]\,\hat{n}_1+\mathrm{Cos}[q_1[t]]\,\hat{n}_2,\ \hat{n}_3\right\}$

```
In[108]:= rotA = rot0[].rotW;
          AtoN = rotA.{n[1], n[2], n[3]}
```

Out[109]= $\left\{\mathrm{Cos}[q_1[t]]\,\hat{n}_1+\mathrm{Sin}[q_1[t]]\,\hat{n}_2,\ -\mathrm{Sin}[q_1[t]]\,\hat{n}_1+\mathrm{Cos}[q_1[t]]\,\hat{n}_2,\ \hat{n}_3\right\}$

```
In[110]:= TranAtoN[x_] := x //. {a[1] → AtoN[[1]], a[2] → AtoN[[2]], a[3] → AtoN[[3]]}
```

```
In[111]:= rotB = rot0[].rotA;
          BtoN = rotB.{n[1], n[2], n[3]}
```

Out[112]= $\left\{\mathrm{Cos}[q_1[t]]\,\hat{n}_1+\mathrm{Sin}[q_1[t]]\,\hat{n}_2,\ -\mathrm{Sin}[q_1[t]]\,\hat{n}_1+\mathrm{Cos}[q_1[t]]\,\hat{n}_2,\ \hat{n}_3\right\}$

```
In[113]:= TranBtoN[x_] := x //. {b[1] → BtoN[[1]], b[2] → BtoN[[2]], b[3] → BtoN[[3]]}
```

```
In[114]:= rotC = rot3[q2[t]].rotB;
          CtoN = rotC.{n[1], n[2], n[3]}
```

Out[115]=
$$\begin{aligned}\{&(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]]-\mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\hat{n}_1+\\&(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]+\mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\hat{n}_2,\\&(-\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]-\mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\hat{n}_1+\\&(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]]-\mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\hat{n}_2,\ \hat{n}_3\}\end{aligned}$$

```
In[116]:= TranCtoN[x_] := x //. {c[1] → CtoN[[1]], c[2] → CtoN[[2]], c[3] → CtoN[[3]]}
```

```
In[117]:= rotD = rot2[q3[t]].rotC;
          DtoN = rotD.{n[1], n[2], n[3]}
```

Out[118]=
$$\begin{aligned}\{&\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]]-\mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\hat{n}_1+\\&\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]+\mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\hat{n}_2-\mathrm{Sin}[q_3[t]]\,\hat{n}_3,\\&(-\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]-\mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\hat{n}_1+\\&(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]]-\mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\hat{n}_2,\\&(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]]-\mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\hat{n}_1+\\&(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]+\mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\hat{n}_2+\mathrm{Cos}[q_3[t]]\,\hat{n}_3\}\end{aligned}$$

```
In[119]:= TranDtoN[x_] := x //. {d[1] → DtoN[[1]], d[2] → DtoN[[2]], d[3] → DtoN[[3]]}
```

```
In[120]:= rotE = rot2[q4[t]].rotD;
          EtoN = rotE.{n[1], n[2], n[3]}
```

$$\text{Out[121]=}\ \{(\text{Cos}[q_3[t]]\ \text{Cos}[q_4[t]]\ (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]]) - (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]]\ \text{Sin}[q_4[t]])\ \hat{n}_1 + (\text{Cos}[q_3[t]]\ \text{Cos}[q_4[t]]\ (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]]) - (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]]\ \text{Sin}[q_4[t]])\ \hat{n}_2 + (-\text{Cos}[q_4[t]]\ \text{Sin}[q_3[t]] - \text{Cos}[q_3[t]]\ \text{Sin}[q_4[t]])\ \hat{n}_3,$$
$$(-\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] - \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \hat{n}_1 + (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \hat{n}_2,$$
$$(\text{Cos}[q_4[t]]\ (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]] + \text{Cos}[q_3[t]]\ (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_4[t]])\ \hat{n}_1 + (\text{Cos}[q_4[t]]\ (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]] + \text{Cos}[q_3[t]]\ (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_4[t]])\ \hat{n}_2 + (\text{Cos}[q_3[t]]\ \text{Cos}[q_4[t]] - \text{Sin}[q_3[t]]\ \text{Sin}[q_4[t]])\ \hat{n}_3\}$$

```
In[122]:= TranEtoN[x_] := x //. {e[1] → EtoN[[1]], e[2] → EtoN[[2]], e[3] → EtoN[[3]]}
```

```
In[123]:= rotF = rot1[q5[t]].rotE;
          FtoN = rotF.{n[1], n[2], n[3]}
```

$$\text{Out[124]=}\ \{(\text{Cos}[q_3[t]]\ \text{Cos}[q_4[t]]\ (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]]) - (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]]\ \text{Sin}[q_4[t]])\ \hat{n}_1 + (\text{Cos}[q_3[t]]\ \text{Cos}[q_4[t]]\ (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]]) - (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]]\ \text{Sin}[q_4[t]])\ \hat{n}_2 + (-\text{Cos}[q_4[t]]\ \text{Sin}[q_3[t]] - \text{Cos}[q_3[t]]\ \text{Sin}[q_4[t]])\ \hat{n}_3,$$
$$(\text{Cos}[q_5[t]]\ (-\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] - \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]]) + (\text{Cos}[q_4[t]]\ (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]] + \text{Cos}[q_3[t]]\ (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_4[t]])\ \text{Sin}[q_5[t]])\ \hat{n}_1 + (\text{Cos}[q_5[t]]\ (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]]) + (\text{Cos}[q_4[t]]\ (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]] + \text{Cos}[q_3[t]]\ (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_4[t]])\ \text{Sin}[q_5[t]])\ \hat{n}_2 + (\text{Cos}[q_3[t]]\ \text{Cos}[q_4[t]] - \text{Sin}[q_3[t]]\ \text{Sin}[q_4[t]])\ \text{Sin}[q_5[t]]\ \hat{n}_3,$$
$$(\text{Cos}[q_5[t]]\ (\text{Cos}[q_4[t]]\ (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]] + \text{Cos}[q_3[t]]\ (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_4[t]]) - (-\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] - \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_5[t]])\ \hat{n}_1 + (\text{Cos}[q_5[t]]\ (\text{Cos}[q_4[t]]\ (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_3[t]] + \text{Cos}[q_3[t]]\ (\text{Cos}[q_2[t]]\ \text{Sin}[q_1[t]] + \text{Cos}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_4[t]]) - (\text{Cos}[q_1[t]]\ \text{Cos}[q_2[t]] - \text{Sin}[q_1[t]]\ \text{Sin}[q_2[t]])\ \text{Sin}[q_5[t]])\ \hat{n}_2 + \text{Cos}[q_5[t]]\ (\text{Cos}[q_3[t]]\ \text{Cos}[q_4[t]] - \text{Sin}[q_3[t]]\ \text{Sin}[q_4[t]])\ \hat{n}_3\}$$

```
In[125]:= TranFtoN[x_] := x //. {f[1] → FtoN[[1]], f[2] → FtoN[[2]], f[3] → FtoN[[3]]}
```

In[126]:=
```
rotG = rot2[q6[t]].rotF;
GtoN = rotG.{n[1], n[2], n[3]}
```

Out[127]=
```
{(Cos[q6[t]] (Cos[q3[t]] Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) -
        (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] Sin[q4[t]]) -
     (Cos[q5[t]] (Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] +
          Cos[q3[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q4[t]]) -
        (-Cos[q2[t]] Sin[q1[t]] - Cos[q1[t]] Sin[q2[t]]) Sin[q5[t]]) Sin[q6[t]]) n̂1 +
   (Cos[q6[t]] (Cos[q3[t]] Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) -
        (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] Sin[q4[t]]) -
     (Cos[q5[t]] (Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] +
          Cos[q3[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q4[t]]) -
        (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q5[t]]) Sin[q6[t]]) n̂2 +
   (Cos[q6[t]] (-Cos[q4[t]] Sin[q3[t]] - Cos[q3[t]] Sin[q4[t]]) -
     Cos[q5[t]] (Cos[q3[t]] Cos[q4[t]] - Sin[q3[t]] Sin[q4[t]]) Sin[q6[t]]) n̂3,
  (Cos[q5[t]] (-Cos[q2[t]] Sin[q1[t]] - Cos[q1[t]] Sin[q2[t]]) +
     (Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] +
        Cos[q3[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q4[t]]) Sin[q5[t]])
   n̂1 + (Cos[q5[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) +
     (Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] +
        Cos[q3[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q4[t]]) Sin[q5[t]])
   n̂2 + (Cos[q3[t]] Cos[q4[t]] - Sin[q3[t]] Sin[q4[t]]) Sin[q5[t]]
   n̂3,
  (Cos[q6[t]] (Cos[q5[t]] (Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[
                t]] + Cos[q3[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q4[t]]) -
        (-Cos[q2[t]] Sin[q1[t]] - Cos[q1[t]] Sin[q2[t]]) Sin[q5[t]]) +
     (Cos[q3[t]] Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) -
        (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] Sin[q4[t]]) Sin[q6[t]])
   n̂1 + (Cos[q6[t]] (Cos[q5[t]] (Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]])
              Sin[q3[t]] + Cos[q3[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]])
              Sin[q4[t]]) - (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q5[t]]) +
     (Cos[q3[t]] Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) -
        (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] Sin[q4[t]]) Sin[q6[t]])
   n̂2 + (Cos[q5[t]] Cos[q6[t]] (Cos[q3[t]] Cos[q4[t]] - Sin[q3[t]] Sin[q4[t]]) +
     (-Cos[q4[t]] Sin[q3[t]] - Cos[q3[t]] Sin[q4[t]]) Sin[q6[t]]) n̂3}
```

In[128]:=
```
TranGtoN[x_] := x //. {g[1] → GtoN[[1]], g[2] → GtoN[[2]], g[3] → GtoN[[3]]}
```

In[129]:=
```
rotH = rot1[q7[t]].rotG;
HtoN = rotH.{n[1], n[2], n[3]}
```

Out[130]=
```
{(Cos[q6[t]] (Cos[q3[t]] Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) -
        (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] Sin[q4[t]]) -
     (Cos[q5[t]] (Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] +
          Cos[q3[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q4[t]]) -
        (-Cos[q2[t]] Sin[q1[t]] - Cos[q1[t]] Sin[q2[t]]) Sin[q5[t]]) Sin[q6[t]]) n̂1 +
   (Cos[q6[t]] (Cos[q3[t]] Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) -
        (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] Sin[q4[t]]) -
     (Cos[q5[t]] (Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] +
          Cos[q3[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q4[t]]) -
        (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q5[t]]) Sin[q6[t]]) n̂2 +
   (Cos[q6[t]] (-Cos[q4[t]] Sin[q3[t]] - Cos[q3[t]] Sin[q4[t]]) -
```

```
     Cos[q5[t]] (Cos[q3[t]] Cos[q4[t]] - Sin[q3[t]] Sin[q4[t]]) Sin[q6[t]]) n̂3,
 (Cos[q7[t]] (Cos[q5[t]] (-Cos[q2[t]] Sin[q1[t]] - Cos[q1[t]] Sin[q2[t]]) +
       (Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] + Cos[q3[t]]
          (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q4[t]]) Sin[q5[t]]) +
    (Cos[q6[t]] (Cos[q5[t]] (Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]])
             Sin[q3[t]] + Cos[q3[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[
              q4[t]]) - (-Cos[q2[t]] Sin[q1[t]] - Cos[q1[t]] Sin[q2[t]]) Sin[q5[t]]) +
       (Cos[q3[t]] Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) -
          (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]])
           Sin[q3[t]] Sin[q4[t]]) Sin[q6[t]]) Sin[q7[t]]) n̂1 +
  (Cos[q7[t]] (Cos[q5[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) +
       (Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] + Cos[q3[t]]
          (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q4[t]]) Sin[q5[t]]) +
    (Cos[q6[t]] (Cos[q5[t]] (Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]])
             Sin[q3[t]] + Cos[q3[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[
              q4[t]]) - (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q5[t]]) +
       (Cos[q3[t]] Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) -
          (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]])
           Sin[q3[t]] Sin[q4[t]]) Sin[q6[t]]) Sin[q7[t]]) n̂2 +
  (Cos[q7[t]] (Cos[q3[t]] Cos[q4[t]] - Sin[q3[t]] Sin[q4[t]]) Sin[q5[t]] +
    (Cos[q5[t]] Cos[q6[t]] (Cos[q3[t]] Cos[q4[t]] - Sin[q3[t]] Sin[q4[t]]) +
       (-Cos[q4[t]] Sin[q3[t]] - Cos[q3[t]] Sin[q4[t]]) Sin[q6[t]]) Sin[q7[t]]) n̂3,
 (Cos[q7[t]] (Cos[q6[t]] (Cos[q5[t]] (Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] -
                Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] +
              Cos[q3[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q4[t]]) -
          (-Cos[q2[t]] Sin[q1[t]] - Cos[q1[t]] Sin[q2[t]]) Sin[q5[t]]) +
       (Cos[q3[t]] Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) -
          (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] Sin[q4[t]])
        Sin[q6[t]]) - (Cos[q5[t]] (-Cos[q2[t]] Sin[q1[t]] - Cos[q1[t]] Sin[q2[t]]) +
       (Cos[q4[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q3[t]] + Cos[q3[t]]
          (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q4[t]]) Sin[q5[t]])
     Sin[q7[t]]) n̂1 + (Cos[q7[t]] (Cos[q6[t]] (Cos[q5[t]]
          (Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] +
            Cos[q3[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q4[t]]) -
          (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) Sin[q5[t]]) +
       (Cos[q3[t]] Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) -
          (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] Sin[q4[t]])
        Sin[q6[t]]) - (Cos[q5[t]] (Cos[q1[t]] Cos[q2[t]] - Sin[q1[t]] Sin[q2[t]]) +
       (Cos[q4[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]]) Sin[q3[t]] +
         Cos[q3[t]] (Cos[q2[t]] Sin[q1[t]] + Cos[q1[t]] Sin[q2[t]])
          Sin[q4[t]]) Sin[q5[t]]) Sin[q7[t]]) n̂2 +
  (Cos[q7[t]] (Cos[q5[t]] Cos[q6[t]] (Cos[q3[t]] Cos[q4[t]] - Sin[q3[t]] Sin[q4[t]]) +
       (-Cos[q4[t]] Sin[q3[t]] - Cos[q3[t]] Sin[q4[t]]) Sin[q6[t]]) -
    (Cos[q3[t]] Cos[q4[t]] - Sin[q3[t]] Sin[q4[t]]) Sin[q5[t]] Sin[q7[t]]) n̂3}
```

```
In[131]:= TranHtoN[x_] := x //. {h[1] → HtoN[[1]], h[2] → HtoN[[2]], h[3] → HtoN[[3]]}
```

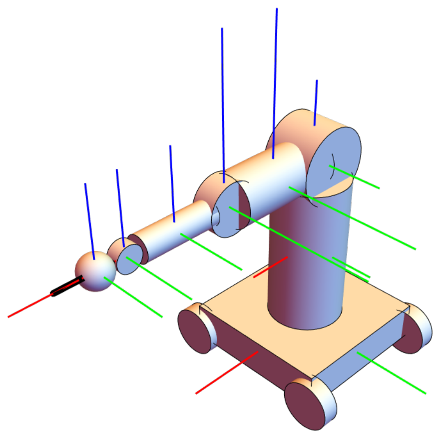
Relative position vectors

Now lets create vectors to the reference frames of each body relative to the previous body or frame.

See composite robot graphic

In[132]:=
```
Show[Graphics3D[robotGraphic, ViewPoint -> {1, 1, 1}, ViewVertical -> {0, 0, 1},
  ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]]
```

Out[132]=



In[133]:=
```
OrWo = x[t] × n[1] + y[t] × n[2] + halfHeightWheel n[3]
```

Out[133]= $\frac{\hat{n}_3}{9} + \hat{n}_1 x[t] + \hat{n}_2 y[t]$

In[134]:=
```
WorAo = w[1] + w[2] + (halfHeightWheel + 1 / 2 heightBase) w[3]
```

Out[134]= $\hat{w}_1 + \hat{w}_2 + \frac{13 \hat{w}_3}{36}$

Riser

In[135]:=
```
AorBo = (1 / 2 heightBase + halfHeightRiser) a[3]
```

Out[135]= $\frac{5 \hat{a}_3}{4}$

Shoulder

In[136]:=
```
BorCo = (halfHeightRiser + 1 / 2 shoulderRadius) b[3]
```

Out[136]= $\frac{5 \hat{b}_3}{4}$

Arm1

In[137]:=
```
CorDo = 1.5 lengthArm1 d[1]
```
Out[137]= $1.05\,\hat{d}_1$

Arm2

In[138]:=
```
DorEo = (lengthArm2 ) e[1] + lengthArm1 d[1]
```
Out[138]= $0.7\,\hat{d}_1 + 0.15\,\hat{e}_1$

Arm3

In[139]:=
```
EorFo = 2 * lengthArm2 e[1] + halfHeightArm3 f[1]
```
Out[139]= $0.3\,\hat{e}_1 + \frac{\hat{f}_1}{2}$

Wrist1

In[140]:=
```
ForGo = (halfHeightArm3 + wrist1Radius) f[1]
```
Out[140]= $\frac{2\,\hat{f}_1}{3}$

Wrist2

In[141]:=
```
GorHo = (halfHeightWrist2 ) g[1]
```
Out[141]= $\frac{\hat{g}_1}{5}$

Pointer

In[142]:=
```
HorP = ( halfHeightWrist2 + pointerLength) h[1]
```
Out[142]= $\frac{7\,\hat{h}_1}{10}$

Animation Example

Absolute position vectors and coordinates in Newtonian frame

Coordinates for Ao, base.

In[143]:=
```
xWo = OrWo . n[1] // TranWtoN
yWo = OrWo . n[2] // TranWtoN
zWo = OrWo . n[3] // TranWtoN
```
Out[143]= x[t]

Out[144]= y[t]

Out[145]= $\frac{1}{9}$

In[146]:=
```
xAo = (OrWo + WorAo ) . n[1] // TranWtoN // TranAtoN
yAo = (OrWo + WorAo) . n[2] // TranWtoN // TranAtoN
zAo = (OrWo + WorAo) . n[3] // TranWtoN // TranAtoN
```

Out[146]= $\mathrm{Cos}[q_1[t]] - \mathrm{Sin}[q_1[t]] + x[t]$

Out[147]= $\mathrm{Cos}[q_1[t]] + \mathrm{Sin}[q_1[t]] + y[t]$

Out[148]= $\frac{17}{36}$

Coordinates for Bo, riser.

In[149]:=
```
xBo = (OrWo + WorAo + AorBo) . n[1] // TranWtoN // TranAtoN // TranBtoN
yBo = (OrWo + WorAo + AorBo) . n[2] // TranWtoN // TranAtoN // TranBtoN
zBo = (OrWo + WorAo + AorBo) . n[3] // TranWtoN // TranAtoN // TranBtoN
```

Out[149]= $\mathrm{Cos}[q_1[t]] - \mathrm{Sin}[q_1[t]] + x[t]$

Out[150]= $\mathrm{Cos}[q_1[t]] + \mathrm{Sin}[q_1[t]] + y[t]$

Out[151]= $\frac{31}{18}$

Coordinates for Co, shoulder.

In[152]:=
```
xCo = (OrWo + WorAo + AorBo + BorCo) . n[1] // TranWtoN // TranAtoN // TranBtoN // TranCtoN
yCo = (OrWo + WorAo + AorBo + BorCo) . n[2] // TranWtoN // TranAtoN // TranBtoN // TranCtoN
zCo = (OrWo + WorAo + AorBo + BorCo) . n[3] // TranWtoN // TranAtoN // TranBtoN // TranCtoN
```

Out[152]= $\mathrm{Cos}[q_1[t]] - \mathrm{Sin}[q_1[t]] + x[t]$

Out[153]= $\mathrm{Cos}[q_1[t]] + \mathrm{Sin}[q_1[t]] + y[t]$

Out[154]= $\frac{107}{36}$

Coordinates for Do, arm1.

In[155]:=
```
xDo = (OrWo + WorAo + AorBo + BorCo + CorDo) . n[1] // TranWtoN // TranAtoN // TranBtoN //
   TranCtoN // TranDtoN
yDo = (OrWo + WorAo + AorBo + BorCo + CorDo) . n[2] // TranWtoN // TranAtoN // TranBtoN //
   TranCtoN // TranDtoN
zDo = (OrWo + WorAo + AorBo + BorCo + CorDo) . n[3] // TranWtoN // TranAtoN // TranBtoN //
   TranCtoN // TranDtoN
```

Out[155]= $\mathrm{Cos}[q_1[t]] - \mathrm{Sin}[q_1[t]] + 1.05\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + x[t]$

Out[156]= $\mathrm{Cos}[q_1[t]] + \mathrm{Sin}[q_1[t]] + 1.05\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + y[t]$

Out[157]= $\frac{107}{36} - 1.05\,\mathrm{Sin}[q_3[t]]$

Coordinates for Eo, arm2.

In[158]:=
```
xEo =
  (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo) . n[1] // TranWtoN // TranAtoN // TranBtoN //
    TranCtoN // TranDtoN // TranEtoN
yEo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo) . n[2] // TranWtoN // TranAtoN //
    TranBtoN // TranCtoN // TranDtoN // TranEtoN
zEo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo) . n[3] // TranWtoN // TranAtoN //
    TranBtoN // TranCtoN // TranDtoN // TranEtoN
```

Out[158]= $\mathrm{Cos}[q_1[t]] - \mathrm{Sin}[q_1[t]] + 1.75\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + 0.15\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) + x[t]$

Out[159]= $\mathrm{Cos}[q_1[t]] + \mathrm{Sin}[q_1[t]] + 1.75\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + 0.15\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) + y[t]$

Out[160]= $\frac{107}{36} - 1.75\,\mathrm{Sin}[q_3[t]] + 0.15\,(-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]] - \mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_4[t]])$

Coordinates for Fo, arm3.

In[161]:=
```
xFo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo) . n[1] // TranWtoN // TranAtoN //
      TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN
yFo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo) . n[2] // TranWtoN // TranAtoN //
      TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN
zFo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo) . n[3] // TranWtoN // TranAtoN //
      TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN
```

Out[161]= $\mathrm{Cos}[q_1[t]] - \mathrm{Sin}[q_1[t]] + 1.75\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + 0.95\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) + x[t]$

Out[162]= $\mathrm{Cos}[q_1[t]] + \mathrm{Sin}[q_1[t]] + 1.75\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + 0.95\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) + y[t]$

Out[163]= $\frac{107}{36} - 1.75\,\mathrm{Sin}[q_3[t]] + 0.95\,(-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]] - \mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_4[t]])$

Coordinates for Go, wrist1.

In[164]:=
```
xGo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo + ForGo) . n[1] // TranWtoN //
      TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN // TranGtoN
yGo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo + ForGo) . n[2] // TranWtoN //
      TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN // TranGtoN
zGo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo + ForGo) . n[3] // TranWtoN //
      TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN // TranGtoN
```

Out[164]= $\mathrm{Cos}[q_1[t]] - \mathrm{Sin}[q_1[t]] + 1.75\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + 1.61667\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) + x[t]$

Out[165]= $\mathrm{Cos}[q_1[t]] + \mathrm{Sin}[q_1[t]] + 1.75\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + 1.61667\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) + y[t]$

Out[166]= $\frac{107}{36} - 1.75\,\mathrm{Sin}[q_3[t]] + 1.61667\,(-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]] - \mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_4[t]])$

Coordinates for Ho, wrist2.

In[167]:=
```
xHo =
  (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo + ForGo + GorHo) . n[1] // TranWtoN //
        TranAtoN // TranBtoN // TranCtoN //
      TranDtoN // TranEtoN // TranFtoN // TranGtoN // TranHtoN
yHo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo + ForGo + GorHo) . n[2] //
        TranWtoN // TranAtoN // TranBtoN // TranCtoN //
      TranDtoN // TranEtoN // TranFtoN // TranGtoN // TranHtoN
zHo = (OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo + ForGo + GorHo) . n[3] //
        TranWtoN // TranAtoN // TranBtoN // TranCtoN //
      TranDtoN // TranEtoN // TranFtoN // TranGtoN // TranHtoN
```

Out[167]=
$$\mathrm{Cos}[q_1[t]] - \mathrm{Sin}[q_1[t]] + 1.75\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + 1.61667\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) + \frac{1}{5}(\mathrm{Cos}[q_6[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) - (\mathrm{Cos}[q_5[t]]\,(\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]] + \mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_4[t]]) - (-\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] - \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_5[t]])\,\mathrm{Sin}[q_6[t]]) + x[t]$$

Out[168]=
$$\mathrm{Cos}[q_1[t]] + \mathrm{Sin}[q_1[t]] + 1.75\,\mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) + 1.61667\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) + \frac{1}{5}(\mathrm{Cos}[q_6[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]) - (\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) - (\mathrm{Cos}[q_5[t]]\,(\mathrm{Cos}[q_4[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_3[t]] + \mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]] + \mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_4[t]]) - (\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]] - \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]])\,\mathrm{Sin}[q_5[t]])\,\mathrm{Sin}[q_6[t]]) + y[t]$$

Out[169]=
$$\frac{107}{36} - 1.75\,\mathrm{Sin}[q_3[t]] + 1.61667\,(-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]] - \mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) + \frac{1}{5}(\mathrm{Cos}[q_6[t]]\,(-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]] - \mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_4[t]]) - \mathrm{Cos}[q_5[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]] - \mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]])\,\mathrm{Sin}[q_6[t]])$$

Animation

Create functions for the coordinates for demonstration purposes.

In[170]:=
```
A = 3; B = 1; Cc = 0; ω = 2 Pi (.1);
```

In[171]:=
```
x[t_] := A Cos[ω t]
y[t_] := A Sin[ω t]
q1[t_] := B t + Cc
q2[t_] := B t + Cc
q3[t_] := B t + Cc
q4[t_] := B t + Cc
q5[t_] := B t + Cc
q6[t_] := B t + Cc
q7[t_] := B t + Cc
```

Create composite graphic out of parts that have been rotated and translated

In[180]:=
```
robotGraphicAnim = {
   Translate[GeometricTransformation[wheelsGraphic, Transpose[rotW]], {xWo, yWo, zWo}],
   (*Base graphic*)
   Translate[GeometricTransformation[baseGraphic, Transpose[rotA]], {xAo, yAo, zAo}],
   (*Riser graphic*)
   Translate[GeometricTransformation[riserGraphic, Transpose[rotB]], {xBo, yBo, zBo}],
   (*Shoulder graphic*)
   Translate[
    GeometricTransformation[shoulderGraphic, Transpose[rotC]], {xCo, yCo, zCo}],
   (*Arm1 graphic*)
   Translate[GeometricTransformation[arm1Graphic, Transpose[rotD]], {xDo, yDo, zDo}],
   (*Arm2 graphic*)
   Translate[GeometricTransformation[arm2Graphic, Transpose[rotE]], {xEo, yEo, zEo}],
   (*Arm3 graphic*)
   Translate[GeometricTransformation[arm3Graphic, Transpose[rotF]], {xFo, yFo, zFo}],
   (*Wrist1 graphic*)
   Translate[GeometricTransformation[wrist1Graphic, Transpose[rotG]], {xGo, yGo, zGo}],
   (*Wrist2 graphic*)
   Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]
  };
```

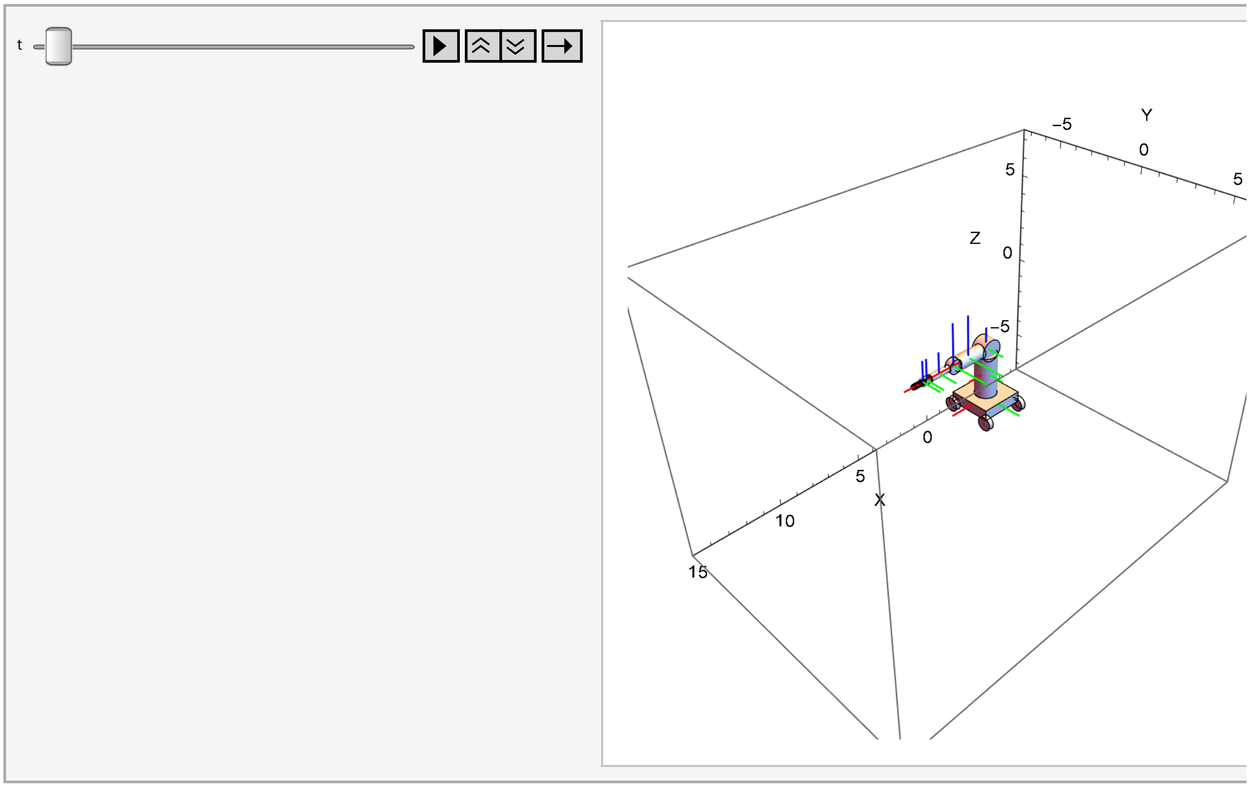
Make it a function of to so it can be looped over time

In[181]:=
```
robotGraphicAnimT[t_] = robotGraphicAnim;
```

In[182]:=
```
tf = 10;
scale = 2.5 A;
```

In[184]:=
```
Animate[Show[Graphics3D[robotGraphicAnimT[t], ViewPoint -> {1, 1, 1},
   ViewVertical -> {0, 0, 1}, ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> True, Axes -> True,
   PlotRange -> {{-scale, 2 scale}, {-scale, scale}, {-scale, scale}}, AspectRatio -> 1,
   AxesLabel → {"X", "Y", "Z"}]], {t, 0, tf, tf / 500}, AnimationRunning → False]
```

Out[184]=



Inverse Kinematics

We need to set up nonlinear equations to be solved to find angles and positions given desired pointer tip location and the tool frame orientation.

First clear all the variables of the kinematics. Sometimes this may cause an error if they have not been assigned numbers yet. Ignore the error and proceed.

In[185]:=
```
x[t_] =.
y[t_] =.
q1[t_] =.
q2[t_] =.
q3[t_] =.
q4[t_] =.
q5[t_] =.
q6[t_] =.
q7[t_] =.
```

Desired and actual tool orientation

Using the generic rotations from above we will assume an Euler: roll-pitch-yaw squence or an Euler 1-2-3 sequence to construct the desired tool orientation.

In[194]:=
```
Cdes[roll_, pitch_ , yaw_] := rot3[yaw].rot2[pitch].rot1[roll]
```

Here is an example

In[195]:=
```
MatrixForm[Cdes[Pi, Pi / 2, Pi / 3]]
```

Out[195]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 1 & 0 & 0 \end{pmatrix}$$

The actual rotation matrix in terms of our robot parameters is given as follows with the time dependence removed for simplicity

In[196]:=
```
Cact = rotH //. {qn_[t] → Qn};
```

Desired and actual tool tip position

The desired position is just a set of three numbers (X_{des},Y_{des},Z_{des}). The actual position vector out to the tool or pointer is given as follows with the time dependence removed

In[197]:=
```
OrP = OrWo + WorAo + AorBo + BorCo + CorDo + DorEo + EorFo + ForGo + GorHo + HorP //.
   {x[t] → Xbase, y[t] → Ybase}
```

Out[197]=

$$\frac{5\,\hat{b}_3}{4} + \frac{5\,\hat{a}_3}{4} + 1.75\,\hat{d}_1 + 0.45\,\hat{e}_1 + \frac{7\,\hat{f}_1}{6} + \frac{\hat{g}_1}{5} + \frac{7\,\hat{h}_1}{10} + X_{base}\,\hat{n}_1 + Y_{base}\,\hat{n}_2 + \frac{\hat{n}_3}{9} + \hat{w}_1 + \hat{w}_2 + \frac{13\,\hat{w}_3}{36}$$

The Newtonian X,Y,Z position of the point is given as follows with the time dependence removed

In[198]:=
```
Xact = (OrP.n[1] // TranWtoN // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN //
     TranFtoN // TranGtoN // TranHtoN) //. {qn_[t] → Qn}
```

Out[198]=

$$\mathrm{Cos}[Q_1] - \mathrm{Sin}[Q_1] + 1.75\,\mathrm{Cos}[Q_3]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]) + 1.61667\,(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]) - (\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]) + \frac{9}{10}\,(\mathrm{Cos}[Q_6]\,(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]) - (\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]) - (\mathrm{Cos}[Q_5]\,(\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3] + \mathrm{Cos}[Q_3]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_4]) - (-\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1] - \mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_5])\,\mathrm{Sin}[Q_6]) + X_{base}$$

In[199]:=
```
Yact = (OrP.n[2] // TranWtoN // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN //
     TranFtoN // TranGtoN // TranHtoN) //. {qn_[t] → Qn}
```

Out[199]=

$$\mathrm{Cos}[Q_1] + \mathrm{Sin}[Q_1] + 1.75\,\mathrm{Cos}[Q_3]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2]) + 1.61667\,(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2]) - (\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]) + \frac{9}{10}\,(\mathrm{Cos}[Q_6]\,(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2]) - (\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]) - (\mathrm{Cos}[Q_5]\,(\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3] + \mathrm{Cos}[Q_3]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_4]) - (\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_5])\,\mathrm{Sin}[Q_6]) + Y_{base}$$

In[200]:=
```
Zact = (OrP.n[3] // TranWtoN // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN //
     TranFtoN // TranGtoN // TranHtoN) //. {qn_[t] → Qn}
```

Out[200]=

$$\frac{107}{36} - 1.75\,\mathrm{Sin}[Q_3] + 1.61667\,(-\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_3] - \mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_4]) + \frac{9}{10}\,(\mathrm{Cos}[Q_6]\,(-\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_3] - \mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_4]) - \mathrm{Cos}[Q_5]\,(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4] - \mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4])\,\mathrm{Sin}[Q_6])$$

Create the equations for the actual angles and positions

This robot has 8 degrees of freedom, so we need at least 8 equations.

First enter desired values.

In[201]:=
```
Xdes := 3;
Ydes := 3;
Zdes := 5;
θr := Pi / 6;
θp := Pi / 3;
θy := Pi / 3;
```

Lets see if we can reach this point. Since the base is mobile we need only check the Z direction when the arm is straight up

In[207]:=
```
Zact //. {Q1 → 0, Q2 → 0, Q3 → -Pi / 2, Q4 → 0, Q5 → 0, Q6 → 0, Q7 → 0}
```
Out[207]= 7.23889

In[208]:=
```
Zdes <= Zact //. {Q1 → 0, Q2 → 0, Q3 → -Pi / 2, Q4 → 0, Q5 → 0, Q6 → 0, Q7 → 0}
```
Out[208]= True

Here is the current desired tool orientation

In[209]:=
```
MatrixForm[Cdes[θr, θp, θy]]
```
Out[209]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4} & \frac{3}{4}+\frac{\sqrt{3}}{8} & -\frac{3}{8}+\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & -\frac{3}{8}+\frac{\sqrt{3}}{4} & \frac{1}{4}+\frac{3\sqrt{3}}{8} \\ \frac{\sqrt{3}}{2} & -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix}$$

Using the three positions first, we have

In[210]:=
```
eq1 = X_act - X_des // distributeScalars
eq2 = Y_act - Y_des // distributeScalars
eq3 = Z_act - Z_des // distributeScalars
```

Out[210]=

$$-3 + \text{Cos}[Q_1] + 1.75\,\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Cos}[Q_3] + 1.61667\,\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4] + \frac{9}{10}\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6] - \text{Sin}[Q_1] - 1.75\,\text{Cos}[Q_3]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2] - 1.61667\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2] - \frac{9}{10}\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2] - 1.61667\,\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4] - \frac{9}{10}\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Cos}[Q_6]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4] + 1.61667\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4] + \frac{9}{10}\text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4] - \frac{9}{10}\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,(\text{Cos}[Q_1]\,\text{Cos}[Q_2] - \text{Sin}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_3]\,\text{Sin}[Q_6] - \frac{9}{10}\text{Cos}[Q_3]\,\text{Cos}[Q_5]\,(\text{Cos}[Q_1]\,\text{Cos}[Q_2] - \text{Sin}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_4]\,\text{Sin}[Q_6] - \frac{9}{10}\text{Cos}[Q_2]\,\text{Sin}[Q_1]\,\text{Sin}[Q_5]\,\text{Sin}[Q_6] - \frac{9}{10}\text{Cos}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_5]\,\text{Sin}[Q_6] + X_{base}$$

Out[211]=

$$-3 + \text{Cos}[Q_1] + \text{Sin}[Q_1] + 1.75\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Sin}[Q_1] + 1.61667\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_1] + \frac{9}{10}\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1] + 1.75\,\text{Cos}[Q_1]\,\text{Cos}[Q_3]\,\text{Sin}[Q_2] + 1.61667\,\text{Cos}[Q_1]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_2] + \frac{9}{10}\text{Cos}[Q_1]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2] - 1.61667\,\text{Cos}[Q_2]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4] - \frac{9}{10}\text{Cos}[Q_2]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4] - 1.61667\,\text{Cos}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4] - \frac{9}{10}\text{Cos}[Q_1]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4] - \frac{9}{10}\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,(\text{Cos}[Q_2]\,\text{Sin}[Q_1] + \text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_3]\,\text{Sin}[Q_6] - \frac{9}{10}\text{Cos}[Q_3]\,\text{Cos}[Q_5]\,(\text{Cos}[Q_2]\,\text{Sin}[Q_1] + \text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_4]\,\text{Sin}[Q_6] + \frac{9}{10}\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Sin}[Q_5]\,\text{Sin}[Q_6] - \frac{9}{10}\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_5]\,\text{Sin}[Q_6] + Y_{base}$$

Out[212]=

$$-\frac{73}{36} - 1.75\,\text{Sin}[Q_3] - 1.61667\,\text{Cos}[Q_4]\,\text{Sin}[Q_3] - \frac{9}{10}\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_3] - 1.61667\,\text{Cos}[Q_3]\,\text{Sin}[Q_4] - \frac{9}{10}\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_4] - \frac{9}{10}\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Sin}[Q_6] + \frac{9}{10}\text{Cos}[Q_5]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]$$

where the equations will be set to zero below. The remaining equations will be selected from the C matrices being equated element by element (set to zero below)

In[213]:=
```
eq4 = C_act[[1]][[1]] - C_des[θ_r, θ_p, θ_y][[1]][[1]];
eq5 = C_act[[1]][[2]] - C_des[θ_r, θ_p, θ_y][[1]][[2]];
eq6 = C_act[[1]][[3]] - C_des[θ_r, θ_p, θ_y][[1]][[3]];

eq7 = C_act[[2]][[1]] - C_des[θ_r, θ_p, θ_y][[2]][[1]];
eq8 = C_act[[2]][[2]] - C_des[θ_r, θ_p, θ_y][[2]][[2]];
eq9 = C_act[[2]][[3]] - C_des[θ_r, θ_p, θ_y][[2]][[3]];

eq10 = C_act[[3]][[1]] - C_des[θ_r, θ_p, θ_y][[3]][[1]];
eq11 = C_act[[3]][[2]] - C_des[θ_r, θ_p, θ_y][[3]][[2]];
eq12 = C_act[[3]][[3]] - C_des[θ_r, θ_p, θ_y][[3]][[3]];
```

Solve the equations for best first guess at angles and base position

The strategy to solve the inverse kinematics depends on the design of the robot. There are several closed form solutions for industrial robots, see Ch4 of the class textbook by Craig.

First try to lock in initial estimates of the base location and angles ,

X_{base},Y_{base}, Q_1, Q_2, Q_3, Q_4, Q_5, and Q_6. Since the base can move this has many possible solutions.

In[221]:=
```
eq1temp = eq1
```

Out[221]=

$$-3+\mathrm{Cos}[Q_1]+1.75\,\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]+1.61667\,\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]+\frac{9}{10}\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_6]-\mathrm{Sin}[Q_1]-1.75\,\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]-1.61667\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]-\frac{9}{10}\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]-1.61667\,\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]-\frac{9}{10}\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]+1.61667\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]+\frac{9}{10}\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]-\frac{9}{10}\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_5]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_6]-\frac{9}{10}\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_6]-\frac{9}{10}\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6]-\frac{9}{10}\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6]+X_{base}$$

In[222]:= **eq2temp = eq2**

Out[222]=
$$-3+\text{Cos}[Q_1]+\text{Sin}[Q_1]+1.75\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Sin}[Q_1]+1.61667\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_1]+\frac{9}{10}\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]+1.75\,\text{Cos}[Q_1]\,\text{Cos}[Q_3]\,\text{Sin}[Q_2]+1.61667\,\text{Cos}[Q_1]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_2]+\frac{9}{10}\text{Cos}[Q_1]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]-1.61667\,\text{Cos}[Q_2]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]-\frac{9}{10}\text{Cos}[Q_2]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]-1.61667\,\text{Cos}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]-\frac{9}{10}\text{Cos}[Q_1]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]-\frac{9}{10}\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,(\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_3]\,\text{Sin}[Q_6]-\frac{9}{10}\text{Cos}[Q_3]\,\text{Cos}[Q_5]\,(\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]+\frac{9}{10}\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Sin}[Q_5]\,\text{Sin}[Q_6]-\frac{9}{10}\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_5]\,\text{Sin}[Q_6]+Y_{base}$$

In[223]:= **eq3temp = eq3**

Out[223]=
$$-\frac{73}{36}-1.75\,\text{Sin}[Q_3]-1.61667\,\text{Cos}[Q_4]\,\text{Sin}[Q_3]-\frac{9}{10}\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_3]-1.61667\,\text{Cos}[Q_3]\,\text{Sin}[Q_4]-\frac{9}{10}\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_4]-\frac{9}{10}\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Sin}[Q_6]+\frac{9}{10}\text{Cos}[Q_5]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]$$

We need something to drive the base to a position that will not have the robot all tied up on itself or outstretched to far. So we will try to align the tool axes with the desired axes is some optimal sense. Here we want the dot products to be 1 for the main components.

In[224]:= **desTool1 = $C_{des}[\theta_r, \theta_p, \theta_y]$[[1]][[1]] n[1] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[1]][[2]] n[2] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[1]][[3]] n[3]**

Out[224]=
$$\frac{\hat{n}_1}{4}+\left(\frac{3}{4}+\frac{\sqrt{3}}{8}\right)\hat{n}_2+\left(-\frac{3}{8}+\frac{\sqrt{3}}{4}\right)\hat{n}_3$$

In[225]:=
```
eqV1temp = 1 == (h[1].desTool1 // TranHtoN) //. {q_n_[t] → Q_n}
```

Out[225]=
$$1 == \left(-\frac{3}{8}+\frac{\sqrt{3}}{4}\right) (\mathrm{Cos}[Q_6] (-\mathrm{Cos}[Q_4] \mathrm{Sin}[Q_3] - \mathrm{Cos}[Q_3] \mathrm{Sin}[Q_4]) - \mathrm{Cos}[Q_5] (\mathrm{Cos}[Q_3] \mathrm{Cos}[Q_4] - \mathrm{Sin}[Q_3] \mathrm{Sin}[Q_4]) \mathrm{Sin}[Q_6]) + \frac{1}{4} (\mathrm{Cos}[Q_6] (\mathrm{Cos}[Q_3] \mathrm{Cos}[Q_4] (\mathrm{Cos}[Q_1] \mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1] \mathrm{Sin}[Q_2]) - (\mathrm{Cos}[Q_1] \mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1] \mathrm{Sin}[Q_2]) \mathrm{Sin}[Q_3] \mathrm{Sin}[Q_4]) - (\mathrm{Cos}[Q_5] (\mathrm{Cos}[Q_4] (\mathrm{Cos}[Q_1] \mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1] \mathrm{Sin}[Q_2]) \mathrm{Sin}[Q_3] + \mathrm{Cos}[Q_3] (\mathrm{Cos}[Q_1] \mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1] \mathrm{Sin}[Q_2]) \mathrm{Sin}[Q_4]) - (-\mathrm{Cos}[Q_2] \mathrm{Sin}[Q_1] - \mathrm{Cos}[Q_1] \mathrm{Sin}[Q_2]) \mathrm{Sin}[Q_5]) \mathrm{Sin}[Q_6]) + \left(\frac{3}{4}+\frac{\sqrt{3}}{8}\right) (\mathrm{Cos}[Q_6] (\mathrm{Cos}[Q_3] \mathrm{Cos}[Q_4] (\mathrm{Cos}[Q_2] \mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1] \mathrm{Sin}[Q_2]) - (\mathrm{Cos}[Q_2] \mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1] \mathrm{Sin}[Q_2]) \mathrm{Sin}[Q_3] \mathrm{Sin}[Q_4]) - (\mathrm{Cos}[Q_5] (\mathrm{Cos}[Q_4] (\mathrm{Cos}[Q_2] \mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1] \mathrm{Sin}[Q_2]) \mathrm{Sin}[Q_3] + \mathrm{Cos}[Q_3] (\mathrm{Cos}[Q_2] \mathrm{Sin}[Q_1] + \mathrm{Cos}[Q_1] \mathrm{Sin}[Q_2]) \mathrm{Sin}[Q_4]) - (\mathrm{Cos}[Q_1] \mathrm{Cos}[Q_2] - \mathrm{Sin}[Q_1] \mathrm{Sin}[Q_2]) \mathrm{Sin}[Q_5]) \mathrm{Sin}[Q_6])$$

In[226]:=
```
desTool2 = C_des[θ_r, θ_p, θ_y][[2]][[1]] n[1] +
  C_des[θ_r, θ_p, θ_y][[2]][[2]] n[2] + C_des[θ_r, θ_p, θ_y][[2]][[3]] n[3]
```

Out[226]=
$$-\frac{1}{4}\sqrt{3}\,\hat{n}_1 + \left(-\frac{3}{8}+\frac{\sqrt{3}}{4}\right)\hat{n}_2 + \left(\frac{1}{4}+\frac{3\sqrt{3}}{8}\right)\hat{n}_3$$

In[227]:= **eqV2temp = 1 == (h[2].desTool2 // TranHtoN) //. {q_n_[t] → Q_n}**

Out[227]=
$$1 == \left(\frac{1}{4}+\frac{3\sqrt{3}}{8}\right)(\text{Cos}[Q_7](\text{Cos}[Q_3]\,\text{Cos}[Q_4]-\text{Sin}[Q_3]\,\text{Sin}[Q_4])\,\text{Sin}[Q_5]+(\text{Cos}[Q_5]\,\text{Cos}[Q_6](\text{Cos}[Q_3]\,\text{Cos}[Q_4]-\text{Sin}[Q_3]\,\text{Sin}[Q_4])+(-\text{Cos}[Q_4]\,\text{Sin}[Q_3]-\text{Cos}[Q_3]\,\text{Sin}[Q_4])\,\text{Sin}[Q_6])\,\text{Sin}[Q_7])+\left(-\frac{3}{8}+\frac{\sqrt{3}}{4}\right)(\text{Cos}[Q_7](\text{Cos}[Q_5](\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\text{Sin}[Q_1]\,\text{Sin}[Q_2])+(\text{Cos}[Q_4](\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_3]+\text{Cos}[Q_3](\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_4])\,\text{Sin}[Q_5])+(\text{Cos}[Q_6](\text{Cos}[Q_5](\text{Cos}[Q_4](\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_3]+\text{Cos}[Q_3](\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_4])-(\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\text{Sin}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_5])+(\text{Cos}[Q_3]\,\text{Cos}[Q_4](\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\text{Cos}[Q_1]\,\text{Sin}[Q_2])-(\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_3]\,\text{Sin}[Q_4])\,\text{Sin}[Q_6])\,\text{Sin}[Q_7])-\frac{1}{4}\sqrt{3}\,(\text{Cos}[Q_7](\text{Cos}[Q_5](-\text{Cos}[Q_2]\,\text{Sin}[Q_1]-\text{Cos}[Q_1]\,\text{Sin}[Q_2])+(\text{Cos}[Q_4](\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\text{Sin}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_3]+\text{Cos}[Q_3](\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\text{Sin}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_4])\,\text{Sin}[Q_5])+(\text{Cos}[Q_6](\text{Cos}[Q_5](\text{Cos}[Q_4](\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\text{Sin}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_3]+\text{Cos}[Q_3](\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\text{Sin}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_4])-(-\text{Cos}[Q_2]\,\text{Sin}[Q_1]-\text{Cos}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_5])+(\text{Cos}[Q_3]\,\text{Cos}[Q_4](\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\text{Sin}[Q_1]\,\text{Sin}[Q_2])-(\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\text{Sin}[Q_1]\,\text{Sin}[Q_2])\,\text{Sin}[Q_3]\,\text{Sin}[Q_4])\,\text{Sin}[Q_6])\,\text{Sin}[Q_7])$$

In[228]:= **desTool3 = $C_{des}[\theta_r, \theta_p, \theta_y]$[[3]][[1]] n[1] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[3]][[2]] n[2] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[3]][[3]] n[3]**

Out[228]=
$$\frac{1}{2}\sqrt{3}\,\hat{n}_1-\frac{\hat{n}_2}{4}+\frac{1}{4}\sqrt{3}\,\hat{n}_3$$

```
In[229]:= eqV3temp = 1 == (h[3].desTool3 // TranHtoN) //. {q_n_[t] → Q_n}
```

$$1 == \frac{1}{4}\sqrt{3}\,(\mathrm{Cos}[Q_7]\,(\mathrm{Cos}[Q_5]\,\mathrm{Cos}[Q_6]\,(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]-\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4])+(-\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_3]-\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_4])\,\mathrm{Sin}[Q_6])-(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]-\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4])\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_7])+$$
$$\frac{1}{4}(-\mathrm{Cos}[Q_7]\,(\mathrm{Cos}[Q_6]\,(\mathrm{Cos}[Q_5]\,(\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]+\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]+\mathrm{Cos}[Q_3]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]+\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_4])-(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_5])+(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]+\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])-(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]+\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4])\,\mathrm{Sin}[Q_6])+(\mathrm{Cos}[Q_5]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])+(\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]+\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]+\mathrm{Cos}[Q_3]\,(\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]+\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_4])\,\mathrm{Sin}[Q_5])\,\mathrm{Sin}[Q_7])+$$
$$\frac{1}{2}\sqrt{3}\,(\mathrm{Cos}[Q_7]\,(\mathrm{Cos}[Q_6]\,(\mathrm{Cos}[Q_5]\,(\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]+\mathrm{Cos}[Q_3]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_4])-(-\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]-\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_5])+(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])-(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4])\,\mathrm{Sin}[Q_6])-(\mathrm{Cos}[Q_5]\,(-\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]-\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2])+(\mathrm{Cos}[Q_4]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_3]+\mathrm{Cos}[Q_3]\,(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2])\,\mathrm{Sin}[Q_4])\,\mathrm{Sin}[Q_5])\,\mathrm{Sin}[Q_7])$$

The robot reach is defined on a sphere about the base coordinate origin. Need to get the base within reach. The minimum base position with respect to reach sphere is found subject to the constraints that the tool axes should be closely aligned with the desired axes and that the joint angles have physical limits due to collisions with other parts, etc.

First we calculate the radius of the sphere of reach based on just the straight reach of the robot. The scaled pointerLength is subtracted to get closer if needed.

```
In[230]:= scalePointer = 1 ;

In[231]:= radius = (OrP.n[1] // TranWtoN // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN //
               TranFtoN // TranGtoN // TranHtoN) //. {q_n_[t] → 0, X_base → 0, Y_base → 0}

Out[231]= 5.26667

In[232]:= eqReach = (eq1temp)^2 + (eq2temp)^2 + (eq3temp)^2 - (radius - scalePointer pointerLength)^2;

In[233]:= minQXY = Minimize[{eqReach, eqV1temp, eqV2temp, eqV3temp, - 2 Pi ≤ Q_1 ≤ 2 Pi,
             - .9 Pi ≤ Q_2 ≤ .9 Pi, - Pi / 2 ≤ Q_3 ≤ Pi / 6, - Pi / 2 ≤ Q_4 ≤ Pi / 6, -.9 Pi ≤ Q_5 ≤ .9 Pi,
             - Pi / 3 ≤ Q_6 ≤ Pi / 3, -2 Pi ≤ Q_7 ≤ 2 Pi}, {Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, X_base, Y_base}]

Out[233]= {-22.7211, {Q_1 → 0.945725, Q_2 → -0.556028, Q_3 → -0.604811, Q_4 → -0.0467296,
             Q_5 → -1.94838, Q_6 → -1.03403, Q_7 → 3.4308, X_base → 0.479696, Y_base → -0.301112}}
```

Here are the initial guess at the base coordinates and angles

In[234]:=
```
solX = minQXY[[2]][[8]]
```
Out[234]= $X_{base} \to 0.479696$

In[235]:=
```
solY = minQXY[[2]][[9]]
```
Out[235]= $Y_{base} \to -0.301112$

In[236]:=
```
solQ1 = minQXY[[2]][[1]]
```
Out[236]= $Q_1 \to 0.945725$

In[237]:=
```
solQ2 = minQXY[[2]][[2]]
```
Out[237]= $Q_2 \to -0.556028$

In[238]:=
```
solQ3 = minQXY[[2]][[3]]
```
Out[238]= $Q_3 \to -0.604811$

In[239]:=
```
solQ4 = minQXY[[2]][[4]]
```
Out[239]= $Q_4 \to -0.0467296$

In[240]:=
```
solQ5 = minQXY[[2]][[5]]
```
Out[240]= $Q_5 \to -1.94838$

In[241]:=
```
solQ6 = minQXY[[2]][[6]]
```
Out[241]= $Q_6 \to -1.03403$

In[242]:=
```
solQ7 = minQXY[[2]][[7]]
```
Out[242]= $Q_7 \to 3.4308$

Solve the full equations for angles with base positions known

Initial guesses at solution and root finder algorithm to refine the initial solutions. The optimal search above is too slow for real-time operations, but if we have good initial guesses, they can be refined with this operation and then this result can be used to start the next solution if the next desired location is near this one.

In[243]:=

```
q1o = Q1 /. solQ1;
q2o = Q2 /. solQ2;
q3o = Q3 /. solQ3;
q4o = Q4 /. solQ4;
q5o = Q5 /. solQ5;
q6o = Q6 /. solQ6;
q7o = Q7 /. solQ7;
invKinSol = FindRoot[{(eq1 /. solX) == 0, (eq2 /. solY) == 0, (eq3) == 0, (eq4) == 0,
    (eq6) == 0, (eq8) == 0, (eq12) == 0}, {Q1, q1o, -2 Pi, 2 Pi}, {Q2, q2o, -.9 Pi, .9 Pi},
   {Q3, q3o, - Pi / 2, Pi / 6}, {Q4, q4o, - Pi / 2, Pi / 6}, {Q5, q5o, -.9 Pi, .9 Pi},
   {Q6, q6o, - Pi / 3, Pi / 3}, {Q7, q7o, - 2 Pi, 2 Pi}, MaxIterations → 10000]
```

FindRoot: Encountered a singular Jacobian at the point $\{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7\}$ =
{0.945725, –0.556028, –0.604811, –0.0467296, –1.94838, –1.03403, 3.4308}. Try perturbing the initial point(s).

Out[250]= $\{Q_1 \to 0.945725, Q_2 \to -0.556028, Q_3 \to -0.604811, Q_4 \to -0.0467296, Q_5 \to -1.94838, Q_6 \to -1.03403, Q_7 \to 3.4308\}$

Compare desired to actual orientations

In[251]:=

```
Cdes[θr, θp, θy] // MatrixForm
Cact //. invKinSol // Chop // MatrixForm
```

Out[251]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4} & \frac{3}{4}+\frac{\sqrt{3}}{8} & -\frac{3}{8}+\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & -\frac{3}{8}+\frac{\sqrt{3}}{4} & \frac{1}{4}+\frac{3\sqrt{3}}{8} \\ \frac{\sqrt{3}}{2} & -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix}$$

Out[252]//MatrixForm=

$$\begin{pmatrix} 0.250359 & 0.966404 & 0.0581661 \\ -0.432412 & 0.0578633 & 0.899818 \\ 0.866222 & -0.250429 & 0.432371 \end{pmatrix}$$

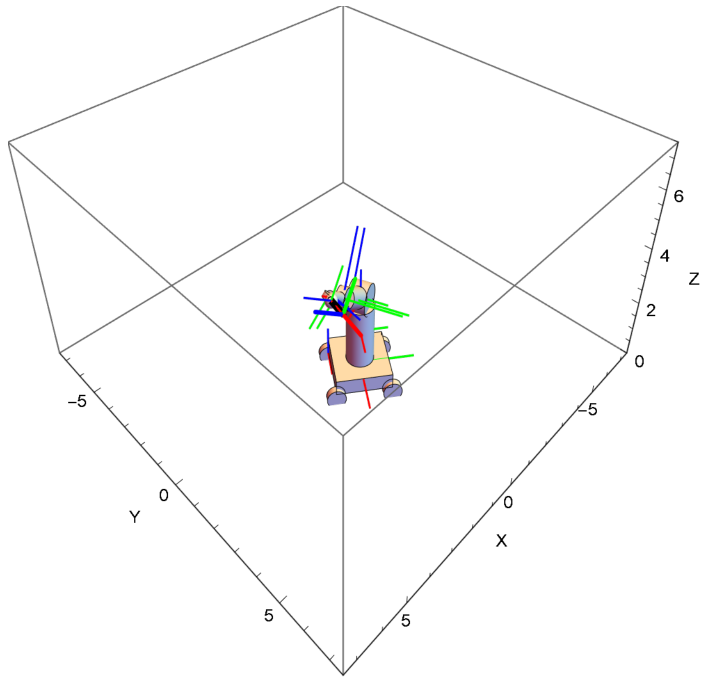
See if these solutions work. Create composite graphic to check the inverse solution feasibility

In[253]:=

```
x[t_] = Xbase /. solX;
y[t_] = Ybase /. solY;
q1[t_] = Q1 /. invKinSol;
q2[t_] = Q2 /. invKinSol;
q3[t_] = Q3 /. invKinSol;
q4[t_] = Q4 /. invKinSol;
q5[t_] = Q5 /. invKinSol;
q6[t_] = Q6 /. invKinSol;
q7[t_] = Q7 /. invKinSol;
```

```
In[262]:= robotGraphicInvKin = {
    (*desired point*)
    {PointSize[.01], Point[{Xdes, Ydes, Zdes}]},

    (*desired tool orientation*)
    Translate[GeometricTransformation[
      {{AbsoluteThickness[2], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
       {AbsoluteThickness[2], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
       {AbsoluteThickness[2], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}},
      Transpose[Cdes[θr, θp, θy]]], {Xdes, Ydes, Zdes}],
    Translate[GeometricTransformation[wheelsGraphic, Transpose[rotW]], {xWo, yWo, zWo}],
    (*Base graphic*)
    Translate[GeometricTransformation[baseGraphic, Transpose[rotA]], {xAo, yAo, zAo}],

    (*Riser graphic*)
    Translate[GeometricTransformation[riserGraphic, Transpose[rotB]], {xBo, yBo, zBo}],

    (*Shoulder graphic*)
    Translate[
     GeometricTransformation[shoulderGraphic, Transpose[rotC]], {xCo, yCo, zCo}],

    (*Arm1 graphic*)
    Translate[GeometricTransformation[arm1Graphic, Transpose[rotD]], {xDo, yDo, zDo}],

    (*Arm2 graphic*)
    Translate[GeometricTransformation[arm2Graphic, Transpose[rotE]], {xEo, yEo, zEo}],

    (*Arm3 graphic*)
    Translate[GeometricTransformation[arm3Graphic, Transpose[rotF]], {xFo, yFo, zFo}],

    (*Wrist1 graphic*)
    Translate[GeometricTransformation[wrist1Graphic, Transpose[rotG]], {xGo, yGo, zGo}],

    (*Wrist2 graphic*)
    Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]
   };
Show[Graphics3D[robotGraphicInvKin, ViewPoint -> {1, 1, 1},
  ViewVertical -> {0, 0, 1}, ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed → True,
  Axes -> True, PlotRange -> {{-scale, scale}, {-scale, scale}, {0, scale}},
  AspectRatio -> 1, AxesLabel → {"X", "Y", "Z"}]]
```

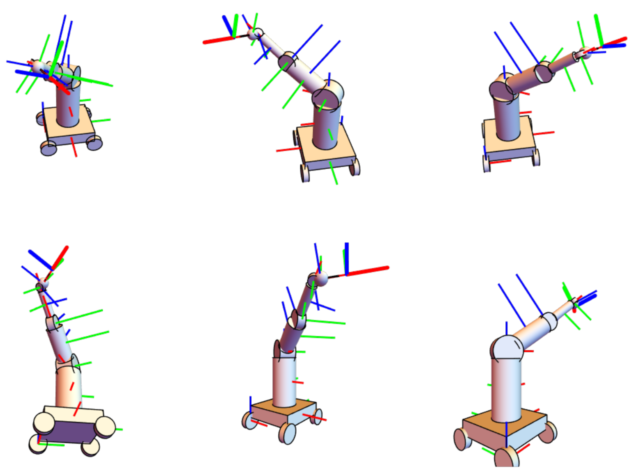
Out[263]=



Various views. Do they look doable, no collisions between parts, etc.

In[264]:=
```
Show[GraphicsGrid[
  {{Graphics3D[robotGraphicInvKin, ViewPoint → {1, 1, 1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicInvKin, ViewPoint → {-1, 1, 1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicInvKin, ViewPoint → {1, -1, 1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
   {Graphics3D[robotGraphicInvKin, ViewPoint → {1, 1, -1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicInvKin, ViewPoint → {1, 0, 0}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicInvKin, ViewPoint → {0, -1, 0}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

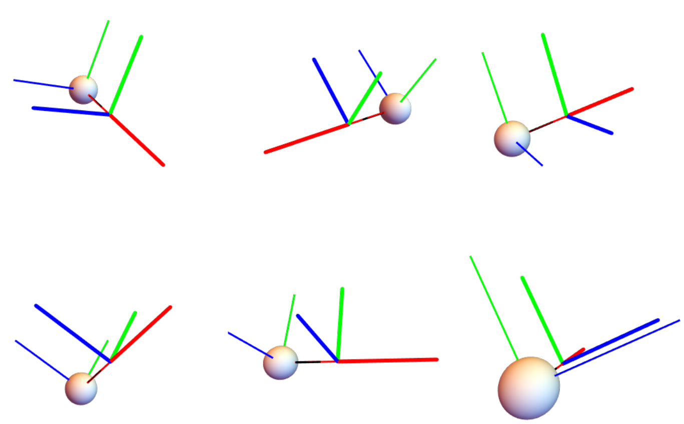
Out[264]=



Here is just the tool, look for frames to line-up

In[265]:=
```
toolGraphicInvKin =
  {{PointSize[0.01`], Point[{Xdes, Ydes, Zdes}]}, Translate[GeometricTransformation[
     {{AbsoluteThickness[2], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
      {AbsoluteThickness[2], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
      {AbsoluteThickness[2], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}},
     Transpose[Cdes[θr, θp, θy]]], {Xdes, Ydes, Zdes}],
   Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]};
Show[GraphicsGrid[{{Graphics3D[toolGraphicInvKin, ViewPoint → {1, 1, 1},
      ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
     Graphics3D[toolGraphicInvKin, ViewPoint → {-1, 1, 1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
     Graphics3D[toolGraphicInvKin, ViewPoint → {1, -1, 1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
    {Graphics3D[toolGraphicInvKin, ViewPoint → {1, 1, -1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
     Graphics3D[toolGraphicInvKin, ViewPoint → {1, 0, 0}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
     Graphics3D[toolGraphicInvKin, ViewPoint → {0, -1, 0}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

Out[266]=



Inverse Kinematics Animation

Here we create a simple path plan to get form the rest position to the desired position.

In[267]:=
```
x[t_] =.
y[t_] =.
q₁[t_] =.
q₂[t_] =.
q₃[t_] =.
q₄[t_] =.
q₅[t_] =.
q₆[t_] =.
q₇[t_] =.
tf =.
```

In[277]:=
```
x[t_] = Xbase t / tf /. solX;
y[t_] = Ybase t / tf /. solY;
q₁[t_] = Q₁ t / tf /. invKinSol;
q₂[t_] = Q₂ t / tf /. invKinSol;
q₃[t_] = Q₃ t / tf /. invKinSol;
q₄[t_] = Q₄ t / tf /. invKinSol;
q₅[t_] = Q₅ t / tf /. invKinSol;
q₆[t_] = Q₆ t / tf /. invKinSol;
q₇[t_] = Q₇ t / tf /. invKinSol;
```

Create composite graphic out of parts that have been rotated and translated

In[286]:=
```
robotGraphicInvKinAnim = {
    (*desired point*)
    {PointSize[.01], Point[{Xdes, Ydes, Zdes}]},

    (*desired tool orientation*)
    Translate[GeometricTransformation[
      {{AbsoluteThickness[2], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
       {AbsoluteThickness[2], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
       {AbsoluteThickness[2], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}},
      Transpose[Cdes[θr, θp, θy]]], {Xdes, Ydes, Zdes}],

    Translate[GeometricTransformation[wheelsGraphic, Transpose[rotW]], {xWo, yWo, zWo}],
    (*Base graphic*)
    Translate[GeometricTransformation[baseGraphic, Transpose[rotA]], {xAo, yAo, zAo}],

    (*Riser graphic*)
    Translate[GeometricTransformation[riserGraphic, Transpose[rotB]], {xBo, yBo, zBo}],

    (*Shoulder graphic*)
    Translate[
     GeometricTransformation[shoulderGraphic, Transpose[rotC]], {xCo, yCo, zCo}],

    (*Arm1 graphic*)
    Translate[GeometricTransformation[arm1Graphic, Transpose[rotD]], {xDo, yDo, zDo}],

    (*Arm2 graphic*)
    Translate[GeometricTransformation[arm2Graphic, Transpose[rotE]], {xEo, yEo, zEo}],

    (*Arm3 graphic*)
    Translate[GeometricTransformation[arm3Graphic, Transpose[rotF]], {xFo, yFo, zFo}],

    (*Wrist1 graphic*)
    Translate[GeometricTransformation[wrist1Graphic, Transpose[rotG]], {xGo, yGo, zGo}],

    (*Wrist2 graphic*)
    Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]
   };
```

In[287]:=
```
robotGraphicInvKinAnimT[t_] = robotGraphicInvKinAnim;
```

Loop over time

In[288]:=
```
tf = 2;
```

In[289]:=
```
Animate[Show[Graphics3D[robotGraphicInvKinAnimT[t], ViewPoint -> {1, 1, 1},
    ViewVertical -> {0, 0, 1}, ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> True, Axes -> True,
    PlotRange -> {{-scale, scale}, {-scale, scale}, {-scale, scale}}, AspectRatio -> 1,
    AxesLabel → {"X", "Y", "Z"}]], {t, 0, tf, tf / 500}, AnimationRunning → False]
```

Out[289]=