MCHE 470 - Robotics Dynamics and Control

Loading vector operation tools

In[1]:=
```
<< "C:\\Users\\ambik\\OneDrive - University of Louisiana
    Lafayette\\Documents\\Wolfram Mathematica\\vectorDefsMM30.m"
```

```
These Engineering Vector algorithms are copyright Alan A. Barhorst
```

Typical rotations

Generic 0 rotation (Identity)

In[2]:=
```
rot0[q_ : 1] = {{1, 0, 0}, {0, 1, 0},
                {0, 0, 1}};
MatrixForm [rot0[ ]]
```

Out[2]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Generic 1-rotation

In[3]:=
```
rot1[q_] = {{1, 0, 0}, {0, Cos[q], Sin[q]},
            {0, -Sin[q], Cos[q]}};
MatrixForm [rot1[q1[t]]]
```

Out[3]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[q_1[t]] & \mathrm{Sin}[q_1[t]] \\ 0 & -\mathrm{Sin}[q_1[t]] & \mathrm{Cos}[q_1[t]] \end{pmatrix}$$

Generic 2-rotation

In[4]:=
```
rot2[q_] = {{Cos[q], 0, -Sin[q]}, {0, 1, 0},
            {Sin[q], 0, Cos[q]}};
MatrixForm [rot2[q2[t]]]
```

Out[4]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[q_2[t]] & 0 & -\mathrm{Sin}[q_2[t]] \\ 0 & 1 & 0 \\ \mathrm{Sin}[q_2[t]] & 0 & \mathrm{Cos}[q_2[t]] \end{pmatrix}$$

Generic 3-rotation

In[5]:=
```
rot3[q_] = {{Cos[q], Sin[q], 0},
             {-Sin[q], Cos[q], 0},
             {0, 0, 1}};
MatrixForm[rot3[q3[t]]]
```

Out[5]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[q_3[t]] & \text{Sin}[q_3[t]] & 0 \\ -\text{Sin}[q_3[t]] & \text{Cos}[q_3[t]] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Define the symbols used for Unit Vectors and Unit Dyads

Define Unit Vectors and Unit Dyads for however many frames we need for the system. For this example we will use three frames of reference, with the frame **N** being the Newtonian frame. The header **unitVector** must be included. The arguments are **[frame, symbol, direction]**. So unit vector **b[1]=unitVector[B,b,1]** is the vector in the **B** frame in the **1** direction. The unitDyads are double vectors used to describe inertia properties.

In[6]:=
```
a[x_]:=unitVector[A,a,x]
b[x_]:=unitVector[B,b,x]
c[x_]:=unitVector[C,c,x]
d[x_]:=unitVector[D,d,x]
e[x_]:=unitVector[E,e,x]
f[x_]:=unitVector[F,f,x]
g[x_]:=unitVector[G,g,x]
h[x_]:=unitVector[H,h,x]
n[x_]:=unitVector[N,n,x]
aa[x_,y_]:=unitDyad[a[x],a[y]]
bb[x_,y_]:=unitDyad[b[x],b[y]]
cc[x_,y_]:=unitDyad[c[x],c[y]]
dd[x_,y_]:=unitDyad[d[x],d[y]]
ee[x_,y_]:=unitDyad[e[x],e[y]]
ff[x_,y_]:=unitDyad[f[x],f[y]]
gg[x_,y_]:=unitDyad[g[x],g[y]]
hh[x_,y_]:=unitDyad[h[x],h[y]]
```

Graphical construction of robot (uses graphic primitives from *Mathematica* v6 and above)

Base platform

Draw the base platform from regular polygons

In[23]:=
```
vecL = 2;
```

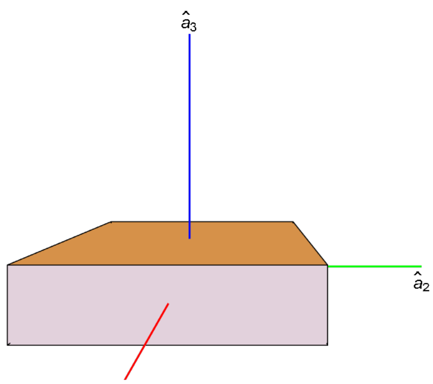
In[24]:=
```
widthBase = 2; depthBase = 2; heightBase = 1 / 2;
```

In[25]:=
```
baseShape = Cuboid[
   {-widthBase / 2, -depthBase / 2, -heightBase / 2},
   {widthBase / 2, depthBase / 2, heightBase / 2}];
```

In[26]:=
```
baseGraphicF = {baseShape,
   {Text[â1, {vecL, 0, 0}, {0, 1}],
    Text[â2, {0, vecL, 0}, {0, 1}], Text[â3, {0, 0, vecL}, {0, -1}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}}};
```

In[27]:=
```
Show[Graphics3D[baseGraphicF], ViewPoint -> {1, 0, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[27]=



In[28]:=
```
baseGraphic = {baseShape,
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Riser cylinder

Draw the riser as a cylinder

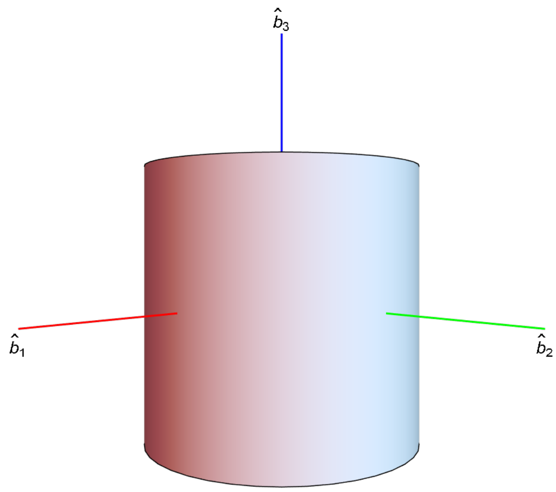
In[29]:=
```
vecL = 1;
```

In[30]:=
```
halfHeightRiser = 1 / 2;
riserBase = {0, 0, -halfHeightRiser};
riserTop = {0, 0, halfHeightRiser};
riserRadius = 1 / 2;
```

In[34]:=
```
riserGraphicF = {Cylinder[{riserBase, riserTop}, riserRadius],
        {AbsoluteThickness[1], {Text[b̂1, {vecL, 0, 0}, {0, 1}],
     Text[b̂2, {0, vecL, 0}, {0, 1}], Text[b̂3, {0, 0, vecL}, {0, -1}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
     {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
     {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}}}};
```

In[35]:=
```
Show[Graphics3D[riserGraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[35]=



In[36]:=
```
riserGraphic = {Cylinder[{riserBase, riserTop}, riserRadius],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Shoulder cylinder

Draw the shoulder

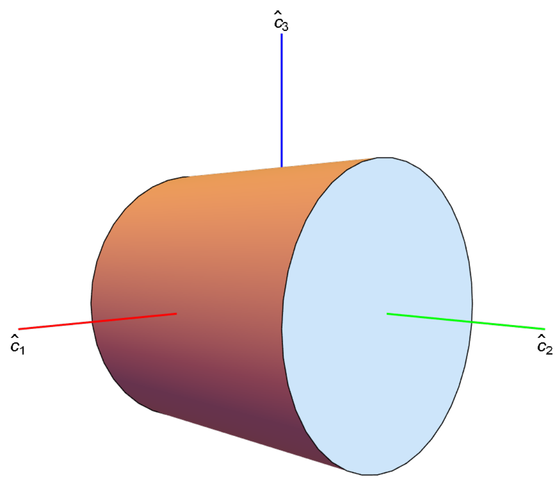
In[37]:=
```
vecL = 1;
```

In[38]:=
```
halfHeightShoulder = 1 / 4 + 1 / 4;
shoulderBase = {0, 0, -halfHeightShoulder};
shoulderTop = {0, 0, halfHeightShoulder};
shoulderRadius = 1 / 2;
```

In[42]:=
```
shoulderGraphicF =
  {Rotate[Cylinder[{shoulderBase, shoulderTop}, shoulderRadius], Pi / 2, {1, 0, 0}],
   Text[ĉ1, {vecL, 0, 0}, {0, 1}], Text[ĉ2, {0, vecL, 0}, {0, 1}],
   Text[ĉ3, {0, 0, vecL}, {0, -1}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

In[43]:=
```
Show[Graphics3D[shoulderGraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[43]=



In[44]:=
```
shoulderGraphic =
  {Rotate[Cylinder[{shoulderBase, shoulderTop}, shoulderRadius], Pi / 2, {1, 0, 0}],
      {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Arm segment 1

Draw the first arm from polygons

In[45]:=
```
vecL = 2;
```

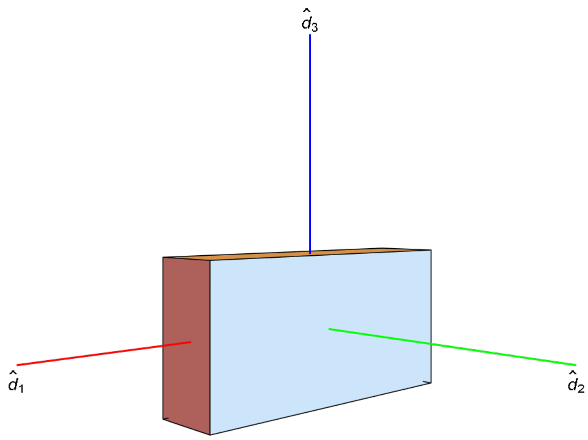
In[46]:=
```
lengthArm1 = 2; depthArm1 = 1 / 2; heightArm1 = 2 shoulderRadius;
```

In[47]:=
```
arm1Shape = Cuboid[
   {-lengthArm1 / 2, -depthArm1 / 2, -heightArm1 / 2},
   {lengthArm1 / 2, depthArm1 / 2, heightArm1 / 2}];
```

In[48]:=
```
arm1GraphicF = {arm1Shape,
   {Text[d̂1, {vecL, 0, 0}, {0, 1}],
    Text[d̂2, {0, vecL, 0}, {0, 1}], Text[d̂3, {0, 0, vecL}, {0, -1}],
      {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}}};
```

In[49]:=
```
Show[Graphics3D[arm1GraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[49]=



In[50]:=
```
arm1Graphic = {arm1Shape,
      {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Arm segment 2

Draw the second arm from polygons

In[51]:=
```
vecL = 2;
```

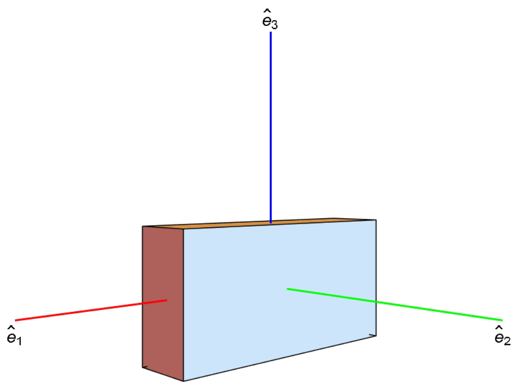
In[52]:=
```
lengthArm2 = 1; depthArm2 = 1 / 3; heightArm2 = shoulderRadius;
```

In[53]:=
```
arm2Shape = Cuboid[
   {-lengthArm2 / 2, -depthArm2 / 2, -heightArm2 / 2},
   {lengthArm2 / 2, depthArm2 / 2, heightArm2 / 2}];
```

In[54]:=
```
arm2GraphicF = {arm1Shape,
   {Text[ê1, {vecL, 0, 0}, {0, 1}],
    Text[ê2, {0, vecL, 0}, {0, 1}], Text[ê3, {0, 0, vecL}, {0, -1}],
       {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
    {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}}};
```

In[55]:=
```
Show[Graphics3D[arm2GraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[55]=



In[56]:=
```
arm2Graphic = {arm2Shape,
      {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Arm segment 3

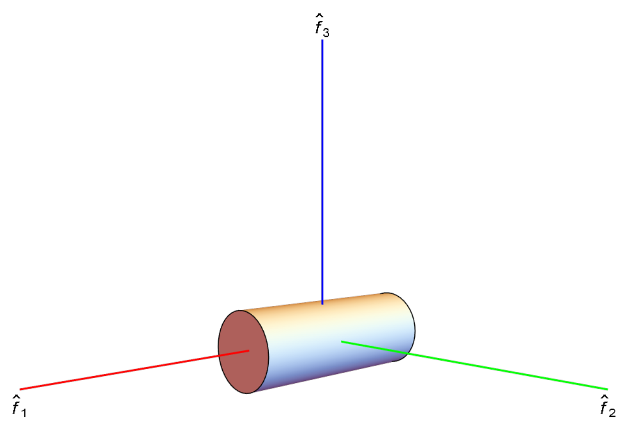
Draw the third arm as a cylinder

In[57]:=
```
vecL = 1;
```

In[58]:=
```
halfHeightArm3 = 1 / 3;
arm3Base = {0, 0, -halfHeightArm3};
arm3Top = {0, 0, halfHeightArm3};
arm3Radius = 1 / 8;
```

In[62]:=
```
arm3GraphicF = {Rotate[Cylinder[{arm3Base, arm3Top}, arm3Radius], Pi / 2, {0, 1, 0}],
   Text[f̂1, {vecL, 0, 0}, {0, 1}],
   Text[f̂2, {0, vecL, 0}, {0, 1}], Text[f̂3, {0, 0, vecL}, {0, -1}],
      {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

In[63]:=
```
Show[Graphics3D[arm3GraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[63]=


In[64]:=
```
arm3Graphic = {Rotate[Cylinder[{arm3Base, arm3Top}, arm3Radius], Pi / 2, {0, 1, 0}],
      {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Wrist1

Draw the wrist and tool pointer as a cylinder and dark line

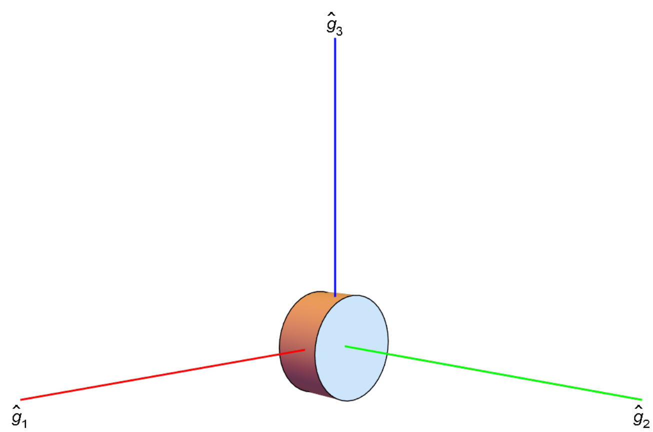
In[65]:=
```
vecL = 1;
```

In[66]:=
```
halfHeightWrist1 = 1 / 12;
wrist1Base = {0, 0, -halfHeightWrist1};
wrist1Top = {0, 0, halfHeightWrist1};
wrist1Radius = 1 / 6;
```

In[70]:=
```
wristGraphicF = {Rotate[Cylinder[{wrist1Base, wrist1Top}, wrist1Radius], Pi / 2, {1, 0, 0}],
   Text[ĝ1, {vecL, 0, 0}, {0, 1}],
   Text[ĝ2, {0, vecL, 0}, {0, 1}], Text[ĝ3, {0, 0, vecL}, {0, -1}],
      {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

In[71]:=
```
Show[Graphics3D[wristGraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[71]=



In[72]:=
```
wrist1Graphic = {Rotate[Cylinder[{wrist1Base, wrist1Top}, wrist1Radius], Pi / 2, {1, 0, 0}],
   {Thickness[.01], Line[{{wrist1Radius, 0, 0}, {wrist1Radius + pointerLength, 0, 0}}]},
       {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Wrist2 and pointer

Draw the wrist and tool pointer as a cylinder and dark line

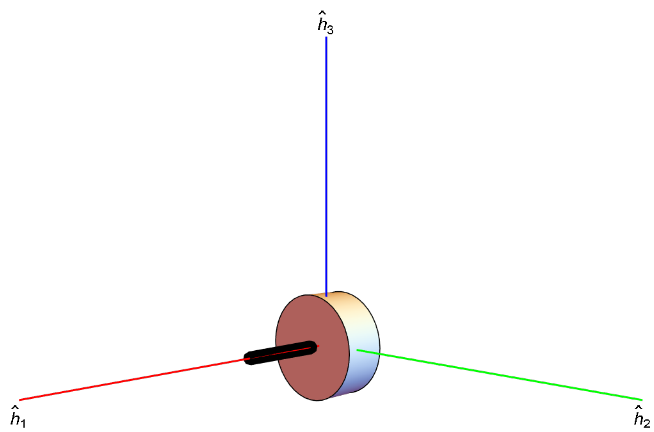
In[73]:=
```
vecL = 1;
```

In[74]:=
```
halfHeightWrist2 = 1 / 14;
wrist2Base = {0, 0, -halfHeightWrist2};
wrist2Top = {0, 0, halfHeightWrist2};
wrist2Radius = 1 / 6;
pointerLength = 1 / 4;
```

In[78]:=
```
wrist2GraphicF =
  {Rotate[Cylinder[{wrist2Base, wrist2Top}, wrist2Radius], Pi / 2, {0, 1, 0}],
   {Thickness[.02], Line[{{halfHeightWrist2, 0, 0},
      {halfHeightWrist2 + pointerLength, 0, 0}}]},
   Text[ĥ1, {vecL, 0, 0}, {0, 1}], Text[ĥ2, {0, vecL, 0}, {0, 1}],
   Text[ĥ3, {0, 0, vecL}, {0, -1}],
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

In[79]:=
```
Show[Graphics3D[wrist2GraphicF], ViewPoint -> {1, 1, 0}, ViewVertical -> {0, 0, 1},
 ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]
```

Out[79]=



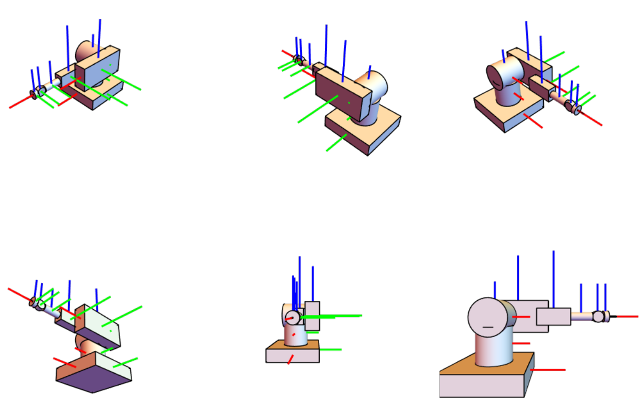
In[80]:=
```
wrist2Graphic = {Rotate[Cylinder[{wrist2Base, wrist2Top}, wrist2Radius], Pi / 2, {0, 1, 0}],
   {Thickness[.01],
    Line[{{halfHeightWrist2, 0, 0}, {halfHeightWrist2 + pointerLength, 0, 0}}]},
        {AbsoluteThickness[1], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
   {AbsoluteThickness[1], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}};
```

Entire robot

In[81]:=
```
robotGraphic =
  (*Base graphic*)
  {Translate[baseGraphic, {0, 0, 1 / 2 heightBase}],

   (*Riser graphic*)
   Translate[riserGraphic, {0, 0, heightBase + halfHeightRiser}],

   (*Shoulder graphic*)
   Translate[shoulderGraphic,
    {0, 0, heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Arm1 graphic*)
   Translate[arm1Graphic, {1 / 4 lengthArm1, halfHeightShoulder + 1 / 2 depthArm1,
     heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Arm2 graphic*)
   Translate[arm2Graphic, {lengthArm1 - 1 / 3 lengthArm2, halfHeightShoulder -
      1 / 2 depthArm2 , heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Arm3 graphic*)
   Translate[arm3Graphic,
    {3 / 4 lengthArm1 + 2 / 3 lengthArm2 + halfHeightArm3, halfHeightShoulder -
      1 / 2 depthArm2 , heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Wrist1 graphic*)
   Translate[wrist1Graphic, {3 / 4 lengthArm1 + 2 / 3 lengthArm2 +
      2 halfHeightArm3 + wrist1Radius, halfHeightShoulder - 1 / 2 depthArm2 ,
     heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}],

   (*Wrist2 graphic*)
   Translate[wrist2Graphic, {3 / 4 lengthArm1 + 2 / 3 lengthArm2 + 2 halfHeightArm3 +
      2 wrist1Radius + halfHeightWrist2, halfHeightShoulder - 1 / 2 depthArm2 ,
     heightBase + 2 halfHeightRiser + 1 / 2 shoulderRadius}]};
```

In[82]:=
```
Show[
 GraphicsGrid[{{Graphics3D[robotGraphic, ViewPoint → {1, 1, 1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
     Graphics3D[robotGraphic, ViewPoint → {-1, 1, 1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
     Graphics3D[robotGraphic, ViewPoint → {1, -1, 1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
    {Graphics3D[robotGraphic, ViewPoint → {1, 1, -1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
     Graphics3D[robotGraphic, ViewPoint → {1, 0, 0}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
     Graphics3D[robotGraphic, ViewPoint → {0, -1, 0}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

Out[82]=



Rotations

Lets assume the robot has a moving base, shoulder, and three link arm, with wrist1 and 2. It has a 3-2-2-1-2-1 rotation sequence. Starting from the Newtonian frame N we have a 0-rotation to A, then a 0-rotation to B, then a 3-rotation to C, then a 2-rotation to D, then a 2-rotation to E, then a 1-rotation to F, then a 2-rotation to G, then a 1-rotation to H and the tool pointer

In[83]:=
```
rotA = rot0[];
AtoN = rotA.{n[1], n[2], n[3]}
```

Out[84]= $\{\hat{n}_1, \hat{n}_2, \hat{n}_3\}$

In[85]:=
```
TranAtoN[x_] := x //. {a[1] → AtoN[[1]], a[2] → AtoN[[2]], a[3] → AtoN[[3]]}
```

In[86]:=
```
rotB = rot0[];
BtoN = rotB.{n[1], n[2], n[3]}
```

Out[87]= $\{\hat{n}_1, \hat{n}_2, \hat{n}_3\}$

In[88]:=
```
TranBtoN[x_] := x //. {b[1] → BtoN[[1]], b[2] → BtoN[[2]], b[3] → BtoN[[3]]}
```

In[89]:=
```
rotC = rot3[q1[t]];
CtoN = rotC.{n[1], n[2], n[3]}
```

Out[90]= $\{\mathrm{Cos}[q_1[t]]\,\hat{n}_1 + \mathrm{Sin}[q_1[t]]\,\hat{n}_2,\ -\mathrm{Sin}[q_1[t]]\,\hat{n}_1 + \mathrm{Cos}[q_1[t]]\,\hat{n}_2,\ \hat{n}_3\}$

In[91]:=
```
TranCtoN[x_] := x //. {c[1] → CtoN[[1]], c[2] → CtoN[[2]], c[3] → CtoN[[3]]}
```

In[92]:=
```
rotD = rot2[q2[t]].rot3[q1[t]];
DtoN = rotD.{n[1], n[2], n[3]}
```

Out[93]= $\{\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_2[t]]\,\hat{n}_1 + \mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]\,\hat{n}_2 - \mathrm{Sin}[q_2[t]]\,\hat{n}_3,$
$-\mathrm{Sin}[q_1[t]]\,\hat{n}_1 + \mathrm{Cos}[q_1[t]]\,\hat{n}_2,$
$\mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_2[t]]\,\hat{n}_1 + \mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_2[t]]\,\hat{n}_2 + \mathrm{Cos}[q_2[t]]\,\hat{n}_3\}$

In[94]:=
```
TranDtoN[x_] := x //. {d[1] → DtoN[[1]], d[2] → DtoN[[2]], d[3] → DtoN[[3]]}
```

In[95]:=
```
rotE = rot2[q3[t]].rot2[q2[t]].rot3[q1[t]];
EtoN = rotE.{n[1], n[2], n[3]}
```

Out[96]= $\{\mathrm{Cos}[q_1[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]] - \mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_1 +$
$\mathrm{Sin}[q_1[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]] - \mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_2 +$
$(-\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_2[t]] - \mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_3,\ -\mathrm{Sin}[q_1[t]]\,\hat{n}_1 + \mathrm{Cos}[q_1[t]]\,\hat{n}_2,$
$\mathrm{Cos}[q_1[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_2[t]] + \mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_1 +$
$\mathrm{Sin}[q_1[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_2[t]] + \mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_2 +$
$(\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]] - \mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_3\}$

In[97]:=
```
TranEtoN[x_] := x //. {e[1] → EtoN[[1]], e[2] → EtoN[[2]], e[3] → EtoN[[3]]}
```

In[98]:=
```
rotF = rot1[q_4[t]].rot2[q_3[t]].rot2[q_2[t]].rot3[q_1[t]];
FtoN = rotF.{n[1], n[2], n[3]}
```

Out[99]=
$$\begin{aligned}
\{&\mathrm{Cos}[q_1[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]-\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_1+\\
&\mathrm{Sin}[q_1[t]]\,(\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]-\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_2+\\
&(-\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_2[t]]-\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_3,\\
&(-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_1[t]]+\mathrm{Cos}[q_1[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_4[t]]+\\
&\quad\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]))\,\hat{n}_1+(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_4[t]]+\\
&\quad\mathrm{Sin}[q_1[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_4[t]]+\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]))\,\hat{n}_2+\\
&(\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_4[t]]-\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]])\,\hat{n}_3,\\
&(\mathrm{Cos}[q_1[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_2[t]]+\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]])+\\
&\quad\mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_4[t]])\,\hat{n}_1+\\
&(\mathrm{Sin}[q_1[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_2[t]]+\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]])-\\
&\quad\mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_4[t]])\,\hat{n}_2+\\
&(\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]])\,\hat{n}_3\}
\end{aligned}$$

In[100]:=
```
TranFtoN[x_] := x //. {f[1] → FtoN[[1]], f[2] → FtoN[[2]], f[3] → FtoN[[3]]}
```

In[101]:=
```
rotG = rot2[q_5[t]].rot1[q_4[t]].rot2[q_3[t]].rot2[q_2[t]].rot3[q_1[t]];
GtoN = rotG.{n[1], n[2], n[3]}
```

Out[102]=
$$\begin{aligned}
\{&(-\mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_5[t]]+\\
&\quad\mathrm{Cos}[q_1[t]]\,(\mathrm{Sin}[q_2[t]]\,(-\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]]-\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_5[t]])+\\
&\qquad\mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]]-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]])))\,\hat{n}_1+\\
&(\mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_5[t]]+\mathrm{Sin}[q_1[t]]\\
&\quad(\mathrm{Sin}[q_2[t]]\,(-\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]]-\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_5[t]])+\\
&\qquad\mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]]-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]])))\,\hat{n}_2+\\
&(\mathrm{Cos}[q_2[t]]\,(-\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]]-\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_5[t]])-\\
&\quad\mathrm{Sin}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]]-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]]))\,\hat{n}_3,\\
&(-\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_1[t]]+\mathrm{Cos}[q_1[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_4[t]]+\\
&\quad\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]))\,\hat{n}_1+(\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_4[t]]+\\
&\quad\mathrm{Sin}[q_1[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_4[t]]+\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]))\,\hat{n}_2+\\
&(\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_4[t]]-\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]])\,\hat{n}_3,\\
&(\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_4[t]]+\\
&\quad\mathrm{Cos}[q_1[t]]\,(\mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]]+\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_5[t]])+\\
&\qquad\mathrm{Sin}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]-\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]])))\,\hat{n}_1+\\
&(-\mathrm{Cos}[q_1[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_4[t]]+\mathrm{Sin}[q_1[t]]\\
&\quad(\mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]]+\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_5[t]])+\\
&\qquad\mathrm{Sin}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]-\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]])))\,\hat{n}_2+\\
&(-\mathrm{Sin}[q_2[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]]+\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_5[t]])+\\
&\quad\mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]-\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]]))\,\hat{n}_3\}
\end{aligned}$$

In[103]:=
```
TranGtoN[x_] := x //. {g[1] → GtoN[[1]], g[2] → GtoN[[2]], g[3] → GtoN[[3]]}
```

In[104]:=
```
rotH = rot1[q6[t]].rot2[q5[t]].rot1[q4[t]].rot2[q3[t]].rot2[q2[t]].rot3[q1[t]];
HtoN = rotH.{n[1], n[2], n[3]}
```

Out[105]=
$$\{(-\mathrm{Sin}[q_1[t]]\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_5[t]] + \mathrm{Cos}[q_1[t]]\,(\mathrm{Sin}[q_2[t]]\,(-\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]] - \mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_5[t]]) + \mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]] - \mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]])))\,\hat{n}_1 + (\mathrm{Cos}[q_1[t]]\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_5[t]] + \mathrm{Sin}[q_1[t]]\,(\mathrm{Sin}[q_2[t]]\,(-\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]] - \mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_5[t]]) + \mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]] - \mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]])))\,\hat{n}_2 + (\mathrm{Cos}[q_2[t]]\,(-\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]] - \mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_5[t]]) - \mathrm{Sin}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]] - \mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]]))\,\hat{n}_3,$$
$$(-\mathrm{Sin}[q_1[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_6[t]] - \mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]]) + \mathrm{Cos}[q_1[t]]\,(\mathrm{Sin}[q_2[t]]\,(-\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]]\,\mathrm{Sin}[q_6[t]] + \mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_4[t]] + \mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_6[t]])) + \mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_5[t]]\,\mathrm{Sin}[q_6[t]] + \mathrm{Sin}[q_3[t]]\,(\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_4[t]] + \mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_6[t]]))))\,\hat{n}_1 + (\mathrm{Cos}[q_1[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_6[t]] - \mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]]) + \mathrm{Sin}[q_1[t]]\,(\mathrm{Sin}[q_2[t]]\,(-\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]]\,\mathrm{Sin}[q_6[t]] + \mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_4[t]] + \mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_6[t]])) + \mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_5[t]]\,\mathrm{Sin}[q_6[t]] + \mathrm{Sin}[q_3[t]]\,(\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_4[t]] + \mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_6[t]]))))\,\hat{n}_2 + (\mathrm{Cos}[q_2[t]]\,(-\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]]\,\mathrm{Sin}[q_6[t]] + \mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_4[t]] + \mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_6[t]])) - \mathrm{Sin}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_5[t]]\,\mathrm{Sin}[q_6[t]] + \mathrm{Sin}[q_3[t]]\,(\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_4[t]] + \mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_6[t]])))\,\hat{n}_3,$$
$$(-\mathrm{Sin}[q_1[t]]\,(-\mathrm{Cos}[q_5[t]]\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_4[t]] - \mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_6[t]]) + \mathrm{Cos}[q_1[t]]\,(\mathrm{Sin}[q_2[t]]\,(-\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]] + \mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Cos}[q_6[t]] - \mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]])) + \mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_5[t]] + \mathrm{Sin}[q_3[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Cos}[q_6[t]] - \mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]]))))\,\hat{n}_1 + (\mathrm{Cos}[q_1[t]]\,(-\mathrm{Cos}[q_5[t]]\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_4[t]] - \mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_6[t]]) + \mathrm{Sin}[q_1[t]]\,(\mathrm{Sin}[q_2[t]]\,(-\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]] + \mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Cos}[q_6[t]] - \mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]])) + \mathrm{Cos}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_5[t]] + \mathrm{Sin}[q_3[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Cos}[q_6[t]] - \mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]]))))\,\hat{n}_2 + (\mathrm{Cos}[q_2[t]]\,(-\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]] + \mathrm{Cos}[q_3[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Cos}[q_6[t]] - \mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]])) - \mathrm{Sin}[q_2[t]]\,(\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_5[t]] + \mathrm{Sin}[q_3[t]]\,(\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Cos}[q_6[t]] - \mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]])))\,\hat{n}_3\}$$

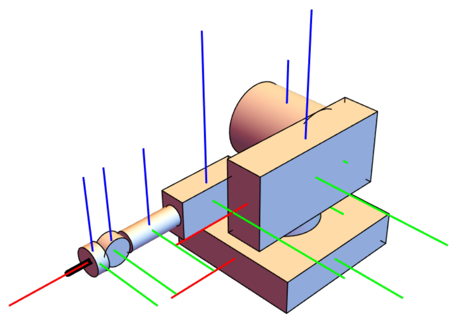
In[106]:=
```
TranHtoN[x_] := x //. {h[1] → HtoN[[1]], h[2] → HtoN[[2]], h[3] → HtoN[[3]]}
```

Relative position vectors

Now lets create vectors to the reference frames of each body relative to the previous body or frame.
See composite robot graphic

In[107]:=
```
Show[Graphics3D[robotGraphic, ViewPoint -> {1, 1, 1}, ViewVertical -> {0, 0, 1},
  ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]]
```

Out[107]=



Base

In[108]:=
```
OrAo = x[t] × n[1] + y[t] × n[2] + 1 / 2 heightBase n[3]
```

Out[108]= $\frac{\hat{n}_3}{4} + \hat{n}_1 \, x[t] + \hat{n}_2 \, y[t]$

Riser

In[109]:=
```
AorBo = (1 / 2 heightBase + halfHeightRiser) a[3]
```

Out[109]= $\frac{3 \, \hat{a}_3}{4}$

Shoulder

In[110]:=
```
BorCo = (halfHeightRiser + 1 / 2 shoulderRadius) b[3]
```

Out[110]= $\frac{3 \, \hat{b}_3}{4}$

Arm1

In[111]:=
```
CorDo = halfHeightShoulder c[2] + 1 / 2 depthArm1 d[2] + 1 / 4 lengthArm1 d[1]
```

Out[111]= $\frac{\hat{c}_2}{2} + \frac{\hat{d}_1}{2} + \frac{\hat{d}_2}{4}$

Arm2

In[112]:=
```
DorEo = 1 / 2 lengthArm1 d[1] -
  (1 / 2 depthArm1 + 1 / 2 depthArm2) e[2] + 1 / 2 lengthArm2 e[1] - 1 / 3 lengthArm2 e[1]
```

Out[112]= $\hat{d}_1 + \frac{\hat{e}_1}{6} - \frac{5\,\hat{e}_2}{12}$

Arm3

In[113]:=
```
EorFo = 1 / 2 lengthArm2 e[1] + halfHeightArm3 f[1]
```

Out[113]= $\frac{\hat{e}_1}{2} + \frac{\hat{f}_1}{3}$

Wrist1

In[114]:=
```
ForGo = (halfHeightArm3 + wrist1Radius) f[1]
```

Out[114]= $\frac{\hat{f}_1}{2}$

Wrist2

In[115]:=
```
GorHo = (wrist1Radius + halfHeightWrist2) g[1]
```

Out[115]= $\frac{5\,\hat{g}_1}{21}$

Pointer

In[116]:=
```
HorP = (halfHeightWrist2 + pointerLength) h[1]
```

Out[116]= $\frac{9\,\hat{h}_1}{28}$

Graphic transformation function

This function will be used to rotate the graphic primatives using the home-built transformations. **Do not mess with it!!!!**

In[117]:=
```
myRotateShape[shape_, myRotationMatrix_] := Block[{rotmat = myRotationMatrix},
  shape /. {poly : Polygon[_] :→ Map[(rotmat.#) &, poly, {2}], line : Line[_] :→
     Map[(rotmat.#) &, line, {2}], point : Point[_] :→ Map[(rotmat.#) &, point, {1}]}]
```

Homogeneous Transformation for the Robot

Animation Example

Absolute position vectors and coordinates in Newtonian frame

Animation

Create functions for the coordinates for demonstration purposes.

In[169]:=
```
A = 3; B = 1; Cc = 0; ω = 2 Pi (.1);
```

In[170]:=
```
x[t_] := A Cos[ω t]
y[t_] := A Sin[ω t]
q1[t_] := B t + Cc
q2[t_] := B t + Cc
q3[t_] := B t + Cc
q4[t_] := B t + Cc
q5[t_] := B t + Cc
q6[t_] := B t + Cc
```

Create composite graphic out of parts that have been rotated and translated

In[178]:=
```
robotGraphicAnim = {
   (*Base graphic*)
   Translate[GeometricTransformation[baseGraphic, Transpose[rotA]], {xAo, yAo, zAo}],
   (*Riser graphic*)
   Translate[GeometricTransformation[riserGraphic, Transpose[rotB]], {xBo, yBo, zBo}],
   (*Shoulder graphic*)
   Translate[
    GeometricTransformation[shoulderGraphic, Transpose[rotC]], {xCo, yCo, zCo}],
   (*Arm1 graphic*)
   Translate[GeometricTransformation[arm1Graphic, Transpose[rotD]], {xDo, yDo, zDo}],
   (*Arm2 graphic*)
   Translate[GeometricTransformation[arm2Graphic, Transpose[rotE]], {xEo, yEo, zEo}],
   (*Arm3 graphic*)
   Translate[GeometricTransformation[arm3Graphic, Transpose[rotF]], {xFo, yFo, zFo}],
   (*Wrist1 graphic*)
   Translate[GeometricTransformation[wrist1Graphic, Transpose[rotG]], {xGo, yGo, zGo}],
   (*Wrist2 graphic*)
   Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]
  };
```

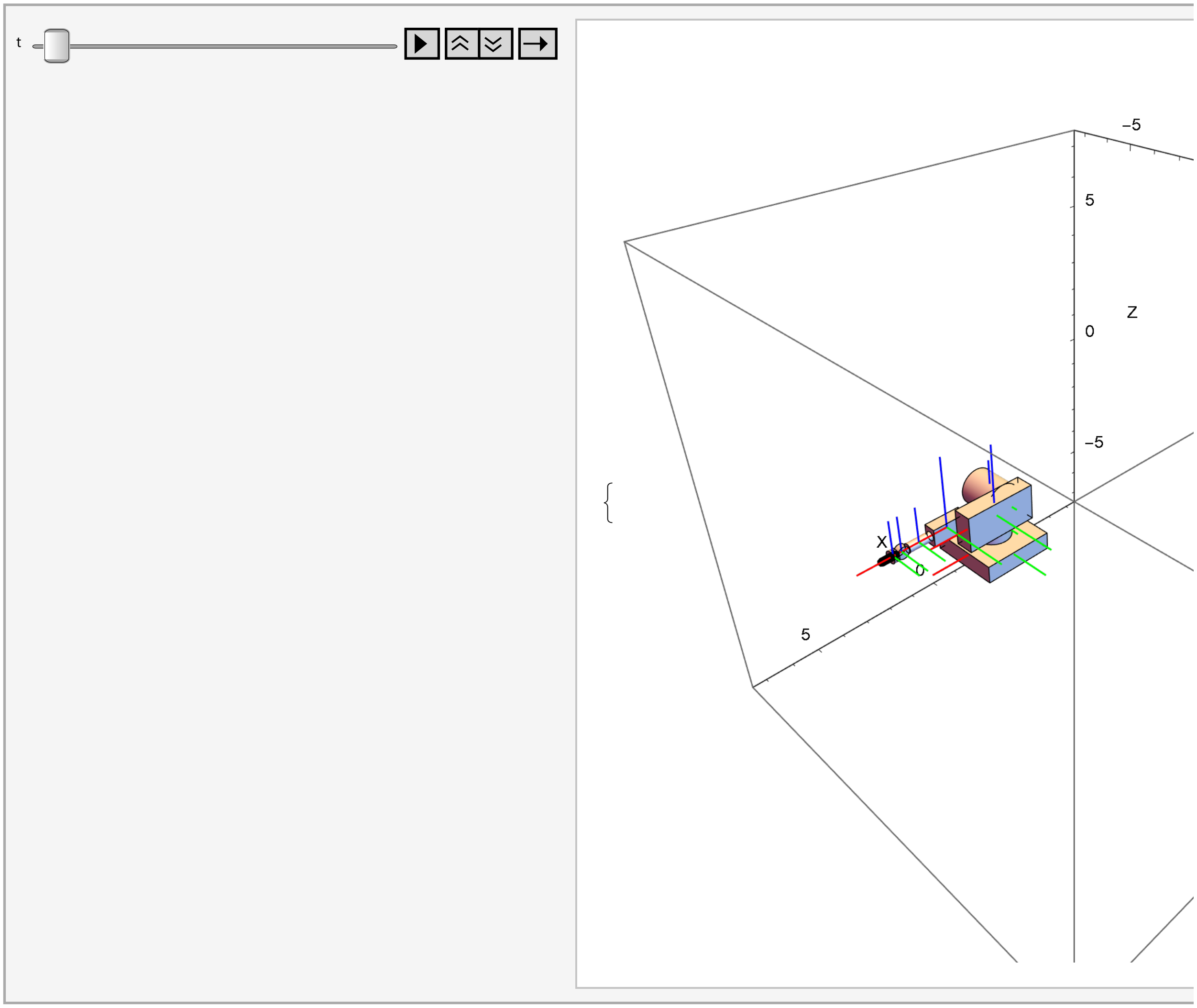
Make it a function of to so it can be looped over time

In[179]:=
```
robotGraphicAnimT[t_] = robotGraphicAnim;
NtPp[t_] = NtP;
```

In[181]:=
```
tf = 10;
scale = 2.5 A;
```

In[183]:=
```
Animate[
 {Show[Graphics3D[robotGraphicAnimT[t], ViewPoint -> {1, 1, 1}, ViewVertical -> {0, 0, 1},
    ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> True, Axes -> True,
    PlotRange -> {{-scale, scale}, {-scale, scale}, {-scale, scale}},
    AspectRatio -> 1, AxesLabel → {"X", "Y", "Z"}], ImageSize → 500],
  MatrixForm[NtPp[t] // N]}, {t, 0, tf, tf / 500}, AnimationRunning → False]
```

Out[183]=



This is the old style way to do animation with stop action frames.

For[t = 0, t <= tf, t += tf/100,

Print[Show[Graphics3D[robotGraphicAnim, ViewPoint -> {1, 1, 1}, ViewVertical -> {0, 0, 1}, ViewCenter -> {1/2, 1/2, 1/2}, Boxed -> True, Axes -> True, PlotRange -> {{-scale, scale}, {-scale, scale}, {-scale, scale}}, AspectRatio -> 1, AxesLabel -> {"X", "Y", "Z"}]]]]

t =.;

Inverse Kinematics

We need to set up nonlinear equations to be solved to find angles and positions given desired pointer tip location and the tool frame orientation.

First clear all the variables of the kinematics. Sometimes this may cause an error if they have not been assigned numbers yet. Ignore the error and proceed.

In[184]:=
```
x[t_] =.
y[t_] =.
q₁[t_] =.
q₂[t_] =.
q₃[t_] =.
q₄[t_] =.
q₅[t_] =.
q₆[t_] =.
```

Desired and actual tool orientation

Using the generic rotations from above we will assume an Euler: roll-pitch-yaw sequence or an Euler 1-2-3 sequence to construct the desired tool orientation.

In[192]:=
```
Cdes[roll_, pitch_ , yaw_] := rot3[yaw].rot2[pitch].rot1[roll]
```

Here is an example

In[193]:=
```
MatrixForm[Cdes[Pi, Pi / 2, Pi / 3]]
```

Out[193]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 1 & 0 & 0 \end{pmatrix}$$

The actual rotation matrix in terms of our robot parameters is given as follows with the time dependence removed for simplicity

In[194]:=
```
Cact = rotH //. {qn_[t] → Qn};
```

Desired and actual tool tip position

The desired position is just a set of three numbers (X_{des},Y_{des},Z_{des}). The actual position vector out to the tool or pointer is given as follows with the time dependence removed

In[195]:=
```
OrP = OrAo + AorBo + BorCo + CorDo + DorEo + EorFo + ForGo + GorHo + HorP //.
   {x[t] → Xbase, y[t] → Ybase}
```

Out[195]=

$$\frac{3\,\hat{b}_3}{4} + \frac{3\,\hat{a}_3}{4} + \frac{\hat{c}_2}{2} + \frac{3\,\hat{d}_1}{2} + \frac{\hat{d}_2}{4} + \frac{2\,\hat{e}_1}{3} - \frac{5\,\hat{e}_2}{12} + \frac{5\,\hat{f}_1}{6} + \frac{5\,\hat{g}_1}{21} + \frac{9\,\hat{h}_1}{28} + X_{base}\,\hat{n}_1 + Y_{base}\,\hat{n}_2 + \frac{\hat{n}_3}{4}$$

The Newtonian X,Y,Z position of the point is given as follows with the time dependence removed

In[196]:=
```
X_act = (OrP.n[1] // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN //
    TranGtoN // TranHtoN) //. {q_n_[t] → Q_n}
```

Out[196]=
$$\frac{3}{2}\text{Cos}[Q_1]\ \text{Cos}[Q_2] - \frac{\text{Sin}[Q_1]}{3} + \frac{3}{2}\text{Cos}[Q_1]\ (\text{Cos}[Q_2]\ \text{Cos}[Q_3] - \text{Sin}[Q_2]\ \text{Sin}[Q_3]) + \frac{47}{84}(-\text{Sin}[Q_1]\ \text{Sin}[Q_4]\ \text{Sin}[Q_5] + \text{Cos}[Q_1]\ (\text{Sin}[Q_2]\ (-\text{Cos}[Q_5]\ \text{Sin}[Q_3] - \text{Cos}[Q_3]\ \text{Cos}[Q_4]\ \text{Sin}[Q_5]) + \text{Cos}[Q_2]\ (\text{Cos}[Q_3]\ \text{Cos}[Q_5] - \text{Cos}[Q_4]\ \text{Sin}[Q_3]\ \text{Sin}[Q_5]))) + X_{base}$$

In[197]:=
```
Y_act = (OrP.n[2] // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN //
    TranGtoN // TranHtoN) //. {q_n_[t] → Q_n}
```

Out[197]=
$$\frac{\text{Cos}[Q_1]}{3} + \frac{3}{2}\text{Cos}[Q_2]\ \text{Sin}[Q_1] + \frac{3}{2}\text{Sin}[Q_1]\ (\text{Cos}[Q_2]\ \text{Cos}[Q_3] - \text{Sin}[Q_2]\ \text{Sin}[Q_3]) + \frac{47}{84}(\text{Cos}[Q_1]\ \text{Sin}[Q_4]\ \text{Sin}[Q_5] + \text{Sin}[Q_1]\ (\text{Sin}[Q_2]\ (-\text{Cos}[Q_5]\ \text{Sin}[Q_3] - \text{Cos}[Q_3]\ \text{Cos}[Q_4]\ \text{Sin}[Q_5]) + \text{Cos}[Q_2]\ (\text{Cos}[Q_3]\ \text{Cos}[Q_5] - \text{Cos}[Q_4]\ \text{Sin}[Q_3]\ \text{Sin}[Q_5]))) + Y_{base}$$

In[198]:=
```
Z_act = (OrP.n[3] // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN //
    TranGtoN // TranHtoN) //. {q_n_[t] → Q_n}
```

Out[198]=
$$\frac{7}{4} - \frac{3\ \text{Sin}[Q_2]}{2} + \frac{3}{2}(-\text{Cos}[Q_3]\ \text{Sin}[Q_2] - \text{Cos}[Q_2]\ \text{Sin}[Q_3]) + \frac{47}{84}(\text{Cos}[Q_2]\ (-\text{Cos}[Q_5]\ \text{Sin}[Q_3] - \text{Cos}[Q_3]\ \text{Cos}[Q_4]\ \text{Sin}[Q_5]) - \text{Sin}[Q_2]\ (\text{Cos}[Q_3]\ \text{Cos}[Q_5] - \text{Cos}[Q_4]\ \text{Sin}[Q_3]\ \text{Sin}[Q_5]))$$

Create the equations for the actual angles and positions

This robot has 8 degrees of freedom, so we need at least 8 equations.

First enter desired values.

In[199]:=
```
X_des := 1;
Y_des := 1;
Z_des := 3;
θ_r := Pi / 2;
θ_p := Pi / 4;
θ_y := Pi / 4;
```

Lets see if we can reach this point. Since the base is mobile we need only check the Z direction when the arm is straight up

In[205]:=
```
Z_des <= Z_act //. {Q_1 → 0, Q_2 → -Pi / 2, Q_3 → 0, Q_4 → 0, Q_5 → 0, Q_6 → 0}
```

Out[205]= True

Here is the current desired tool orientation

In[206]:= `MatrixForm[C_des[θ_r, θ_p, θ_y]]`

Out[206]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

Using the three positions first, we have

In[207]:=
```
eq1 = X_act - X_des // distributeScalars
eq2 = Y_act - Y_des // distributeScalars
eq3 = Z_act - Z_des // distributeScalars
```

ReplaceRepeated: Exiting after

$-1+\frac{3}{2}\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\frac{\text{Sin}[Q_1]}{3}+\frac{3}{2}\text{Cos}[Q_1]\,(\text{Cos}[Q_2]\,\text{Cos}[Q_3]-\text{Sin}[Q_2]\,\text{Sin}[Q_3])+\frac{47}{84}(-\text{Sin}[Q_1]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]+\text{Cos}[Q_1]\,(\text{Sin}[$
Subscript[≪2≫]] (Times[≪3≫] + Times[≪4≫]) + Cos[Subscript[≪2≫]] (Times[≪2≫] + Times[≪4≫]))) +
X_{base} scanned 3 times.

Out[207]=
$$-1+\frac{3}{2}\text{Cos}[Q_1]\,\text{Cos}[Q_2]+\frac{3}{2}\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]+\frac{47}{84}\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_5]-\frac{\text{Sin}[Q_1]}{3}-\frac{3}{2}\text{Cos}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]-\frac{47}{84}\text{Cos}[Q_1]\,\text{Cos}[Q_5]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]-\frac{47}{84}\text{Cos}[Q_1]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_2]\,\text{Sin}[Q_5]-\frac{47}{84}\text{Cos}[Q_1]\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]-\frac{47}{84}\text{Sin}[Q_1]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]+X_{base}$$

ReplaceRepeated: Exiting after

$-1+\frac{\text{Cos}[Q_1]}{3}+\frac{3}{2}\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\frac{3}{2}\text{Sin}[Q_1]\,(\text{Cos}[Q_2]\,\text{Cos}[Q_3]-\text{Sin}[Q_2]\,\text{Sin}[Q_3])+\frac{47}{84}(\text{Cos}[Q_1]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]+\text{Sin}[Q_1]\,(\text{Sin}[$
Subscript[≪2≫]] (Times[≪3≫] + Times[≪4≫]) + Cos[Subscript[≪2≫]] (Times[≪2≫] + Times[≪4≫]))) +
Y_{base} scanned 3 times.

Out[208]=
$$-1+\frac{\text{Cos}[Q_1]}{3}+\frac{3}{2}\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\frac{3}{2}\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Sin}[Q_1]+\frac{47}{84}\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_5]\,\text{Sin}[Q_1]-\frac{3}{2}\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]-\frac{47}{84}\text{Cos}[Q_5]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]-\frac{47}{84}\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_5]-\frac{47}{84}\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]+\frac{47}{84}\text{Cos}[Q_1]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]+Y_{base}$$

ReplaceRepeated: Exiting after

$-\frac{5}{4}-\frac{3\,\text{Sin}[Q_2]}{2}+\frac{3}{2}(-\text{Cos}[Q_3]\,\text{Sin}[Q_2]-\text{Cos}[Q_2]\,\text{Sin}[Q_3])+\frac{47}{84}(\text{Cos}[Q_2]\,(-\text{Cos}[\text{Subscript}[\ll2\gg]]\,\text{Sin}[\text{Subscript}[\ll2\gg]]-\text{Cos}[$
Subscript[≪2≫]] Cos[Subscript[≪2≫]] Sin[Subscript[≪2≫]]) – Sin[Q₂] (Cos[Subscript[≪2≫]] Cos[Subscript[
≪2≫]] – Cos[Subscript[≪2≫]] Sin[Subscript[≪2≫]] Sin[Subscript[≪2≫]]))
scanned 2 times.

Out[209]=
$$-\frac{5}{4}-\frac{3\,\text{Sin}[Q_2]}{2}-\frac{3}{2}\text{Cos}[Q_3]\,\text{Sin}[Q_2]-\frac{47}{84}\text{Cos}[Q_3]\,\text{Cos}[Q_5]\,\text{Sin}[Q_2]-\frac{3}{2}\text{Cos}[Q_2]\,\text{Sin}[Q_3]-\frac{47}{84}\text{Cos}[Q_2]\,\text{Cos}[Q_5]\,\text{Sin}[Q_3]-\frac{47}{84}\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_5]+\frac{47}{84}\text{Cos}[Q_4]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]$$

where the equations will be set to zero below. The remaining equations will be selected from the C matrices being equated element by element (set to zero below)

In[210]:= $eq4 = C_{act}[[1]][[1]] - C_{des}[\theta_r, \theta_p, \theta_y][[1]][[1]];$
$eq5 = C_{act}[[1]][[2]] - C_{des}[\theta_r, \theta_p, \theta_y][[1]][[2]];$
$eq6 = C_{act}[[1]][[3]] - C_{des}[\theta_r, \theta_p, \theta_y][[1]][[3]];$

$eq7 = C_{act}[[2]][[1]] - C_{des}[\theta_r, \theta_p, \theta_y][[2]][[1]];$
$eq8 = C_{act}[[2]][[2]] - C_{des}[\theta_r, \theta_p, \theta_y][[2]][[2]];$
$eq9 = C_{act}[[2]][[3]] - C_{des}[\theta_r, \theta_p, \theta_y][[2]][[3]];$

$eq10 = C_{act}[[3]][[1]] - C_{des}[\theta_r, \theta_p, \theta_y][[3]][[1]];$
$eq11 = C_{act}[[3]][[2]] - C_{des}[\theta_r, \theta_p, \theta_y][[3]][[2]];$
$eq12 = C_{act}[[3]][[3]] - C_{des}[\theta_r, \theta_p, \theta_y][[3]][[3]];$

Solve the equations for best first guess at angles and base position

The strategy to solve the inverse kinematics depends on the design of the robot. There are several closed form solutions for industrial robots, see Ch4 of the class textbook by Craig.

First try to lock in initial estimates of the base location and angles ,
X_{base},Y_{base}, Q_1, Q_2, Q_3, Q_4, Q_5, and Q_6. Since the base can move this has many possible solutions.

In[218]:= **eq1temp = eq1**

Out[218]=
$$-1 + \frac{3}{2}\operatorname{Cos}[Q_1]\operatorname{Cos}[Q_2] + \frac{3}{2}\operatorname{Cos}[Q_1]\operatorname{Cos}[Q_2]\operatorname{Cos}[Q_3] + \frac{47}{84}\operatorname{Cos}[Q_1]\operatorname{Cos}[Q_2]\operatorname{Cos}[Q_3]\operatorname{Cos}[Q_5] - \frac{\operatorname{Sin}[Q_1]}{3} - \frac{3}{2}\operatorname{Cos}[Q_1]\operatorname{Sin}[Q_2]\operatorname{Sin}[Q_3] - \frac{47}{84}\operatorname{Cos}[Q_1]\operatorname{Cos}[Q_5]\operatorname{Sin}[Q_2]\operatorname{Sin}[Q_3] - \frac{47}{84}\operatorname{Cos}[Q_1]\operatorname{Cos}[Q_3]\operatorname{Cos}[Q_4]\operatorname{Sin}[Q_2]\operatorname{Sin}[Q_5] - \frac{47}{84}\operatorname{Cos}[Q_1]\operatorname{Cos}[Q_2]\operatorname{Cos}[Q_4]\operatorname{Sin}[Q_3]\operatorname{Sin}[Q_5] - \frac{47}{84}\operatorname{Sin}[Q_1]\operatorname{Sin}[Q_4]\operatorname{Sin}[Q_5] + X_{base}$$

In[219]:= **eq2temp = eq2**

Out[219]=
$$-1 + \frac{\operatorname{Cos}[Q_1]}{3} + \frac{3}{2}\operatorname{Cos}[Q_2]\operatorname{Sin}[Q_1] + \frac{3}{2}\operatorname{Cos}[Q_2]\operatorname{Cos}[Q_3]\operatorname{Sin}[Q_1] + \frac{47}{84}\operatorname{Cos}[Q_2]\operatorname{Cos}[Q_3]\operatorname{Cos}[Q_5]\operatorname{Sin}[Q_1] - \frac{3}{2}\operatorname{Sin}[Q_1]\operatorname{Sin}[Q_2]\operatorname{Sin}[Q_3] - \frac{47}{84}\operatorname{Cos}[Q_5]\operatorname{Sin}[Q_1]\operatorname{Sin}[Q_2]\operatorname{Sin}[Q_3] - \frac{47}{84}\operatorname{Cos}[Q_3]\operatorname{Cos}[Q_4]\operatorname{Sin}[Q_1]\operatorname{Sin}[Q_2]\operatorname{Sin}[Q_5] - \frac{47}{84}\operatorname{Cos}[Q_2]\operatorname{Cos}[Q_4]\operatorname{Sin}[Q_1]\operatorname{Sin}[Q_3]\operatorname{Sin}[Q_5] + \frac{47}{84}\operatorname{Cos}[Q_1]\operatorname{Sin}[Q_4]\operatorname{Sin}[Q_5] + Y_{base}$$

In[220]:= **eq3temp = eq3**

Out[220]= $-\frac{5}{4}-\frac{3\,\mathrm{Sin}[Q_2]}{2}-\frac{3}{2}\,\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_2]-\frac{47}{84}\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_2]-\frac{3}{2}\,\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_3]-\frac{47}{84}\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_3]-\frac{47}{84}\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_5]+\frac{47}{84}\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]$

We need something to drive the base to a position that will not have the robot all tied up on itself or outstretched to far. So we will try to align the tool axes with the desired axes is some optimal sense. Here we want the dot products to be 1 for the main components.

In[221]:= **desTool1 = $C_{des}[\theta_r, \theta_p, \theta_y]$[[1]][[1]] n[1] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[1]][[2]] n[2] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[1]][[3]] n[3]**

Out[221]= $\frac{\hat{n}_1}{2}+\frac{\hat{n}_2}{2}+\frac{\hat{n}_3}{\sqrt{2}}$

In[222]:= **eqV1temp = 1 == (h[1].desTool1 // TranHtoN) //. $\{q_{n\_}[t] \to Q_n\}$**

Out[222]= $1 == \frac{1}{\sqrt{2}}(\mathrm{Cos}[Q_2]\,(-\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_3]-\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_5])-\mathrm{Sin}[Q_2]\,(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]-\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]))+\frac{1}{2}\,(-\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_5]+\mathrm{Cos}[Q_1]\,(\mathrm{Sin}[Q_2]\,(-\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_3]-\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_5])+\mathrm{Cos}[Q_2]\,(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]-\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5])))+\frac{1}{2}\,(\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_5]+\mathrm{Sin}[Q_1]\,(\mathrm{Sin}[Q_2]\,(-\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_3]-\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_5])+\mathrm{Cos}[Q_2]\,(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]-\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5])))$

In[223]:= **desTool2 = $C_{des}[\theta_r, \theta_p, \theta_y]$[[2]][[1]] n[1] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[2]][[2]] n[2] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[2]][[3]] n[3]**

Out[223]= $-\frac{\hat{n}_1}{2}-\frac{\hat{n}_2}{2}+\frac{\hat{n}_3}{\sqrt{2}}$

In[224]:= **eqV2temp = 1 == (h[2].desTool2 // TranHtoN) //. $\{q_{n\_}[t] \to Q_n\}$**

Out[224]=
$$1 == \frac{1}{\sqrt{2}}(\mathrm{Cos}[Q_2]\ (-\mathrm{Sin}[Q_3]\ \mathrm{Sin}[Q_5]\ \mathrm{Sin}[Q_6] + \mathrm{Cos}[Q_3]\ (\mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_4] + \mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Sin}[Q_6])) - \mathrm{Sin}[Q_2]\ (\mathrm{Cos}[Q_3]\ \mathrm{Sin}[Q_5]\ \mathrm{Sin}[Q_6] + \mathrm{Sin}[Q_3]\ (\mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_4] + \mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Sin}[Q_6]))) + \frac{1}{2}(\mathrm{Sin}[Q_1]\ (\mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_6] - \mathrm{Cos}[Q_5]\ \mathrm{Sin}[Q_4]\ \mathrm{Sin}[Q_6]) - \mathrm{Cos}[Q_1]\ (\mathrm{Sin}[Q_2]\ (-\mathrm{Sin}[Q_3]\ \mathrm{Sin}[Q_5]\ \mathrm{Sin}[Q_6] + \mathrm{Cos}[Q_3]\ (\mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_4] + \mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Sin}[Q_6])) + \mathrm{Cos}[Q_2]\ (\mathrm{Cos}[Q_3]\ \mathrm{Sin}[Q_5]\ \mathrm{Sin}[Q_6] + \mathrm{Sin}[Q_3]\ (\mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_4] + \mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Sin}[Q_6])))) + \frac{1}{2}(-\mathrm{Cos}[Q_1]\ (\mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_6] - \mathrm{Cos}[Q_5]\ \mathrm{Sin}[Q_4]\ \mathrm{Sin}[Q_6]) - \mathrm{Sin}[Q_1]\ (\mathrm{Sin}[Q_2]\ (-\mathrm{Sin}[Q_3]\ \mathrm{Sin}[Q_5]\ \mathrm{Sin}[Q_6] + \mathrm{Cos}[Q_3]\ (\mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_4] + \mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Sin}[Q_6])) + \mathrm{Cos}[Q_2]\ (\mathrm{Cos}[Q_3]\ \mathrm{Sin}[Q_5]\ \mathrm{Sin}[Q_6] + \mathrm{Sin}[Q_3]\ (\mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_4] + \mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Sin}[Q_6]))))$$

In[225]:= **desTool3 = $C_{des}[\theta_r, \theta_p, \theta_y]$[[3]][[1]] n[1] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[3]][[2]] n[2] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[3]][[3]] n[3]**

Out[225]=
$$\frac{\hat{n}_1}{\sqrt{2}} - \frac{\hat{n}_2}{\sqrt{2}}$$

In[226]:= **eqV3temp = 1 == (h[3].desTool3 // TranHtoN) //. $\{q_{n\_}[t] \to Q_n\}$**

Out[226]=
$$1 == \frac{1}{\sqrt{2}}(-\mathrm{Sin}[Q_1]\ (-\mathrm{Cos}[Q_5]\ \mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_4] - \mathrm{Cos}[Q_4]\ \mathrm{Sin}[Q_6]) + \mathrm{Cos}[Q_1]\ (\mathrm{Sin}[Q_2]\ (-\mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_3]\ \mathrm{Sin}[Q_5] + \mathrm{Cos}[Q_3]\ (\mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Cos}[Q_6] - \mathrm{Sin}[Q_4]\ \mathrm{Sin}[Q_6])) + \mathrm{Cos}[Q_2]\ (\mathrm{Cos}[Q_3]\ \mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_5] + \mathrm{Sin}[Q_3]\ (\mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Cos}[Q_6] - \mathrm{Sin}[Q_4]\ \mathrm{Sin}[Q_6])))) - \frac{1}{\sqrt{2}}(\mathrm{Cos}[Q_1]\ (-\mathrm{Cos}[Q_5]\ \mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_4] - \mathrm{Cos}[Q_4]\ \mathrm{Sin}[Q_6]) + \mathrm{Sin}[Q_1]\ (\mathrm{Sin}[Q_2]\ (-\mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_3]\ \mathrm{Sin}[Q_5] + \mathrm{Cos}[Q_3]\ (\mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Cos}[Q_6] - \mathrm{Sin}[Q_4]\ \mathrm{Sin}[Q_6])) + \mathrm{Cos}[Q_2]\ (\mathrm{Cos}[Q_3]\ \mathrm{Cos}[Q_6]\ \mathrm{Sin}[Q_5] + \mathrm{Sin}[Q_3]\ (\mathrm{Cos}[Q_4]\ \mathrm{Cos}[Q_5]\ \mathrm{Cos}[Q_6] - \mathrm{Sin}[Q_4]\ \mathrm{Sin}[Q_6]))))$$

The robot reach is defined on a sphere about the base coordinate origin. Need to get the base within reach. The minimum base position with respect to reach sphere is found subject to the constraints that the tool axes should be closely aligned with the desired axes and that the joint angles have physical limits due to collisions with other parts, etc.

First we calculate the radius of the sphere of reach based on just the straight reach of the robot. The scaled pointerLength is subtracted to get closer if needed.

In[227]:= **scalePointer = 1 ;**

In[228]:= **radius = (OrP.n[1] // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN // TranGtoN // TranHtoN) //. $\{q_{n\_}[t] \to 0, X_{base} \to 0, Y_{base} \to 0\}$**

Out[228]=
$$\frac{299}{84}$$

```
In[229]:= eqReach = (eq1temp)^2 + (eq2temp)^2 + (eq3temp)^2 - (radius - scalePointer pointerLength)^2;

In[230]:= minQXY = Minimize[{eqReach, eqV1temp, eqV2temp, eqV3temp, - .9 Pi ≤ Q1 ≤ .9 Pi,
            - (Pi + Pi / 4 ) ≤ Q2 ≤ Pi / 4, - Pi / 2 ≤ Q3 ≤ Pi / 2, -2 Pi ≤ Q4 ≤ 2 Pi,
            - Pi ≤ Q5 ≤ Pi, -2 Pi ≤ Q6 ≤ 2 Pi}, {Q1, Q2, Q3, Q4, Q5, Q6, Xbase, Ybase}]

Out[230]= {-10.9529, {Q1 → 1.75072, Q2 → -1.14191, Q3 → 1.48865,
            Q4 → 0.627561, Q5 → -1.43217, Q6 → 0.674693, Xbase → 1.41219, Ybase → -1.22177}}
```

Here are the initial guess at the base coordinates and angles

```
In[231]:= solX = minQXY[[2]][[7]]

Out[231]= Xbase → 1.41219

In[232]:= solY = minQXY[[2]][[8]]

Out[232]= Ybase → -1.22177

In[233]:= solQ1 = minQXY[[2]][[1]]

Out[233]= Q1 → 1.75072

In[234]:= solQ2 = minQXY[[2]][[2]]

Out[234]= Q2 → -1.14191

In[235]:= solQ3 = minQXY[[2]][[3]]

Out[235]= Q3 → 1.48865

In[236]:= solQ4 = minQXY[[2]][[4]]

Out[236]= Q4 → 0.627561

In[237]:= solQ5 = minQXY[[2]][[5]]

Out[237]= Q5 → -1.43217

In[238]:= solQ6 = minQXY[[2]][[6]]

Out[238]= Q6 → 0.674693
```

Solve the full equations for angles with base positions known

Initial guesses at solution and root finder algorithm to refine the initial solutions. The optimal search above is too slow for real-time operations, but if we have good initial guesses, they can be refined with this operation and then this result can be used to start the next solution if the next desired location is near this one.

In[239]:=
```
q1o = Q1 /. solQ1;
q2o = Q2 /. solQ2;
q3o = Q3 /. solQ3;
q4o = Q4 /. solQ4;
q5o = Q5 /. solQ5;
q6o = Q6 /. solQ6;
invKinSol = FindRoot[
  {(eq1 /. solX) == 0, (eq2 /. solY) == 0, (eq3) == 0, (eq4) == 0, (eq8) == 0, (eq12) == 0},
  {{Q1, q1o, -.9 Pi, .9 Pi}, {Q2, q2o, - (Pi + Pi / 4 ), Pi / 4}, {Q3, q3o, - Pi / 2, Pi / 2},
   {Q4, q4o, -2 Pi, 2 Pi}, {Q5, q5o, - Pi, Pi}, {Q6, q6o, - 2 Pi, 2 Pi}}, MaxIterations → 10 000]
```

Out[245]= $\{Q_1 \to 1.75068,\ Q_2 \to -1.14193,\ Q_3 \to 1.48871,\ Q_4 \to 0.627375,\ Q_5 \to -1.43216,\ Q_6 \to 0.674921\}$

Compare desired to actual orientations

In[246]:=
```
Cdes[θr, θp, θy] // MatrixForm
Cact //. invKinSol // Chop // MatrixForm
Cerr = (Cdes[θr, θp, θy] - Cact) //. invKinSol // Chop // MatrixForm
```

Out[246]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

Out[247]//MatrixForm=

$$\begin{pmatrix} 0.5 & 0.5 & 0.707107 \\ -0.5 & -0.5 & 0.707107 \\ 0.707107 & -0.707107 & 0 \end{pmatrix}$$

Out[248]//MatrixForm=

$$\begin{pmatrix} 0 & 1.95905 \times 10^{-8} & -1.38526 \times 10^{-8} \\ 1.95905 \times 10^{-8} & 0 & 1.38526 \times 10^{-8} \\ 1.38526 \times 10^{-8} & 1.38526 \times 10^{-8} & 0 \end{pmatrix}$$

Compare desired and actual position

In[249]:=
```
Xerr = eq1 //. solX //. invKinSol // Chop
Yerr = eq2 //. solY //. invKinSol // Chop
Zerr = eq3 //. invKinSol // Chop
```

Out[249]= 0

Out[250]= 0

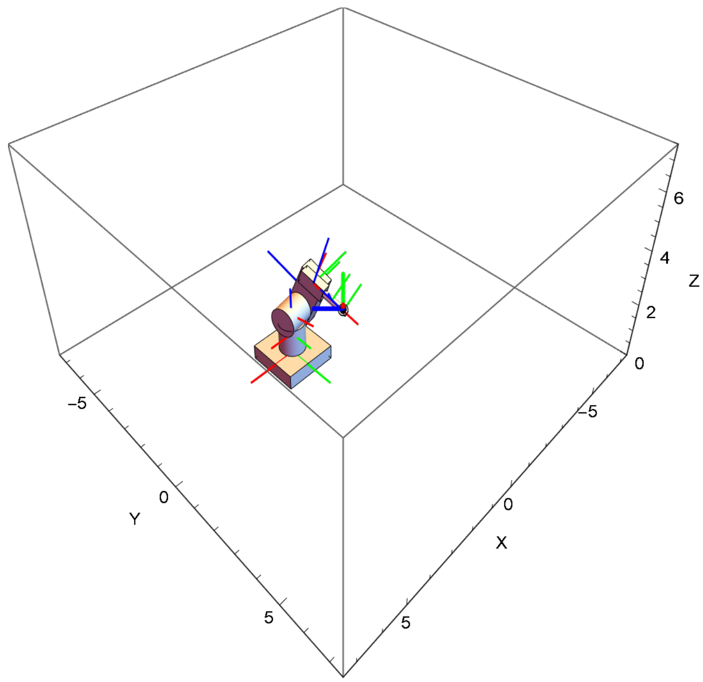
Out[251]= 0

See if these solutions work. Create composite graphic to check the inverse solution feasibility

In[252]:=
```
x[t_] = Xbase /. solX;
y[t_] = Ybase /. solY;
q1[t_] = Q1 /. invKinSol;
q2[t_] = Q2 /. invKinSol;
q3[t_] = Q3 /. invKinSol;
q4[t_] = Q4 /. invKinSol;
q5[t_] = Q5 /. invKinSol;
q6[t_] = Q6 /. invKinSol;
```

In[260]:=
```
robotGraphicInvKin = {
   (*desired point*)
   {PointSize[.01], Point[{Xdes, Ydes, Zdes}]},

   (*desired tool orientation*)
   Translate[GeometricTransformation[
     {{AbsoluteThickness[2], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
      {AbsoluteThickness[2], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
      {AbsoluteThickness[2], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}},
     Transpose[Cdes[θr, θp, θy]]], {Xdes, Ydes, Zdes}],

   (*Base graphic*)
   Translate[GeometricTransformation[baseGraphic, Transpose[rotA]], {xAo, yAo, zAo}],

   (*Riser graphic*)
   Translate[GeometricTransformation[riserGraphic, Transpose[rotB]], {xBo, yBo, zBo}],

   (*Shoulder graphic*)
   Translate[
    GeometricTransformation[shoulderGraphic, Transpose[rotC]], {xCo, yCo, zCo}],

   (*Arm1 graphic*)
   Translate[GeometricTransformation[arm1Graphic, Transpose[rotD]], {xDo, yDo, zDo}],

   (*Arm2 graphic*)
   Translate[GeometricTransformation[arm2Graphic, Transpose[rotE]], {xEo, yEo, zEo}],

   (*Arm3 graphic*)
   Translate[GeometricTransformation[arm3Graphic, Transpose[rotF]], {xFo, yFo, zFo}],

   (*Wrist1 graphic*)
   Translate[GeometricTransformation[wrist1Graphic, Transpose[rotG]], {xGo, yGo, zGo}],

   (*Wrist2 graphic*)
   Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]
  };
Show[Graphics3D[robotGraphicInvKin, ViewPoint -> {1, 1, 1},
  ViewVertical -> {0, 0, 1}, ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed → True,
  Axes -> True, PlotRange -> {{-scale, scale}, {-scale, scale}, {0, scale}},
  AspectRatio -> 1, AxesLabel → {"X", "Y", "Z"}]]
```

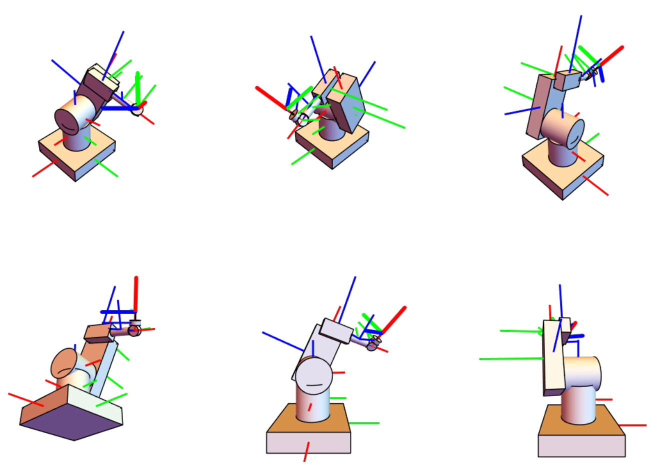
Out[261]=



Various views. Do they look doable, no collisions between parts, etc.

In[262]:=
```
Show[GraphicsGrid[
  {{Graphics3D[robotGraphicInvKin, ViewPoint → {1, 1, 1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicInvKin, ViewPoint → {-1, 1, 1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicInvKin, ViewPoint → {1, -1, 1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
   {Graphics3D[robotGraphicInvKin, ViewPoint → {1, 1, -1}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicInvKin, ViewPoint → {1, 0, 0}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicInvKin, ViewPoint → {0, -1, 0}, ViewVertical → {0, 0, 1},
     ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

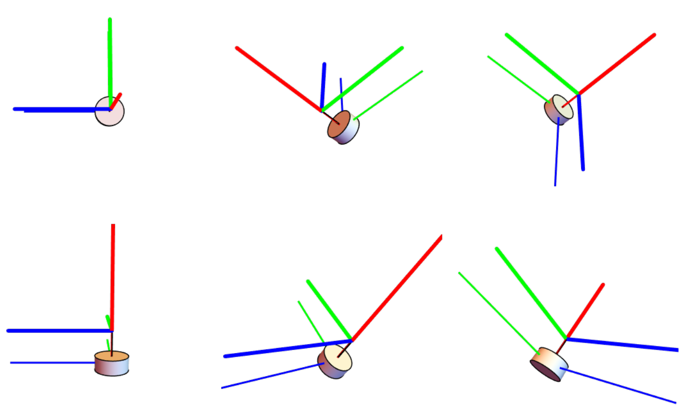
Out[262]=



Here is just the tool, look for frames to line-up

In[263]:=
```
toolGraphicInvKin =
  {{PointSize[0.01`], Point[{Xdes, Ydes, Zdes}]}, Translate[GeometricTransformation[
     {{AbsoluteThickness[2], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
      {AbsoluteThickness[2], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
      {AbsoluteThickness[2], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}},
     Transpose[Cdes[θr, θp, θy]]], {Xdes, Ydes, Zdes}],
   Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]};
Show[GraphicsGrid[{{Graphics3D[toolGraphicInvKin, ViewPoint → {1, 1, 1},
      ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[toolGraphicInvKin, ViewPoint → {-1, 1, 1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[toolGraphicInvKin, ViewPoint → {1, -1, 1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
   {Graphics3D[toolGraphicInvKin, ViewPoint → {1, 1, -1}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[toolGraphicInvKin, ViewPoint → {1, 0, 0}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[toolGraphicInvKin, ViewPoint → {0, -1, 0}, ViewVertical → {0, 0, 1},
      ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

Out[264]=



Inverse Kinematics Animation

Here we create a simple path plan to get form the rest position to the desired position.

In[265]:=
```
x[t_] =.
y[t_] =.
q1[t_] =.
q2[t_] =.
q3[t_] =.
q4[t_] =.
q5[t_] =.
q6[t_] =.
tf =.
```

In[274]:=
```
x[t_] = Xbase t / tf /. solX;
y[t_] = Ybase t / tf /. solY;
q1[t_] = Q1 t / tf /. invKinSol;
q2[t_] = Q2 t / tf /. invKinSol;
q3[t_] = Q3 t / tf /. invKinSol;
q4[t_] = Q4 t / tf /. invKinSol;
q5[t_] = Q5 t / tf /. invKinSol;
q6[t_] = Q6 t / tf /. invKinSol;
```

Create composite graphic out of parts that have been rotated and translated

In[282]:=
```
robotGraphicInvKinAnim = {
   (*desired point*)
   {PointSize[.01], Point[{Xdes, Ydes, Zdes}]},

   (*desired tool orientation*)
   Translate[GeometricTransformation[
     {{AbsoluteThickness[2], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
      {AbsoluteThickness[2], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
      {AbsoluteThickness[2], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}},
     Transpose[Cdes[θr, θp, θy]]], {Xdes, Ydes, Zdes}],

   (*Base graphic*)
   Translate[GeometricTransformation[baseGraphic, Transpose[rotA]], {xAo, yAo, zAo}],

   (*Riser graphic*)
   Translate[GeometricTransformation[riserGraphic, Transpose[rotB]], {xBo, yBo, zBo}],

   (*Shoulder graphic*)
   Translate[
    GeometricTransformation[shoulderGraphic, Transpose[rotC]], {xCo, yCo, zCo}],

   (*Arm1 graphic*)
   Translate[GeometricTransformation[arm1Graphic, Transpose[rotD]], {xDo, yDo, zDo}],

   (*Arm2 graphic*)
   Translate[GeometricTransformation[arm2Graphic, Transpose[rotE]], {xEo, yEo, zEo}],

   (*Arm3 graphic*)
   Translate[GeometricTransformation[arm3Graphic, Transpose[rotF]], {xFo, yFo, zFo}],

   (*Wrist1 graphic*)
   Translate[GeometricTransformation[wrist1Graphic, Transpose[rotG]], {xGo, yGo, zGo}],

   (*Wrist2 graphic*)
   Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]
  };
```

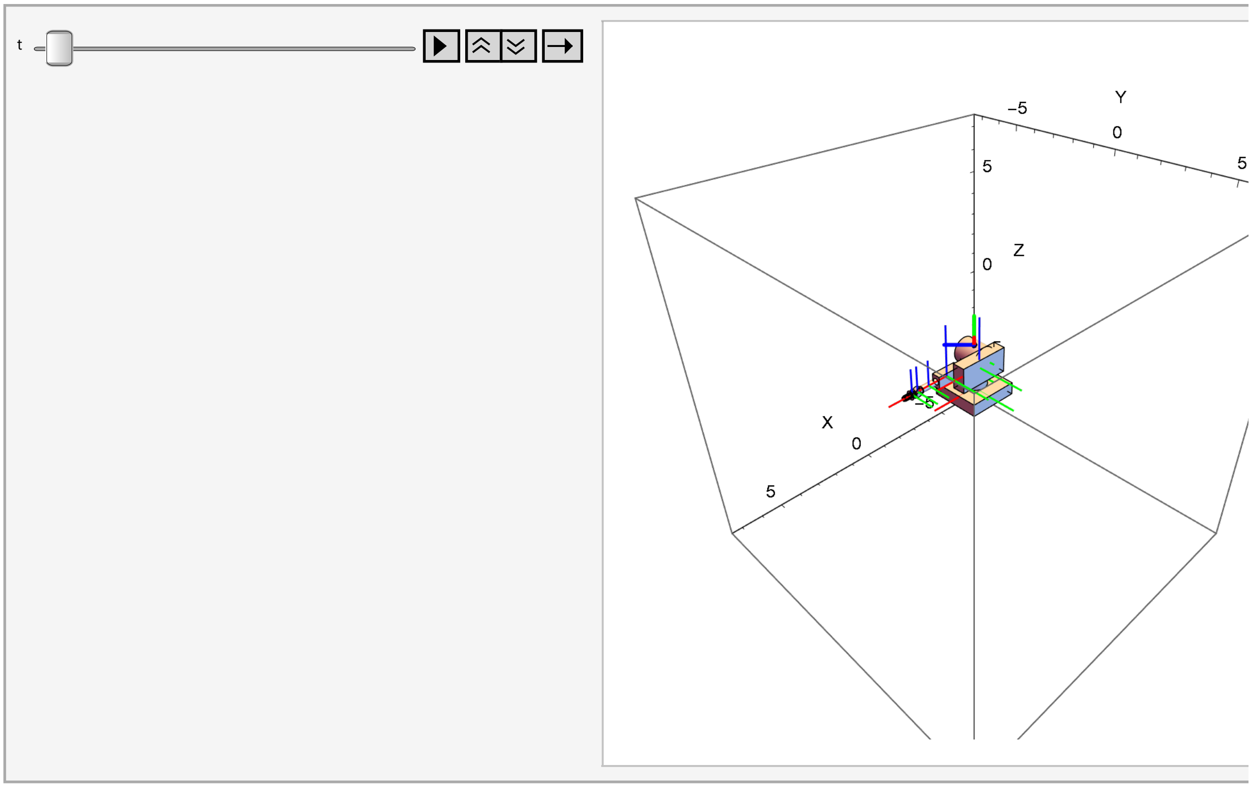
In[283]:=
```
robotGraphicInvKinAnimT[t_] = robotGraphicInvKinAnim;
```

Loop over time

In[284]:=
```
tf = 2;
```

In[285]:=
```
Animate[Show[Graphics3D[robotGraphicInvKinAnimT[t], ViewPoint -> {1, 1, 1},
    ViewVertical -> {0, 0, 1}, ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> True, Axes -> True,
    PlotRange -> {{-scale, scale}, {-scale, scale}, {-scale, scale}}, AspectRatio -> 1,
    AxesLabel → {"X", "Y", "Z"}]], {t, 0, tf, tf / 500}, AnimationRunning → False]
```

Out[285]=



We need to set up nonlinear equations to be solved to find angles and positions given desired pointer tip location and the tool frame orientation.

First clear all the variables of the kinematics. Sometimes this may cause an error if they have not been assigned numbers yet. Ignore the error and proceed.

In[286]:=
```
x[t_] =.
y[t_] =.
q1[t_] =.
q2[t_] =.
q3[t_] =.
q4[t_] =.
q5[t_] =.
q6[t_] =.
```

Desired and actual tool orientation

Using the generic rotations from above we will assume an Euler: roll-pitch-yaw squence or an Euler 1-2-3 sequence to construct the desired tool orientation.

In[294]:=
```
Cdes[roll_, pitch_ , yaw_] := rot3[yaw].rot2[pitch].rot1[roll]
```

Here is an example

In[295]:=
```
MatrixForm[C_des[Pi, Pi / 2, Pi / 3]]
```

Out[295]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 1 & 0 & 0 \end{pmatrix}$$

The actual rotation matrix in terms of our robot parameters is given as follows with the time dependence removed for simplicity

In[296]:=
```
C_act = rotH //. {q_n_[t] → Q_n};
```

Desired and actual tool tip position

The desired position is just a set of three numbers (X_{des},Y_{des},Z_{des}). The actual position vector out to the tool or pointer is given as follows with the time dependence removed

In[297]:=
```
OrP = OrAo + AorBo + BorCo + CorDo + DorEo + EorFo + ForGo + GorHo + HorP //.
   {x[t] → X_base, y[t] → Y_base}
```

Out[297]=
$$\frac{3\,\hat{b}_3}{4}+\frac{3\,\hat{a}_3}{4}+\frac{\hat{c}_2}{2}+\frac{3\,\hat{d}_1}{2}+\frac{\hat{d}_2}{4}+\frac{2\,\hat{e}_1}{3}-\frac{5\,\hat{e}_2}{12}+\frac{5\,\hat{f}_1}{6}+\frac{5\,\hat{g}_1}{21}+\frac{9\,\hat{h}_1}{28}+X_{base}\,\hat{n}_1+Y_{base}\,\hat{n}_2+\frac{\hat{n}_3}{4}$$

The Newtonian X,Y,Z position of the point is given as follows with the time dependence removed

In[298]:=
```
X_act = (OrP.n[1] // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN //
     TranGtoN // TranHtoN) //. {q_n_[t] → Q_n}
```

Out[298]=
$$\frac{3}{2}\text{Cos}[Q_1]\,\text{Cos}[Q_2]-\frac{\text{Sin}[Q_1]}{3}+\frac{3}{2}\text{Cos}[Q_1]\,(\text{Cos}[Q_2]\,\text{Cos}[Q_3]-\text{Sin}[Q_2]\,\text{Sin}[Q_3])+$$
$$\frac{47}{84}(-\text{Sin}[Q_1]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]+\text{Cos}[Q_1]\,(\text{Sin}[Q_2]\,(-\text{Cos}[Q_5]\,\text{Sin}[Q_3]-\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_5])+$$
$$\text{Cos}[Q_2]\,(\text{Cos}[Q_3]\,\text{Cos}[Q_5]-\text{Cos}[Q_4]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5])))+X_{base}$$

In[299]:=
```
Y_act = (OrP.n[2] // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN //
     TranGtoN // TranHtoN) //. {q_n_[t] → Q_n}
```

Out[299]=
$$\frac{\text{Cos}[Q_1]}{3}+\frac{3}{2}\text{Cos}[Q_2]\,\text{Sin}[Q_1]+\frac{3}{2}\text{Sin}[Q_1]\,(\text{Cos}[Q_2]\,\text{Cos}[Q_3]-\text{Sin}[Q_2]\,\text{Sin}[Q_3])+$$
$$\frac{47}{84}(\text{Cos}[Q_1]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]+\text{Sin}[Q_1]\,(\text{Sin}[Q_2]\,(-\text{Cos}[Q_5]\,\text{Sin}[Q_3]-\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_5])+$$
$$\text{Cos}[Q_2]\,(\text{Cos}[Q_3]\,\text{Cos}[Q_5]-\text{Cos}[Q_4]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5])))+Y_{base}$$

In[300]:=
```
Z_act = (OrP.n[3] // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN //
    TranGtoN // TranHtoN) //. {q_n_[t] → Q_n}
```

Out[300]=
$$\frac{7}{4}-\frac{3\,\mathrm{Sin}[Q_2]}{2}+\frac{3}{2}\left(-\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_2]-\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_3]\right)+\frac{47}{84}\left(\mathrm{Cos}[Q_2]\left(-\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_3]-\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_5]\right)-\mathrm{Sin}[Q_2]\left(\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]-\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]\right)\right)$$

Create the equations for the actual angles and positions

Solve the equations for best first guess at angles and base position

The strategy to solve the inverse kinematics depends on the design of the robot. There are several closed form solutions for industrial robots, see Ch4 of the class textbook by Craig.

First try to lock in initial estimates of the base location and angles ,
X_{base},Y_{base}, Q_1, Q_2, Q_3, Q_4, Q_5, and Q_6. Since the base can move this has many possible solutions.

In[320]:=
```
eq1temp = eq1
```

Out[320]=
$$-1+\frac{3}{2}\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]+\frac{3}{2}\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]+\frac{47}{84}\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]-\frac{\mathrm{Sin}[Q_1]}{3}-\frac{3}{2}\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]-\frac{47}{84}\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]-\frac{47}{84}\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_5]-\frac{47}{84}\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]-\frac{47}{84}\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_5]+X_{base}$$

In[321]:=
```
eq2temp = eq2
```

Out[321]=
$$-1+\frac{\mathrm{Cos}[Q_1]}{3}+\frac{3}{2}\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]+\frac{3}{2}\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_1]+\frac{47}{84}\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_1]-\frac{3}{2}\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]-\frac{47}{84}\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]-\frac{47}{84}\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_5]-\frac{47}{84}\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]+\frac{47}{84}\mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_5]+Y_{base}$$

In[322]:=
```
eq3temp = eq3
```

Out[322]=
$$-\frac{5}{4}-\frac{3\,\mathrm{Sin}[Q_2]}{2}-\frac{3}{2}\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_2]-\frac{47}{84}\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_2]-\frac{3}{2}\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_3]-\frac{47}{84}\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_3]-\frac{47}{84}\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_5]+\frac{47}{84}\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]$$

We need something to drive the base to a position that will not have the robot all tied up on itself or outstretched to far. So we will try to align the tool axes with the desired axes is some optimal sense. Here we want the dot products to be 1 for the main components.

In[323]:= desTool1 = $C_{des}[\theta_r, \theta_p, \theta_y]$[[1]][[1]] n[1] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[1]][[2]] n[2] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[1]][[3]] n[3]

Out[323]= $$\frac{\hat{n}_1}{2} + \frac{\hat{n}_2}{2} + \frac{\hat{n}_3}{\sqrt{2}}$$

In[324]:= eqV1temp = 1 == (h[1].desTool1 // TranHtoN) //. $\{q_{n\_}[t] \to Q_n\}$

Out[324]=
$$1 == \frac{1}{\sqrt{2}}(\text{Cos}[Q_2]\ (-\text{Cos}[Q_5]\ \text{Sin}[Q_3] - \text{Cos}[Q_3]\ \text{Cos}[Q_4]\ \text{Sin}[Q_5]) - \text{Sin}[Q_2]\ (\text{Cos}[Q_3]\ \text{Cos}[Q_5] - \text{Cos}[Q_4]\ \text{Sin}[Q_3]\ \text{Sin}[Q_5])) + \frac{1}{2}(-\text{Sin}[Q_1]\ \text{Sin}[Q_4]\ \text{Sin}[Q_5] + \text{Cos}[Q_1]\ (\text{Sin}[Q_2]\ (-\text{Cos}[Q_5]\ \text{Sin}[Q_3] - \text{Cos}[Q_3]\ \text{Cos}[Q_4]\ \text{Sin}[Q_5]) + \text{Cos}[Q_2]\ (\text{Cos}[Q_3]\ \text{Cos}[Q_5] - \text{Cos}[Q_4]\ \text{Sin}[Q_3]\ \text{Sin}[Q_5]))) + \frac{1}{2}(\text{Cos}[Q_1]\ \text{Sin}[Q_4]\ \text{Sin}[Q_5] + \text{Sin}[Q_1]\ (\text{Sin}[Q_2]\ (-\text{Cos}[Q_5]\ \text{Sin}[Q_3] - \text{Cos}[Q_3]\ \text{Cos}[Q_4]\ \text{Sin}[Q_5]) + \text{Cos}[Q_2]\ (\text{Cos}[Q_3]\ \text{Cos}[Q_5] - \text{Cos}[Q_4]\ \text{Sin}[Q_3]\ \text{Sin}[Q_5])))$$

In[325]:= desTool2 = $C_{des}[\theta_r, \theta_p, \theta_y]$[[2]][[1]] n[1] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[2]][[2]] n[2] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[2]][[3]] n[3]

Out[325]= $$-\frac{\hat{n}_1}{2} - \frac{\hat{n}_2}{2} + \frac{\hat{n}_3}{\sqrt{2}}$$

In[326]:= eqV2temp = 1 == (h[2].desTool2 // TranHtoN) //. $\{q_{n\_}[t] \to Q_n\}$

Out[326]=
$$1 == \frac{1}{\sqrt{2}}(\text{Cos}[Q_2]\ (-\text{Sin}[Q_3]\ \text{Sin}[Q_5]\ \text{Sin}[Q_6] + \text{Cos}[Q_3]\ (\text{Cos}[Q_6]\ \text{Sin}[Q_4] + \text{Cos}[Q_4]\ \text{Cos}[Q_5]\ \text{Sin}[Q_6])) - \text{Sin}[Q_2]\ (\text{Cos}[Q_3]\ \text{Sin}[Q_5]\ \text{Sin}[Q_6] + \text{Sin}[Q_3]\ (\text{Cos}[Q_6]\ \text{Sin}[Q_4] + \text{Cos}[Q_4]\ \text{Cos}[Q_5]\ \text{Sin}[Q_6]))) + \frac{1}{2}(\text{Sin}[Q_1]\ (\text{Cos}[Q_4]\ \text{Cos}[Q_6] - \text{Cos}[Q_5]\ \text{Sin}[Q_4]\ \text{Sin}[Q_6]) - \text{Cos}[Q_1]\ (\text{Sin}[Q_2]\ (-\text{Sin}[Q_3]\ \text{Sin}[Q_5]\ \text{Sin}[Q_6] + \text{Cos}[Q_3]\ (\text{Cos}[Q_6]\ \text{Sin}[Q_4] + \text{Cos}[Q_4]\ \text{Cos}[Q_5]\ \text{Sin}[Q_6])) + \text{Cos}[Q_2]\ (\text{Cos}[Q_3]\ \text{Sin}[Q_5]\ \text{Sin}[Q_6] + \text{Sin}[Q_3]\ (\text{Cos}[Q_6]\ \text{Sin}[Q_4] + \text{Cos}[Q_4]\ \text{Cos}[Q_5]\ \text{Sin}[Q_6])))) + \frac{1}{2}(-\text{Cos}[Q_1]\ (\text{Cos}[Q_4]\ \text{Cos}[Q_6] - \text{Cos}[Q_5]\ \text{Sin}[Q_4]\ \text{Sin}[Q_6]) - \text{Sin}[Q_1]\ (\text{Sin}[Q_2]\ (-\text{Sin}[Q_3]\ \text{Sin}[Q_5]\ \text{Sin}[Q_6] + \text{Cos}[Q_3]\ (\text{Cos}[Q_6]\ \text{Sin}[Q_4] + \text{Cos}[Q_4]\ \text{Cos}[Q_5]\ \text{Sin}[Q_6])) + \text{Cos}[Q_2]\ (\text{Cos}[Q_3]\ \text{Sin}[Q_5]\ \text{Sin}[Q_6] + \text{Sin}[Q_3]\ (\text{Cos}[Q_6]\ \text{Sin}[Q_4] + \text{Cos}[Q_4]\ \text{Cos}[Q_5]\ \text{Sin}[Q_6]))))$$

In[327]:= desTool3 = $C_{des}[\theta_r, \theta_p, \theta_y]$[[3]][[1]] n[1] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[3]][[2]] n[2] + $C_{des}[\theta_r, \theta_p, \theta_y]$[[3]][[3]] n[3]

Out[327]= $$\frac{\hat{n}_1}{\sqrt{2}} - \frac{\hat{n}_2}{\sqrt{2}}$$

In[328]:= `eqV3temp = 1 == (h[3].desTool3 // TranHtoN) //. {q_n_[t] → Q_n}`

Out[328]=
$$1 == \frac{1}{\sqrt{2}} (-\text{Sin}[Q_1] (-\text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_4] - \text{Cos}[Q_4] \text{Sin}[Q_6]) + \text{Cos}[Q_1] (\text{Sin}[Q_2] (-\text{Cos}[Q_6] \text{Sin}[Q_3] \text{Sin}[Q_5] + \text{Cos}[Q_3] (\text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] - \text{Sin}[Q_4] \text{Sin}[Q_6])) + \text{Cos}[Q_2] (\text{Cos}[Q_3] \text{Cos}[Q_6] \text{Sin}[Q_5] + \text{Sin}[Q_3] (\text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] - \text{Sin}[Q_4] \text{Sin}[Q_6])))) - \frac{1}{\sqrt{2}} (\text{Cos}[Q_1] (-\text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_4] - \text{Cos}[Q_4] \text{Sin}[Q_6]) + \text{Sin}[Q_1] (\text{Sin}[Q_2] (-\text{Cos}[Q_6] \text{Sin}[Q_3] \text{Sin}[Q_5] + \text{Cos}[Q_3] (\text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] - \text{Sin}[Q_4] \text{Sin}[Q_6])) + \text{Cos}[Q_2] (\text{Cos}[Q_3] \text{Cos}[Q_6] \text{Sin}[Q_5] + \text{Sin}[Q_3] (\text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] - \text{Sin}[Q_4] \text{Sin}[Q_6]))))$$

The robot reach is defined on a sphere about the base coordinate origin. Need to get the base within reach. The minimum base position with respect to reach sphere is found subject to the constraints that the tool axes should be closely aligned with the desired axes and that the joint angles have physical limits due to collisions with other parts, etc.

First we calculate the radius of the sphere of reach based on just the straight reach of the robot. The scaled pointerLength is subtracted to get closer if needed.

In[329]:= `scalePointer = 1 ;`

In[330]:=
```
radius = (OrP.n[1] // TranAtoN // TranBtoN // TranCtoN // TranDtoN // TranEtoN // TranFtoN //
    TranGtoN // TranHtoN) //. {q_n_[t] → 0, X_base → 0, Y_base → 0}
```

Out[330]= $\frac{299}{84}$

In[331]:= `eqReach = (eq1temp)^2 + (eq2temp)^2 + (eq3temp)^2 - (radius - scalePointer pointerLength)^2;`

In[332]:=
```
minQXY = Minimize[{eqReach, eqV1temp, eqV2temp, eqV3temp, - .9 Pi ≤ Q_1 ≤ .9 Pi,
   - (Pi + Pi / 4 ) ≤ Q_2 ≤ Pi / 4, - Pi / 2 ≤ Q_3 ≤ Pi / 2, -2 Pi ≤ Q_4 ≤ 2 Pi,
   - Pi ≤ Q_5 ≤ Pi, -2 Pi ≤ Q_6 ≤ 2 Pi}, {Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, X_base, Y_base}]
```

Out[332]= $\{-10.9529, \{Q_1 \to 1.75072, Q_2 \to -1.14191, Q_3 \to 1.48865, Q_4 \to 0.627561, Q_5 \to -1.43217, Q_6 \to 0.674693, X_{base} \to 1.41219, Y_{base} \to -1.22177\}\}$

Here are the initial guess at the base coordinates and angles

In[333]:= `solX = minQXY[[2]][[7]]`

Out[333]= $X_{base} \to 1.41219$

In[334]:= `solY = minQXY[[2]][[8]]`

Out[334]= $Y_{base} \to -1.22177$

In[335]:= `solQ1 = minQXY[[2]][[1]]`

Out[335]= $Q_1 \to 1.75072$

In[336]:= `solQ2 = minQXY[[2]][[2]]`

Out[336]= $Q_2 \to -1.14191$

In[337]:=
```
solQ3 = minQXY[[2]][[3]]
```
Out[337]= $Q_3 \to 1.48865$

In[338]:=
```
solQ4 = minQXY[[2]][[4]]
```
Out[338]= $Q_4 \to 0.627561$

In[339]:=
```
solQ5 = minQXY[[2]][[5]]
```
Out[339]= $Q_5 \to -1.43217$

In[340]:=
```
solQ6 = minQXY[[2]][[6]]
```
Out[340]= $Q_6 \to 0.674693$

Solve the full equations for angles with base positions known

Inverse Kinematics Animation

Here we create a simple path plan to get form the rest position to the desired position.

In[363]:=
```
x[t_] =.
y[t_] =.
q1[t_] =.
q2[t_] =.
q3[t_] =.
q4[t_] =.
q5[t_] =.
q6[t_] =.
tf =.
```

In[372]:=
```
x[t_] = Xbase t / tf /. solX;
y[t_] = Ybase t / tf /. solY;
q1[t_] = Q1 t / tf /. invKinSol;
q2[t_] = Q2 t / tf /. invKinSol;
q3[t_] = Q3 t / tf /. invKinSol;
q4[t_] = Q4 t / tf /. invKinSol;
q5[t_] = Q5 t / tf /. invKinSol;
q6[t_] = Q6 t / tf /. invKinSol;
```

Create composite graphic out of parts that have been rotated and translated

In[380]:=
```
robotGraphicInvKinAnim = {
    (*desired point*)
    {PointSize[.01], Point[{Xdes, Ydes, Zdes}]},

    (*desired tool orientation*)
    Translate[GeometricTransformation[
      {{AbsoluteThickness[2], RGBColor[1, 0, 0], Line[{{0, 0, 0}, {vecL, 0, 0}}]},
       {AbsoluteThickness[2], RGBColor[0, 1, 0], Line[{{0, 0, 0}, {0, vecL, 0}}]},
       {AbsoluteThickness[2], RGBColor[0, 0, 1], Line[{{0, 0, 0}, {0, 0, vecL}}]}},
      Transpose[Cdes[θr, θp, θy]]], {Xdes, Ydes, Zdes}],

    (*Base graphic*)
    Translate[GeometricTransformation[baseGraphic, Transpose[rotA]], {xAo, yAo, zAo}],

    (*Riser graphic*)
    Translate[GeometricTransformation[riserGraphic, Transpose[rotB]], {xBo, yBo, zBo}],

    (*Shoulder graphic*)
    Translate[
     GeometricTransformation[shoulderGraphic, Transpose[rotC]], {xCo, yCo, zCo}],

    (*Arm1 graphic*)
    Translate[GeometricTransformation[arm1Graphic, Transpose[rotD]], {xDo, yDo, zDo}],

    (*Arm2 graphic*)
    Translate[GeometricTransformation[arm2Graphic, Transpose[rotE]], {xEo, yEo, zEo}],

    (*Arm3 graphic*)
    Translate[GeometricTransformation[arm3Graphic, Transpose[rotF]], {xFo, yFo, zFo}],

    (*Wrist1 graphic*)
    Translate[GeometricTransformation[wrist1Graphic, Transpose[rotG]], {xGo, yGo, zGo}],

    (*Wrist2 graphic*)
    Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]
   };
```

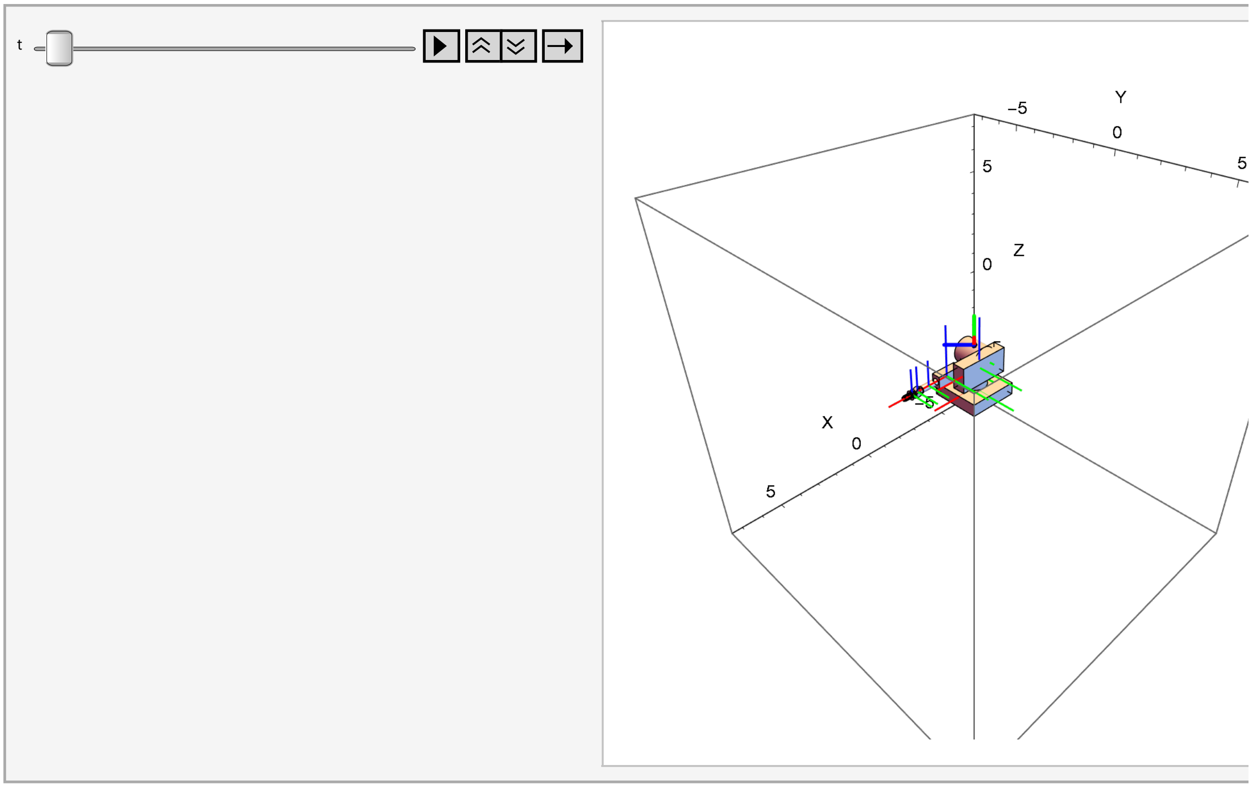
In[381]:=
```
robotGraphicInvKinAnimT[t_] = robotGraphicInvKinAnim;
```

Loop over time

In[382]:=
```
tf = 2;
```

In[383]:=
```
Animate[Show[Graphics3D[robotGraphicInvKinAnimT[t], ViewPoint -> {1, 1, 1},
   ViewVertical -> {0, 0, 1}, ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> True, Axes -> True,
   PlotRange -> {{-scale, scale}, {-scale, scale}, {-scale, scale}}, AspectRatio -> 1,
   AxesLabel → {"X", "Y", "Z"}]], {t, 0, tf, tf / 500}, AnimationRunning → False]
```

Out[383]=



Dynamic Equations of Motion

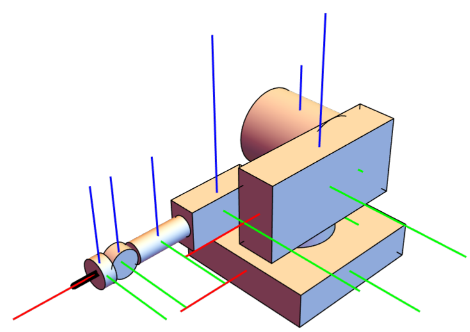
In this section we will generate the equations of motion for the system. We will use this to calculate loads that must be equilibriated to maintain static equilibrium, as well we will use the equations to predict motion given motor forces and torques. The method we will used is based on the projection method called Kane's equations, or Kane's form of the Gibbs-Appell equations.

First clear all the variables of the kinematics. Sometimes this may cause an error if they have not been assigned numbers yet. Ignore the error and proceed.

In[384]:=
```
x[t_] =.
y[t_] =.
q1[t_] =.
q2[t_] =.
q3[t_] =.
q4[t_] =.
q5[t_] =.
q6[t_] =.
A =.
B =.
t =.
```

In[395]:=
```
Show[Graphics3D[robotGraphic, ViewPoint -> {1, 1, 1}, ViewVertical -> {0, 0, 1},
  ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]]
```

Out[395]=



Create the needed terms for each body

Base platform

First we express all applied forces and torques to the given body. Do not include reactions forces unless they are active forces, like this body is pushing out the next body. The forces F_{motx} and F_{moty} are from the positional motors.

In[396]:= $F_{app}[1] = -M_1\, g\, n[3] + F_{motx}\, a[1] + F_{moty}\, a[2]$

Out[396]= $F_{motx}\, \hat{a}_1 + F_{moty}\, \hat{a}_2 - g\, M_1\, \hat{n}_3$

The torques on this body must include the reaction from the next body if it is applied in this body. Do not include constraint reactions.

In[397]:= $T_{app}[1] = 0$

Out[397]= 0

Next we need position vectors from the Newtonian frame origin to current frame origin. We also need the vector from the frame origin to the current body's center of mass.

In[398]:=
```
OrB[1] = OrAo
BrCM[1] = 0
```

Out[398]= $\frac{\hat{n}_3}{4} + \hat{n}_1\, x[t] + \hat{n}_2\, y[t]$

Out[399]= 0

We calculate the position to the center of current body's center of mass from the Newtonian origin.

In[400]:= **OrCM[1] = OrB[1] + BrCM[1]**

Out[400]= $\frac{\hat{n}_3}{4} + \hat{n}_1\, x[t] + \hat{n}_2\, y[t]$

Next we develop the expression for the angular velocity of the current body. We use simple angular velocity to calculate this.

In[401]:= **NωB[1] = omega[N, A] = 0**

Out[401]= 0

Now we calculate the velocity and acceleration of the origin and center of mass for this body.

In[402]:= **OvB[1] = DvDt[N, OrB[1]]**

Out[402]= $\hat{n}_1\, x'[t] + \hat{n}_2\, y'[t]$

In[403]:= **OvCM[1] = DvDt[N, OrCM[1]]**

Out[403]= $\hat{n}_1\, x'[t] + \hat{n}_2\, y'[t]$

In[404]:= **OxB[1] = DvDt[N, OvB[1]]**

Out[404]= $\hat{n}_1\, x''[t] + \hat{n}_2\, y''[t]$

In[405]:= **OxCM[1] = DvDt[N, OvCM[1]]**

Out[405]= $\hat{n}_1\, x''[t] + \hat{n}_2\, y''[t]$

We also need the angular acceleration.

In[406]:= **NαB[1] = DvDt[N, NωB[1]]**

Out[406]= 0

Now we express the inertia dyad for this body about its frame origin point b. This may have more terms if the body axis is not aligned with the principal axes.

In[407]:= $\mathbf{I_b[1] = Ai_{11}\, aa[1, 1] + Ai_{22}\, aa[2, 2] + Ai_{33}\, aa[3, 3]}$

Out[407]= $Ai_{11}\left(\hat{a}_1\hat{a}_1\right) + Ai_{22}\left(\hat{a}_2\hat{a}_2\right) + Ai_{33}\left(\hat{a}_3\hat{a}_3\right)$

Next we develop a transformation from this body to the previous body or frame, not all the way back to the Newtonian frame in general.

In[408]:= **relT[1] = rot0[].{n[1], n[2], n[3]};**

In[409]:= **relTran[1] = {a[1] → relT[1][[1]], a[2] → relT[1][[2]], a[3] → relT[1][[3]]}**

Out[409]= $\{\hat{a}_1 \to \hat{n}_1,\ \hat{a}_2 \to \hat{n}_2,\ \hat{a}_3 \to \hat{n}_3\}$

Now we calculate the inertia force and inertia torque for this body.

In[410]:= $\mathbf{I_f[1] = M_1\, OxCM[1]\ //\ distributeScalars}$

Out[410]= $M_1\, \hat{n}_1\, x''[t] + M_1\, \hat{n}_2\, y''[t]$

In[411]:= $I_t[1] = (M_1\, BrCM[1] \times OxB[1] + I_b[1] . N\alpha B[1] + (N\omega B[1] \times I_b[1]) . N\omega B[1])$

Out[411]= 0

Now we calculate the potential and kinetic energy for this robot part. This can be used to check numerical integration and can be used in Lagrange' s equations for the EOM if so desired.

In[412]:= $PE[1] = M_1\, g\, n[3] . OrCM[1]$

Out[412]= $\frac{g M_1}{4}$

This kinetic energy is for general case when the point b is not at the center of mass. The middle term goes away when they coincide at the center of mass.

In[413]:= $KE[1] = \frac{1}{2} M_1\, OvB[1] . OvB[1] + M_1\, OvB[1] . (N\omega B[1] \times BrCM[1]) + \frac{1}{2} (N\omega B[1] . I_b[1]) . N\omega B[1]$ // Expand

Out[413]= $\frac{1}{2} M_1\, x'[t]^2 + \frac{1}{2} M_1\, y'[t]^2$

Riser cylinder

First we express all applied forces and torques to the given body. Do not include reactions forces unless they are active forces, like this body is pushing out the next body.

In[414]:= $F_{app}[2] = - M_2\, g\, n[3]$

Out[414]= $-g M_2\, \hat{n}_3$

The torques on this body must include the reaction from the next body if it is applied in this body. Do not include constraint reactions.

In[415]:= $T_{app}[2] = -T_{mot}[1]\, b[3]$

Out[415]= $-\hat{b}_3\, T_{mot}[1]$

Next we need position vectors from the Newtonian frame origin to current frame origin. We also need the vector from the frame origin to the curent body's center of mass.

In[416]:= OrB[2] = OrAo + AorBo
BrCM[2] = 0

Out[416]= $\frac{3 \hat{a}_3}{4} + \frac{\hat{n}_3}{4} + \hat{n}_1\, x[t] + \hat{n}_2\, y[t]$

Out[417]= 0

We calculate the position to the center of current body's center of mass from the Newtonian origin.

In[418]:= OrCM[2] = OrB[2] + BrCM[2]

Out[418]= $\frac{3 \hat{a}_3}{4} + \frac{\hat{n}_3}{4} + \hat{n}_1\, x[t] + \hat{n}_2\, y[t]$

Next we develop the expression for the angular velocity of the current body. We use simple angular velocity to calculate this.

In[419]:=
```
NωB[2] = omega[N, B] = 0
```
Out[419]= 0

Now we calculate the velocity and acceleration of the orign and center of mass for this body.

In[420]:=
```
OvB[2] = DvDt[N, OrB[2]]
```
Out[420]= $\hat{n}_1\, x'[t] + \hat{n}_2\, y'[t]$

In[421]:=
```
OvCM[2] = DvDt[N, OrCM[2]]
```
Out[421]= $\hat{n}_1\, x'[t] + \hat{n}_2\, y'[t]$

In[422]:=
```
OxB[2] = DvDt[N, OvB[2]]
```
Out[422]= $\hat{n}_1\, x''[t] + \hat{n}_2\, y''[t]$

In[423]:=
```
OxCM[2] = DvDt[N, OvCM[2]]
```
Out[423]= $\hat{n}_1\, x''[t] + \hat{n}_2\, y''[t]$

We also need the angular acceleration.

In[424]:=
```
NαB[2] = DvDt[N, NωB[2]]
```
Out[424]= 0

Now we express the inertia dyad for this body about its frame origin point b. This may have more terms if the body axis is not aligned with the principal axes.

In[425]:= $I_b[2] = Bi_{11}\, bb[1, 1] + Bi_{22}\, bb[2, 2] + Bi_{33}\, bb[3, 3]$

Out[425]= $Bi_{11}\left(\hat{b}_1\hat{b}_1\right) + Bi_{22}\left(\hat{b}_2\hat{b}_2\right) + Bi_{33}\left(\hat{b}_3\hat{b}_3\right)$

Next we develop a transformation from this body to the previous body or frame.

In[426]:=
```
relT[2] = rot0[].{a[1], a[2], a[3]};
```
In[427]:=
```
relTran[2] = {b[1] → relT[2][[1]], b[2] → relT[2][[2]], b[3] → relT[2][[3]]}
```
Out[427]= $\left\{\hat{b}_1 \to \hat{a}_1,\ \hat{b}_2 \to \hat{a}_2,\ \hat{b}_3 \to \hat{a}_3\right\}$

Now we calculate the inertia force and inertia torque for this body.

In[428]:= $I_f[2] = M_2\, OxCM[2]$ // distributeScalars

Out[428]= $M_2\, \hat{n}_1\, x''[t] + M_2\, \hat{n}_2\, y''[t]$

In[429]:= $I_t[2] = (M_2\, BrCM[2] \times OxB[2] + I_b[2].N\alpha B[2] + (N\omega B[2] \times I_b[2]).N\omega B[2])$

Out[429]= 0

Now we calculate the potential and kinetic energy for this robot part. This can be used to check numerical integration and can be used in Lagrange' s equations for the EOM if so desired.

In[430]:=
```
PE[2] = M2 g n[3].OrCM[2] // Expand
```

Out[430]= $\frac{g M_2}{4} + \frac{3}{4} g\, \hat{a}_3.\hat{n}_3 M_2$

This kinetic energy is for general case when the point b is not at the center of mass. The middle term goes away when they coincide at the center of mass.

In[431]:=
```
KE[2] = 1/2 M2 OvB[2].OvB[2] + M2 OvB[2].(NωB[2] × BrCM[2]) + 1/2 (NωB[2].Ib[2]).NωB[2] // Expand
```

Out[431]= $\frac{1}{2} M_2 x'[t]^2 + \frac{1}{2} M_2 y'[t]^2$

Shoulder cylinder

First we express all applied forces and torques to the given body. Do not include reactions forces unless they are active forces, like this body is pushing out the next body.

In[432]:=
```
Fapp[3] = - M3 g n[3]
```

Out[432]= $-g M_3 \hat{n}_3$

The torques on this body must include the reaction from the next body if it is applied in this body. Do not include constraint reactions.

In[433]:=
```
Tapp[3] = Tmot[1] c[3] - Tmot[2] c[2]
```

Out[433]= $\hat{c}_3 T_{mot}[1] - \hat{c}_2 T_{mot}[2]$

Next we need position vectors from the Newtonian frame origin to current frame origin. We also need the vector from the frame origin to the curent body's center of mass.

In[434]:=
```
OrB[3] = OrAo + AorBo + BorCo
BrCM[3] = 0
```

Out[434]= $\frac{3 \hat{a}_3}{4} + \frac{3 \hat{b}_3}{4} + \frac{\hat{n}_3}{4} + \hat{n}_1 x[t] + \hat{n}_2 y[t]$

Out[435]= 0

We calculate the position to the center of current body's center of mass from the Newtonian origin.

In[436]:=
```
OrCM[3] = OrB[3] + BrCM[3]
```

Out[436]= $\frac{3 \hat{a}_3}{4} + \frac{3 \hat{b}_3}{4} + \frac{\hat{n}_3}{4} + \hat{n}_1 x[t] + \hat{n}_2 y[t]$

Next we develop the expression for the angular velocity of the current body. We use simple angular velocity to calculate this.

In[437]:=
```
NωB[3] = omega[N, C] = q1'[t] c[3]
```

Out[437]= $\hat{c}_3 q_1'[t]$

Now we calculate the velocity and acceleration of the orign and center of mass for this body.

In[438]:= `OvB[3] = DvDt[N, OrB[3]]`

Out[438]= $\hat{n}_1\,x'[t] + \hat{n}_2\,y'[t]$

In[439]:= `OvCM[3] = DvDt[N, OrCM[3]]`

Out[439]= $\hat{n}_1\,x'[t] + \hat{n}_2\,y'[t]$

In[440]:= `OxB[3] = DvDt[N, OvB[3]]`

Out[440]= $\hat{n}_1\,x''[t] + \hat{n}_2\,y''[t]$

In[441]:= `OxCM[3] = DvDt[N, OvCM[3]]`

Out[441]= $\hat{n}_1\,x''[t] + \hat{n}_2\,y''[t]$

We also need the angular acceleration.

In[442]:= `NαB[3] = DvDt[N, NωB[3]]`

Out[442]= $\hat{c}_3\,q_1''[t]$

Now we express the inertia dyad for this body about its frame origin point b. This may have more terms if the body axis is not aligned with the principal axes.

In[443]:= $\mathbf{I_b[3] = Ci_{11}\,cc[1, 1] + Ci_{22}\,cc[2, 2] + Ci_{33}\,cc[3, 3]}$

Out[443]= $Ci_{11}\,(\hat{c}_1\hat{c}_1) + Ci_{22}\,(\hat{c}_2\hat{c}_2) + Ci_{33}\,(\hat{c}_3\hat{c}_3)$

Next we develop a transformation from this body to the previous body or frame.

In[444]:= $\mathbf{relT[3] = rot3[q_1[t]].\{b[1], b[2], b[3]\};}$

In[445]:= `relTran[3] = {c[1] → relT[3][[1]], c[2] → relT[3][[2]], c[3] → relT[3][[3]]}`

Out[445]= $\{\hat{c}_1 \to Cos[q_1[t]]\,\hat{b}_1 + Sin[q_1[t]]\,\hat{b}_2,\ \hat{c}_2 \to -Sin[q_1[t]]\,\hat{b}_1 + Cos[q_1[t]]\,\hat{b}_2,\ \hat{c}_3 \to \hat{b}_3\}$

Now we calculate the inertia force and inertia torque for this body.

In[446]:= $\mathbf{I_f[3] = M_3\,OxCM[3]\ //\ distributeScalars}$

Out[446]= $M_3\,\hat{n}_1\,x''[t] + M_3\,\hat{n}_2\,y''[t]$

In[447]:= $\mathbf{I_t[3] = (M_3\,BrCM[3] \times OxB[3] + I_b[3].N\alpha B[3] + (N\omega B[3] \times I_b[3]).N\omega B[3])}$

Out[447]= $Ci_{33}\,\hat{c}_3\,q_1''[t]$

Now we calculate the potential and kinetic energy for this robot part. This can be used to check numerical integration and can be used in Lagrange' s equations for the EOM if so desired.

In[448]:= $\mathbf{PE[3] = M_3\,g\,n[3].OrCM[3]\ //\ Expand}$

Out[448]= $\frac{g\,M_3}{4} + \frac{3}{4}\,g\,\hat{a}_3.\hat{n}_3\,M_3 + \frac{3}{4}\,g\,\hat{b}_3.\hat{n}_3\,M_3$

This kinetic energy is for general case when the point b is not at the center of mass. The middle term goes away when they coincide at the center of mass.

In[449]:= $KE[3] = \frac{1}{2} M_3\, OvB[3].OvB[3] + M_3\, OvB[3].(N\omega B[3] \times BrCM[3]) + \frac{1}{2} (N\omega B[3].I_b[3]).N\omega B[3]$ // Expand

Out[449]= $\frac{1}{2} M_3\, x'[t]^2 + \frac{1}{2} M_3\, y'[t]^2 + \frac{1}{2} Ci_{33}\, q_1'[t]^2$

Arm segment 1

Arm segment 2

Arm segment 3

Wrist1

Wrist2 and tool

Equations of motion

For this robot we have:

In[543]:= $N_{bodies} = 8;$
$N_{dof} = 8;$

These next equations are written for each degree of freedom or coordinate used.

For x[t] the equations of motion is
(they will be set to zero when solution is required).

In[545]:= eq[1] =

$$\sum_{ii=1}^{N_{bodies}} \left(\left(F_{app}[ii] - I_f[ii]\right).Pvel[OvB[ii], x'[t]] + \left(T_{app}[ii] - I_t[ii]\right).Pvel[N\omega B[ii], x'[t]]\right)$$

Out[545]=
$$\hat{a}_1.\hat{n}_1 F_{motx} + \hat{a}_2.\hat{n}_1 F_{moty} - \hat{h}_1.\hat{n}_1 F_{toolx} - \hat{h}_2.\hat{n}_1 F_{tooly} - \hat{h}_3.\hat{n}_1 F_{toolz} + \frac{1}{2} \hat{c}_1 \times \hat{d}_1.\hat{n}_1 M_4 q_1'[t] q_2'[t] +$$
$$\frac{1}{4} \hat{c}_1 \times \hat{d}_2.\hat{n}_1 M_4 q_1'[t] q_2'[t] + \frac{3}{2} \hat{c}_1 \times \hat{d}_1.\hat{n}_1 M_5 q_1'[t] q_2'[t] + \frac{1}{4} \hat{c}_1 \times \hat{d}_2.\hat{n}_1 M_5 q_1'[t] q_2'[t] +$$
$$\frac{1}{6} \hat{c}_1 \times \hat{e}_1.\hat{n}_1 M_5 q_1'[t] q_2'[t] - \frac{5}{12} \hat{c}_1 \times \hat{e}_2.\hat{n}_1 M_5 q_1'[t] q_2'[t] + \frac{3}{2} \hat{c}_1 \times \hat{d}_1.\hat{n}_1 M_6 q_1'[t] q_2'[t] +$$
$$\frac{1}{4} \hat{c}_1 \times \hat{d}_2.\hat{n}_1 M_6 q_1'[t] q_2'[t] + \frac{2}{3} \hat{c}_1 \times \hat{e}_1.\hat{n}_1 M_6 q_1'[t] q_2'[t] - \frac{5}{12} \hat{c}_1 \times \hat{e}_2.\hat{n}_1 M_6 q_1'[t] q_2'[t] +$$
$$\frac{1}{3} \hat{c}_1 \times \hat{f}_1.\hat{n}_1 M_6 q_1'[t] q_2'[t] + \frac{3}{2} \hat{c}_1 \times \hat{d}_1.\hat{n}_1 M_7 q_1'[t] q_2'[t] + \frac{1}{4} \hat{c}_1 \times \hat{d}_2.\hat{n}_1 M_7 q_1'[t] q_2'[t] +$$
$$\frac{2}{3} \hat{c}_1 \times \hat{e}_1.\hat{n}_1 M_7 q_1'[t] q_2'[t] - \frac{5}{12} \hat{c}_1 \times \hat{e}_2.\hat{n}_1 M_7 q_1'[t] q_2'[t] + \frac{5}{6} \hat{c}_1 \times \hat{f}_1.\hat{n}_1 M_7 q_1'[t] q_2'[t] +$$
$$\frac{3}{2} \hat{c}_1 \times \hat{d}_1.\hat{n}_1 M_8 q_1'[t] q_2'[t] + \frac{1}{4} \hat{c}_1 \times \hat{d}_2.\hat{n}_1 M_8 q_1'[t] q_2'[t] + \frac{2}{3} \hat{c}_1 \times \hat{e}_1.\hat{n}_1 M_8 q_1'[t] q_2'[t] -$$
$$\frac{5}{12} \hat{c}_1 \times \hat{e}_2.\hat{n}_1 M_8 q_1'[t] q_2'[t] + \frac{5}{6} \hat{c}_1 \times \hat{f}_1.\hat{n}_1 M_8 q_1'[t] q_2'[t] + \frac{5}{21} \hat{c}_1 \times \hat{g}_1.\hat{n}_1 M_8 q_1'[t] q_2'[t] + \hat{c}_3.\hat{n}_1$$

$$\left(-\frac{1}{2}\hat{c}_3.\hat{d}_1 M_4 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_4 q_1'[t]^2-\frac{1}{2}\hat{c}_2.\hat{d}_1 M_4 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_4 q_1'[t]\,q_2'[t]\right)+$$

$$\hat{c}_3.\hat{n}_1\left(-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_5 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_5 q_1'[t]^2-\frac{1}{6}\hat{c}_3.\hat{e}_1 M_5 q_1'[t]^2+\frac{5}{12}\hat{c}_3.\hat{e}_2 M_5 q_1'[t]^2-\frac{3}{2}\hat{c}_2.\hat{d}_1 M_5 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_5 q_1'[t]\,q_2'[t]-\frac{1}{6}\hat{c}_2.\hat{e}_1 M_5 q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_2.\hat{e}_2 M_5 q_1'[t]\,q_2'[t]\right)+$$

$$\hat{c}_3.\hat{n}_1\left(-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_6 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_6 q_1'[t]^2-\frac{2}{3}\hat{c}_3.\hat{e}_1 M_6 q_1'[t]^2+\frac{5}{12}\hat{c}_3.\hat{e}_2 M_6 q_1'[t]^2-\frac{1}{3}\hat{c}_3.\hat{f}_1 M_6 q_1'[t]^2-\frac{3}{2}\hat{c}_2.\hat{d}_1 M_6 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_6 q_1'[t]\,q_2'[t]-\frac{2}{3}\hat{c}_2.\hat{e}_1 M_6 q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_2.\hat{e}_2 M_6 q_1'[t]\,q_2'[t]-\frac{1}{3}\hat{c}_2.\hat{f}_1 M_6 q_1'[t]\,q_2'[t]\right)+$$

$$\hat{c}_3.\hat{n}_1\left(-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_7 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_7 q_1'[t]^2-\frac{2}{3}\hat{c}_3.\hat{e}_1 M_7 q_1'[t]^2+\frac{5}{12}\hat{c}_3.\hat{e}_2 M_7 q_1'[t]^2-\frac{5}{6}\hat{c}_3.\hat{f}_1 M_7 q_1'[t]^2-\frac{3}{2}\hat{c}_2.\hat{d}_1 M_7 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_7 q_1'[t]\,q_2'[t]-\frac{2}{3}\hat{c}_2.\hat{e}_1 M_7 q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_2.\hat{e}_2 M_7 q_1'[t]\,q_2'[t]-\frac{5}{6}\hat{c}_2.\hat{f}_1 M_7 q_1'[t]\,q_2'[t]\right)+$$

$$\hat{c}_3.\hat{n}_1\left(-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_8 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_8 q_1'[t]^2-\frac{2}{3}\hat{c}_3.\hat{e}_1 M_8 q_1'[t]^2+\frac{5}{12}\hat{c}_3.\hat{e}_2 M_8 q_1'[t]^2-\frac{5}{6}\hat{c}_3.\hat{f}_1 M_8 q_1'[t]^2-\frac{5}{21}\hat{c}_3.\hat{g}_1 M_8 q_1'[t]^2-\frac{3}{2}\hat{c}_2.\hat{d}_1 M_8 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_8 q_1'[t]\,q_2'[t]-\frac{2}{3}\hat{c}_2.\hat{e}_1 M_8 q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_2.\hat{e}_2 M_8 q_1'[t]\,q_2'[t]-\frac{5}{6}\hat{c}_2.\hat{f}_1 M_8 q_1'[t]\,q_2'[t]-\frac{5}{21}\hat{c}_2.\hat{g}_1 M_8 q_1'[t]\,q_2'[t]\right)+$$

$$\hat{d}_2.\hat{n}_1\left(\frac{1}{4}M_4 q_1'[t]^2+\frac{1}{4}M_4 q_2'[t]^2\right)+\hat{d}_1.\hat{n}_1\left(\frac{1}{2}M_4 q_1'[t]^2+\frac{1}{2}M_4 q_2'[t]^2\right)+$$

$$\hat{c}_2.\hat{n}_1\left(\frac{1}{2}M_4 q_1'[t]^2-\frac{1}{2}\hat{c}_3.\hat{d}_1 M_4 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_4 q_1'[t]\,q_2'[t]-\frac{1}{2}\hat{c}_2.\hat{d}_1 M_4 q_2'[t]^2-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_4 q_2'[t]^2\right)+\hat{d}_1.\hat{n}_1\left(\frac{3}{2}M_5 q_1'[t]^2+\frac{3}{2}M_5 q_2'[t]^2\right)+$$

$$\hat{c}_2.\hat{n}_1\left(\frac{1}{2}M_5 q_1'[t]^2-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_5 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_5 q_1'[t]\,q_2'[t]-\frac{1}{6}\hat{c}_3.\hat{e}_1 M_5 q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_3.\hat{e}_2 M_5 q_1'[t]\,q_2'[t]-\frac{3}{2}\hat{c}_2.\hat{d}_1 M_5 q_2'[t]^2-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_5 q_2'[t]^2-\frac{1}{6}\hat{c}_2.\hat{e}_1 M_5 q_2'[t]^2+\frac{5}{12}\hat{c}_2.\hat{e}_2 M_5 q_2'[t]^2\right)+\hat{d}_1.\hat{n}_1\left(\frac{3}{2}M_6 q_1'[t]^2+\frac{3}{2}M_6 q_2'[t]^2\right)+$$

$$\hat{c}_2.\hat{n}_1\left(\frac{1}{2}M_6 q_1'[t]^2-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_6 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_6 q_1'[t]\,q_2'[t]-\frac{2}{3}\hat{c}_3.\hat{e}_1 M_6 q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_3.\hat{e}_2 M_6 q_1'[t]\,q_2'[t]-\frac{1}{3}\hat{c}_3.\hat{f}_1 M_6 q_1'[t]\,q_2'[t]-\frac{3}{2}\hat{c}_2.\hat{d}_1 M_6 q_2'[t]^2-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_6 q_2'[t]^2-\frac{2}{3}\hat{c}_2.\hat{e}_1 M_6 q_2'[t]^2+\frac{5}{12}\hat{c}_2.\hat{e}_2 M_6 q_2'[t]^2-\frac{1}{3}\hat{c}_2.\hat{f}_1 M_6 q_2'[t]^2\right)+$$

$$\hat{d}_1.\hat{n}_1\left(\frac{3}{2}M_7\,q_1'[t]^2+\frac{3}{2}M_7\,q_2'[t]^2\right)+\hat{c}_2.\hat{n}_1$$
$$\left(\frac{1}{2}M_7\,q_1'[t]^2-\frac{3}{2}\hat{c}_3.\hat{d}_1M_7\,q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_3.\hat{d}_2M_7\,q_1'[t]\,q_2'[t]-\frac{2}{3}\hat{c}_3.\hat{e}_1M_7\,q_1'[t]\,q_2'[t]+\right.$$
$$\frac{5}{12}\hat{c}_3.\hat{e}_2M_7\,q_1'[t]\,q_2'[t]-\frac{5}{6}\hat{c}_3.\hat{f}_1M_7\,q_1'[t]\,q_2'[t]-\frac{3}{2}\hat{c}_2.\hat{d}_1M_7\,q_2'[t]^2-$$
$$\left.\frac{1}{4}\hat{c}_2.\hat{d}_2M_7\,q_2'[t]^2-\frac{2}{3}\hat{c}_2.\hat{e}_1M_7\,q_2'[t]^2+\frac{5}{12}\hat{c}_2.\hat{e}_2M_7\,q_2'[t]^2-\frac{5}{6}\hat{c}_2.\hat{f}_1M_7\,q_2'[t]^2\right)+$$
$$\hat{d}_1.\hat{n}_1\left(\frac{3}{2}M_8\,q_1'[t]^2+\frac{3}{2}M_8\,q_2'[t]^2\right)+\hat{c}_2.\hat{n}_1\left(\frac{1}{2}M_8\,q_1'[t]^2-\frac{3}{2}\hat{c}_3.\hat{d}_1M_8\,q_1'[t]\,q_2'[t]-\right.$$
$$\frac{1}{4}\hat{c}_3.\hat{d}_2M_8\,q_1'[t]\,q_2'[t]-\frac{2}{3}\hat{c}_3.\hat{e}_1M_8\,q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_3.\hat{e}_2M_8\,q_1'[t]\,q_2'[t]-$$
$$\frac{5}{6}\hat{c}_3.\hat{f}_1M_8\,q_1'[t]\,q_2'[t]-\frac{5}{21}\hat{c}_3.\hat{g}_1M_8\,q_1'[t]\,q_2'[t]-\frac{3}{2}\hat{c}_2.\hat{d}_1M_8\,q_2'[t]^2-\frac{1}{4}\hat{c}_2.\hat{d}_2M_8\,q_2'[t]^2-$$
$$\left.\frac{2}{3}\hat{c}_2.\hat{e}_1M_8\,q_2'[t]^2+\frac{5}{12}\hat{c}_2.\hat{e}_2M_8\,q_2'[t]^2-\frac{5}{6}\hat{c}_2.\hat{f}_1M_8\,q_2'[t]^2-\frac{5}{21}\hat{c}_2.\hat{g}_1M_8\,q_2'[t]^2\right)+$$
$$\hat{e}_2.\hat{n}_1\left(-\frac{5}{12}M_5\,q_1'[t]^2-\frac{5}{12}M_5\,q_2'[t]^2-\frac{5}{6}\hat{c}_3.\hat{d}_2M_5\,q_1'[t]\,q_3'[t]-\right.$$
$$\left.\frac{5}{6}\hat{c}_2.\hat{d}_2M_5\,q_2'[t]\,q_3'[t]-\frac{5}{12}M_5\,q_3'[t]^2\right)+\hat{e}_1.\hat{n}_1$$
$$\left(\frac{1}{6}M_5\,q_1'[t]^2+\frac{1}{6}M_5\,q_2'[t]^2+\frac{1}{3}\hat{c}_3.\hat{d}_2M_5\,q_1'[t]\,q_3'[t]+\frac{1}{3}\hat{c}_2.\hat{d}_2M_5\,q_2'[t]\,q_3'[t]+\frac{1}{6}M_5\,q_3'[t]^2\right)+$$
$$\hat{d}_2.\hat{n}_1\left(\frac{1}{4}M_5\,q_1'[t]^2+\frac{1}{4}M_5\,q_2'[t]^2-\frac{1}{3}\hat{c}_3.\hat{e}_1M_5\,q_1'[t]\,q_3'[t]+\frac{5}{6}\hat{c}_3.\hat{e}_2M_5\,q_1'[t]\,q_3'[t]-\right.$$
$$\left.\frac{1}{3}\hat{c}_2.\hat{e}_1M_5\,q_2'[t]\,q_3'[t]+\frac{5}{6}\hat{c}_2.\hat{e}_2M_5\,q_2'[t]\,q_3'[t]-\frac{1}{6}\hat{d}_2.\hat{e}_1M_5\,q_3'[t]^2+\frac{5}{12}\hat{d}_2.\hat{e}_2M_5\,q_3'[t]^2\right)+$$
$$\hat{e}_2.\hat{n}_1\left(-\frac{5}{12}M_6\,q_1'[t]^2-\frac{5}{12}M_6\,q_2'[t]^2-\frac{5}{6}\hat{c}_3.\hat{d}_2M_6\,q_1'[t]\,q_3'[t]-\right.$$
$$\left.\frac{5}{6}\hat{c}_2.\hat{d}_2M_6\,q_2'[t]\,q_3'[t]-\frac{5}{12}M_6\,q_3'[t]^2\right)+$$
$$\hat{d}_2.\hat{n}_1\left(\frac{1}{4}M_6\,q_1'[t]^2+\frac{1}{4}M_6\,q_2'[t]^2-\frac{4}{3}\hat{c}_3.\hat{e}_1M_6\,q_1'[t]\,q_3'[t]+\frac{5}{6}\hat{c}_3.\hat{e}_2M_6\,q_1'[t]\,q_3'[t]-\right.$$
$$\frac{2}{3}\hat{c}_3.\hat{f}_1M_6\,q_1'[t]\,q_3'[t]-\frac{4}{3}\hat{c}_2.\hat{e}_1M_6\,q_2'[t]\,q_3'[t]+\frac{5}{6}\hat{c}_2.\hat{e}_2M_6\,q_2'[t]\,q_3'[t]-$$
$$\left.\frac{2}{3}\hat{c}_2.\hat{f}_1M_6\,q_2'[t]\,q_3'[t]-\frac{2}{3}\hat{d}_2.\hat{e}_1M_6\,q_3'[t]^2+\frac{5}{12}\hat{d}_2.\hat{e}_2M_6\,q_3'[t]^2-\frac{1}{3}\hat{d}_2.\hat{f}_1M_6\,q_3'[t]^2\right)+$$
$$\hat{e}_2.\hat{n}_1\left(-\frac{5}{12}M_7\,q_1'[t]^2-\frac{5}{12}M_7\,q_2'[t]^2-\frac{5}{6}\hat{c}_3.\hat{d}_2M_7\,q_1'[t]\,q_3'[t]-\right.$$
$$\left.\frac{5}{6}\hat{c}_2.\hat{d}_2M_7\,q_2'[t]\,q_3'[t]-\frac{5}{12}M_7\,q_3'[t]^2\right)+$$
$$\hat{d}_2.\hat{n}_1\left(\frac{1}{4}M_7\,q_1'[t]^2+\frac{1}{4}M_7\,q_2'[t]^2-\frac{4}{3}\hat{c}_3.\hat{e}_1M_7\,q_1'[t]\,q_3'[t]+\frac{5}{6}\hat{c}_3.\hat{e}_2M_7\,q_1'[t]\,q_3'[t]-\right.$$
$$\frac{5}{3}\hat{c}_3.\hat{f}_1M_7\,q_1'[t]\,q_3'[t]-\frac{4}{3}\hat{c}_2.\hat{e}_1M_7\,q_2'[t]\,q_3'[t]+\frac{5}{6}\hat{c}_2.\hat{e}_2M_7\,q_2'[t]\,q_3'[t]-$$

$$\frac{5}{3}\,\hat{c}_2.\hat{f}_1\,M_7\,q_2'[t]\,q_3'[t]-\frac{2}{3}\,\hat{d}_2.\hat{e}_1\,M_7\,q_3'[t]^2+\frac{5}{12}\,\hat{d}_2.\hat{e}_2\,M_7\,q_3'[t]^2-\frac{5}{6}\,\hat{d}_2.\hat{f}_1\,M_7\,q_3'[t]^2\Big)+$$

$$\hat{e}_2.\hat{n}_1\,\Big(-\frac{5}{12}\,M_8\,q_1'[t]^2-\frac{5}{12}\,M_8\,q_2'[t]^2-\frac{5}{6}\,\hat{c}_3.\hat{d}_2\,M_8\,q_1'[t]\,q_3'[t]-$$
$$\frac{5}{6}\,\hat{c}_2.\hat{d}_2\,M_8\,q_2'[t]\,q_3'[t]-\frac{5}{12}\,M_8\,q_3'[t]^2\Big)+$$

$$\hat{d}_2.\hat{n}_1\,\Big(\frac{1}{4}\,M_8\,q_1'[t]^2+\frac{1}{4}\,M_8\,q_2'[t]^2-\frac{4}{3}\,\hat{c}_3.\hat{e}_1\,M_8\,q_1'[t]\,q_3'[t]+\frac{5}{6}\,\hat{c}_3.\hat{e}_2\,M_8\,q_1'[t]\,q_3'[t]-$$
$$\frac{5}{3}\,\hat{c}_3.\hat{f}_1\,M_8\,q_1'[t]\,q_3'[t]-\frac{10}{21}\,\hat{c}_3.\hat{g}_1\,M_8\,q_1'[t]\,q_3'[t]-\frac{4}{3}\,\hat{c}_2.\hat{e}_1\,M_8\,q_2'[t]\,q_3'[t]+$$
$$\frac{5}{6}\,\hat{c}_2.\hat{e}_2\,M_8\,q_2'[t]\,q_3'[t]-\frac{5}{3}\,\hat{c}_2.\hat{f}_1\,M_8\,q_2'[t]\,q_3'[t]-\frac{10}{21}\,\hat{c}_2.\hat{g}_1\,M_8\,q_2'[t]\,q_3'[t]-$$
$$\frac{2}{3}\,\hat{d}_2.\hat{e}_1\,M_8\,q_3'[t]^2+\frac{5}{12}\,\hat{d}_2.\hat{e}_2\,M_8\,q_3'[t]^2-\frac{5}{6}\,\hat{d}_2.\hat{f}_1\,M_8\,q_3'[t]^2-\frac{5}{21}\,\hat{d}_2.\hat{g}_1\,M_8\,q_3'[t]^2\Big)+$$

$$\hat{f}_1.\hat{n}_1\,\Big(\frac{1}{3}\,M_6\,q_1'[t]^2+\frac{1}{3}\,M_6\,q_2'[t]^2+\frac{2}{3}\,\hat{c}_3.\hat{d}_2\,M_6\,q_1'[t]\,q_3'[t]+\frac{2}{3}\,\hat{c}_2.\hat{d}_2\,M_6\,q_2'[t]\,q_3'[t]+\frac{1}{3}\,M_6\,q_3'[t]^2+$$
$$\frac{2}{3}\,\hat{c}_3.\hat{e}_1\,M_6\,q_1'[t]\,q_4'[t]+\frac{2}{3}\,\hat{c}_2.\hat{e}_1\,M_6\,q_2'[t]\,q_4'[t]+\frac{2}{3}\,\hat{d}_2.\hat{e}_1\,M_6\,q_3'[t]\,q_4'[t]+\frac{1}{3}\,M_6\,q_4'[t]^2\Big)+$$

$$\hat{e}_1.\hat{n}_1\,\Big(\frac{2}{3}\,M_6\,q_1'[t]^2+\frac{2}{3}\,M_6\,q_2'[t]^2+\frac{4}{3}\,\hat{c}_3.\hat{d}_2\,M_6\,q_1'[t]\,q_3'[t]+\frac{4}{3}\,\hat{c}_2.\hat{d}_2\,M_6\,q_2'[t]\,q_3'[t]+$$
$$\frac{2}{3}\,M_6\,q_3'[t]^2-\frac{2}{3}\,\hat{c}_3.\hat{f}_1\,M_6\,q_1'[t]\,q_4'[t]-\frac{2}{3}\,\hat{c}_2.\hat{f}_1\,M_6\,q_2'[t]\,q_4'[t]-$$
$$\frac{2}{3}\,\hat{d}_2.\hat{f}_1\,M_6\,q_3'[t]\,q_4'[t]-\frac{1}{3}\,\hat{e}_1.\hat{f}_1\,M_6\,q_4'[t]^2\Big)+$$

$$\hat{f}_1.\hat{n}_1\,\Big(\frac{5}{6}\,M_7\,q_1'[t]^2+\frac{5}{6}\,M_7\,q_2'[t]^2+\frac{5}{3}\,\hat{c}_3.\hat{d}_2\,M_7\,q_1'[t]\,q_3'[t]+\frac{5}{3}\,\hat{c}_2.\hat{d}_2\,M_7\,q_2'[t]\,q_3'[t]+\frac{5}{6}\,M_7\,q_3'[t]^2+$$
$$\frac{5}{3}\,\hat{c}_3.\hat{e}_1\,M_7\,q_1'[t]\,q_4'[t]+\frac{5}{3}\,\hat{c}_2.\hat{e}_1\,M_7\,q_2'[t]\,q_4'[t]+\frac{5}{3}\,\hat{d}_2.\hat{e}_1\,M_7\,q_3'[t]\,q_4'[t]+\frac{5}{6}\,M_7\,q_4'[t]^2\Big)+$$

$$\hat{e}_1.\hat{n}_1\,\Big(\frac{2}{3}\,M_7\,q_1'[t]^2+\frac{2}{3}\,M_7\,q_2'[t]^2+\frac{4}{3}\,\hat{c}_3.\hat{d}_2\,M_7\,q_1'[t]\,q_3'[t]+\frac{4}{3}\,\hat{c}_2.\hat{d}_2\,M_7\,q_2'[t]\,q_3'[t]+$$
$$\frac{2}{3}\,M_7\,q_3'[t]^2-\frac{5}{3}\,\hat{c}_3.\hat{f}_1\,M_7\,q_1'[t]\,q_4'[t]-\frac{5}{3}\,\hat{c}_2.\hat{f}_1\,M_7\,q_2'[t]\,q_4'[t]-$$
$$\frac{5}{3}\,\hat{d}_2.\hat{f}_1\,M_7\,q_3'[t]\,q_4'[t]-\frac{5}{6}\,\hat{e}_1.\hat{f}_1\,M_7\,q_4'[t]^2\Big)+$$

$$\hat{f}_1.\hat{n}_1\,\Big(\frac{5}{6}\,M_8\,q_1'[t]^2+\frac{5}{6}\,M_8\,q_2'[t]^2+\frac{5}{3}\,\hat{c}_3.\hat{d}_2\,M_8\,q_1'[t]\,q_3'[t]+\frac{5}{3}\,\hat{c}_2.\hat{d}_2\,M_8\,q_2'[t]\,q_3'[t]+\frac{5}{6}\,M_8\,q_3'[t]^2+$$
$$\frac{5}{3}\,\hat{c}_3.\hat{e}_1\,M_8\,q_1'[t]\,q_4'[t]+\frac{5}{3}\,\hat{c}_2.\hat{e}_1\,M_8\,q_2'[t]\,q_4'[t]+\frac{5}{3}\,\hat{d}_2.\hat{e}_1\,M_8\,q_3'[t]\,q_4'[t]+\frac{5}{6}\,M_8\,q_4'[t]^2\Big)+$$

$$\hat{e}_1.\hat{n}_1\,\Big(\frac{2}{3}\,M_8\,q_1'[t]^2+\frac{2}{3}\,M_8\,q_2'[t]^2+\frac{4}{3}\,\hat{c}_3.\hat{d}_2\,M_8\,q_1'[t]\,q_3'[t]+\frac{4}{3}\,\hat{c}_2.\hat{d}_2\,M_8\,q_2'[t]\,q_3'[t]+$$
$$\frac{2}{3}\,M_8\,q_3'[t]^2-\frac{5}{3}\,\hat{c}_3.\hat{f}_1\,M_8\,q_1'[t]\,q_4'[t]-\frac{10}{21}\,\hat{c}_3.\hat{g}_1\,M_8\,q_1'[t]\,q_4'[t]-$$
$$\frac{5}{3}\,\hat{c}_2.\hat{f}_1\,M_8\,q_2'[t]\,q_4'[t]-\frac{10}{21}\,\hat{c}_2.\hat{g}_1\,M_8\,q_2'[t]\,q_4'[t]-\frac{5}{3}\,\hat{d}_2.\hat{f}_1\,M_8\,q_3'[t]\,q_4'[t]-$$

$$\frac{10}{21}\hat{d}_2.\hat{g}_1 M_8 q_3'[t]\, q_4'[t] - \frac{5}{6}\hat{e}_1.\hat{f}_1 M_8 q_4'[t]^2 - \frac{5}{21}\hat{e}_1.\hat{g}_1 M_8 q_4'[t]^2\Big) +$$

$$\hat{g}_1.\hat{n}_1 \Big(\frac{5}{21} M_8 q_1'[t]^2 + \frac{5}{21} M_8 q_2'[t]^2 + \frac{10}{21}\hat{c}_3.\hat{d}_2 M_8 q_1'[t]\, q_3'[t] + \frac{10}{21}\hat{c}_2.\hat{d}_2 M_8 q_2'[t]\, q_3'[t] +$$

$$\frac{5}{21} M_8 q_3'[t]^2 + \frac{10}{21}\hat{c}_3.\hat{e}_1 M_8 q_1'[t]\, q_4'[t] + \frac{10}{21}\hat{c}_2.\hat{e}_1 M_8 q_2'[t]\, q_4'[t] + \frac{10}{21}\hat{d}_2.\hat{e}_1 M_8 q_3'[t]\, q_4'[t] +$$

$$\frac{5}{21} M_8 q_4'[t]^2 + \frac{10}{21}\hat{c}_3.\hat{f}_2 M_8 q_1'[t]\, q_5'[t] + \frac{10}{21}\hat{c}_2.\hat{f}_2 M_8 q_2'[t]\, q_5'[t] +$$

$$\frac{10}{21}\hat{d}_2.\hat{f}_2 M_8 q_3'[t]\, q_5'[t] + \frac{10}{21}\hat{e}_1.\hat{f}_2 M_8 q_4'[t]\, q_5'[t] + \frac{5}{21} M_8 q_5'[t]^2\Big) +$$

$$\hat{f}_2.\hat{n}_1 \Big(-\frac{10}{21}\hat{c}_3.\hat{g}_1 M_8 q_1'[t]\, q_5'[t] - \frac{10}{21}\hat{c}_2.\hat{g}_1 M_8 q_2'[t]\, q_5'[t] - \frac{10}{21}\hat{d}_2.\hat{g}_1 M_8 q_3'[t]\, q_5'[t] -$$

$$\frac{10}{21}\hat{e}_1.\hat{g}_1 M_8 q_4'[t]\, q_5'[t] - \frac{5}{21}\hat{f}_2.\hat{g}_1 M_8 q_5'[t]^2\Big) -$$

$$M_1 x''[t] - M_2 x''[t] - M_3 x''[t] - M_4 x''[t] - M_5 x''[t] - M_6 x''[t] - M_7 x''[t] -$$

$$M_8 x''[t] - \frac{1}{2}\hat{c}_3 \times \hat{d}_1.\hat{n}_1 M_4 q_1''[t] -$$

$$\frac{1}{4}\hat{c}_3 \times \hat{d}_2.\hat{n}_1 M_4 q_1''[t] + \frac{1}{2}\hat{c}_1.\hat{n}_1 M_4 q_1''[t] -$$

$$\frac{3}{2}\hat{c}_3 \times \hat{d}_1.\hat{n}_1 M_5 q_1''[t] - \frac{1}{4}\hat{c}_3 \times \hat{d}_2.\hat{n}_1 M_5 q_1''[t] -$$

$$\frac{1}{6}\hat{c}_3 \times \hat{e}_1.\hat{n}_1 M_5 q_1''[t] + \frac{5}{12}\hat{c}_3 \times \hat{e}_2.\hat{n}_1 M_5 q_1''[t] +$$

$$\frac{1}{2}\hat{c}_1.\hat{n}_1 M_5 q_1''[t] - \frac{3}{2}\hat{c}_3 \times \hat{d}_1.\hat{n}_1 M_6 q_1''[t] -$$

$$\frac{1}{4}\hat{c}_3 \times \hat{d}_2.\hat{n}_1 M_6 q_1''[t] - \frac{2}{3}\hat{c}_3 \times \hat{e}_1.\hat{n}_1 M_6 q_1''[t] +$$

$$\frac{5}{12}\hat{c}_3 \times \hat{e}_2.\hat{n}_1 M_6 q_1''[t] - \frac{1}{3}\hat{c}_3 \times \hat{f}_1.\hat{n}_1 M_6 q_1''[t] +$$

$$\frac{1}{2}\hat{c}_1.\hat{n}_1 M_6 q_1''[t] - \frac{3}{2}\hat{c}_3 \times \hat{d}_1.\hat{n}_1 M_7 q_1''[t] -$$

$$\frac{1}{4}\hat{c}_3 \times \hat{d}_2.\hat{n}_1 M_7 q_1''[t] - \frac{2}{3}\hat{c}_3 \times \hat{e}_1.\hat{n}_1 M_7 q_1''[t] +$$

$$\frac{5}{12}\hat{c}_3 \times \hat{e}_2.\hat{n}_1 M_7 q_1''[t] - \frac{5}{6}\hat{c}_3 \times \hat{f}_1.\hat{n}_1 M_7 q_1''[t] +$$

$$\frac{1}{2}\hat{c}_1.\hat{n}_1 M_7 q_1''[t] - \frac{3}{2}\hat{c}_3 \times \hat{d}_1.\hat{n}_1 M_8 q_1''[t] -$$

$$\frac{1}{4}\hat{c}_3 \times \hat{d}_2.\hat{n}_1 M_8 q_1''[t] - \frac{2}{3}\hat{c}_3 \times \hat{e}_1.\hat{n}_1 M_8 q_1''[t] +$$

$$\frac{5}{12}\hat{c}_3 \times \hat{e}_2.\hat{n}_1 M_8 q_1''[t] - \frac{5}{6}\hat{c}_3 \times \hat{f}_1.\hat{n}_1 M_8 q_1''[t] -$$

$$\frac{5}{21}\hat{c}_3 \times \hat{g}_1.\hat{n}_1 M_8 q_1''[t] + \frac{1}{2}\hat{c}_1.\hat{n}_1 M_8 q_1''[t] -$$

$$\frac{1}{2}\hat{c}_2 \times \hat{d}_1.\hat{n}_1 M_4 q_2''[t] - \frac{1}{4}\hat{c}_2 \times \hat{d}_2.\hat{n}_1 M_4 q_2''[t] -$$

$$\frac{3}{2}\hat{c}_2\times\hat{d}_1.\hat{n}_1 M_5 q_2''[t]-\frac{1}{4}\hat{c}_2\times\hat{d}_2.\hat{n}_1 M_5 q_2''[t]-$$
$$\frac{1}{6}\hat{c}_2\times\hat{e}_1.\hat{n}_1 M_5 q_2''[t]+\frac{5}{12}\hat{c}_2\times\hat{e}_2.\hat{n}_1 M_5 q_2''[t]-$$
$$\frac{3}{2}\hat{c}_2\times\hat{d}_1.\hat{n}_1 M_6 q_2''[t]-\frac{1}{4}\hat{c}_2\times\hat{d}_2.\hat{n}_1 M_6 q_2''[t]-$$
$$\frac{2}{3}\hat{c}_2\times\hat{e}_1.\hat{n}_1 M_6 q_2''[t]+\frac{5}{12}\hat{c}_2\times\hat{e}_2.\hat{n}_1 M_6 q_2''[t]-$$
$$\frac{1}{3}\hat{c}_2\times\hat{f}_1.\hat{n}_1 M_6 q_2''[t]-\frac{3}{2}\hat{c}_2\times\hat{d}_1.\hat{n}_1 M_7 q_2''[t]-$$
$$\frac{1}{4}\hat{c}_2\times\hat{d}_2.\hat{n}_1 M_7 q_2''[t]-\frac{2}{3}\hat{c}_2\times\hat{e}_1.\hat{n}_1 M_7 q_2''[t]+$$
$$\frac{5}{12}\hat{c}_2\times\hat{e}_2.\hat{n}_1 M_7 q_2''[t]-\frac{5}{6}\hat{c}_2\times\hat{f}_1.\hat{n}_1 M_7 q_2''[t]-$$
$$\frac{3}{2}\hat{c}_2\times\hat{d}_1.\hat{n}_1 M_8 q_2''[t]-\frac{1}{4}\hat{c}_2\times\hat{d}_2.\hat{n}_1 M_8 q_2''[t]-$$
$$\frac{2}{3}\hat{c}_2\times\hat{e}_1.\hat{n}_1 M_8 q_2''[t]+\frac{5}{12}\hat{c}_2\times\hat{e}_2.\hat{n}_1 M_8 q_2''[t]-$$
$$\frac{5}{6}\hat{c}_2\times\hat{f}_1.\hat{n}_1 M_8 q_2''[t]-\frac{5}{21}\hat{c}_2\times\hat{g}_1.\hat{n}_1 M_8 q_2''[t]-$$
$$\frac{1}{6}\hat{d}_2\times\hat{e}_1.\hat{n}_1 M_5 q_3''[t]+\frac{5}{12}\hat{d}_2\times\hat{e}_2.\hat{n}_1 M_5 q_3''[t]-$$
$$\frac{2}{3}\hat{d}_2\times\hat{e}_1.\hat{n}_1 M_6 q_3''[t]+\frac{5}{12}\hat{d}_2\times\hat{e}_2.\hat{n}_1 M_6 q_3''[t]-$$
$$\frac{1}{3}\hat{d}_2\times\hat{f}_1.\hat{n}_1 M_6 q_3''[t]-\frac{2}{3}\hat{d}_2\times\hat{e}_1.\hat{n}_1 M_7 q_3''[t]+$$
$$\frac{5}{12}\hat{d}_2\times\hat{e}_2.\hat{n}_1 M_7 q_3''[t]-\frac{5}{6}\hat{d}_2\times\hat{f}_1.\hat{n}_1 M_7 q_3''[t]-$$
$$\frac{2}{3}\hat{d}_2\times\hat{e}_1.\hat{n}_1 M_8 q_3''[t]+\frac{5}{12}\hat{d}_2\times\hat{e}_2.\hat{n}_1 M_8 q_3''[t]-$$
$$\frac{5}{6}\hat{d}_2\times\hat{f}_1.\hat{n}_1 M_8 q_3''[t]-\frac{5}{21}\hat{d}_2\times\hat{g}_1.\hat{n}_1 M_8 q_3''[t]-$$
$$\frac{1}{3}\hat{e}_1\times\hat{f}_1.\hat{n}_1 M_6 q_4''[t]-\frac{5}{6}\hat{e}_1\times\hat{f}_1.\hat{n}_1 M_7 q_4''[t]-\frac{5}{6}\hat{e}_1\times\hat{f}_1.\hat{n}_1 M_8 q_4''[t]-$$
$$\frac{5}{21}\hat{e}_1\times\hat{g}_1.\hat{n}_1 M_8 q_4''[t]-\frac{5}{21}\hat{f}_2\times\hat{g}_1.\hat{n}_1 M_8 q_5''[t]$$

For y[t] the equations of motion is
(they will be set to zero when solution is required).

In[546]:= **eq[2] =**

$$\sum_{ii=1}^{N_{bodies}}\left(\left(F_{app}[ii]-I_f[ii]\right).\text{Pvel}[\text{OvB}[ii],y'[t]]+\left(T_{app}[ii]-I_t[ii]\right).\text{Pvel}[\text{N}\omega\text{B}[ii],y'[t]]\right)$$

Out[546]= $\hat{a}_1.\hat{n}_2 F_{motx}+\hat{a}_2.\hat{n}_2 F_{moty}-\hat{h}_1.\hat{n}_2 F_{toolx}-\hat{h}_2.\hat{n}_2 F_{tooly}-\hat{h}_3.\hat{n}_2 F_{toolz}+\frac{1}{2}\hat{c}_1\times\hat{d}_1.\hat{n}_2 M_4 q_1'[t]\, q_2'[t]+$

$$\frac{1}{4}\hat{c}_1\times\hat{d}_2.\hat{n}_2 M_4 q_1'[t]\,q_2'[t]+\frac{3}{2}\hat{c}_1\times\hat{d}_1.\hat{n}_2 M_5 q_1'[t]\,q_2'[t]+\frac{1}{4}\hat{c}_1\times\hat{d}_2.\hat{n}_2 M_5 q_1'[t]\,q_2'[t]+$$
$$\frac{1}{6}\hat{c}_1\times\hat{e}_1.\hat{n}_2 M_5 q_1'[t]\,q_2'[t]-\frac{5}{12}\hat{c}_1\times\hat{e}_2.\hat{n}_2 M_5 q_1'[t]\,q_2'[t]+\frac{3}{2}\hat{c}_1\times\hat{d}_1.\hat{n}_2 M_6 q_1'[t]\,q_2'[t]+$$
$$\frac{1}{4}\hat{c}_1\times\hat{d}_2.\hat{n}_2 M_6 q_1'[t]\,q_2'[t]+\frac{2}{3}\hat{c}_1\times\hat{e}_1.\hat{n}_2 M_6 q_1'[t]\,q_2'[t]-\frac{5}{12}\hat{c}_1\times\hat{e}_2.\hat{n}_2 M_6 q_1'[t]\,q_2'[t]+$$
$$\frac{1}{3}\hat{c}_1\times\hat{f}_1.\hat{n}_2 M_6 q_1'[t]\,q_2'[t]+\frac{3}{2}\hat{c}_1\times\hat{d}_1.\hat{n}_2 M_7 q_1'[t]\,q_2'[t]+\frac{1}{4}\hat{c}_1\times\hat{d}_2.\hat{n}_2 M_7 q_1'[t]\,q_2'[t]+$$
$$\frac{2}{3}\hat{c}_1\times\hat{e}_1.\hat{n}_2 M_7 q_1'[t]\,q_2'[t]-\frac{5}{12}\hat{c}_1\times\hat{e}_2.\hat{n}_2 M_7 q_1'[t]\,q_2'[t]+\frac{5}{6}\hat{c}_1\times\hat{f}_1.\hat{n}_2 M_7 q_1'[t]\,q_2'[t]+$$
$$\frac{3}{2}\hat{c}_1\times\hat{d}_1.\hat{n}_2 M_8 q_1'[t]\,q_2'[t]+\frac{1}{4}\hat{c}_1\times\hat{d}_2.\hat{n}_2 M_8 q_1'[t]\,q_2'[t]+\frac{2}{3}\hat{c}_1\times\hat{e}_1.\hat{n}_2 M_8 q_1'[t]\,q_2'[t]-$$
$$\frac{5}{12}\hat{c}_1\times\hat{e}_2.\hat{n}_2 M_8 q_1'[t]\,q_2'[t]+\frac{5}{6}\hat{c}_1\times\hat{f}_1.\hat{n}_2 M_8 q_1'[t]\,q_2'[t]+\frac{5}{21}\hat{c}_1\times\hat{g}_1.\hat{n}_2 M_8 q_1'[t]\,q_2'[t]+\hat{c}_3.\hat{n}_2$$
$$\left(-\frac{1}{2}\hat{c}_3.\hat{d}_1 M_4 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_4 q_1'[t]^2-\frac{1}{2}\hat{c}_2.\hat{d}_1 M_4 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_4 q_1'[t]\,q_2'[t]\right)+$$
$$\hat{c}_3.\hat{n}_2\left(-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_5 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_5 q_1'[t]^2-\frac{1}{6}\hat{c}_3.\hat{e}_1 M_5 q_1'[t]^2+\frac{5}{12}\hat{c}_3.\hat{e}_2 M_5 q_1'[t]^2-\frac{3}{2}\hat{c}_2.\hat{d}_1 M_5\right.$$
$$\left.q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_5 q_1'[t]\,q_2'[t]-\frac{1}{6}\hat{c}_2.\hat{e}_1 M_5 q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_2.\hat{e}_2 M_5 q_1'[t]\,q_2'[t]\right)+$$
$$\hat{c}_3.\hat{n}_2\left(-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_6 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_6 q_1'[t]^2-\frac{2}{3}\hat{c}_3.\hat{e}_1 M_6 q_1'[t]^2+\frac{5}{12}\hat{c}_3.\hat{e}_2 M_6 q_1'[t]^2-\right.$$
$$\frac{1}{3}\hat{c}_3.\hat{f}_1 M_6 q_1'[t]^2-\frac{3}{2}\hat{c}_2.\hat{d}_1 M_6 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_6 q_1'[t]\,q_2'[t]-$$
$$\left.\frac{2}{3}\hat{c}_2.\hat{e}_1 M_6 q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_2.\hat{e}_2 M_6 q_1'[t]\,q_2'[t]-\frac{1}{3}\hat{c}_2.\hat{f}_1 M_6 q_1'[t]\,q_2'[t]\right)+$$
$$\hat{c}_3.\hat{n}_2\left(-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_7 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_7 q_1'[t]^2-\frac{2}{3}\hat{c}_3.\hat{e}_1 M_7 q_1'[t]^2+\frac{5}{12}\hat{c}_3.\hat{e}_2 M_7 q_1'[t]^2-\right.$$
$$\frac{5}{6}\hat{c}_3.\hat{f}_1 M_7 q_1'[t]^2-\frac{3}{2}\hat{c}_2.\hat{d}_1 M_7 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_7 q_1'[t]\,q_2'[t]-$$
$$\left.\frac{2}{3}\hat{c}_2.\hat{e}_1 M_7 q_1'[t]\,q_2'[t]+\frac{5}{12}\hat{c}_2.\hat{e}_2 M_7 q_1'[t]\,q_2'[t]-\frac{5}{6}\hat{c}_2.\hat{f}_1 M_7 q_1'[t]\,q_2'[t]\right)+$$
$$\hat{c}_3.\hat{n}_2\left(-\frac{3}{2}\hat{c}_3.\hat{d}_1 M_8 q_1'[t]^2-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_8 q_1'[t]^2-\frac{2}{3}\hat{c}_3.\hat{e}_1 M_8 q_1'[t]^2+\right.$$
$$\frac{5}{12}\hat{c}_3.\hat{e}_2 M_8 q_1'[t]^2-\frac{5}{6}\hat{c}_3.\hat{f}_1 M_8 q_1'[t]^2-\frac{5}{21}\hat{c}_3.\hat{g}_1 M_8 q_1'[t]^2-$$
$$\frac{3}{2}\hat{c}_2.\hat{d}_1 M_8 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_2.\hat{d}_2 M_8 q_1'[t]\,q_2'[t]-\frac{2}{3}\hat{c}_2.\hat{e}_1 M_8 q_1'[t]\,q_2'[t]+$$
$$\left.\frac{5}{12}\hat{c}_2.\hat{e}_2 M_8 q_1'[t]\,q_2'[t]-\frac{5}{6}\hat{c}_2.\hat{f}_1 M_8 q_1'[t]\,q_2'[t]-\frac{5}{21}\hat{c}_2.\hat{g}_1 M_8 q_1'[t]\,q_2'[t]\right)+$$
$$\hat{d}_2.\hat{n}_2\left(\frac{1}{4}M_4 q_1'[t]^2+\frac{1}{4}M_4 q_2'[t]^2\right)+\hat{d}_1.\hat{n}_2\left(\frac{1}{2}M_4 q_1'[t]^2+\frac{1}{2}M_4 q_2'[t]^2\right)+$$
$$\hat{c}_2.\hat{n}_2\left(\frac{1}{2}M_4 q_1'[t]^2-\frac{1}{2}\hat{c}_3.\hat{d}_1 M_4 q_1'[t]\,q_2'[t]-\frac{1}{4}\hat{c}_3.\hat{d}_2 M_4 q_1'[t]\,q_2'[t]-\right.$$

$$\frac{1}{2}\hat{c}_2.\hat{d}_1 M_4 q_2'[t]^2 - \frac{1}{4}\hat{c}_2.\hat{d}_2 M_4 q_2'[t]^2\Big) + \hat{d}_1.\hat{n}_2\left(\frac{3}{2}M_5 q_1'[t]^2 + \frac{3}{2}M_5 q_2'[t]^2\right) +$$

$$\hat{c}_2.\hat{n}_2\Big(\frac{1}{2}M_5 q_1'[t]^2 - \frac{3}{2}\hat{c}_3.\hat{d}_1 M_5 q_1'[t]\, q_2'[t] - \frac{1}{4}\hat{c}_3.\hat{d}_2 M_5 q_1'[t]\, q_2'[t] -$$

$$\frac{1}{6}\hat{c}_3.\hat{e}_1 M_5 q_1'[t]\, q_2'[t] + \frac{5}{12}\hat{c}_3.\hat{e}_2 M_5 q_1'[t]\, q_2'[t] - \frac{3}{2}\hat{c}_2.\hat{d}_1 M_5 q_2'[t]^2 - \frac{1}{4}\hat{c}_2.\hat{d}_2 M_5 q_2'[t]^2 -$$

$$\frac{1}{6}\hat{c}_2.\hat{e}_1 M_5 q_2'[t]^2 + \frac{5}{12}\hat{c}_2.\hat{e}_2 M_5 q_2'[t]^2\Big) + \hat{d}_1.\hat{n}_2\left(\frac{3}{2}M_6 q_1'[t]^2 + \frac{3}{2}M_6 q_2'[t]^2\right) +$$

$$\hat{c}_2.\hat{n}_2\Big(\frac{1}{2}M_6 q_1'[t]^2 - \frac{3}{2}\hat{c}_3.\hat{d}_1 M_6 q_1'[t]\, q_2'[t] - \frac{1}{4}\hat{c}_3.\hat{d}_2 M_6 q_1'[t]\, q_2'[t] - \frac{2}{3}\hat{c}_3.\hat{e}_1 M_6 q_1'[t]\, q_2'[t] +$$

$$\frac{5}{12}\hat{c}_3.\hat{e}_2 M_6 q_1'[t]\, q_2'[t] - \frac{1}{3}\hat{c}_3.\hat{f}_1 M_6 q_1'[t]\, q_2'[t] - \frac{3}{2}\hat{c}_2.\hat{d}_1 M_6 q_2'[t]^2 -$$

$$\frac{1}{4}\hat{c}_2.\hat{d}_2 M_6 q_2'[t]^2 - \frac{2}{3}\hat{c}_2.\hat{e}_1 M_6 q_2'[t]^2 + \frac{5}{12}\hat{c}_2.\hat{e}_2 M_6 q_2'[t]^2 - \frac{1}{3}\hat{c}_2.\hat{f}_1 M_6 q_2'[t]^2\Big) +$$

$$\hat{d}_1.\hat{n}_2\left(\frac{3}{2}M_7 q_1'[t]^2 + \frac{3}{2}M_7 q_2'[t]^2\right) + \hat{c}_2.\hat{n}_2$$

$$\Big(\frac{1}{2}M_7 q_1'[t]^2 - \frac{3}{2}\hat{c}_3.\hat{d}_1 M_7 q_1'[t]\, q_2'[t] - \frac{1}{4}\hat{c}_3.\hat{d}_2 M_7 q_1'[t]\, q_2'[t] - \frac{2}{3}\hat{c}_3.\hat{e}_1 M_7 q_1'[t]\, q_2'[t] +$$

$$\frac{5}{12}\hat{c}_3.\hat{e}_2 M_7 q_1'[t]\, q_2'[t] - \frac{5}{6}\hat{c}_3.\hat{f}_1 M_7 q_1'[t]\, q_2'[t] - \frac{3}{2}\hat{c}_2.\hat{d}_1 M_7 q_2'[t]^2 -$$

$$\frac{1}{4}\hat{c}_2.\hat{d}_2 M_7 q_2'[t]^2 - \frac{2}{3}\hat{c}_2.\hat{e}_1 M_7 q_2'[t]^2 + \frac{5}{12}\hat{c}_2.\hat{e}_2 M_7 q_2'[t]^2 - \frac{5}{6}\hat{c}_2.\hat{f}_1 M_7 q_2'[t]^2\Big) +$$

$$\hat{d}_1.\hat{n}_2\left(\frac{3}{2}M_8 q_1'[t]^2 + \frac{3}{2}M_8 q_2'[t]^2\right) + \hat{c}_2.\hat{n}_2\Big(\frac{1}{2}M_8 q_1'[t]^2 - \frac{3}{2}\hat{c}_3.\hat{d}_1 M_8 q_1'[t]\, q_2'[t] -$$

$$\frac{1}{4}\hat{c}_3.\hat{d}_2 M_8 q_1'[t]\, q_2'[t] - \frac{2}{3}\hat{c}_3.\hat{e}_1 M_8 q_1'[t]\, q_2'[t] + \frac{5}{12}\hat{c}_3.\hat{e}_2 M_8 q_1'[t]\, q_2'[t] -$$

$$\frac{5}{6}\hat{c}_3.\hat{f}_1 M_8 q_1'[t]\, q_2'[t] - \frac{5}{21}\hat{c}_3.\hat{g}_1 M_8 q_1'[t]\, q_2'[t] - \frac{3}{2}\hat{c}_2.\hat{d}_1 M_8 q_2'[t]^2 - \frac{1}{4}\hat{c}_2.\hat{d}_2 M_8 q_2'[t]^2 -$$

$$\frac{2}{3}\hat{c}_2.\hat{e}_1 M_8 q_2'[t]^2 + \frac{5}{12}\hat{c}_2.\hat{e}_2 M_8 q_2'[t]^2 - \frac{5}{6}\hat{c}_2.\hat{f}_1 M_8 q_2'[t]^2 - \frac{5}{21}\hat{c}_2.\hat{g}_1 M_8 q_2'[t]^2\Big) +$$

$$\hat{e}_2.\hat{n}_2\Big(-\frac{5}{12}M_5 q_1'[t]^2 - \frac{5}{12}M_5 q_2'[t]^2 - \frac{5}{6}\hat{c}_3.\hat{d}_2 M_5 q_1'[t]\, q_3'[t] -$$

$$\frac{5}{6}\hat{c}_2.\hat{d}_2 M_5 q_2'[t]\, q_3'[t] - \frac{5}{12}M_5 q_3'[t]^2\Big) + \hat{e}_1.\hat{n}_2$$

$$\left(\frac{1}{6}M_5 q_1'[t]^2 + \frac{1}{6}M_5 q_2'[t]^2 + \frac{1}{3}\hat{c}_3.\hat{d}_2 M_5 q_1'[t]\, q_3'[t] + \frac{1}{3}\hat{c}_2.\hat{d}_2 M_5 q_2'[t]\, q_3'[t] + \frac{1}{6}M_5 q_3'[t]^2\right) +$$

$$\hat{d}_2.\hat{n}_2\Big(\frac{1}{4}M_5 q_1'[t]^2 + \frac{1}{4}M_5 q_2'[t]^2 - \frac{1}{3}\hat{c}_3.\hat{e}_1 M_5 q_1'[t]\, q_3'[t] + \frac{5}{6}\hat{c}_3.\hat{e}_2 M_5 q_1'[t]\, q_3'[t] -$$

$$\frac{1}{3}\hat{c}_2.\hat{e}_1 M_5 q_2'[t]\, q_3'[t] + \frac{5}{6}\hat{c}_2.\hat{e}_2 M_5 q_2'[t]\, q_3'[t] - \frac{1}{6}\hat{d}_2.\hat{e}_1 M_5 q_3'[t]^2 + \frac{5}{12}\hat{d}_2.\hat{e}_2 M_5 q_3'[t]^2\Big) +$$

$$\hat{e}_2.\hat{n}_2\Big(-\frac{5}{12}M_6 q_1'[t]^2 - \frac{5}{12}M_6 q_2'[t]^2 - \frac{5}{6}\hat{c}_3.\hat{d}_2 M_6 q_1'[t]\, q_3'[t] -$$

$$\frac{5}{6}\hat{c}_2.\hat{d}_2 M_6 q_2'[t]\, q_3'[t] - \frac{5}{12}M_6 q_3'[t]^2\Big) +$$

$$\hat{d}_2.\hat{n}_2\ \Big(\frac{1}{4} M_6\, q_1'[t]^2+\frac{1}{4} M_6\, q_2'[t]^2-\frac{4}{3}\,\hat{c}_3.\hat{e}_1 M_6\, q_1'[t]\, q_3'[t]+\frac{5}{6}\,\hat{c}_3.\hat{e}_2 M_6\, q_1'[t]\, q_3'[t]-$$

$$\frac{2}{3}\,\hat{c}_3.\hat{f}_1 M_6\, q_1'[t]\, q_3'[t]-\frac{4}{3}\,\hat{c}_2.\hat{e}_1 M_6\, q_2'[t]\, q_3'[t]+\frac{5}{6}\,\hat{c}_2.\hat{e}_2 M_6\, q_2'[t]\, q_3'[t]-$$

$$\frac{2}{3}\,\hat{c}_2.\hat{f}_1 M_6\, q_2'[t]\, q_3'[t]-\frac{2}{3}\,\hat{d}_2.\hat{e}_1 M_6\, q_3'[t]^2+\frac{5}{12}\,\hat{d}_2.\hat{e}_2 M_6\, q_3'[t]^2-\frac{1}{3}\,\hat{d}_2.\hat{f}_1 M_6\, q_3'[t]^2\Big)+$$

$$\hat{e}_2.\hat{n}_2\ \Big(-\frac{5}{12} M_7\, q_1'[t]^2-\frac{5}{12} M_7\, q_2'[t]^2-\frac{5}{6}\,\hat{c}_3.\hat{d}_2 M_7\, q_1'[t]\, q_3'[t]-$$

$$\frac{5}{6}\,\hat{c}_2.\hat{d}_2 M_7\, q_2'[t]\, q_3'[t]-\frac{5}{12} M_7\, q_3'[t]^2\Big)+$$

$$\hat{d}_2.\hat{n}_2\ \Big(\frac{1}{4} M_7\, q_1'[t]^2+\frac{1}{4} M_7\, q_2'[t]^2-\frac{4}{3}\,\hat{c}_3.\hat{e}_1 M_7\, q_1'[t]\, q_3'[t]+\frac{5}{6}\,\hat{c}_3.\hat{e}_2 M_7\, q_1'[t]\, q_3'[t]-$$

$$\frac{5}{3}\,\hat{c}_3.\hat{f}_1 M_7\, q_1'[t]\, q_3'[t]-\frac{4}{3}\,\hat{c}_2.\hat{e}_1 M_7\, q_2'[t]\, q_3'[t]+\frac{5}{6}\,\hat{c}_2.\hat{e}_2 M_7\, q_2'[t]\, q_3'[t]-$$

$$\frac{5}{3}\,\hat{c}_2.\hat{f}_1 M_7\, q_2'[t]\, q_3'[t]-\frac{2}{3}\,\hat{d}_2.\hat{e}_1 M_7\, q_3'[t]^2+\frac{5}{12}\,\hat{d}_2.\hat{e}_2 M_7\, q_3'[t]^2-\frac{5}{6}\,\hat{d}_2.\hat{f}_1 M_7\, q_3'[t]^2\Big)+$$

$$\hat{e}_2.\hat{n}_2\ \Big(-\frac{5}{12} M_8\, q_1'[t]^2-\frac{5}{12} M_8\, q_2'[t]^2-\frac{5}{6}\,\hat{c}_3.\hat{d}_2 M_8\, q_1'[t]\, q_3'[t]-$$

$$\frac{5}{6}\,\hat{c}_2.\hat{d}_2 M_8\, q_2'[t]\, q_3'[t]-\frac{5}{12} M_8\, q_3'[t]^2\Big)+$$

$$\hat{d}_2.\hat{n}_2\ \Big(\frac{1}{4} M_8\, q_1'[t]^2+\frac{1}{4} M_8\, q_2'[t]^2-\frac{4}{3}\,\hat{c}_3.\hat{e}_1 M_8\, q_1'[t]\, q_3'[t]+\frac{5}{6}\,\hat{c}_3.\hat{e}_2 M_8\, q_1'[t]\, q_3'[t]-$$

$$\frac{5}{3}\,\hat{c}_3.\hat{f}_1 M_8\, q_1'[t]\, q_3'[t]-\frac{10}{21}\,\hat{c}_3.\hat{g}_1 M_8\, q_1'[t]\, q_3'[t]-\frac{4}{3}\,\hat{c}_2.\hat{e}_1 M_8\, q_2'[t]\, q_3'[t]+$$

$$\frac{5}{6}\,\hat{c}_2.\hat{e}_2 M_8\, q_2'[t]\, q_3'[t]-\frac{5}{3}\,\hat{c}_2.\hat{f}_1 M_8\, q_2'[t]\, q_3'[t]-\frac{10}{21}\,\hat{c}_2.\hat{g}_1 M_8\, q_2'[t]\, q_3'[t]-$$

$$\frac{2}{3}\,\hat{d}_2.\hat{e}_1 M_8\, q_3'[t]^2+\frac{5}{12}\,\hat{d}_2.\hat{e}_2 M_8\, q_3'[t]^2-\frac{5}{6}\,\hat{d}_2.\hat{f}_1 M_8\, q_3'[t]^2-\frac{5}{21}\,\hat{d}_2.\hat{g}_1 M_8\, q_3'[t]^2\Big)+$$

$$\hat{f}_1.\hat{n}_2\ \Big(\frac{1}{3} M_6\, q_1'[t]^2+\frac{1}{3} M_6\, q_2'[t]^2+\frac{2}{3}\,\hat{c}_3.\hat{d}_2 M_6\, q_1'[t]\, q_3'[t]+\frac{2}{3}\,\hat{c}_2.\hat{d}_2 M_6\, q_2'[t]\, q_3'[t]+\frac{1}{3} M_6\, q_3'[t]^2+$$

$$\frac{2}{3}\,\hat{c}_3.\hat{e}_1 M_6\, q_1'[t]\, q_4'[t]+\frac{2}{3}\,\hat{c}_2.\hat{e}_1 M_6\, q_2'[t]\, q_4'[t]+\frac{2}{3}\,\hat{d}_2.\hat{e}_1 M_6\, q_3'[t]\, q_4'[t]+\frac{1}{3} M_6\, q_4'[t]^2\Big)+$$

$$\hat{e}_1.\hat{n}_2\ \Big(\frac{2}{3} M_6\, q_1'[t]^2+\frac{2}{3} M_6\, q_2'[t]^2+\frac{4}{3}\,\hat{c}_3.\hat{d}_2 M_6\, q_1'[t]\, q_3'[t]+\frac{4}{3}\,\hat{c}_2.\hat{d}_2 M_6\, q_2'[t]\, q_3'[t]+$$

$$\frac{2}{3} M_6\, q_3'[t]^2-\frac{2}{3}\,\hat{c}_3.\hat{f}_1 M_6\, q_1'[t]\, q_4'[t]-\frac{2}{3}\,\hat{c}_2.\hat{f}_1 M_6\, q_2'[t]\, q_4'[t]-$$

$$\frac{2}{3}\,\hat{d}_2.\hat{f}_1 M_6\, q_3'[t]\, q_4'[t]-\frac{1}{3}\,\hat{e}_1.\hat{f}_1 M_6\, q_4'[t]^2\Big)+$$

$$\hat{f}_1.\hat{n}_2\ \Big(\frac{5}{6} M_7\, q_1'[t]^2+\frac{5}{6} M_7\, q_2'[t]^2+\frac{5}{3}\,\hat{c}_3.\hat{d}_2 M_7\, q_1'[t]\, q_3'[t]+\frac{5}{3}\,\hat{c}_2.\hat{d}_2 M_7\, q_2'[t]\, q_3'[t]+\frac{5}{6} M_7\, q_3'[t]^2+$$

$$\frac{5}{3}\,\hat{c}_3.\hat{e}_1 M_7\, q_1'[t]\, q_4'[t]+\frac{5}{3}\,\hat{c}_2.\hat{e}_1 M_7\, q_2'[t]\, q_4'[t]+\frac{5}{3}\,\hat{d}_2.\hat{e}_1 M_7\, q_3'[t]\, q_4'[t]+\frac{5}{6} M_7\, q_4'[t]^2\Big)+$$

$$\hat{e}_1.\hat{n}_2\ \Big(\frac{2}{3} M_7\, q_1'[t]^2+\frac{2}{3} M_7\, q_2'[t]^2+\frac{4}{3}\,\hat{c}_3.\hat{d}_2 M_7\, q_1'[t]\, q_3'[t]+\frac{4}{3}\,\hat{c}_2.\hat{d}_2 M_7\, q_2'[t]\, q_3'[t]+$$

$$\frac{2}{3} M_7 q_3'[t]^2 - \frac{5}{3} \hat{c}_3.\hat{f}_1 M_7 q_1'[t]\, q_4'[t] - \frac{5}{3} \hat{c}_2.\hat{f}_1 M_7 q_2'[t]\, q_4'[t] -$$

$$\left.\frac{5}{3} \hat{d}_2.\hat{f}_1 M_7 q_3'[t]\, q_4'[t] - \frac{5}{6} \hat{e}_1.\hat{f}_1 M_7 q_4'[t]^2\right) +$$

$$\hat{f}_1.\hat{n}_2 \left(\frac{5}{6} M_8 q_1'[t]^2 + \frac{5}{6} M_8 q_2'[t]^2 + \frac{5}{3} \hat{c}_3.\hat{d}_2 M_8 q_1'[t]\, q_3'[t] + \frac{5}{3} \hat{c}_2.\hat{d}_2 M_8 q_2'[t]\, q_3'[t] + \frac{5}{6} M_8 q_3'[t]^2 +\right.$$

$$\left.\frac{5}{3} \hat{c}_3.\hat{e}_1 M_8 q_1'[t]\, q_4'[t] + \frac{5}{3} \hat{c}_2.\hat{e}_1 M_8 q_2'[t]\, q_4'[t] + \frac{5}{3} \hat{d}_2.\hat{e}_1 M_8 q_3'[t]\, q_4'[t] + \frac{5}{6} M_8 q_4'[t]^2\right) +$$

$$\hat{e}_1.\hat{n}_2 \left(\frac{2}{3} M_8 q_1'[t]^2 + \frac{2}{3} M_8 q_2'[t]^2 + \frac{4}{3} \hat{c}_3.\hat{d}_2 M_8 q_1'[t]\, q_3'[t] + \frac{4}{3} \hat{c}_2.\hat{d}_2 M_8 q_2'[t]\, q_3'[t] +\right.$$

$$\frac{2}{3} M_8 q_3'[t]^2 - \frac{5}{3} \hat{c}_3.\hat{f}_1 M_8 q_1'[t]\, q_4'[t] - \frac{10}{21} \hat{c}_3.\hat{g}_1 M_8 q_1'[t]\, q_4'[t] -$$

$$\frac{5}{3} \hat{c}_2.\hat{f}_1 M_8 q_2'[t]\, q_4'[t] - \frac{10}{21} \hat{c}_2.\hat{g}_1 M_8 q_2'[t]\, q_4'[t] - \frac{5}{3} \hat{d}_2.\hat{f}_1 M_8 q_3'[t]\, q_4'[t] -$$

$$\left.\frac{10}{21} \hat{d}_2.\hat{g}_1 M_8 q_3'[t]\, q_4'[t] - \frac{5}{6} \hat{e}_1.\hat{f}_1 M_8 q_4'[t]^2 - \frac{5}{21} \hat{e}_1.\hat{g}_1 M_8 q_4'[t]^2\right) +$$

$$\hat{g}_1.\hat{n}_2 \left(\frac{5}{21} M_8 q_1'[t]^2 + \frac{5}{21} M_8 q_2'[t]^2 + \frac{10}{21} \hat{c}_3.\hat{d}_2 M_8 q_1'[t]\, q_3'[t] + \frac{10}{21} \hat{c}_2.\hat{d}_2 M_8 q_2'[t]\, q_3'[t] +\right.$$

$$\frac{5}{21} M_8 q_3'[t]^2 + \frac{10}{21} \hat{c}_3.\hat{e}_1 M_8 q_1'[t]\, q_4'[t] + \frac{10}{21} \hat{c}_2.\hat{e}_1 M_8 q_2'[t]\, q_4'[t] + \frac{10}{21} \hat{d}_2.\hat{e}_1 M_8 q_3'[t]\, q_4'[t] +$$

$$\frac{5}{21} M_8 q_4'[t]^2 + \frac{10}{21} \hat{c}_3.\hat{f}_2 M_8 q_1'[t]\, q_5'[t] + \frac{10}{21} \hat{c}_2.\hat{f}_2 M_8 q_2'[t]\, q_5'[t] +$$

$$\left.\frac{10}{21} \hat{d}_2.\hat{f}_2 M_8 q_3'[t]\, q_5'[t] + \frac{10}{21} \hat{e}_1.\hat{f}_2 M_8 q_4'[t]\, q_5'[t] + \frac{5}{21} M_8 q_5'[t]^2\right) +$$

$$\hat{f}_2.\hat{n}_2 \left(-\frac{10}{21} \hat{c}_3.\hat{g}_1 M_8 q_1'[t]\, q_5'[t] - \frac{10}{21} \hat{c}_2.\hat{g}_1 M_8 q_2'[t]\, q_5'[t] - \frac{10}{21} \hat{d}_2.\hat{g}_1 M_8 q_3'[t]\, q_5'[t] -\right.$$

$$\left.\frac{10}{21} \hat{e}_1.\hat{g}_1 M_8 q_4'[t]\, q_5'[t] - \frac{5}{21} \hat{f}_2.\hat{g}_1 M_8 q_5'[t]^2\right) -$$

$$M_1 y''[t] - M_2 y''[t] - M_3 y''[t] - M_4 y''[t] - M_5 y''[t] - M_6 y''[t] - M_7 y''[t] -$$

$$M_8 y''[t] - \frac{1}{2} \hat{c}_3 \times \hat{d}_1.\hat{n}_2 M_4 q_1''[t] -$$

$$\frac{1}{4} \hat{c}_3 \times \hat{d}_2.\hat{n}_2 M_4 q_1''[t] + \frac{1}{2} \hat{c}_1.\hat{n}_2 M_4 q_1''[t] -$$

$$\frac{3}{2} \hat{c}_3 \times \hat{d}_1.\hat{n}_2 M_5 q_1''[t] - \frac{1}{4} \hat{c}_3 \times \hat{d}_2.\hat{n}_2 M_5 q_1''[t] -$$

$$\frac{1}{6} \hat{c}_3 \times \hat{e}_1.\hat{n}_2 M_5 q_1''[t] + \frac{5}{12} \hat{c}_3 \times \hat{e}_2.\hat{n}_2 M_5 q_1''[t] +$$

$$\frac{1}{2} \hat{c}_1.\hat{n}_2 M_5 q_1''[t] - \frac{3}{2} \hat{c}_3 \times \hat{d}_1.\hat{n}_2 M_6 q_1''[t] -$$

$$\frac{1}{4} \hat{c}_3 \times \hat{d}_2.\hat{n}_2 M_6 q_1''[t] - \frac{2}{3} \hat{c}_3 \times \hat{e}_1.\hat{n}_2 M_6 q_1''[t] +$$

$$\frac{5}{12} \hat{c}_3 \times \hat{e}_2.\hat{n}_2 M_6 q_1''[t] - \frac{1}{3} \hat{c}_3 \times \hat{f}_1.\hat{n}_2 M_6 q_1''[t] +$$

$$\frac{1}{2} \hat{c}_1.\hat{n}_2 M_6 q_1''[t] - \frac{3}{2} \hat{c}_3 \times \hat{d}_1.\hat{n}_2 M_7 q_1''[t] -$$

$$\frac{1}{4}\hat{c}_3\times\hat{d}_2.\hat{n}_2 M_7 q_1''[t] - \frac{2}{3}\hat{c}_3\times\hat{e}_1.\hat{n}_2 M_7 q_1''[t] +$$
$$\frac{5}{12}\hat{c}_3\times\hat{e}_2.\hat{n}_2 M_7 q_1''[t] - \frac{5}{6}\hat{c}_3\times\hat{f}_1.\hat{n}_2 M_7 q_1''[t] +$$
$$\frac{1}{2}\hat{c}_1.\hat{n}_2 M_7 q_1''[t] - \frac{3}{2}\hat{c}_3\times\hat{d}_1.\hat{n}_2 M_8 q_1''[t] -$$
$$\frac{1}{4}\hat{c}_3\times\hat{d}_2.\hat{n}_2 M_8 q_1''[t] - \frac{2}{3}\hat{c}_3\times\hat{e}_1.\hat{n}_2 M_8 q_1''[t] +$$
$$\frac{5}{12}\hat{c}_3\times\hat{e}_2.\hat{n}_2 M_8 q_1''[t] - \frac{5}{6}\hat{c}_3\times\hat{f}_1.\hat{n}_2 M_8 q_1''[t] -$$
$$\frac{5}{21}\hat{c}_3\times\hat{g}_1.\hat{n}_2 M_8 q_1''[t] + \frac{1}{2}\hat{c}_1.\hat{n}_2 M_8 q_1''[t] -$$
$$\frac{1}{2}\hat{c}_2\times\hat{d}_1.\hat{n}_2 M_4 q_2''[t] - \frac{1}{4}\hat{c}_2\times\hat{d}_2.\hat{n}_2 M_4 q_2''[t] -$$
$$\frac{3}{2}\hat{c}_2\times\hat{d}_1.\hat{n}_2 M_5 q_2''[t] - \frac{1}{4}\hat{c}_2\times\hat{d}_2.\hat{n}_2 M_5 q_2''[t] -$$
$$\frac{1}{6}\hat{c}_2\times\hat{e}_1.\hat{n}_2 M_5 q_2''[t] + \frac{5}{12}\hat{c}_2\times\hat{e}_2.\hat{n}_2 M_5 q_2''[t] -$$
$$\frac{3}{2}\hat{c}_2\times\hat{d}_1.\hat{n}_2 M_6 q_2''[t] - \frac{1}{4}\hat{c}_2\times\hat{d}_2.\hat{n}_2 M_6 q_2''[t] -$$
$$\frac{2}{3}\hat{c}_2\times\hat{e}_1.\hat{n}_2 M_6 q_2''[t] + \frac{5}{12}\hat{c}_2\times\hat{e}_2.\hat{n}_2 M_6 q_2''[t] -$$
$$\frac{1}{3}\hat{c}_2\times\hat{f}_1.\hat{n}_2 M_6 q_2''[t] - \frac{3}{2}\hat{c}_2\times\hat{d}_1.\hat{n}_2 M_7 q_2''[t] -$$
$$\frac{1}{4}\hat{c}_2\times\hat{d}_2.\hat{n}_2 M_7 q_2''[t] - \frac{2}{3}\hat{c}_2\times\hat{e}_1.\hat{n}_2 M_7 q_2''[t] +$$
$$\frac{5}{12}\hat{c}_2\times\hat{e}_2.\hat{n}_2 M_7 q_2''[t] - \frac{5}{6}\hat{c}_2\times\hat{f}_1.\hat{n}_2 M_7 q_2''[t] -$$
$$\frac{3}{2}\hat{c}_2\times\hat{d}_1.\hat{n}_2 M_8 q_2''[t] - \frac{1}{4}\hat{c}_2\times\hat{d}_2.\hat{n}_2 M_8 q_2''[t] -$$
$$\frac{2}{3}\hat{c}_2\times\hat{e}_1.\hat{n}_2 M_8 q_2''[t] + \frac{5}{12}\hat{c}_2\times\hat{e}_2.\hat{n}_2 M_8 q_2''[t] -$$
$$\frac{5}{6}\hat{c}_2\times\hat{f}_1.\hat{n}_2 M_8 q_2''[t] - \frac{5}{21}\hat{c}_2\times\hat{g}_1.\hat{n}_2 M_8 q_2''[t] -$$
$$\frac{1}{6}\hat{d}_2\times\hat{e}_1.\hat{n}_2 M_5 q_3''[t] + \frac{5}{12}\hat{d}_2\times\hat{e}_2.\hat{n}_2 M_5 q_3''[t] -$$
$$\frac{2}{3}\hat{d}_2\times\hat{e}_1.\hat{n}_2 M_6 q_3''[t] + \frac{5}{12}\hat{d}_2\times\hat{e}_2.\hat{n}_2 M_6 q_3''[t] -$$
$$\frac{1}{3}\hat{d}_2\times\hat{f}_1.\hat{n}_2 M_6 q_3''[t] - \frac{2}{3}\hat{d}_2\times\hat{e}_1.\hat{n}_2 M_7 q_3''[t] +$$
$$\frac{5}{12}\hat{d}_2\times\hat{e}_2.\hat{n}_2 M_7 q_3''[t] - \frac{5}{6}\hat{d}_2\times\hat{f}_1.\hat{n}_2 M_7 q_3''[t] -$$
$$\frac{2}{3}\hat{d}_2\times\hat{e}_1.\hat{n}_2 M_8 q_3''[t] + \frac{5}{12}\hat{d}_2\times\hat{e}_2.\hat{n}_2 M_8 q_3''[t] -$$

$$\frac{5}{6}\,\hat{d}_2\times\hat{f}_1.\hat{n}_2\,M_8\,q_3''[t]-\frac{5}{21}\,\hat{d}_2\times\hat{g}_1.\hat{n}_2\,M_8\,q_3''[t]-$$
$$\frac{1}{3}\,\hat{e}_1\times\hat{f}_1.\hat{n}_2\,M_6\,q_4''[t]-\frac{5}{6}\,\hat{e}_1\times\hat{f}_1.\hat{n}_2\,M_7\,q_4''[t]-\frac{5}{6}\,\hat{e}_1\times\hat{f}_1.\hat{n}_2\,M_8\,q_4''[t]-$$
$$\frac{5}{21}\,\hat{e}_1\times\hat{g}_1.\hat{n}_2\,M_8\,q_4''[t]-\frac{5}{21}\,\hat{f}_2\times\hat{g}_1.\hat{n}_2\,M_8\,q_5''[t]$$

For $q_1[t]$ the equations of motion is (they will be set to zero when solution is required).

In[547]:= $$\mathbf{eq[3]} = \sum_{ii=1}^{N_{bodies}}\left(\left(F_{app}[ii]-I_f[ii]\right).\mathrm{Pvel}[\mathrm{OvB}[ii],q_1'[t]]+\left(T_{app}[ii]-I_t[ii]\right).\mathrm{Pvel}[\mathrm{N\omega B}[ii],q_1'[t]]\right)$$

Out[547]= $$-\frac{3}{2}\,\hat{c}_3\times\hat{d}_1.\hat{h}_1\,F_{toolx}-\frac{1}{4}\,\hat{c}_3\times\hat{d}_2.\hat{h}_1\,F_{toolx}-\frac{2}{3}\,\hat{c}_3\times\hat{e}_1.\hat{h}_1\,F_{toolx}+\frac{5}{12}\,\hat{c}_3\times\hat{e}_2.\hat{h}_1\,F_{toolx}-\frac{5}{6}\,\hat{c}_3\times\hat{f}_1.\hat{h}_1\,F_{toolx}-\frac{5}{21}\,\hat{c}_3\times\hat{g}_1.\hat{h}_1\,F_{toolx}+\langle\!\langle 1151 \rangle\!\rangle+\hat{c}_3.\hat{g}_1\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_3.\hat{g}_2\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_3.\hat{g}_3\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_3.\hat{h}_1\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_3.\hat{h}_2\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_3.\hat{h}_3\,(\langle\!\langle 1 \rangle\!\rangle)$$

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For $q_2[t]$ the equations of motion is (they will be set to zero when solution is required).

In[548]:= $$\mathbf{eq[4]} = \sum_{ii=1}^{N_{bodies}}\left(\left(F_{app}[ii]-I_f[ii]\right).\mathrm{Pvel}[\mathrm{OvB}[ii],q_2'[t]]+\left(T_{app}[ii]-I_t[ii]\right).\mathrm{Pvel}[\mathrm{N\omega B}[ii],q_2'[t]]\right)$$

Out[548]= $$-\frac{3}{2}\,\hat{c}_2\times\hat{d}_1.\hat{h}_1\,F_{toolx}-\frac{1}{4}\,\hat{c}_2\times\hat{d}_2.\hat{h}_1\,F_{toolx}-\frac{2}{3}\,\hat{c}_2\times\hat{e}_1.\hat{h}_1\,F_{toolx}+\langle\!\langle 1027 \rangle\!\rangle+\hat{c}_2.\hat{g}_1\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_2.\hat{g}_2\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_2.\hat{g}_3\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_2.\hat{h}_1\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_2.\hat{h}_2\,(\langle\!\langle 1 \rangle\!\rangle)+\hat{c}_2.\hat{h}_3\,(\langle\!\langle 1 \rangle\!\rangle)$$

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For $q_3[t]$ the equations of motion is (they will be set to zero when solution is required).

In[549]:= $$\mathbf{eq[5]} = \sum_{ii=1}^{N_{bodies}} \Big(\big(F_{app}[ii] - I_f[ii]\big).\mathrm{Pvel}[\mathrm{OvB}[ii], q_3{}'[t]] + \big(T_{app}[ii] - I_t[ii]\big).\mathrm{Pvel}[\mathrm{N}\omega\mathrm{B}[ii], q_3{}'[t]]\Big)$$

Out[549]= $$-\frac{2}{3}\,\hat{d}_2\times\hat{e}_1.\hat{h}_1\,F_{toolx} + \frac{5}{12}\,\hat{d}_2\times\hat{e}_2.\hat{h}_1\,F_{toolx} - \frac{5}{6}\,\hat{d}_2\times\hat{f}_1.\hat{h}_1\,F_{toolx} - \frac{5}{21}\,\hat{d}_2\times\hat{g}_1.\hat{h}_1\,F_{toolx} - \frac{2}{3}\,\hat{d}_2\times\hat{e}_1.\hat{h}_2\,F_{tooly} + \cdots 630 \cdots + \hat{d}_2.\hat{g}_1\,(\cdots 1 \cdots) + \hat{d}_2.\hat{g}_2\,(\cdots 1 \cdots) + \hat{d}_2.\hat{g}_3\,(\cdots 1 \cdots) + \hat{d}_2.\hat{h}_1\,(\cdots 1 \cdots) + \hat{d}_2.\hat{h}_2\,(\cdots 1 \cdots) + \hat{d}_2.\hat{h}_3\,(\cdots 1 \cdots)$$

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For $q_4[t]$ the equations of motion is (they will be set to zero when solution is required).

In[550]:= $$\mathbf{eq[6]} = \sum_{ii=1}^{N_{bodies}} \Big(\big(F_{app}[ii] - I_f[ii]\big).\mathrm{Pvel}[\mathrm{OvB}[ii], q_4{}'[t]] + \big(T_{app}[ii] - I_t[ii]\big).\mathrm{Pvel}[\mathrm{N}\omega\mathrm{B}[ii], q_4{}'[t]]\Big)$$

Out[550]= $$\cdots 206 \cdots + \hat{e}_1.\hat{g}_1\,(\cdots 1 \cdots) + \hat{e}_1.\hat{g}_2\,(\cdots 1 \cdots) + \hat{e}_1.\hat{g}_3\,(\cdots 1 \cdots) + \hat{e}_1.\hat{h}_1\,(\cdots 1 \cdots) + \hat{e}_1.\hat{h}_2\,(\cdots 1 \cdots) + \hat{e}_1.\hat{h}_3\,(\cdots 1 \cdots)$$

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For $q_5[t]$ the equations of motion is (they will be set to zero when solution is required).

In[551]:= $$\mathbf{eq[7]} = \sum_{ii=1}^{N_{bodies}} \Big(\big(F_{app}[ii] - I_f[ii]\big).\mathrm{Pvel}[\mathrm{OvB}[ii], q_5{}'[t]] + \big(T_{app}[ii] - I_t[ii]\big).\mathrm{Pvel}[\mathrm{N}\omega\mathrm{B}[ii], q_5{}'[t]]\Big)$$

Out[551]= $$-\hat{f}_2.\hat{h}_1\,T_{toolx} - \hat{f}_2.\hat{h}_2\,T_{tooly} - \hat{f}_2.\hat{h}_3\,T_{toolz} + \hat{f}_2.\hat{g}_2\,T_{mot}[5] - \hat{f}_2.\hat{g}_1\,T_{mot}[6] + \hat{f}_2.\hat{h}_1\,T_{mot}[6] - \hat{c}_3\times\hat{f}_2.\hat{g}_1\,\Big(-\hat{c}_3.\hat{g}_1\,Gi_{11}\,q_1{}'[t]^2 - \hat{c}_2.\hat{g}_1\,Gi_{11}\,q_1{}'[t]\,q_2{}'[t] - \hat{d}_2.\hat{g}_1\,Gi_{11}\,q_1{}'[t]\,q_3{}'[t] - \hat{e}_1.\hat{g}_1\,Gi_{11}\,q_1{}'[t]\,q_4{}'[t] - \hat{f}_2.\hat{g}_1\,Gi_{11}\,q_1{}'[t]\,q_5{}'[t]\Big) - \hat{c}_3\times\hat{f}_2.\hat{g}_2\,\Big(-\hat{c}_3.\hat{g}_2\,Gi_{22}\,q_1{}'[t]^2 - \hat{c}_2.\hat{g}_2\,Gi_{22}\,q_1{}'[t]\,q_2{}'[t] - \hat{d}_2.\hat{g}_2\,Gi_{22}\,q_1{}'[t]\,q_3{}'[t] - \hat{e}_1.\hat{g}_2\,Gi_{22}\,q_1{}'[t]\,q_4{}'[t] - \hat{f}_2.\hat{g}_2\,Gi_{22}\,q_1{}'[t]\,q_5{}'[t]\Big) - \hat{c}_3\times\hat{f}_2.\hat{g}_3\,\Big(-\hat{c}_3.\hat{g}_3\,Gi_{33}\,q_1{}'[t]^2 - \hat{c}_2.\hat{g}_3\,Gi_{33}\,q_1{}'[t]\,q_2{}'[t] - \hat{d}_2.\hat{g}_3\,Gi_{33}\,q_1{}'[t]\,q_3{}'[t] - \hat{e}_1.\hat{g}_3\,Gi_{33}\,q_1{}'[t]\,q_4{}'[t] - \hat{f}_2.\hat{g}_3\,Gi_{33}\,q_1{}'[t]\,q_5{}'[t]\Big) - \hat{c}_2\times\hat{f}_2.\hat{g}_1\,\Big(-\hat{c}_3.\hat{g}_1\,Gi_{11}\,q_1{}'[t]\,q_2{}'[t] - \hat{c}_2.\hat{g}_1\,Gi_{11}\,q_2{}'[t]^2 - \hat{d}_2.\hat{g}_1\,Gi_{11}\,q_2{}'[t]\,q_3{}'[t] - \hat{e}_1.\hat{g}_1\,Gi_{11}\,q_2{}'[t]\,q_4{}'[t] - \hat{f}_2.\hat{g}_1\,Gi_{11}\,q_2{}'[t]\,q_5{}'[t]\Big) -$$

$$\hat{c}_2\times\hat{f}_2.\hat{g}_2\left(-\hat{c}_3.\hat{g}_2\,Gi_{22}\,q_1'[t]\,q_2'[t]-\hat{c}_2.\hat{g}_2\,Gi_{22}\,q_2'[t]^2-\hat{d}_2.\hat{g}_2\,Gi_{22}\,q_2'[t]\,q_3'[t]-\hat{e}_1.\hat{g}_2\,Gi_{22}\,q_2'[t]\,q_4'[t]-\hat{f}_2.\hat{g}_2\,Gi_{22}\,q_2'[t]\,q_5'[t]\right)-$$

$$\hat{c}_2\times\hat{f}_2.\hat{g}_3\left(-\hat{c}_3.\hat{g}_3\,Gi_{33}\,q_1'[t]\,q_2'[t]-\hat{c}_2.\hat{g}_3\,Gi_{33}\,q_2'[t]^2-\hat{d}_2.\hat{g}_3\,Gi_{33}\,q_2'[t]\,q_3'[t]-\hat{e}_1.\hat{g}_3\,Gi_{33}\,q_2'[t]\,q_4'[t]-\hat{f}_2.\hat{g}_3\,Gi_{33}\,q_2'[t]\,q_5'[t]\right)-$$

$$\hat{d}_2\times\hat{f}_2.\hat{g}_1\left(-\hat{c}_3.\hat{g}_1\,Gi_{11}\,q_1'[t]\,q_3'[t]-\hat{c}_2.\hat{g}_1\,Gi_{11}\,q_2'[t]\,q_3'[t]-\hat{d}_2.\hat{g}_1\,Gi_{11}\,q_3'[t]^2-\hat{e}_1.\hat{g}_1\,Gi_{11}\,q_3'[t]\,q_4'[t]-\hat{f}_2.\hat{g}_1\,Gi_{11}\,q_3'[t]\,q_5'[t]\right)-$$

$$\hat{d}_2\times\hat{f}_2.\hat{g}_2\left(-\hat{c}_3.\hat{g}_2\,Gi_{22}\,q_1'[t]\,q_3'[t]-\hat{c}_2.\hat{g}_2\,Gi_{22}\,q_2'[t]\,q_3'[t]-\hat{d}_2.\hat{g}_2\,Gi_{22}\,q_3'[t]^2-\hat{e}_1.\hat{g}_2\,Gi_{22}\,q_3'[t]\,q_4'[t]-\hat{f}_2.\hat{g}_2\,Gi_{22}\,q_3'[t]\,q_5'[t]\right)-$$

$$\hat{d}_2\times\hat{f}_2.\hat{g}_3\left(-\hat{c}_3.\hat{g}_3\,Gi_{33}\,q_1'[t]\,q_3'[t]-\hat{c}_2.\hat{g}_3\,Gi_{33}\,q_2'[t]\,q_3'[t]-\hat{d}_2.\hat{g}_3\,Gi_{33}\,q_3'[t]^2-\hat{e}_1.\hat{g}_3\,Gi_{33}\,q_3'[t]\,q_4'[t]-\hat{f}_2.\hat{g}_3\,Gi_{33}\,q_3'[t]\,q_5'[t]\right)-$$

$$\hat{e}_1\times\hat{f}_2.\hat{g}_1\left(-\hat{c}_3.\hat{g}_1\,Gi_{11}\,q_1'[t]\,q_4'[t]-\hat{c}_2.\hat{g}_1\,Gi_{11}\,q_2'[t]\,q_4'[t]-\hat{d}_2.\hat{g}_1\,Gi_{11}\,q_3'[t]\,q_4'[t]-\hat{e}_1.\hat{g}_1\,Gi_{11}\,q_4'[t]^2-\hat{f}_2.\hat{g}_1\,Gi_{11}\,q_4'[t]\,q_5'[t]\right)-$$

$$\hat{e}_1\times\hat{f}_2.\hat{g}_2\left(-\hat{c}_3.\hat{g}_2\,Gi_{22}\,q_1'[t]\,q_4'[t]-\hat{c}_2.\hat{g}_2\,Gi_{22}\,q_2'[t]\,q_4'[t]-\hat{d}_2.\hat{g}_2\,Gi_{22}\,q_3'[t]\,q_4'[t]-\hat{e}_1.\hat{g}_2\,Gi_{22}\,q_4'[t]^2-\hat{f}_2.\hat{g}_2\,Gi_{22}\,q_4'[t]\,q_5'[t]\right)-$$

$$\hat{e}_1\times\hat{f}_2.\hat{g}_3\left(-\hat{c}_3.\hat{g}_3\,Gi_{33}\,q_1'[t]\,q_4'[t]-\hat{c}_2.\hat{g}_3\,Gi_{33}\,q_2'[t]\,q_4'[t]-\hat{d}_2.\hat{g}_3\,Gi_{33}\,q_3'[t]\,q_4'[t]-\hat{e}_1.\hat{g}_3\,Gi_{33}\,q_4'[t]^2-\hat{f}_2.\hat{g}_3\,Gi_{33}\,q_4'[t]\,q_5'[t]\right)-$$

$$\hat{c}_3\times\hat{f}_2.\hat{h}_1\left(-\hat{c}_3.\hat{h}_1\,Hi_{11}\,q_1'[t]^2-\hat{c}_2.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_2'[t]-\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_3'[t]-\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_4'[t]-\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_6'[t]\right)-$$

$$\hat{c}_3\times\hat{f}_2.\hat{h}_2\left(-\hat{c}_3.\hat{h}_2\,Hi_{22}\,q_1'[t]^2-\hat{c}_2.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_2'[t]-\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_3'[t]-\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_4'[t]-\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_6'[t]\right)-$$

$$\hat{c}_3\times\hat{f}_2.\hat{h}_3\left(-\hat{c}_3.\hat{h}_3\,Hi_{33}\,q_1'[t]^2-\hat{c}_2.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_2'[t]-\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_3'[t]-\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_4'[t]-\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_6'[t]\right)-$$

$$\hat{c}_2\times\hat{f}_2.\hat{h}_1\left(-\hat{c}_3.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_2'[t]-\hat{c}_2.\hat{h}_1\,Hi_{11}\,q_2'[t]^2-\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_3'[t]-\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_4'[t]-\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_6'[t]\right)-$$

$$\hat{c}_2\times\hat{f}_2.\hat{h}_2\left(-\hat{c}_3.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_2'[t]-\hat{c}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]^2-\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_3'[t]-\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_4'[t]-\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_6'[t]\right)-$$

$$\hat{c}_2\times\hat{f}_2.\hat{h}_3\left(-\hat{c}_3.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_2'[t]-\hat{c}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]^2-\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_3'[t]-\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_4'[t]-\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_6'[t]\right)-$$

$$\hat{d}_2\times\hat{f}_2.\hat{h}_1\left(-\hat{c}_3.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_3'[t]-\hat{c}_2.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_3'[t]-\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_3'[t]^2-\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_4'[t]-\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_6'[t]\right)-$$

$$\hat{d}_2\times\hat{f}_2.\hat{h}_2\left(-\hat{c}_3.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_3'[t]-\hat{c}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_3'[t]-\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_3'[t]^2-\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_4'[t]-\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_6'[t]\right)-$$

$$\hat{d}_2\times\hat{f}_2.\hat{h}_3\,\Big(-\hat{c}_3.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_3'[t]-\hat{c}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_3'[t]-\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_3'[t]^2-$$
$$\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_4'[t]-\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_6'[t]\Big)-$$
$$\hat{e}_1\times\hat{f}_2.\hat{h}_1\,\Big(-\hat{c}_3.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_4'[t]-\hat{c}_2.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_4'[t]-\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_4'[t]-$$
$$\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_4'[t]^2-\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_4'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_4'[t]\,q_6'[t]\Big)-$$
$$\hat{e}_1\times\hat{f}_2.\hat{h}_2\,\Big(-\hat{c}_3.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_4'[t]-\hat{c}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_4'[t]-\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_4'[t]-$$
$$\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_4'[t]^2-\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_4'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_4'[t]\,q_6'[t]\Big)-$$
$$\hat{e}_1\times\hat{f}_2.\hat{h}_3\,\Big(-\hat{c}_3.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_4'[t]-\hat{c}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_4'[t]-\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_4'[t]-$$
$$\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_4'[t]^2-\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_4'[t]\,q_5'[t]-\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_4'[t]\,q_6'[t]\Big)+$$
$$\hat{f}_2\times\hat{g}_1.\hat{h}_1\,\Big(-\hat{c}_3.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_6'[t]-\hat{c}_2.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_6'[t]-\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_6'[t]-$$
$$\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_4'[t]\,q_6'[t]-\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_5'[t]\,q_6'[t]-\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_6'[t]^2\Big)+$$
$$\hat{f}_2\times\hat{g}_1.\hat{h}_2\,\Big(-\hat{c}_3.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_6'[t]-\hat{c}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_6'[t]-\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_6'[t]-$$
$$\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_4'[t]\,q_6'[t]-\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_5'[t]\,q_6'[t]-\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_6'[t]^2\Big)+$$
$$\hat{f}_2\times\hat{g}_1.\hat{h}_3\,\Big(-\hat{c}_3.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_6'[t]-\hat{c}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_6'[t]-\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_6'[t]-$$
$$\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_4'[t]\,q_6'[t]-\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_5'[t]\,q_6'[t]-\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_6'[t]^2\Big)+$$
$$\hat{f}_2.\hat{g}_1\,\Big(\hat{c}_1.\hat{g}_1\,Gi_{11}\,q_1'[t]\,q_2'[t]-\hat{c}_3\times\hat{d}_2.\hat{g}_1\,Gi_{11}\,q_1'[t]\,q_3'[t]-\hat{c}_2\times\hat{d}_2.\hat{g}_1\,Gi_{11}\,q_2'[t]\,q_3'[t]-$$
$$\hat{c}_3\times\hat{e}_1.\hat{g}_1\,Gi_{11}\,q_1'[t]\,q_4'[t]-\hat{c}_2\times\hat{e}_1.\hat{g}_1\,Gi_{11}\,q_2'[t]\,q_4'[t]-$$
$$\hat{d}_2\times\hat{e}_1.\hat{g}_1\,Gi_{11}\,q_3'[t]\,q_4'[t]-\hat{c}_3\times\hat{f}_2.\hat{g}_1\,Gi_{11}\,q_1'[t]\,q_5'[t]-\hat{c}_2\times\hat{f}_2.\hat{g}_1\,Gi_{11}\,q_2'[t]\,q_5'[t]-$$
$$\hat{d}_2\times\hat{f}_2.\hat{g}_1\,Gi_{11}\,q_3'[t]\,q_5'[t]-\hat{e}_1\times\hat{f}_2.\hat{g}_1\,Gi_{11}\,q_4'[t]\,q_5'[t]-\hat{c}_3.\hat{g}_1\,Gi_{11}\,q_1''[t]-$$
$$\hat{c}_2.\hat{g}_1\,Gi_{11}\,q_2''[t]-\hat{d}_2.\hat{g}_1\,Gi_{11}\,q_3''[t]-\hat{e}_1.\hat{g}_1\,Gi_{11}\,q_4''[t]-\hat{f}_2.\hat{g}_1\,Gi_{11}\,q_5''[t]\Big)+$$
$$\hat{f}_2.\hat{g}_2\,\Big(\hat{c}_1.\hat{g}_2\,Gi_{22}\,q_1'[t]\,q_2'[t]-\hat{c}_3\times\hat{d}_2.\hat{g}_2\,Gi_{22}\,q_1'[t]\,q_3'[t]-\hat{c}_2\times\hat{d}_2.\hat{g}_2\,Gi_{22}\,q_2'[t]\,q_3'[t]-$$
$$\hat{c}_3\times\hat{e}_1.\hat{g}_2\,Gi_{22}\,q_1'[t]\,q_4'[t]-\hat{c}_2\times\hat{e}_1.\hat{g}_2\,Gi_{22}\,q_2'[t]\,q_4'[t]-$$
$$\hat{d}_2\times\hat{e}_1.\hat{g}_2\,Gi_{22}\,q_3'[t]\,q_4'[t]-\hat{c}_3\times\hat{f}_2.\hat{g}_2\,Gi_{22}\,q_1'[t]\,q_5'[t]-\hat{c}_2\times\hat{f}_2.\hat{g}_2\,Gi_{22}\,q_2'[t]\,q_5'[t]-$$
$$\hat{d}_2\times\hat{f}_2.\hat{g}_2\,Gi_{22}\,q_3'[t]\,q_5'[t]-\hat{e}_1\times\hat{f}_2.\hat{g}_2\,Gi_{22}\,q_4'[t]\,q_5'[t]-\hat{c}_3.\hat{g}_2\,Gi_{22}\,q_1''[t]-$$
$$\hat{c}_2.\hat{g}_2\,Gi_{22}\,q_2''[t]-\hat{d}_2.\hat{g}_2\,Gi_{22}\,q_3''[t]-\hat{e}_1.\hat{g}_2\,Gi_{22}\,q_4''[t]-\hat{f}_2.\hat{g}_2\,Gi_{22}\,q_5''[t]\Big)+$$
$$\hat{f}_2.\hat{g}_3\,\Big(\hat{c}_1.\hat{g}_3\,Gi_{33}\,q_1'[t]\,q_2'[t]-\hat{c}_3\times\hat{d}_2.\hat{g}_3\,Gi_{33}\,q_1'[t]\,q_3'[t]-\hat{c}_2\times\hat{d}_2.\hat{g}_3\,Gi_{33}\,q_2'[t]\,q_3'[t]-$$
$$\hat{c}_3\times\hat{e}_1.\hat{g}_3\,Gi_{33}\,q_1'[t]\,q_4'[t]-\hat{c}_2\times\hat{e}_1.\hat{g}_3\,Gi_{33}\,q_2'[t]\,q_4'[t]-$$
$$\hat{d}_2\times\hat{e}_1.\hat{g}_3\,Gi_{33}\,q_3'[t]\,q_4'[t]-\hat{c}_3\times\hat{f}_2.\hat{g}_3\,Gi_{33}\,q_1'[t]\,q_5'[t]-\hat{c}_2\times\hat{f}_2.\hat{g}_3\,Gi_{33}\,q_2'[t]\,q_5'[t]-$$
$$\hat{d}_2\times\hat{f}_2.\hat{g}_3\,Gi_{33}\,q_3'[t]\,q_5'[t]-\hat{e}_1\times\hat{f}_2.\hat{g}_3\,Gi_{33}\,q_4'[t]\,q_5'[t]-\hat{c}_3.\hat{g}_3\,Gi_{33}\,q_1''[t]-$$
$$\hat{c}_2.\hat{g}_3\,Gi_{33}\,q_2''[t]-\hat{d}_2.\hat{g}_3\,Gi_{33}\,q_3''[t]-\hat{e}_1.\hat{g}_3\,Gi_{33}\,q_4''[t]-\hat{f}_2.\hat{g}_3\,Gi_{33}\,q_5''[t]\Big)+$$
$$\frac{5}{21}\,\Big(-\hat{f}_2\times\hat{g}_1.\hat{h}_1\,F_{toolx}-\hat{f}_2\times\hat{g}_1.\hat{h}_2\,F_{tooly}-\hat{f}_2\times\hat{g}_1.\hat{h}_3\,F_{toolz}-g\,\hat{f}_2\times\hat{g}_1.\hat{n}_3\,M_8+$$
$$\frac{3}{2}\,\hat{c}_1\times\hat{d}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_1'[t]\,q_2'[t]+\frac{1}{4}\,\hat{c}_1\times\hat{d}_2.\hat{f}_2\times\hat{g}_1\,M_8\,q_1'[t]\,q_2'[t]+$$
$$\frac{2}{3}\,\hat{c}_1\times\hat{e}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_1'[t]\,q_2'[t]-\frac{5}{12}\,\hat{c}_1\times\hat{e}_2.\hat{f}_2\times\hat{g}_1\,M_8\,q_1'[t]\,q_2'[t]+$$

$$\frac{5}{6}\,\hat{c}_1\times\hat{f}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_1'[t]\,q_2'[t]+\frac{5}{21}\,\hat{c}_1\times\hat{g}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_1'[t]\,q_2'[t]+$$

$$\hat{c}_3\times\hat{f}_2.\hat{g}_1\left(-\frac{3}{2}\,\hat{c}_3.\hat{d}_1\,M_8\,q_1'[t]^2-\frac{1}{4}\,\hat{c}_3.\hat{d}_2\,M_8\,q_1'[t]^2-\frac{2}{3}\,\hat{c}_3.\hat{e}_1\,M_8\,q_1'[t]^2+\frac{5}{12}\,\hat{c}_3.\hat{e}_2\,M_8\,q_1'[t]^2-\right.$$

$$\frac{5}{6}\,\hat{c}_3.\hat{f}_1\,M_8\,q_1'[t]^2-\frac{5}{21}\,\hat{c}_3.\hat{g}_1\,M_8\,q_1'[t]^2-\frac{3}{2}\,\hat{c}_2.\hat{d}_1\,M_8\,q_1'[t]\,q_2'[t]-\frac{1}{4}\,\hat{c}_2.\hat{d}_2\,M_8\,q_1'[t]\,q_2'[t]-$$

$$\frac{2}{3}\,\hat{c}_2.\hat{e}_1\,M_8\,q_1'[t]\,q_2'[t]+\frac{5}{12}\,\hat{c}_2.\hat{e}_2\,M_8\,q_1'[t]\,q_2'[t]-\frac{5}{6}\,\hat{c}_2.\hat{f}_1\,M_8\,q_1'[t]\,q_2'[t]-$$

$$\left.\frac{5}{21}\,\hat{c}_2.\hat{g}_1\,M_8\,q_1'[t]\,q_2'[t]\right)+\hat{d}_1\times\hat{f}_2.\hat{g}_1\left(\frac{3}{2}\,M_8\,q_1'[t]^2+\frac{3}{2}\,M_8\,q_2'[t]^2\right)+$$

$$\hat{c}_2\times\hat{f}_2.\hat{g}_1\left(\frac{1}{2}\,M_8\,q_1'[t]^2-\frac{3}{2}\,\hat{c}_3.\hat{d}_1\,M_8\,q_1'[t]\,q_2'[t]-\frac{1}{4}\,\hat{c}_3.\hat{d}_2\,M_8\,q_1'[t]\,q_2'[t]-\right.$$

$$\frac{2}{3}\,\hat{c}_3.\hat{e}_1\,M_8\,q_1'[t]\,q_2'[t]+\frac{5}{12}\,\hat{c}_3.\hat{e}_2\,M_8\,q_1'[t]\,q_2'[t]-\frac{5}{6}\,\hat{c}_3.\hat{f}_1\,M_8\,q_1'[t]\,q_2'[t]-$$

$$\frac{5}{21}\,\hat{c}_3.\hat{g}_1\,M_8\,q_1'[t]\,q_2'[t]-\frac{3}{2}\,\hat{c}_2.\hat{d}_1\,M_8\,q_2'[t]^2-\frac{1}{4}\,\hat{c}_2.\hat{d}_2\,M_8\,q_2'[t]^2-\frac{2}{3}\,\hat{c}_2.\hat{e}_1\,M_8\,q_2'[t]^2+$$

$$\left.\frac{5}{12}\,\hat{c}_2.\hat{e}_2\,M_8\,q_2'[t]^2-\frac{5}{6}\,\hat{c}_2.\hat{f}_1\,M_8\,q_2'[t]^2-\frac{5}{21}\,\hat{c}_2.\hat{g}_1\,M_8\,q_2'[t]^2\right)+\hat{e}_2\times\hat{f}_2.\hat{g}_1\left(-\frac{5}{12}\,M_8\,q_1'[t]^2-\right.$$

$$\left.\frac{5}{12}\,M_8\,q_2'[t]^2-\frac{5}{6}\,\hat{c}_3.\hat{d}_2\,M_8\,q_1'[t]\,q_3'[t]-\frac{5}{6}\,\hat{c}_2.\hat{d}_2\,M_8\,q_2'[t]\,q_3'[t]-\frac{5}{12}\,M_8\,q_3'[t]^2\right)+$$

$$\hat{d}_2\times\hat{f}_2.\hat{g}_1\left(\frac{1}{4}\,M_8\,q_1'[t]^2+\frac{1}{4}\,M_8\,q_2'[t]^2-\frac{4}{3}\,\hat{c}_3.\hat{e}_1\,M_8\,q_1'[t]\,q_3'[t]+\frac{5}{6}\,\hat{c}_3.\hat{e}_2\,M_8\,q_1'[t]\,q_3'[t]-\right.$$

$$\frac{5}{3}\,\hat{c}_3.\hat{f}_1\,M_8\,q_1'[t]\,q_3'[t]-\frac{10}{21}\,\hat{c}_3.\hat{g}_1\,M_8\,q_1'[t]\,q_3'[t]-\frac{4}{3}\,\hat{c}_2.\hat{e}_1\,M_8\,q_2'[t]\,q_3'[t]+$$

$$\frac{5}{6}\,\hat{c}_2.\hat{e}_2\,M_8\,q_2'[t]\,q_3'[t]-\frac{5}{3}\,\hat{c}_2.\hat{f}_1\,M_8\,q_2'[t]\,q_3'[t]-\frac{10}{21}\,\hat{c}_2.\hat{g}_1\,M_8\,q_2'[t]\,q_3'[t]-$$

$$\left.\frac{2}{3}\,\hat{d}_2.\hat{e}_1\,M_8\,q_3'[t]^2+\frac{5}{12}\,\hat{d}_2.\hat{e}_2\,M_8\,q_3'[t]^2-\frac{5}{6}\,\hat{d}_2.\hat{f}_1\,M_8\,q_3'[t]^2-\frac{5}{21}\,\hat{d}_2.\hat{g}_1\,M_8\,q_3'[t]^2\right)+\hat{f}_3.\hat{g}_1$$

$$\left(\frac{5}{6}\,M_8\,q_1'[t]^2+\frac{5}{6}\,M_8\,q_2'[t]^2+\frac{5}{3}\,\hat{c}_3.\hat{d}_2\,M_8\,q_1'[t]\,q_3'[t]+\frac{5}{3}\,\hat{c}_2.\hat{d}_2\,M_8\,q_2'[t]\,q_3'[t]+\frac{5}{6}\,M_8\,q_3'[t]^2+\right.$$

$$\left.\frac{5}{3}\,\hat{c}_3.\hat{e}_1\,M_8\,q_1'[t]\,q_4'[t]+\frac{5}{3}\,\hat{c}_2.\hat{e}_1\,M_8\,q_2'[t]\,q_4'[t]+\frac{5}{3}\,\hat{d}_2.\hat{e}_1\,M_8\,q_3'[t]\,q_4'[t]+\frac{5}{6}\,M_8\,q_4'[t]^2\right)+$$

$$\hat{e}_1\times\hat{f}_2.\hat{g}_1\left(\frac{2}{3}\,M_8\,q_1'[t]^2+\frac{2}{3}\,M_8\,q_2'[t]^2+\frac{4}{3}\,\hat{c}_3.\hat{d}_2\,M_8\,q_1'[t]\,q_3'[t]+\frac{4}{3}\,\hat{c}_2.\hat{d}_2\,M_8\,q_2'[t]\,q_3'[t]+\right.$$

$$\frac{2}{3}\,M_8\,q_3'[t]^2-\frac{5}{3}\,\hat{c}_3.\hat{f}_1\,M_8\,q_1'[t]\,q_4'[t]-\frac{10}{21}\,\hat{c}_3.\hat{g}_1\,M_8\,q_1'[t]\,q_4'[t]-$$

$$\frac{5}{3}\,\hat{c}_2.\hat{f}_1\,M_8\,q_2'[t]\,q_4'[t]-\frac{10}{21}\,\hat{c}_2.\hat{g}_1\,M_8\,q_2'[t]\,q_4'[t]-\frac{5}{3}\,\hat{d}_2.\hat{f}_1\,M_8\,q_3'[t]\,q_4'[t]-$$

$$\left.\frac{10}{21}\,\hat{d}_2.\hat{g}_1\,M_8\,q_3'[t]\,q_4'[t]-\frac{5}{6}\,\hat{e}_1.\hat{f}_1\,M_8\,q_4'[t]^2-\frac{5}{21}\,\hat{e}_1.\hat{g}_1\,M_8\,q_4'[t]^2\right)-$$

$$\hat{f}_2\times\hat{g}_1.\hat{n}_1\,M_8\,x''[t]-\hat{f}_2\times\hat{g}_1.\hat{n}_2\,M_8\,y''[t]+\frac{1}{2}\,\hat{c}_1\times\hat{f}_2.\hat{g}_1\,M_8\,q_1''[t]-\frac{3}{2}\,\hat{c}_3\times\hat{d}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_1''[t]-$$

$$\frac{1}{4}\,\hat{c}_3\times\hat{d}_2.\hat{f}_2\times\hat{g}_1\,M_8\,q_1''[t]-\frac{2}{3}\,\hat{c}_3\times\hat{e}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_1''[t]+\frac{5}{12}\,\hat{c}_3\times\hat{e}_2.\hat{f}_2\times\hat{g}_1\,M_8\,q_1''[t]-$$

$$\frac{5}{6}\,\hat{c}_3\times\hat{f}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_1''[t]-\frac{5}{21}\,\hat{c}_3\times\hat{g}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_1''[t]-\frac{3}{2}\,\hat{c}_2\times\hat{d}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_2''[t]-$$
$$\frac{1}{4}\,\hat{c}_2\times\hat{d}_2.\hat{f}_2\times\hat{g}_1\,M_8\,q_2''[t]-\frac{2}{3}\,\hat{c}_2\times\hat{e}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_2''[t]+\frac{5}{12}\,\hat{c}_2\times\hat{e}_2.\hat{f}_2\times\hat{g}_1\,M_8\,q_2''[t]-$$
$$\frac{5}{6}\,\hat{c}_2\times\hat{f}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_2''[t]-\frac{5}{21}\,\hat{c}_2\times\hat{g}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_2''[t]-\frac{2}{3}\,\hat{d}_2\times\hat{e}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_3''[t]+$$
$$\frac{5}{12}\,\hat{d}_2\times\hat{e}_2.\hat{f}_2\times\hat{g}_1\,M_8\,q_3''[t]-\frac{5}{6}\,\hat{d}_2\times\hat{f}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_3''[t]-\frac{5}{21}\,\hat{d}_2\times\hat{g}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_3''[t]-$$
$$\left.\frac{5}{6}\,\hat{e}_1\times\hat{f}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_4''[t]-\frac{5}{21}\,\hat{e}_1\times\hat{g}_1.\hat{f}_2\times\hat{g}_1\,M_8\,q_4''[t]-\frac{5}{21}\,M_8\,q_5''[t]\right)+$$

$$\hat{f}_2.\hat{h}_1\left(\hat{c}_1.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_2'[t]-\hat{c}_3\times\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_3'[t]-\hat{c}_2\times\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_3'[t]-\right.$$
$$\hat{c}_3\times\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_4'[t]-\hat{c}_2\times\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_4'[t]-$$
$$\hat{d}_2\times\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_4'[t]-\hat{c}_3\times\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_5'[t]-\hat{c}_2\times\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_5'[t]-$$
$$\hat{d}_2\times\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_5'[t]-\hat{e}_1\times\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_4'[t]\,q_5'[t]-\hat{c}_3\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_6'[t]-$$
$$\hat{c}_2\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_6'[t]-\hat{d}_2\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_6'[t]-\hat{e}_1\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_4'[t]\,q_6'[t]-$$
$$\hat{f}_2\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_5'[t]\,q_6'[t]-\hat{c}_3.\hat{h}_1\,Hi_{11}\,q_1''[t]-\hat{c}_2.\hat{h}_1\,Hi_{11}\,q_2''[t]-$$
$$\left.\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_3''[t]-\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_4''[t]-\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_5''[t]-\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_6''[t]\right)+$$

$$\hat{f}_2.\hat{h}_2\left(\hat{c}_1.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_2'[t]-\hat{c}_3\times\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_3'[t]-\hat{c}_2\times\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_3'[t]-\right.$$
$$\hat{c}_3\times\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_4'[t]-\hat{c}_2\times\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_4'[t]-$$
$$\hat{d}_2\times\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_4'[t]-\hat{c}_3\times\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_5'[t]-\hat{c}_2\times\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_5'[t]-$$
$$\hat{d}_2\times\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_5'[t]-\hat{e}_1\times\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_4'[t]\,q_5'[t]-\hat{c}_3\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_6'[t]-$$
$$\hat{c}_2\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_6'[t]-\hat{d}_2\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_6'[t]-\hat{e}_1\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_4'[t]\,q_6'[t]-$$
$$\hat{f}_2\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_5'[t]\,q_6'[t]-\hat{c}_3.\hat{h}_2\,Hi_{22}\,q_1''[t]-\hat{c}_2.\hat{h}_2\,Hi_{22}\,q_2''[t]-$$
$$\left.\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_3''[t]-\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_4''[t]-\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_5''[t]-\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_6''[t]\right)+$$

$$\hat{f}_2.\hat{h}_3\left(\hat{c}_1.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_2'[t]-\hat{c}_3\times\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_3'[t]-\hat{c}_2\times\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_3'[t]-\right.$$
$$\hat{c}_3\times\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_4'[t]-\hat{c}_2\times\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_4'[t]-$$
$$\hat{d}_2\times\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_4'[t]-\hat{c}_3\times\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_5'[t]-\hat{c}_2\times\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_5'[t]-$$
$$\hat{d}_2\times\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_5'[t]-\hat{e}_1\times\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_4'[t]\,q_5'[t]-\hat{c}_3\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_6'[t]-$$
$$\hat{c}_2\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_6'[t]-\hat{d}_2\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_6'[t]-\hat{e}_1\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_4'[t]\,q_6'[t]-$$
$$\hat{f}_2\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_5'[t]\,q_6'[t]-\hat{c}_3.\hat{h}_3\,Hi_{33}\,q_1''[t]-\hat{c}_2.\hat{h}_3\,Hi_{33}\,q_2''[t]-$$
$$\left.\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_3''[t]-\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_4''[t]-\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_5''[t]-\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_6''[t]\right)$$

For $q_6[t]$ the equations of motion is
(they will be set to zero when solution is required).

In[552]:=
$$\mathbf{eq[8]}=\sum_{ii=1}^{N_{bodies}}\left(\left(\mathbf{F_{app}[ii]-I_f[ii]}\right).\mathbf{Pvel[OvB[ii],\,q_6'[t]]}+\left(\mathbf{T_{app}[ii]-I_t[ii]}\right).\mathbf{Pvel[N\omega B[ii],\,q_6'[t]]}\right)$$

Out[552]=
$$-\hat{g}_1.\hat{h}_1\,T_{toolx}-\hat{g}_1.\hat{h}_2\,T_{tooly}-\hat{g}_1.\hat{h}_3\,T_{toolz}+\hat{g}_1.\hat{h}_1\,T_{mot}[6]-$$
$$\hat{c}_3\times\hat{g}_1.\hat{h}_1\left(-\hat{c}_3.\hat{h}_1\,Hi_{11}\,q_1'[t]^2-\hat{c}_2.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_2'[t]-\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_3'[t]-\right.$$

$$\hat{e}_1.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_4'[t] - \hat{f}_2.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_6'[t]\Big) -$$

$$\hat{c}_3 \times \hat{g}_1.\hat{h}_2 \Big(-\hat{c}_3.\hat{h}_2 \, Hi_{22} \, q_1'[t]^2 - \hat{c}_2.\hat{h}_2 \, Hi_{22} \, q_1'[t] \, q_2'[t] - \hat{d}_2.\hat{h}_2 \, Hi_{22} \, q_1'[t] \, q_3'[t] -$$

$$\hat{e}_1.\hat{h}_2 \, Hi_{22} \, q_1'[t] \, q_4'[t] - \hat{f}_2.\hat{h}_2 \, Hi_{22} \, q_1'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_2 \, Hi_{22} \, q_1'[t] \, q_6'[t]\Big) -$$

$$\hat{c}_3 \times \hat{g}_1.\hat{h}_3 \Big(-\hat{c}_3.\hat{h}_3 \, Hi_{33} \, q_1'[t]^2 - \hat{c}_2.\hat{h}_3 \, Hi_{33} \, q_1'[t] \, q_2'[t] - \hat{d}_2.\hat{h}_3 \, Hi_{33} \, q_1'[t] \, q_3'[t] -$$

$$\hat{e}_1.\hat{h}_3 \, Hi_{33} \, q_1'[t] \, q_4'[t] - \hat{f}_2.\hat{h}_3 \, Hi_{33} \, q_1'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_3 \, Hi_{33} \, q_1'[t] \, q_6'[t]\Big) -$$

$$\hat{c}_2 \times \hat{g}_1.\hat{h}_1 \Big(-\hat{c}_3.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_2'[t] - \hat{c}_2.\hat{h}_1 \, Hi_{11} \, q_2'[t]^2 - \hat{d}_2.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_3'[t] -$$

$$\hat{e}_1.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_4'[t] - \hat{f}_2.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_6'[t]\Big) -$$

$$\hat{c}_2 \times \hat{g}_1.\hat{h}_2 \Big(-\hat{c}_3.\hat{h}_2 \, Hi_{22} \, q_1'[t] \, q_2'[t] - \hat{c}_2.\hat{h}_2 \, Hi_{22} \, q_2'[t]^2 - \hat{d}_2.\hat{h}_2 \, Hi_{22} \, q_2'[t] \, q_3'[t] -$$

$$\hat{e}_1.\hat{h}_2 \, Hi_{22} \, q_2'[t] \, q_4'[t] - \hat{f}_2.\hat{h}_2 \, Hi_{22} \, q_2'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_2 \, Hi_{22} \, q_2'[t] \, q_6'[t]\Big) -$$

$$\hat{c}_2 \times \hat{g}_1.\hat{h}_3 \Big(-\hat{c}_3.\hat{h}_3 \, Hi_{33} \, q_1'[t] \, q_2'[t] - \hat{c}_2.\hat{h}_3 \, Hi_{33} \, q_2'[t]^2 - \hat{d}_2.\hat{h}_3 \, Hi_{33} \, q_2'[t] \, q_3'[t] -$$

$$\hat{e}_1.\hat{h}_3 \, Hi_{33} \, q_2'[t] \, q_4'[t] - \hat{f}_2.\hat{h}_3 \, Hi_{33} \, q_2'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_3 \, Hi_{33} \, q_2'[t] \, q_6'[t]\Big) -$$

$$\hat{d}_2 \times \hat{g}_1.\hat{h}_1 \Big(-\hat{c}_3.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_3'[t] - \hat{c}_2.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_3'[t] - \hat{d}_2.\hat{h}_1 \, Hi_{11} \, q_3'[t]^2 -$$

$$\hat{e}_1.\hat{h}_1 \, Hi_{11} \, q_3'[t] \, q_4'[t] - \hat{f}_2.\hat{h}_1 \, Hi_{11} \, q_3'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_1 \, Hi_{11} \, q_3'[t] \, q_6'[t]\Big) -$$

$$\hat{d}_2 \times \hat{g}_1.\hat{h}_2 \Big(-\hat{c}_3.\hat{h}_2 \, Hi_{22} \, q_1'[t] \, q_3'[t] - \hat{c}_2.\hat{h}_2 \, Hi_{22} \, q_2'[t] \, q_3'[t] - \hat{d}_2.\hat{h}_2 \, Hi_{22} \, q_3'[t]^2 -$$

$$\hat{e}_1.\hat{h}_2 \, Hi_{22} \, q_3'[t] \, q_4'[t] - \hat{f}_2.\hat{h}_2 \, Hi_{22} \, q_3'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_2 \, Hi_{22} \, q_3'[t] \, q_6'[t]\Big) -$$

$$\hat{d}_2 \times \hat{g}_1.\hat{h}_3 \Big(-\hat{c}_3.\hat{h}_3 \, Hi_{33} \, q_1'[t] \, q_3'[t] - \hat{c}_2.\hat{h}_3 \, Hi_{33} \, q_2'[t] \, q_3'[t] - \hat{d}_2.\hat{h}_3 \, Hi_{33} \, q_3'[t]^2 -$$

$$\hat{e}_1.\hat{h}_3 \, Hi_{33} \, q_3'[t] \, q_4'[t] - \hat{f}_2.\hat{h}_3 \, Hi_{33} \, q_3'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_3 \, Hi_{33} \, q_3'[t] \, q_6'[t]\Big) -$$

$$\hat{e}_1 \times \hat{g}_1.\hat{h}_1 \Big(-\hat{c}_3.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_4'[t] - \hat{c}_2.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_4'[t] - \hat{d}_2.\hat{h}_1 \, Hi_{11} \, q_3'[t] \, q_4'[t] -$$

$$\hat{e}_1.\hat{h}_1 \, Hi_{11} \, q_4'[t]^2 - \hat{f}_2.\hat{h}_1 \, Hi_{11} \, q_4'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_1 \, Hi_{11} \, q_4'[t] \, q_6'[t]\Big) -$$

$$\hat{e}_1 \times \hat{g}_1.\hat{h}_2 \Big(-\hat{c}_3.\hat{h}_2 \, Hi_{22} \, q_1'[t] \, q_4'[t] - \hat{c}_2.\hat{h}_2 \, Hi_{22} \, q_2'[t] \, q_4'[t] - \hat{d}_2.\hat{h}_2 \, Hi_{22} \, q_3'[t] \, q_4'[t] -$$

$$\hat{e}_1.\hat{h}_2 \, Hi_{22} \, q_4'[t]^2 - \hat{f}_2.\hat{h}_2 \, Hi_{22} \, q_4'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_2 \, Hi_{22} \, q_4'[t] \, q_6'[t]\Big) -$$

$$\hat{e}_1 \times \hat{g}_1.\hat{h}_3 \Big(-\hat{c}_3.\hat{h}_3 \, Hi_{33} \, q_1'[t] \, q_4'[t] - \hat{c}_2.\hat{h}_3 \, Hi_{33} \, q_2'[t] \, q_4'[t] - \hat{d}_2.\hat{h}_3 \, Hi_{33} \, q_3'[t] \, q_4'[t] -$$

$$\hat{e}_1.\hat{h}_3 \, Hi_{33} \, q_4'[t]^2 - \hat{f}_2.\hat{h}_3 \, Hi_{33} \, q_4'[t] \, q_5'[t] - \hat{g}_1.\hat{h}_3 \, Hi_{33} \, q_4'[t] \, q_6'[t]\Big) -$$

$$\hat{f}_2 \times \hat{g}_1.\hat{h}_1 \Big(-\hat{c}_3.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_5'[t] - \hat{c}_2.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_5'[t] - \hat{d}_2.\hat{h}_1 \, Hi_{11} \, q_3'[t] \, q_5'[t] -$$

$$\hat{e}_1.\hat{h}_1 \, Hi_{11} \, q_4'[t] \, q_5'[t] - \hat{f}_2.\hat{h}_1 \, Hi_{11} \, q_5'[t]^2 - \hat{g}_1.\hat{h}_1 \, Hi_{11} \, q_5'[t] \, q_6'[t]\Big) -$$

$$\hat{f}_2 \times \hat{g}_1.\hat{h}_2 \Big(-\hat{c}_3.\hat{h}_2 \, Hi_{22} \, q_1'[t] \, q_5'[t] - \hat{c}_2.\hat{h}_2 \, Hi_{22} \, q_2'[t] \, q_5'[t] - \hat{d}_2.\hat{h}_2 \, Hi_{22} \, q_3'[t] \, q_5'[t] -$$

$$\hat{e}_1.\hat{h}_2 \, Hi_{22} \, q_4'[t] \, q_5'[t] - \hat{f}_2.\hat{h}_2 \, Hi_{22} \, q_5'[t]^2 - \hat{g}_1.\hat{h}_2 \, Hi_{22} \, q_5'[t] \, q_6'[t]\Big) -$$

$$\hat{f}_2 \times \hat{g}_1.\hat{h}_3 \Big(-\hat{c}_3.\hat{h}_3 \, Hi_{33} \, q_1'[t] \, q_5'[t] - \hat{c}_2.\hat{h}_3 \, Hi_{33} \, q_2'[t] \, q_5'[t] - \hat{d}_2.\hat{h}_3 \, Hi_{33} \, q_3'[t] \, q_5'[t] -$$

$$\hat{e}_1.\hat{h}_3 \, Hi_{33} \, q_4'[t] \, q_5'[t] - \hat{f}_2.\hat{h}_3 \, Hi_{33} \, q_5'[t]^2 - \hat{g}_1.\hat{h}_3 \, Hi_{33} \, q_5'[t] \, q_6'[t]\Big) +$$

$$\hat{g}_1.\hat{h}_1 \Big(\hat{c}_1.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_2'[t] - \hat{c}_3 \times \hat{d}_2.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_3'[t] - \hat{c}_2 \times \hat{d}_2.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_3'[t] -$$

$$\hat{c}_3 \times \hat{e}_1.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_4'[t] - \hat{c}_2 \times \hat{e}_1.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_4'[t] -$$

$$\hat{d}_2 \times \hat{e}_1.\hat{h}_1 \, Hi_{11} \, q_3'[t] \, q_4'[t] - \hat{c}_3 \times \hat{f}_2.\hat{h}_1 \, Hi_{11} \, q_1'[t] \, q_5'[t] - \hat{c}_2 \times \hat{f}_2.\hat{h}_1 \, Hi_{11} \, q_2'[t] \, q_5'[t] -$$

$$\hat{d}_2\times\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_5'[t]-\hat{e}_1\times\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_4'[t]\,q_5'[t]-\hat{c}_3\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_1'[t]\,q_6'[t]-$$
$$\hat{c}_2\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_2'[t]\,q_6'[t]-\hat{d}_2\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_3'[t]\,q_6'[t]-\hat{e}_1\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_4'[t]\,q_6'[t]-$$
$$\hat{f}_2\times\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_5'[t]\,q_6'[t]-\hat{c}_3.\hat{h}_1\,Hi_{11}\,q_1''[t]-\hat{c}_2.\hat{h}_1\,Hi_{11}\,q_2''[t]-$$
$$\left.\hat{d}_2.\hat{h}_1\,Hi_{11}\,q_3''[t]-\hat{e}_1.\hat{h}_1\,Hi_{11}\,q_4''[t]-\hat{f}_2.\hat{h}_1\,Hi_{11}\,q_5''[t]-\hat{g}_1.\hat{h}_1\,Hi_{11}\,q_6''[t]\right)+$$
$$\hat{g}_1.\hat{h}_2\,\left(\hat{c}_1.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_2'[t]-\hat{c}_3\times\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_3'[t]-\hat{c}_2\times\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_3'[t]-\right.$$
$$\hat{c}_3\times\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_4'[t]-\hat{c}_2\times\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_4'[t]-$$
$$\hat{d}_2\times\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_4'[t]-\hat{c}_3\times\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_5'[t]-\hat{c}_2\times\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_5'[t]-$$
$$\hat{d}_2\times\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_5'[t]-\hat{e}_1\times\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_4'[t]\,q_5'[t]-\hat{c}_3\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_1'[t]\,q_6'[t]-$$
$$\hat{c}_2\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_2'[t]\,q_6'[t]-\hat{d}_2\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_3'[t]\,q_6'[t]-\hat{e}_1\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_4'[t]\,q_6'[t]-$$
$$\hat{f}_2\times\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_5'[t]\,q_6'[t]-\hat{c}_3.\hat{h}_2\,Hi_{22}\,q_1''[t]-\hat{c}_2.\hat{h}_2\,Hi_{22}\,q_2''[t]-$$
$$\left.\hat{d}_2.\hat{h}_2\,Hi_{22}\,q_3''[t]-\hat{e}_1.\hat{h}_2\,Hi_{22}\,q_4''[t]-\hat{f}_2.\hat{h}_2\,Hi_{22}\,q_5''[t]-\hat{g}_1.\hat{h}_2\,Hi_{22}\,q_6''[t]\right)+$$
$$\hat{g}_1.\hat{h}_3\,\left(\hat{c}_1.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_2'[t]-\hat{c}_3\times\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_3'[t]-\hat{c}_2\times\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_3'[t]-\right.$$
$$\hat{c}_3\times\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_4'[t]-\hat{c}_2\times\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_4'[t]-$$
$$\hat{d}_2\times\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_4'[t]-\hat{c}_3\times\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_5'[t]-\hat{c}_2\times\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_5'[t]-$$
$$\hat{d}_2\times\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_5'[t]-\hat{e}_1\times\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_4'[t]\,q_5'[t]-\hat{c}_3\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_1'[t]\,q_6'[t]-$$
$$\hat{c}_2\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_2'[t]\,q_6'[t]-\hat{d}_2\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_3'[t]\,q_6'[t]-\hat{e}_1\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_4'[t]\,q_6'[t]-$$
$$\hat{f}_2\times\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_5'[t]\,q_6'[t]-\hat{c}_3.\hat{h}_3\,Hi_{33}\,q_1''[t]-\hat{c}_2.\hat{h}_3\,Hi_{33}\,q_2''[t]-$$
$$\left.\hat{d}_2.\hat{h}_3\,Hi_{33}\,q_3''[t]-\hat{e}_1.\hat{h}_3\,Hi_{33}\,q_4''[t]-\hat{f}_2.\hat{h}_3\,Hi_{33}\,q_5''[t]-\hat{g}_1.\hat{h}_3\,Hi_{33}\,q_6''[t]\right)$$

Transform these equations recursively

In[553]:=
```
tRules = Join[relTran[8], relTran[7], relTran[6],
  relTran[5], relTran[4], relTran[3], relTran[2], relTran[1]]
```

Out[553]=

$$\{\hat{h}_1\to\hat{g}_1,\ \hat{h}_2\to \mathrm{Cos}[q_6[t]]\,\hat{g}_2+\mathrm{Sin}[q_6[t]]\,\hat{g}_3,\ \hat{h}_3\to -\mathrm{Sin}[q_6[t]]\,\hat{g}_2+\mathrm{Cos}[q_6[t]]\,\hat{g}_3,$$
$$\hat{g}_1\to \mathrm{Cos}[q_5[t]]\,\hat{f}_1-\mathrm{Sin}[q_5[t]]\,\hat{f}_3,\ \hat{g}_2\to\hat{f}_2,\ \hat{g}_3\to \mathrm{Sin}[q_5[t]]\,\hat{f}_1+\mathrm{Cos}[q_5[t]]\,\hat{f}_3,$$
$$\hat{f}_1\to\hat{e}_1,\ \hat{f}_2\to \mathrm{Cos}[q_4[t]]\,\hat{e}_2+\mathrm{Sin}[q_4[t]]\,\hat{e}_3,\ \hat{f}_3\to -\mathrm{Sin}[q_4[t]]\,\hat{e}_2+\mathrm{Cos}[q_4[t]]\,\hat{e}_3,$$
$$\hat{e}_1\to \mathrm{Cos}[q_3[t]]\,\hat{d}_1-\mathrm{Sin}[q_3[t]]\,\hat{d}_3,\ \hat{e}_2\to\hat{d}_2,\ \hat{e}_3\to \mathrm{Sin}[q_3[t]]\,\hat{d}_1+\mathrm{Cos}[q_3[t]]\,\hat{d}_3,$$
$$\hat{d}_1\to \mathrm{Cos}[q_2[t]]\,\hat{c}_1-\mathrm{Sin}[q_2[t]]\,\hat{c}_3,\ \hat{d}_2\to\hat{c}_2,\ \hat{d}_3\to \mathrm{Sin}[q_2[t]]\,\hat{c}_1+\mathrm{Cos}[q_2[t]]\,\hat{c}_3,$$
$$\hat{c}_1\to \mathrm{Cos}[q_1[t]]\,\hat{b}_1+\mathrm{Sin}[q_1[t]]\,\hat{b}_2,\ \hat{c}_2\to -\mathrm{Sin}[q_1[t]]\,\hat{b}_1+\mathrm{Cos}[q_1[t]]\,\hat{b}_2,$$
$$\hat{c}_3\to\hat{b}_3,\ \hat{b}_1\to\hat{a}_1,\ \hat{b}_2\to\hat{a}_2,\ \hat{b}_3\to\hat{a}_3,\ \hat{a}_1\to\hat{n}_1,\ \hat{a}_2\to\hat{n}_2,\ \hat{a}_3\to\hat{n}_3\}$$

In[554]:=
```
relTransform[x_] := x //. tRules
```

These may take a while to complete, be patient!

In[555]:=
```
eqT[1] = eq[1] // relTransform;
```

In[556]:=
```
eqT[2] = eq[2] // relTransform;
```

In[557]:=
```
eqT[3] = eq[3] // relTransform;
```

In[558]:=
```
eqT[4] = eq[4] // relTransform;
```

In[559]:=
```
eqT[5] = eq[5] // relTransform;
```

In[560]:=
```
eqT[6] = eq[6] // relTransform;
```

In[561]:=
```
eqT[7] = eq[7] // relTransform;
```

In[562]:=
```
eqT[8] = eq[8] // relTransform;
```

Total Energy

Potential Energy

In[563]:= $\text{PEt} = \left(\sum_{i=1}^{N_{bodies}} \text{PE}[i]\right)$ // relTransform

Out[563]=
$$\frac{g M_1}{4} + g M_2 + \frac{7 g M_3}{4} + g\left(\frac{7}{4} - \frac{1}{2}\text{Sin}[q_2[t]]\right) M_4 + g\left(\frac{7}{4} - \frac{3}{2}\text{Sin}[q_2[t]] + \frac{1}{6}(-\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]] - \text{Cos}[q_2[t]]\,\text{Sin}[q_3[t]])\right) M_5 + g\left(\frac{7}{4} - \frac{3}{2}\text{Sin}[q_2[t]] - \text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]] - \text{Cos}[q_2[t]]\,\text{Sin}[q_3[t]]\right) M_6 + \frac{7 g M_7}{4} - \frac{3}{2} g\,\text{Sin}[q_2[t]]\, M_7 + \frac{3}{2} g\,(-\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]] - \text{Cos}[q_2[t]]\,\text{Sin}[q_3[t]])\, M_7 + \frac{7 g M_8}{4} - \frac{3}{2} g\,\text{Sin}[q_2[t]]\, M_8 + \frac{3}{2} g\,(-\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]] - \text{Cos}[q_2[t]]\,\text{Sin}[q_3[t]])\, M_8 + \frac{5}{21} g\,(\text{Cos}[q_5[t]]\,(-\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]] - \text{Cos}[q_2[t]]\,\text{Sin}[q_3[t]]) - \text{Cos}[q_4[t]]\,(\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]] - \text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]])\,\text{Sin}[q_5[t]])\, M_8$$

Kinetic Energy

In[564]:= $\text{KEt} = \left(\sum_{i=1}^{N_{bodies}} \text{KE}[i]\right)$ // relTransform

Out[564]=
$$\frac{1}{2} M_1 x'[t]^2 + \frac{1}{2} M_2 x'[t]^2 + \frac{1}{2} M_3 x'[t]^2 + \frac{1}{2} M_4 x'[t]^2 + \frac{1}{2} M_5 x'[t]^2 + \frac{1}{2} M_6 \langle\!\langle 1 \rangle\!\rangle^2 + \langle\!\langle 576 \rangle\!\rangle + \langle\!\langle 1 \rangle\!\rangle + \langle\!\langle 1 \rangle\!\rangle + (\langle\!\langle 1 \rangle\!\rangle)\,\langle\!\langle 3 \rangle\!\rangle\, q_6'[t] + \frac{1}{2}\left(\text{Cos}[q_5[t]]\,(\langle\!\langle 1 \rangle\!\rangle) - \text{Sin}[q_5[t]]\,(\langle\!\langle 1 \rangle\!\rangle)\right)^2 \text{Hi}_{11}\, q_6'[t]^2 + \frac{1}{2}\left(\langle\!\langle 5 \rangle\!\rangle + \langle\!\langle 1 \rangle\!\rangle\right)^2 \text{Hi}_{22}\, q_6'[t]^2 + \frac{1}{2}\left(-\text{Cos}[q_6[t]]\,\text{Sin}[q_5[t]]\,(\langle\!\langle 1 \rangle\!\rangle) + \langle\!\langle 1 \rangle\!\rangle - \langle\!\langle 1 \rangle\!\rangle + \left(-\text{Cos}[q_4[t]]\,(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2)\,\text{Sin}[q_4[t]] + \text{Cos}[q_4[t]]\,(\text{Cos}[q_3[t]]\,(\langle\!\langle 1 \rangle\!\rangle^2\,\text{Cos}[\langle\!\langle 1 \rangle\!\rangle] + \langle\!\langle 4 \rangle\!\rangle)\,\text{Sin}[q_4[t]] + \langle\!\langle 1 \rangle\!\rangle)\right)\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\right)^2 \text{Hi}_{33}\, q_6'[t]^2$$

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Total Energy

In[565]:=
```
TEt = PEt + KEt;
```

Static equilibrium equations (gravity loads and braking torques/forces)

In this section we set up the equations to find the gravity loads. These loads will make the robot collapse when the motors are not powered, so we must determine the braking torques/forces required to keep the robot safe.

Here is a graphic to use to see what configurations are found when maximum brake forces are required.

In[566]:=
```
robotGraphicBrakes = {

   (*Base graphic*)
   Translate[GeometricTransformation[baseGraphic, Transpose[rotA]], {xAo, yAo, zAo}],

   (*Riser graphic*)
   Translate[GeometricTransformation[riserGraphic, Transpose[rotB]], {xBo, yBo, zBo}],

   (*Shoulder graphic*)
   Translate[
    GeometricTransformation[shoulderGraphic, Transpose[rotC]], {xCo, yCo, zCo}],

   (*Arm1 graphic*)
   Translate[GeometricTransformation[arm1Graphic, Transpose[rotD]], {xDo, yDo, zDo}],

   (*Arm2 graphic*)
   Translate[GeometricTransformation[arm2Graphic, Transpose[rotE]], {xEo, yEo, zEo}],

   (*Arm3 graphic*)
   Translate[GeometricTransformation[arm3Graphic, Transpose[rotF]], {xFo, yFo, zFo}],

   (*Wrist1 graphic*)
   Translate[
    GeometricTransformation[wrist1Graphic, Transpose[rotG]], {xGo, yGo, zGo}],

   (*Wrist2 graphic*)
   Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]
  } //. {q_n_[t] → Q_n, x[t] → 0, y[t] → 0};
```

First we set all derivatives to zero (this is a static analysis). Distribute scalars and loosen dots as many times as needed to get the equations evaluated.

In[567]:=
```
eqS[1] = eqT[1] //. Derivative[_][_][t] → 0
```

Out[567]=
```
Fmotx - (Cos[q5[t]] (Cos[q1[t]] Cos[q2[t]] Cos[q3[t]] - Cos[q1[t]] Sin[q2[t]] Sin[q3[t]]) -
     (Cos[q4[t]] (Cos[q1[t]] Cos[q3[t]] Sin[q2[t]] + Cos[q1[t]] Cos[q2[t]] Sin[q3[t]]) +
        Sin[q1[t]] Sin[q4[t]]) Sin[q5[t]]) Ftoolx - (Cos[q6[t]] (-Cos[q4[t]] Sin[q1[t]] +
        (Cos[q1[t]] Cos[q3[t]] Sin[q2[t]] + Cos[q1[t]] Cos[q2[t]] Sin[q3[t]]) Sin[q4[t]]) +
     (Cos[q5[t]] (Cos[q4[t]] (Cos[q1[t]] Cos[q3[t]] Sin[q2[t]] +
              Cos[q1[t]] Cos[q2[t]] Sin[q3[t]]) + Sin[q1[t]] Sin[q4[t]]) +
        (Cos[q1[t]] Cos[q2[t]] Cos[q3[t]] - Cos[q1[t]] Sin[q2[t]] Sin[q3[t]])
         Sin[q5[t]]) Sin[q6[t]]) Ftooly -
  (Cos[q6[t]] (Cos[q5[t]] (Cos[q4[t]] (Cos[q1[t]] Cos[q3[t]] Sin[q2[t]] +
              Cos[q1[t]] Cos[q2[t]] Sin[q3[t]]) + Sin[q1[t]] Sin[q4[t]]) +
        (Cos[q1[t]] Cos[q2[t]] Cos[q3[t]] - Cos[q1[t]] Sin[q2[t]] Sin[q3[t]]) Sin[q5[t]]) -
     (-Cos[q4[t]] Sin[q1[t]] + (Cos[q1[t]] Cos[q3[t]] Sin[q2[t]] +
           Cos[q1[t]] Cos[q2[t]] Sin[q3[t]]) Sin[q4[t]]) Sin[q6[t]]) Ftoolz
```

In[568]:=
```
eqS[2] = eqT[2] //. Derivative[_][_][t] → 0
```

Out[568]=
```
Fmoty - (Cos[q5[t]] (Cos[q2[t]] Cos[q3[t]] Sin[q1[t]] - Sin[q1[t]] Sin[q2[t]] Sin[q3[t]]) -
     (Cos[q4[t]] (Cos[q3[t]] Sin[q1[t]] Sin[q2[t]] + Cos[q2[t]] Sin[q1[t]] Sin[q3[t]]) -
        Cos[q1[t]] Sin[q4[t]]) Sin[q5[t]]) Ftoolx - (Cos[q6[t]] (Cos[q1[t]] Cos[q4[t]] +
        (Cos[q3[t]] Sin[q1[t]] Sin[q2[t]] + Cos[q2[t]] Sin[q1[t]] Sin[q3[t]]) Sin[q4[t]]) +
     (Cos[q5[t]] (Cos[q4[t]] (Cos[q3[t]] Sin[q1[t]] Sin[q2[t]] +
              Cos[q2[t]] Sin[q1[t]] Sin[q3[t]]) - Cos[q1[t]] Sin[q4[t]]) +
        (Cos[q2[t]] Cos[q3[t]] Sin[q1[t]] - Sin[q1[t]] Sin[q2[t]] Sin[q3[t]])
         Sin[q5[t]]) Sin[q6[t]]) Ftooly -
  (Cos[q6[t]] (Cos[q5[t]] (Cos[q4[t]] (Cos[q3[t]] Sin[q1[t]] Sin[q2[t]] +
              Cos[q2[t]] Sin[q1[t]] Sin[q3[t]]) - Cos[q1[t]] Sin[q4[t]]) +
        (Cos[q2[t]] Cos[q3[t]] Sin[q1[t]] - Sin[q1[t]] Sin[q2[t]] Sin[q3[t]]) Sin[q5[t]]) -
     (Cos[q1[t]] Cos[q4[t]] + (Cos[q3[t]] Sin[q1[t]] Sin[q2[t]] +
           Cos[q2[t]] Sin[q1[t]] Sin[q3[t]]) Sin[q4[t]]) Sin[q6[t]]) Ftoolz
```

In[569]:=
```
eqS[3] = (eqT[3] //. Derivative[_][_][t] → 0) // distributeScalars // distributeScalars //
  distributeScalars
```

ReplaceRepeated: Exiting after ≪17≫ + ≪15≫ scanned 4 times.

Out[569]=
```
1/3 Cos[q1[t]]^2 Cos[q2[t]] Cos[q3[t]] Cos[q5[t]] Ftoolx +
 1/3 Cos[q2[t]] Cos[q3[t]] Cos[q5[t]] Sin[q1[t]]^2 Ftoolx -
 1/3 Cos[q1[t]]^2 Cos[q5[t]] Sin[q2[t]] Sin[q3[t]] Ftoolx -
 1/3 Cos[q5[t]] Sin[q1[t]]^2 Sin[q2[t]] Sin[q3[t]] Ftoolx -
 1/3 Cos[q1[t]]^2 Cos[q3[t]] Cos[q4[t]] Sin[q2[t]] Sin[q5[t]] Ftoolx -
 1/3 Cos[q3[t]] Cos[q4[t]] Sin[q1[t]]^2 Sin[q2[t]] Sin[q5[t]] Ftoolx -
 1/3 Cos[q1[t]]^2 Cos[q2[t]] Cos[q4[t]] Sin[q3[t]] Sin[q5[t]] Ftoolx -
```

$\frac{1}{3}$ Cos[q_2[t]] Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_3[t]] Sin[q_5[t]] F_{toolx} −
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]] Sin[q_4[t]] Sin[q_5[t]] F_{toolx} −
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]] Cos[q_3[t]] Sin[q_4[t]] Sin[q_5[t]] F_{toolx} −
$\frac{3}{2}$ Cos[q_2[t]] Sin[q_1[t]]^2 Sin[q_4[t]] Sin[q_5[t]] F_{toolx} −
$\frac{3}{2}$ Cos[q_2[t]] Cos[q_3[t]] Sin[q_1[t]]^2 Sin[q_4[t]] Sin[q_5[t]] F_{toolx} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Sin[q_2[t]] Sin[q_3[t]] Sin[q_4[t]] Sin[q_5[t]] F_{toolx} +
$\frac{3}{2}$ Sin[q_1[t]]^2 Sin[q_2[t]] Sin[q_3[t]] Sin[q_4[t]] Sin[q_5[t]] F_{toolx} −
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]] Cos[q_4[t]] Cos[q_6[t]] F_{tooly} −
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]] Cos[q_3[t]] Cos[q_4[t]] Cos[q_6[t]] F_{tooly} −
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]] Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] F_{tooly} −
$\frac{3}{2}$ Cos[q_2[t]] Cos[q_4[t]] Cos[q_6[t]] Sin[q_1[t]]^2 F_{tooly} −
$\frac{3}{2}$ Cos[q_2[t]] Cos[q_3[t]] Cos[q_4[t]] Cos[q_6[t]] Sin[q_1[t]]^2 F_{tooly} −
$\frac{5}{21}$ Cos[q_2[t]] Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 F_{tooly} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_4[t]] Cos[q_6[t]] Sin[q_2[t]] Sin[q_3[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_2[t]] Sin[q_3[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_4[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]] Sin[q_3[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]] Sin[q_3[t]] F_{tooly} +
$\frac{1}{3}$ Cos[q_1[t]]^2 Cos[q_3[t]] Cos[q_6[t]] Sin[q_2[t]] Sin[q_4[t]] F_{tooly} +
$\frac{1}{3}$ Cos[q_3[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]] Sin[q_4[t]] F_{tooly} +
$\frac{1}{3}$ Cos[q_1[t]]^2 Cos[q_2[t]] Cos[q_6[t]] Sin[q_3[t]] Sin[q_4[t]] F_{tooly} +
$\frac{1}{3}$ Cos[q_2[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_3[t]] Sin[q_4[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]] Cos[q_4[t]]^2 Cos[q_6[t]] Sin[q_2[t]] Sin[q_5[t]] F_{tooly} +

$$\frac{5}{21} \text{Cos}[q_3[t]] \text{Cos}[q_4[t]]^2 \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_5[t]] F_{tooly} +$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_4[t]]^2 \text{Cos}[q_6[t]] \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] F_{tooly} +$$
$$\frac{5}{21} \text{Cos}[q_2[t]] \text{Cos}[q_4[t]]^2 \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] F_{tooly} +$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]] \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]] F_{tooly} +$$
$$\frac{5}{21} \text{Cos}[q_3[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]] F_{tooly} +$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_6[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]] F_{tooly} +$$
$$\frac{5}{21} \text{Cos}[q_2[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]] F_{tooly} +$$
$$\frac{1}{3} \text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_2[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{1}{3} \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{1}{3} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_3[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{1}{3} \text{Cos}[q_2[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_5[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_5[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{3}{2} \text{Cos}[q_2[t]] \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{3}{2} \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{5}{21} \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} -$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_5[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} -$$
$$\frac{3}{2} \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} -$$
$$\frac{5}{21} \text{Cos}[q_5[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{tooly} +$$
$$\frac{1}{3} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] F_{tooly} +$$

$$\frac{1}{3}\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,F_{tooly}-$$
$$\frac{1}{3}\,\text{Cos}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,F_{tooly}-$$
$$\frac{1}{3}\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,F_{tooly}+$$
$$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}+$$
$$\frac{5}{21}\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}-$$
$$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}-$$
$$\frac{5}{21}\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}+$$
$$\frac{1}{3}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,F_{toolz}+$$
$$\frac{1}{3}\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,F_{toolz}+$$
$$\frac{1}{3}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]\,F_{toolz}+$$
$$\frac{1}{3}\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,F_{toolz}+$$
$$\frac{3}{2}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_4[t]]\,F_{toolz}+$$
$$\frac{3}{2}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_4[t]]\,F_{toolz}+$$
$$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_4[t]]\,F_{toolz}+$$
$$\frac{3}{2}\,\text{Cos}[q_2[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,F_{toolz}+$$
$$\frac{3}{2}\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,F_{toolz}+$$
$$\frac{5}{21}\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,F_{toolz}-$$
$$\frac{3}{2}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,F_{toolz}-$$
$$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,F_{toolz}-$$
$$\frac{3}{2}\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,F_{toolz}-$$
$$\frac{5}{21}\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,F_{toolz}+$$
$$\frac{1}{3}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_5[t]]\,F_{toolz}+$$

$$\frac{1}{3}\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_5[t]]\,F_{toolz}-$$
$$\frac{1}{3}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,F_{toolz}-$$
$$\frac{1}{3}\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,F_{toolz}+$$
$$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]^2\,F_{toolz}+$$
$$\frac{5}{21}\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]^2\,F_{toolz}-$$
$$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]^2\,F_{toolz}-$$
$$\frac{5}{21}\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]^2\,F_{toolz}+$$
$$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}+$$
$$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}+$$
$$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}+$$
$$\frac{3}{2}\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_6[t]]\,F_{toolz}+$$
$$\frac{3}{2}\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_6[t]]\,F_{toolz}+$$
$$\frac{5}{21}\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_6[t]]\,F_{toolz}-$$
$$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$$
$$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$$
$$\frac{3}{2}\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$$
$$\frac{5}{21}\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$$
$$\frac{1}{3}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$$
$$\frac{1}{3}\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$$
$$\frac{1}{3}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$$
$$\frac{1}{3}\text{Cos}[q_2[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$$
$$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$$

$$\frac{5}{21} \text{Cos}[q_3[t]] \text{Cos}[q_4[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_4[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_2[t]] \text{Cos}[q_4[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]] \text{Sin}[q_2[t]] \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_3[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_2[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] F_{toolz} +$$
$$\text{Cos}[q_3[t]] \text{Cos}[q_5[t]] \text{Sin}[q_2[t]] T_{toolx} + \text{Cos}[q_2[t]] \text{Cos}[q_5[t]] \text{Sin}[q_3[t]] T_{toolx} +$$
$$\text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Sin}[q_5[t]] T_{toolx} -$$
$$\text{Cos}[q_4[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] T_{toolx} -$$
$$\text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_6[t]] \text{Sin}[q_4[t]] T_{tooly} +$$
$$\text{Cos}[q_6[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] T_{tooly} -$$
$$\text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_6[t]] T_{tooly} +$$
$$\text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_6[t]] T_{tooly} +$$
$$\text{Cos}[q_3[t]] \text{Sin}[q_2[t]] \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] T_{tooly} +$$
$$\text{Cos}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] \text{Sin}[q_6[t]] T_{tooly} -$$
$$\text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] T_{toolz} +$$
$$\text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] T_{toolz} +$$
$$\text{Cos}[q_3[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]] \text{Sin}[q_5[t]] T_{toolz} +$$
$$\text{Cos}[q_2[t]] \text{Cos}[q_6[t]] \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] T_{toolz} +$$
$$\text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] T_{toolz} -$$
$$\text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] T_{toolz} + T_{mot}[1]$$

In[570]:=
```
eqS[4] = (eqT[4] //. Derivative[_][_][t] → 0) // distributeScalars // distributeScalars //
  distributeScalars
```

ReplaceRepeated: Exiting after ≪18≫ + ≪27≫ scanned 4 times.

Out[570]=
$$-\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_3[t]] F_{toolx} -$$
$$\frac{3}{2} \text{Cos}[q_2[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]] F_{toolx} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] F_{toolx} -$$
$$\frac{3}{2} \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] F_{toolx} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Sin}[q_5[t]] F_{toolx} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Sin}[q_5[t]] F_{toolx} -$$

$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_3[t]] Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_5[t]] F_{toolx} −

$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_5[t]] F_{toolx} −

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_3[t]] Cos[q_4[t]] Sin[q_2[t]]^2 Sin[q_5[t]] F_{toolx} −

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Sin[q_2[t]]^2 Sin[q_5[t]] F_{toolx} −

$\frac{3}{2}$ Cos[q_3[t]] Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_5[t]] F_{toolx} −

$\frac{3}{2}$ Cos[q_3[t]]^2 Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_5[t]] F_{toolx} −

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_4[t]] Sin[q_3[t]]^2 Sin[q_5[t]] F_{toolx} −

$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]] F_{toolx} −

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_4[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]] F_{toolx} −

$\frac{3}{2}$ Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]] F_{toolx} +

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]] Cos[q_6[t]] Sin[q_4[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_6[t]] Sin[q_4[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_5[t]] Cos[q_6[t]] Sin[q_4[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_3[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_4[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_4[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_4[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_3[t]] Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_4[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_4[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_5[t]] Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_4[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_3[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_4[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_3[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_4[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_3[t]]^2 Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_4[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_6[t]] Sin[q_3[t]]^2 Sin[q_4[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_5[t]] Cos[q_6[t]] Sin[q_3[t]]^2 Sin[q_4[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_5[t]] Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Sin[q_6[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Sin[q_1[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Sin[q_1[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Sin[q_1[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Sin[q_2[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Sin[q_2[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Sin[q_2[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Sin[q_3[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Sin[q_3[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_4[t]] Cos[q_5[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_4[t]] Cos[q_5[t]]^2 Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_6[t]] F_{tooly} −

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Sin[q_3[t]] Sin[q_5[t]] Sin[q_6[t]] F_{tooly} −

$\frac{3}{2}$ Cos[q_2[t]]^2 Sin[q_1[t]]^2 Sin[q_3[t]] Sin[q_5[t]] Sin[q_6[t]] F_{tooly} −

$\frac{3}{2}$ Cos[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]] Sin[q_5[t]] Sin[q_6[t]] F_{tooly} −

$\frac{3}{2}$ Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]] Sin[q_5[t]] Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Sin[q_2[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_3[t]]^2 Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_4[t]] Sin[q_3[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_4[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{5}{21}$ Cos[q_4[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] F_{toolz} +

$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] F_{toolz} +

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Cos[q_6[t]] F_{toolz} +

$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 F_{toolz} +

$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 F_{toolz} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_2[t]]^2 F_{toolz} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_2[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 F_{toolz} +
$\frac{3}{2}$ Cos[q_3[t]] Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 F_{toolz} +
$\frac{3}{2}$ Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 F_{toolz} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_3[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_3[t]]^2 F_{toolz} +
$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 F_{toolz} +
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_4[t]] Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 F_{toolz} +
$\frac{3}{2}$ Cos[q_4[t]] Cos[q_5[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_4[t]] Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 F_{toolz} −
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_6[t]] Sin[q_3[t]] Sin[q_5[t]] F_{toolz} −
$\frac{3}{2}$ Cos[q_2[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_3[t]] Sin[q_5[t]] F_{toolz} −
$\frac{3}{2}$ Cos[q_1[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_3[t]] Sin[q_5[t]] F_{toolz} −
$\frac{3}{2}$ Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]] Sin[q_5[t]] F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_6[t]] Sin[q_5[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]] Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_5[t]]^2 F_{toolz} +

$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_5[t]]^2 F_{toolz} +$$
$$\frac{5}{21} \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_5[t]]^2 F_{toolz} +$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]]^2 F_{toolz} +$$
$$\frac{5}{21} \text{Cos}[q_2[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]]^2 F_{toolz} +$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]]^2 F_{toolz} +$$
$$\frac{5}{21} \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]]^2 F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_3[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_3[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{3}{2} \text{Cos}[q_2[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$
$$\frac{5}{21} \text{Cos}[q_2[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{toolz} -$$

$$\frac{3}{2}\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]^2\,\mathrm{Sin}[q_3[t]]^2\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]]\,F_{\mathrm{toolz}}-$$
$$\frac{5}{21}\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_2[t]]^2\,\mathrm{Sin}[q_3[t]]^2\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]]\,F_{\mathrm{toolz}}-$$
$$\frac{3}{2}\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]^2\,\mathrm{Sin}[q_3[t]]^2\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]]\,F_{\mathrm{toolz}}-$$
$$\frac{5}{21}\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]^2\,\mathrm{Sin}[q_3[t]]^2\,\mathrm{Sin}[q_4[t]]\,\mathrm{Sin}[q_6[t]]\,F_{\mathrm{toolz}}+$$
$$\frac{1}{2}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,M_4+\frac{1}{2}\,g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_4+$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,M_5+\frac{1}{6}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,M_5+$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_5+\frac{1}{6}\,g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_5-$$
$$\frac{1}{6}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_5-\frac{1}{6}\,g\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_5+$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,M_6+g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,M_6+$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_6+g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_6-$$
$$g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_6-g\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_6+$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,M_7+\frac{3}{2}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,M_7+$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_7+\frac{3}{2}\,g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_7-$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_7-\frac{3}{2}\,g\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_7+$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,M_8+\frac{3}{2}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,M_8+$$
$$\frac{5}{21}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]]\,M_8+$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_8+\frac{3}{2}\,g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_8+$$
$$\frac{5}{21}\,g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_8-$$
$$\frac{3}{2}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_8-$$
$$\frac{5}{21}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_8-\frac{3}{2}\,g\,\mathrm{Sin}[q_1[t]]^2$$
$$\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_8-\frac{5}{21}\,g\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_8-$$
$$\frac{5}{21}\,g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_5[t]]\,M_8-$$

$$\frac{5}{21} g \operatorname{Cos}[q_3[t]] \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_2[t]] \operatorname{Sin}[q_5[t]] M_8 -$$
$$\frac{5}{21} g \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_2[t]] \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_3[t]] \operatorname{Sin}[q_5[t]] M_8 -$$
$$\frac{5}{21} g \operatorname{Cos}[q_2[t]] \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_3[t]] \operatorname{Sin}[q_5[t]] M_8 -$$
$$\operatorname{Cos}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] \operatorname{Sin}[q_5[t]] T_{toolx} - \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] \operatorname{Sin}[q_5[t]] T_{toolx} -$$
$$\operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Cos}[q_6[t]] T_{tooly} - \operatorname{Cos}[q_4[t]] \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 T_{tooly} +$$
$$\operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_5[t]] \operatorname{Sin}[q_4[t]] \operatorname{Sin}[q_6[t]] T_{tooly} +$$
$$\operatorname{Cos}[q_5[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] \operatorname{Sin}[q_6[t]] T_{tooly} +$$
$$\operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_5[t]] \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_4[t]] T_{toolz} +$$
$$\operatorname{Cos}[q_5[t]] \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] T_{toolz} + \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_6[t]] T_{toolz} +$$
$$\operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_6[t]] T_{toolz} + \operatorname{Cos}[q_1[t]]^2 T_{mot}[2] + \operatorname{Sin}[q_1[t]]^2 T_{mot}[2]$$

In[571]:=
```
eqS[5] = (eqT[5] //. Derivative[_][_][t] → 0) // distributeScalars // distributeScalars //
    loosenDots // distributeScalars // distributeScalars
```

ReplaceRepeated: Exiting after ≪17≫ + ≪18≫ scanned 4 times.

Out[571]=
$$-\frac{3}{2} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_5[t]] F_{toolx} -$$
$$\frac{3}{2} \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_5[t]] F_{toolx} -$$
$$\frac{3}{2} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_5[t]] F_{toolx} -$$
$$\frac{3}{2} \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_5[t]] F_{toolx} -$$
$$\frac{3}{2} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_3[t]]^2 \operatorname{Sin}[q_5[t]] F_{toolx} -$$
$$\frac{3}{2} \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_3[t]]^2 \operatorname{Sin}[q_5[t]] F_{toolx} -$$
$$\frac{3}{2} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_3[t]]^2 \operatorname{Sin}[q_5[t]] F_{toolx} -$$
$$\frac{3}{2} \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_3[t]]^2 \operatorname{Sin}[q_5[t]] F_{toolx} +$$
$$\frac{3}{2} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_4[t]] F_{tooly} +$$
$$\frac{5}{21} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_5[t]] \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_4[t]] F_{tooly} +$$
$$\frac{3}{2} \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] F_{tooly} +$$
$$\frac{5}{21} \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_5[t]] \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] F_{tooly} +$$
$$\frac{3}{2} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_4[t]] F_{tooly} +$$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{3}{2}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{tooly}$ +

$\frac{5}{21}\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{3}{2}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{3}{2}\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{5}{21}\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}\,+$

$\frac{3}{2}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,F_{\text{toolz}}\,+$

$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,F_{\text{toolz}}\,+$

$\frac{3}{2}\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,F_{\text{toolz}}\,+$

$\frac{5}{21}\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,F_{\text{toolz}}\,+$

$\frac{3}{2}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,F_{\text{toolz}}\,+$

$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,F_{\text{toolz}}\,+$

$\frac{3}{2}\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,F_{\text{toolz}}\,+$

$\frac{5}{21}\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,F_{\text{toolz}}\,+$

$\frac{3}{2}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,F_{\text{toolz}}\,+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,F_{toolz}+$

$\frac{3}{2}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,F_{toolz}+$

$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,F_{toolz}+$

$\frac{3}{2}\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_5[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_5[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,F_{toolz}-$

$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$

$\frac{3}{2}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$

$\frac{5}{21}\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$

$\frac{3}{2}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$

$\frac{3}{2}\text{Cos}[q_3[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,F_{toolz}-$

$$\frac{5}{21} \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{\text{toolz}} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{\text{toolz}} -$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{\text{toolz}} -$$
$$\frac{3}{2} \text{Cos}[q_2[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{\text{toolz}} -$$
$$\frac{5}{21} \text{Cos}[q_2[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{\text{toolz}} -$$
$$\frac{3}{2} \text{Cos}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{\text{toolz}} -$$
$$\frac{5}{21} \text{Cos}[q_1[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{\text{toolz}} -$$
$$\frac{3}{2} \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{\text{toolz}} -$$
$$\frac{5}{21} \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_6[t]] F_{\text{toolz}} +$$
$$\frac{1}{6} g \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] M_5 +$$
$$\frac{1}{6} g \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Sin}[q_1[t]]^2 M_5 - \frac{1}{6} g \text{Cos}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] M_5 -$$
$$\frac{1}{6} g \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] M_5 + g \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] M_6 +$$
$$g \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Sin}[q_1[t]]^2 M_6 - g \text{Cos}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] M_6 -$$
$$g \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] M_6 + \frac{3}{2} g \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] M_7 +$$
$$\frac{3}{2} g \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Sin}[q_1[t]]^2 M_7 - \frac{3}{2} g \text{Cos}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] M_7 -$$
$$\frac{3}{2} g \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] M_7 + \frac{3}{2} g \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] M_8 +$$
$$\frac{5}{21} g \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_5[t]] M_8 +$$
$$\frac{3}{2} g \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Sin}[q_1[t]]^2 M_8 + \frac{5}{21} g \text{Cos}[q_2[t]] \text{Cos}[q_3[t]]$$
$$\text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 M_8 - \frac{3}{2} g \text{Cos}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] M_8 -$$
$$\frac{5}{21} g \text{Cos}[q_1[t]]^2 \text{Cos}[q_5[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] M_8 - \frac{3}{2} g \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]$$
$$\text{Sin}[q_3[t]] M_8 - \frac{5}{21} g \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] M_8 -$$
$$\frac{5}{21} g \text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Sin}[q_2[t]] \text{Sin}[q_5[t]] M_8 -$$

$$\frac{5}{21} g \operatorname{Cos}[q_3[t]] \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_2[t]] \operatorname{Sin}[q_5[t]] M_8 -$$
$$\frac{5}{21} g \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_2[t]] \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_3[t]] \operatorname{Sin}[q_5[t]] M_8 -$$
$$\frac{5}{21} g \operatorname{Cos}[q_2[t]] \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_3[t]] \operatorname{Sin}[q_5[t]] M_8 -$$
$$\operatorname{Cos}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] \operatorname{Sin}[q_5[t]] T_{toolx} - \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] \operatorname{Sin}[q_5[t]] T_{toolx} -$$
$$\operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Cos}[q_6[t]] T_{tooly} - \operatorname{Cos}[q_4[t]] \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 T_{tooly} +$$
$$\operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_5[t]] \operatorname{Sin}[q_4[t]] \operatorname{Sin}[q_6[t]] T_{tooly} +$$
$$\operatorname{Cos}[q_5[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] \operatorname{Sin}[q_6[t]] T_{tooly} +$$
$$\operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_5[t]] \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_4[t]] T_{toolz} +$$
$$\operatorname{Cos}[q_5[t]] \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_4[t]] T_{toolz} + \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_6[t]] T_{toolz} +$$
$$\operatorname{Cos}[q_4[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_6[t]] T_{toolz} + \operatorname{Cos}[q_1[t]]^2 T_{mot}[3] + \operatorname{Sin}[q_1[t]]^2 T_{mot}[3]$$

In[572]:=
```
eqS[6] = (eqT[6] //. Derivative[_][_][t] → 0) // distributeScalars // distributeScalars //
    loosenDots // distributeScalars // distributeScalars
```

••• ReplaceRepeated: Exiting after ≪19≫ + ≪5≫ scanned 4 times.

••• ReplaceRepeated: Exiting after $\frac{5}{6}$ Cos[q1[t]] Cos[q2[t]] Cos[q3[t]]^2 Cos[q5[t]] Sin[q1[t]] Sin[q2[t]]^2 Sin[q3[t]] F_{toolx} + ≪14≫ + ≪144≫ scanned 10 times.

Out[572]=
$$-\frac{5}{21} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_4[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_4[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_4[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_4[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_4[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_3[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_4[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_3[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_4[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_3[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_4[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_3[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_4[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_2[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_4[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_1[t]]^2 \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_4[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$
$$\frac{5}{21} \operatorname{Cos}[q_3[t]]^2 \operatorname{Cos}[q_6[t]] \operatorname{Sin}[q_1[t]]^2 \operatorname{Sin}[q_2[t]]^2 \operatorname{Sin}[q_4[t]]^2 \operatorname{Sin}[q_5[t]] F_{tooly} -$$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Cos}[q_2[t]]^2\text{Cos}[q_6[t]]\text{Sin}[q_3[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]F_{tooly}-$

$\frac{5}{21}\text{Cos}[q_2[t]]^2\text{Cos}[q_6[t]]\text{Sin}[q_1[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]F_{tooly}-$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Cos}[q_6[t]]\text{Sin}[q_2[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]F_{tooly}-$

$\frac{5}{21}\text{Cos}[q_6[t]]\text{Sin}[q_1[t]]^2\text{Sin}[q_2[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]F_{tooly}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Cos}[q_2[t]]^2\text{Cos}[q_3[t]]^2\text{Cos}[q_4[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_2[t]]^2\text{Cos}[q_3[t]]^2\text{Cos}[q_4[t]]^2\text{Sin}[q_1[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Cos}[q_3[t]]^2\text{Cos}[q_4[t]]^2\text{Sin}[q_2[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_3[t]]^2\text{Cos}[q_4[t]]^2\text{Sin}[q_1[t]]^2\text{Sin}[q_2[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Cos}[q_2[t]]^2\text{Cos}[q_4[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_2[t]]^2\text{Cos}[q_4[t]]^2\text{Sin}[q_1[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Cos}[q_4[t]]^2\text{Sin}[q_2[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_4[t]]^2\text{Sin}[q_1[t]]^2\text{Sin}[q_2[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Cos}[q_2[t]]^2\text{Cos}[q_3[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_2[t]]^2\text{Cos}[q_3[t]]^2\text{Sin}[q_1[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Cos}[q_3[t]]^2\text{Sin}[q_2[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_3[t]]^2\text{Sin}[q_1[t]]^2\text{Sin}[q_2[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Cos}[q_2[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_2[t]]^2\text{Sin}[q_1[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Cos}[q_1[t]]^2\text{Sin}[q_2[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}+$

$\frac{5}{21}\text{Sin}[q_1[t]]^2\text{Sin}[q_2[t]]^2\text{Sin}[q_3[t]]^2\text{Sin}[q_4[t]]^2\text{Sin}[q_5[t]]\text{Sin}[q_6[t]]F_{toolz}-$

$\frac{5}{21}g\,\text{Cos}[q_1[t]]^2\text{Cos}[q_2[t]]\text{Cos}[q_3[t]]\text{Sin}[q_4[t]]\text{Sin}[q_5[t]]M_8-$

$\frac{5}{21}g\,\text{Cos}[q_2[t]]\text{Cos}[q_3[t]]\text{Sin}[q_1[t]]^2\text{Sin}[q_4[t]]\text{Sin}[q_5[t]]M_8+$

$$\frac{5}{21} g \, \mathrm{Cos}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_4[t]] \, \mathrm{Sin}[q_5[t]] \, M_8 +$$
$$\frac{5}{21} g \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_4[t]] \, \mathrm{Sin}[q_5[t]] \, M_8 -$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_5[t]] \, T_{toolx} -$$
$$\mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_1[t]]^2 \, T_{toolx} -$$
$$\mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_2[t]]^2 \, T_{toolx} -$$
$$2 \, \mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]] \, T_{toolx} +$$
$$2 \, \mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]] \, T_{toolx} +$$
$$2 \, \mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]] \, T_{toolx} -$$
$$\mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_3[t]]^2 \, T_{toolx} -$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_2[t]]^2 \, \mathrm{Sin}[q_3[t]]^2 \, T_{toolx} -$$
$$\mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]]^2 \, \mathrm{Sin}[q_3[t]]^2 \, T_{toolx} -$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_5[t]] \, T_{toolx} +$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_5[t]] \, T_{toolx} +$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_5[t]] \, T_{toolx} -$$
$$\mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_5[t]] \, T_{toolx} +$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_5[t]] \, T_{toolx} +$$
$$\mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_5[t]] \, T_{toolx} +$$
$$\mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_2[t]]^2 \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_5[t]] \, T_{toolx} -$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_2[t]]^2 \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_5[t]] \, T_{toolx} -$$
$$\mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]]^2 \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_5[t]] \, T_{toolx} +$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]]^2 \, \mathrm{Sin}[q_5[t]] \, T_{toolx} -$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]]^2 \, \mathrm{Sin}[q_5[t]] \, T_{toolx} -$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]]^2 \, \mathrm{Sin}[q_5[t]] \, T_{toolx} +$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_4[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_4[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_4[t]] \, T_{tooly} +$$
$$\mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_4[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_4[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_4[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_2[t]]^2 \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_4[t]] \, T_{tooly} +$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_2[t]]^2 \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_4[t]] \, T_{tooly} +$$
$$\mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]]^2 \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_4[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]]^2 \, \mathrm{Sin}[q_4[t]] \, T_{tooly} +$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]]^2 \, \mathrm{Sin}[q_4[t]] \, T_{tooly} +$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_6[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_3[t]]^2 \, \mathrm{Sin}[q_4[t]] \, T_{tooly} +$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_4[t]] \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_6[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_4[t]] \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_6[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_2[t]] \, \mathrm{Cos}[q_3[t]]^2 \, \mathrm{Cos}[q_4[t]] \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_2[t]] \, \mathrm{Sin}[q_6[t]] \, T_{tooly} +$$
$$\mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_6[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_1[t]]^2 \, \mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_6[t]] \, T_{tooly} -$$
$$\mathrm{Cos}[q_2[t]]^2 \, \mathrm{Cos}[q_3[t]] \, \mathrm{Cos}[q_4[t]] \, \mathrm{Cos}[q_5[t]] \, \mathrm{Sin}[q_1[t]]^2 \, \mathrm{Sin}[q_3[t]] \, \mathrm{Sin}[q_6[t]] \, T_{tooly} -$$

$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_3[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,T_{toolz}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,T_{toolz}-$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,T_{toolz}+$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]\,T_{toolz}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]\,T_{toolz}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,T_{toolz}-$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,T_{toolz}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,T_{toolz}+$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,T_{toolz}-$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,T_{toolz}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,T_{toolz}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,T_{toolz}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_5[t]]\,T_{toolz}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_5[t]]\,T_{toolz}-$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]\,T_{toolz}-$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{toolz}+$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{toolz}+$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{toolz}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{toolz}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{toolz}-$
$\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{toolz}-$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz}+$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz}+$
$\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz}-$

$\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}}+$
$\text{Cos}[q_2[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}}-$
$\text{Cos}[q_2[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,T_{\text{mot}}[4]+\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_1[t]]^2\,T_{\text{mot}}[4]+$
$\text{Cos}[q_3[t]]^2\,\text{Sin}[q_2[t]]^2\,T_{\text{mot}}[4]+2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,T_{\text{mot}}[4]-$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,T_{\text{mot}}[4]-$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,T_{\text{mot}}[4]+$
$\text{Cos}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,T_{\text{mot}}[4]+\text{Cos}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,T_{\text{mot}}[4]+$
$\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,T_{\text{mot}}[4]$

In[573]:=
```
eqS[7] = (eqT[7] //. Derivative[_][_][t] → 0) // distributeScalars // distributeScalars //
    loosenDots // distributeScalars // distributeScalars // distributeScalars
```

••• ReplaceRepeated: Exiting after ≪10≫ + (Cos[q₄[t]] (Cos[Subscript[≪2≫][≪1≫]]^2 + Sin[Subscript[≪2≫][≪1≫]]^2) Sin[q₄[t]] Sin[q₅[t]] + Sin[q₄[t]] (Cos[Subscript[≪2≫][≪1≫]] Sin[Subscript[≪2≫][≪1≫]] (Times[≪3≫] + Times[≪2≫]) + Cos[Subscript[≪2≫][≪1≫]] Cos[Subscript[≪2≫][≪1≫]] (Times[≪2≫] + Times[≪2≫]) − Cos[Subscript[≪2≫][≪1≫]] (Times[≪3≫] + Times[≪3≫]) Sin[Subscript[≪2≫][≪1≫]] − Sin[Subscript[≪2≫][≪1≫]] (Times[≪3≫] + Times[≪4≫]) Sin[Subscript[≪2≫][≪1≫]])) $T_{\text{mot}}[6]$ scanned 3 times.

••• ReplaceRepeated: Exiting after ≪18≫ + ≪32≫ scanned 8 times.

Out[573]=
$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$
$\frac{5}{21}\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,F_{\text{tooly}}+$

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_5[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_5[t]]^2 Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_5[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_5[t]]^2 Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]]^2 Sin[q_1[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]]^2 Sin[q_2[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_3[t]]^2 Cos[q_4[t]]^2 Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_4[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_4[t]]^2 Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_4[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_4[t]]^2 Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Sin[q_4[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Sin[q_1[t]]^2 Sin[q_4[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Sin[q_2[t]]^2 Sin[q_4[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_3[t]]^2 Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_4[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 Sin[q_5[t]]^2 Sin[q_6[t]] F_{tooly} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] F_{toolz} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 F_{toolz} +

$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_3[t]]^2 Cos[q_4[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_4[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_3[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_4[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_4[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_4[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_4[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_4[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_4[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_3[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_4[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_3[t]]^2 Sin[q_4[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_5[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_4[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]]^2 Cos[q_6[t]] Sin[q_5[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_5[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_3[t]]^2 Cos[q_4[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_5[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_3[t]]^2 Cos[q_4[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_5[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_2[t]]^2 Cos[q_4[t]]^2 Cos[q_6[t]] Sin[q_3[t]]^2 Sin[q_5[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_2[t]]^2 Cos[q_4[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_1[t]]^2 Cos[q_4[t]]^2 Cos[q_6[t]] Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 F_{toolz} +
$\frac{5}{21}$ Cos[q_4[t]]^2 Cos[q_6[t]] Sin[q_1[t]]^2 Sin[q_2[t]]^2 Sin[q_3[t]]^2 Sin[q_5[t]]^2 F_{toolz} +

$\frac{5}{21}\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]^2\,\mathrm{Cos}[q_3[t]]^2\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]^2\,F_{toolz}$ +
$\frac{5}{21}\mathrm{Cos}[q_2[t]]^2\,\mathrm{Cos}[q_3[t]]^2\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]^2\,F_{toolz}$ +
$\frac{5}{21}\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_3[t]]^2\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_2[t]]^2\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]^2\,F_{toolz}$ +
$\frac{5}{21}\mathrm{Cos}[q_3[t]]^2\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]^2\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]^2\,F_{toolz}$ +
$\frac{5}{21}\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]^2\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_3[t]]^2\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]^2\,F_{toolz}$ +
$\frac{5}{21}\mathrm{Cos}[q_2[t]]^2\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_3[t]]^2\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]^2\,F_{toolz}$ +
$\frac{5}{21}\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_2[t]]^2\,\mathrm{Sin}[q_3[t]]^2\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]^2\,F_{toolz}$ +
$\frac{5}{21}\mathrm{Cos}[q_6[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]^2\,\mathrm{Sin}[q_3[t]]^2\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]^2\,F_{toolz}$ +
$\frac{5}{21}g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,M_8$ +
$\frac{5}{21}g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_1[t]]^2\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_4[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_5[t]]\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_4[t]]^2\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_5[t]]\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_4[t]]^2\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]]\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_4[t]]^2\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_5[t]]\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_3[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]\,M_8$ −
$\frac{5}{21}g\,\mathrm{Cos}[q_2[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]^2\,\mathrm{Sin}[q_5[t]]\,M_8$ +
$\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]^2\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_4[t]]\,T_{toolx}$ −
$\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]^2\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_4[t]]\,T_{toolx}$ −
$\mathrm{Cos}[q_2[t]]\,\mathrm{Cos}[q_3[t]]^2\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_1[t]]^2\,\mathrm{Sin}[q_2[t]]\,\mathrm{Sin}[q_4[t]]\,T_{toolx}$ +
$\mathrm{Cos}[q_2[t]]^2\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]\,T_{toolx}$ −
$\mathrm{Cos}[q_1[t]]^2\,\mathrm{Cos}[q_2[t]]^2\,\mathrm{Cos}[q_3[t]]\,\mathrm{Cos}[q_5[t]]\,\mathrm{Sin}[q_3[t]]\,\mathrm{Sin}[q_4[t]]\,T_{toolx}$ −

$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,T_{\text{toolx}}-$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,T_{\text{toolx}}-$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,T_{\text{toolx}}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}-$
$\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}-$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}+$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]$
$\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}+2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]$
$\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}+$
$\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolx}}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_6[t]]\,T_{\text{tooly}}-\text{Cos}[q_4[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,T_{\text{tooly}}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_4[t]]^2\,T_{\text{tooly}}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]^2\,T_{\text{tooly}}-$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]^2\,T_{\text{tooly}}+$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]^2\,T_{\text{tooly}}-$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]^2\,T_{\text{tooly}}-$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]^2\,T_{\text{tooly}}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]^2\,T_{\text{tooly}}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]^2\,T_{\text{tooly}}-$
$\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]^2\,T_{\text{tooly}}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}+$
$\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}-$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}+$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]$
$\text{Sin}[q_6[t]]\,T_{\text{tooly}}-2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]$
$\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}-2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]$
$\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}-$
$\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}}-$

$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} -$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} -$
$\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} +$
$\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} -$
$\text{Cos}[q_2[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} +$
$\text{Cos}[q_2[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{\text{tooly}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} +$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]$
$\text{Sin}[q_4[t]]\,T_{\text{toolz}} - 2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]$
$\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} - 2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]$
$\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} + \text{Cos}[q_4[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_3[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} -$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} + \text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]^2\,T_{\text{mot}}[5] +$
$\text{Cos}[q_4[t]]^2\,\text{Sin}[q_1[t]]^2\,T_{\text{mot}}[5] + \text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Sin}[q_4[t]]^2\,T_{\text{mot}}[5] +$

$\text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]]^2 T_{mot}[5] +$
$\text{Cos}[q_3[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]]^2 T_{mot}[5] -$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]]^2 T_{mot}[5] +$
$2 \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]]^2 T_{mot}[5] +$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]]^2 T_{mot}[5] +$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]]^2 T_{mot}[5] +$
$\text{Cos}[q_2[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]]^2 T_{mot}[5] +$
$\text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]]^2 T_{mot}[5]$

In[574]:=
```
eqS[8] = (eqT[8] //. Derivative[_][_][t] → 0) // distributeScalars // distributeScalars //
   loosenDots // distributeScalars
```

... ReplaceRepeated: Exiting after
−((Cos[q_5[t]] (Cos[Subscript[≪2≫][≪1≫]] (Times[≪2≫] + Times[≪3≫]) − (Times[≪4≫] + Times[≪3≫]) Sin[Subscript[≪2≫][≪1≫]]) − Sin[q_5[t]] (Cos[Subscript[≪2≫][≪1≫]] Cos[Subscript[≪2≫][≪1≫]] (Times[≪2≫] + Times[≪2≫]) − Cos[Subscript[≪2≫][≪1≫]] (Times[≪3≫] + Times[≪3≫]) Sin[Subscript[≪2≫][≪1≫]] − (Power[≪2≫] + Power[≪2≫]) Sin[≪1≫]^2 Sin[Subscript[≪2≫][≪1≫]])) T_{toolx}) − (≪1≫ − ≪1≫ − ≪1≫ + ≪1≫) ≪1≫ ≪1≫ ≪1≫ + (Cos[q_5[t]] (Cos[Subscript[≪2≫][≪1≫]] (Times[≪2≫] + Times[≪3≫]) − (Times[≪4≫] + Times[≪3≫]) Sin[Subscript[≪2≫][≪1≫]]) − Sin[q_5[t]] (≪1≫ − ≪1≫ − ≪1≫)) T_{mot}[6]
scanned 3 times.

... ReplaceRepeated: Exiting after
−Cos[q_3[t]] Cos[q_5[t]]^2 (Cos[q_2[t]] Cos[q_3[t]] (Cos[≪1≫]^2 Cos[Subscript[≪2≫][≪1≫]] + Cos[Subscript[≪2≫][≪1≫]] Sin[≪1≫]^2) + Cos[q_3[t]] Sin[q_2[t]]^2 + Cos[q_2[t]] Sin[q_2[t]] Sin[q_3[t]] − Cos[q_2[t]] (Cos[≪1≫]^2 Sin[Subscript[≪2≫][≪1≫]] + Sin[≪1≫]^2 Sin[Subscript[≪2≫][≪1≫]]) Sin[q_3[t]]) T_{toolx} + ≪12≫ + ≪22≫ scanned 7 times.

Out[574]=
$-\text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]]^2 T_{toolx} -$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]]^2 \text{Sin}[q_1[t]]^2 T_{toolx} -$
$\text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]]^2 \text{Sin}[q_2[t]]^2 T_{toolx} -$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] T_{toolx} +$
$2 \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] T_{toolx} +$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] T_{toolx} -$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_5[t]]^2 \text{Sin}[q_3[t]]^2 T_{toolx} -$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_5[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 T_{toolx} -$
$\text{Cos}[q_5[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 T_{toolx} -$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_2[t]] \text{Sin}[q_5[t]] T_{toolx} +$
$2 \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_2[t]] \text{Sin}[q_5[t]] T_{toolx} +$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_5[t]] T_{toolx} -$
$2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] T_{toolx} +$
$2 \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] T_{toolx} +$
$2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] T_{toolx} +$
$2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] T_{toolx} -$
$2 \text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] T_{toolx} -$
$2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_5[t]] T_{toolx} +$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]] T_{toolx} -$
$2 \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]] T_{toolx} -$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]] T_{toolx} -$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]]^2 \text{Sin}[q_5[t]]^2 T_{toolx} -$

$\text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_5[t]]^2 T_{\text{toolx}} -$
$\text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_5[t]]^2 T_{\text{toolx}} +$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_4[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_5[t]]^2 T_{\text{toolx}} -$
$2 \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_4[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_5[t]]^2 T_{\text{toolx}} -$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_4[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_5[t]]^2 T_{\text{toolx}} -$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_4[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]]^2 T_{\text{toolx}} -$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_4[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]]^2 T_{\text{toolx}} -$
$\text{Cos}[q_4[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_5[t]]^2 T_{\text{toolx}} -$
$\text{Cos}[q_1[t]]^2 \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]]^2 T_{\text{toolx}} - \text{Sin}[q_1[t]]^2 \text{Sin}[q_4[t]]^2 \text{Sin}[q_5[t]]^2 T_{\text{toolx}} +$
$\text{Cos}[q_2[t]] \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]] \text{Sin}[q_4[t]] T_{\text{tooly}} -$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]] \text{Sin}[q_4[t]] T_{\text{tooly}} -$
$\text{Cos}[q_2[t]] \text{Cos}[q_3[t]]^2 \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_4[t]] T_{\text{tooly}} +$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] T_{\text{tooly}} -$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] T_{\text{tooly}} -$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] T_{\text{tooly}} -$
$\text{Cos}[q_3[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] T_{\text{tooly}} +$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] T_{\text{tooly}} +$
$\text{Cos}[q_3[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] T_{\text{tooly}} -$
$\text{Cos}[q_2[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] T_{\text{tooly}} +$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] T_{\text{tooly}} +$
$\text{Cos}[q_2[t]] \text{Cos}[q_5[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] T_{\text{tooly}} -$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} +$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} -$
$\text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} +$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} +$
$\text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} -$
$2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]]$
$\text{Sin}[q_5[t]] T_{\text{tooly}} + 2 \text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_6[t]]$
$\text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} + 2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]]$
$\text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_3[t]] \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} +$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} +$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} +$
$\text{Cos}[q_4[t]] \text{Cos}[q_6[t]] \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]]^2 \text{Sin}[q_4[t]] \text{Sin}[q_5[t]] T_{\text{tooly}} +$
$\text{Cos}[q_2[t]] \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_6[t]] T_{\text{tooly}} -$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]] \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_6[t]] T_{\text{tooly}} -$
$\text{Cos}[q_2[t]] \text{Cos}[q_3[t]]^2 \text{Cos}[q_4[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]] \text{Sin}[q_6[t]] T_{\text{tooly}} +$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_6[t]] T_{\text{tooly}} -$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_6[t]] T_{\text{tooly}} -$
$\text{Cos}[q_2[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_6[t]] T_{\text{tooly}} -$
$\text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_6[t]] T_{\text{tooly}} +$
$\text{Cos}[q_1[t]]^2 \text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_6[t]] T_{\text{tooly}} +$
$\text{Cos}[q_3[t]] \text{Cos}[q_4[t]] \text{Cos}[q_5[t]]^2 \text{Sin}[q_1[t]]^2 \text{Sin}[q_2[t]]^2 \text{Sin}[q_3[t]] \text{Sin}[q_6[t]] T_{\text{tooly}} -$

$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]$
$\text{Sin}[q_6[t]]\,T_{tooly}-2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]$
$\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]$
$\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+2\,\text{Cos}[q_2[t]]$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]$
$\text{Sin}[q_6[t]]\,T_{tooly}-\text{Cos}[q_2[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}-$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,\text{Sin}[q_6[t]]\,T_{tooly}+$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,T_{toolz}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,T_{toolz}-$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,T_{toolz}+$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]\,T_{toolz}-$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]\,T_{toolz}-$

$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]^2\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,T_{\text{toolz}} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]$
$\text{Sin}[q_5[t]]\,T_{\text{toolz}} - 2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]$
$\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} + 2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]$
$\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} + 2\,\text{Cos}[q_2[t]]$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]$
$\text{Sin}[q_5[t]]\,T_{\text{toolz}} - \text{Cos}[q_2[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_4[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_5[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]^2\,\text{Sin}[q_5[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} +$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} -$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_6[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} -$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{\text{toolz}} +$

$\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} +$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_2[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} +$
$\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} +$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]$
$\text{Sin}[q_6[t]]\,T_{toolz} - 2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]$
$\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} - 2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]$
$\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} -$
$\text{Cos}[q_4[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_4[t]]\,\text{Sin}[q_5[t]]\,\text{Sin}[q_6[t]]\,T_{toolz} +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]^2\,T_{mot}[6] +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,T_{mot}[6] +$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]^2\,T_{mot}[6] +$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,T_{mot}[6] -$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,T_{mot}[6] -$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,T_{mot}[6] +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_3[t]]^2\,T_{mot}[6] +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_5[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,T_{mot}[6] +$
$\text{Cos}[q_5[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]^2\,T_{mot}[6] +$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]\,T_{mot}[6] -$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]\,T_{mot}[6] -$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_5[t]]\,T_{mot}[6] +$
$2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{mot}[6] -$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{mot}[6] -$
$2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{mot}[6] -$
$2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{mot}[6] +$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{mot}[6] +$
$2\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]\,T_{mot}[6] -$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{mot}[6] +$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{mot}[6] +$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_4[t]]\,\text{Cos}[q_5[t]]\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]\,T_{mot}[6] +$
$\text{Cos}[q_2[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Sin}[q_5[t]]^2\,T_{mot}[6] +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]^2\,T_{mot}[6] +$
$\text{Cos}[q_3[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]^2\,\text{Sin}[q_5[t]]^2\,T_{mot}[6] -$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,T_{mot}[6] +$
$2\,\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,T_{mot}[6] +$
$2\,\text{Cos}[q_2[t]]\,\text{Cos}[q_3[t]]\,\text{Cos}[q_4[t]]^2\,\text{Sin}[q_1[t]]^2\,\text{Sin}[q_2[t]]\,\text{Sin}[q_3[t]]\,\text{Sin}[q_5[t]]^2\,T_{mot}[6] +$
$\text{Cos}[q_1[t]]^2\,\text{Cos}[q_2[t]]^2\,\text{Cos}[q_4[t]]^2\,\text{Sin}[q_3[t]]^2\,\text{Sin}[q_5[t]]^2\,T_{mot}[6] +$

```
Cos[q2[t]]^2 Cos[q4[t]]^2 Sin[q1[t]]^2 Sin[q3[t]]^2 Sin[q5[t]]^2 Tmot[6] +
Cos[q4[t]]^2 Sin[q2[t]]^2 Sin[q3[t]]^2 Sin[q5[t]]^2 Tmot[6] +
Cos[q1[t]]^2 Sin[q4[t]]^2 Sin[q5[t]]^2 Tmot[6] + Sin[q1[t]]^2 Sin[q4[t]]^2 Sin[q5[t]]^2 Tmot[6]
```

Now solve for the force and torque expressions

```
In[575]:= brakeSol = Solve[{eqS[1] == 0, eqS[2] == 0, eqS[3] == 0, eqS[4] == 0, eqS[5] == 0, eqS[6] == 0,
    eqS[7] == 0, eqS[8] == 0}, {Fmotx, Fmoty, Tmot[1], Tmot[2], Tmot[3], Tmot[4], Tmot[5], Tmot[6]}]

Out[575]= {{Fmotx → (Cos[q1[t]] Cos[q2[t]] Cos[q3[t]] Cos[q5[t]] - Cos[q1[t]] Cos[q5[t]] Sin[q2[t]]
        Sin[q3[t]] - Cos[q1[t]] Cos[q3[t]] Cos[q4[t]] Sin[q2[t]] Sin[q5[t]] -
       Cos[q1[t]] Cos[q2[t]] Cos[q4[t]] Sin[q3[t]] Sin[q5[t]] -
       Sin[q1[t]] Sin[q4[t]] Sin[q5[t]]) Ftoolx - (… 13 … + … 1 …) Ftooly +
     (Cos[q1[t]] Cos[q3[t]] Cos[q4[t]] Cos[q5[t]] Cos[q6[t]] Sin[q2[t]] +
       Cos[q1[t]] Cos[q2[t]] Cos[q4[t]] Cos[… 1 …] Cos[q6[t]] Sin[q3[t]] + … 9 …)
     Ftoolz, … 6 …, Tmot[6] → (… 1 …)/(… 1 …)}}

large output | show less | show more | show all | set size limit...
```

Now we will maximize these functions of the angles. First I will set the expected maximum tool static force in each direction. I will also set values for my masses.

```
In[576]:= Ftoolmax = 100;
Ttoolmax = 50;
g = 9.81;
M1 = 20;
M2 = 20;
M3 = 10;
M4 = 10;
M5 = 5;
M6 = 2;
M7 = 1;
M8 = 1;
```

x-Brake force at base

Here is the brake force in the x -direction

In[577]:= $F_{brakex} = F_{motx}$ /. brakeSol //. $\{q_{n\_}[t] \to Q_n\}$ // First

Out[577]= $(\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_5] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5] - \mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_5])\,F_{toolx} - (\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_1] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_4] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_6] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_6] - \mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_6] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6] + \mathrm{Cos}[Q_1]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6])\,F_{tooly} + (\mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_5]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_2] + \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_5]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_3] + \mathrm{Cos}[Q_5]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_4] + \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_5] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5] + \mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_1]\,\mathrm{Sin}[Q_6] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_6] - \mathrm{Cos}[Q_1]\,\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_6])\,F_{toolz}$

The maximum force is the first item and it occurs at the angles shown.

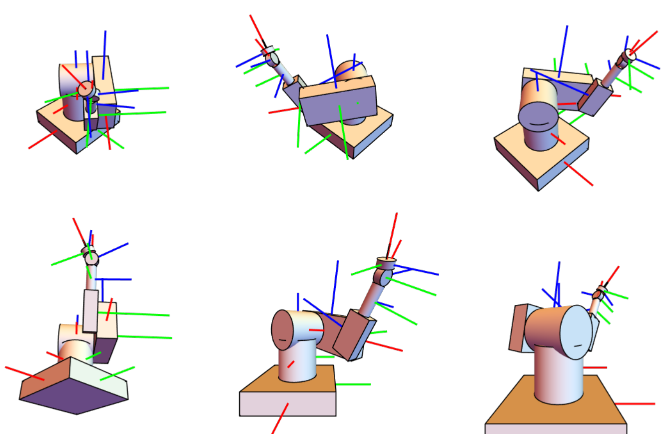
In[578]:= **xBrakeSol = Maximize**$[\{F_{brakex}, -.9\,\mathrm{Pi} \le Q_1 \le .9\,\mathrm{Pi}, -(\mathrm{Pi} + \mathrm{Pi}/4) \le Q_2 \le \mathrm{Pi}/4, -\mathrm{Pi}/2 \le Q_3 \le \mathrm{Pi}/2, -2\,\mathrm{Pi} \le Q_4 \le 2\,\mathrm{Pi}, -\mathrm{Pi} \le Q_5 \le \mathrm{Pi}, -2\,\mathrm{Pi} \le Q_6 \le 2\,\mathrm{Pi}, 0 \le F_{toolx} \le F_{toolmax}, 0 \le F_{tooly} \le F_{toolmax}, 0 \le F_{toolz} \le F_{toolmax}\}, \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, F_{toolx}, F_{tooly}, F_{toolz}\}]$

Out[578]= $\{173.205, \{Q_1 \to 0.849595, Q_2 \to 0.157084, Q_3 \to -1.20927, Q_4 \to -1.71678, Q_5 \to 0.440622, Q_6 \to 4.90373, F_{toolx} \to 100., F_{tooly} \to 100., F_{toolz} \to 100.\}\}$

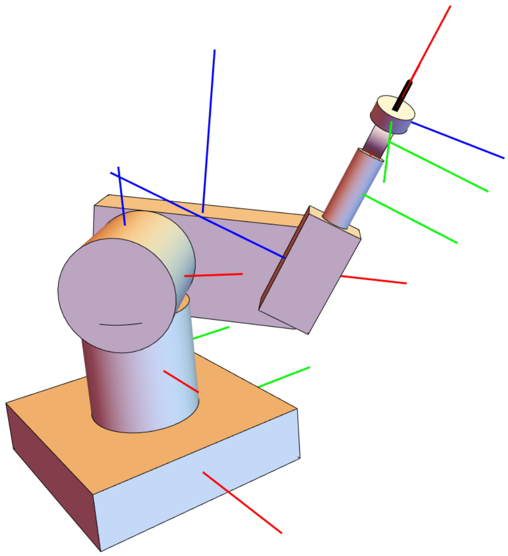
Here is a picture of this robot orientation for this maximum brake force.

In[579]:=
```
Show[GraphicsGrid[{{Graphics3D[robotGraphicBrakes //.xBrakeSol[[2]], ViewPoint → {1, 1, 1},
    ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
   Graphics3D[robotGraphicBrakes //.xBrakeSol[[2]], ViewPoint → {-1, 1, 1},
    ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
   Graphics3D[robotGraphicBrakes //.xBrakeSol[[2]], ViewPoint → {1, -1, 1},
    ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
  {Graphics3D[robotGraphicBrakes //.xBrakeSol[[2]], ViewPoint → {1, 1, -1},
    ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
   Graphics3D[robotGraphicBrakes //.xBrakeSol[[2]], ViewPoint → {1, 0, 0},
    ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
   Graphics3D[robotGraphicBrakes //.xBrakeSol[[2]], ViewPoint → {0, -1, 0},
    ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

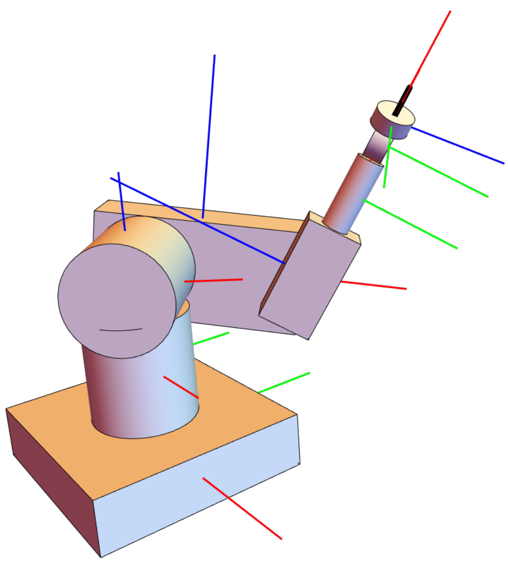
Out[579]=



In[580]:=



Out[580]=



y-Brake force at base

In[581]:= $F_{brakey} = F_{moty}$ /. brakeSol //. $\{q_{n\_}[t] \to Q_n\}$ // First

Out[581]= $(\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_5]\,\text{Sin}[Q_1] - \text{Cos}[Q_5]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3] - \text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_5] - \text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5] + \text{Cos}[Q_1]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5])\,F_{toolx} + (\text{Cos}[Q_1]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6] + \text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_4] + \text{Cos}[Q_2]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4] + \text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_6] + \text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3]\,\text{Sin}[Q_6] - \text{Cos}[Q_1]\,\text{Cos}[Q_5]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6] + \text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Sin}[Q_1]\,\text{Sin}[Q_5]\,\text{Sin}[Q_6] - \text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]\,\text{Sin}[Q_6])\,F_{tooly} + (\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2] + \text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3] - \text{Cos}[Q_1]\,\text{Cos}[Q_5]\,\text{Cos}[Q_6]\,\text{Sin}[Q_4] + \text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_5] - \text{Cos}[Q_6]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5] - \text{Cos}[Q_1]\,\text{Cos}[Q_4]\,\text{Sin}[Q_6] - \text{Cos}[Q_3]\,\text{Sin}[Q_1]\,\text{Sin}[Q_2]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6] - \text{Cos}[Q_2]\,\text{Sin}[Q_1]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6])\,F_{toolz}$

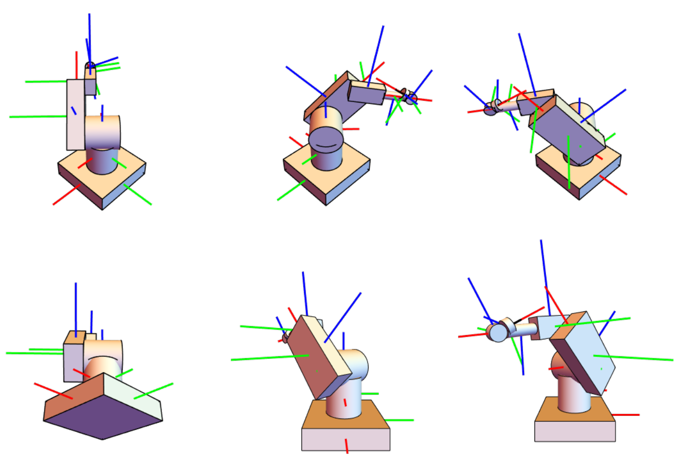
In[582]:= yBrakeSol =
Maximize$[\{F_{brakey}, -.9\,\text{Pi} \le Q_1 \le .9\,\text{Pi}, -(\text{Pi} + \text{Pi}/4) \le Q_2 \le \text{Pi}/4, -\text{Pi}/2 \le Q_3 \le \text{Pi}/2, -2\,\text{Pi} \le Q_4 \le 2\,\text{Pi}, -\text{Pi} \le Q_5 \le \text{Pi}, -2\,\text{Pi} \le Q_6 \le 2\,\text{Pi}, 0 \le F_{toolx} \le F_{toolmax}, 0 \le F_{tooly} \le F_{toolmax}, 0 \le F_{toolz} \le F_{toolmax}\}, \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, F_{toolx}, F_{tooly}, F_{toolz}\}]$

Out[582]= $\{173.205, \{Q_1 \to -2.33719, Q_2 \to -0.824831, Q_3 \to 0.979922, Q_4 \to -3.29165, Q_5 \to 2.7522, Q_6 \to -1.39567, F_{toolx} \to 100., F_{tooly} \to 100., F_{toolz} \to 100.\}\}$

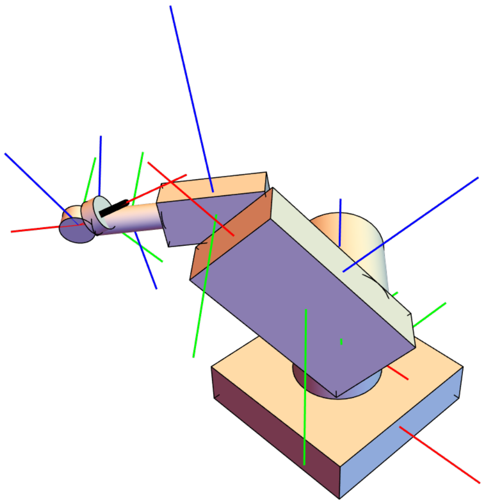
Here is a picture of this robot orientation for this maximum brake force.

In[583]:=
```
Show[GraphicsGrid[{{Graphics3D[robotGraphicBrakes //.yBrakeSol[[2]], ViewPoint → {1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.yBrakeSol[[2]], ViewPoint → {-1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.yBrakeSol[[2]], ViewPoint → {1, -1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
   {Graphics3D[robotGraphicBrakes //.yBrakeSol[[2]], ViewPoint → {1, 1, -1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.yBrakeSol[[2]], ViewPoint → {1, 0, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.yBrakeSol[[2]], ViewPoint → {0, -1, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

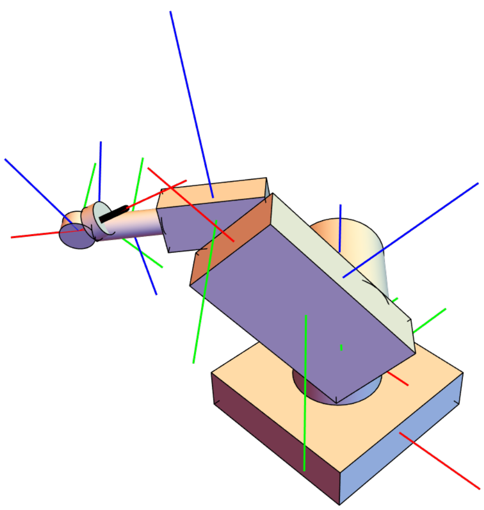
Out[583]=



In[584]:=



Out[584]=



motor1-Brake torque

In[585]:= T_{brake1} = T_{mot}[1] /. brakeSol //. {$q_{n\_}$[t] → Q_n} // First // Chop

Out[585]=
$$\frac{1}{42}\Big(-14\,\mathrm{Cos}[Q_1]^2\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]\,F_{toolx} -$$
$$14\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_1]^2\,F_{toolx} + 14\,\mathrm{Cos}[Q_1]^2\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,F_{toolx} +$$

$14\,\text{Cos}[Q_5]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,F_{\text{toolx}} + 14\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_2]\,\text{Sin}[Q_5]\,F_{\text{toolx}} +$
$14\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_5]\,F_{\text{toolx}} + 14\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]$
$\text{Sin}[Q_3]\,\text{Sin}[Q_5]\,F_{\text{toolx}} + 14\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]\,F_{\text{toolx}} +$
$63\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]\,F_{\text{toolx}} + 63\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]\,F_{\text{toolx}} +$
$63\,\text{Cos}[Q_2]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]\,F_{\text{toolx}} + 63\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]\,F_{\text{toolx}} -$
$63\,\text{Cos}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]\,F_{\text{toolx}} -$
$63\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_5]\,F_{\text{toolx}} +$
$63\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,F_{\text{tooly}} + 63\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,F_{\text{tooly}} +$
$10\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Cos}[Q_6]\,F_{\text{tooly}} +$
$63\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,F_{\text{tooly}} + 63\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,F_{\text{tooly}} +$
$10\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,F_{\text{tooly}} -$
$63\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,F_{\text{tooly}} -$
$63\,\text{Cos}[Q_4]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,F_{\text{tooly}} -$
$14\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_4]\,F_{\text{tooly}} -$
$14\,\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_4]\,F_{\text{tooly}} -$
$14\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_6]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,F_{\text{tooly}} -$
$14\,\text{Cos}[Q_2]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]^2\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_5]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]^2\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_5]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]^2\,\text{Cos}[Q_6]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]^2\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_4]^2\,\text{Sin}[Q_5]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_4]^2\,\text{Sin}[Q_5]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_6]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]^2\,\text{Sin}[Q_5]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_2]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]^2\,\text{Sin}[Q_5]\,F_{\text{tooly}} -$
$14\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Sin}[Q_2]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$
$14\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$
$14\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Sin}[Q_3]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$
$14\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_3]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$
$63\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_5]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$
$63\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_5]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_5]^2\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$
$63\,\text{Cos}[Q_2]\,\text{Cos}[Q_5]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$
$63\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_5]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$
$10\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_5]^2\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} +$
$63\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_5]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} +$
$10\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_5]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} +$
$63\,\text{Cos}[Q_5]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} +$
$10\,\text{Cos}[Q_5]^2\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,F_{\text{tooly}} -$

$14 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Sin}[Q_5] \, \text{Sin}[Q_6] \, F_{\text{tooly}} -$
$14 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_5] \, \text{Sin}[Q_6] \, F_{\text{tooly}} + 14 \, \text{Cos}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3]$
$\text{Sin}[Q_5] \, \text{Sin}[Q_6] \, F_{\text{tooly}} + 14 \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_5] \, \text{Sin}[Q_6] \, F_{\text{tooly}} -$
$10 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Sin}[Q_4] \, \text{Sin}[Q_5]^2 \, \text{Sin}[Q_6] \, F_{\text{tooly}} -$
$10 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_4] \, \text{Sin}[Q_5]^2 \, \text{Sin}[Q_6] \, F_{\text{tooly}} +$
$10 \, \text{Cos}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, \text{Sin}[Q_5]^2 \, \text{Sin}[Q_6] \, F_{\text{tooly}} +$
$10 \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, \text{Sin}[Q_5]^2 \, \text{Sin}[Q_6] \, F_{\text{tooly}} -$
$14 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_3] \, \text{Cos}[Q_4] \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_2] \, F_{\text{toolz}} -$
$14 \, \text{Cos}[Q_3] \, \text{Cos}[Q_4] \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, F_{\text{toolz}} -$
$14 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_4] \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_3] \, F_{\text{toolz}} -$
$14 \, \text{Cos}[Q_2] \, \text{Cos}[Q_4] \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_3] \, F_{\text{toolz}} -$
$63 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_4] \, F_{\text{toolz}} -$
$63 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_4] \, F_{\text{toolz}} -$
$10 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_5]^2 \, \text{Cos}[Q_6] \, \text{Sin}[Q_4] \, F_{\text{toolz}} -$
$63 \, \text{Cos}[Q_2] \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_4] \, F_{\text{toolz}} -$
$63 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_4] \, F_{\text{toolz}} -$
$10 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_5]^2 \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_4] \, F_{\text{toolz}} +$
$63 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_5]^2 \, \text{Cos}[Q_6] \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, F_{\text{toolz}} +$
$63 \, \text{Cos}[Q_5] \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_5]^2 \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, F_{\text{toolz}} -$
$14 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_6] \, \text{Sin}[Q_5] \, F_{\text{toolz}} -$
$14 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_5] \, F_{\text{toolz}} +$
$14 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_6] \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_5] \, F_{\text{toolz}} +$
$14 \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_5] \, F_{\text{toolz}} -$
$10 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_6] \, \text{Sin}[Q_4] \, \text{Sin}[Q_5]^2 \, F_{\text{toolz}} -$
$10 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_4] \, \text{Sin}[Q_5]^2 \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_6] \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, \text{Sin}[Q_5]^2 \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_6] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, \text{Sin}[Q_5]^2 \, F_{\text{toolz}} -$
$63 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_4] \, \text{Sin}[Q_6] \, F_{\text{toolz}} - 63 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_4] \, \text{Sin}[Q_6] \, F_{\text{toolz}} -$
$10 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_4] \, \text{Cos}[Q_5] \, \text{Sin}[Q_6] \, F_{\text{toolz}} -$
$63 \, \text{Cos}[Q_2] \, \text{Cos}[Q_4] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_6] \, F_{\text{toolz}} - 63 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_4] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_6] \, F_{\text{toolz}} -$
$10 \, \text{Cos}[Q_2] \, \text{Cos}[Q_3] \, \text{Cos}[Q_4] \, \text{Cos}[Q_5] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$63 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_4] \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_4] \, \text{Cos}[Q_5] \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$63 \, \text{Cos}[Q_4] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_4] \, \text{Cos}[Q_5] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$14 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_3] \, \text{Sin}[Q_2] \, \text{Sin}[Q_4] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$14 \, \text{Cos}[Q_3] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_4] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$14 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$14 \, \text{Cos}[Q_2] \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_3] \, \text{Sin}[Q_4] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_3] \, \text{Cos}[Q_4]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_5] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_3] \, \text{Cos}[Q_4]^2 \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_2] \, \text{Sin}[Q_5] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_1]^2 \, \text{Cos}[Q_2] \, \text{Cos}[Q_4]^2 \, \text{Sin}[Q_3] \, \text{Sin}[Q_5] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$
$10 \, \text{Cos}[Q_2] \, \text{Cos}[Q_4]^2 \, \text{Sin}[Q_1]^2 \, \text{Sin}[Q_3] \, \text{Sin}[Q_5] \, \text{Sin}[Q_6] \, F_{\text{toolz}} +$

$10\,\mathrm{Cos}[Q_1]^2\,\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_4]^2\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6]\,F_{toolz}+$
$10\,\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_1]^2\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_4]^2\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6]\,F_{toolz}+$
$10\,\mathrm{Cos}[Q_1]^2\,\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]^2\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6]\,F_{toolz}+$
$10\,\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_1]^2\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]^2\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6]\,F_{toolz}-42\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_2]\,T_{toolx}-$
$42\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_3]\,T_{toolx}-42\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_5]\,T_{toolx}+$
$42\,\mathrm{Cos}[Q_4]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]\,T_{toolx}+42\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_4]\,T_{tooly}-$
$42\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]\,T_{tooly}+42\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_6]\,T_{tooly}-$
$42\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_5]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_6]\,T_{tooly}-42\,\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6]\,T_{tooly}-$
$42\,\mathrm{Cos}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]\,\mathrm{Sin}[Q_6]\,T_{tooly}+42\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_5]\,\mathrm{Cos}[Q_6]\,T_{toolz}-$
$42\,\mathrm{Cos}[Q_4]\,\mathrm{Cos}[Q_5]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,T_{toolz}-$
$42\,\mathrm{Cos}[Q_3]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_5]\,T_{toolz}-42\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_6]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_5]\,T_{toolz}-$
$42\,\mathrm{Cos}[Q_2]\,\mathrm{Cos}[Q_3]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_6]\,T_{toolz}+42\,\mathrm{Sin}[Q_2]\,\mathrm{Sin}[Q_3]\,\mathrm{Sin}[Q_4]\,\mathrm{Sin}[Q_6]\,T_{toolz})$

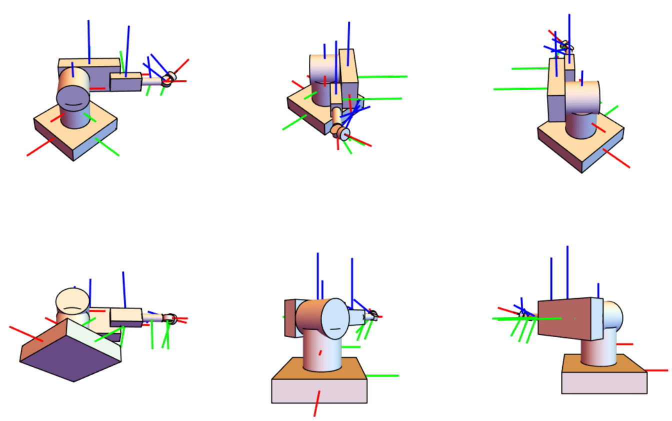
In[586]:= **mot1BrakeSol =**
Maximize[{T_{brake1}, - .9 Pi ≤ Q_1 ≤ .9 Pi, - (Pi + Pi / 4) ≤ Q_2 ≤ Pi / 4, - Pi / 2 ≤ Q_3 ≤ Pi / 2,
-2 Pi ≤ Q_4 ≤ 2 Pi, - Pi ≤ Q_5 ≤ Pi, -2 Pi ≤ Q_6 ≤ 2 Pi, 0 ≤ F_{toolx} ≤ $F_{toolmax}$,
0 ≤ F_{tooly} ≤ $F_{toolmax}$, 0 ≤ F_{toolz} ≤ $F_{toolmax}$, 0 ≤ T_{toolx} ≤ $T_{toolmax}$, 0 ≤ T_{tooly} ≤ $T_{toolmax}$,
0 ≤ T_{toolz} ≤ $T_{toolmax}$}, {Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, F_{toolx}, F_{tooly}, F_{toolz}, T_{toolx}, T_{tooly}, T_{toolz}}]

Out[586]= {597.59, {$Q_1 \to 2.38032$, $Q_2 \to 1.95885\times10^{-8}$, $Q_3 \to -7.89188\times10^{-7}$, $Q_4 \to 5.13566$, $Q_5 \to -0.771017$,
$Q_6 \to -5.89815$, $F_{toolx} \to 100.$, $F_{tooly} \to 100.$, $F_{toolz} \to 100.$, $T_{toolx} \to 50.$, $T_{tooly} \to 0.$, $T_{toolz} \to 50.$}}

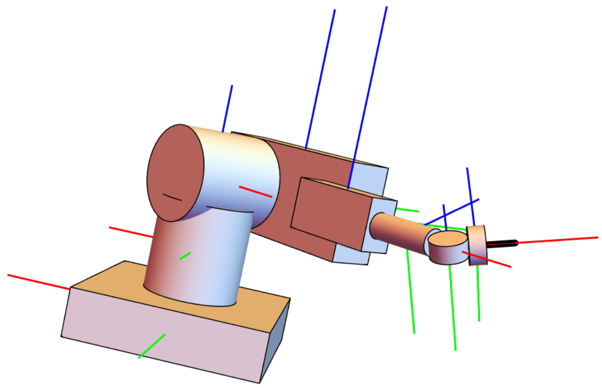
Here is a picture of this robot orientation for this maximum brake force.

In[587]:=
```
Show[
 GraphicsGrid[{{Graphics3D[robotGraphicBrakes //.mot1BrakeSol[[2]], ViewPoint → {1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot1BrakeSol[[2]], ViewPoint → {-1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot1BrakeSol[[2]], ViewPoint → {1, -1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
   {Graphics3D[robotGraphicBrakes //.mot1BrakeSol[[2]], ViewPoint → {1, 1, -1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot1BrakeSol[[2]], ViewPoint → {1, 0, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot1BrakeSol[[2]], ViewPoint → {0, -1, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

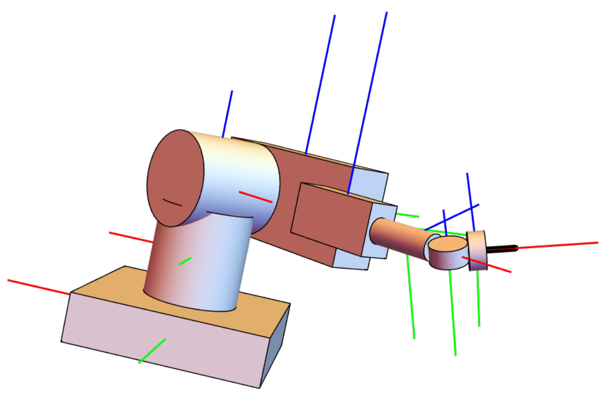
Out[587]=



In[588]:=



Out[588]=



motor2-Brake torque

In[589]:= $T_{brake2} = T_{mot}[2]$ /. brakeSol //. $\{q_{n\_}[t] \to Q_n\}$ // First // Chop

Out[589]=
$$\frac{1}{42}\Big(-7622.37\,\text{Cos}[Q_2] - 2403.45\,\text{Cos}[Q_2]\,\text{Cos}[Q_3] -$$
$$98.1\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_5] + 2403.45\,\text{Sin}[Q_2]\,\text{Sin}[Q_3] + 98.1\,\text{Cos}[Q_5]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3] +$$
$$98.1\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_2]\,\text{Sin}[Q_5] + 98.1\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5] +$$
$$63\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_5]\,\text{Sin}[Q_3]\,F_{toolx} + 63\,\text{Cos}[Q_5]\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_3]\,F_{toolx} +$$
$$63\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_5]\,F_{toolx} + 63\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_3]^2\,\text{Cos}[Q_4]\,\text{Sin}[Q_5]\,F_{toolx} +$$
$$63\,\text{Cos}[Q_3]\,\text{Cos}[Q_4]\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_5]\,F_{toolx} + 63\,\text{Cos}[Q_3]^2\,\text{Cos}[Q_4]\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_5]\,F_{toolx} +$$
$$63\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_4]\,\text{Sin}[Q_3]^2\,\text{Sin}[Q_5]\,F_{toolx} + 63\,\text{Cos}[Q_4]\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_3]^2\,\text{Sin}[Q_5]\,F_{toolx} -$$

$63 Cos[Q_2]^2 Cos[Q_3] Cos[Q_6] Sin[Q_4] F_{tooly} - 63 Cos[Q_2]^2 Cos[Q_3]^2 Cos[Q_6] Sin[Q_4] F_{tooly} -$
$10 Cos[Q_2]^2 Cos[Q_3]^2 Cos[Q_5] Cos[Q_6] Sin[Q_4] F_{tooly} -$
$63 Cos[Q_3] Cos[Q_6] Sin[Q_2]^2 Sin[Q_4] F_{tooly} - 63 Cos[Q_3]^2 Cos[Q_6] Sin[Q_2]^2 Sin[Q_4] F_{tooly} -$
$10 Cos[Q_3]^2 Cos[Q_5] Cos[Q_6] Sin[Q_2]^2 Sin[Q_4] F_{tooly} - 63 Cos[Q_2]^2 Cos[Q_6]$
$Sin[Q_3]^2 Sin[Q_4] F_{tooly} - 10 Cos[Q_2]^2 Cos[Q_5] Cos[Q_6] Sin[Q_3]^2 Sin[Q_4] F_{tooly} -$
$63 Cos[Q_6] Sin[Q_2]^2 Sin[Q_3]^2 Sin[Q_4] F_{tooly} - 10 Cos[Q_5] Cos[Q_6] Sin[Q_2]^2$
$Sin[Q_3]^2 Sin[Q_4] F_{tooly} - 63 Cos[Q_2]^2 Cos[Q_3] Cos[Q_4] Cos[Q_5] Sin[Q_6] F_{tooly} -$
$63 Cos[Q_2]^2 Cos[Q_3]^2 Cos[Q_4] Cos[Q_5] Sin[Q_6] F_{tooly} - 10 Cos[Q_2]^2 Cos[Q_3]^2 Cos[Q_4]$
$Cos[Q_5]^2 Sin[Q_6] F_{tooly} - 63 Cos[Q_3] Cos[Q_4] Cos[Q_5] Sin[Q_2]^2 Sin[Q_6] F_{tooly} -$
$63 Cos[Q_3]^2 Cos[Q_4] Cos[Q_5] Sin[Q_2]^2 Sin[Q_6] F_{tooly} - 10 Cos[Q_3]^2 Cos[Q_4] Cos[Q_5]^2$
$Sin[Q_2]^2 Sin[Q_6] F_{tooly} - 63 Cos[Q_2]^2 Cos[Q_4] Cos[Q_5] Sin[Q_3]^2 Sin[Q_6] F_{tooly} -$
$10 Cos[Q_2]^2 Cos[Q_4] Cos[Q_5]^2 Sin[Q_3]^2 Sin[Q_6] F_{tooly} - 63 Cos[Q_4] Cos[Q_5] Sin[Q_2]^2$
$Sin[Q_3]^2 Sin[Q_6] F_{tooly} - 10 Cos[Q_4] Cos[Q_5]^2 Sin[Q_2]^2 Sin[Q_3]^2 Sin[Q_6] F_{tooly} +$
$63 Cos[Q_2]^2 Sin[Q_3] Sin[Q_5] Sin[Q_6] F_{tooly} + 63 Sin[Q_2]^2 Sin[Q_3] Sin[Q_5] Sin[Q_6] F_{tooly} -$
$10 Cos[Q_2]^2 Cos[Q_3]^2 Cos[Q_4] Sin[Q_5]^2 Sin[Q_6] F_{tooly} -$
$10 Cos[Q_3]^2 Cos[Q_4] Sin[Q_2]^2 Sin[Q_5]^2 Sin[Q_6] F_{tooly} -$
$10 Cos[Q_2]^2 Cos[Q_4] Sin[Q_3]^2 Sin[Q_5]^2 Sin[Q_6] F_{tooly} -$
$10 Cos[Q_4] Sin[Q_2]^2 Sin[Q_3]^2 Sin[Q_5]^2 Sin[Q_6] F_{tooly} -$
$63 Cos[Q_2]^2 Cos[Q_3] Cos[Q_4] Cos[Q_5] Cos[Q_6] F_{toolz} -$
$63 Cos[Q_2]^2 Cos[Q_3]^2 Cos[Q_4] Cos[Q_5] Cos[Q_6] F_{toolz} -$
$10 Cos[Q_2]^2 Cos[Q_3]^2 Cos[Q_4] Cos[Q_5]^2 Cos[Q_6] F_{toolz} -$
$63 Cos[Q_3] Cos[Q_4] Cos[Q_5] Cos[Q_6] Sin[Q_2]^2 F_{toolz} -$
$63 Cos[Q_3]^2 Cos[Q_4] Cos[Q_5] Cos[Q_6] Sin[Q_2]^2 F_{toolz} -$
$10 Cos[Q_3]^2 Cos[Q_4] Cos[Q_5]^2 Cos[Q_6] Sin[Q_2]^2 F_{toolz} -$
$63 Cos[Q_2]^2 Cos[Q_4] Cos[Q_5] Cos[Q_6] Sin[Q_3]^2 F_{toolz} -$
$10 Cos[Q_2]^2 Cos[Q_4] Cos[Q_5]^2 Cos[Q_6] Sin[Q_3]^2 F_{toolz} -$
$63 Cos[Q_4] Cos[Q_5] Cos[Q_6] Sin[Q_2]^2 Sin[Q_3]^2 F_{toolz} -$
$10 Cos[Q_4] Cos[Q_5]^2 Cos[Q_6] Sin[Q_2]^2 Sin[Q_3]^2 F_{toolz} +$
$63 Cos[Q_2]^2 Cos[Q_6] Sin[Q_3] Sin[Q_5] F_{toolz} + 63 Cos[Q_6] Sin[Q_2]^2 Sin[Q_3] Sin[Q_5] F_{toolz} -$
$10 Cos[Q_2]^2 Cos[Q_3]^2 Cos[Q_4] Cos[Q_6] Sin[Q_5]^2 F_{toolz} -$
$10 Cos[Q_3]^2 Cos[Q_4] Cos[Q_6] Sin[Q_2]^2 Sin[Q_5]^2 F_{toolz} -$
$10 Cos[Q_2]^2 Cos[Q_4] Cos[Q_6] Sin[Q_3]^2 Sin[Q_5]^2 F_{toolz} -$
$10 Cos[Q_4] Cos[Q_6] Sin[Q_2]^2 Sin[Q_3]^2 Sin[Q_5]^2 F_{toolz} +$
$63 Cos[Q_2]^2 Cos[Q_3] Sin[Q_4] Sin[Q_6] F_{toolz} + 63 Cos[Q_2]^2 Cos[Q_3]^2 Sin[Q_4] Sin[Q_6] F_{toolz} +$
$10 Cos[Q_2]^2 Cos[Q_3]^2 Cos[Q_5] Sin[Q_4] Sin[Q_6] F_{toolz} +$
$63 Cos[Q_3] Sin[Q_2]^2 Sin[Q_4] Sin[Q_6] F_{toolz} + 63 Cos[Q_3]^2 Sin[Q_2]^2 Sin[Q_4] Sin[Q_6] F_{toolz} +$
$10 Cos[Q_3]^2 Cos[Q_5] Sin[Q_2]^2 Sin[Q_4] Sin[Q_6] F_{toolz} +$
$63 Cos[Q_2]^2 Sin[Q_3]^2 Sin[Q_4] Sin[Q_6] F_{toolz} + 10 Cos[Q_2]^2 Cos[Q_5] Sin[Q_3]^2$
$Sin[Q_4] Sin[Q_6] F_{toolz} + 63 Sin[Q_2]^2 Sin[Q_3]^2 Sin[Q_4] Sin[Q_6] F_{toolz} +$
$10 Cos[Q_5] Sin[Q_2]^2 Sin[Q_3]^2 Sin[Q_4] Sin[Q_6] F_{toolz} + 42 Sin[Q_4] Sin[Q_5] T_{toolx} +$
$42 Cos[Q_4] Cos[Q_6] T_{tooly} - 42 Cos[Q_5] Sin[Q_4] Sin[Q_6] T_{tooly} -$
$42 Cos[Q_5] Cos[Q_6] Sin[Q_4] T_{toolz} - 42 Cos[Q_4] Sin[Q_6] T_{toolz})$

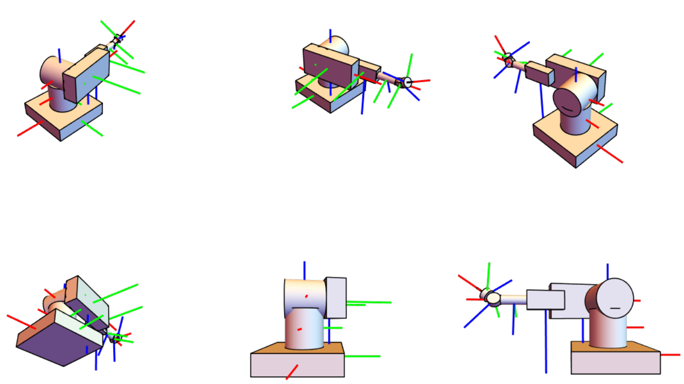
In[590]:=
```
mot2BrakeSol =
 Maximize[{T_brake2, - .9 Pi ≤ Q1 ≤  .9 Pi, - (Pi + Pi / 4) ≤ Q2 ≤  Pi / 4, - Pi / 2 ≤ Q3 ≤  Pi / 2,
   -2 Pi ≤ Q4 ≤ 2 Pi, - Pi ≤ Q5 ≤ Pi, -2 Pi ≤ Q6 ≤ 2 Pi, 0 ≤ F_toolx ≤ F_toolmax,
   0 ≤ F_tooly ≤ F_toolmax, 0 ≤ F_toolz ≤ F_toolmax, 0 ≤ T_toolx ≤ T_toolmax, 0 ≤ T_tooly ≤ T_toolmax,
   0 ≤ T_toolz ≤ T_toolmax}, {Q1, Q2, Q3, Q4, Q5, Q6, F_toolx, F_tooly, F_toolz, T_toolx, T_tooly, T_toolz}]
```

Out[590]= {834.954, {Q1 → 0.208042, Q2 → -3.14648, Q3 → -0.00192161, Q4 → -5.81231, Q5 → 0.667294,
 Q6 → -2.8196, F_toolx → 100., F_tooly → 100., F_toolz → 100., T_toolx → 50., T_tooly → 0., T_toolz → 50.}}

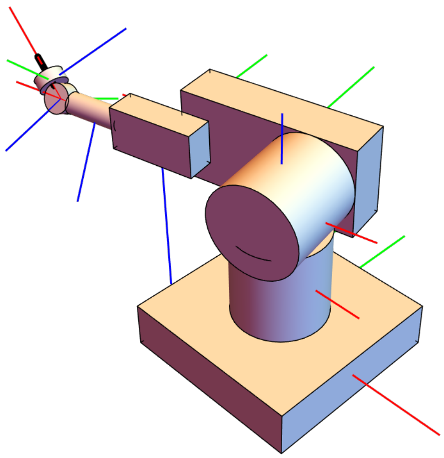
Here is a picture of this robot orientation for this maximum brake force.

In[591]:=
```
Show[
 GraphicsGrid[{{Graphics3D[robotGraphicBrakes //.mot2BrakeSol[[2]], ViewPoint → {1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot2BrakeSol[[2]], ViewPoint → {-1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot2BrakeSol[[2]], ViewPoint → {1, -1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
   {Graphics3D[robotGraphicBrakes //.mot2BrakeSol[[2]], ViewPoint → {1, 1, -1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot2BrakeSol[[2]], ViewPoint → {1, 0, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot2BrakeSol[[2]], ViewPoint → {0, -1, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

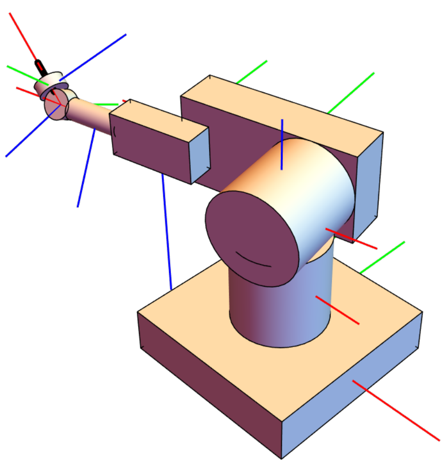
Out[591]=



In[592]:=



Out[592]=



motor3-Brake torque

In[593]:= $T_{brake3} = T_{mot}[3]$ /. brakeSol //. $\{q_{n\_}[t] \to Q_n\}$ // First // Chop

Out[593]=

$$\begin{aligned}
&\frac{1}{42}\Big(-2403.45 \cos[Q_2] \cos[Q_3] - 98.1 \cos[Q_2] \cos[Q_3] \cos[Q_5] + 2403.45 \sin[Q_2] \sin[Q_3] + \\
&98.1 \cos[Q_5] \sin[Q_2] \sin[Q_3] + 98.1 \cos[Q_3] \cos[Q_4] \sin[Q_2] \sin[Q_5] + \\
&98.1 \cos[Q_2] \cos[Q_4] \sin[Q_3] \sin[Q_5] + 63 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_4] \sin[Q_5] F_{toolx} + \\
&63 \cos[Q_3]^2 \cos[Q_4] \sin[Q_2]^2 \sin[Q_5] F_{toolx} + 63 \cos[Q_2]^2 \cos[Q_4] \sin[Q_3]^2 \sin[Q_5] F_{toolx} + \\
&63 \cos[Q_4] \sin[Q_2]^2 \sin[Q_3]^2 \sin[Q_5] F_{toolx} - 63 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_6] \sin[Q_4] F_{tooly} - \\
&10 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_5] \cos[Q_6] \sin[Q_4] F_{tooly} - \\
&63 \cos[Q_3]^2 \cos[Q_6] \sin[Q_2]^2 \sin[Q_4] F_{tooly} - \\
&10 \cos[Q_3]^2 \cos[Q_5] \cos[Q_6] \sin[Q_2]^2 \sin[Q_4] F_{tooly} - 63 \cos[Q_2]^2 \cos[Q_6] \\
&\quad \sin[Q_3]^2 \sin[Q_4] F_{tooly} - 10 \cos[Q_2]^2 \cos[Q_5] \cos[Q_6] \sin[Q_3]^2 \sin[Q_4] F_{tooly} - \\
&63 \cos[Q_6] \sin[Q_2]^2 \sin[Q_3]^2 \sin[Q_4] F_{tooly} - 10 \cos[Q_5] \cos[Q_6] \sin[Q_2]^2 \\
&\quad \sin[Q_3]^2 \sin[Q_4] F_{tooly} - 63 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_4] \cos[Q_5] \sin[Q_6] F_{tooly} - \\
&10 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_4] \cos[Q_5]^2 \sin[Q_6] F_{tooly} - 63 \cos[Q_3]^2 \cos[Q_4] \cos[Q_5] \\
&\quad \sin[Q_2]^2 \sin[Q_6] F_{tooly} - 10 \cos[Q_3]^2 \cos[Q_4] \cos[Q_5]^2 \sin[Q_2]^2 \sin[Q_6] F_{tooly} - \\
&63 \cos[Q_2]^2 \cos[Q_4] \cos[Q_5] \sin[Q_3]^2 \sin[Q_6] F_{tooly} - 10 \cos[Q_2]^2 \cos[Q_4] \cos[Q_5]^2 \\
&\quad \sin[Q_3]^2 \sin[Q_6] F_{tooly} - 63 \cos[Q_4] \cos[Q_5] \sin[Q_2]^2 \sin[Q_3]^2 \sin[Q_6] F_{tooly} - \\
&10 \cos[Q_4] \cos[Q_5]^2 \sin[Q_2]^2 \sin[Q_3]^2 \sin[Q_6] F_{tooly} - 10 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_4] \\
&\quad \sin[Q_5]^2 \sin[Q_6] F_{tooly} - 10 \cos[Q_3]^2 \cos[Q_4] \sin[Q_2]^2 \sin[Q_5]^2 \sin[Q_6] F_{tooly} - \\
&10 \cos[Q_2]^2 \cos[Q_4] \sin[Q_3]^2 \sin[Q_5]^2 \sin[Q_6] F_{tooly} - 10 \cos[Q_4] \sin[Q_2]^2 \sin[Q_3]^2 \\
&\quad \sin[Q_5]^2 \sin[Q_6] F_{tooly} - 63 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_4] \cos[Q_5] \cos[Q_6] F_{toolz} - \\
&10 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_4] \cos[Q_5]^2 \cos[Q_6] F_{toolz} - 63 \cos[Q_3]^2 \cos[Q_4] \cos[Q_5] \\
&\quad \cos[Q_6] \sin[Q_2]^2 F_{toolz} - 10 \cos[Q_3]^2 \cos[Q_4] \cos[Q_5]^2 \cos[Q_6] \sin[Q_2]^2 F_{toolz} - \\
&63 \cos[Q_2]^2 \cos[Q_4] \cos[Q_5] \cos[Q_6] \sin[Q_3]^2 F_{toolz} - 10 \cos[Q_2]^2 \cos[Q_4] \cos[Q_5]^2 \\
&\quad \cos[Q_6] \sin[Q_3]^2 F_{toolz} - 63 \cos[Q_4] \cos[Q_5] \cos[Q_6] \sin[Q_2]^2 \sin[Q_3]^2 F_{toolz} - \\
&10 \cos[Q_4] \cos[Q_5]^2 \cos[Q_6] \sin[Q_2]^2 \sin[Q_3]^2 F_{toolz} - 10 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_4] \\
&\quad \cos[Q_6] \sin[Q_5]^2 F_{toolz} - 10 \cos[Q_3]^2 \cos[Q_4] \cos[Q_6] \sin[Q_2]^2 \sin[Q_5]^2 F_{toolz} - \\
&10 \cos[Q_2]^2 \cos[Q_4] \cos[Q_6] \sin[Q_3]^2 \sin[Q_5]^2 F_{toolz} - 10 \cos[Q_4] \cos[Q_6] \sin[Q_2]^2 \\
&\quad \sin[Q_3]^2 \sin[Q_5]^2 F_{toolz} + 63 \cos[Q_2]^2 \cos[Q_3]^2 \sin[Q_4] \sin[Q_6] F_{toolz} + \\
&10 \cos[Q_2]^2 \cos[Q_3]^2 \cos[Q_5] \sin[Q_4] \sin[Q_6] F_{toolz} + 63 \cos[Q_3]^2 \sin[Q_2]^2 \\
&\quad \sin[Q_4] \sin[Q_6] F_{toolz} + 10 \cos[Q_3]^2 \cos[Q_5] \sin[Q_2]^2 \sin[Q_4] \sin[Q_6] F_{toolz} + \\
&63 \cos[Q_2]^2 \sin[Q_3]^2 \sin[Q_4] \sin[Q_6] F_{toolz} + 10 \cos[Q_2]^2 \cos[Q_5] \sin[Q_3]^2 \\
&\quad \sin[Q_4] \sin[Q_6] F_{toolz} + 63 \sin[Q_2]^2 \sin[Q_3]^2 \sin[Q_4] \sin[Q_6] F_{toolz} + \\
&10 \cos[Q_5] \sin[Q_2]^2 \sin[Q_3]^2 \sin[Q_4] \sin[Q_6] F_{toolz} + 42 \sin[Q_4] \sin[Q_5] T_{toolx} + \\
&42 \cos[Q_4] \cos[Q_6] T_{tooly} - 42 \cos[Q_5] \sin[Q_4] \sin[Q_6] T_{tooly} - \\
&42 \cos[Q_5] \cos[Q_6] \sin[Q_4] T_{toolz} - 42 \cos[Q_4] \sin[Q_6] T_{toolz}\Big)
\end{aligned}$$

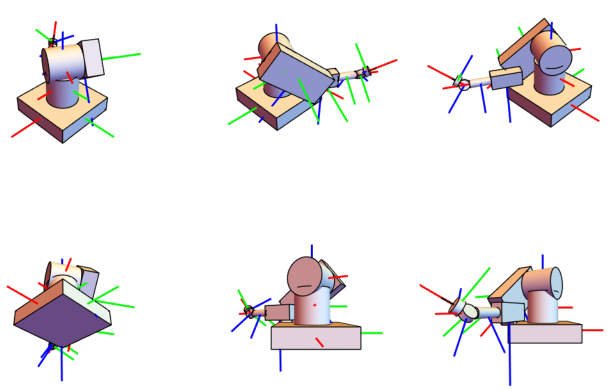
In[594]:=
```
mot3BrakeSol =
 Maximize[{Tbrake3, - .9 Pi ≤ Q1 ≤ .9 Pi, - (Pi + Pi / 4) ≤ Q2 ≤ Pi / 4, - Pi / 2 ≤ Q3 ≤ Pi / 2,
   -2 Pi ≤ Q4 ≤ 2 Pi, - Pi ≤ Q5 ≤ Pi, -2 Pi ≤ Q6 ≤ 2 Pi, 0 ≤ Ftoolx ≤ Ftoolmax,
   0 ≤ Ftooly ≤ Ftoolmax, 0 ≤ Ftoolz ≤ Ftoolmax, 0 ≤ Ttoolx ≤ Ttoolmax, 0 ≤ Ttooly ≤ Ttoolmax,
   0 ≤ Ttoolz ≤ Ttoolmax}, {Q1, Q2, Q3, Q4, Q5, Q6, Ftoolx, Ftooly, Ftoolz, Ttoolx, Ttooly, Ttoolz}]
```

Out[594]= $\{395.847, \{Q_1 \to 1.04618, Q_2 \to -3.79726, Q_3 \to 0.634849, Q_4 \to -5.73533, Q_5 \to 0.664603, Q_6 \to -2.82217, F_{toolx} \to 100., F_{tooly} \to 100., F_{toolz} \to 100., T_{toolx} \to 50., T_{tooly} \to 0., T_{toolz} \to 50.\}\}$

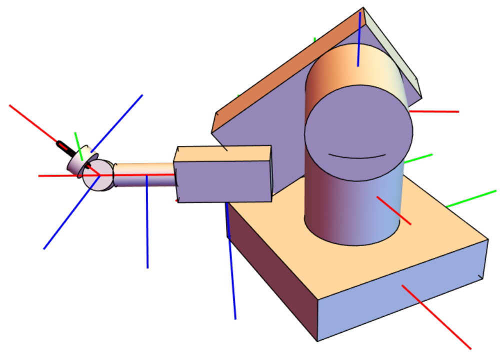
Here is a picture of this robot orientation for this maximum brake force.

In[595]:=
```
Show[
 GraphicsGrid[{{Graphics3D[robotGraphicBrakes //.mot3BrakeSol[[2]], ViewPoint → {1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot3BrakeSol[[2]], ViewPoint → {-1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot3BrakeSol[[2]], ViewPoint → {1, -1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
   {Graphics3D[robotGraphicBrakes //.mot3BrakeSol[[2]], ViewPoint → {1, 1, -1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot3BrakeSol[[2]], ViewPoint → {1, 0, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot3BrakeSol[[2]], ViewPoint → {0, -1, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

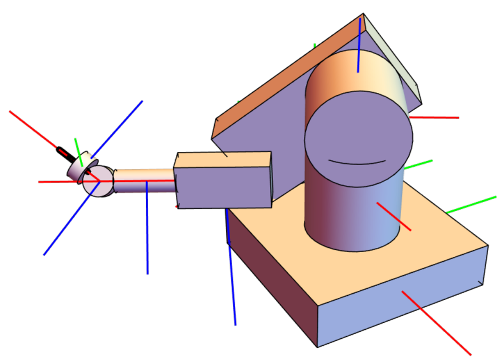
Out[595]=



In[596]:=



Out[596]=



motor4-Brake torque

In[597]:= $T_{brake4} = T_{mot}[4]$ /. brakeSol //. $\{q_{n\_}[t] \to Q_n\}$ // First // Chop

Out[597]= $(49.05\,Cos[Q_1]^2\,Cos[Q_2]\,Cos[Q_3]\,Sin[Q_4]\,Sin[Q_5] +$
$49.05\,Cos[Q_2]\,Cos[Q_3]\,Sin[Q_1]^2\,Sin[Q_4]\,Sin[Q_5] - 49.05\,Cos[Q_1]^2\,Sin[Q_2]$
$Sin[Q_3]\,Sin[Q_4]\,Sin[Q_5] - 49.05\,Sin[Q_1]^2\,Sin[Q_2]\,Sin[Q_3]\,Sin[Q_4]\,Sin[Q_5] +$
$5\,Cos[Q_1]^2\,Cos[Q_2]^2\,Cos[Q_3]^2\,Cos[Q_4]^2\,Cos[Q_6]\,Sin[Q_5]\,F_{tooly} +$
$5\,Cos[Q_2]^2\,Cos[Q_3]^2\,Cos[Q_4]^2\,Cos[Q_6]\,Sin[Q_1]^2\,Sin[Q_5]\,F_{tooly} +$
$5\,Cos[Q_1]^2\,Cos[Q_3]^2\,Cos[Q_4]^2\,Cos[Q_6]\,Sin[Q_2]^2\,Sin[Q_5]\,F_{tooly} +$
$5\,Cos[Q_3]^2\,Cos[Q_4]^2\,Cos[Q_6]\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_5]\,F_{tooly} +$

$5\,Cos[Q_1]^2\,Cos[Q_2]^2\,Cos[Q_4]^2\,Cos[Q_6]\,Sin[Q_3]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_2]^2\,Cos[Q_4]^2\,Cos[Q_6]\,Sin[Q_1]^2\,Sin[Q_3]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_1]^2\,Cos[Q_4]^2\,Cos[Q_6]\,Sin[Q_2]^2\,Sin[Q_3]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_4]^2\,Cos[Q_6]\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_3]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_1]^2\,Cos[Q_2]^2\,Cos[Q_3]^2\,Cos[Q_6]\,Sin[Q_4]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_2]^2\,Cos[Q_3]^2\,Cos[Q_6]\,Sin[Q_1]^2\,Sin[Q_4]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_1]^2\,Cos[Q_3]^2\,Cos[Q_6]\,Sin[Q_2]^2\,Sin[Q_4]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_3]^2\,Cos[Q_6]\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_4]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_1]^2\,Cos[Q_2]^2\,Cos[Q_6]\,Sin[Q_3]^2\,Sin[Q_4]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_2]^2\,Cos[Q_6]\,Sin[Q_1]^2\,Sin[Q_3]^2\,Sin[Q_4]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_1]^2\,Cos[Q_6]\,Sin[Q_2]^2\,Sin[Q_3]^2\,Sin[Q_4]^2\,Sin[Q_5]\,F_{tooly}\,+$
$5\,Cos[Q_6]\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_3]^2\,Sin[Q_4]^2\,Sin[Q_5]\,F_{tooly}\,-$
$5\,Cos[Q_1]^2\,Cos[Q_2]^2\,Cos[Q_3]^2\,Cos[Q_4]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_2]^2\,Cos[Q_3]^2\,Cos[Q_4]^2\,Sin[Q_1]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_1]^2\,Cos[Q_3]^2\,Cos[Q_4]^2\,Sin[Q_2]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_3]^2\,Cos[Q_4]^2\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_1]^2\,Cos[Q_2]^2\,Cos[Q_4]^2\,Sin[Q_3]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_2]^2\,Cos[Q_4]^2\,Sin[Q_1]^2\,Sin[Q_3]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_1]^2\,Cos[Q_4]^2\,Sin[Q_2]^2\,Sin[Q_3]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_4]^2\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_3]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_1]^2\,Cos[Q_2]^2\,Cos[Q_3]^2\,Sin[Q_4]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_2]^2\,Cos[Q_3]^2\,Sin[Q_1]^2\,Sin[Q_4]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_1]^2\,Cos[Q_3]^2\,Sin[Q_2]^2\,Sin[Q_4]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_3]^2\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_4]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_1]^2\,Cos[Q_2]^2\,Sin[Q_3]^2\,Sin[Q_4]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_2]^2\,Sin[Q_1]^2\,Sin[Q_3]^2\,Sin[Q_4]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Cos[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_3]^2\,Sin[Q_4]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,-$
$5\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_3]^2\,Sin[Q_4]^2\,Sin[Q_5]\,Sin[Q_6]\,F_{toolz}\,+$
$21\,Cos[Q_1]^2\,Cos[Q_2]^2\,Cos[Q_3]^2\,Cos[Q_5]\,T_{toolx} + 21\,Cos[Q_2]^2\,Cos[Q_3]^2\,Cos[Q_5]\,Sin[Q_1]^2\,T_{toolx}\,+$
$21\,Cos[Q_3]^2\,Cos[Q_5]\,Sin[Q_2]^2\,T_{toolx} + 42\,Cos[Q_2]\,Cos[Q_3]\,Cos[Q_5]\,Sin[Q_2]\,Sin[Q_3]\,T_{toolx}\,-$
$42\,Cos[Q_1]^2\,Cos[Q_2]\,Cos[Q_3]\,Cos[Q_5]\,Sin[Q_2]\,Sin[Q_3]\,T_{toolx}\,-$
$42\,Cos[Q_2]\,Cos[Q_3]\,Cos[Q_5]\,Sin[Q_1]^2\,Sin[Q_2]\,Sin[Q_3]\,T_{toolx} + 21\,Cos[Q_2]^2\,Cos[Q_5]\,Sin[Q_3]^2\,T_{toolx}\,+$
$21\,Cos[Q_1]^2\,Cos[Q_5]\,Sin[Q_2]^2\,Sin[Q_3]^2\,T_{toolx} + 21\,Cos[Q_5]\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_3]^2\,T_{toolx}\,+$
$21\,Cos[Q_2]\,Cos[Q_3]^2\,Cos[Q_4]\,Sin[Q_2]\,Sin[Q_5]\,T_{toolx}\,-$
$21\,Cos[Q_1]^2\,Cos[Q_2]\,Cos[Q_3]^2\,Cos[Q_4]\,Sin[Q_2]\,Sin[Q_5]\,T_{toolx}\,-$
$21\,Cos[Q_2]\,Cos[Q_3]^2\,Cos[Q_4]\,Sin[Q_1]^2\,Sin[Q_2]\,Sin[Q_5]\,T_{toolx}\,+$
$21\,Cos[Q_2]^2\,Cos[Q_3]\,Cos[Q_4]\,Sin[Q_3]\,Sin[Q_5]\,T_{toolx}\,-$
$21\,Cos[Q_1]^2\,Cos[Q_2]^2\,Cos[Q_3]\,Cos[Q_4]\,Sin[Q_3]\,Sin[Q_5]\,T_{toolx}\,-$
$21\,Cos[Q_2]^2\,Cos[Q_3]\,Cos[Q_4]\,Sin[Q_1]^2\,Sin[Q_3]\,Sin[Q_5]\,T_{toolx}\,-$
$21\,Cos[Q_3]\,Cos[Q_4]\,Sin[Q_2]^2\,Sin[Q_3]\,Sin[Q_5]\,T_{toolx}\,+$
$21\,Cos[Q_1]^2\,Cos[Q_3]\,Cos[Q_4]\,Sin[Q_2]^2\,Sin[Q_3]\,Sin[Q_5]\,T_{toolx}\,+$
$21\,Cos[Q_3]\,Cos[Q_4]\,Sin[Q_1]^2\,Sin[Q_2]^2\,Sin[Q_3]\,Sin[Q_5]\,T_{toolx}\,-$
$21\,Cos[Q_2]\,Cos[Q_4]\,Sin[Q_2]\,Sin[Q_3]^2\,Sin[Q_5]\,T_{toolx}\,+$
$21\,Cos[Q_1]^2\,Cos[Q_2]\,Cos[Q_4]\,Sin[Q_2]\,Sin[Q_3]^2\,Sin[Q_5]\,T_{toolx}\,+$

$21 \text{Cos}[Q_2] \text{Cos}[Q_4] \text{Sin}[Q_1]^2 \text{Sin}[Q_2] \text{Sin}[Q_3]^2 \text{Sin}[Q_5] T_{toolx} -$
$21 \text{Cos}[Q_2] \text{Cos}[Q_3]^2 \text{Cos}[Q_6] \text{Sin}[Q_2] \text{Sin}[Q_4] T_{tooly} +$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_2] \text{Cos}[Q_3]^2 \text{Cos}[Q_6] \text{Sin}[Q_2] \text{Sin}[Q_4] T_{tooly} +$
$21 \text{Cos}[Q_2] \text{Cos}[Q_3]^2 \text{Cos}[Q_6] \text{Sin}[Q_1]^2 \text{Sin}[Q_2] \text{Sin}[Q_4] T_{tooly} -$
$21 \text{Cos}[Q_2]^2 \text{Cos}[Q_3] \text{Cos}[Q_6] \text{Sin}[Q_3] \text{Sin}[Q_4] T_{tooly} +$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_2]^2 \text{Cos}[Q_3] \text{Cos}[Q_6] \text{Sin}[Q_3] \text{Sin}[Q_4] T_{tooly} +$
$21 \text{Cos}[Q_2]^2 \text{Cos}[Q_3] \text{Cos}[Q_6] \text{Sin}[Q_1]^2 \text{Sin}[Q_3] \text{Sin}[Q_4] T_{tooly} +$
$21 \text{Cos}[Q_3] \text{Cos}[Q_6] \text{Sin}[Q_2]^2 \text{Sin}[Q_3] \text{Sin}[Q_4] T_{tooly} -$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_3] \text{Cos}[Q_6] \text{Sin}[Q_2]^2 \text{Sin}[Q_3] \text{Sin}[Q_4] T_{tooly} -$
$21 \text{Cos}[Q_3] \text{Cos}[Q_6] \text{Sin}[Q_1]^2 \text{Sin}[Q_2]^2 \text{Sin}[Q_3] \text{Sin}[Q_4] T_{tooly} +$
$21 \text{Cos}[Q_2] \text{Cos}[Q_6] \text{Sin}[Q_2] \text{Sin}[Q_3]^2 \text{Sin}[Q_4] T_{tooly} -$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_2] \text{Cos}[Q_6] \text{Sin}[Q_2] \text{Sin}[Q_3]^2 \text{Sin}[Q_4] T_{tooly} -$
$21 \text{Cos}[Q_2] \text{Cos}[Q_6] \text{Sin}[Q_1]^2 \text{Sin}[Q_2] \text{Sin}[Q_3]^2 \text{Sin}[Q_4] T_{tooly} -$
$21 \text{Cos}[Q_2] \text{Cos}[Q_3]^2 \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_2] \text{Sin}[Q_6] T_{tooly} +$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_2] \text{Cos}[Q_3]^2 \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_2] \text{Sin}[Q_6] T_{tooly} +$
$21 \text{Cos}[Q_2] \text{Cos}[Q_3]^2 \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_1]^2 \text{Sin}[Q_2] \text{Sin}[Q_6] T_{tooly} -$
$21 \text{Cos}[Q_2]^2 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_3] \text{Sin}[Q_6] T_{tooly} +$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_2]^2 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_3] \text{Sin}[Q_6] T_{tooly} +$
$21 \text{Cos}[Q_2]^2 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_1]^2 \text{Sin}[Q_3] \text{Sin}[Q_6] T_{tooly} +$
$21 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_2]^2 \text{Sin}[Q_3] \text{Sin}[Q_6] T_{tooly} -$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_2]^2 \text{Sin}[Q_3] \text{Sin}[Q_6] T_{tooly} -$
$21 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_1]^2 \text{Sin}[Q_2]^2 \text{Sin}[Q_3] \text{Sin}[Q_6] T_{tooly} +$
$21 \text{Cos}[Q_2] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_2] \text{Sin}[Q_3]^2 \text{Sin}[Q_6] T_{tooly} -$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_2] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_2] \text{Sin}[Q_3]^2 \text{Sin}[Q_6] T_{tooly} -$
$21 \text{Cos}[Q_2] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Sin}[Q_1]^2 \text{Sin}[Q_2] \text{Sin}[Q_3]^2 \text{Sin}[Q_6] T_{tooly} +$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_2]^2 \text{Cos}[Q_3]^2 \text{Sin}[Q_5] \text{Sin}[Q_6] T_{tooly} +$
$21 \text{Cos}[Q_2]^2 \text{Cos}[Q_3]^2 \text{Sin}[Q_1]^2 \text{Sin}[Q_5] \text{Sin}[Q_6] T_{tooly} + 21 \text{Cos}[Q_3]^2 \text{Sin}[Q_2]^2 \text{Sin}[Q_5]$
$\text{Sin}[Q_6] T_{tooly} + 42 \text{Cos}[Q_2] \text{Cos}[Q_3] \text{Sin}[Q_2] \text{Sin}[Q_3] \text{Sin}[Q_5] \text{Sin}[Q_6] T_{tooly} -$
$42 \text{Cos}[Q_1]^2 \text{Cos}[Q_2] \text{Cos}[Q_3] \text{Sin}[Q_2] \text{Sin}[Q_3] \text{Sin}[Q_5] \text{Sin}[Q_6] T_{tooly} -$
$42 \text{Cos}[Q_2] \text{Cos}[Q_3] \text{Sin}[Q_1]^2 \text{Sin}[Q_2] \text{Sin}[Q_3] \text{Sin}[Q_5] \text{Sin}[Q_6] T_{tooly} +$
$21 \text{Cos}[Q_2]^2 \text{Sin}[Q_3]^2 \text{Sin}[Q_5] \text{Sin}[Q_6] T_{tooly} + 21 \text{Cos}[Q_1]^2 \text{Sin}[Q_2]^2 \text{Sin}[Q_3]^2$
$\text{Sin}[Q_5] \text{Sin}[Q_6] T_{tooly} + 21 \text{Sin}[Q_1]^2 \text{Sin}[Q_2]^2 \text{Sin}[Q_3]^2 \text{Sin}[Q_5] \text{Sin}[Q_6] T_{tooly} -$
$21 \text{Cos}[Q_2] \text{Cos}[Q_3]^2 \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_2] T_{toolz} +$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_2] \text{Cos}[Q_3]^2 \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_2] T_{toolz} +$
$21 \text{Cos}[Q_2] \text{Cos}[Q_3]^2 \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_1]^2 \text{Sin}[Q_2] T_{toolz} -$
$21 \text{Cos}[Q_2]^2 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_3] T_{toolz} +$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_2]^2 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_3] T_{toolz} +$
$21 \text{Cos}[Q_2]^2 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_1]^2 \text{Sin}[Q_3] T_{toolz} +$
$21 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_2]^2 \text{Sin}[Q_3] T_{toolz} -$
$21 \text{Cos}[Q_1]^2 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_2]^2 \text{Sin}[Q_3] T_{toolz} -$
$21 \text{Cos}[Q_3] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_1]^2 \text{Sin}[Q_2]^2 \text{Sin}[Q_3] T_{toolz} +$
$21 \text{Cos}[Q_2] \text{Cos}[Q_4] \text{Cos}[Q_5] \text{Cos}[Q_6] \text{Sin}[Q_2] \text{Sin}[Q_3]^2 T_{toolz} -$

$21\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]^2\,T_{toolz} -$
$21\,\text{Cos}[Q_2]\,\text{Cos}[Q_4]\,\text{Cos}[Q_5]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]^2\,T_{toolz} +$
$21\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_3]^2\,\text{Cos}[Q_6]\,\text{Sin}[Q_5]\,T_{toolz} +$
$21\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_3]^2\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_5]\,T_{toolz} + 21\,\text{Cos}[Q_3]^2\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]^2$
$\text{Sin}[Q_5]\,T_{toolz} + 42\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]\,T_{toolz} -$
$42\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]\,T_{toolz} -$
$42\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]\,\text{Sin}[Q_5]\,T_{toolz} +$
$21\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_6]\,\text{Sin}[Q_3]^2\,\text{Sin}[Q_5]\,T_{toolz} +$
$21\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_6]\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_3]^2\,\text{Sin}[Q_5]\,T_{toolz} +$
$21\,\text{Cos}[Q_6]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_3]^2\,\text{Sin}[Q_5]\,T_{toolz} + 21\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]^2\,\text{Sin}[Q_2]$
$\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} - 21\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} -$
$21\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]^2\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} + 21\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_3]\,\text{Sin}[Q_3]$
$\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} - 21\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_3]\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} -$
$21\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_3]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} - 21\,\text{Cos}[Q_3]\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_3]$
$\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} + 21\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_3]\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} +$
$21\,\text{Cos}[Q_3]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_3]\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} - 21\,\text{Cos}[Q_2]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]^2$
$\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} + 21\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]^2\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz} +$
$21\,\text{Cos}[Q_2]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3]^2\,\text{Sin}[Q_4]\,\text{Sin}[Q_6]\,T_{toolz}\Big) \Big/$
$\Big(21\,\Big(\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]^2\,\text{Cos}[Q_3]^2 + \text{Cos}[Q_2]^2\,\text{Cos}[Q_3]^2\,\text{Sin}[Q_1]^2 + \text{Cos}[Q_3]^2\,\text{Sin}[Q_2]^2 +$
$2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3] - 2\,\text{Cos}[Q_1]^2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Sin}[Q_2]\,\text{Sin}[Q_3] -$
$2\,\text{Cos}[Q_2]\,\text{Cos}[Q_3]\,\text{Sin}[Q_1]^2\,\text{Sin}[Q_2]\,\text{Sin}[Q_3] + \text{Cos}[Q_2]^2\,\text{Sin}[Q_3]^2 +$
$\text{Cos}[Q_1]^2\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_3]^2 + \text{Sin}[Q_1]^2\,\text{Sin}[Q_2]^2\,\text{Sin}[Q_3]^2\Big)\Big)$

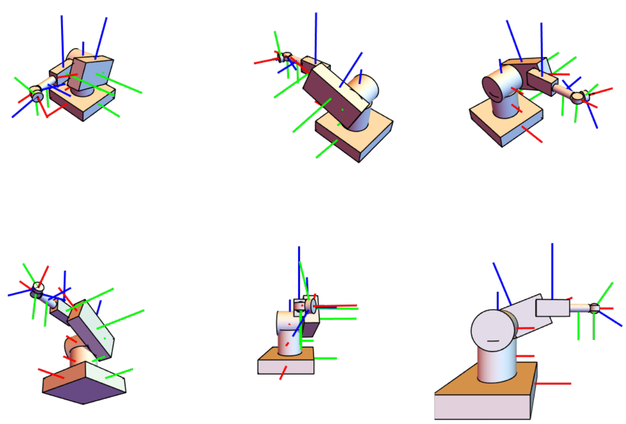
In[598]:= **mot4BrakeSol =**
$\textbf{Maximize}\big[\{T_{brake4},\ -.9\,Pi \le Q_1 \le .9\,Pi,\ -(Pi + Pi/4) \le Q_2 \le Pi/4,\ -Pi/2 \le Q_3 \le Pi/2,$
$-2\,Pi \le Q_4 \le 2\,Pi,\ -Pi \le Q_5 \le Pi,\ -2\,Pi \le Q_6 \le 2\,Pi,\ 0 \le F_{toolx} \le F_{toolmax},$
$0 \le F_{tooly} \le F_{toolmax},\ 0 \le F_{toolz} \le F_{toolmax},\ 0 \le T_{toolx} \le T_{toolmax},\ 0 \le T_{tooly} \le T_{toolmax},$
$0 \le T_{toolz} \le T_{toolmax}\},\ \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, F_{toolx}, F_{tooly}, F_{toolz}, T_{toolx}, T_{tooly}, T_{toolz}\}\big]$

Out[598]= $\{104.258,$
$\{Q_1 \to 0.116383,\ Q_2 \to -0.390279,\ Q_3 \to 0.390279,\ Q_4 \to -1.5708,\ Q_5 \to -1.07062,\ Q_6 \to 3.737,$
$F_{toolx} \to 54.5962,\ F_{tooly} \to 100.,\ F_{toolz} \to 5.93691 \times 10^{-9},\ T_{toolx} \to 50.,\ T_{tooly} \to 50.,\ T_{toolz} \to 50.\}\}$

Here is a picture of this robot orientation for this maximum brake force.

In[599]:=
```
Show[
 GraphicsGrid[{{Graphics3D[robotGraphicBrakes //.mot4BrakeSol[[2]], ViewPoint → {1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot4BrakeSol[[2]], ViewPoint → {-1, 1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot4BrakeSol[[2]], ViewPoint → {1, -1, 1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]},
   {Graphics3D[robotGraphicBrakes //.mot4BrakeSol[[2]], ViewPoint → {1, 1, -1},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot4BrakeSol[[2]], ViewPoint → {1, 0, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All],
    Graphics3D[robotGraphicBrakes //.mot4BrakeSol[[2]], ViewPoint → {0, -1, 0},
     ViewVertical → {0, 0, 1}, ViewCenter → {1/2, 1/2, 1/2}, Boxed → False, PlotRange → All]}}]]
```

Out[599]=



motor5-Brake torque

motor6-Brake torque

Dynamic simulation

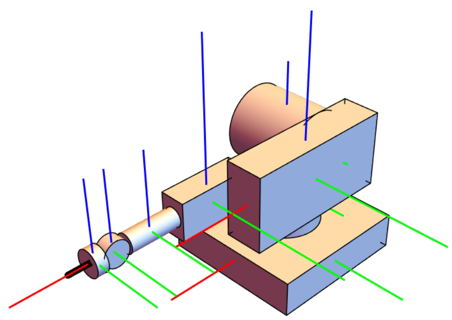
First we will need to input or calculate the mass inertia dyad for each of the bodies of the robot. For my robot I will assume the bodies are hollow and made of aluminum. Here is the density of Al.

In[607]:= ρ_{Al} = 2700 (*Kg/m^3*)

Out[607]= 2700

In[608]:=
```
Show[Graphics3D[robotGraphic, ViewPoint -> {1, 1, 1}, ViewVertical -> {0, 0, 1},
  ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> False, PlotRange -> All]]
```

Out[608]=



Base mass and mass inertia dyad

The base is a hollow cuboid. Using the previous information of the shape we have the following principal inertia terms. Note that if you do not have symmetry in your robots parts you will need the full inertia dyad here and above.

This is the percentage of void desired per unit length.

In[609]:=
```
voidScale = .98;
```

Here are the solid and void volumes.

In[610]:= V_{solid} = widthBase depthBase heightBase
V_{void} = $voidScale^3$ widthBase depthBase heightBase

Out[610]= 2

Out[611]= 1.88238

Here is the mass of the solid and void.

In[612]:= $M_{solid} = \rho_{Al} V_{solid}$
$M_{void} = \rho_{Al} V_{void}$

Out[612]= 5400

Out[613]= 5082.44

Here is the computed mass.

In[614]:=
```
M_1 = M_solid - M_void
```

Out[614]= 317.563

Here are the principal inertias.

In[615]:=
$$Ai_{11} = \frac{1}{12} M_{solid} \left(depthBase^2 + heightBase^2\right) - \frac{1}{12} M_{void} \left((voidScale\ depthBase)^2 + (voidScale\ heightBase)^2\right)$$
$$Ai_{22} = \frac{1}{12} M_{solid} \left(widthBase^2 + heightBase^2\right) - \frac{1}{12} M_{void} \left((voidScale\ widthBase)^2 + (voidScale\ heightBase)^2\right)$$
$$Ai_{33} = \frac{1}{12} M_{solid} \left(depthBase^2 + widthBase^2\right) - \frac{1}{12} M_{void} \left((voidScale\ depthBase)^2 + (voidScale\ widthBase)^2\right)$$

Out[615]= 183.751

Out[616]= 183.751

Out[617]= 345.885

Riser mass and mass inertia dyad

The riser is a hollow cylinder. Using the previous information of the shape we have the following principal inertia terms. Note that if you do not have symmetry in your robots parts you will need the full inertia dyad here and above.

This is the percentage of void desired per unit length.

In[618]:=
```
voidScale = .98;
```

Here are the solid and void volumes.

In[619]:=
$$V_{solid} = \pi\ riserRadius^2\ (2\ halfHeightRiser)$$
$$V_{void} = voidScale^3 \left(\pi\ riserRadius^2\ (2\ halfHeightRiser)\right)$$

Out[619]= $\frac{\pi}{4}$

Out[620]= 0.73921

Here is the mass of the solid and void.

In[621]:= $M_{solid} = \rho_{Al} V_{solid}$
$M_{void} = \rho_{Al} V_{void}$

Out[621]= $675\,\pi$

Out[622]= 1995.87

Here is the computed mass.

In[623]:= $M_2 = M_{solid} - M_{void}$

Out[623]= 124.707

Here are the principal inertias.

In[624]:=
$$Bi_{11} = \frac{1}{4} M_{solid}\, riserRadius^2 + \frac{1}{12} M_{solid}\, (2\, halfHeightRiser)^2 - \left(\frac{1}{4} M_{void}\, (voidScale\, riserRadius)^2 + \frac{1}{12} M_{void}\, (voidScale\, (2\, halfHeightRiser))^2\right)$$
$$Bi_{22} = Bi_{11}$$
$$Bi_{33} = \frac{1}{2} M_{solid}\, riserRadius^2 - \frac{1}{2} M_{void}\, (voidScale\, riserRadius)^2$$

Out[624]= 29.7125

Out[625]= 29.7125

Out[626]= 25.4679

Shoulder mass and mass inertia dyad

The riser is a hollow cylinder. Using the previous information of the shape we have the following principal inertia terms. Note that if you do not have symmetry in your robots parts you will need the full inertia dyad here and above. I will assume the endcap for the shoulder is very thin and ignore it here.

This is the percentage of void desired per unit length.

In[627]:= voidScale = .98;

Here are the solid and void volumes.

In[628]:= $V_{solid} = \pi\, shoulderRadius^2\, (2\, halfHeightShoulder)$
$V_{void} = voidScale^3 \left(\pi\, shoulderRadius^2\, (2\, halfHeightShoulder)\right)$

Out[628]= $\frac{\pi}{4}$

Out[629]= 0.73921

Here is the mass of the solid and void.

In[630]:= $M_{solid} = \rho_{Al} V_{solid}$
$M_{void} = \rho_{Al} V_{void}$

Out[630]= $675\,\pi$

Out[631]= 1995.87

Here is the computed mass.

In[632]:= $M_3 = M_{solid} - M_{void}$

Out[632]= 124.707

Here are the principal inertias.

In[633]:=
$$Ci_{11} = \frac{1}{4} M_{solid}\, shoulderRadius^2 + \frac{1}{12} M_{solid}\, (2\, halfHeightShoulder)^2 - \left(\frac{1}{4} M_{void}\, (voidScale\, shoulderRadius)^2 + \frac{1}{12} M_{void}\, (voidScale\, (2\, halfHeightShoulder))^2\right)$$
$$Ci_{22} = \frac{1}{2} M_{solid}\, shoulderRadius^2 - \frac{1}{2} M_{void}\, (voidScale\, shoulderRadius)^2$$
$$Ci_{33} = Ci_{11}$$

Out[633]= 29.7125

Out[634]= 25.4679

Out[635]= 29.7125

Arm1 mass and mass inertia dyad

The base is a hollow cuboid. Using the previous information of the shape we have the following principal inertia terms. Note that if you do not have symmetry in your robots parts you will need the full inertia dyad here and above.

This is the percentage of void desired per unit length.

In[636]:= `voidScale = .98;`

Here are the solid and void volumes.

In[637]:= $V_{solid} = lengthArm1\, depthArm1\, heightArm1$
$V_{void} = voidScale^3\, lengthArm1\, depthArm1\, heightArm1$

Out[637]= 1

Out[638]= 0.941192

Here is the mass of the solid and void.

In[639]:= $M_{solid} = \rho_{Al} V_{solid}$
$M_{void} = \rho_{Al} V_{void}$

Out[639]= 2700

Out[640]= 2541.22

Here is the computed mass.

In[641]:= $M_4 = M_{solid} - M_{void}$

Out[641]= 158.782

Here are the principal inertias.

In[642]:= $Di_{11} = \frac{1}{12} M_{solid} \left(depthArm1^2 + heightArm1^2\right) - \frac{1}{12} M_{void} \left((voidScale\ depthArm1)^2 + (voidScale\ heightArm1)^2\right)$
$Di_{22} = \frac{1}{12} M_{solid} \left(lengthArm1^2 + heightArm1^2\right) - \frac{1}{12} M_{void} \left((voidScale\ lengthArm1)^2 + (voidScale\ heightArm1)^2\right)$
$Di_{33} = \frac{1}{12} M_{solid} \left(depthArm1^2 + lengthArm1^2\right) - \frac{1}{12} M_{void} \left((voidScale\ depthArm1)^2 + (voidScale\ lengthArm1)^2\right)$

Out[642]= 27.0223

Out[643]= 108.089

Out[644]= 91.8757

Arm2 mass and mass inertia dyad

The base is a hollow cuboid. Using the previous information of the shape we have the following principal inertia terms. Note that if you do not have symmetry in your robots parts you will need the full inertia dyad here and above.

This is the percentage of void desired per unit length.

In[645]:= voidScale = .98;

Here are the solid and void volumes.

In[646]:= $V_{solid} = lengthArm2\ depthArm2\ heightArm2$
$V_{void} = voidScale^3\ lengthArm2\ depthArm2\ heightArm2$

Out[646]= $\frac{1}{6}$

Out[647]= 0.156865

Here is the mass of the solid and void.

In[648]:= $M_{solid} = \rho_{Al} V_{solid}$
$M_{void} = \rho_{Al} V_{void}$

Out[648]= 450

Out[649]= 423.536

Here is the computed mass.

In[650]:= $M_5 = M_{solid} - M_{void}$

Out[650]= 26.4636

Here are the principal inertias.

In[651]:=
$$Ei_{11} = \frac{1}{12} M_{solid} \left(depthArm2^2 + heightArm2^2\right) - \frac{1}{12} M_{void} \left((voidScale\ depthArm2)^2 + (voidScale\ heightArm2)^2\right)$$
$$Ei_{22} = \frac{1}{12} M_{solid} \left(lengthArm2^2 + heightArm2^2\right) - \frac{1}{12} M_{void} \left((voidScale\ lengthArm2)^2 + (voidScale\ heightArm2)^2\right)$$
$$Ei_{33} = \frac{1}{12} M_{solid} \left(depthArm2^2 + lengthArm2^2\right) - \frac{1}{12} M_{void} \left((voidScale\ depthArm2)^2 + (voidScale\ lengthArm2)^2\right)$$

Out[651]= 1.30107

Out[652]= 4.50371

Out[653]= 4.0033

Arm3 mass and mass inertia dyad

The riser is a hollow cylinder. Using the previous information of the shape we have the following principal inertia terms. Note that if you do not have symmetry in your robots parts you will need the full inertia dyad here and above. I will assume the endcap for the shoulder is very thin and ignore it here.

This is the percentage of void desired per unit length.

In[654]:= **voidScale = .98;**

Here are the solid and void volumes.

In[655]:= $V_{solid} = \pi\, arm3Radius^2\ (2\ halfHeightArm3)$
$V_{void} = voidScale^3\ (\pi\, arm3Radius^2\ (2\ halfHeightArm3))$

Out[655]= $\frac{\pi}{96}$

Out[656]= 0.0308004

Here is the mass of the solid and void.

In[657]:= $M_{solid} = \rho_{Al}\ V_{solid}$
$M_{void} = \rho_{Al}\ V_{void}$

Out[657]= $\frac{225\,\pi}{8}$

Out[658]= 83.1612

Here is the computed mass.

In[659]:= $M_6 = M_{solid} - M_{void}$

Out[659]= 5.19612

Here are the principal inertias.

In[660]:= $Fi_{11} = \frac{1}{2} M_{solid}\ arm3Radius^2 - \frac{1}{2} M_{void}\ (voidScale\ arm3Radius)^2$

$Fi_{22} = \frac{1}{4} M_{solid}\ arm3Radius^2 + \frac{1}{12} M_{solid}\ (2\ halfHeightArm3)^2 - \left(\frac{1}{4} M_{void}\ (voidScale\ arm3Radius)^2 + \frac{1}{12} M_{void}\ (voidScale\ (2\ halfHeightArm3))^2\right)$

$Fi_{33} = Fi_{22}$

Out[660]= 0.0663226

Out[661]= 0.34758

Out[662]= 0.34758

Wrist1 mass and mass inertia dyad

The riser is a hollow cylinder. Using the previous information of the shape we have the following principal inertia terms. Note that if you do not have symmetry in your robots parts you will need the full inertia dyad here and above. I will assume the endcap for the shoulder is very thin and ignore it here.

This is the percentage of void desired per unit length.

In[663]:= **voidScale = .98;**

Here are the solid and void volumes.

In[664]:= $V_{solid} = \pi\, \text{wrist1Radius}^2\, (2\, \text{halfHeightWrist1})$
$V_{void} = \text{voidScale}^3\, (\pi\, \text{wrist1Radius}^2\, (2\, \text{halfHeightWrist1}))$

Out[664]= $\frac{\pi}{216}$

Out[665]= 0.0136891

Here is the mass of the solid and void.

In[666]:= $M_{solid} = \rho_{Al}\, V_{solid}$
$M_{void} = \rho_{Al}\, V_{void}$

Out[666]= $\frac{25\,\pi}{2}$

Out[667]= 36.9605

Here is the computed mass.

In[668]:= $M_7 = M_{solid} - M_{void}$

Out[668]= 2.30938

Here are the principal inertias.

In[669]:= $Gi_{11} = \frac{1}{4} M_{solid}\, \text{wrist1Radius}^2 + \frac{1}{12} M_{solid}\, (2\, \text{halfHeightWrist1})^2 - \left(\frac{1}{4} M_{void}\, (\text{voidScale}\, \text{wrist1Radius})^2 + \frac{1}{12} M_{void}\, (\text{voidScale}\, (2\, \text{halfHeightWrist1}))^2\right)$
$Gi_{22} = \frac{1}{2} M_{solid}\, \text{wrist1Radius}^2 - \frac{1}{2} M_{void}\, (\text{voidScale}\, \text{wrist1Radius})^2$
$Gi_{33} = Gi_{11}$

Out[669]= 0.0349354

Out[670]= 0.0524031

Out[671]= 0.0349354

Wrist2 and tool mass and mass inertia dyad

The riser is a hollow cylinder. Using the previous information of the shape we have the following principal inertia terms. Note that if you do not have symmetry in your robots parts you will need the full inertia dyad here and above. I will assume the endcap for the shoulder is very thin and ignore it here.

This is the percentage of void desired per unit length.

In[672]:= `voidScale = .98;`

Here are the solid and void volumes.

In[673]:= $V_{solid} = \pi\, \text{wrist2Radius}^2\ (2\ \text{halfHeightWrist2})$

$V_{void} = \text{voidScale}^3\ (\pi\, \text{wrist2Radius}^2\ (2\ \text{halfHeightWrist2}))$

Out[673]= $\frac{\pi}{252}$

Out[674]= 0.0117335

Here is the mass of the solid and void.

In[675]:= $M_{solid} = \rho_{Al}\ V_{solid}$

$M_{void} = \rho_{Al}\ V_{void}$

Out[675]= $\frac{75\,\pi}{7}$

Out[676]= 31.6804

Here is the computed mass.

In[677]:= $M_8 = M_{solid} - M_{void}$

Out[677]= 1.97947

Here are the principal inertias.

In[678]:= $Hi_{11} = \frac{1}{2} M_{solid}\, \text{wrist2Radius}^2 - \frac{1}{2} M_{void}\ (\text{voidScale wrist2Radius})^2$

$Hi_{22} = \frac{1}{4} M_{solid}\, \text{wrist2Radius}^2 + \frac{1}{12} M_{solid}\ (2\ \text{halfHeightWrist2})^2 - \left(\frac{1}{4} M_{void}\ (\text{voidScale wrist2Radius})^2 + \frac{1}{12} M_{void}\ (\text{voidScale}\ (2\ \text{halfHeightWrist2}))^2\right)$

$Hi_{33} = Hi_{22}$

Out[678]= 0.0449169

Out[679]= 0.0279585

Out[680]= 0.0279585

Verification of the simulation

Now we see if the simulation makes sense by running some simple tests. We will turn off the brakes and motors and see if the system will rotate down to the gravity neutral position.

In[681]:=
```
F_motx = 0;
F_moty = 0;
T_mot[1] = 0;
T_mot[2] = 0;
T_mot[3] = 0;
T_mot[4] = 0;
T_mot[5] = 0;
T_mot[6] = 0;
F_toolx = 0;
F_tooly = 0;
F_toolz = 0;
T_toolx = 0;
T_tooly = 0;
T_toolz = 0;
```

Clear the variables after each simulation and animation test.

In[695]:=
```
x[t_] =.
y[t_] =.
q1[t_] =.
q2[t_] =.
q3[t_] =.
q4[t_] =.
q5[t_] =.
q6[t_] =.
tf =.
```

Unset: Assignment on x for x[t_] not found.

Out[695]= $Failed

Unset: Assignment on y for y[t_] not found.

Out[696]= $Failed

Unset: Assignment on Subscript for q_1[t_] not found.

Out[697]= $Failed

Unset: Assignment on Subscript for q_2[t_] not found.

Out[698]= $Failed

Unset: Assignment on Subscript for q_3[t_] not found.

Out[699]= $Failed

Unset: Assignment on Subscript for q_4[t_] not found.

Out[700]= $Failed

Unset: Assignment on Subscript for q_5[t_] not found.

Out[701]= $Failed

Unset: Assignment on Subscript for q_6[t_] not found.

Out[702]= $Failed

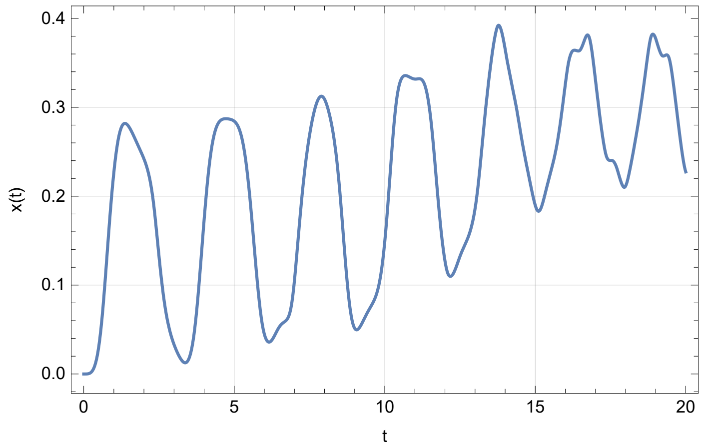
Simulate the response with the indicated initial conditions.

```
In[704]:= tf = 20;
solTest =
 Monitor[NDSolve[{eqT[1] == 0, eqT[2] == 0, eqT[3] == 0, eqT[4] == 0, eqT[5] == 0, eqT[6] == 0,
     eqT[7] == 0, eqT[8] == 0, x'[0] == 0, x[0] == 0, y'[0] == 0, y[0] == 0,
     q1'[0] == 0, q1[0] == 0, q2'[0] == 0, q2[0] == 0, q3'[0] == 0, q3[0] == 0,
     q4'[0] == 0, q4[0] == 0, q5'[0] == 0, q5[0] == 0, q6'[0] == 0, q6[0] == 0},
    {x, y, q1, q2, q3, q4, q5, q6}, {t, 0, tf}, StepMonitor :→ (time = t;),
    MaxSteps → 50000, Method -> {"EquationSimplification" -> "Residual"}],
   ProgressIndicator[time, {0, tf}]] // First
```

```
Out[705]= {x → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ],
 y → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ],
 q1 → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ],
 q2 → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ],
 q3 → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ],
 q4 → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ],
 q5 → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ],
 q6 → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ]}
```

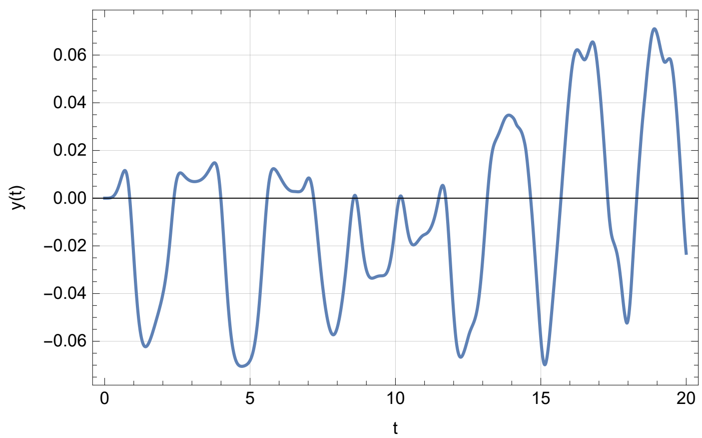
In[706]:=
```
Plot[x[t] /. solTest, {t, 0, tf}, PlotRange → All,
 FrameLabel → {"t", "x(t)"}, Frame → True, GridLines → Automatic]
```

Out[706]=



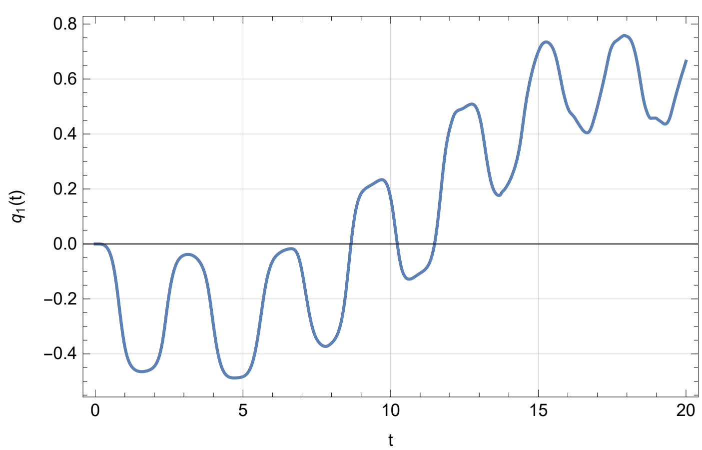
In[707]:=
```
Plot[y[t] /. solTest, {t, 0, tf}, PlotRange → All,
 FrameLabel → {"t", "y(t)"}, Frame → True, GridLines → Automatic]
```

Out[707]=

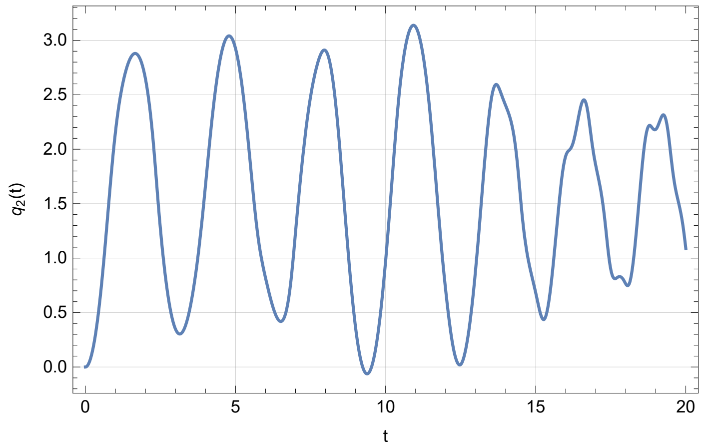


In[708]:=
```
Plot[q1[t] /. solTest, {t, 0, tf}, PlotRange → All,
 FrameLabel → {"t", "q1(t)"}, Frame → True, GridLines → Automatic]
```

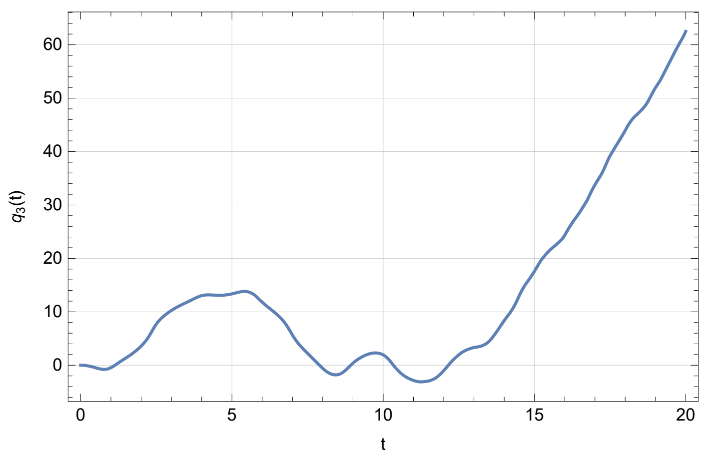
Out[708]=



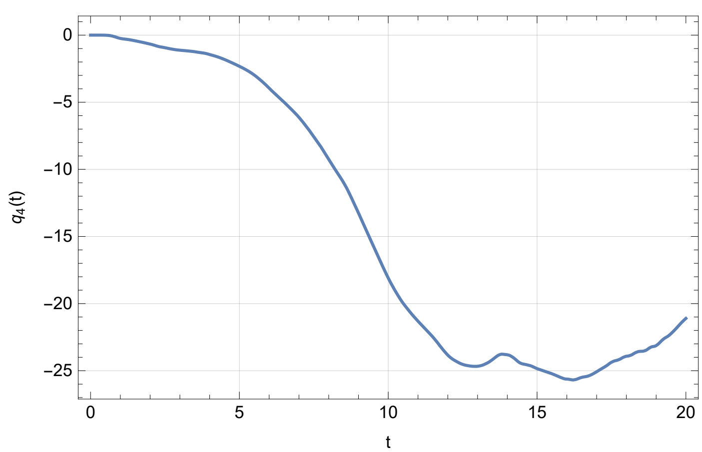
In[709]:=
```
Plot[q2[t] /. solTest, {t, 0, tf}, PlotRange → All,
  FrameLabel → {"t", "q2(t)"}, Frame → True, GridLines → Automatic]
```

Out[709]=


In[710]:=
```
Plot[q3[t] /. solTest, {t, 0, tf}, PlotRange → All,
  FrameLabel → {"t", "q3(t)"}, Frame → True, GridLines → Automatic]
```

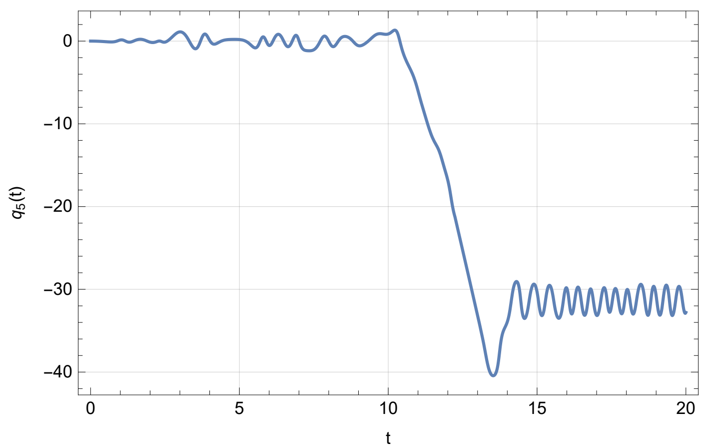
Out[710]=


In[711]:=
```
Plot[q4[t] /. solTest, {t, 0, tf}, PlotRange → All,
  FrameLabel → {"t", "q4(t)"}, Frame → True, GridLines → Automatic]
```

Out[711]=


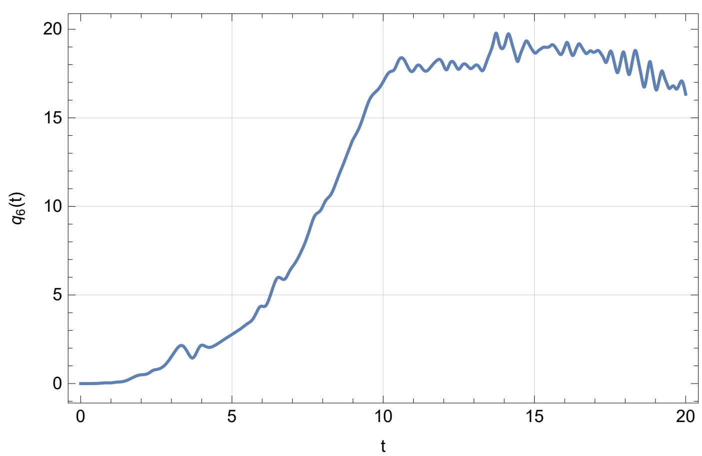
In[712]:=
```
Plot[q5[t] /. solTest, {t, 0, tf}, PlotRange → All,
  FrameLabel → {"t", "q5(t)"}, Frame → True, GridLines → Automatic]
```

Out[712]=



In[713]:=
```
Plot[q6[t] /. solTest, {t, 0, tf}, PlotRange → All,
  FrameLabel → {"t", "q6(t)"}, Frame → True, GridLines → Automatic]
```

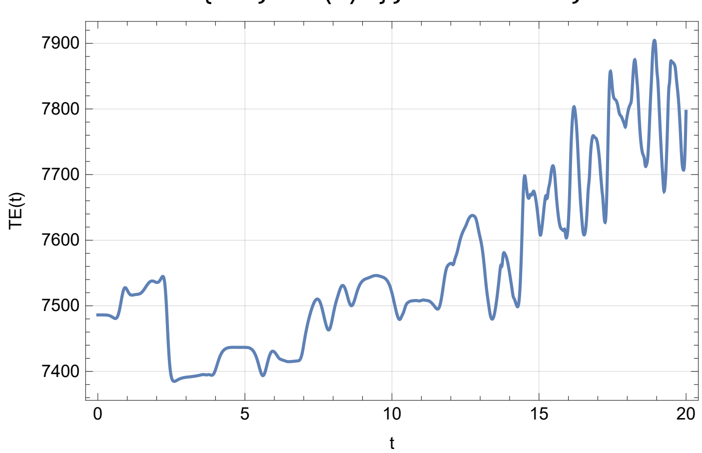
Out[713]=



This plot may take some time to complete.

In[714]:=
```
Plot[TEt /. solTest, {t, 0, tf}, PlotRange → All,
  FrameLabel → {"t", "TE(t)"}, Frame → True, GridLines → Automatic]
```

Out[714]=



In[715]:=
```
x[t_] = x[t] /. solTest;
y[t_] = y[t] /. solTest;
q1[t_] = q1[t] /. solTest;
q2[t_] = q2[t] /. solTest;
q3[t_] = q3[t] /. solTest;
q4[t_] = q4[t] /. solTest;
q5[t_] = q5[t] /. solTest;
q6[t_] = q6[t] /. solTest;
```

Create composite graphic out of parts that have been rotated and translated

In[723]:=
```
robotGraphicDynAnim = {
   (*Base graphic*)
   Translate[GeometricTransformation[baseGraphic, Transpose[rotA]], {xAo, yAo, zAo}],

   (*Riser graphic*)
   Translate[GeometricTransformation[riserGraphic, Transpose[rotB]], {xBo, yBo, zBo}],

   (*Shoulder graphic*)
   Translate[
    GeometricTransformation[shoulderGraphic, Transpose[rotC]], {xCo, yCo, zCo}],

   (*Arm1 graphic*)
   Translate[GeometricTransformation[arm1Graphic, Transpose[rotD]], {xDo, yDo, zDo}],

   (*Arm2 graphic*)
   Translate[GeometricTransformation[arm2Graphic, Transpose[rotE]], {xEo, yEo, zEo}],

   (*Arm3 graphic*)
   Translate[GeometricTransformation[arm3Graphic, Transpose[rotF]], {xFo, yFo, zFo}],

   (*Wrist1 graphic*)
   Translate[GeometricTransformation[wrist1Graphic, Transpose[rotG]], {xGo, yGo, zGo}],

   (*Wrist2 graphic*)
   Translate[GeometricTransformation[wrist2Graphic, Transpose[rotH]], {xHo, yHo, zHo}]
  };
```

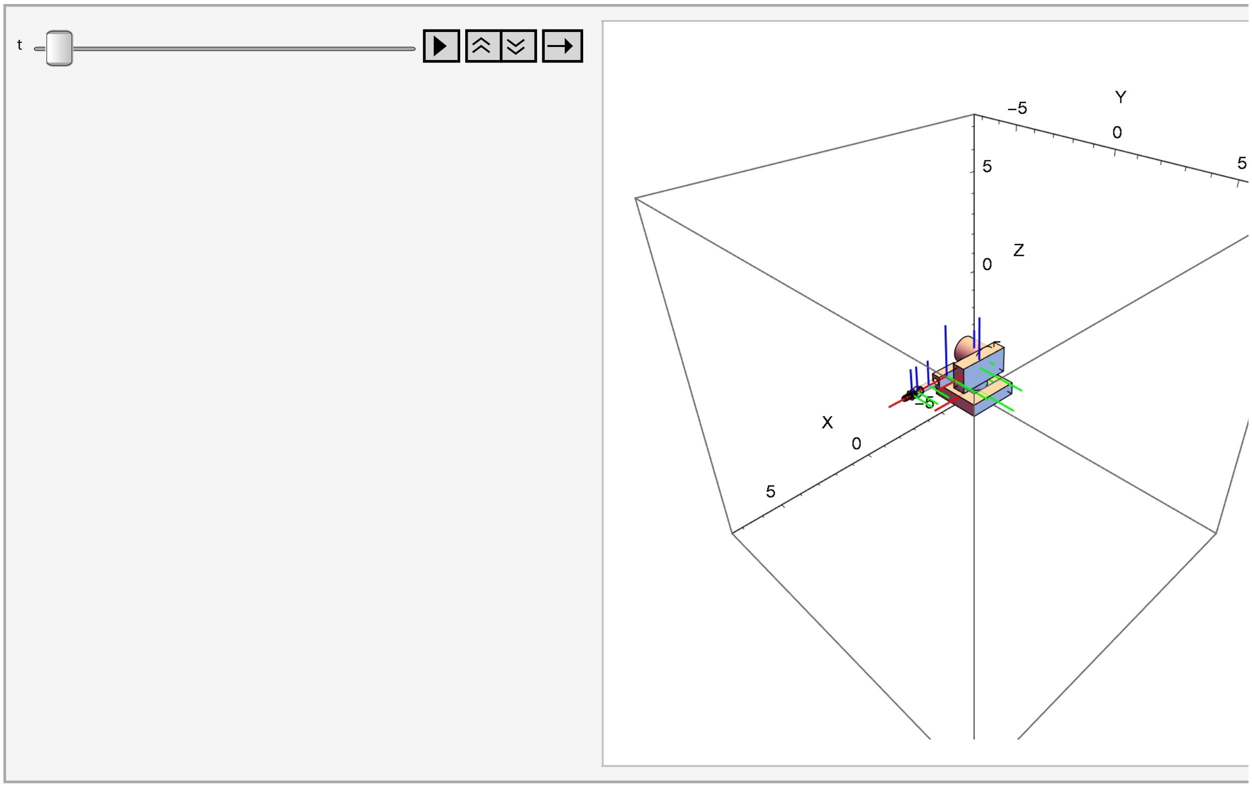
In[724]:=
```
robotGraphicDynAnimT[t_] = robotGraphicDynAnim;
```

Loop over time

In[725]:=
```
tf = 20;
```

In[726]:=
```
Animate[Show[Graphics3D[robotGraphicDynAnimT[t], ViewPoint -> {1, 1, 1},
    ViewVertical -> {0, 0, 1}, ViewCenter -> {1 / 2, 1 / 2, 1 / 2}, Boxed -> True, Axes -> True,
    PlotRange -> {{-scale, scale}, {-scale, scale}, {-scale, scale}}, AspectRatio -> 1,
    AxesLabel → {"X", "Y", "Z"}]], {t, 0, tf, tf / 500}, AnimationRunning → False]
```

Out[726]=



Controller design

We have decided to try a proportional plus integral plus derivative (PID) control law for each motor. To implement this we will treat each motor torque/force as a state. So the integration will be performed by integrating the state equations, this implies that we must take two derivatives of the error, which can be problematic if the error is noisy. This implies our desired path must be smooth up to second derivatives in time.

Control laws and control equations of motion

Clear everything again.

In[727]:=
```
x[t_] =.
y[t_] =.
q1[t_] =.
q2[t_] =.
q3[t_] =.
q4[t_] =.
q5[t_] =.
q6[t_] =.
```

In[735]:= F_{motx} =.
F_{moty} =.
T_{mot}[1] =.
T_{mot}[2] =.
T_{mot}[3] =.
T_{mot}[4] =.
T_{mot}[5] =.
T_{mot}[6] =.
F_{toolx} =.
F_{tooly} =.
F_{toolz} =.
T_{toolx} =.
T_{tooly} =.
T_{toolz} =.

In[749]:= ω =.

Here are the state equations for motor force and torque, desired minus actual is the error in a given state. I will replace the place holder symbols for motor forces and torques with the states defined here later.

Base x-motor

In[750]:= err[1] = x_d[t] - x[t];
eqM[1] = Fm_x'[t] - K_{Int}[1] err[1] - K_{prop}[1] D[err[1], t] - K_{deriv}[1] D[err[1], {t, 2}]

Out[751]= $-K_{Int}[1]\,(-x[t]+x_d[t]) + Fm_x{}'[t] - K_{prop}[1]\,(-x'[t]+x_d{}'[t]) - K_{deriv}[1]\,(-x''[t]+x_d{}''[t])$

Base y-motor

In[752]:= err[2] = y_d[t] - y[t];
eqM[2] = Fm_y'[t] - K_{Int}[2] err[2] - K_{prop}[2] D[err[2], t] - K_{deriv}[2] D[err[2], {t, 2}]

Out[753]= $-K_{Int}[2]\,(-y[t]+y_d[t]) + Fm_y{}'[t] - K_{prop}[2]\,(-y'[t]+y_d{}'[t]) - K_{deriv}[2]\,(-y''[t]+y_d{}''[t])$

q_1-motor

In[754]:= err[3] = q_{d1}[t] - q_1[t];
eqM[3] = Tm_1'[t] - K_{Int}[3] err[3] - K_{prop}[3] D[err[3], t] - K_{deriv}[3] D[err[3], {t, 2}]

Out[755]= $-K_{Int}[3]\,(-q_1[t]+q_{d1}[t]) - K_{prop}[3]\,(-q_1{}'[t]+q_{d1}{}'[t]) + Tm_1{}'[t] - K_{deriv}[3]\,(-q_1{}''[t]+q_{d1}{}''[t])$

q_2-motor

In[756]:= err[4] = q_{d2}[t] - q_2[t];
eqM[4] = Tm_2'[t] - K_{Int}[4] err[4] - K_{prop}[4] D[err[4], t] - K_{deriv}[4] D[err[4], {t, 2}]

Out[757]= $-K_{Int}[4]\,(-q_2[t]+q_{d2}[t]) - K_{prop}[4]\,(-q_2{}'[t]+q_{d2}{}'[t]) + Tm_2{}'[t] - K_{deriv}[4]\,(-q_2{}''[t]+q_{d2}{}''[t])$

q_3-motor

In[758]:= err[5] = q_{d3}[t] - q_3[t];
eqM[5] = Tm_3'[t] - K_{Int}[5] err[5] - K_{prop}[5] D[err[5], t] - K_{deriv}[5] D[err[5], {t, 2}]

Out[759]= $-K_{Int}[5]\,(-q_3[t]+q_{d3}[t]) - K_{prop}[5]\,(-q_3{}'[t]+q_{d3}{}'[t]) + Tm_3{}'[t] - K_{deriv}[5]\,(-q_3{}''[t]+q_{d3}{}''[t])$

q_4-motor

```
In[760]:= err[6] = qd4[t] - q4[t];
eqM[6] = Tm4'[t] - KInt[6] err[6] - Kprop[6] D[err[6], t] - Kderiv[6] D[err[6], {t, 2}]
```

Out[761]= $-K_{Int}[6]\ (-q_4[t] + q_{d4}[t]) - K_{prop}[6]\ (-q_4'[t] + q_{d4}'[t]) + Tm_4'[t] - K_{deriv}[6]\ (-q_4''[t] + q_{d4}''[t])$

q_5-motor

```
In[762]:= err[7] = qd5[t] - q5[t];
eqM[7] = Tm5'[t] - KInt[7] err[7] - Kprop[7] D[err[7], t] - Kderiv[7] D[err[7], {t, 2}]
```

Out[763]= $-K_{Int}[7]\ (-q_5[t] + q_{d5}[t]) - K_{prop}[7]\ (-q_5'[t] + q_{d5}'[t]) + Tm_5'[t] - K_{deriv}[7]\ (-q_5''[t] + q_{d5}''[t])$

q_6-motor

```
In[764]:= err[8] = qd6[t] - q6[t];
eqM[8] = Tm6'[t] - KInt[8] err[8] - Kprop[8] D[err[8], t] - Kderiv[8] D[err[8], {t, 2}]
```

Out[765]= $-K_{Int}[8]\ (-q_6[t] + q_{d6}[t]) - K_{prop}[8]\ (-q_6'[t] + q_{d6}'[t]) + Tm_6'[t] - K_{deriv}[8]\ (-q_6''[t] + q_{d6}''[t])$

Replace the place holders with the states for motor forces and torques.

```
In[766]:= eqC[1] = eqT[1] //. {Fmotx → Fmx[t], Fmoty → Fmy[t], Tmot[n_] → Tmn[t]};
In[767]:= eqC[2] = eqT[2] //. {Fmotx → Fmx[t], Fmoty → Fmy[t], Tmot[n_] → Tmn[t]};
In[768]:= eqC[3] = eqT[3] //. {Fmotx → Fmx[t], Fmoty → Fmy[t], Tmot[n_] → Tmn[t]};
In[769]:= eqC[4] = eqT[4] //. {Fmotx → Fmx[t], Fmoty → Fmy[t], Tmot[n_] → Tmn[t]};
In[770]:= eqC[5] = eqT[5] //. {Fmotx → Fmx[t], Fmoty → Fmy[t], Tmot[n_] → Tmn[t]};
In[771]:= eqC[6] = eqT[6] //. {Fmotx → Fmx[t], Fmoty → Fmy[t], Tmot[n_] → Tmn[t]};
In[772]:= eqC[7] = eqT[7] //. {Fmotx → Fmx[t], Fmoty → Fmy[t], Tmot[n_] → Tmn[t]};
In[773]:= eqC[8] = eqT[8] //. {Fmotx → Fmx[t], Fmoty → Fmy[t], Tmot[n_] → Tmn[t]};
```

Now we want to design the controller gains based on the step response of the system at a given motor. We will do this by holding all the other states constant for the given motor control constant determination. We will also linearize the system equations so we can then use pole placement techniques to size the gains.

Control gains base x-motor

Base x-motor. Here we fixate all the other movements.

In[774]:=
```
eqG[1] =
  eqC[1] //. {y[t] → 0, y'[t] → 0, y''[t] → 0, q1[t] → 0, q1'[t] → 0, q1''[t] → 0, q2[t] → 0,
      q2'[t] → 0, q2''[t] → 0, q3[t] → 0, q3'[t] → 0, q3''[t] → 0, q4[t] → 0, q4'[t] → 0,
      q4''[t] → 0, q5[t] → 0, q5'[t] → 0, q5''[t] → 0, q6[t] → 0, q6'[t] → 0, q6''[t] → 0} //.
    {Fmy[t] → 0, Tm1[t] → 0, Tm2[t] → 0, Tm3[t] → 0, Tm4[t] → 0, Tm5[t] → 0, Tm6[t] → 0} //.
   {Ftoolx → 0, Ftooly → 0, Ftoolz → 0, Ttoolx → 0, Ttooly → 0, Ttoolz → 0}
```

Out[774]= $0. + Fm_x[t] - 761.707\, x''[t]$

Linearize the equation.

In[775]:=
```
eqL[1] = Normal[Series[eqG[1], {x[t], 0, 1}, {x'[t], 0, 1}]]
```

Out[775]= $0. + Fm_x[t] - 761.707\, x''[t]$

We take a derivative to allow substitution of the motor control state equation.

In[776]:=
```
eqLd[1] = D[eqL[1], t]
```

Out[776]= $Fm_x'[t] - 761.707\, x^{(3)}[t]$

Solve the motor state equation for the motor force/torque derivative.

In[777]:=
```
solM[1] = Solve[eqM[1] == 0, Fmx'[t]] // First
```

Out[777]= $\{Fm_x'[t] \to -x[t]\, K_{Int}[1] + K_{Int}[1]\, x_d[t] - K_{prop}[1]\, x'[t] + K_{prop}[1]\, x_d'[t] - K_{deriv}[1]\, x''[t] + K_{deriv}[1]\, x_d''[t]\}$

Here is the complete linear equation for this motor in terms of the control gains.

In[778]:=
```
eqLf[1] = eqLd[1] //. solM[1]
```

Out[778]= $-x[t]\, K_{Int}[1] + K_{Int}[1]\, x_d[t] - K_{prop}[1]\, x'[t] + K_{prop}[1]\, x_d'[t] - K_{deriv}[1]\, x''[t] + K_{deriv}[1]\, x_d''[t] - 761.707\, x^{(3)}[t]$

This is the characteristic polynomial.

In[779]:=
```
charEq[1] =
  eqLf[1] //. {Derivative[n_][x][t] → s^n, x[t] → 1, xd[t] → 0, xd'[t] → 0, xd''[t] → 0}
```

Out[779]= $-761.707\, s^3 - s^2 K_{deriv}[1] - K_{Int}[1] - s\, K_{prop}[1]$

Put the characteristic equation in standard form.

In[780]:=
```
charEq[1] = charEq[1] / Coefficient[charEq[1], s^3] // Expand
```

Out[780]= $1.\, s^3 + 0.00131284\, s^2 K_{deriv}[1] + 0.00131284\, K_{Int}[1] + 0.00131284\, s\, K_{prop}[1]$

The desired characteristic polynomial is given by the following system with a first order pole and a second order pole pair.

In[781]:=
```
charDesired[1] = Expand[(s + a1) (s^2 + 2 ζ1 ωn1 s + ωn1^2)]
```

Out[781]= $s^3 + s^2 a_1 + 2 s^2 \zeta_1 \omega_{n1} + 2 s\, a_1 \zeta_1 \omega_{n1} + s\, \omega_{n1}^2 + a_1 \omega_{n1}^2$

The time to peak is defined as

In[782]:=
```
eqTp[1] = t_p1 == π / (ω_n1 √(1 - ζ_1^2));
```

The two percent settling time is given by

In[783]:=
```
eqTs[1] = t_s1 == 4 / (ζ_1 ω_n1);
```

In[784]:=
```
solZW[1] = Solve[{eqTp[1], eqTs[1]}, {ζ_1, ω_n1}][[2]]
```

Out[784]= $\left\{\zeta_1 \to (4\, t_{p1}) \Big/ \left(\sqrt{(16\, t_{p1}^2 + \pi^2\, t_{s1}^2)}\right),\ \omega_{n1} \to \left(\sqrt{(16\, t_{p1}^2 + \pi^2\, t_{s1}^2)}\right) \Big/ (t_{p1}\, t_{s1})\right\}$

Set up equations for gains by equating coefficients of the characteristic polynomials.

In[785]:=
```
eqGain1[1] = Coefficient[charEq[1], s^2] == Coefficient[charDesired[1], s^2] /. solZW[1]
```

Out[785]= $0.00131284\, K_{deriv}[1] == a_1 + 8 / t_{s1}$

In[786]:=
```
eqGain2[1] = Coefficient[charEq[1], s] == Coefficient[charDesired[1], s] /. solZW[1]
```

Out[786]= $0.00131284\, K_{prop}[1] == (8\, a_1) / t_{s1} + (16\, t_{p1}^2 + \pi^2\, t_{s1}^2) \Big/ (t_{p1}^2\, t_{s1}^2)$

In[787]:=
```
eqGain3[1] = (charEq[1] /. s → 0) == (charDesired[1] /. s → 0) /. solZW[1]
```

Out[787]= $0. + 0.00131284\, K_{Int}[1] == \left(a_1\, (16\, t_{p1}^2 + \pi^2\, t_{s1}^2)\right) \Big/ (t_{p1}^2\, t_{s1}^2)$

Solve for the controller gains.

In[788]:=
```
solGains[1] =
 Solve[{eqGain1[1], eqGain2[1], eqGain3[1]}, {K_prop[1], K_Int[1], K_deriv[1]}] // First
```

Out[788]= $\{K_{prop}[1] \to 761.707\, \left((8.\, a_1) / t_{s1} + (1.\, (16.\, t_{p1}^2 + 9.8696\, t_{s1}^2)) \Big/ (t_{p1}^2\, t_{s1}^2)\right),$
$K_{Int}[1] \to 761.707\, \left(0. + (1.\, a_1\, (16.\, t_{p1}^2 + 9.8696\, t_{s1}^2)) \Big/ (t_{p1}^2\, t_{s1}^2)\right),$
$K_{deriv}[1] \to 761.707\, (1.\, a_1 + 8. / t_{s1})\}$

Now set the time constants.

In[789]:=
```
tConstRules[1] = {a_1 → 100, t_p1 → 1 / 4, t_s1 → 1 / 2};
```

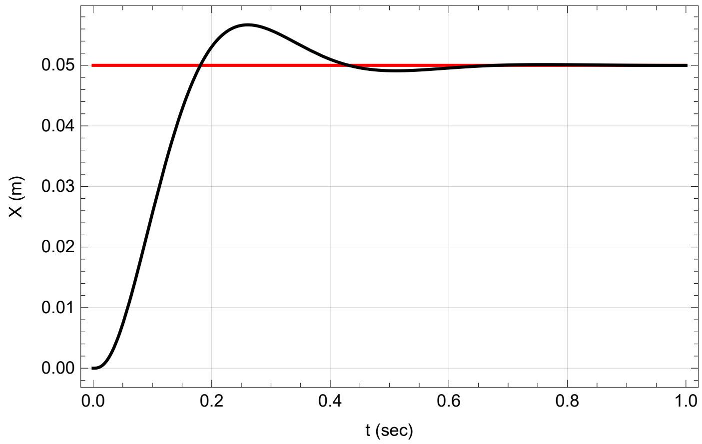
Check the step response

In[790]:=
```
t_0 = -.00000001;
t_f = 1;
stepMag = 5 / 100; (*5 cm*)
x_d[t_] := stepMag UnitStep[t]
stepSol[1] = NDSolve[{(eqG[1] /. solGains[1] /. tConstRules[1]) == 0,
   (eqM[1] /. solGains[1] /. tConstRules[1]) == 0,
   x'[t_0] == 0, x[t_0] == 0, Fm_x[t_0] == 0}, {x, Fm_x}, {t, t_0, t_f}]
```

Out[794]= {{x → InterpolatingFunction[ Domain: $\{\{-1. \times 10^{-8}, 1.\}\}$ Output: scalar],

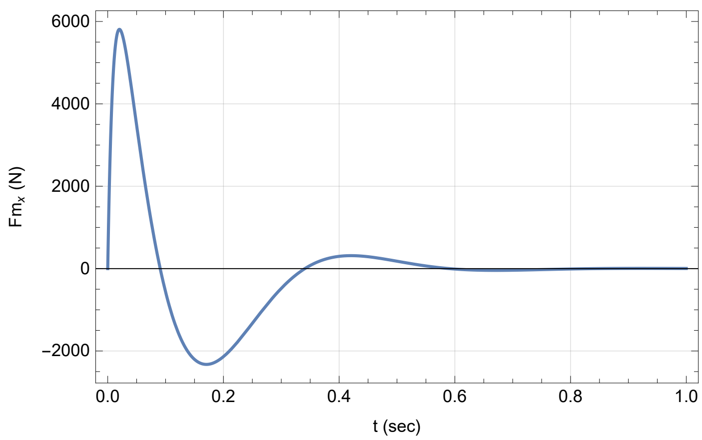
Fm_x → InterpolatingFunction[ Domain: $\{\{-1. \times 10^{-8}, 1.\}\}$ Output: scalar]}}

In[795]:=
```
Plot[{x_d[t], x[t] /. stepSol[1]}, {t, t_0, t_f}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "X (m)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

Out[795]=


We can see that the system displacement response is good. The desired value is in red.

In[796]:=
```
Plot[Fm_x[t] /. stepSol[1], {t, t_0, t_f}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Fm_x (N)"}, PlotRange → All]
```

Out[796]=


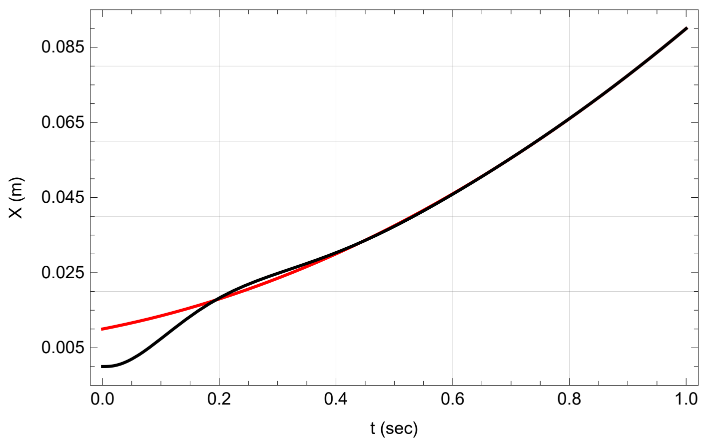
This is the force needed to do this step maneuver. It is roughly 1350 pounds peak load.

Check the tracking of a polynomial.

In[797]:=
```
t0 = -.00000001;
tf = 1;
xd[t_] := (5 t^2 + 3 t + 1) / 100 (* cm*)
trackSol[1] = NDSolve[{(eqG[1] /. solGains[1] /. tConstRules[1]) == 0,
   (eqM[1] /. solGains[1] /. tConstRules[1]) == 0,
   x'[t0] == 0, x[t0] == 0, Fmx[t0] == 0}, {x, Fmx}, {t, t0, tf}]
```

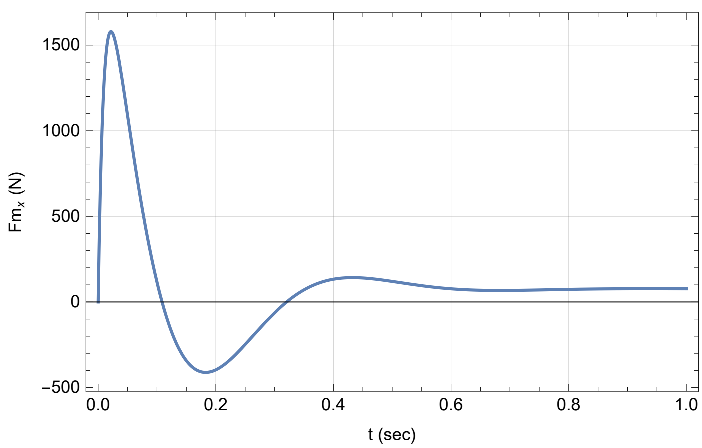
Out[800]=
```
{{x → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Fmx → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

In[801]:=
```
Plot[{xd[t], x[t] /. trackSol[1]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "X (m)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

Out[801]=


We can see that the system tracks a parabola fairly well, could be better initially, however this initial error is due to the step change at t=0. So if we are nearer to the initial tracking curve, we should be ok.

In[802]:=
```
Plot[Fmx[t] /. trackSol[1], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Fmx (N)"}, PlotRange → All]
```

Out[802]=


Clear the desired variable.

In[803]:=
```
xd[t_] =.
```

Control gains base y-motor

Base y-motor. Here we fixate all the other movements.

In[804]:=
```
eqG[2] =
  eqC[2] //. {x[t] → 0, x''[t] → 0, x''[t] → 0, q1[t] → 0, q1'[t] → 0, q1''[t] → 0, q2[t] → 0,
      q2'[t] → 0, q2''[t] → 0, q3[t] → 0, q3'[t] → 0, q3''[t] → 0, q4[t] → 0, q4'[t] → 0,
      q4''[t] → 0, q5[t] → 0, q5'[t] → 0, q5''[t] → 0, q6[t] → 0, q6'[t] → 0, q6''[t] → 0} //.
    {Fmx[t] → 0, Tm1[t] → 0, Tm2[t] → 0, Tm3[t] → 0, Tm4[t] → 0, Tm5[t] → 0, Tm6[t] → 0} //.
   {Ftoolx → 0, Ftooly → 0, Ftoolz → 0, Ttoolx → 0, Ttooly → 0, Ttoolz → 0}
```

Out[804]= $0. + Fm_y[t] - 761.707\, y''[t]$

Linearize the equation.

In[805]:=
```
eqL[2] = Normal[Series[eqG[2], {y[t], 0, 1}, {y'[t], 0, 1}]]
```

Out[805]= $0. + Fm_y[t] - 761.707\, y''[t]$

We take a derivative to allow substitution of the motor control state equation.

In[806]:=
```
eqLd[2] = D[eqL[2], t]
```

Out[806]= $Fm_y'[t] - 761.707\, y^{(3)}[t]$

Solve the motor state equation for the motor force/torque derivative.

In[807]:=
```
solM[2] = Solve[eqM[2] == 0, Fmy'[t]] // First
```

Out[807]= $\{Fm_y'[t] \to$
$-y[t]\, K_{Int}[2] + K_{Int}[2]\, y_d[t] - K_{prop}[2]\, y'[t] + K_{prop}[2]\, y_d'[t] - K_{deriv}[2]\, y''[t] + K_{deriv}[2]\, y_d''[t]\}$

Here is the complete linear equation for this motor in terms of the control gains.

In[808]:=
```
eqLf[2] = eqLd[2] //. solM[2]
```

Out[808]= $-y[t]\, K_{Int}[2] + K_{Int}[2]\, y_d[t] - K_{prop}[2]\, y'[t] +$
$K_{prop}[2]\, y_d'[t] - K_{deriv}[2]\, y''[t] + K_{deriv}[2]\, y_d''[t] - 761.707\, y^{(3)}[t]$

This is the characteristic polynomial.

In[809]:=
```
charEq[2] =
  eqLf[2] //. {Derivative[n_][y][t] → s^n, y[t] → 1, yd[t] → 0, yd'[t] → 0, yd''[t] → 0}
```

Out[809]= $-761.707\, s^3 - s^2\, K_{deriv}[2] - K_{Int}[2] - s\, K_{prop}[2]$

Put the characteristic equation in standard form.

In[810]:=
```
charEq[2] = charEq[2] / Coefficient[charEq[2], s^3] // Expand
```

Out[810]= $1.\, s^3 + 0.00131284\, s^2\, K_{deriv}[2] + 0.00131284\, K_{Int}[2] + 0.00131284\, s\, K_{prop}[2]$

The desired characteristic polynomial is given by the following system with a first order pole and a second order pole pair.

In[811]:=
```
charDesired[2] = Expand[(s + a2) (s^2 + 2 ζ2 ωn2 s + ωn2^2)]
```

Out[811]= $s^3 + s^2 a_2 + 2 s^2 \zeta_2 \omega_{n2} + 2 s a_2 \zeta_2 \omega_{n2} + s \omega_{n2}^2 + a_2 \omega_{n2}^2$

The time to peak is defined as

In[812]:=
```
eqTp[2] = tp2 == π / (ωn2 √(1 - ζ2^2));
```

The two percent settling time is given by

In[813]:=
```
eqTs[2] = ts2 == 4 / (ζ2 ωn2);
```

In[814]:=
```
solZW[2] = Solve[{eqTp[2], eqTs[2]}, {ζ2, ωn2}][[2]]
```

Out[814]= $\left\{\zeta_2 \to (4 t_{p2}) \Big/ \left(\sqrt{\left(16 t_{p2}^2 + \pi^2 t_{s2}^2\right)}\right), \omega_{n2} \to \left(\sqrt{\left(16 t_{p2}^2 + \pi^2 t_{s2}^2\right)}\right) \Big/ (t_{p2} t_{s2})\right\}$

Set up equations for gains by equating coefficients of the characterstic polynomials.

In[815]:=
```
eqGain1[2] = Coefficient[charEq[2], s^2] == Coefficient[charDesired[2], s^2] /. solZW[2]
```

Out[815]= $0.00131284\, K_{deriv}[2] == a_2 + 8 / t_{s2}$

In[816]:=
```
eqGain2[2] = Coefficient[charEq[2], s] == Coefficient[charDesired[2], s] /. solZW[2]
```

Out[816]= $0.00131284\, K_{prop}[2] == (8 a_2) / t_{s2} + \left(16 t_{p2}^2 + \pi^2 t_{s2}^2\right) \Big/ \left(t_{p2}^2 t_{s2}^2\right)$

In[817]:=
```
eqGain3[2] = (charEq[2] /. s → 0) == (charDesired[2] /. s → 0) /. solZW[2]
```

Out[817]= $0. + 0.00131284\, K_{Int}[2] == \left(a_2 \left(16 t_{p2}^2 + \pi^2 t_{s2}^2\right)\right) \Big/ \left(t_{p2}^2 t_{s2}^2\right)$

Solve for the controller gains.

In[818]:=
```
solGains[2] =
  Solve[{eqGain1[2], eqGain2[2], eqGain3[2]}, {Kprop[2], KInt[2], Kderiv[2]}] // First
```

Out[818]= $\{K_{prop}[2] \to 761.707 \left((8.\, a_2) / t_{s2} + \left(1.\left(16.\, t_{p2}^2 + 9.8696\, t_{s2}^2\right)\right) \Big/ \left(t_{p2}^2 t_{s2}^2\right)\right),$
$K_{Int}[2] \to 761.707 \left(0. + \left(1.\, a_2 \left(16.\, t_{p2}^2 + 9.8696\, t_{s2}^2\right)\right) \Big/ \left(t_{p2}^2 t_{s2}^2\right)\right),$
$K_{deriv}[2] \to 761.707\, (1.\, a_2 + 8. / t_{s2})\}$

Now set the time constants.

In[819]:=
```
tConstRules[2] = {a2 → 100, tp2 → 1 / 4, ts2 → 1 / 2};
```

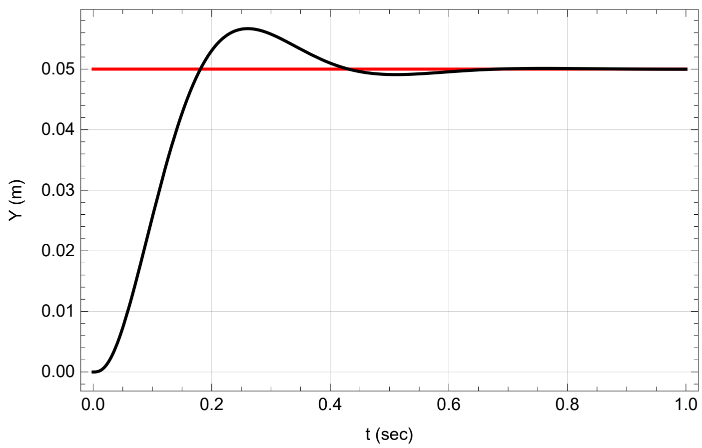
Check the step response

In[820]:=
```
t0 = -.00000001;
tf = 1;
stepMag = 5 / 100; (*5 cm*)
yd[t_] := stepMag UnitStep[t]
stepSol[2] = NDSolve[{(eqG[2] /. solGains[2] /. tConstRules[2]) == 0,
   (eqM[2] /. solGains[2] /. tConstRules[2]) == 0,
   y'[t0] == 0, y[t0] == 0, Fmy[t0] == 0}, {y, Fmy}, {t, t0, tf}]
```

Out[824]=
```
{{y → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Fmy → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

In[825]:=
```
Plot[{yd[t], y[t] /. stepSol[2]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Y (m)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

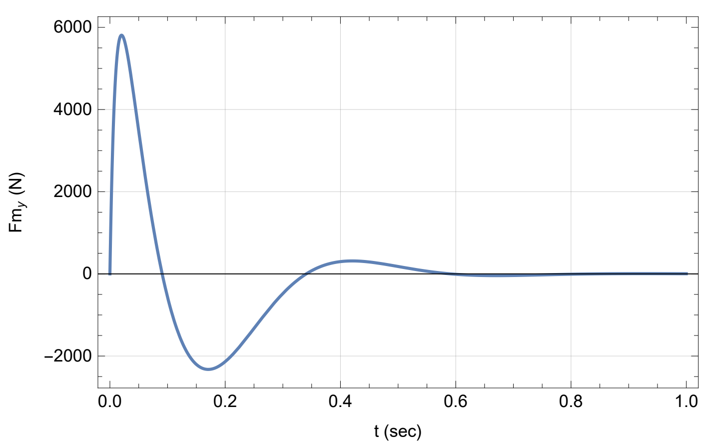
Out[825]=



We can see that the system displacement response is good. The desired value is in red.

In[826]:=
```
Plot[Fmy[t] /. stepSol[2], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Fmy (N)"}, PlotRange → All]
```

Out[826]=



This is the force needed to do this step maneuver. It is roughly 1350 pounds peak load.

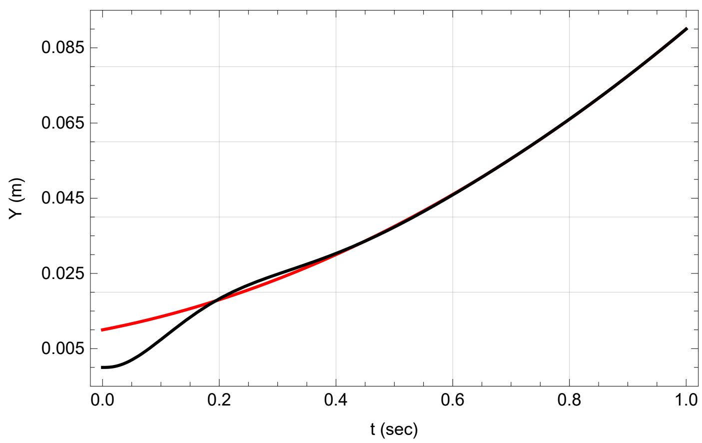
Check the tracking of a polynomial.

In[827]:=
```
t0 = -.00000001;
tf = 1;
yd[t_] := (5 t^2 + 3 t + 1) / 100 (* cm*)
trackSol[2] = NDSolve[{(eqG[2] /. solGains[2] /. tConstRules[2]) == 0,
   (eqM[2] /. solGains[2] /. tConstRules[2]) == 0,
   y'[t0] == 0, y[t0] == 0, Fmy[t0] == 0}, {y, Fmy}, {t, t0, tf}]
```

Out[830]=
```
{{y → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Fmy → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

In[831]:=
```
Plot[{yd[t], y[t] /. trackSol[2]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Y (m)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

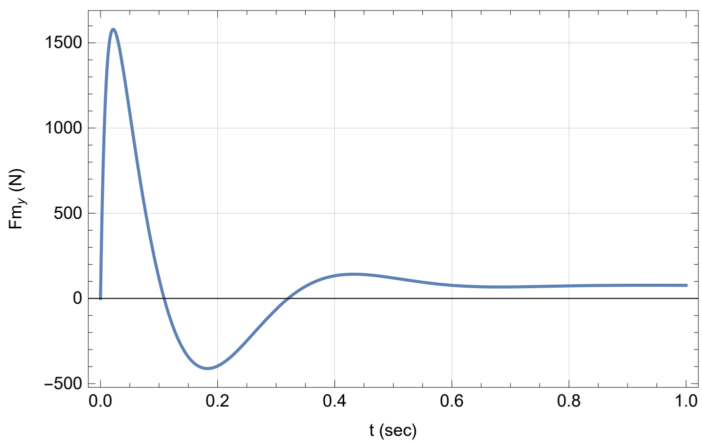
Out[831]=



We can see that the system tracks a parabola fairly well, could be better initially, however this initial error is due to the step change at t=0. So if we are nearer to the initial tracking curve, we should be ok.

In[832]:=
```
Plot[Fmy[t] /. trackSol[2], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Fmy (N)"}, PlotRange → All]
```

Out[832]=



Clear the desired variable.

In[833]:=
```
yd[t_] =.
```

Control gains q_1-motor

q_1-motor. Here we fixate all the other movements.

In[834]:=
```
eqG[3] =
 eqC[3] //. {x[t] → 0, x''[t] → 0, x''[t] → 0, y[t] → 0, y'[t] → 0, y''[t] → 0, q2[t] → 0,
     q2'[t] → 0, q2''[t] → 0, q3[t] → 0, q3'[t] → 0, q3''[t] → 0, q4[t] → 0, q4'[t] → 0,
     q4''[t] → 0, q5[t] → 0, q5'[t] → 0, q5''[t] → 0, q6[t] → 0, q6'[t] → 0, q6''[t] → 0} //.
   {Fmx[t] → 0, Fmy[t] → 0, Tm2[t] → 0, Tm3[t] → 0, Tm4[t] → 0, Tm5[t] → 0, Tm6[t] → 0} //.
  {Ftoolx → 0, Ftooly → 0, Ftoolz → 0, Ttoolx → 0, Ttooly → 0, Ttoolz → 0}
```

Out[834]=
$$
\begin{aligned}
&0. + \text{Tm}_1[t] + (1/4)\left(0. - 79.3908\, q_1'[t]^2\right) + (1/4)\left(0. - 39.6954\, q_1'[t]^2\right) + \\
&(1/6)\left(0. - 11.0265\, q_1'[t]^2\right) + (3/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. - 11.0265\, q_1'[t]^2\right) + \\
&(1/4)\left(0. - 7.79417\, q_1'[t]^2\right) + (1/4)\left(0. - 3.46408\, q_1'[t]^2\right) + (1/4)\left(0. - 2.96921\, q_1'[t]^2\right) + \\
&(2/3)\left(0. - 2.16505\, q_1'[t]^2\right) - (1/3)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. - 2.16505\, q_1'[t]^2\right) + \\
&(3/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. - 2.16505\, q_1'[t]^2\right) + (2/3)\left(0. - 0.962244\, q_1'[t]^2\right) - \\
&(5/6)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. - 0.962244\, q_1'[t]^2\right) + \\
&(3/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. - 0.962244\, q_1'[t]^2\right) + (2/3)\left(0. - 0.82478\, q_1'[t]^2\right) - \\
&(15/14)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. - 0.82478\, q_1'[t]^2\right) + \\
&(3/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. - 0.82478\, q_1'[t]^2\right) - \\
&(1/6)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 0.471303\, q_1'[t]^2\right) - \\
&(1/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 0.471303\, q_1'[t]^2\right) + (3/2)\left(0. + 0.494868\, q_1'[t]^2\right) - \\
&(73/42)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 0.494868\, q_1'[t]^2\right) + \\
&(3/2)\left(0. + 0.577346\, q_1'[t]^2\right) - (3/2)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 0.577346\, q_1'[t]^2\right) + \\
&(68/21)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 0.989736\, q_1'[t]^2\right) + \\
&3\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 1.15469\, q_1'[t]^2\right) + (3/2)\left(0. + 1.29903\, q_1'[t]^2\right) - \\
&\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 1.29903\, q_1'[t]^2\right) + (5/12)\left(0. + 1.31965\, q_1'[t]^2\right) + \\
&(1/4)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 1.31965\, q_1'[t]^2\right) - \\
&(1/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 1.31965\, q_1'[t]^2\right) + (5/12)\left(0. + 1.53959\, q_1'[t]^2\right) + \\
&(1/4)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 1.53959\, q_1'[t]^2\right) - \\
&(1/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 1.53959\, q_1'[t]^2\right) - \\
&(1/6)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 1.64956\, q_1'[t]^2\right) - \\
&(1/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 1.64956\, q_1'[t]^2\right) - \\
&(1/6)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 1.73204\, q_1'[t]^2\right) - \\
&(1/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 1.73204\, q_1'[t]^2\right) - \\
&(1/6)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 1.92449\, q_1'[t]^2\right) - \\
&(1/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 1.92449\, q_1'[t]^2\right) + \\
&(5/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 2.59806\, q_1'[t]^2\right) - \\
&(1/12)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 2.96921\, q_1'[t]^2\right) - \\
&(1/12)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 3.46408\, q_1'[t]^2\right) + (5/12)\left(0. + 3.46408\, q_1'[t]^2\right) + \\
&(1/4)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 3.46408\, q_1'[t]^2\right) - \\
&(1/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 3.46408\, q_1'[t]^2\right) + (5/12)\left(0. + 4.4106\, q_1'[t]^2\right) + \\
&(1/4)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 4.4106\, q_1'[t]^2\right) - \\
&(1/2)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 4.4106\, q_1'[t]^2\right) + (3/2)\left(0. + 6.6159\, q_1'[t]^2\right) - \\
&(1/6)\left(-\text{Cos}[q_1[t]]^2 - \text{Sin}[q_1[t]]^2\right)\left(0. + 6.6159\, q_1'[t]^2\right) - \\
&(1/12)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 7.79417\, q_1'[t]^2\right) + \\
&(5/3)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 13.2318\, q_1'[t]^2\right) + (1/2)\left(0. + 39.6954\, q_1'[t]^2\right) - \\
&(1/12)\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right)\left(0. + 39.6954\, q_1'[t]^2\right) - \\
&322.295\, q_1''[t] - 84.2373\left(\text{Cos}[q_1[t]]^2 + \text{Sin}[q_1[t]]^2\right) q_1''[t]
\end{aligned}
$$

Linearize the equation.

In[835]:= `eqL[3] = Normal[Series[eqG[3], {q1[t], 0, 1}, {q1'[t], 0, 1}]]`

Out[835]= $Tm_1[t] - 406.532\, q_1''[t]$

We take a derivative to allow substitution of the motor control state equation.

In[836]:= `eqLd[3] = D[eqL[3], t]`

Out[836]= $Tm_1'[t] - 406.532\, q_1^{(3)}[t]$

Solve the motor state equation for the motor force/torque derivative.

In[837]:= `solM[3] = Solve[eqM[3] == 0, Tm1'[t]] // First`

Out[837]= $\{Tm_1'[t] \to -K_{Int}[3]\, q_1[t] + K_{Int}[3]\, q_{d1}[t] - K_{prop}[3]\, q_1'[t] + K_{prop}[3]\, q_{d1}'[t] - K_{deriv}[3]\, q_1''[t] + K_{deriv}[3]\, q_{d1}''[t]\}$

Here is the complete linear equation for this motor in terms of the control gains.

In[838]:= `eqLf[3] = eqLd[3] //. solM[3]`

Out[838]= $-K_{Int}[3]\, q_1[t] + K_{Int}[3]\, q_{d1}[t] - K_{prop}[3]\, q_1'[t] + K_{prop}[3]\, q_{d1}'[t] - K_{deriv}[3]\, q_1''[t] + K_{deriv}[3]\, q_{d1}''[t] - 406.532\, q_1^{(3)}[t]$

This is the characteristic polynomial.

In[839]:= `charEq[3] = eqLf[3] //. {Derivative[n_][q1][t] → s^n, q1[t] → 1, qd1[t] → 0, qd1'[t] → 0, qd1''[t] → 0}`

Out[839]= $-406.532\, s^3 - s^2 K_{deriv}[3] - K_{Int}[3] - s\, K_{prop}[3]$

Put the characteristic equation in standard form.

In[840]:= `charEq[3] = charEq[3] / Coefficient[charEq[3], s^3] // Expand`

Out[840]= $1.\, s^3 + 0.00245983\, s^2 K_{deriv}[3] + 0.00245983\, K_{Int}[3] + 0.00245983\, s\, K_{prop}[3]$

The desired characteristic polynomial is given by the following system with a first order pole and a second order pole pair.

In[841]:= `charDesired[3] = Expand[(s + a3) (s^2 + 2 ζ3 ωn3 s + ωn3^2)]`

Out[841]= $s^3 + s^2 a_3 + 2 s^2 \zeta_3 \omega_{n3} + 2\, s\, a_3 \zeta_3 \omega_{n3} + s\, \omega_{n3}^2 + a_3 \omega_{n3}^2$

The time to peak is defined as

In[842]:= `eqTp[3] = tp3 == π / (ωn3 √(1 - ζ3^2));`

The two percent settling time is given by

In[843]:= `eqTs[3] = ts3 == 4 / (ζ3 ωn3);`

In[844]:= `solZW[3] = Solve[{eqTp[3], eqTs[3]}, {ζ3, ωn3}][[2]]`

Out[844]= $\left\{\zeta_3 \to (4\, t_{p3}) \Big/ \left(\sqrt{(16\, t_{p3}^2 + \pi^2 t_{s3}^2)}\right),\ \omega_{n3} \to \left(\sqrt{(16\, t_{p3}^2 + \pi^2 t_{s3}^2)}\right) \Big/ (t_{p3}\, t_{s3})\right\}$

Set up equations for gains by equating coefficients of the characterstic polynomials.

In[845]:=
```
eqGain1[3] = Coefficient[charEq[3], s^2] == Coefficient[charDesired[3], s^2] /. solZW[3]
```
Out[845]= $0.00245983\, K_{deriv}[3] == a_3 + 8 / t_{s3}$

In[846]:=
```
eqGain2[3] = Coefficient[charEq[3], s] == Coefficient[charDesired[3], s] /. solZW[3]
```
Out[846]= $0.00245983\, K_{prop}[3] == (8\, a_3) / t_{s3} + \left(16\, t_{p3}^2 + \pi^2\, t_{s3}^2\right) / \left(t_{p3}^2\, t_{s3}^2\right)$

In[847]:=
```
eqGain3[3] = (charEq[3] /. s → 0) == (charDesired[3] /. s → 0) /. solZW[3]
```
Out[847]= $0. + 0.00245983\, K_{Int}[3] == \left(a_3\, \left(16\, t_{p3}^2 + \pi^2\, t_{s3}^2\right)\right) / \left(t_{p3}^2\, t_{s3}^2\right)$

Solve for the controller gains.

In[848]:=
```
solGains[3] =
 Solve[{eqGain1[3], eqGain2[3], eqGain3[3]}, {Kprop[3], KInt[3], Kderiv[3]}] // First
```
Out[848]= $\{K_{prop}[3] \to 406.532\, \left((8.\, a_3) / t_{s3} + \left(1.\, \left(16.\, t_{p3}^2 + 9.8696\, t_{s3}^2\right)\right) / \left(t_{p3}^2\, t_{s3}^2\right)\right),$
$K_{Int}[3] \to 406.532\, \left(0. + \left(1.\, a_3\, \left(16.\, t_{p3}^2 + 9.8696\, t_{s3}^2\right)\right) / \left(t_{p3}^2\, t_{s3}^2\right)\right),$
$K_{deriv}[3] \to 406.532\, (1.\, a_3 + 8. / t_{s3})\}$

Now set the time constants.

In[849]:=
```
tConstRules[3] = {a3 → 100, tp3 → 1 / 4, ts3 → 1 / 2};
```

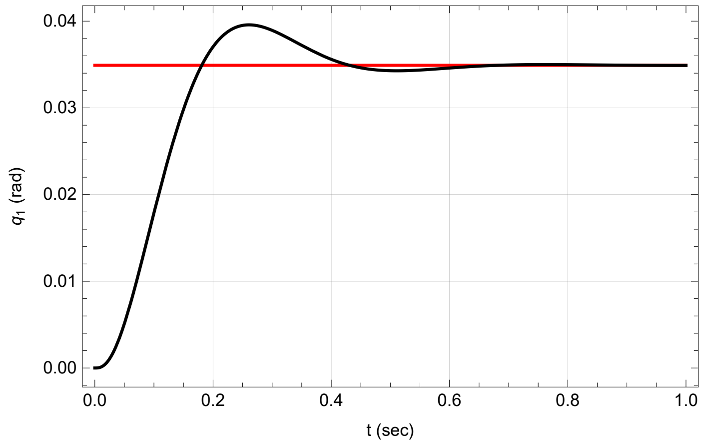
Check the step response

In[850]:=
```
t0 = -.00000001;
tf = 1;
stepMag = 2 π / 180; (*2 degrees*)
qd1[t_] := stepMag UnitStep[t]
stepSol[3] = NDSolve[{(eqG[3] /. solGains[3] /. tConstRules[3]) == 0,
    (eqM[3] /. solGains[3] /. tConstRules[3]) == 0,
    q1'[t0] == 0, q1[t0] == 0, Tm1[t0] == 0}, {q1, Tm1}, {t, t0, tf}]
```

Out[854]= {{q_1 → InterpolatingFunction[ Domain: {{$-1. \times 10^{-8}$, 1.}} Output: scalar],

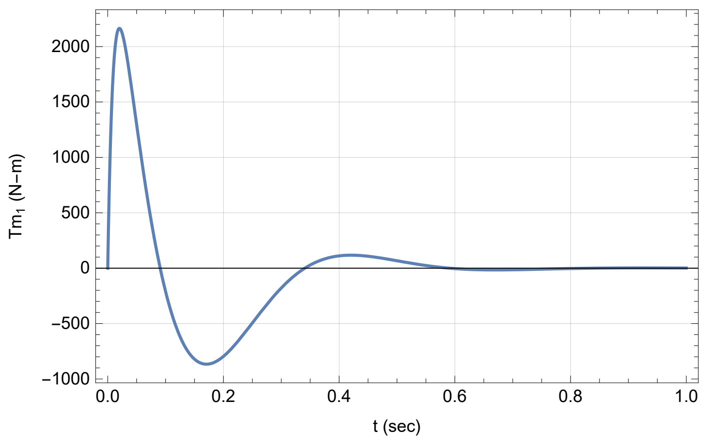
Tm$_1$ → InterpolatingFunction[ Domain: {{$-1. \times 10^{-8}$, 1.}} Output: scalar]}}

In[855]:=
```
Plot[{qd1[t], q1[t] /. stepSol[3]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q1 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

Out[855]=


We can see that the system displacement response is good. The desired value is in red.

In[856]:=
```
Plot[Tm1[t] /. stepSol[3], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm1 (N-m)"}, PlotRange → All]
```

Out[856]=


This is the force needed to do this step maneuver. It is roughly 1850 ft-lb peak torque.

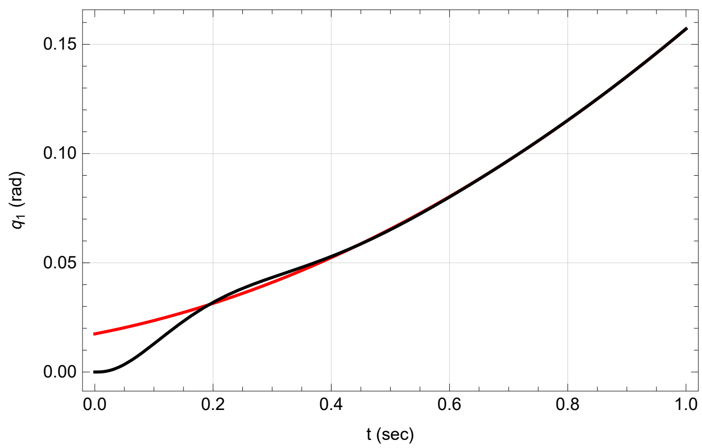
Check the tracking of a polynomial.

In[857]:=
```
t0 = -.00000001;
tf = 1;
qd1[t_] := (5 t^2 + 3 t + 1) π / 180 (*rad*)
trackSol[3] = NDSolve[{(eqG[3] /. solGains[3] /. tConstRules[3]) == 0,
   (eqM[3] /. solGains[3] /. tConstRules[3]) == 0,
   q1'[t0] == 0, q1[t0] == 0, Tm1[t0] == 0}, {q1, Tm1}, {t, t0, tf}]
```

Out[860]=
```
{{q1 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Tm1 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

In[861]:=
```
Plot[{qd1[t], q1[t] /. trackSol[3]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q1 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

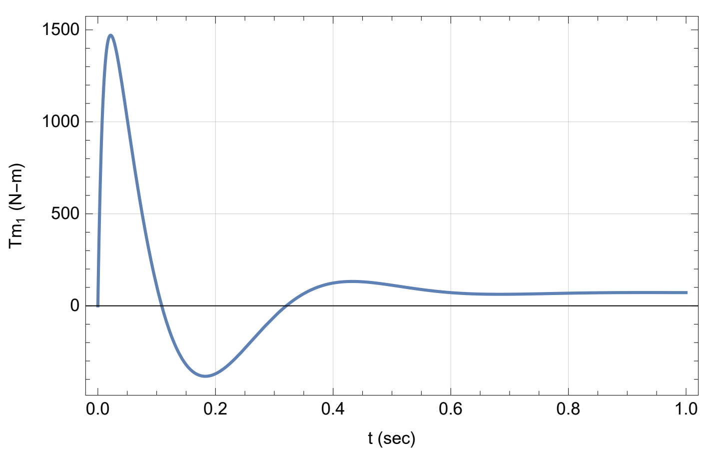
Out[861]=



We can see that the system tracks a parabola fairly well, could be better initially, however this initial error is due to the step change at t=0. So if we are nearer to the initial tracking curve, we should be ok.

In[862]:=
```
Plot[Tm1[t] /. trackSol[3], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm1 (N-m)"}, PlotRange → All]
```

Out[862]=



Clear the desired variable.

In[863]:=
```
qd1[t_] =.
```

Control gains q_2-motor

q_2-motor. Here we fixate all the other movements.

In[864]:=
```
eqG[4] =
  eqC[4] //. {x[t] → 0, x''[t] → 0, x''[t] → 0, y[t] → 0, y'[t] → 0, y''[t] → 0, q1[t] → 0,
      q1'[t] → 0, q1''[t] → 0, q3[t] → 0, q3'[t] → 0, q3''[t] → 0, q4[t] → 0, q4'[t] → 0,
      q4''[t] → 0, q5[t] → 0, q5'[t] → 0, q5''[t] → 0, q6[t] → 0, q6'[t] → 0, q6''[t] → 0} //.
    {Fmx[t] → 0, Fmy[t] → 0, Tm1[t] → 0, Tm3[t] → 0, Tm4[t] → 0, Tm5[t] → 0, Tm6[t] → 0} //.
   {Ftoolx → 0, Ftooly → 0, Ftoolz → 0, Ttoolx → 0, Ttooly → 0, Ttoolz → 0}
```

Out[864]= $0. + 1469.78\,\mathrm{Cos}[q_2[t]] + \mathrm{Tm}_2[t] - 260.631\,q_2''[t] - 58.0226\left(\mathrm{Cos}[q_2[t]]^2 + \mathrm{Sin}[q_2[t]]^2\right) q_2''[t]$

Linearize the equation.

In[865]:=
```
eqL[4] = Normal[Series[eqG[4], {q2[t], 0, 1}, {q2'[t], 0, 1}]]
```

Out[865]= $1469.78 + \mathrm{Tm}_2[t] - 318.653\,q_2''[t]$

We take a derivative to allow substitution of the motor control state equation.

In[866]:=
```
eqLd[4] = D[eqL[4], t]
```

Out[866]= $\mathrm{Tm}_2'[t] - 318.653\,q_2^{(3)}[t]$

Solve the motor state equation for the motor force/torque derivative.

In[867]:=
```
solM[4] = Solve[eqM[4] == 0, Tm2'[t]] // First
```

Out[867]= $\{\mathrm{Tm}_2'[t] \to -K_{\mathrm{Int}}[4]\,q_2[t] + K_{\mathrm{Int}}[4]\,q_{d2}[t] - K_{\mathrm{prop}}[4]\,q_2'[t] + K_{\mathrm{prop}}[4]\,q_{d2}'[t] - K_{\mathrm{deriv}}[4]\,q_2''[t] + K_{\mathrm{deriv}}[4]\,q_{d2}''[t]\}$

Here is the complete linear equation for this motor in terms of the control gains.

In[868]:=
```
eqLf[4] = eqLd[4] //. solM[4]
```

Out[868]= $-K_{\mathrm{Int}}[4]\,q_2[t] + K_{\mathrm{Int}}[4]\,q_{d2}[t] - K_{\mathrm{prop}}[4]\,q_2'[t] + K_{\mathrm{prop}}[4]\,q_{d2}'[t] - K_{\mathrm{deriv}}[4]\,q_2''[t] + K_{\mathrm{deriv}}[4]\,q_{d2}''[t] - 318.653\,q_2^{(3)}[t]$

This is the characteristic polynomial.

In[869]:=
```
charEq[4] =
  eqLf[4] //. {Derivative[n_][q2][t] → s^n, q2[t] → 1, qd2[t] → 0, qd2'[t] → 0, qd2''[t] → 0}
```

Out[869]= $-318.653\,s^3 - s^2\,K_{\mathrm{deriv}}[4] - K_{\mathrm{Int}}[4] - s\,K_{\mathrm{prop}}[4]$

Put the characteristic equation in standard form.

In[870]:=
```
charEq[4] = charEq[4] / Coefficient[charEq[4], s^3] // Expand
```

Out[870]= $1.\,s^3 + 0.00313821\,s^2\,K_{\mathrm{deriv}}[4] + 0.00313821\,K_{\mathrm{Int}}[4] + 0.00313821\,s\,K_{\mathrm{prop}}[4]$

The desired characteristic polynomial is given by the following system with a first order pole and a second order pole pair.

In[871]:=
```
charDesired[4] = Expand[(s + a4) (s^2 + 2 ζ4 ωn4 s + ωn4^2)]
```

Out[871]= $s^3 + s^2 a_4 + 2 s^2 \zeta_4 \omega_{n4} + 2 s a_4 \zeta_4 \omega_{n4} + s \omega_{n4}^2 + a_4 \omega_{n4}^2$

The time to peak is defined as

In[872]:=
```
eqTp[4] = tp4 == π / (ωn4 √(1 - ζ4^2));
```

The two percent settling time is given by

In[873]:=
```
eqTs[4] = ts4 == 4 / (ζ4 ωn4);
```

In[874]:=
```
solZW[4] = Solve[{eqTp[4], eqTs[4]}, {ζ4, ωn4}][[2]]
```

Out[874]= $\left\{\zeta_4 \to (4 t_{p4}) / \left(\sqrt{\left(16 t_{p4}^2 + \pi^2 t_{s4}^2\right)}\right), \omega_{n4} \to \left(\sqrt{\left(16 t_{p4}^2 + \pi^2 t_{s4}^2\right)}\right) / (t_{p4} t_{s4})\right\}$

Set up equations for gains by equating coefficients of the characteristic polynomials.

In[875]:=
```
eqGain1[4] = Coefficient[charEq[4], s^2] == Coefficient[charDesired[4], s^2] /. solZW[4]
```

Out[875]= $0.00313821 K_{deriv}[4] == a_4 + 8 / t_{s4}$

In[876]:=
```
eqGain2[4] = Coefficient[charEq[4], s] == Coefficient[charDesired[4], s] /. solZW[4]
```

Out[876]= $0.00313821 K_{prop}[4] == (8 a_4) / t_{s4} + \left(16 t_{p4}^2 + \pi^2 t_{s4}^2\right) / \left(t_{p4}^2 t_{s4}^2\right)$

In[877]:=
```
eqGain3[4] = (charEq[4] /. s → 0) == (charDesired[4] /. s → 0) /. solZW[4]
```

Out[877]= $0. + 0.00313821 K_{Int}[4] == \left(a_4 \left(16 t_{p4}^2 + \pi^2 t_{s4}^2\right)\right) / \left(t_{p4}^2 t_{s4}^2\right)$

Solve for the controller gains.

In[878]:=
```
solGains[4] =
  Solve[{eqGain1[4], eqGain2[4], eqGain3[4]}, {Kprop[4], KInt[4], Kderiv[4]}] // First
```

Out[878]= $\{K_{prop}[4] \to 318.653 \left((8.\, a_4) / t_{s4} + \left(1. \left(16.\, t_{p4}^2 + 9.8696 t_{s4}^2\right)\right) / \left(t_{p4}^2 t_{s4}^2\right)\right),$
$K_{Int}[4] \to 318.653 \left(0. + \left(1.\, a_4 \left(16.\, t_{p4}^2 + 9.8696 t_{s4}^2\right)\right) / \left(t_{p4}^2 t_{s4}^2\right)\right),$
$K_{deriv}[4] \to 318.653 (1.\, a_4 + 8. / t_{s4})\}$

Now set the time constants.

In[879]:=
```
tConstRules[4] = {a4 → 10, tp4 → 1 / 4, ts4 → 1 / 2};
```

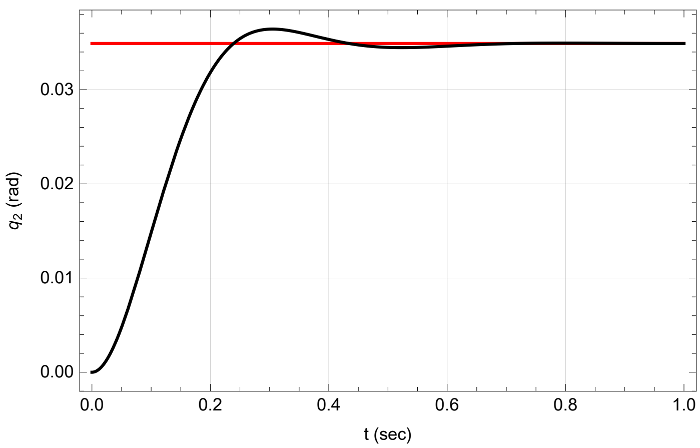
Check the step response

In[880]:=
```
t0 = -.00000001;
tf = 1;
stepMag = 2 π / 180; (*2 degrees*)
qd2[t_] := stepMag UnitStep[t]
stepSol[4] = NDSolve[{(eqG[4] /. solGains[4] /. tConstRules[4]) == 0,
    (eqM[4] /. solGains[4] /. tConstRules[4]) == 0,
    q2'[t0] == 0, q2[t0] == 0, Tm2[t0] == 0}, {q2, Tm2}, {t, t0, tf}]
```

Out[884]=
```
{{q2 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Tm2 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

In[885]:=
```
Plot[{qd2[t], q2[t] /. stepSol[4]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q2 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

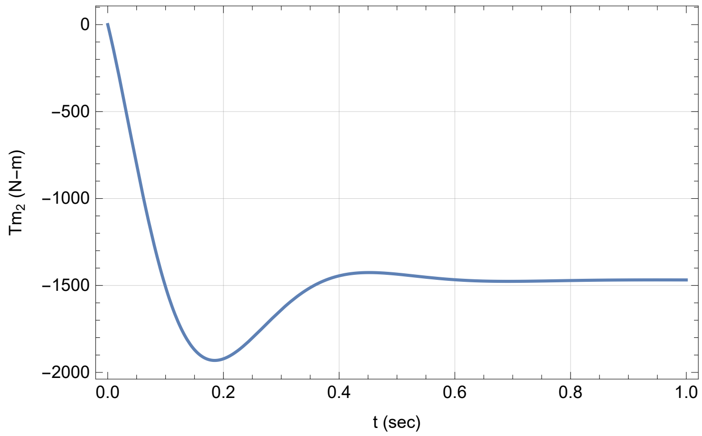
Out[885]=



We can see that the system displacement response is good. The desired value is in red. Notice here how the gravity load affects the settling time. There is more undershoot than for the motors wothout gravity load.

In[886]:=
```
Plot[Tm2[t] /. stepSol[4], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm2 (N-m)"}, PlotRange → All]
```

Out[886]=



This is the force needed to do this step maneuver. It is roughly 1600 ft-lb peak torque.

Check the tracking of a polynomial.

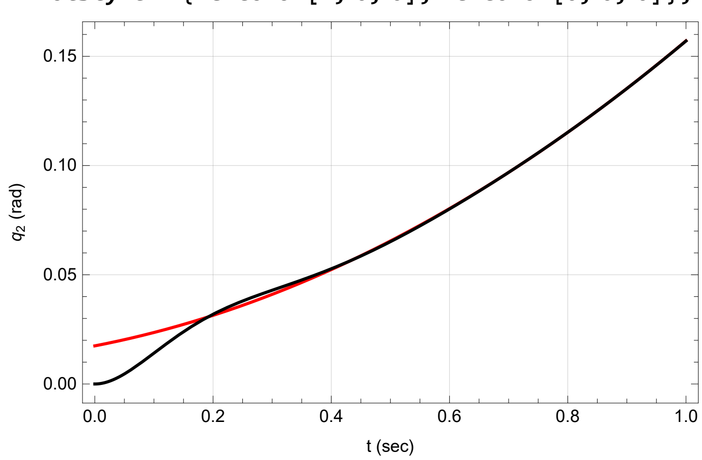
In[887]:=
```
t0 = -.00000001;
tf = 1;
qd2[t_] := (5 t^2 + 3 t + 1) π / 180 (*rad*)
trackSol[4] = NDSolve[{(eqG[4] /. solGains[4] /. tConstRules[4]) == 0,
   (eqM[4] /. solGains[4] /. tConstRules[4]) == 0,
   q2'[t0] == 0, q2[t0] == 0, Tm2[t0] == 0}, {q2, Tm2}, {t, t0, tf}]
```

Out[890]= {{q_2 → InterpolatingFunction[Domain: {{$-1. \times 10^{-8}$, 1.}} Output: scalar],

Tm$_2$ → InterpolatingFunction[Domain: {{$-1. \times 10^{-8}$, 1.}} Output: scalar]}}

In[891]:=
```
Plot[{qd2[t], q2[t] /. trackSol[4]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q2 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

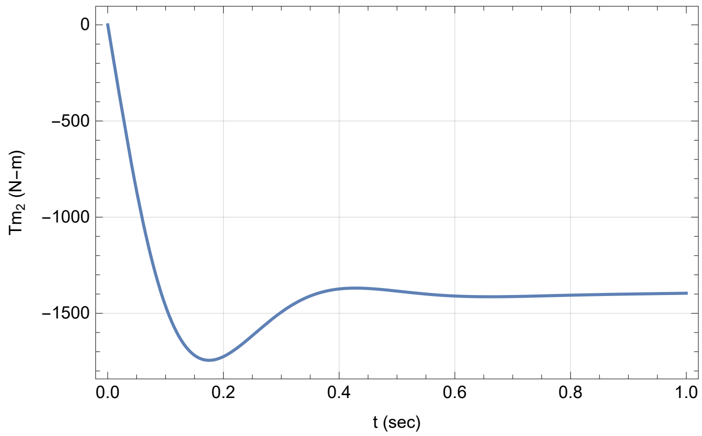
Out[891]=



We can see that the system tracks a parabola fairly well, could be better initially, however this initial error is due to the step change at t=0. So if we are nearer to the initial tracking curve, we should be ok.

In[892]:=
```
Plot[Tm2[t] /. trackSol[4], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm2 (N-m)"}, PlotRange → All]
```

Out[892]=



Clear the desired variable.

In[893]:=
```
qd2[t_] =.
```

Control gains q_3-motor

q_3-motor. Here we fixate all the other movements.

In[894]:=
```
eqG[5] =
 eqC[5] //. {x[t] → 0, x''[t] → 0, x''[t] → 0, y[t] → 0, y'[t] → 0, y''[t] → 0, q1[t] → 0,
     q1'[t] → 0, q1''[t] → 0, q2[t] → 0, q2'[t] → 0, q2''[t] → 0, q4[t] → 0, q4'[t] → 0,
     q4''[t] → 0, q5[t] → 0, q5'[t] → 0, q5''[t] → 0, q6[t] → 0, q6'[t] → 0, q6''[t] → 0} //.
   {Fmx[t] → 0, Fmy[t] → 0, Tm1[t] → 0, Tm2[t] → 0, Tm4[t] → 0, Tm5[t] → 0, Tm6[t] → 0} //.
  {Ftoolx → 0, Ftooly → 0, Ftoolz → 0, Ttoolx → 0, Ttooly → 0, Ttoolz → 0}
```

Out[894]= $0. + 161.976\,\mathrm{Cos}[q_3[t]] + Tm_3[t] - 19.7913\,q_3''[t] - 8.48869\,\left(\mathrm{Cos}[q_3[t]]^2 + \mathrm{Sin}[q_3[t]]^2\right) q_3''[t]$

Linearize the equation.

In[895]:=
```
eqL[5] = Normal[Series[eqG[5], {q3[t], 0, 1}, {q3'[t], 0, 1}]]
```

Out[895]= $161.976 + Tm_3[t] - 28.28\,q_3''[t]$

We take a derivative to allow substitution of the motor control state equation.

In[896]:=
```
eqLd[5] = D[eqL[5], t]
```

Out[896]= $Tm_3'[t] - 28.28\,q_3^{(3)}[t]$

Solve the motor state equation for the motor force/torque derivative.

In[897]:=
```
solM[5] = Solve[eqM[5] == 0, Tm3'[t]] // First
```

Out[897]= $\{Tm_3'[t] \to -K_{Int}[5]\,q_3[t] + K_{Int}[5]\,q_{d3}[t] - K_{prop}[5]\,q_3'[t] + K_{prop}[5]\,q_{d3}'[t] - K_{deriv}[5]\,q_3''[t] + K_{deriv}[5]\,q_{d3}''[t]\}$

Here is the complete linear equation for this motor in terms of the control gains.

In[898]:=
```
eqLf[5] = eqLd[5] //. solM[5]
```

Out[898]= $-K_{Int}[5]\, q_3[t] + K_{Int}[5]\, q_{d3}[t] - K_{prop}[5]\, q_3'[t] + K_{prop}[5]\, q_{d3}'[t] - K_{deriv}[5]\, q_3''[t] + K_{deriv}[5]\, q_{d3}''[t] - 28.28\, q_3^{(3)}[t]$

This is the characteristic polynomial.

In[899]:=
```
charEq[5] =
  eqLf[5] //. {Derivative[n_][q3][t] → s^n, q3[t] → 1, qd3[t] → 0, qd3'[t] → 0, qd3''[t] → 0}
```

Out[899]= $-28.28\, s^3 - s^2 K_{deriv}[5] - K_{Int}[5] - s\, K_{prop}[5]$

Put the characteristic equation in standard form.

In[900]:=
```
charEq[5] = charEq[5] / Coefficient[charEq[5], s^3] // Expand
```

Out[900]= $1.\, s^3 + 0.0353607\, s^2 K_{deriv}[5] + 0.0353607\, K_{Int}[5] + 0.0353607\, s\, K_{prop}[5]$

The desired characteristic polynomial is given by the following system with a first order pole and a second order pole pair.

In[901]:=
```
charDesired[5] = Expand[(s + a5) (s^2 + 2 ζ5 ωn5 s + ωn5^2)]
```

Out[901]= $s^3 + s^2 a_5 + 2 s^2 \zeta_5 \omega_{n5} + 2 s\, a_5 \zeta_5 \omega_{n5} + s\, \omega_{n5}^2 + a_5 \omega_{n5}^2$

The time to peak is defined as

In[902]:=
```
eqTp[5] = tp5 == π / (ωn5 √(1 - ζ5^2));
```

The two percent settling time is given by

In[903]:=
```
eqTs[5] = ts5 == 4 / (ζ5 ωn5);
```

In[904]:=
```
solZW[5] = Solve[{eqTp[5], eqTs[5]}, {ζ5, ωn5}][[2]]
```

Out[904]= $\left\{\zeta_5 \to (4\, t_{p5}) \Big/ \left(\sqrt{\left(16\, t_{p5}^2 + \pi^2 t_{s5}^2\right)}\right),\ \omega_{n5} \to \left(\sqrt{\left(16\, t_{p5}^2 + \pi^2 t_{s5}^2\right)}\right) \Big/ (t_{p5}\, t_{s5})\right\}$

Set up equations for gains by equating coefficients of the characterstic polynomials.

In[905]:=
```
eqGain1[5] = Coefficient[charEq[5], s^2] == Coefficient[charDesired[5], s^2] /. solZW[5]
```

Out[905]= $0.0353607\, K_{deriv}[5] == a_5 + 8 / t_{s5}$

In[906]:=
```
eqGain2[5] = Coefficient[charEq[5], s] == Coefficient[charDesired[5], s] /. solZW[5]
```

Out[906]= $0.0353607\, K_{prop}[5] == (8\, a_5) / t_{s5} + \left(16\, t_{p5}^2 + \pi^2 t_{s5}^2\right) / \left(t_{p5}^2\, t_{s5}^2\right)$

In[907]:=
```
eqGain3[5] = (charEq[5] /. s → 0) == (charDesired[5] /. s → 0) /. solZW[5]
```

Out[907]= $0. + 0.0353607\, K_{Int}[5] == \left(a_5 \left(16\, t_{p5}^2 + \pi^2 t_{s5}^2\right)\right) / \left(t_{p5}^2\, t_{s5}^2\right)$

Solve for the controller gains.

In[908]:=
```
solGains[5] =
 Solve[{eqGain1[5], eqGain2[5], eqGain3[5]}, {K_prop[5], K_Int[5], K_deriv[5]}] // First
```

Out[908]= $\{K_{prop}[5] \to 28.28\ ((8.\ a_5)/t_{s5} + (1.\ (16.\ t_{p5}^2 + 9.8696\ t_{s5}^2))/(t_{p5}^2\ t_{s5}^2)),$
$K_{Int}[5] \to 28.28\ (0. + (1.\ a_5\ (16.\ t_{p5}^2 + 9.8696\ t_{s5}^2))/(t_{p5}^2\ t_{s5}^2)),\ K_{deriv}[5] \to 28.28\ (1.\ a_5 + 8./t_{s5})\}$

Now set the time constants.

In[909]:=
```
tConstRules[5] = {a_5 → 10, t_p5 → 1/4, t_s5 → 1/2};
```

Check the step response

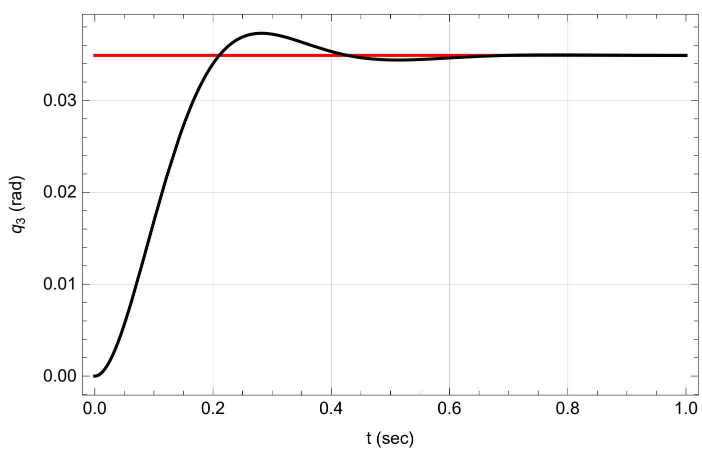
In[910]:=
```
t_0 = -.00000001;
t_f = 1;
stepMag = 2 π / 180; (*2 degrees*)
q_d3[t_] := stepMag UnitStep[t]
stepSol[5] = NDSolve[{(eqG[5] /. solGains[5] /. tConstRules[5]) == 0,
   (eqM[5] /. solGains[5] /. tConstRules[5]) == 0,
   q_3'[t_0] == 0, q_3[t_0] == 0, Tm_3[t_0] == 0}, {q_3, Tm_3}, {t, t_0, t_f}]
```

Out[914]= $\{\{q_3 \to$ InterpolatingFunction[Domain: $\{\{-1. \times 10^{-8}, 1.\}\}$ Output: scalar],

$Tm_3 \to$ InterpolatingFunction[Domain: $\{\{-1. \times 10^{-8}, 1.\}\}$ Output: scalar]$\}\}$

In[915]:=
```
Plot[{q_d3[t], q_3[t] /. stepSol[5]}, {t, t_0, t_f}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q_3 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

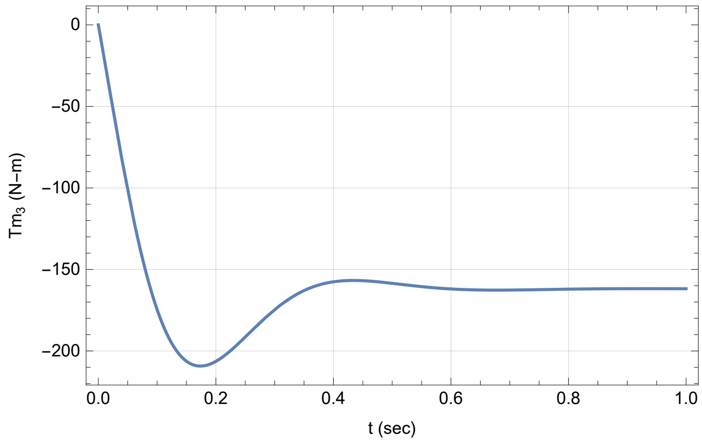
Out[915]=



We can see that the system displacement response is good. The desired value is in red. Notice here how the gravity load affects the settling time. There is more undershoot than for the motors wothout gravity load.

In[916]:=
```
Plot[Tm3[t] /. stepSol[5], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm3 (N-m)"}, PlotRange → All]
```

Out[916]=



This is the force needed to do this step maneuver. It is roughly 175 ft-lb peak torque.

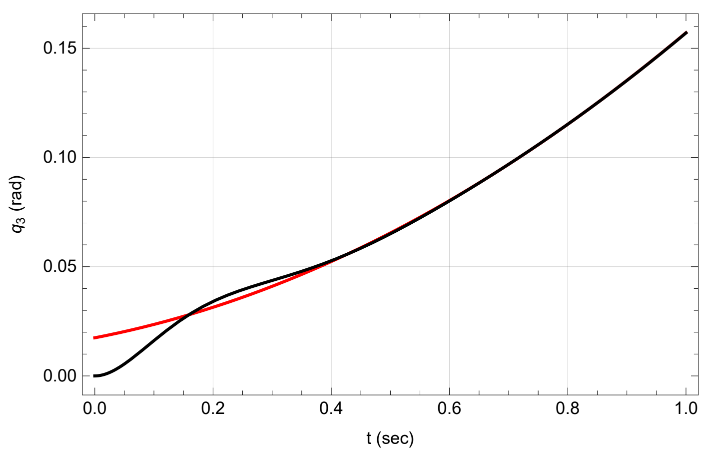
Check the tracking of a polynomial.

In[917]:=
```
t0 = -.00000001;
tf = 1;
qd3[t_] := (5 t^2 + 3 t + 1) π / 180 (*rad*)
trackSol[5] = NDSolve[{(eqG[5] /. solGains[5] /. tConstRules[5]) == 0,
   (eqM[5] /. solGains[5] /. tConstRules[5]) == 0,
   q3'[t0] == 0, q3[t0] == 0, Tm3[t0] == 0}, {q3, Tm3}, {t, t0, tf}]
```

Out[920]=
```
{{q3 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Tm3 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

In[921]:=
```
Plot[{qd3[t], q3[t] /. trackSol[5]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q3 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

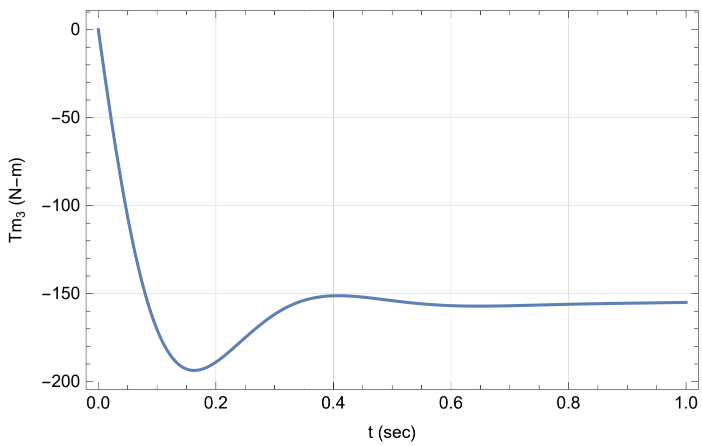
Out[921]=



We can see that the system tracks a parabola fairly well, could be better initially, however this initial error is due to the step change at t=0. So if we are nearer to the initial tracking curve, we should be ok.

In[922]:=
```
Plot[Tm3[t] /. trackSol[5], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm3 (N-m)"}, PlotRange → All]
```

Out[922]=



Clear the desired variable.

In[923]:=
```
qd3[t_] =.
```

Control gains q_4-motor

q_4-motor. Here we fixate all the other movements.

In[924]:=
```
eqG[6] =
 eqC[6] //. {x[t] → 0, x''[t] → 0, x''[t] → 0, y[t] → 0, y'[t] → 0, y''[t] → 0, q1[t] → 0,
     q1'[t] → 0, q1''[t] → 0, q2[t] → 0, q2'[t] → 0, q2''[t] → 0, q3[t] → 0, q3'[t] → 0,
     q3''[t] → 0, q5[t] → 0, q5'[t] → 0, q5''[t] → 0, q6[t] → 0, q6'[t] → 0, q6''[t] → 0} //.
   {Fmx[t] → 0, Fmy[t] → 0, Tm1[t] → 0, Tm2[t] → 0, Tm3[t] → 0, Tm5[t] → 0, Tm6[t] → 0} //.
  {Ftoolx → 0, Ftooly → 0, Ftoolz → 0, Ttoolx → 0, Ttooly → 0, Ttoolz → 0}
```

Out[924]= $0. + Tm_4[t] + (5/6)\,(0. - 1.92449\,q_4''[t]) + (1/3)\,(0. - 1.73204\,q_4''[t]) - 1.63302\,q_4''[t]$

Linearize the equation.

In[925]:=
```
eqL[6] = Normal[Series[eqG[6], {q4[t], 0, 1}, {q4'[t], 0, 1}]]
```

Out[925]= $0. + Tm_4[t] + (5/6)\,(0. - 1.92449\,q_4''[t]) + (1/3)\,(0. - 1.73204\,q_4''[t]) - 1.63302\,q_4''[t]$

We take a derivative to allow substitution of the motor control state equation.

In[926]:=
```
eqLd[6] = D[eqL[6], t]
```

Out[926]= $Tm_4'[t] - 3.81411\,q_4^{(3)}[t]$

Solve the motor state equation for the motor force/torque derivative.

In[927]:=
```
solM[6] = Solve[eqM[6] == 0, Tm4'[t]] // First
```

Out[927]= $\{Tm_4'[t] \to -K_{Int}[6]\,q_4[t] + K_{Int}[6]\,q_{d4}[t] - K_{prop}[6]\,q_4'[t] + K_{prop}[6]\,q_{d4}'[t] - K_{deriv}[6]\,q_4''[t] + K_{deriv}[6]\,q_{d4}''[t]\}$

Here is the complete linear equation for this motor in terms of the control gains.

In[928]:= `eqLf[6] = eqLd[6] //. solM[6]`

Out[928]= $-K_{Int}[6]\, q_4[t] + K_{Int}[6]\, q_{d4}[t] - K_{prop}[6]\, q_4'[t] + K_{prop}[6]\, q_{d4}'[t] - K_{deriv}[6]\, q_4''[t] + K_{deriv}[6]\, q_{d4}''[t] - 3.81411\, q_4^{(3)}[t]$

This is the characteristic polynomial.

In[929]:= `charEq[6] = eqLf[6] //. {Derivative[n_][q4][t] → s^n, q4[t] → 1, qd4[t] → 0, qd4'[t] → 0, qd4''[t] → 0}`

Out[929]= $-3.81411\, s^3 - s^2 K_{deriv}[6] - K_{Int}[6] - s\, K_{prop}[6]$

Put the characteristic equation in standard form.

In[930]:= `charEq[6] = charEq[6] / Coefficient[charEq[6], s^3] // Expand`

Out[930]= $1.\, s^3 + 0.262184\, s^2 K_{deriv}[6] + 0.262184\, K_{Int}[6] + 0.262184\, s\, K_{prop}[6]$

The desired characteristic polynomial is given by the following system with a first order pole and a second order pole pair.

In[931]:= `charDesired[6] = Expand[(s + a6) (s^2 + 2 ζ6 ωn6 s + ωn6^2)]`

Out[931]= $s^3 + s^2 a_6 + 2\, s^2 \zeta_6\, \omega_{n6} + 2\, s\, a_6\, \zeta_6\, \omega_{n6} + s\, \omega_{n6}^2 + a_6\, \omega_{n6}^2$

The time to peak is defined as

In[932]:= `eqTp[6] = tp6 == π / (ωn6 √(1 - ζ6^2));`

The two percent settling time is given by

In[933]:= `eqTs[6] = ts6 == 4 / (ζ6 ωn6);`

In[934]:= `solZW[6] = Solve[{eqTp[6], eqTs[6]}, {ζ6, ωn6}][[2]]`

Out[934]= $\left\{\zeta_6 \to (4\, t_{p6}) \Big/ \left(\sqrt{\left(16\, t_{p6}^2 + \pi^2 t_{s6}^2\right)}\right),\ \omega_{n6} \to \left(\sqrt{\left(16\, t_{p6}^2 + \pi^2 t_{s6}^2\right)}\right) \Big/ (t_{p6}\, t_{s6})\right\}$

Set up equations for gains by equating coefficients of the characterstic polynomials.

In[935]:= `eqGain1[6] = Coefficient[charEq[6], s^2] == Coefficient[charDesired[6], s^2] /. solZW[6]`

Out[935]= $0.262184\, K_{deriv}[6] == a_6 + 8 / t_{s6}$

In[936]:= `eqGain2[6] = Coefficient[charEq[6], s] == Coefficient[charDesired[6], s] /. solZW[6]`

Out[936]= $0.262184\, K_{prop}[6] == (8\, a_6) / t_{s6} + \left(16\, t_{p6}^2 + \pi^2 t_{s6}^2\right) \Big/ \left(t_{p6}^2\, t_{s6}^2\right)$

In[937]:= `eqGain3[6] = (charEq[6] /. s → 0) == (charDesired[6] /. s → 0) /. solZW[6]`

Out[937]= $0. + 0.262184\, K_{Int}[6] == \left(a_6 \left(16\, t_{p6}^2 + \pi^2 t_{s6}^2\right)\right) \Big/ \left(t_{p6}^2\, t_{s6}^2\right)$

Solve for the controller gains.

In[938]:=
```
solGains[6] =
  Solve[{eqGain1[6], eqGain2[6], eqGain3[6]}, {Kprop[6], KInt[6], Kderiv[6]}] // First
```

Out[938]=
$$\left\{K_{prop}[6] \to 3.81411\left((8.\,a_6)/t_{s6} + \left(1.\left(16.\,t_{p6}^2 + 9.8696\,t_{s6}^2\right)\right)\Big/\left(t_{p6}^2\,t_{s6}^2\right)\right),\right.$$
$$K_{Int}[6] \to 3.81411\left(0. + \left(1.\,a_6\left(16.\,t_{p6}^2 + 9.8696\,t_{s6}^2\right)\right)\Big/\left(t_{p6}^2\,t_{s6}^2\right)\right),$$
$$\left.K_{deriv}[6] \to 3.81411\,(1.\,a_6 + 8./t_{s6})\right\}$$

Now set the time constants.

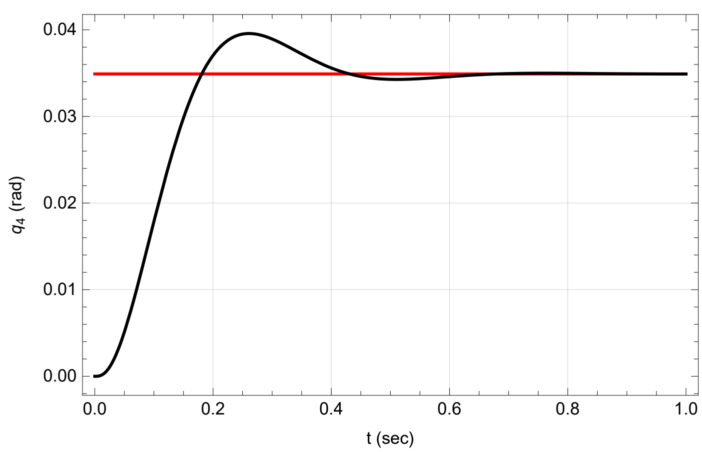
In[939]:=
```
tConstRules[6] = {a6 → 100, tp6 → 1/4, ts6 → 1/2};
```

Check the step response

In[940]:=
```
t0 = -.00000001;
tf = 1;
stepMag = 2 π / 180; (*2 degrees*)
qd4[t_] := stepMag UnitStep[t]
stepSol[6] = NDSolve[{(eqG[6] /. solGains[6] /. tConstRules[6]) == 0,
   (eqM[6] /. solGains[6] /. tConstRules[6]) == 0,
   q4'[t0] == 0, q4[t0] == 0, Tm4[t0] == 0}, {q4, Tm4}, {t, t0, tf}]
```

Out[944]=
```
{{q4 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Tm4 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

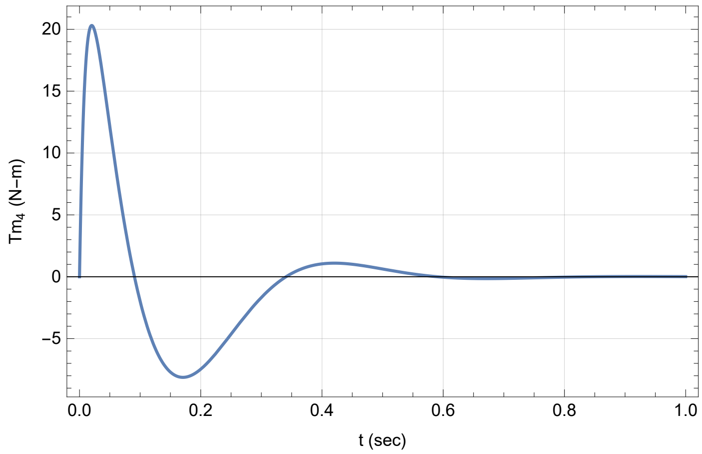
In[945]:=
```
Plot[{qd4[t], q4[t] /. stepSol[6]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q4 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

Out[945]=


We can see that the system displacement response is good. The desired value is in red.

In[946]:=
```
Plot[Tm4[t] /. stepSol[6], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm4 (N-m)"}, PlotRange → All]
```

Out[946]=



This is the force needed to do this step maneuver. It is roughly 7.5 ft-lb peak torque.

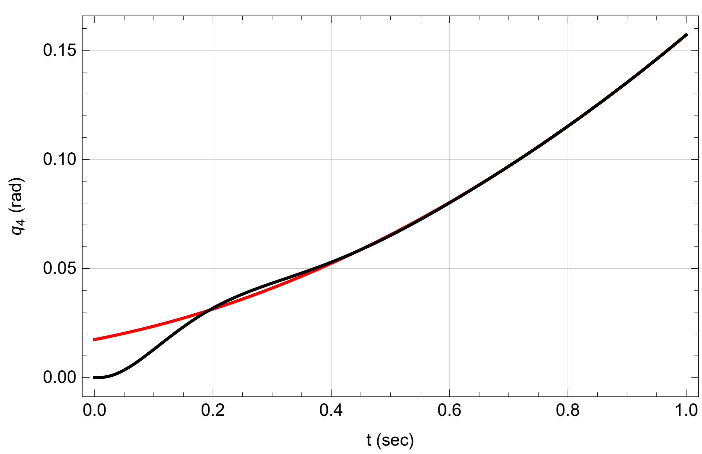
Check the tracking of a polynomial.

In[947]:=
```
t0 = -.00000001;
tf = 1;
qd4[t_] := (5 t^2 + 3 t + 1) π / 180 (*rad*)
trackSol[6] = NDSolve[{(eqG[6] /. solGains[6] /. tConstRules[6]) == 0,
   (eqM[6] /. solGains[6] /. tConstRules[6]) == 0,
   q4'[t0] == 0, q4[t0] == 0, Tm4[t0] == 0}, {q4, Tm4}, {t, t0, tf}]
```

Out[950]=
```
{{q4 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Tm4 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

In[951]:=
```
Plot[{qd4[t], q4[t] /. trackSol[6]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q4 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

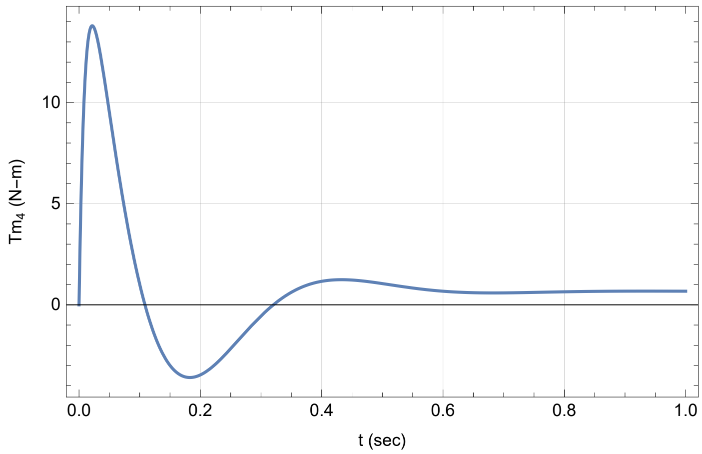
Out[951]=



We can see that the system tracks a parabola fairly well, could be better initially, however this initial error is due to the step change at t=0. So if we are nearer to the initial tracking curve, we should be ok.

In[952]:=
```
Plot[Tm4[t] /. trackSol[6], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm4 (N-m)"}, PlotRange → All]
```

Out[952]=



Clear the desired variable.

In[953]:=
```
qd4[t_] =.
```

Control gains q_5-motor

q_5-motor. Here we fixate all the other movements.

In[954]:=
```
eqG[7] =
 eqC[7] //. {x[t] → 0, x'[t] → 0, x''[t] → 0, y[t] → 0, y'[t] → 0, y''[t] → 0, q1[t] → 0,
     q1'[t] → 0, q1''[t] → 0, q2[t] → 0, q2'[t] → 0, q2''[t] → 0, q3[t] → 0, q3'[t] → 0,
     q3''[t] → 0, q4[t] → 0, q4'[t] → 0, q4''[t] → 0, q6[t] → 0, q6'[t] → 0, q6''[t] → 0} //.
   {Fmx[t] → 0, Fmy[t] → 0, Tm1[t] → 0, Tm2[t] → 0, Tm3[t] → 0, Tm4[t] → 0, Tm6[t] → 0} //.
  {Ftoolx → 0, Ftooly → 0, Ftoolz → 0, Ttoolx → 0, Ttooly → 0, Ttoolz → 0}
```

Out[954]= $0. + Tm_5[t] + (5/21)\,(0. + 19.4186\,\mathrm{Cos}[q_5[t]] - 0.471303\,q_5''[t]) - 0.0803616\,q_5''[t]$

Linearize the equation.

In[955]:=
```
eqL[7] = Normal[Series[eqG[7], {q5[t], 0, 1}, {q5'[t], 0, 1}]]
```

Out[955]= $4.62348 + Tm_5[t] - 0.192577\,q_5''[t]$

We take a derivative to allow substitution of the motor control state equation.

In[956]:=
```
eqLd[7] = D[eqL[7], t]
```

Out[956]= $Tm_5'[t] - 0.192577\,q_5^{(3)}[t]$

Solve the motor state equation for the motor force/torque derivative.

In[957]:=
```
solM[7] = Solve[eqM[7] == 0, Tm5'[t]] // First
```

Out[957]= $\{Tm_5'[t] \to -K_{Int}[7]\,q_5[t] + K_{Int}[7]\,q_{d5}[t] - K_{prop}[7]\,q_5'[t] + K_{prop}[7]\,q_{d5}'[t] - K_{deriv}[7]\,q_5''[t] + K_{deriv}[7]\,q_{d5}''[t]\}$

Here is the complete linear equation for this motor in terms of the control gains.

In[958]:= `eqLf[7] = eqLd[7] //. solM[7]`

Out[958]= $-K_{Int}[7]\, q_5[t] + K_{Int}[7]\, q_{d5}[t] - K_{prop}[7]\, q_5'[t] + K_{prop}[7]\, q_{d5}'[t] - K_{deriv}[7]\, q_5''[t] + K_{deriv}[7]\, q_{d5}''[t] - 0.192577\, q_5^{(3)}[t]$

This is the characteristic polynomial.

In[959]:= `charEq[7] =` `eqLf[7] //. {Derivative[n_][q5][t] → s^n, q5[t] → 1, qd5[t] → 0, qd5'[t] → 0, qd5''[t] → 0}`

Out[959]= $-0.192577\, s^3 - s^2 K_{deriv}[7] - K_{Int}[7] - s\, K_{prop}[7]$

Put the characteristic equation in standard form.

In[960]:= `charEq[7] = charEq[7] / Coefficient[charEq[7], s^3] // Expand`

Out[960]= $1.\, s^3 + 5.19274\, s^2 K_{deriv}[7] + 5.19274\, K_{Int}[7] + 5.19274\, s\, K_{prop}[7]$

The desired characteristic polynomial is given by the following system with a first order pole and a second order pole pair.

In[961]:= `charDesired[7] = Expand[(s + a7) (s^2 + 2 ζ7 ωn7 s + ωn7^2)]`

Out[961]= $s^3 + s^2 a_7 + 2\, s^2 \zeta_7\, \omega_{n7} + 2\, s\, a_7\, \zeta_7\, \omega_{n7} + s\, \omega_{n7}^2 + a_7\, \omega_{n7}^2$

The time to peak is defined as

In[962]:= `eqTp[7] = tp7 == π / (ωn7 √(1 - ζ7^2));`

The two percent settling time is given by

In[963]:= `eqTs[7] = ts7 == 4 / (ζ7 ωn7);`

In[964]:= `solZW[7] = Solve[{eqTp[7], eqTs[7]}, {ζ7, ωn7}][[2]]`

Out[964]= $\left\{\zeta_7 \to (4\, t_{p7}) \Big/ \left(\sqrt{\left(16\, t_{p7}^2 + \pi^2 t_{s7}^2\right)}\right),\ \omega_{n7} \to \left(\sqrt{\left(16\, t_{p7}^2 + \pi^2 t_{s7}^2\right)}\right) \Big/ (t_{p7}\, t_{s7})\right\}$

Set up equations for gains by equating coefficients of the characterstic polynomials.

In[965]:= `eqGain1[7] = Coefficient[charEq[7], s^2] == Coefficient[charDesired[7], s^2] /. solZW[7]`

Out[965]= $5.19274\, K_{deriv}[7] == a_7 + 8 / t_{s7}$

In[966]:= `eqGain2[7] = Coefficient[charEq[7], s] == Coefficient[charDesired[7], s] /. solZW[7]`

Out[966]= $5.19274\, K_{prop}[7] == (8\, a_7) / t_{s7} + \left(16\, t_{p7}^2 + \pi^2 t_{s7}^2\right) \Big/ \left(t_{p7}^2\, t_{s7}^2\right)$

In[967]:= `eqGain3[7] = (charEq[7] /. s → 0) == (charDesired[7] /. s → 0) /. solZW[7]`

Out[967]= $0. + 5.19274\, K_{Int}[7] == \left(a_7 \left(16\, t_{p7}^2 + \pi^2 t_{s7}^2\right)\right) \Big/ \left(t_{p7}^2\, t_{s7}^2\right)$

Solve for the controller gains.

In[968]:=
```
solGains[7] =
  Solve[{eqGain1[7], eqGain2[7], eqGain3[7]}, {Kprop[7], KInt[7], Kderiv[7]}] // First
```

Out[968]=
$$\left\{K_{prop}[7] \to 0.192577\left((8.\,a_7)/t_{s7} + \left(1.\left(16.\,t_{p7}^2 + 9.8696\,t_{s7}^2\right)\right)/\left(t_{p7}^2\,t_{s7}^2\right)\right),\right.$$
$$K_{Int}[7] \to 0.192577\left(0. + \left(1.\,a_7\left(16.\,t_{p7}^2 + 9.8696\,t_{s7}^2\right)\right)/\left(t_{p7}^2\,t_{s7}^2\right)\right),$$
$$\left.K_{deriv}[7] \to 0.192577\,(1.\,a_7 + 8./t_{s7})\right\}$$

Now set the time constants. For this smaller system I had to make the first order pole mush faster to decrease overshoot and tracking error.

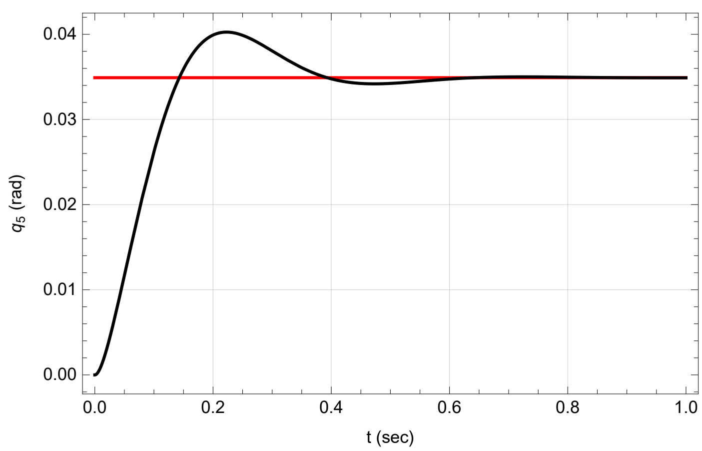
In[969]:=
```
tConstRules[7] = {a7 → 100, tp7 → 1/4, ts7 → 1/2};
```

Check the step response

In[970]:=
```
t0 = -.00000001;
tf = 1;
stepMag = 2 π / 180; (*2 degrees*)
qd5[t_] := stepMag UnitStep[t]
stepSol[7] = NDSolve[{(eqG[7] /. solGains[7] /. tConstRules[7]) == 0,
    (eqM[7] /. solGains[7] /. tConstRules[7]) == 0,
   q5'[t0] == 0, q5[t0] == 0, Tm5[t0] == 0}, {q5, Tm5}, {t, t0, tf}]
```

Out[974]=
```
{{q5 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Tm5 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

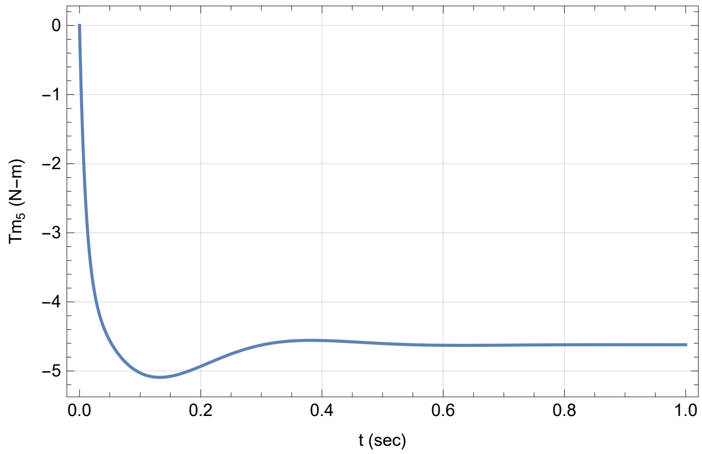
In[975]:=
```
Plot[{qd5[t], q5[t] /. stepSol[7]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q5 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

Out[975]=


We can see that the system displacement response is good. The desired value is in red.

In[976]:=
```
Plot[Tm5[t] /. stepSol[7], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm5 (N-m)"}, PlotRange → All]
```

Out[976]=



This is the force needed to do this step maneuver. It is roughly 4.5 ft-lb peak torque.

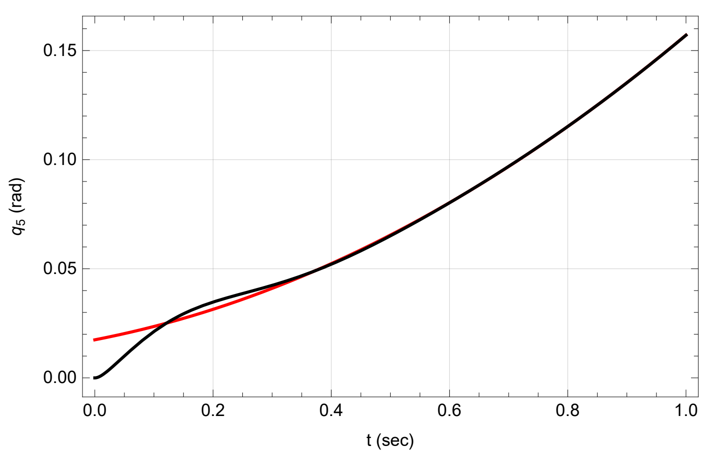
Check the tracking of a polynomial.

In[977]:=
```
t0 = -.00000001;
tf = 1;
qd5[t_] := (5 t^2 + 3 t + 1) π / 180 (*rad*)
trackSol[7] = NDSolve[{(eqG[7] /. solGains[7] /. tConstRules[7]) == 0,
   (eqM[7] /. solGains[7] /. tConstRules[7]) == 0,
   q5'[t0] == 0, q5[t0] == 0, Tm5[t0] == 0}, {q5, Tm5}, {t, t0, tf}]
```

Out[980]=
```
{{q5 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Tm5 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

In[981]:=
```
Plot[{qd5[t], q5[t] /. trackSol[7]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q5 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

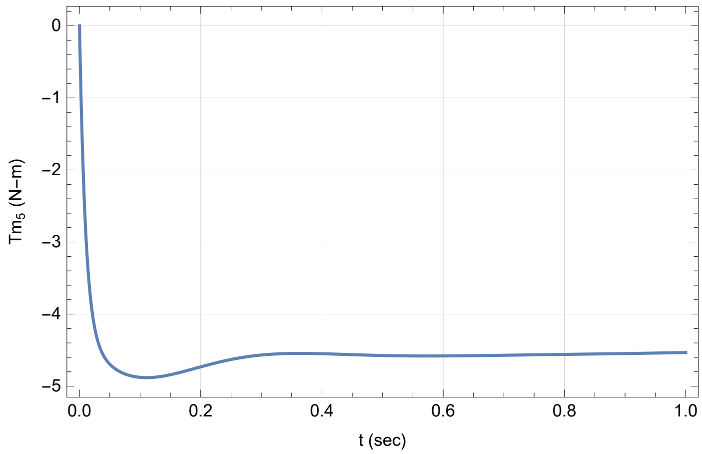
Out[981]=



We can see that the system tracks a parabola fairly well, could be better initially, however this initial error is due to the step change at t=0. So if we are nearer to the initial tracking curve, we should be ok.

In[982]:=
```
Plot[Tm5[t] /. trackSol[7], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm5 (N-m)"}, PlotRange → All]
```

Out[982]=



Clear the desired variable.

In[983]:=
```
qd5[t_] =.
```

Control gains q_6-motor

q_6-motor. Here we fixate all the other movements.

In[984]:=
```
eqG[8] =
 eqC[8] //. {x[t] → 0, x'[t] → 0, x''[t] → 0, y[t] → 0, y'[t] → 0, y''[t] → 0, q1[t] → 0,
     q1'[t] → 0, q1''[t] → 0, q2[t] → 0, q2'[t] → 0, q2''[t] → 0, q3[t] → 0, q3'[t] → 0,
     q3''[t] → 0, q4[t] → 0, q4'[t] → 0, q4''[t] → 0, q5[t] → 0, q5'[t] → 0, q5''[t] → 0} //.
   {Fmx[t] → 0, Fmy[t] → 0, Tm1[t] → 0, Tm2[t] → 0, Tm3[t] → 0, Tm4[t] → 0, Tm5[t] → 0} //.
  {Ftoolx → 0, Ftooly → 0, Ftoolz → 0, Ttoolx → 0, Ttooly → 0, Ttoolz → 0}
```

Out[984]= $0. + Tm_6[t] - 0.0449169\, q_6''[t]$

Linearize the equation.

In[985]:=
```
eqL[8] = Normal[Series[eqG[8], {q6[t], 0, 1}, {q6'[t], 0, 1}]]
```

Out[985]= $0. + Tm_6[t] - 0.0449169\, q_6''[t]$

We take a derivative to allow substitution of the motor control state equation.

In[986]:=
```
eqLd[8] = D[eqL[8], t]
```

Out[986]= $Tm_6'[t] - 0.0449169\, q_6^{(3)}[t]$

Solve the motor state equation for the motor force/torque derivative.

In[987]:=
```
solM[8] = Solve[eqM[8] == 0, Tm6'[t]] // First
```

Out[987]= $\{Tm_6'[t] \to -K_{Int}[8]\, q_6[t] + K_{Int}[8]\, q_{d6}[t] - K_{prop}[8]\, q_6'[t] + K_{prop}[8]\, q_{d6}'[t] - K_{deriv}[8]\, q_6''[t] + K_{deriv}[8]\, q_{d6}''[t]\}$

Here is the complete linear equation for this motor in terms of the control gains.

In[988]:= `eqLf[8] = eqLd[8] //. solM[8]`

Out[988]= $-K_{Int}[8]\, q_6[t] + K_{Int}[8]\, q_{d6}[t] - K_{prop}[8]\, q_6'[t] + K_{prop}[8]\, q_{d6}'[t] - K_{deriv}[8]\, q_6''[t] + K_{deriv}[8]\, q_{d6}''[t] - 0.0449169\, q_6^{(3)}[t]$

This is the characteristic polynomial.

In[989]:= `charEq[8] = eqLf[8] //. {Derivative[n_][q6][t] → s^n, q6[t] → 1, qd6[t] → 0, qd6'[t] → 0, qd6''[t] → 0}`

Out[989]= $-0.0449169\, s^3 - s^2 K_{deriv}[8] - K_{Int}[8] - s\, K_{prop}[8]$

Put the characteristic equation in standard form.

In[990]:= `charEq[8] = charEq[8] / Coefficient[charEq[8], s^3] // Expand`

Out[990]= $1.\, s^3 + 22.2633\, s^2 K_{deriv}[8] + 22.2633\, K_{Int}[8] + 22.2633\, s\, K_{prop}[8]$

The desired characteristic polynomial is given by the following system with a first order pole and a second order pole pair.

In[991]:= `charDesired[8] = Expand[(s + a8) (s^2 + 2 ζ8 ωn8 s + ωn8^2)]`

Out[991]= $s^3 + s^2 a_8 + 2 s^2 \zeta_8 \omega_{n8} + 2 s\, a_8 \zeta_8 \omega_{n8} + s\, \omega_{n8}^2 + a_8 \omega_{n8}^2$

The time to peak is defined as

In[992]:= `eqTp[8] = tp8 == π / (ωn8 √(1 - ζ8^2));`

The two percent settling time is given by

In[993]:= `eqTs[8] = ts8 == 4 / (ζ8 ωn8);`

In[994]:= `solZW[8] = Solve[{eqTp[8], eqTs[8]}, {ζ8, ωn8}][[2]]`

Out[994]= $\left\{\zeta_8 \to (4\, t_{p8}) \Big/ \left(\sqrt{\left(16\, t_{p8}^2 + \pi^2 t_{s8}^2\right)}\right),\ \omega_{n8} \to \left(\sqrt{\left(16\, t_{p8}^2 + \pi^2 t_{s8}^2\right)}\right) \Big/ \left(t_{p8}\, t_{s8}\right)\right\}$

Set up equations for gains by equating coefficients of the characterstic polynomials.

In[995]:= `eqGain1[8] = Coefficient[charEq[8], s^2] == Coefficient[charDesired[8], s^2] /. solZW[8]`

Out[995]= $22.2633\, K_{deriv}[8] == a_8 + 8 / t_{s8}$

In[996]:= `eqGain2[8] = Coefficient[charEq[8], s] == Coefficient[charDesired[8], s] /. solZW[8]`

Out[996]= $22.2633\, K_{prop}[8] == (8\, a_8) / t_{s8} + \left(16\, t_{p8}^2 + \pi^2 t_{s8}^2\right) / \left(t_{p8}^2\, t_{s8}^2\right)$

In[997]:= `eqGain3[8] = (charEq[8] /. s → 0) == (charDesired[8] /. s → 0) /. solZW[8]`

Out[997]= $0. + 22.2633\, K_{Int}[8] == \left(a_8 \left(16\, t_{p8}^2 + \pi^2 t_{s8}^2\right)\right) / \left(t_{p8}^2\, t_{s8}^2\right)$

Solve for the controller gains.

In[998]:=
```
solGains[8] =
 Solve[{eqGain1[8], eqGain2[8], eqGain3[8]}, {K_prop[8], K_Int[8], K_deriv[8]}] // First
```

Out[998]=
$$\{K_{prop}[8] \to 0.0449169\ \left((8.\ a_8)/t_{s8} + \left(1.\ (16.\ t_{p8}^2 + 9.8696\ t_{s8}^2)\right) \Big/ \left(t_{p8}^2\ t_{s8}^2\right)\right),$$
$$K_{Int}[8] \to 0.0449169\ \left(0. + \left(1.\ a_8\ (16.\ t_{p8}^2 + 9.8696\ t_{s8}^2)\right) \Big/ \left(t_{p8}^2\ t_{s8}^2\right)\right),$$
$$K_{deriv}[8] \to 0.0449169\ (1.\ a_8 + 8./t_{s8})\}$$

Now set the time constants.

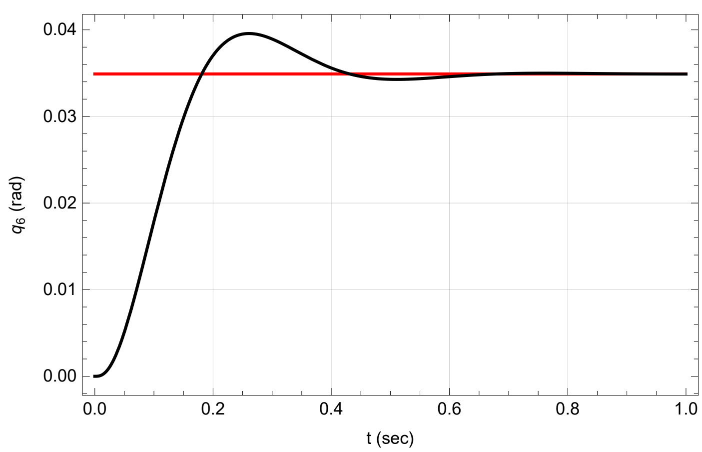
In[999]:=
```
tConstRules[8] = {a_8 → 100, t_p8 → 1 / 4, t_s8 → 1 / 2};
```

Check the step response

In[1000]:=
```
t_0 = -.00000001;
t_f = 1;
stepMag = 2 π / 180; (*2 degrees*)
q_d6[t_] := stepMag UnitStep[t]
stepSol[8] = NDSolve[{(eqG[8] /. solGains[8] /. tConstRules[8]) == 0,
   (eqM[8] /. solGains[8] /. tConstRules[8]) == 0,
   q_6'[t_0] == 0, q_6[t_0] == 0, Tm_6[t_0] == 0}, {q_6, Tm_6}, {t, t_0, t_f}]
```

Out[1004]=
```
{{q_6 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Tm_6 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

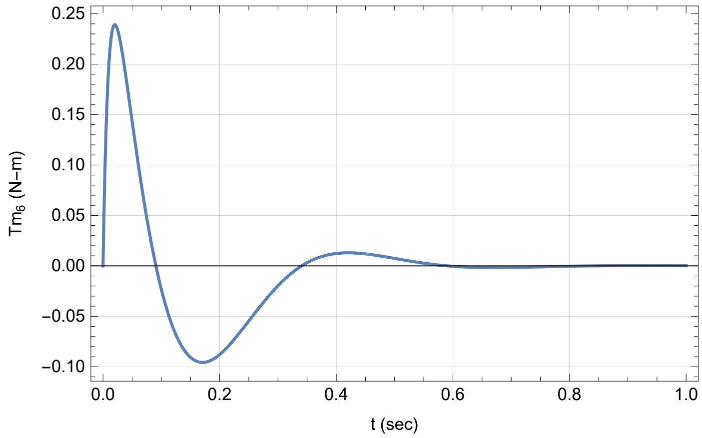
In[1005]:=
```
Plot[{q_d6[t], q_6[t] /. stepSol[8]}, {t, t_0, t_f}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q_6 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

Out[1005]=


We can see that the system displacement response is good. The desired value is in red.

In[1006]:=
```
Plot[Tm6[t] /. stepSol[8], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm6 (N-m)"}, PlotRange → All]
```

Out[1006]=



This is the force needed to do this step maneuver. It is roughly 0.2 ft-lb peak torque.

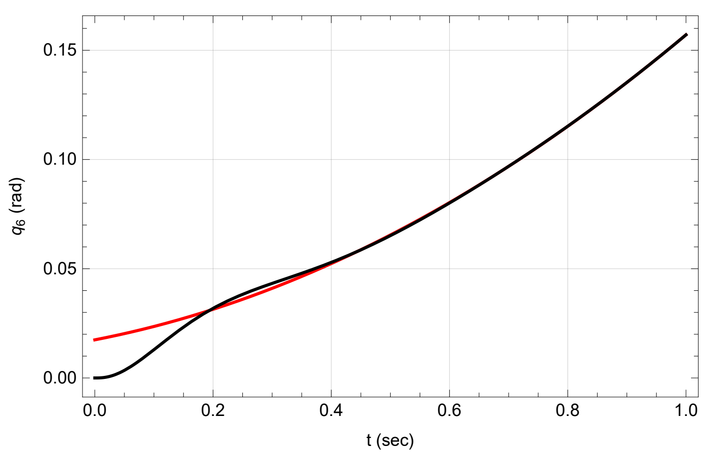
Check the tracking of a polynomial.

In[1007]:=
```
t0 = -.00000001;
tf = 1;
qd6[t_] := (5 t^2 + 3 t + 1) π / 180 (*rad*)
trackSol[8] = NDSolve[{(eqG[8] /. solGains[8] /. tConstRules[8]) == 0,
   (eqM[8] /. solGains[8] /. tConstRules[8]) == 0,
   q6'[t0] == 0, q6[t0] == 0, Tm6[t0] == 0}, {q6, Tm6}, {t, t0, tf}]
```

Out[1010]=
```
{{q6 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ],
  Tm6 → InterpolatingFunction[ Domain: {{-1.×10^-8, 1.}} Output: scalar ]}}
```

In[1011]:=
```
Plot[{qd6[t], q6[t] /. trackSol[8]}, {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "q6 (rad)"},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}, PlotRange → All]
```

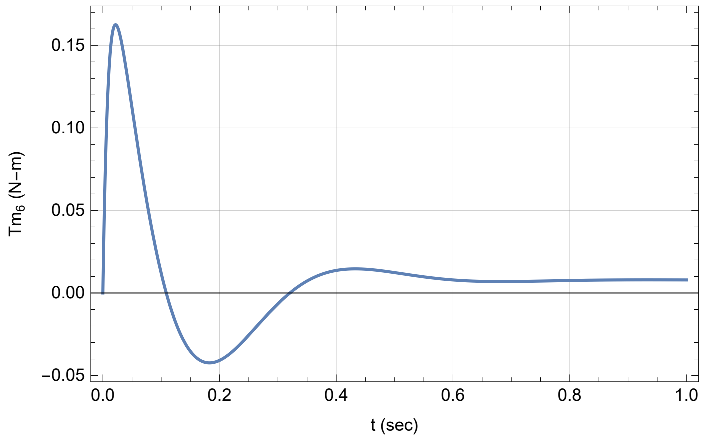
Out[1011]=



We can see that the system tracks a parabola fairly well, could be better initially, however this initial error is due to the step change at t=0. So if we are nearer to the initial tracking curve, we should be ok.

In[1012]:=
```
Plot[Tm6[t] /. trackSol[8], {t, t0, tf}, Frame → True,
 GridLines → Automatic, FrameLabel → {"t (sec)", "Tm6 (N-m)"}, PlotRange → All]
```

Out[1012]=



Clear the desired variable.

In[1013]:=
```
qd6[t_] =.
```