

Discrete Mathematics

Peergrade assignment 1

1. Solve for x the following equations. Present the result in the simplest form.

(a) $\frac{5}{3}x + \frac{2}{5} = 7$
(b) $\frac{4}{1+x} + \frac{15}{4} = 5$
(c) $|\frac{4x}{5}| = (\frac{1}{2} - \frac{1}{3})^{-1}$
(d) $\frac{x-3}{3} = 2(\frac{2+x}{5} + 1)$

Note: you need to show *how* you obtained the solution. Mark each step with a justification.

2. Show that \rightarrow and \sim form a *functionally complete* set of logical operators, that is that we can express other logical operators using *only* these two.

To do that, find formulas written using only p , q , \rightarrow and \sim , which encode

- (a) disjunction¹ $p \vee q$,
(b) conjunction $p \wedge q$ and
(c) biconditional $p \leftrightarrow q$

In each step, use a truth table to show that your formula is logically equivalent to the operator it encodes.

3. Use the logical equivalences in Figure 1 below to verify the following logical equivalences step by step. Supply a reason for each step. i.e. state, which of the equivalences from Figure 1 you used.

- (a) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
(b) $(p \wedge \sim q) \rightarrow r \equiv (p \wedge \sim r) \rightarrow q$
(c) $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$
(d) $\sim((p \rightarrow q) \wedge \sim q) \vee \sim p \equiv \mathbf{t}$

¹Hint: start by looking at Figure 1, particularly the Conditional (12) law.

Given any statement variables p , q , and r , a tautology \mathbf{t} and a contradiction \mathbf{c} , the following logical equivalences hold:

1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. Negation laws:	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. Double negative law:	$\sim(\sim p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. Universal bound laws:	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. De Morgan's laws:	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of \mathbf{t} and \mathbf{c} :	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$
12. Conditional:	$\sim p \vee q \equiv p \rightarrow q$	$p \rightarrow q \equiv \sim q \rightarrow \sim p$
13. Biconditional:	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

Figure 1: Logical equivalences.

For example, to verify the equivalence $(p \wedge q) \wedge q \equiv p \wedge q$, you would write:

$$\begin{array}{llll}
 (p \wedge q) \wedge q & & & (p \wedge q) \wedge q \\
 \equiv p \wedge (q \wedge q) & (2) & \text{or} & \stackrel{(2)}{\equiv} p \wedge (q \wedge q) \\
 \equiv p \wedge q & (7) & & \stackrel{(7)}{\equiv} p \wedge q
 \end{array}$$