Peergrade assignment 1

September 9, 2024

Exercise 1

a. $\frac{5}{3}x + \frac{2}{5} = 7$

$15 \cdot \frac{5}{3}x + 15 \cdot \frac{2}{5} = 15 \cdot 7$ Multiply all terms by the LCM[1] of denominators

$5 \cdot 5x + 3 \cdot 2 = 15 \cdot 7$ Reduce fractions

$25x + 6 = 105$ Perform multiplications

$(25x + 6) - 6 = (105) - 6$ Subtract 6 from each side

$25x = 99$ Perform subtraction

$\frac{25x}{25} = \frac{99}{25}$ Divide each side by 25

$x = \frac{99}{25}$ Reduce fractions where possible

b. $\frac{4}{1+x} + \frac{15}{4} = 5$

$4 \cdot (1+x) \cdot \frac{4}{1+x} + 4 \cdot (1+x) \cdot \frac{15}{4} = 4 \cdot (1+x) \cdot 5$ Multiply all terms by the LCM of the denominators

$4 \cdot 4 + (1+x) \cdot 15 = 4 \cdot (1+x) \cdot 5$ Reduce fractions

$16 + 15 + 15x = 20 + 20x$ Perform multiplications (use distributive property)

$31 + 15x = 20 + 20x$ Add terms where appropriate

$(31 + 15x) - 20 = (20 + 20x) - 20$ Subtract 20 from each side

[1] Least Common Multiple

$(11 + 15x) - 15x = (20x) - 15x$ Subtract 15x from each side

$11 = 5x$ Perform subtraction

$\frac{11}{5} = \frac{5x}{5}$ Divide each side by 5

$\frac{11}{5} = x$ Reduce fractions where possible

c. $|\frac{4x}{5}| = (\frac{1}{2} - \frac{1}{3})^{-1}$

$(\frac{1}{2} - \frac{1}{3})^{-1}$ Isolate right side

$(\frac{3}{3} \cdot \frac{1}{2} - \frac{2}{2} \cdot \frac{1}{3})^{-1}$ Multiply each fraction with the denominator of the other (also expressed as fraction)

$(\frac{3}{6} - \frac{2}{6})^{-1}$ Perform multiplications

$(\frac{1}{6})^{-1}$ Perform subtraction

6 A negative exponent indicates the reciprocal of the base

$|\frac{4x}{5}| = 6$ Rewrite the initial equation

$\frac{4x}{5} = 6$ or $\frac{4x}{5} = -6$ Remove absolute

$5 \cdot \frac{4x}{5} = 5 \cdot 6$ or $5 \cdot \frac{4x}{5} = 5 \cdot (-6)$ Multiply each side by 5

$4x = 30$ or $4x = -30$ Perform multiplications and reduce fractions

$\frac{4x}{4} = \frac{30}{4}$ or $\frac{4x}{4} = -\frac{30}{4}$ Divide each side by 4

$x = \frac{15}{2}$ or $x = -\frac{15}{2}$ Reduce fractions

d. $\frac{x-3}{3} = 2(\frac{2+x}{5} + 1)$

$\frac{x-3}{3} = \frac{4+2x}{5} + 2$ Apply distributive property on the right side

$15 \cdot (\frac{x-3}{3}) = 15 \cdot (\frac{4+2x}{5}) + 15 \cdot 2$ Multiply all terms by the LCM of the denominators

$5(x-3) = 3(4+2x) + 15 \cdot 2$ Reduce fractions

$5x - 15 = 12 + 6x + 30$ Perform multiplications (apply distributive property)

$(5x-15) + 15 = (12 + 6x + 30) + 15$ Add 15 to each side

$5x = 6x + 57$ Add terms where appropriate

$(5x) - 5x = (6x + 57) - 5x$ Subtract 5x from each side

$0 = x + 57$ Perform subtraction

$-57 = (x + 57) - 57$ Subtract 57 from each side

$-57 = x$ Perform subtraction

Exercise 2

a. $p \vee q$

$\equiv \neg(\neg p) \vee q$ by the double negative law (6)

$\equiv \neg p \to q$ by the conditional logical equivalence (12)

p	q	$\neg p$	$p \vee q$	$\neg(\neg p)$	$\neg(\neg p) \vee q$	$\neg p \to q$
T	T	F	T	T	T	T
T	F	F	T	T	T	T
F	T	T	T	F	T	T
F	F	T	F	F	F	F

From the columns 5 and 6 of the truth table it is clear that the formula in every step is logically equivalent to the initial formula (column 3).

b. $p \wedge q$

$\equiv \neg(\neg p \vee \neg q)$ by De Morgan's law for $\vee$ (9)

$\equiv \neg(p \to \neg q)$ by the conditional logical equivalence (12)

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$	$p \rightarrow \neg q$	$\neg(p \rightarrow \neg q)$
T	T	F	F	T	F	T	F	T
T	F	F	T	F	T	F	T	F
F	T	T	F	F	T	F	T	F
F	F	T	T	F	T	F	T	F

From the columns 6 and 8 of the truth table it is clear that the formula in every step is logically equivalent to the initial formula (column 4).

c. $p \leftrightarrow q$

$\equiv (p \rightarrow q) \wedge (q \rightarrow p)$ by the biconditional logical equivalence (13)

$\equiv \neg(\neg(p \rightarrow q) \vee \neg(q \rightarrow p))$ by De Morgan's law for $\wedge$ (9)

$\equiv \neg((p \rightarrow q) \rightarrow \neg(q \rightarrow p))$ by the conditional logical equivalence (12)

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$\neg(p \rightarrow q)$	$\neg(q \rightarrow p)$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$\neg(p \rightarrow q) \vee \neg(q \rightarrow p)$
T	T	T	T	T	T	F	F	F
T	F	F	T	F	F	T	F	T
f	T	T	F	F	F	F	T	T
F	F	T	T	T	T	F	F	F

$\neg(\neg(p \rightarrow q) \vee \neg(q \rightarrow p)$	$(p \rightarrow q) \rightarrow \neg(q \rightarrow p)$	$\neg((p \rightarrow q) \rightarrow \neg(q \rightarrow p))$
T	F	T
F	T	F
F	T	F
T	F	T

From the columns 7, 9 and 11 of the truth table it is clear that the formula in every step is logically equivalent to the initial formula (column 4).

Exercise 3

a. $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow r) \wedge (q \rightarrow r)$ rewrite left side

$\equiv (\neg p \vee r) \wedge (\neg q \vee r)$ by the conditional logical equivalence (12) on $(p \rightarrow r)$

$\equiv (r \vee \neg p) \wedge (r \vee \neg q)$ by the commutative law for $\vee$ (1)

$\equiv r \vee (\neg p \wedge \neg q)$ by the distributive law for $\vee$ (3)

$\equiv r \vee \neg(p \vee q)$ by De Morgan's law (9)

$\equiv \neg(p \vee q) \vee r$ by the commutative law for $\vee$ (1)

$\equiv (p \vee q) \to r$ by the conditional logical equivalence (12)

b. $(p \wedge \neg q) \to r \equiv (p \wedge \neg r) \to q$

$(p \wedge \neg q) \to r$ rewrite left side

$\equiv \neg(p \wedge \neg q) \vee r$ by the conditional logical equivalence (12)

$\equiv r \vee \neg(p \wedge \neg q)$ by the commutative law for $\vee$ (1)

$\equiv r \vee \neg(\neg(\neg p \vee q))$ by De Morgan's law (9)

$\equiv r \vee (\neg p \vee q)$ by the double negative law (6)

$\equiv (r \vee \neg p) \vee q$ by the associative law for $\vee$ (2)

$\equiv \neg(\neg r \wedge p) \vee q$ by De Morgan's law (9)

$\equiv \neg(p \wedge \neg r) \vee q$ by the commutative law for $\wedge$ (1)

$\equiv \neg(\neg(p \wedge \neg r)) \to q$ by the conditional logical equivalence (12)

$\equiv (p \wedge \neg r) \to q$ by the double negative law (6)

c. $(p \wedge q) \to r \equiv p \to (q \to r)$

$(p \wedge q) \to r$ rewrite left side

$\equiv \neg(p \wedge q) \vee r$ by the conditional logical equivalence (12)

$\equiv \neg(\neg(\neg p \vee \neg q)) \vee r$ by De Morgan's law for $\wedge$ (9)

$\equiv (\neg p \vee \neg q) \vee r$ by the double negative law (6)

$\equiv \neg p(\neg q \vee r)$ by the associative law for $\vee$ (2)

$\equiv \neg p \vee (q \rightarrow r)$ by the conditional logical equivalence (12) on $(\neg q \vee r)$

$\equiv p \rightarrow (q \rightarrow r)$ by the conditional logical equivalence (12)

d. $\neg((p \rightarrow q) \wedge \neg q) \vee \neg p \equiv t$

$\neg((p \rightarrow q) \wedge \neg q) \vee \neg p$ rewrite left side

$\equiv \neg((\neg p \vee q) \wedge \neg q) \vee \neg p$ by the conditional logical equivalence (12) on $(p \rightarrow q)$

$\equiv \neg(\neg q \wedge (\neg p \vee q)) \vee \neg p$ by the commutative law for $\wedge$ (1)

$\equiv \neg((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \vee \neg p$ by the distributive law for $\wedge$ (3)

$\equiv \neg((\neg q \wedge \neg p) \vee c) \vee \neg p$ by the negation law for $\wedge$ (5)

$\equiv \neg(\neg q \wedge \neg p) \vee \neg p$ by the identity law for $\vee$ (4)

$\equiv \neg(\neg(q \vee p)) \vee \neg p$ by De Morgan's law for $\wedge$ (9)

$\equiv (q \vee p) \vee \neg p$ by the double negative law (6)

$\equiv (p \vee q) \vee \neg p$ by the commutative law for $\vee$ (1)

$\equiv \neg p \vee (p \vee q)$ by the commutative law for $\vee$ (1)

$\equiv (\neg p \vee p) \vee q$ by the associative law for $\vee$ (2)

$\equiv t \vee q$ by the negation law for $\vee$ (5)

$\equiv t$ by the universal bound law for $\vee$ (8)