Peergrade assignment 2

September 19, 2024

Exercise 1

a. $(A \cup C) \cap B = \{1, 2, 4\}$

b. $A \cup (C - B) = \{2, 3, 4, 5\}$

c. $C \times A = \{(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4)\}$

d. $\mathcal{P}(A) = \{\emptyset, \{2\}, \{4\}, \{2, 4\}\}$

e. $B^{\complement} \cap C = \{3, 5\}$

Exercise 2

$$\begin{array}{r} 2 \\ 2346 \,\overline{)\,6256} \\ 4692 \\ \hline 1564 \end{array}$$

Divide 6256 by 2346

$6256 = 2346 \cdot 2 + 1564$ — by the Quotient-Remainder Theorem (Theorem 4.5.1 of textbook)

gcd(6256, 2346) = gcd(2346, 1564) — by Lemma 4.10.2 of textbook

$$\begin{array}{r} 1 \\ 1564 \,\overline{)\,2346} \\ 1564 \\ \hline 782 \end{array}$$

Divide 2346 by 1564

$2346 = 1564 \cdot 1 + 782$ — by the Quotient-Remainder Theorem

gcd(2346, 1564) = gcd(1564, 782) — by Lemma 4.10.2 of textbook

$$
\begin{array}{r}
2 \\
782\,\overline{)\,1564} \\
1564 \\
\hline
0
\end{array}
$$

Divide 1564 by 782

$1564 = 782 \cdot 2 + 0$ by the Quotient-Remainder Theorem

gcd(1564, 782) = gcd(782, 0) by Lemma 4.10.2 of textbook

gcd(6256, 2346) = gcd(2346, 1564) Put together all equations
= gcd(1564, 782)
= gcd(782, 0)
= 782 by Lemma 4.10.1 of textbook

Therefore, gcd(6256, 2346) = 782.

Exercise 3

a. For all integers a and b, if a is odd and b is odd, then $a + b$ is even.

Let a and b be odd integers.
By the first fact given by the assingment, since a and b are odd, there is an integer k such that

$$a = 2k + 1 \tag{1}$$

and there is also an integer r such that

$$b = 2r + 1 \tag{2}$$

From (1) and (2), the sum of a and b is

$$\begin{aligned} a + b &= (2k + 1) + (2r + 1) \\ &= 2k + 2r + 2 \end{aligned}$$

And by the distributive law

$$a + b = 2 \cdot (k + r + 1)$$

where $(k + r + 1)$ is an integer (let's name it n) since it is the sum of integers.

Thus, $a + b = 2 \cdot n$ and by the second fact given by the assignment, $a + b$ is even.

b. For all integers a, b and c, if $a \nmid bc$ then $a \nmid b$.

Contrapositive: For all integers a,b and c, if $a \mid b$ then $a \mid bc$.

By definition of divisibility, since $a \mid b$, there is an integer k such that

$$b = a \cdot k$$

By multiplying each side by c and ,

$$bc = (a \cdot k) \cdot c$$

By the associative law for multiplication $(a \cdot k) \cdot c = a \cdot (k \cdot c)$. Hence

$$bc = a \cdot (k \cdot c)$$

where $(k \cdot c)$ is an integer (let's name it n) since it is the product of integers.

Thus, $bc = a \cdot n$ and by the definition of divisibility, $a \mid bc$.

Since the statement "if $a \mid b$ then $a \mid bc$" is true, by contraposition, the statement "if $a \nmid bc$ then $a \nmid b$." is also true.