Assignment #1

CS 500, Fall 2010

Due Tuesday, October 5

Part I

1. Let $f(x, y, z) \doteq 5xy - yz + z^2 + 7$, and suppose $\mathcal{S}$ is the zero set of f; i.e., the point P is in $\mathcal{S}$ if $f(P) = 0$.

   (a) Find the nearest intersection point P between $\mathcal{S}$ and the ray with origin $P_0 = (-1, 3, 1)$ and direction $\mathbf{d}_0 = (0, 1, 2)$.

   (b) Find a surface normal to $\mathcal{S}$ at the point P found in part (a).

2. Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be the rear half–spaces defined by the points $C_1 = (-2, 2, -5)$, $C_2 = (2, 5, 0)$ and outwardly pointing normals $\mathbf{n}_1 = (5, 3, 0)$, $\mathbf{n}_2 = (-3, 2, 1)$ (respectively). Find the intersection interval between $\mathcal{H}_1 \cap \mathcal{H}_2$ and the ray with origin $P_0 = (2, 0, 1)$ and direction $\mathbf{d}_0 = (1, -1, 2)$.

3. Does the ray with origin $P_0 = (-14, -18, 19)$ and direction $\mathbf{d}_0 = (9, 6, 3)$ intersect the quadrilateral with vertices $V_0 = (2, 5, 3)$, $V_1 = (18, 5, 43)$, $V_2 = (26, -3, 59)$, and $V_3 = (10, -3, 19)$? If so, at what point?

4. The *Blivet of Angband* is defined to be the image of the parametrized surface $h : \mathbb{R}^2 \to \mathbb{R}^3$, where

$$h(u, v) \doteq (2u^3 + uv, u^2v^2 - u^3v, u^5 + u^2v^3).$$

   (a) Find a vector normal to this surface at the point $h(1, 2)$. *[Hint: find two vectors that are tangent to the surface at the given point.]*

   (b) Assume that h gives the surface in *object space*. If we place it in *world space* using the affine transformation $\tau : \mathbb{R}^3 \to \mathbb{R}^3$, given in homogeneous coordinates by the matrix

$$\begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & 3 & -2 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

   find a vector normal to the Blivet of Angband in world coordinates at the point $P = (6, -12, 3)$. Note that $P = \tau(h(1, 2))$.

5. Recall that the *gradient* of a real–valued function $f : \mathbb{R}^3 \to \mathbb{R}$ is the vector–valued function

$$\nabla f(x, yz) \doteq \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right).$$

Let $f : \mathbb{R}^3 \to \mathbb{R}$ be the function $f(x, y, z) = 7x^3y^2 + 2xz^4 - 3xyz + x^2 - 25$.

   (a) Compute ∇f.

   (b) Find a normal to the implicit surface $f(x, y, z) = 0$ at the point $(1, 2, 1)$.

6. If we write $P = (x, y, z)$, and if $C = (C_x, C_y, C_z)$ is constant, show that

$$\nabla |P - C| = \frac{P - C}{|P - C|}.$$

7. Recall that an ellipse is the set of all points in the plane such that the sum of the distances from two fixed points (the *focii*) is constant. Analogously, an *ellipsoid of revolution* can be defined as follows. Let F_a and F_b be two points in space (the focii), and let d be a positive number. A point P lies on the ellipsoid of revolution $\mathcal{E}$ if and only if the sum of the distances between P and the two focii F_a and F_b is d.

   (a) Write down an equation for the point P.

   (b) Write down an algorithm (in pseudocode) for finding the nearest point of intersection between the ray $\rho(t) = P_0 + t\mathbf{d}_0$, $t \geq 0$, and the ellipsoid of revolution $\mathcal{E}$. Hint: show that

$$|P - F_b|^2 - |P - F_a|^2 = -2P \cdot (F_b - F_a) + |F_b|^2 - |F_a|^2.$$

   (c) Show that the (unnormalized) outwardly pointing surface normal to $\mathcal{E}$ at the point P can be taken to be the sum of the two vectors

$$\mathbf{n} = |P - F_b|\,(P - F_a) + |P - F_a|\,(P - F_b).$$

8. A convex optical lens can be Mathematically modeled as the intersection of two spheres. Given two spheres $\mathcal{S}_1$ and $\mathcal{S}_2$, with centers C_1, C_2 and radii r_1, r_2 (respectively), write down an algorithm (in pseudocode) for determining the nearest point of intersection between the ray $\rho(t) = P_0 + t\mathbf{d}_0$, $t \geq 0$, and the lens formed by the intersection of $\mathcal{S}_1$ and $\mathcal{S}_2$. You do not need to rederive the intersection computation for a ray and a sphere done in class. *[Hint: review how we determined the intersection interval between a ray and a box.]*

Part II

Write a ray caster. In particular, your program will read a file containing data about a scene to be rendered, and then render the scene. This assignment will form the backbone of the remaining assignments for the semester, so you should design your ray caster with care and with a view towards the ray *tracing* assignments that will follow. Remember that you are not allowed to use DirectX or OpenGL.

Details and Requirements

- You should provide two versions of your program: (1) one with a graphical interface, and (2) anohter with a command line interface.

  1. The graphical version should include a simple, but well–behaved, graphical front end to display the rendered image in a window. The program must be able to read in a scene file that the user specifies; you may implement this using either a drag and drop interface, or a file open dialog box (or both). You may name the executable as you see fit. However, if you use Microsoft Visual Studio, you must name your solution file `raycast.sln`; otherwise you must include a make file named `raycast.mak`.
  2. The command line version must be named `raycast.exe`, and must take three command line arguments: the first is the input scene file name, the second is the output file name *without the file extension,* and the third is the *horizontal* pixel width of the output image (the height is determined by the aspect ratio of the camera in the scene file). The output file may be in any standard graphics file format: BMP, JPG, or PNG; your program will supply the appropriate file extension of the output file name. For example, if the user types in

     ```
     raycast ctest1.txt ctest1_out 400
     ```

     on the command line, then your program should produce the single output image file `ctest1_out.bmp` (the extension will depend on your choice of output file format) of size 400×400 pixels (the camera in `ctest1.txt` has an aspect ratio of unity).

  Note that your solution or make file should produce *both* versions of the program; that is, it should produce two executables.

- Your programs must be able to render objects of type `SPHERE`, `BOX`, `POLYGON`, and `ELLIPSOID`, should ignore objects of an unrecognized type, as well as any illumination properties of the scene, such as `AIR` and `AMBIENT`.

- Each rendered object should be given the color (c_r, c_g, c_b), where

$$c_r = (\mathbf{n} \cdot \mathbf{v}) D_r, \quad c_g = (\mathbf{n} \cdot \mathbf{v}) D_g, \quad c_b = (\mathbf{n} \cdot \mathbf{v}) D_b.$$

  Here (D_r, D_g, D_b) is the object's diffuse reflection coefficient (given in the scene file), $\mathbf{n}$ is the outwardly pointing *unit* surface normal at the point of intersection, and $\mathbf{v}$ is the *unit* vector from the intersection point to the viewer's eye (that is, $\mathbf{v}$ is opposite the ray direction).

- A portion of your grade for this assignment will determined by the efficiency of your ray caster!