1. **Problem**

   For the matrix

   $$A = \begin{pmatrix} 16 & -12 & -12 & -16 \\ -12 & 25 & 1 & -4 \\ -12 & 1 & 17 & 14 \\ -16 & -4 & 14 & 57 \end{pmatrix}.$$

   compute the matrix $L = (\ell_{ij})_{1 \leq i,j \leq 4}$ from the Cholesky decomposition $A = LL^\top$.

   Which of the following statements are true?

   (a) $\ell_{41} \geq -4$

   (b) $\ell_{33} \geq 2$

   (c) $\ell_{11} \leq 4$

   (d) $\ell_{31} \geq -3$

   (e) $\ell_{32} \geq -2$

   Solution

   The decomposition yields

   $$L = \begin{pmatrix} 4 & 0 & 0 & 0 \\ -3 & 4 & 0 & 0 \\ -3 & -2 & 2 & 0 \\ -4 & -4 & -3 & 4 \end{pmatrix}$$

   and hence:

   (a) True. $\ell_{41} = -4$

   (b) True. $\ell_{33} = 2$

   (c) True. $\ell_{11} = 4$

   (d) True. $\ell_{31} = -3$

   (e) True. $\ell_{32} = -2$