Demostración 2

5. Derivación

$$D^4 f(x_j) \approx \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4}$$

A partir del Polinomio de Taylor:

$$f(x+2h) = f(x) + 2hf'(x) + \frac{(2h)^2}{2}f''(x) + \frac{(2h)^3}{3!}f^{(3)}(x) + \frac{(2h)^4}{4!}f^{(4)}(x)$$

$$f(x-2h) = f(x) - 2hf'(x) + \frac{(2h)^2}{2}f''(x) - \frac{(2h)^3}{3!}f^{(3)}(x) + \frac{(2h)^4}{4!}f^{(4)}(x)$$

Sumando ambas expresiones tenemos que:

$$f(x+2h) + f(x-2h) = f(x) + \cancel{2hf'(x)} + \frac{(2h)^2}{2}f''(x) + \cancel{\frac{(2h)^3}{3!}f^{(3)}(x)} + \frac{(2h)^4}{4}f^{(4)}(x) +$$
$$f(x) - \cancel{2hf'(x)} + \frac{(2h)^2}{2}f''(x) - \cancel{\frac{(2h)^3}{3!}f^{(3)}(x)} + \frac{(2h)^4}{4}f^{(4)}(x)$$

$$f(x+2h) + f(x-2h) = 2f(x) + 2\left(\frac{(2h)^2}{2}f''(x)\right) + 2\left(\frac{(2h)^4}{4!}f^{(4)}(x)\right)$$
$$= 2f(x) + (2h)^2 f''(x) + \frac{(2h)^4}{12}f^{(4)}(x)$$
$$= 2f(x) + 4h^2 f''(x) + \frac{4h^4}{3}f^{(4)}(x)$$

Recordemos que:

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$f(x+2h) + f(x-2h) = 2f(x) + 4h^2\left(\frac{f(x+h) - 2f(x) + f(x-h)}{h^2}\right) + \frac{4h^4}{3}f^{(4)}(x)$$

$$f(x+2h) + f(x-2h) = 2f(x) + 4f(x+h) - 8f(x) + 4f(x-h) + \frac{4}{3}h^4 f^{(4)}(x)$$

Despejamos $f^{(4)}(x)$

$$\frac{3}{4}\left(\frac{f(x+2h) - 4f(x+h) + 6f(x) - 4f(x-h)}{h^4}\right) = f^{(4)}(x)$$

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reemplazamos $x_{i+1} = x+h$ $x_{i-1} = x-h$ $x_{i+2} = x+2h$ $x_{i-2} = x-2h$

$$f^{(4)}(x) = \frac{3}{4}\left(\frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_i) - 4f(x_i - 1) + f(x_{i-2})}{h^4}\right)$$

$$f^{(4)}(x) \simeq \frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4}$$

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