$f \approx P_2(x)$

$$P_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)$$

$$= \int P_2(x)\,dx$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \int_a^b (x-b)(x-x_m)\,dx + \frac{f(x_m)}{(x_m-a)(x_m-b)} \int_a^b (x-a)(x-b)\,dx + \frac{f(b)}{(b-a)(b-x_m)} \int_a^b (x-a)(x-x_m)\,dx$$

Por medio de $\int u\,dv = u \cdot v \int v\,du$

$$\int_a^b (x-b)(x-x_m)\,dx = \frac{-b^3 + 3b^2x_m - 2a^3 + 3a^2x_m + 3a^2b - 6abx_m}{6}$$

$$\int_a^b (x-a)(x-b)\,dx = \frac{-b^3 + 3ab^2 + a^3 - 3a^2b}{6} = \frac{(a-b)^3}{6}$$

$$\int_a^b (x-a)(x-c)\,dx = \frac{a^3 - 3b^2x_m + 2b^3 - 3a^2x_m - 3ab^2 + 6abx_m}{6}$$

$$\frac{3x_m(a^2+b^2-2ab) - b^3 - 2a^3 + 3a^2b}{6} = \frac{3x_m(a-b)^2 - b^3 - 2a^3 + 3a^2b}{6}$$

$$\frac{-3x_m(a^2+b^2-2ab) + a^3 + 2b^3 - 3ab^2}{6} = \frac{-3x_m(a-b)^2 + a^3 + 2b^3 - 3ab^2}{6}$$

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$P_2(x) =$

$$\frac{f(a)}{(a-b)(a-x_m)}\left(\frac{3x_m(a-b)^2-b^3-2a^3+3a^2b}{6}\right)+\frac{f(x_m)}{(x_m-a)(x_m-b)}\left(\frac{(a-b)^3}{6}\right)+$$

$$\frac{f(b)}{(b-a)(b-x_m)}\left(\frac{-3x_m(a-b)^2+a^3+2b^3-3ab^2}{6}\right)$$