Enunciados a y b - Punto 8 Derivación

a. polinomio interpolador del conjunto soporte

Conjunto soporte ⤵

$$\Omega = \{(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2))\}$$

$$L_0(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} = \frac{x^2 - x_2x - x_1x + x_1x_2}{x_0^2 - x_2x_0 - x_1x_0 + x_1x_2}$$

$$= \frac{x^2 + (-x_1 - x_2)x + x_1x_2}{x_0^2 + (-x_1 - x_2)x_0 + x_1x_2}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} = \frac{x^2 - x_0x - x_2x + x_0x_2}{x_1^2 - x_0x_1 - x_2x_1 + x_0x_2}$$

$$= \frac{x^2 + (-x_0 - x_2)x + x_0x_2}{x_1^2 + (-x_0 - x_2)x_1 + x_0x_2}$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} = \frac{x^2 - x_0x - x_1x + x_0x_1}{x_2^2 - x_0x_2 - x_1x_2 + x_0x_1}$$

$$= \frac{x^2 + (-x_0 - x_1)x + x_0x_1}{x_2^2 + (-x_0 - x_1)x_2 + x_0x_1}$$

Polinomio interpolador

$$P(x) = f(x_0)\, L_0(x) + f(x_1)\, L_1(x) + f(x_2)\, L_2(x)$$

$$P(x) = f(x_0) \cdot \frac{x^2+(-x_1-x_2)x+x_1x_2}{x_0^2+(-x_1-x_2)x_0+x_1x_2} + f(x_1) \cdot \ldots$$

$$\ldots \frac{x^2+(-x_0-x_2)x+x_0x_2}{x_1^2+(-x_0-x_2)x_1+x_0x_2} + f(x_2) \cdot \frac{x^2+(-x_0-x_1)x+x_0x_1}{x_2^2+(-x_0-x_1)x_2+x_0x_1}$$

b. Derivada polinomio interpolador

$$\frac{d}{dx} L_0(x) = \frac{2x+(-x_1-x_2)}{x_0^2+(-x_1-x_2)x_0+x_1x_2}$$

$$\frac{d}{dx} L_1(x) = \frac{2x+(-x_0-x_2)}{x_1^2+(-x_0-x_2)x_1+x_0x_2}$$

$$\frac{d}{dx} L_2(x) = \frac{2x+(-x_0-x_1)}{x_2^2+(-x_0-x_1)x_2+x_0x_1}$$

$$\frac{d}{dx} L(x_0) = f(x_0)\, \frac{2x_0+(-x_1-x_2)}{x_0^2+(-x_1-x_2)x_0+x_1x_2}$$

$$+ f(x_1)\, \frac{2x_0+(-x_0-x_2)}{x_1^2+(-x_0-x_2)x_1+x_0x_2} + f(x_2)\, \frac{2x_0+(-x_0-x_1)}{x_2^2+(-x_0-x_1)x_2+x_0x_1}$$

Al ser equiespaciado tenemos que

$$L'(x_0) = \frac{1}{2}\left(-3F(x_0) + 4F(x_1) - F(x_2)\right)$$