$$I = \int_a^b f(x)\,dx \cong \int_a^b P_1(x)\,dx = \frac{b-a}{2}\left(f(a) + f(b)\right)$$

$$f(x) \approx P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$\int_a^b f(x)\,dx = \int_a^b P_1(x)\,dx = \int_a^b \frac{x-b}{a-b} f(a) + \int_a^b \frac{x-a}{b-a} f(b)$$

$$= \int_a^b P_1(x)\,dx = \frac{f(a)}{a-b}\int_a^b x-b + \frac{f(b)}{b-a}\int_a^b x-a$$

$$= \frac{f(a)}{a-b}\left(\frac{x^2}{2} - bx\right)\Bigg|_a^b + \frac{f(b)}{b-a}\left(\frac{x^2}{2} - ax\right)\Bigg|_a^b$$

$$(b-a) = -(a-b)$$
$$= -a+b$$
$$-a = b-a$$

$$\frac{f(a)}{a-b}\left(\left(\frac{x^2}{2}-bx\right)\Bigg|_a^b\right) + \frac{f(b)}{b-a}\left(\frac{x^2}{2}-ax\Bigg|_a^b\right)$$

$$\frac{f(a)}{a-b}\left(\frac{b^2}{2}-b^2-\left(\frac{a^2}{2}-ab\right)\right) + \frac{f(b)}{b-a}\left(\frac{b^2}{2}-ab-\left(\frac{a^2}{2}-a^2\right)\right)$$

$$\left(-\frac{b^2}{2}-\frac{a^2}{2}+ab\right) \qquad \left(\frac{b^2}{2}-ab+\frac{a^2}{2}\right)$$

$$-\frac{f(a)}{a-b}\left(\frac{b^2}{2}+\frac{a^2}{2}-ab\right) + \frac{f(b)}{b-a}\left(\frac{b^2}{2}+\frac{a^2}{2}-ab\right)$$

$$\left(\frac{b^2}{2}+\frac{a^2}{2}-ab\right)\left(\frac{f(b)}{b-a}-\frac{f(a)}{a-b}\right)$$

$$\left(\frac{a^2+b^2}{2}-ab\right)\left(\frac{f(b)}{b-a}-\frac{f(a)}{a-b}\right)$$

$$a-b=-(b-a)$$
$$=-b+a$$
$$a-b=a-b$$

$$\frac{(a^2+b^2-2(ab))}{2}\left(\frac{f(b)}{b-a}-\frac{f(a)}{a-b}\right)$$

$$\frac{a^2-2ab+b^2}{2}\left(\frac{f(b)}{b-a}-\frac{f(a)}{a-b}\right)$$

$$\frac{(b-a)^2}{2}\left(\frac{f(b)}{b-a}-\frac{f(a)}{a-b}\right)$$

$$\frac{(b-a)^2}{2}\left(\frac{f(b)}{b-a}-\frac{f(a)}{-(b-a)}\right)$$

$$\frac{(b-a)^2}{2}\left(\frac{f(b)}{b-a}+\frac{f(a)}{b-a}\right)$$

$$\frac{(b-a)^2}{2}\left(\frac{f(b)+f(a)}{\cancel{b-a}}\right) = \frac{b-a}{2}\left(f(b)+f(a)\right)$$