Enunciados c y d - punto 7 Mínimos cuadrados

c

$$\chi^2(\vec{\theta}) = \sum_{i=1}^{N} \left(\frac{y_1 - M(x_i, \vec{\theta})}{\sigma_i} \right)^2$$

Derivada parcial:

$$\frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} = \frac{\partial \left(\sum_{i=1}^{N} \left(\frac{y_i - M(x_i, \vec{\theta})}{\sigma_i} \right)^2 \right)}{\partial \theta_i}$$

$$= \sum_{i=1}^{N} 2 \left(\frac{y_i - M(x_i, \vec{\theta})}{\sigma_i} \right) \left(\frac{-\partial M(x_i, \vec{\theta})}{\partial \theta_i} \right)$$

$$= -2 \sum_{i=1}^{N} \left(\frac{y_i - M(x_i, \vec{\theta})}{\sigma_i} \right) \left(\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} \right)$$

d

$$\vec{\theta}_{j+1} = \vec{\theta_j} - \gamma \left(-2 \sum_{i=1}^{N} (y_i - M(x_i, \vec{\theta_j})) \nabla_\theta M(x_i, \vec{\theta}_j) \right)$$

Descenso del gradiente → dirección opuesta al gradiente

$$\boxed{x_n = x_{n-1} - \underbrace{\beta}_{\text{distancia}} \nabla_x f(x_n)}$$

$\vec{\theta_j} \to x_{n-1}$ (vector inicial) ; $\gamma \to \beta$ (distancia)

$$\nabla_\theta \chi^2(\vec{\theta}) =$$

$$\left[-2\sum_{i=1}^{N}\left(y_i - M(x_i,\vec{\theta}_0)\right)\left(\frac{\partial M(x_i,\vec{\theta})}{\partial \theta_0}\right), -2\sum_{i=1}^{N}\left(y_i - M(x_i,\vec{\theta}_1)\right)\left(\frac{\partial M(x_i,\vec{\theta})}{\partial \theta_1}\right), -2\sum_{i=1}^{N}\left(y_i - M(x_i,\vec{\theta}_2)\right)\left(\frac{\partial M(x_i,\vec{\theta})}{\partial \theta_2}\right)\right]$$

$$\boxed{\nabla_\theta M(x_i,\vec{\theta}_j) = \left[\frac{\partial M(x_i,\vec{\theta})}{\partial \theta_0}, \frac{\partial M(x_i,\vec{\theta})}{\partial \theta_1}, \frac{\partial M(x_i,\vec{\theta})}{\partial \theta_2}\right]}$$