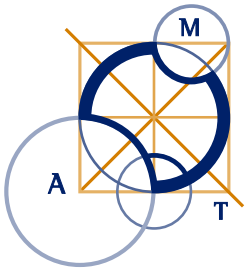

2018 AMOC SENIOR CONTEST

Tuesday, 21 August 2018
Time allowed: 4 hours
No calculators are to be used.
Each question is worth seven points.

1. Determine the maximum possible value of $a + b$, where a and b are two different non-negative real numbers that satisfy

$$a + \sqrt{b} = b + \sqrt{a}.$$

2. Prove that, among any ten consecutive positive integers, there are five numbers such that no two of them have a common factor larger than 1.

3. Fourteen people meet one day to play three matches of netball. For each match, they divide themselves into two teams of seven players. In each match, one team wins while the other team loses. After all three matches, no person has been on a losing team three times.

Prove that there are at least three players who were on the same team as each other for all three matches.

4. Let K_1 and K_2 be circles that intersect at two points A and B. The tangents to K_1 at A and B intersect at a point P inside K_2, and the line BP intersects K_2 again at C. The tangents to K_2 at A and C intersect at a point Q, and the line QA intersects K_1 again at D.

Prove that QP is perpendicular to PD if and only if the centre of K_2 lies on K_1.

5. Determine all functions f defined for positive real numbers and taking positive real numbers as values such that

$$xf(xf(2y)) = y + xyf(x)$$

for all positive real numbers x and y.

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