

The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Senior Paper: Years 11, 12
Northern Spring 2011 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. All faces of a convex polyhedron are similar triangles.
Prove that this polyhedron has two pairs of congruent faces. (3 points)
2. Worms grow at the rate of 1 metre per hour. When they reach their maximum length of 1 metre, they stop growing. If a worm is fully grown, one can dissect it at any point along its length, into two parts, so that two new worms arise, which, since their lengths are now less than 1 metre in length, grow at the rate of 1 metre per hour.
Starting with 1 fully grown worm, can one obtain 10 fully grown worms in less than 1 hour? (4 points)
3. Around a circle are placed 100 white stones, and an integer k such that $1 \leq k \leq 50$ is given. A game is played such that at each move, one can choose any k consecutive stones for which the first and last ones are white, and paint those two stones black.
For which values of k is it possible to make all 100 stones black after several moves? (4 points)
4. Suppose that four altitudes of a convex pentagon concur.
Prove that the fifth altitude is concurrent with the other four. (5 points)
5. There are 100 towns and several roads in a country. Each road connects two towns and roads don't intersect. It is possible to reach any town from any other town driving along the roads.
Prove that it is possible to pave some roads so that an odd number of paved roads emanate from each town. (5 points)