

The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS
AMO TRAINING SESSIONS

2012 Senior Mathematics Contest Problems

1. Let $ABCD$ be a cyclic quadrilateral. Let K_1 be the circle that passes through D and is tangent to AB at A , and let K_2 be the circle that passes through D and is tangent to BC at C . Let P be the point other than D in which K_1 and K_2 intersect.

Prove that P lies on the line through A and C .

2. Show that $(n+1)^{n+1} + (-n)^n$ is not divisible by 9 for any $n \in \mathbb{N}$.
3. Determine all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$(x+y)f(x) + f(y^2) = (x+y)f(y) + f(x^2), \quad \forall x, y \in \mathbb{R}.$$

4. Consider the grid of points in the (x, y) -plane with positive integer coordinates ≤ 100 . If (x_1, y_1) and (x_2, y_2) are two such grid points further satisfying $x_2 \geq x_1$ and $y_2 \geq y_1$, then we say (x_2, y_2) is *stronger than* (x_1, y_1) .

Determine the least $n \in \mathbb{N}$ such that every set of n points in the grid contains three distinct points A, B, C such that

A is stronger than B and B is stronger than C .

5. Determine all pairs (x, y) of positive integers that satisfy

$$x^{2012} = y^x.$$