

The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS
AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Autumn 2010 (A Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. A round coin may be used to construct a circle passing through one or two given points on the plane.

Given a line on the plane, show how to use this coin to construct two points such that they define a line perpendicular to the given line.

Note that the coin may not be used to construct a circle tangent to the given line.

(4 points)

2. Petya has an instrument which can locate the midpoint of a line segment, and also the point which divides the line segment in the ratio $n : (n + 1)$, for any $n \in \mathbb{N}$. Petya claims that with this instrument, he can locate the point which divides a line segment into two segments whose lengths are in any given rational ratio.

Is Petya right?

(5 points)

3. At a circular track, 10 cyclists started from some point at the same time in the same direction with different constant speeds. If any two cyclists are at a point at the same time again, we say that they meet. No three of the cyclists meet at the same time.

Prove that by the time every pair of cyclists have met at least once, each cyclist has had at least 25 meetings.

(8 points)

4. A rectangle is divided into 2×1 and 1×2 dominoes. In each domino, a diagonal is drawn, such that no two diagonals have a common endpoint.

Prove that exactly two corners of the rectangle are endpoints of these diagonals.

(8 points)

5. For each side of a given pentagon, divide its length by the total length of all other sides.

Prove that the sum of all the fractions obtained is less than 2.

(8 points)

6. In acute $\triangle ABC$, an arbitrary point P is chosen on altitude AH , where the foot of the altitude is at H . Points E and F are the midpoints of sides CA and AB , respectively. The perpendiculars from E to CP and from F to BP meet at point K .

Prove that $KB = KC$.

(8 points)

7. Merlin summons the n knights of Camelot for a conference. Each day, he assigns them to the n seats at the Round Table. From the second day on, any two neighbours may interchange their seats if they were not neighbours on the first day. The knights try to sit in some cyclic order which has already occurred before on an earlier day. If they succeed, then the conference comes to an end when the day is over.

What is the maximum number of days for which Merlin can guarantee that the conference will last?

(12 points)