The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

**AMO/TT TRAINING SESSIONS**

# 2009 Australian Intermediate Mathematics Olympiad Problems

1. A point $P$ lies inside a square $ABCD$ of side 120. Let $F$ be a point on $CD$ such that $PF \perp CD$. Given $P$ is equidistant from $A, B, F$, find $PA$.

2. If $935\,712 \times N$ is a perfect cube for some $N \in \mathbb{N}$, find the minimum possible value of $N$.

3. $A, B, C$ are digits. The 3-digit number $(ACB)$ is divisible by 3, $(BAC)$ is divisible by 4, $(BCA)$ is divisible by 5, and $(CBA)$ has an odd number of factors. Find $(ABC)$.

4. While waiting for the bishop to arrive at St Stephen's Anglican Church to lead a service, every person present greeted every other person with a handshake. Arriving late, the bishop shook hands with only some of the people as he made his way in. Altogether, 1933 handshakes had taken place. How many people shook hands with the bishop?

5. In the grid shown, a number is to be placed in each small square so that the product of all three numbers in any row, column, or diagonal is the same positive number. Find the sum of $x$ and $y$.

|   |    |   |
|---|----|---|
|   |    |   |
| $x$ | 32 | $y$ |
| 16 | 8 |   |

6. An integer $a$ has just two digits. When the digits are reversed, the resulting number $b$ is $p\%$ larger than $a$. Given that $p$ is an odd integer find the largest possible value of $p$.

7. Let $a, b, c, d, e$ be a five-term geometric sequence such that $a, b, c, d, e$ are integers and $0 < a < b < c < d < e < 100$. What is the sum of all possible values for $c$?

8. In $\triangle ABC$, $\angle ABC = 138°$ and $\angle ACB = 24°$. Point $D$ is on $AC$ so that $\angle BDC = 60°$, and point $E$ is on $AB$ so that $\angle ADE = 60°$. If $\angle DEC = x°$, find $x$.

9. Suppose $\triangle ABC$ is equilateral with $AB = x$. Extend $BC$ to points $A'$ (on same side as $B$) and $A''$ (on same side as $C$) such that $A'B = CA'' = y$. Similarly, extend $CA$ to points $B'$ (on same side as $C$) and $B''$ (on same side as $A$) such that $B'C = AB'' = y$; and extend $AB$ to points $C'$ (on same side as $A$) and $C''$ (on same side as $B$) such that $C'A = BC''' = y$.

   (a) Prove that the points $A', B'', C', A'', B', C'''$ lie on a circle.

   (b) If $x$ and $y$ are positive integers, determine the smallest value for $R^2$, where $R$ is the radius of that circle.

10. What is the maximum number of terms in an arithmetic sequence of primes with common difference 6?

    *Investigation*

    Find the minimum common difference for an increasing arithmetic sequence of 6 primes.