

The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Junior Paper: Years 8, 9, 10
Northern Autumn 2009 (O Level)

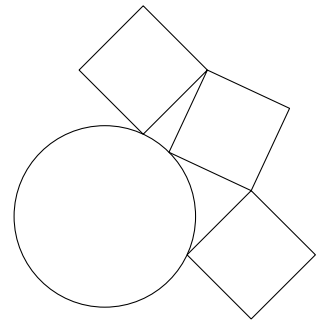
Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Is it possible to cut a square into 9 smaller squares, so that one of the smaller squares is coloured white, three are coloured grey and the remaining five are coloured black, and such that the smaller squares of the same colour have the same size, while the smaller squares that are coloured differently have different size? (3 points)
2. There are 40 weights with masses 1 g, 2 g, \dots , 40 g, respectively. Ten weights with even masses are put on the left pan of some scales, and ten weights with odd masses are put on the right pan of the same scales, such that the scales are balanced.

Prove that on one pan there are two weights whose masses differ by 20 g. (4 points)

3. To the circumference of a disk of radius 5 cm, squares of side length 5 cm are attached, one after another, while possible, such that:
 - (i) one vertex of each square lies on the circumference of the disk;
 - (ii) the squares do not overlap; and
 - (iii) each square has a common vertex with its predecessor.

Determine how many squares can be attached to the disk and prove that the first and last square attached will have a common vertex. (The first three squares of a possible arrangement are shown in the diagram.)



(4 points)

4. A 7-digit code, consisting of seven distinct digits, is called *good*. Suppose the password for a safe is a *good code*, and that the safe can be opened if an entered code is good and a digit of that code and the corresponding digit of the password are the same at some position.

Is there a guaranteed method of opening the safe with fewer than 7 attempts without knowing the password? (5 points)

5. On a new website, 2000 people have registered. Each of them invited 1000 people (from the other people registered) to be their friends. Two people are regarded as friends if and only if each invited the other to be their friend.

What is the least number of pairs of friends possible on the website? (5 points)