The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

Tournament of the Towns Questions III

1. Find all integer solutions to the equation
$$y^k = x^2 + x$$
where k is a natural number greater than 1. (3 points)

2. Find all solutions of
$$2^n + 7 = x^2$$
in which n and x are both integers. Prove that there are no other solutions. (4 points)

3. A set of 1989 numbers is given. It is known that the sum of any 10 of them is positive. Prove that the sum of all of these numbers is positive. (Folklore, 3 points)

4. Find the positive integer solutions of the equation
$$x + \cfrac{1}{y + \cfrac{1}{z}} = \frac{10}{7}$$
(G. Galperin, 3 points)

5. We define $N!!$ to be $N(N-2)(N-4)\ldots 5.3.1$ if N is odd and $N(N-2)(N-4)\ldots 6.4.2$ if N is even. For example, $8!! = 8.6.4.2$ and $9!! = 9.7.5.3.1$. Prove that $1986!! + 1985!!$ is divisible by 1987. (V. V. Proizvolov, Moscow, 5 points)

6. The numbers 2^{1989} and 5^{1989} are written out one after the other (in decimal notation). How many digits are written altogether? (G. Galperin, 3 points)

7. For which natural number k does
$$\frac{k^2}{1.001^k}$$
attain its maximum value? (4 points)

8. For any natural number $n \geq 2$ prove the inequality
$$\sqrt{2\sqrt{3\sqrt{4\ldots\sqrt{(n-1)\sqrt{n}}}}} < 3.$$
(V. Proizvolov, Moscow, 5 points)