2010 Australian Intermediate Mathematics Olympiad Problems

1. A 3-digit number has the following properties.

   - If its tens and units digits are interchanged, its value increases by 36.
   - If its hundreds and units digits are interchanged, its value decreases by 198.

   When its hundreds and tens digits are interchanged, its value decreases.
   By how much?

2. A number in base twelve is 3140. The same number in base b is 320.
   What is b?

3. Starship Conquest is pursuing starship Anarky, which is 12 klongs ahead of Conquest's current position. After Conquest has travelled 45 klongs, Anarky is just 7 klongs ahead.

   Assuming both starships are travelling on the same straight line at constant speeds, how many more klongs will it take for Conquest to catch Anarky?

4. In the diagram, B is a battery, L is a lamp, and S_1, S_2, S_3, S_4 are switches. The probability for each switch being on is $\frac{1}{3}$ and these probabilities are independent. Let p be the probability that the lamp is on.
   Find $729p$.
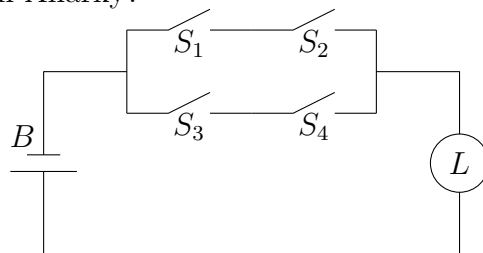
5. Jess has five bank accounts. If three account balances at a time were added, the following amounts would result: $94, 97, 99, 100, 101, 103, 104, 106, 107, 109$.

   What is the sum of the lowest and highest balances?

6. Rectangle $ABCD$ has side lengths $AB = 195$ and $BC = 130$. Point P lies in its interior such that $CP = 117$ and $DP = 156$.

   Determine the length of AP.

7. Suppose $\triangle ABC$ has area 2010, and let X, Y and Z be points on the sides AB, BC and CA, respectively, such that

$$\frac{AX}{XB} = \frac{2}{3}, \quad \frac{BY}{YC} = \frac{32}{35}, \quad \frac{CZ}{ZA} = \frac{1}{5}.$$

   Determine the area of $\triangle XYZ$.

8. If s is the smallest positive integer with the property that its digit sum and the digits sum of $s + 1$ are both divisible by 19, how many digits does s have?

9. Quadrilateral $ABCD$ is such that the midpoint O of AB is the centre of a circle to which AD, DC and CB are tangents; AB is not a diameter of the circle.

   Prove $AB^2 = 4AD \times BC$.

10. A 2-digit number is called *productive* if it is the product of two single digit numbers.

    Find all 9-digit numbers in which all digits are different and each pair of neighbouring digits forms a productive number.

    Investigation

    Find all pairs of consecutive numbers, each consisting of 8 distinct digits and for which each pair of neighbouring digits forms a productive number.