AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems IV

1. Suppose you and your partner attended a party with three other couples. Several handshakes took place. Noone shook hands with himself or herself or with their partner, and noone shook hands with the same person more than once. After all the handshaking was completed, you asked each person, including your partner, how many hands they had shaken. Each person gave a different answer.

   (i) How many hands did you shake?

   (ii) How many hands did your partner shake?

2. A paper rectangle $ABCD$ of area 1 is folded along a straight line so that C coincides with A. Prove that the area of the pentagon thus obtained is less than 3/4. (Tournament, 1995)

3. In the centre of a square swimming pool is a boy, while his teacher (who cannot swim) is standing at one corner of the pool. The teacher can run three times as fast as the boy can swim, but the boy can run faster than the teacher. Can the boy escape from the teacher? (1987 Tournament)

4. A schoolgirl forgot to write a multiplication sign between two 3-digit numbers and wrote them as one number. This 6-digit result proved to be 3 times as great as the product. Find the numbers. (1984 Tournament)

5. Eight students were asked to solve 8 problems (the same set for each of the students).

   (i) Each problem was solved by 5 students. Prove that one can find two students so that each of the problems was solved by at least one of them.

   (ii) If each problem was solved by 4 students, then it is possible that no such pair of students exists. Prove this.

   (1996 Tournament)

6. Prove that from any sequence of 1996 numbers $a_1, a_2, \ldots, a_{1996}$ one can choose one or several consecutive numbers so that their sum differs from an integer by less than 0.001.

   (1996 Tournament)

7. Sixty children participate in a summer camp. Among any 10 of the children, there are three or more who live in the same block. Prove that there must be 15 or more children from the same block. (1994 Tournament)

8. $ABCD$ is a convex quadrilateral inscribed in a circle with centre O, and with mutually perpendicular diagonals. Prove that the broken line AOC divides the quadrilateral into two parts of equal area. (1981 Tournament)

9. Prove that if $a, b, c > 0$ such that

$$a^2 + b^2 - ab = c^2,$$

then $(a - c)(b - c) \leq 0$.   (1996 Tournament)

10. Prove that in any set of 17 distinct natural numbers it is possible to find either a set of 5 numbers such that four of them divide the fifth, or a set of 5 numbers none of which divides the others.   (1983 Tournament)