Even/Odd Lemmas

Definitions

An even number is an integer divisible by 2

An odd number is an integer not divisible by 2

Prove that all even/odd numbers can be simplified into form 2t+x [1]

Let e be an even number

Let o be an odd number

$$2|\mathrm{e} \Rightarrow \exists 2\mathrm{t} = \mathrm{e}$$

$$2 \nmid \mathrm{o} \Rightarrow \exists 2\mathrm{t} + 1 = 0$$

Prove that 2 multiplied by any number is even [2]

Let a be the coefficient of 2.

2a = 2(a), so 2a is divisible by 2, therefore even.

Prove that every number is even or odd. [3]

Let a be an even number.

Let $2\mathrm{a}_1 = \mathrm{a}$.

$\mathrm{a} + 1 = 2\mathrm{a}_1 + 1$, which is the form of odd numbers

Therefore, a being even implies a+1 is odd.

Let b be an odd number.

Let $2\mathrm{b}_1 + 1 = \mathrm{b}$.

$$\mathrm{b} + 1 = 2(\mathrm{b}_1 + 1)$$

[2] proves that this is even

Therefore, b being odd implies b+1 is even.

Combining these 2 statements, there is a cyclic pattern of integers being even, then odd.

This pattern goes on forever and with even integer

Therefore ALL numbers are included in this pattern

Therefore all numbers are even or odd.

Prove that an even number multiplied by any integer will result in an even number [4]

Let a be an even number

Let b be an odd or even number.

Expand a using method described in [1]

Let $2a_1 = a$.

$$ab = 2(a_1)(b)$$

$= 2(a_1 b)$

As proved in [2], this is even.

Prove that 2 even numbers added together results in an even number [5]

Let a, b be even numbers

Expand using method described in [1].

Let $2a_1 = a, 2b_1 = b$.

$$a + b = 2a_1 + 2b_1$$

$= 2(a_1 + b_1)$

As proved in [2], this is even.

Prove that 2 odd numbers added together results in an even number [6]

Let a, b be odd numbers.

Expand:

Let $2a_1 + 1 = a, 2b_1 + 1 = b$

$$a + b = 2a_1 + 1 + 2b_1 + 1$$

$=2(a_1 + b_1 + 1)$

As proved in [2], this is even

Prove that an odd number added to an even number is an odd number [7]

Let a, b be odd and even integers, respectively

Expand:

Let $2a_1 + 1 = a, 2b_1 = b$

$$a + b = 2a_1 + 2b_1 + 1$$

$=2(a_1 + b_1) + 1$

As said in [1], this is the form of an odd number.

Prove that an odd number multiplied by an odd number is an odd number [8]

Let a, b be odd integers.

Expand:

Let $2a_1 + 1 = a, 2b_1 + 1 = b$

$$ab = (2a_1 + 1)(2b_1 + 1)$$

$=4a_1b_1 + 2a_1 + 2b_1 + 1$

$=2(2a_1b_1 + a_1 + b_1) + 1$

As said in [1], this is the form of an odd number

Prove that [6],[7],[8] will yield the same results with subtraction [9]

Let a, b the 2 integers in the equation.

Let c, d be a, b mod 2, respectively.

$$a + b \equiv (c + d) \pmod{2}$$

$$a - b \equiv (c - d) \pmod{2}$$

Therefore we need to prove c+d = c-d

$$(c + d) - (c - d) = 2d$$

Since we only need the result modulo 2, we can simplify to:

$$(\mathrm{C}+\mathrm{d})-(\mathrm{C}-\mathrm{d}) \equiv 0 \pmod 2$$

Therefore are equal in mod 2.

This proves $a+b, a-b$ are the same in mod 2,

So yield the same results (mod 2) for proofs [6,7,8]

Prove that division of even numbers is ambiguous [10]

Let a, b be even numbers.

Let $2a_1 = a, 2b_1 = b.$

$$\frac{a}{b} = \frac{a_1}{b_1}$$

But since we have no way of checking divisibility of a_1, b_1 just from the fact that a,b are even,

we cannot determine if $\frac{a}{b}$ is odd or even

Prove that an odd number / an even number does not return an integer [11]

Let a, b be odd and even numbers, respectively

Let $2a_1 + 1 = a, 2b_1 = b$

$\frac{a}{b}$ is never an integer, as a is even, and for $\frac{a}{b}$ to be an integer, a has to be even, but is odd.

$$\frac{a}{b} = \frac{2a_1}{2b_1 + 1}$$

Prove that an odd number / an odd number is odd [12]

This means that ALL odd numbers * odd numbers are odd.

This has been proven [8]

This also means that NO even numbers * even numbers are odd.

As proven in [4], the result has to been even, so this statement is true.

QED