The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO TRAINING SESSIONS

Tournament of the Towns Problems
Senior Paper: Years 11, 12
Northern Autumn 2006 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Three positive integers are written on the blackboard. Mary records in her notebook the product of any two of them and reduces the third number on the blackboard by 1. With the new trio of numbers, she repeats the process, and continues until one of the numbers on the blackboard becomes zero. What will be the sum of the numbers in Mary's notebook at that point? (4 points)

2. The incircle of the quadrilateral $ABCD$ touches AB, BC, CD and DA at E, F, G and H, respectively. Prove that the line joining the incentres of triangles HAE and FCG is perpendicular to the line joining the incentres of triangles EBF and GDH. (4 points)

3. Each of the numbers $1, 2, 3, \ldots, 2006^2$ is placed at random into a cell of a 2006×2006 board. Prove that there exist two cells which share a common side or a common vertex such that the sum of the numbers in them is divisible by 4. (4 points)

4. Every term of an infinite geometric progression is also a term of a given finite arithmetic progression. Prove that the common ratio of the geometric progression is an integer. (4 points)

5. Can a regular octahedron be inscribed in a cube in such a way that all vertices of the octahedron are on the cube's edges? (5 points)