

The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

AMO/TT TRAINING SESSIONS

Tournament of the Towns Problems
Senior Paper: Years 11, 12
Northern Autumn 2009 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. A 7-digit code, consisting of seven distinct digits, is called *good*. Suppose the password for a safe is a *good code*, and that the safe can be opened if an entered code is good and a digit of that code and the corresponding digit of the password are the same at some position.

Is there a guaranteed method of opening the safe with fewer than 7 attempts without knowing the password? (4 points)

2. A closed broken line $ABCDEF$, which consists of six pieces, is given in space. The opposite pieces of the line are parallel ($AB \parallel DE$, $BC \parallel EF$, and $CD \parallel FA$).

Given that $AB \neq DE$, prove that all pieces of the line lie in the same plane. (4 points)

3. Do there exist $a, b, c, d \in \mathbb{N}$ such that $a^3 + b^3 + c^3 + d^3 = 100^{100}$? (4 points)

4. A point is chosen on each side of a regular 2009-gon. These points are vertices of the 2009-gon with area S . Each point is reflected with respect to the midpoint of the side where that point lies.

Prove that a 2009-gon with vertices in the newly constructed points has the same area S . (4 points)

5. There are two capitals, South and North, and some towns in a country. Some of them are connected by roads, and some of the roads are tollroads. It is known that there are not less than 10 tollroads on any route from the southern capital to the northern one.

Prove that all tollroads can be distributed between 10 companies in a way such that all 10 companies will have their tollroads on any route from the southern capital to the northern one. (5 points)