Лабораторна робота №4
Символьні і числові розв'язки диференціальних рівнянь, числові розв'язки рівнянь з частинними похідними другого порядку в Mathematica і Maxima.

1. Знайдіть символьний розв'язок звичайного диференціального рівняння. Відобразіть розв'язок із використанням константи з індексом.

1. $\dot{y} = x - y^2$,

2. $2(y + \dot{y}) = x + 3$,

3. $\dot{y} = \dfrac{x^2 + y^2}{2} - 1$,

4. $(y^2 + 1)\dot{y} = y - x$,

5. $y\dot{y} + x = 0$,

6. $x\dot{y} = 2y$,

7. $x\dot{y} + y = 0$,

8. $\dot{y} + y = (x - y)^3$,

9. $\dot{y} = x - e^y$,

10. $y(\dot{y} + x) = 1$,

11. $\dot{y} = \dfrac{y - 3x}{x + 3y}$,

12. $\dot{y} = \dfrac{y}{x + y}$,

13. $x^2 + y^2\dot{y} = 1$,

14. $(x^2 + y^2)\dot{y} = 4x$,

15. $x^2 + 3y^2 = 2\dot{y}$,

16. $\dot{y} = 4(x - 5)^2$,

17. $\dot{y} = x^2 + 4e^x$,

18. $x = 5y^2 + 2\dot{y} + 6$,

19. $\dot{y} = (x - 1)^3$,

20. $\dot{y} = \sin(x + 1)$,

21. $y^2 + 3\dot{y} = x^3$,

22. $2\dot{y} = \sin(5x + 1)$,

23. $\dot{y} = e^{3x} + 2$,

24. $(2x + 1)\dot{y} = 4x + 2y$,

25. $x^2\dot{y} + xy + 1 = 0$

26. $y = x(\dot{y} - x\cos x)$,

27. $2x(x^2 + y)x = \dot{y}$

28. $x\dot{y} + (x + 1)y = 3x^2e^{-x}$

29. $(2e^y - x)\dot{y} = 1$,

30. $\dot{y} + 2y = y^2e^x$.

2. Знайдіть розв'язок початкової задачі для звичайного диференціального рівняння. Побудуйте його графік.

1. $\dot{y} - \dfrac{y}{x} = x^2$, $y(1) = 0$,

2. $-\dot{y}\,\mathrm{ctg}\,x = 2x\sin x$, $y\left(\dfrac{\pi}{2}\right) = 1$,

3. $\dot{y} + y\cos y = \dfrac{1}{2}\sin 2x$, $y(0) = 0$,

4. $\dot{y} + y\,\mathrm{tg}\,x = \cos^2 x$, $y\left(\dfrac{\pi}{4}\right) = \dfrac{1}{2}$,

5. $\dot{y} - \dfrac{y}{x + 2} = x^2 + 2x$, $y(-1) = \dfrac{3}{2}$,

6. $\dot{y} - \dfrac{1}{x + 1}y = e^x(x + 1)$, $y(0) = 1$,

7. $\dot{y} - \dfrac{y}{x} = x\sin x$, $y(\dfrac{\pi}{2}) = 1$,

8. $\dot{y} + \dfrac{y}{x} = \sin x$, $y(\pi) = \dfrac{1}{\pi}$,

9. $\dot{y} + \dfrac{y}{2x} = x^2$, $y(1) = 1$,

10. $\dot{y} + \dfrac{2x}{1 + x^2}y = \dfrac{2x^2}{1 + x^2}$, $y(0) = \dfrac{2}{3}$,

11. $\dot{y} - \dfrac{2x - 5}{x^2}y = 5$, $y(2) = 4$,

12. $\dot{y} + \dfrac{y}{x} = \dfrac{x + 1}{x}e^x$, $y(1) = e$,

13. $\dot{y} - \dfrac{y}{x} = -2\dfrac{\ln x}{x}$, $y(1) = 1$,

14. $\dot{y} - \dfrac{y}{x} = -\dfrac{12}{x^3}$, $y(1) = 4$,

15. $\dot{y} + \dfrac{2}{x}y = x^3$, $y(1) - \dfrac{5}{6}$,

16. $\dot{y} + \dfrac{y}{x} = 3x$, $y(1) = 1$,

17. $\dot{y} - \dfrac{2xy}{1 + x^2} = 1 + x^2$, $y(1) = 3$,

18. $\dot{y} + \dfrac{1 - 2x}{x^2}y = 1$, $y(1) = 1$,

19. $\dot{y} + \dfrac{3y}{x} = \dfrac{2}{x^3}$, $y(1) = 1$,

20. $\dot{y} + 2xy = -2x^3$, $y(1) = e^{-1}$,

21. $\dot{y} + \dfrac{xy}{2(1 - x^2)} = \dfrac{x}{2}$, $y(0) = \dfrac{2}{3}$,

22. $\dot{y} + xy = -x^3$, $y(0) = 0$,

23. $\dot{y} - \dfrac{2}{x + 1}y = e^x(x + 1)^2$, $y(0) = 1$,

24. $\dot{y} + 2xy = xe^{-x^2}\sin x,\ y(0) = 1,$

25. $\dot{y} + \dfrac{2y}{x+1} = (x+1)^2,\ y(0) = \dfrac{1}{2},$

26. $\dot{y} - y\cos x = -\sin 2x,\ y(0) = 3,$

27. $\dot{y} - 4xy = -4x^3,\ y(0) = -\dfrac{1}{2},$

28. $\dot{y} - \dfrac{y}{x} = -\dfrac{\ln x}{x},\ y(1) = 1,$

29. $\dot{y} - 3x^2 y = \dfrac{x^2(1+x^3)}{3},\ y(0) = 0,$

30. $\dot{y} - y\cos x = \sin 2x,\ y(0) = -1.$

3. Знайдіть загальний розв'язок диференціального рівняння вищого порядку.

1. $x^2\ddot{y} = \dot{y}^2,$

2. $2x\dot{y}\ddot{y} = \dot{y}^2 - 1,$

3. $y^3\ddot{y} = 1,$

4. $\dot{y}^2 + 2y\ddot{y} = 0,$

5. $\ddot{y} = 2y\dot{y},$

6. $y\ddot{y} + 1 = \dot{y}^2,$

7. $\ddot{y}(e^x + 1) + \dot{y} = 0,$

8. $y\ddot{y} = \dot{y}^2 - \dot{y}^3,$

9. $2y\ddot{y} = y^2 + \dot{y}^2,$

10. $\ddot{y}^3 + x\ddot{y} = 2\dot{y},$

11. $\ddot{y}^2 + \dot{y} = x\ddot{y},$

12. $\ddot{y} + \dot{y}^2 = 2e^{-y},$

13. $\ddot{y}^2 = \dot{y}^2 + 1,$

14. $\ddot{y} = e^y,$

15. $2\dot{y}(\ddot{y} + 2) = x\ddot{y}^2$

16. $y^4 - y^3\ddot{y} = 1,$

17. $\dot{y}^2 = (3y - 2\dot{y})\ddot{y},$

18. $\ddot{y}(2\dot{y} + x) = 1,$

19. $(1 - x^2)\ddot{y} + x\dot{y} = 2,$

20. $y\ddot{y} - 2y\dot{y}\ln(y) = \dot{y}^2,$

21. $(\dot{y} + 2y)\ddot{y} = \dot{y}^2$

22. $x\ddot{y} = \dot{y} + x\sin\dfrac{\dot{y}}{x},$

23. $y\ddot{y} + y = \dot{y}^2,$

24. $x\ddot{y} = \dot{y} + x(\dot{y}^2 + x^2),$

25. $y\ddot{y} + \dot{y} = 1,$

26. $\ddot{y} = x\dot{y} + y + 1,$

27. $x\ddot{y} = 2y\dot{y} - \dot{y},$

28. $x\ddot{y} - \dot{y} = x^y\dot{y},$

29. $x^2 y\ddot{y} + \dot{y}^2 = 0,$

30. $x^2(\dot{y}^2 - 2y\ddot{y}) = y^2.$

4. Знайдіть розв'язок крайової задачі для диференціального рівняння вищого порядку. Побудуйте його графік.

1. $\ddot{y} - y = 2x;\ \ y(0) = 0, y(1) = -1.$

2. $\ddot{y} + \dot{y} = 1;\ \ \dot{y}(0) = 0, y(1) = 1.$

3. $\ddot{y} - \dot{y} = 0;\ \ y(0) = -1, \dot{y}(1) - y(1) = 2.$

4. $\ddot{y} + y = 1;\ \ y(0) = 0, y(\dfrac{\pi}{2}) = 0.$

5. $\ddot{y} + y = 1;\ \ y(0) = 0, y(\pi) = 0.$

6. $\ddot{y} + y = 2x - \pi;\ \ y(0) = 0, y(\pi) = 0.$

7. $\ddot{y} - \dot{y} - 2y = 0;\ \ \dot{y}(0) = 2, y(+\infty) = 0.$

8. $\ddot{y} - y = 0;\ \ y(0) = 2, y(3) = 0.$

9. $x^2\ddot{y} - 6y = 0;\ \ y(0) = 31, y(1) = 62.$

10. $x^2\ddot{y} - 2x\dot{y} + 2y = 0;\ \ y(1) = 2, y(2) = 2.$

11. $\ddot{y} = x^2;\ \ y(0) = 0, y(1) = 3.$

12. $\ddot{y} + y = 3x^2 + x;\ \ \dot{y}(0) = 0, y(\pi) = 0.$

13. $\ddot{y} + \dot{y} = \sin x;\ \ y(0), \dot{y}(1) = 0.$

14. $\ddot{y} - y = 2x;\ \ \dot{y}(0) = 0, \dot{y}(2) + y(2) = 0.$

15. $\ddot{y} + y = (x+1)^2;\ \ y(0) = y(\pi), \dot{y}(0) = \dot{y}(\pi).$

16. $x^2\ddot{y} + 2x\dot{y} = e^x;\ \ y(1) = 0, \dot{y}(0) = 0.$

17. $x(\ddot{y}) - \dot{y} = x^2 + 1;\ \ \dot{y}(1) = 0, y(2) = 0.$

18. $x^2\ddot{y} - 2y = x^3 + 2x;\ \ y(1) = 0, y(2) + 2\dot{y}(2) = 0.$

19. $\ddot{y} - y = 2x;\ \ y(0) = 0, \dot{y}(2) = -1.$

20. $\ddot{y} + \dot{y} = 1;\ \ y(0) = 0, y(5) = 1.$

21. $\ddot{y} - \dot{y} = 0;\ \ y(0) = 0, \dot{y}(3) = 1.$

22. $\ddot{y} + y = 1;\ \ y(0) + \dot{y}(0) = 1, \dot{y}(3) = 1.$

23. $\ddot{y} + y = x;\ \ y(1) + \dot{y}(1) = 1, \dot{y}(3) = 1.$

24. $\ddot{y} + y = 2x - \pi;\ \ y(1) = 3, \dot{y}(\pi) = 1.$

25. $\ddot{y} - \dot{y} - 2y = 0;\ \ \dot{y}(1) = 2, y(2) = 0.$

26. $\ddot{y} - y = 0;\ \ y(0) + \dot{y}(0) = \sin 1, y(3) = 0.$

27. $x^2\ddot{y} - 6y = 0;\ \ y(2) = 4, y(5) = 5.$

28. $x^2\ddot{y} - 2x\dot{y} + 2y = 0\ \ y(1) = 2, y(3) = 3.$

29. $\ddot{y} = x^2;\ \ y(1) = 2, \dot{y}(1) = 2.$

30. $\ddot{y} - y = 2x;\ \ y(1) = 2, y(2) = 2.$

5. Знайдіть загальний розв'язок системи звичайних диференціальних рівнянь.

1. $\begin{cases} \dot{x} = x + z - y, \\ \dot{y} = x + y - z, \\ \dot{z} = 2x - y. \end{cases}$

10. $\begin{cases} \dot{x} = 2x - y - z, \\ \dot{y} = 3x - 2y - 3z, \\ \dot{z} = 2z - x + y. \end{cases}$

19. $\begin{cases} \dot{x} = 5x + y, \\ \dot{y} = 3x + 2y. \end{cases}$

2. $\begin{cases} \dot{x} = x - 2y - z, \\ \dot{y} = y - x + z, \\ \dot{z} = x - z. \end{cases}$

11. $\begin{cases} \dot{x} = y - 2x - 2z, \\ \dot{y} = x - 2y + 2z, \\ \dot{z} = 3x - 3y + 5z. \end{cases}$

20. $\begin{cases} \dot{x} = x + 2y, \\ \dot{y} = 2x + y. \end{cases}$

21. $\begin{cases} \dot{x} = 3x + y, \\ \dot{y} = 2x + 5y. \end{cases}$

3. $\begin{cases} \dot{x} = 2x - y + z, \\ \dot{y} = x + 2y - z, \\ \dot{z} = x - y + 2z. \end{cases}$

12. $\begin{cases} \dot{x} = 3x - 2y - z, \\ \dot{y} = 3x - 4y - 3z, \\ \dot{z} = 2x - 4y. \end{cases}$

22. $\begin{cases} \dot{x} = x - y, \\ \dot{y} = y - 4x. \end{cases}$

4. $\begin{cases} \dot{x} = 3x - y + z, \\ \dot{y} = x + y + z, \\ \dot{z} = 4x - y + 4z. \end{cases}$

13. $\begin{cases} \dot{x} = x - y + z, \\ \dot{y} = x + y - z \\ \dot{z} = 2z - y. \end{cases}$

23. $\begin{cases} \dot{x} + x - 8y = 0, \\ \dot{y} - x - y = 0. \end{cases}$

5. $\begin{cases} \dot{x} = 4y - 2z - 4x, \\ \dot{y} = z + x, \\ \dot{z} = 6x - 6y + 5z. \end{cases}$

14. $\begin{cases} \dot{x} = y - 2z - x, \\ \dot{y} = 4x + y, \\ \dot{z} = 2x + y - z. \end{cases}$

24. $\begin{cases} \dot{x} = x + y, \\ \dot{y} = 3y - 2x. \end{cases}$

25. $\begin{cases} \dot{x} = x - 3y, \\ \dot{y} = 3x + y. \end{cases}$

6. $\begin{cases} \dot{x} = x - y - z, \\ \dot{y} = x + y, \\ \dot{z} = 3x + z. \end{cases}$

15. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 2y + 4z, \\ \dot{z} = x - z. \end{cases}$

26. $\begin{cases} \dot{x} + x + 5y = 0, \\ \dot{y} - x - y = 0. \end{cases}$

7. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = x + 3y - z, \\ \dot{z} = 2y + 3z - x. \end{cases}$

16. $\begin{cases} \dot{x} = 2x - y - z, \\ \dot{y} = 2x - y - 2z, \\ \dot{z} = 2z - x + y. \end{cases}$

27. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 4y - x. \end{cases}$

8. $\begin{cases} \dot{x} = 2x + 2z - y, \\ \dot{y} = x + 2z, \\ \dot{z} = y - 2x - z. \end{cases}$

17. $\begin{cases} \dot{x} = 4x - y, \\ \dot{y} = 3x + y - z, \\ \dot{z} = x + z. \end{cases}$

28. $\begin{cases} \dot{x} = 3x - y, \\ \dot{y} = 4x - y. \end{cases}$

29. $\begin{cases} \dot{x} = 2y - 3x, \\ \dot{y} = y - 2x. \end{cases}$

9. $\begin{cases} \dot{x} = 4x - y - z, \\ \dot{y} = x + 2y - z, \\ \dot{z} = x - y + 2z. \end{cases}$

18. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 3x + 4y. \end{cases}$

30. $\begin{cases} \dot{x} - 5x - 3y = 0, \\ \dot{y} + 3x + y = 0. \end{cases}$

6. Задайте початкову умову для системи з завдання 5. Знайдіть розв'язок початкової задачі для системи звичайних диференціальних рівнянь. Побудуйте його графік.

7. Знайдіть розв'язок диференціального рівняння в частинних похідних першого порядку. Зобразіть розв'язок графічно.

1. $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = 0, y = 1, z = 2x,$

7. $x\dfrac{\partial z}{\partial x} - 2y\dfrac{\partial z}{\partial y} = x^2 + y^2, y = 1, z = x^2,$

2. $\dfrac{\partial z}{\partial x} + (2e^x - y)\dfrac{\partial z}{\partial y} = 0, x = 0, z = y,$

8. $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = z - xy, x = 2, z = y^2 + 1,$

3. $2\sqrt{x}\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = 0, x = 1, z = y^2,$

9. $\operatorname{tg} x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = z, y = x, z = x^3,$

4. $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + 2\dfrac{\partial u}{\partial z} = 0, x = 1, u = yz,$

10. $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = z^2(x - 3y), x = 1, yz + 1 = 0,$

5. $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + xy\dfrac{\partial u}{\partial z}, z = 0, u = x^2 + y^2,$

11. $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = z - x^2 - y^2, y = -2, z = x - x^2,$

6. $y^2\dfrac{\partial z}{\partial x} + xy\dfrac{\partial z}{\partial y} = x, x = 0, z = y^2,$

12. $yz\dfrac{\partial z}{\partial x} + xz\dfrac{\partial z}{\partial y} = xy, x = a, y^2 + z^2 = a^2,$

13. $z\dfrac{\partial z}{\partial x} - xy\dfrac{\partial z}{\partial y} = 2xz, x + y = 2, yz = 1,$

14. $z\dfrac{\partial z}{\partial x} + (z^2 - x^2)\dfrac{\partial z}{\partial y} + x = 0, y = x^2, z = 2x,$

15. $(y - z)\dfrac{\partial z}{\partial x} + (z - x)\dfrac{\partial z}{\partial y} = x - y, z = y = -x,$

16. $x\dfrac{\partial z}{\partial x} + (xz + y)\dfrac{\partial z}{\partial y} = z, x + y = 2z, xz = 1,$

17. $y^2\dfrac{\partial z}{\partial x} + yz\dfrac{\partial z}{\partial y} + z^2 = 0, x - y = 0, x - yz = 1,$

18. $x\dfrac{\partial z}{\partial x} + z\dfrac{\partial z}{\partial y} = y, y = 2z, x + 2y = z,$

19. $(y + 2z^2)\dfrac{\partial z}{\partial x} - 2x^2z\dfrac{\partial z}{\partial y} = x^2, x = z, y = x^2,$

20. $(x-z)\dfrac{\partial z}{\partial x}+(y-z)\dfrac{\partial z}{\partial y} = 2z, x-y = 2, z+2x = 1,$

21. $xy^3\dfrac{\partial z}{\partial x} + x^2z^2\dfrac{\partial z}{\partial y} = y^3z, x = -z^3, y = z^2$

22. $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 2xy, y = x, z = x^2,$

23. $x\dfrac{\partial u}{\partial x} + yz\dfrac{\partial u}{\partial z} = 0, u = x^y, z = 1,$

24. $(z - y)^2\dfrac{\partial u}{\partial x} + z\dfrac{\partial u}{\partial y} + y\dfrac{\partial u}{\partial z} = 0, u = 2y(y - z), x = 0,$

25. $(1 + x^2)\dfrac{\partial z}{\partial x} + xy\dfrac{\partial z}{\partial y} = 0, z = y^2, x = 0,$

26. $y\dfrac{\partial u}{\partial x} + z\dfrac{\partial u}{\partial z} = 0, u = \ln z - \dfrac{1}{y}, x = 1,$

27. $x\dfrac{\partial u}{\partial x} - y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0, u = y + z, x = 1,$

28. $x\dfrac{\partial u}{\partial x} - y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = u, u = y + z, x = 1,$

29. $\dfrac{\partial z}{\partial x} = \dfrac{\partial z}{\partial y}, z = y + 1, x = 0,$

30. $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = u, u = \dfrac{1}{2}(y + z), x = 2,$

31. $yz\dfrac{\partial z}{\partial x} + x\dfrac{\partial z}{\partial y} = 0, z = x^2, y = 1,$

32. $x\dfrac{\partial z}{\partial x} + z\dfrac{\partial z}{\partial y} = 0, z = -y, x = 1,$

33. $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 2z, z = y, x = 1,$

8. Знайдіть розв'язок диференціального рівняння з частинними похідними. Зобразіть розв'язок графічно.

1. $U_{tt}(x,t) = a^2 U_{xx}(x,t), \ 0 < x < l, \ t > 0;$
   $U(0,t) = 0, \ U(l,t) = 0, \ t \geq 0;$
   $U(x,0) = 0, \ U_t(x,0) = \sin\dfrac{2\pi}{l}x, \ 0 \leq x \leq l;$

2. $U_{tt}(x,t) = a^2 U_{xx}(x,t), \ 0 < x < l, \ t > 0;$
   $U(0,t) = 0, U(l,t) = 0, \ t \geq 0;$
   $U(x,0) = \begin{cases} \dfrac{h}{c}x, \ 0 \leq x \leq c, \\ \dfrac{h(x-l)}{c-l}, \ c < x \leq l, \end{cases} \quad U_t(x,0) = 0, \ 0 \leq x \leq l;$

3. $U_{tt}(x,t) = a^2 U_{xx}(x,t), \ 0 < x < l, \ t > 0;$
   $U(0,t) = 0, \ U_x(l,t) = 0, \ t \geq 0;$
   $U(x,0) = \sin\dfrac{5\pi}{2l}x, \ U_t(x,0) = \sin\dfrac{\pi}{2l}x, \ \leq x \leq l;$

4. $U_{tt}(x,t) + 2U_t(x,t) = U_{xx}(x,t) + U(x,t), \ 0 < x < \pi, \ t > 0;$
   $U_x(0,t) = 0, \ U(\pi,t) = 0, t \geq 0;$
   $U(x,0) = 0, \ U_t(x,0) = x, \ 0 \leq x \leq \pi;$

5. $U_{tt}(x,t) = a^2 U_{xx}(x,t), \ 0 < x < l, \ t > 0;$
   $U_x(0,t) = 0, \ U_x(l,t) = 0, \ t \geq 0;$
   $U(x,0) = x, U_t(x,0) = 1, \ 0 \leq x \leq;$

6. $U_{tt}(x,t) = a^2 U_{xx}(x,t), \ 0 < x < l, \ t > 0;$
   $U_x(0,t) = 0, \ U(l,t) = 0, \ t \geq 0;$
   $U(x,0) = \cos\dfrac{\pi}{2l}x, \ U_t(x,0) = \cos\dfrac{3\pi}{2l}x + \cos\dfrac{5\pi}{2l}x, \ 0 \leq x \leq l;$

7. $U_{tt}(x,t) = 9U_{xx}(x,t),\ 0 < x < 4,\ t > 0;$
$U_x(0,t) = 0,\ U(4,t) = 0,\ t \geq;$
$U(x,0),\ U_t(x,0) = 16 - x^2,\ 0 \leq x \leq 4;$

8. $U_{tt}(x,t) = U_{xx}(x,t) + 10U(x,t),\ 0 < x < \pi/2,\ t > 0;$
$U(0,t) = 0,\ U_x(\dfrac{\pi}{2},t) = 0,\ t \geq 0;$
$U(x,0) = \dfrac{1}{9}\sin x + \sin 3x,\ U_t(x,0) = 0,\ 0 \leq x \leq \pi/2;$

9. $U_{tt}(x,t) = a^2 U_{xx}(x,t),\ 0 < x < l,\ t > 0;$
$U(0,t),\ U(l,t) = 0,\ t \geq 0;$
$U(x,0) = x,\ U_t(x,0) = 0,\ 0 \leq x \leq l;$

10. $U_{tt}(x,t) = a^2 U_{xx}(x,t),\ 0 < x < l,\ t > 0;$
$U(0,t) = 0,\ U_x(l,t) = 0,\ t \geq 0;$
$U(x,0) = x,\ U_t(x,0) = \sin\dfrac{\pi}{2l}x + \sin\dfrac{3\pi}{2l}x,\ 0 \leq x \leq l;$

11. $U_{tt}(x,t) = a^2 U_{xx}(x,t), 0 < x < l,\ t > 0;$
$U_x(0,t) = 0,\ U_x(l,t) + hU(l,t) = 0,\ h > 0,\ t \geq 0;$
$U(x,0) = 0,\ U_t(x,0) = 1,\ 0 \leq x \leq l;$

12. $U_{tt}(x,t) = a^2 U_{xx}(x,t), 0 < x < l, t > 0;$
$U_x(0,t) = 0,\ U_x(l,t) = 0,\ t \geq 0;$
$U(x,0) = x(l - x),\ U_t(x,0) = 0,\ 0 \leq x \leq l;$

13. $U_{tt}(x,t) = U_{xx}(x,t), 0 < x < l, t > 0;$
$U(x,t)|_{t=0} = \cos x, U_t(x,t)|_{t=0} = \sin x;$

14. $U_{tt}(x,t) = 4U_{xx}(x,t) + xt, 0 < x < l, t > 0;$
$U(x,t)|_{t=0} = x^2, U_t(x,t)|_{t=0} = x;$

15. $U_{tt}(x,t) = U_{xx}(x,t) + e^x, 0 < x < l, t > 0;$
$U(x,t)|_{t=0} = \sin x, U_t(x,t)|_{t=0} = x + \cos x;$

16. $U_{tt}(x,t) = U_{xx}(x,t), 0 < x < l, t > 0;$
$U(x,t)|_{t=0} = \dfrac{\sin x}{x}, U_t(x,t)|_{t=0} = \dfrac{x}{1 + x^2};$

17. $U_{tt}(x,t) = U_{xx}(x,t), 0 < x < l, t > 0;$
$U(x,t)|_{t=0} = \dfrac{x}{1 + x^2}, U_t(x,t)|_{t=0} = \sin x;$

18. $U_t(x,t) = a^2 U_{xx}(x,t),\ 0 < x < l,\ t > 0,$
$U_t(0,t) = 0,\ U(l,t) = 0,\ t \geq 0,$
$U(x,0) = A(l - x),\ 0 \leq x \leq l;$

19. $U_t(x,t) = U_{xx}(x,t) - 4U(x,t), 0 < x < \pi, t > 0,$
$U(0,t) = 0,\ U(\pi,t) = 0,\ t \geq 0,$
$U(x,0) = x^2 - \pi x,\ 0 \leq x \leq \pi;$

20. $U_t(x,t) = U_{xx}(x,t) - 2U_x(x,t) + x + 2t,\ 0 < x < 1,\ t > 0,$
$U(0,t) = 0, U(1,t) = t,\ t \geq 0,$
$U(x,0) = e^x \sin \pi x,\ 0 \leq x \leq 1;$

21. $U_t(x,t) = U_{xx}(x,t) + U(x,t) + 2\sin 2x \sin x,\ 0 < x < \pi/2,\ t > 0,$
$U_x(0,t) = 0,\ U(\pi/2,t) = 0, t \geq 0,$
$U(x,0) = 0,\ 0 \leq x \leq \pi/2;$

22. $U_t(x,t) = U_{xx}(x,t) + 4U(x,t) + x^2 - 2t - 4x^2 t + 2\cos^2 x,\ 0 < x < \pi,\ t > 0,$
$U_x(0,t) = 0,\ U_x(\pi,t) = 2\pi t,\ t \geq 0,$
$U(x,0) = 0,\ 0 \leq x \leq \pi;$

23. $U_t(x,t) = 3U_{xx}(x,t) - 6U(x,t) + 3x + 6, \ 0 < x < 2, \ t > 0,$
$U(0,t) = 1, \ U(2,t) = 2, \ t \geq 0,$
$U(x,0) = x^2 - \dfrac{3}{2}x + 1, \ 0 \leq x \leq 2;$

24. $U_t(x,t) = U_{xx}(x,t) + U(x,t), \ 0 < x < \pi, \ t > 0,$
$U(0,t) = 0, \ U(\pi,t) = 0, \ t \geq 0,$
$U(x,0) = 3\sin 2x, \ 0 \leq x \leq \pi;$

25. $U_t(x,t) = U_{xx}(x,t) - 4U(x,t), \ 0 < x < \pi, \ t > 0,$
$U(0,t) = 0, \ U(\pi,t) = 0, \ t \geq 0,$
$U(x,0) = 2\sin 3x + 5\sin 7x, \ 0 \leq x \leq \pi;$

26. $U_t(x,t) = U_{xx}(x,t) - U(x,t), \ 0 < x < l, \ t > 0$
$U(0,t) = 0, \ U(l,t) = 0, \ t \geq 0,$
$U(x,0) = 1, \ 0 \leq x \leq l;$

27. $U_t(x,t) = U_{xx}(x,t) + U(x,t) - x + 2\sin 2x \cos x, \ 0 < x < \pi/2, \ t > 0,$
$U(0,t) = 0, \ U_x(\pi/2,t) = 1, \ t \geq 0,$
$U(x,0) = x, \ 0 \leq x \leq \pi/2;$

28. $U_t(x,t) = 16U_{xx}(x,t) + 2, \ 0 < x < 7, \ t > 0,$
$U_x(0,t) = 0, \ U(7,t) = 0, t \geq 0,$
$U(x,0) = 0, \ 0 \leq x \leq 7;$

29. $U_t(x,t) = U_{xx}(x,t), \ 0 < x < l, \ t > 0,$
$U(0,t) = 0, \ U(l,t) = 0, \ t \geq 0,$
$U(x,0) = \begin{cases} x, \ 0 \leq x \leq l/2, \\ l - x, \ l/2 < x \leq l; \end{cases}$

30. $U_t(x,t) = U_{xx}(x,t) + 6U(x,t) + 2t(1 - 3t) - 6x + 2\cos x \cos 2x, \ 0 < x < \pi/2, \ t > 0,$
$U_x(0,t) = 1, \ U(\pi/2,t) = t^2 + \dfrac{\pi}{2}, \ t \geq 0,$
$U(x,0) = x, \ 0 \leq x \leq \pi/2;$

31. $U_t(x,t) = U_{xx}(x,t) + 4U_x(x,t) + x - 4t + 1 + e^{-2x}\cos^2 \pi x, \ 0 < x < 1, \ t > 0,$
$U(0,t) = t, \ U(1,t) = 2t, \ t \geq 0,$
$U(x,0) = 0, \ 0 \leq x \leq 1;$

32. $U_{xx}(x,t) + \dfrac{1}{x}U_x(x,t) + \dfrac{1}{x^2}U_{tt}(x,t) = 0, \ 0 < x < 1, \ 0 < t < \pi/3;$
$U(1,t) = \dfrac{9}{8}\pi t(\dfrac{\pi}{3} - t), \ 0 \leq t \leq \pi/3;$
$U(x,0) = 0, \ U(x,\dfrac{\pi}{3}) = 0, \ 0 \leq x \leq 1;$